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Cost-Efficient Fiber Connection Topology Design for Metropolitan Area WDM Networks

Kyle C. Guan and Vincent W. S. Chan

Abstract—In this paper, we provide some analytical insights into physical architectures that can serve as benchmarks for designing a cost-efficient WDM metropolitan area network (MAN). For uniform all-to-all traffic and regular topologies with nodal symmetry, we identify a class of regular graphs—Generalized Moore Graphs—that have several attractive properties by formulating a first-order cost model and characterizing the tradeoff between fiber and switching resources. Our results show that, in conjunction with minimum hop routing, Moore Graphs achieve the minimum cost and simultaneously use the least number of wavelengths. We also take steps to broaden the scope of our work by addressing irregular network topologies, which represent most existing networks. Our results show that Generalized Moore Graphs can be used to provide useful estimates of the cost of irregular networks and can serve as good reference architectures for the designs of practical networks.

Index Terms—Metropolitan area network (MAN); Optical cross-connect (OXC); Generalized Moore Graph; Regular topology; Routing and wavelength assignment (RWA); Network optimization.

I. INTRODUCTION

Despite significant build-ups in network infrastructures over the past decade to support the increasing digital demands, traffic volume is still expected to have tremendous growth in the foreseeable future. To keep up with the rising traffic volume, carriers have deployed huge capacity in the long-haul networks. Meanwhile, end users' access to higher data rates is still costly [1]. As a viable solution to bridge the gap between the bandwidth glut at the backbone and the high access cost, optical-cross-connect-based lightpath switching and other enabling technologies

are currently incorporated with existing electronic routing and switching to lower both capital expenditures (CapEx) and operational expenditures (OpEx). Nonetheless, only through a careful design of the corresponding network architecture can the potential of these technologies be realized. Since the characteristics of optical devices are quite different from those of electronic devices, the architectures optimized for optical switching paradigms will not be the same as the router-centric network architectures that are widely adopted for today's Internet. As such, efficient optical network architectures that truly take advantage of WDM technology need to be created, especially from the perspective of cost scalability. The ultimate goal is to design networks that not only require a low initial capital investment, but also have good scalability—a decreasing cost-per-node-per-unit-traffic as user number and transaction size increase.

In this paper, we focus on the optimization of WDM network architectures, which are for the most part overlooked by previous research but crucial for the objective of providing high bandwidth to end users at low cost. As a followup to our previous research [2–4], in this work we continue our effort in searching for cost-effective network architectures over the solution space that embodies key aspects of an optical network: fiber connection topologies, physical layer switching, routing and wavelength assignment (RWA), etc.

A. Problem Formulation, Solution Complexity, and Approach

Similar to our previous work [2], we focus on a greenfield design scenario in a metropolitan environment. The question we ask is, given the locations of network nodes and the traffic demand matrix (or a range of matrices), how can we minimize the total network cost (capital investment) over the following design elements:

- Network fiber connection topologies. The cable plant topologies (also known as physical topologies in most research literature, as illustrated in Fig. 1) are determined by factors such as speculated traffic and rights of way. How these fibers

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order cost model and formulate a cost minimization problem for the purpose of characterizing the tradeoffs between fiber and switching resources. Using various optimization techniques, we have found that for regular networks and uniform traffic, the joint design problems of physical topology, dimensioning, and routing can be solved optimally and analytically. We prove that with minimum hop routing, Generalized Moore Graphs, whose average minimum hop distances scale favorably as $\log_{\Delta} N$, achieve the lower bound on network cost and are good reference topologies. We also show that topologies with structures close to Generalized Moore Graphs can achieve close-to-minimum cost. The investigation of the cost scalability further demonstrates the advantage of the Generalized Moore Graphs and their close relatives as benchmark topologies: the minimal normalized cost per unit traffic decreases with increasing network size.

Throughout this paper, we continue to address the theoretical underpinnings of the scalable network design problem with tools of graph theory and convex optimization. The major results are summarized as follows.

We first investigate to what extent wavelength resources can be efficiently dimensioned to support uniform traffic for a given regular network, especially when wavelength conversion is not available. Further exploration of the important properties of Generalized Moore Graphs in conjunction with minimum hop routing indicates that, even without wavelength conversion, supporting a given traffic demand requires a minimum (or near minimum) number of wavelength channels. In other words, wavelengths can also be efficiently provisioned for these Generalized Moore Graphs. Our previous and new results imply that, by routing traffic via minimum hops, Generalized Moore Graphs achieve the minimum cost and simultaneously require the minimum (or near minimum) number of wavelengths.

