

Surface-wave eikonal tomography in a scattering environment

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Abstract

Surface waves are of increasing interest in seismic prospecting. Traveltime tomography based on dispersion measurements is often used to process surface-wave data, but it has limitations due to the *a priori* information it requires. The surface-wave eikonal tomography proposed here does not require such *a priori* information. In complex scattering environments, picking arrivals is difficult because the waveforms are complicated. Working in narrow frequency bands makes it even more difficult as it spreads arrivals in time and introduces overlap. We present here a neighborhood-based cross-correlation picking method that overcomes this difficulty, which then allows for reliable calculation of 2D phase-velocity variation through the eikonal equation.

1 Introduction

Surface-waves are of increasing interest in seismic prospecting, as they provide information about shallow structures (*e.g.*, Campman and Riyanti, 2007; Socco and Boiero, 2008). They are usually processed using linearized (asymptotic) traveltime tomography, to obtain per-frequency phase- or group-velocity maps first, which are then inverted into a S- and P-wave velocity model at shallow depth. Linearized traveltime tomography requires *a priori* information, that is, a starting model, which will be updated in the tomography process. Moreover, it requires a ray tracer to compute traveltimes in the chosen model, which implies some approximations about how waves propagate in the medium. As we will see in the following, eikonal tomography does not need such *a priori* information, nor does it require ray tracing, which makes it a powerful method in complex media where lack of accurate *a priori* information may prevent meaningful linearization.

Surface-wave phase (or group) velocity tomography also requires the fundamental mode of the Rayleigh wave (or Love wave, if using the horizontal transverse component) to be isolated from other arrivals, as its arrival needs to be picked as an input for the method. When dealing with complex shallow subsurface structure, the surface-wave waveforms are composed of multiple arrivals that can be close in time, or overlap. It is then difficult to recognize specific phases or modes, especially in a narrow frequency band, because the arrivals are spread in time and modes may interfere. We present a neighborhood-based cross-correlation method to measure arrival times in such a medium.

2 Theory and method

2.1 Eikonal tomography

Traveltime tomography consists of estimating spatial variations in the propagation speed of seismic waves, from a set of measured phase arrival times between known source and receiver locations. Essentially, the local wavespeeds are integrated (averaged) over the source-receiver paths:

$$t(\mathbf{r}_s, \mathbf{r}_r) = \int_{\Gamma} \frac{1}{c(\mathbf{r})} d\mathbf{r} \quad (1)$$

where t is the traveltime from a source in position \mathbf{r}_s to a receiver in position \mathbf{r}_r , Γ the ray path between \mathbf{r}_s and \mathbf{r}_r , and $c(\mathbf{r})$ the local phase-velocity along the ray. Traveltimes are then inverted, in a de-integration operation, to map phase-velocity in the sampled area. We note that this asymptotic formulation can easily be replaced using finite frequency theory, depending on how the finite frequency measurements are made (Dahlen et al., 2000; de Hoop and van der Hilst, 2005).

Eikonal tomography, as used for instance by Lin et al. (2009) in a global seismology context, represents a different approach. From the same traveltime measurements along available paths, traveltime maps are first built for each source (this usually involves interpolation, as we will see below), and local propagation wavespeeds are directly inferred from the spatial derivative (gradient) of those maps, following the eikonal equation:

$$|\nabla t(\mathbf{r}_s, \mathbf{r})| = \frac{1}{c_s(\mathbf{r})} \quad (2)$$

where \mathbf{r} is the position in the (interpolated) traveltime map, and $c_s(\mathbf{r})$ the local phase-velocity. Notice that $c_s(\mathbf{r})$ may vary with the source, as it may depend on the ray path (anisotropic media), and as the precision of the measurements depends on the source location (in particular its distance from position \mathbf{r}). The inversion process takes place when building the traveltime map, and not in a de-integration step as in traveltime tomography.

Traveltime maps can be obtained efficiently only if receivers are spread in a dense grid, which is usually the case in seismic exploration, because, in principle, the traveltimes have to be known at each point of the study area.

2.2 Traveltime measurements

Because taking the gradient is a numerically unstable operation, measuring the phase arrival times must be done with great care. In a medium with strong scattering, waveforms can be complex, making it difficult to pick phase arrivals with classical methods. We used a neighborhood-based cross-correlation method to measure traveltime differences between close-by receivers. For each source s , the relationship between the traveltime differences and the absolute traveltimes can be written

$$\mathbf{D} \mathbf{t}^s = \Delta \mathbf{t}^s \quad (3)$$

where \mathbf{D} is a differentiation matrix (a sparse matrix with one ‘+1’ and one ‘-1’ per line), $\mathbf{t}^s = (t_i^s)_{i=1\dots N_r}$ is the vector of traveltimes for each receiver (N_r being the number of receivers), and $\Delta \mathbf{t}^s = (\Delta t_{ij}^s = t_j^s - t_i^s)_{i=1\dots N_r, j=1\dots N_r}$ is the vector of measured traveltime differences between neighbors, from cross-correlation of narrow-band filtered (fundamental mode) surface waves.

This is a Bayesian problem, which can be solved with a quasi-Newton method (*e.g.*, Tarantola, 2005):

$$\mathbf{t}^s \approx \tilde{\mathbf{D}}^{-1} \Delta \mathbf{t}^s \quad (4)$$

where $\tilde{\mathbf{D}}^{-1}$ is the pseudo-inverse of the differentiation matrix (that is, $\tilde{\mathbf{D}}^{-1}$ is an integration matrix), which writes

$$\tilde{\mathbf{D}}^{-1} = (\mathbf{D}^T \mathbf{C}_D^{-1} \mathbf{D} + \mathbf{C}_M^{-1})^{-1} \mathbf{D}^T \mathbf{C}_D^{-1} \quad (5)$$

where \mathbf{D}^T denotes the transpose of \mathbf{D} , \mathbf{C}_D is a data covariance matrix, and \mathbf{C}_M is a model covariance matrix. We used a diagonal matrix for \mathbf{C}_D , which elements are proportional to the measurement error in $\Delta \mathbf{t}$. We introduced smoothing in the resulting traveltimes through

$$(\mathbf{C}_M)_{ij} = e^{-(d_{ij}/l)^2} \quad (6)$$

where d_{ij} is the distance between receivers i and j , and l is a correlation length which allows us to control the smoothing.

As $\tilde{\mathbf{D}}^{-1}$ is an integration operator, \mathbf{t}^s is defined modulo a constant of integration. This ambiguity could be resolved by considering the traveltime at zero offset, which has to be zero, but this is difficult because traveltime measurements at short offsets (within one or two wavelengths) are not reliable due to near field effects. However, integration constant is irrelevant, because we will later consider the gradient of \mathbf{t}^s (equation 2).

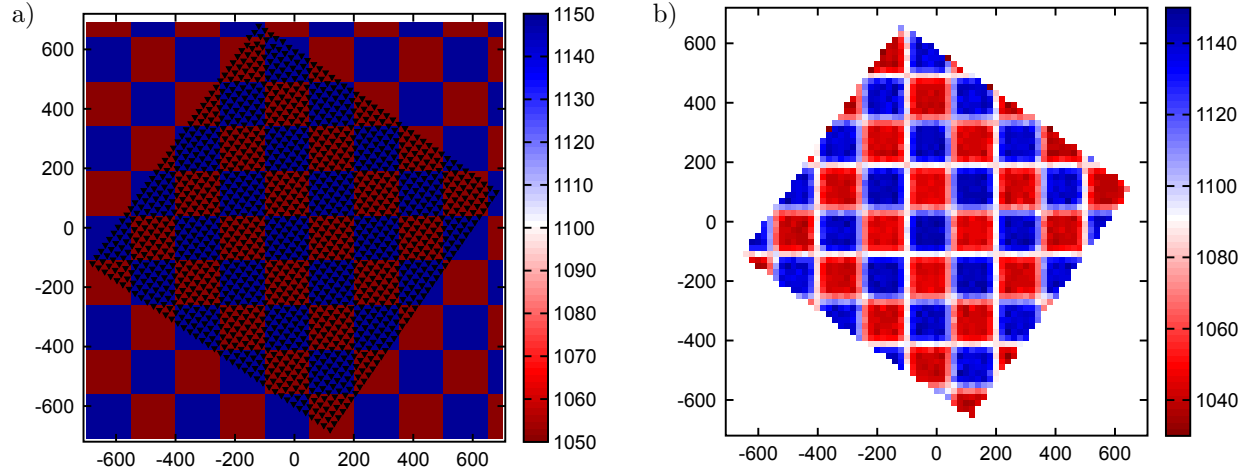


Figure 1: Checkerboard test: (a) True phase-velocity model. Only the receiver array is plotted. The source array is similar, but shifted by half a grid cell in both directions. (b) Result from eikonal tomography.