We also set up a cost model for OEO-switched WDM networks and compare the relative cost benefits of deploying OXC or OEO switches in the network. Our results show that at low data rates, it is economical to use OEO switches; at high data rates, it is more cost-advantageous to use OXC switches.

As a natural extension of our previous work [2], we are also interested in finding a class of regular topologies that provides upper bounds on the average hop distance and network cost. We identify that a class of graphs, (one-sided) Δ -nearest neighbors topologies—first constructed in [2], can provide such bounds.

We further expand our work by looking into irregular network topologies and (static) non-uniform traffic, which represent most existing networks. We show

that if the switching cost is linear with port counts, minimum hop routing is still optimal. The results of Generalized Moore networks can be used to provide useful estimates for the cost of irregular networks. Also, the unique structure of a Generalized Moore Graph—each of its nodes has a full (or almost full) Δ -ary routing spanning tree—can be exploited to suggest improvements for irregular physical topologies. The study of the constructions of Generalized Moore Graphs yields a general yet crucial design guideline (for irregular topology and arbitrary traffic): a cost-effective physical topology should minimize the propagation of large traffic flows. Based on this principle, we also propose a topology design algorithm. The preliminary tests show that the networks generated by the algorithm have minimum or close to minimum cost.

The rest of the paper is organized as follows: In Section II, we first introduce the models for fiber connection topology, with emphasis on reviewing the concept of Generalized Moore Graphs. In Section III, we summarize the network cost model of OXC-switched WDM network and introduce the cost model for OEO-switched WDM networks. In Section IV, we discuss Δ -nearest neighbors topologies as a class of regular topologies that provides upper (worst case) bounds on average hop distance and network cost. We also compare the relative cost benefits of deploying OXC or OEO switches in the network. In Section V, we explore the RWA for Generalized Moore Graphs. We prove that minimum hop routing also minimizes the number of wavelength channels for Moore Graphs. In Section VI, we elaborate on how to extend the results of symmetric regular networks to assess the cost efficiency of irregular networks.

II. NETWORK MODELS

This section briefly describes the graph theoretic models for WDM networks to provide necessary backgrounds and notations before the introduction of the main topics. The materials presented in this section are largely drawn from previously published works [2–5]. More detailed descriptions can be found therein.

A. Cable Plant Topology, Fiber Connection Topology, and Regular Topologies

To first order, the physical architecture of an optical network consists of cable plants, with each cable containing many fibers, and optical switches that are interconnected by the cables, as illustrated in Fig. 1. Such a cable plant layout is called the cable plant topology, which is determined by speculated traffic and target of opportunities for affordable rights of way, as

well as other factors, such as bi-lateral agreements between the carriers. How the fibers within the cables are connected is called the physical (fiber) topology, which is a key design element that is largely up to the network designer.

We follow the practice of representing a WDM mesh network as a (directed or undirected) graph $G(V, E)$. Vertices V (or nodes) represent the optical switches, and (directed or undirected) edges E represent the *fiber connections*. A path from a source node to a destination node consists of several edges. We call the number of edges of a path the number of hops.

The network physical topologies can be broadly classified into two categories: regular and irregular (arbitrary). In our research, we mostly focus on regular topologies, since with their symmetric and well-defined connectivity pattern, they are analytically more tractable than irregular ones. Regular topologies are fair representations for MANs and local area networks (LANs), but can only be used as references for wide area networks (WANs). For an irregular cable plant topology, a regular fiber connection topology can be constructed on top of it by connecting fibers via a static patch panel, as illustrated in Fig. 1. The analysis of such constructed regular topologies can provide estimates for the irregular ones. A detailed study of the irregular topologies by this method will be presented in Section V.

In the graph theory literature, regular topology is defined differently in various contexts. As in [2–4], we provide our definition of regular topology to cover a broad class of topologies that exhibit symmetric and well-defined structures. We say that a topology is regular of node degree Δ , when it satisfies the following conditions:

- There are Δ outgoing edges from and Δ incoming edges to each of its nodes.
- Each node links to Δ other nodes following the same set of (predefined) connectivity rules. In other words, the regular topologies studied in this work have nodal symmetry.
- A topology needs to be Δ -connected. That is, $n(i) \geq \Delta$, $1 \leq i \leq D-1$, as defined in [10]. In this definition, $n(i)$ denotes the number of nodes that are i hops away from a node via minimum hop routing; D denotes the diameter of a topology—the maximum distance among all possible node pairs via minimum hop routing.

Besides node degree, diameter, and connectivity rule, some other parameters are also used to characterize a regular topology:

- The average minimum hop distance H_{\min} between node pairs is an important quality measure for a network. For a regular topology of N nodes, H_{\min} can be expressed as

$$H_{\min} = \frac{1}{N-1} \sum_{i=1}^D in(i). \tag{1}$$

H_{\min} is usually used as an indicator of the propagation delay performance of a network [11]. In [2], we show that it can also be interpreted as a measure for the switching and wavelength resources required for supporting uniform all-to-all traffic. As such, H_{\min} serves as a fundamental parameter.

- The load of an edge is defined as the number of source–destination pairs using this edge. Obviously, for a given network and traffic demand, the load depends on the routing strategies. The maximum load of an edge of $G(V, E)$ provided by a routing is called congestion.

B. Generalized Moore Graph and Moore Graph

Generalized Moore Graphs, with Moore Graphs as special cases, are known to achieve the lower bounds (also called Moore Bounds) on the average hop distance among regular topologies with the same number of nodes and node degrees [12,13]. To get a better understanding as to why Generalized Moore Graphs provide the lower bounds on average hop distances, we study their routing spanning trees (a spanning tree is a connected subgraph that includes all the nodes and has no cycles). Using the Petersen Graph (a Moore Graph, shown in Fig. 2) and the Heawood Graph (a generalized Moore Graph, shown in Fig. 3) as examples, we note that nodes can be efficiently “packed” in the routing spanning trees. For a Moore Graph, each node can reach other nodes in a fully populated Δ -ary minimum hop routing spanning tree; while for a Generalized Moore Graph, the routing spanning tree is full at each level except possibly the last. There exists a rich class of directed [13] and undirected [13] Generalized Moore Graphs. For example, in [14] directed Generalized Moore Graphs with size up to 100 are constructed for $\Delta=3$, $\Delta=4$, and $\Delta=5$. Regular graphs, such as Shuffle Nets [15], de Bruijn Graphs

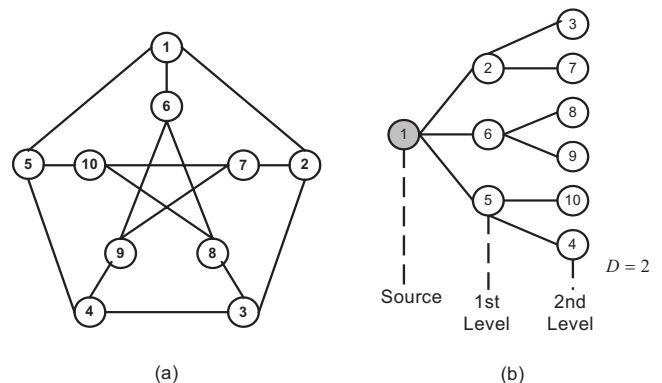


Fig. 2. (a) The Petersen Graph with $N=10$, $\Delta=3$, and $D=2$; (b) the routing spanning tree from node 1.

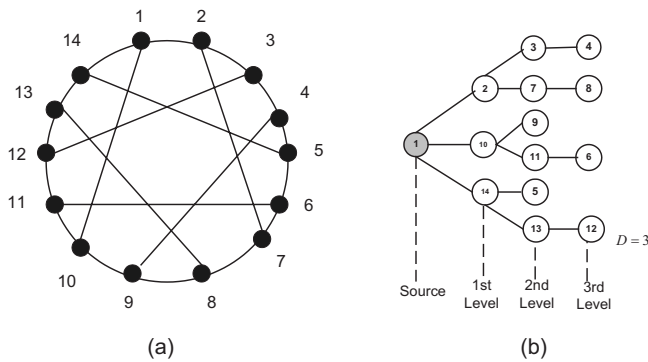


Fig. 3. (a) The Heawood Graph with $N=14$, $\Delta=3$, and $D=3$; (b) the routing spanning tree from node 1.