Equations (4) and (2), may suggest that our method is equivalent to double difference tomography (Zhang and Thurber, 2003). It is different, however, because we do not have to include *a priori* information such as a starting model and, moreover, we do not have to trace any rays, because the ray information is naturally included in the gradient of equation (2). For a more complete discussion on how rays are handled in eikonal tomography we refer to Lin et al. (2009).

A limitation of eikonal tomography is its inability to handle multipathing. The eikonal equation (2) is indeed not valid when multipathing occurs, and, therefore, multipathing is neglected in our study.

3 Application to a scattering environment

3.1 The network

For our study we use data from a high-resolution survey of a 1 km \times 1 km carbonate (karst) area in Northern Oman conducted by Petroleum Development Oman (PDO). Data were acquired with a 40 \times 40 grid of geophones and sources (both with 25 m \times 25 m spacing), but with the source and receiver grids shifted with respect to one another by half a grid distance in both directions (that is, 12.5 m). Seismic vibrator trucks were used as the source on each node of the source grid. Records are 4 s long and the sampling frequency is 125 Hz. For a more complete description of the dataset we refer to Herman and Perkins (2006) and Gouédard et al. (2008).

The complete data set consists of 1600 \times 1600 vertical component source-receiver time-domain signals, and constitutes an exhaustive measurement of the transfer functions of the half-space medium over a 1-km² area. Because of the 2D acquisition geometry, the dataset includes mainly Rayleigh (surface) waves.

3.2 Resolution test

We first made a checkerboard resolution test by computing synthetics with the velocity model presented in Figure 1a, using the actual source/receiver geometry.

Traveltimes are measured using the neighborhood-based approach described above. We considered 9 neighbors for each receiver (neighbor receiver distances are 25 m, or 35 m for diagonal neighbors), and Δt^s are measured using the cross-correlation between the traces windowed around their maximum amplitude. To overcome the one-sample precision in time, data are interpolated to a 1250 Hz sampling before the measurements.

For each of the 1600 sources the travelttime at each receiver is estimated from equation (4) (again, modulo an integration constant), using $l=50$ m in equation (6) and interpolated on a regular grid to account for small misalignment of receivers. The gradient of this travelttime map is computed to obtain the phase-velocity map

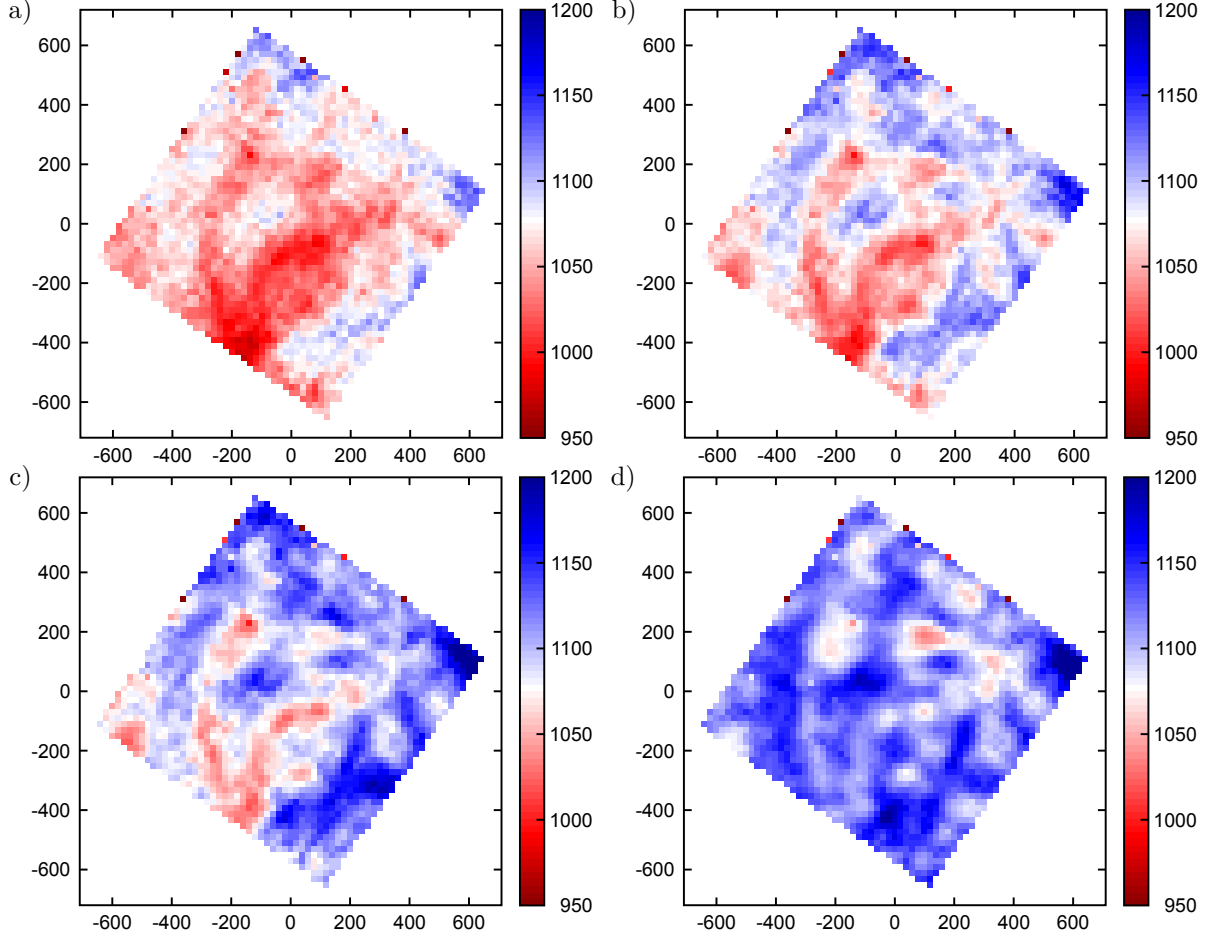


Figure 2: Rayleigh-wave phase-velocity maps obtained from eikonal tomography at (a) 10 Hz; (b) 15 Hz; (c) 20 Hz; and (d) 25 Hz.

for this source, following equation (2). To account for non-reliable traveltime measurements at short offsets, the region within 150 m (≈ 1.5 wavelength) distance from the source is removed from the map. Once this is done for each source, all velocity maps are averaged to give the final velocity model of Figure 1b. In this map, both the geometry and the amplitude of the true model are well retrieved, and small discrepancies are within the uncertainty introduced by smoothing.

3.3 Actual inversion and discussion

To account for the possible data quality reduction due to noise when using real data, we subjected the measured traveltime differences Δt to a quality criterion. The correlation-based approach we used to measure these traveltime differences assumes that the waveforms are similar between neighbors. We thus used the correlation coefficient between these waveforms, windowed around the amplitude maximum to select the fundamental mode Rayleigh wave only, as a quality criterion. We choose a threshold value of 0.98, and any measurement with smaller correlation coefficient is rejected. This procedure typically rejects 30 % of the measurements for each source.

Figures 2a,b,c,d display the resulting phase-velocity maps at 10, 15, 20 and 25 Hz, respectively. As there is no practical limitation to apply this method when filtering the data in narrow bands, maps such as those presented in Figure 2 can be obtained at every frequency (within the effective frequency band of the source pulse). A dispersion curve would then be extracted for each pixel, by combining all maps, and each dispersion curve inverted for a S- and P-wave velocity profile with depth (within a 1D approximation) to get

a pseudo-3D velocity map for the area.

Given the heterogeneity of the medium, as seen in Figure 2, it would have been difficult to compute the ray path if we were to use a traveltimes or double difference tomography approach. Surface-wave eikonal tomography seems an interesting alternative for such a case where shallow structures are complex

Another advantage of eikonal tomography is that it can be used to measure azimuthal anisotropy, as presented by Lin et al. (2009). The vectorial form of equation (2) indeed involves a unit wave number vector, which direction can be used to measure velocity as a function of the propagation direction. The anisotropy study applied to the data presented here is still in progress.

4 Conclusions

We propose an alternative approach for using surface wave tomography in seismic prospecting, namely eikonal tomography, which overcomes an important limitation of traveltimes tomography, *viz* the need of *a priori* information about medium heterogeneity. We also present a way to measure traveltimes in a scattering environment where complex waveforms make phase arrival picking difficult.

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