[15], and Kautz Graphs [15],² are known as close relatives to Generalized Moore Graphs in the sense that they have average hop distances that either satisfy or are very close to those of Generalized Moore Graphs. As such, Generalized Moore Graphs and their close relatives provide sufficient instances that could serve as starting points for the final design of networks.

At this stage, we should note that the existence of (strict) Moore Graphs is nevertheless rare, due to the stringent requirement for their constructions. Directed Moore Graphs exist only for trivial cases where $\Delta=1$ or $D=1$ [16]. Undirected Moore Graphs include full (complete) graphs, rings (with odd number of nodes), the Petersen Graph, and the Hoffman-Singleton Graph (with $N=50$, $\Delta=7$, and $D=2$) [17].³

Moore Graphs also exhibit good properties in load distribution under minimum hop routing and uniform traffic. We will discuss these properties in detail in Section V, in the context of solving RWA problems.

III. COST MODEL

To make this paper self-contained, we summarize the first-order network cost model, part of which was presented in [2–4]. We also introduce a cost model for electronic switches (which was not included in our previous work), for the analysis of relative cost benefits of optical and electronic switching in Section IV.

A. Fiber Connection Cost

As mentioned in Sections I and II, by using fiber patch panels we can set up a fiber connection between two nodes that are not directly linked by a cable. In a metro environment, a fiber connection spans a much shorter distance than that in a wide area network; thus amplifiers, which dominate the long haul fiber connection costs, are not required in general. As such,

²Some de Bruijn Graphs and Kautz Graphs are optimal for Moore Bounds.

³A Moore Graph with $\Delta=57$ and $D=2$ may exist, though its construction has not been realized yet [17].

we can assume that all fiber connections have approximately the same cost. Let C_f denote the cost associated with fiber connections in the network, and we have C_f as a linear function of N and Δ ,

$$C_f = \alpha N \Delta, \quad (2)$$

where the proportional coefficient α is denoted as the marginal cost of a new fiber connection. Note that depending on whether the cable plants pre-exist or not, we assign the marginal cost of a fiber connection differently. For a MAN, the cost for a fiber connection is estimated in the range of \$2 k–\$25 k/km, and a typical fiber length is in the range of 5 to 20 km. Thus α in Eq. (2) is in the range of \$10 k–\$500 k/fiber.

B. Modeling the Costs of OXC and OEO Switches

1) *The Sizes of OXC and OEO Switches*: To first order, the capacity of an OXC is independent of the actual data rate r of each wavelength. As shown in [2], the size of a switch K_o , which equals the sum of the number of lightpaths that pass-through and add-drop at each node, can be obtained as

$$K_o(N, \Delta, t) = t(N-1)[H_{\min}(N, \Delta) + 1]. \quad (3)$$

From Eq. (3), we note that if a wavelength carries a data rate of r Gb/s, the total traffic switched at each node, in the unit of Gb/s, is $t(N-1)r[H_{\min}(N, \Delta) + 1]$. Different from an OXC switch, the port count of an OEO switch depends on the data rate per wavelength r . Derived via a fashion similar to that of Eq. (3), the number of required OEO switching port K_e as a function of rate R and the port utilization η is [5]

$$K_e(N, \Delta, t, r, R, \eta) = \frac{t(N-1)r[H_{\min}(N, \Delta) + 1]}{R\eta}. \quad (4)$$

2) *The Cost Models for OXC and OEO Switches*: We model the cost of an OXC as a function of the number of switching ports required. Since the traffic is all-to-all and the topologies are regular, we can assume that the size of the OXC at each node is the same. If there are K_o lightpaths to be switched, added, and dropped at a node, the OXC needs at least K_o input ports and K_o output ports. For simplicity of analysis, we assume that OXCs have strictly non-blocking switching fabrics. We also temporarily suppress the wavelength continuity constraint [2] so that K_o ports are enough to switch lightpaths without causing any blocking on the network. The cost of OXC scales differently for different types of switching architectures. Table I lists the first-order cost functions corresponding to these switching fabrics: 3-D, multi-stage, and 2-D OXC. In this table, $\zeta(0 < \zeta < 1)$, $\theta(0 < \theta < 1)$, and $\delta(0 < \delta < 1)$ are

TABLE I
COST OF OXC SWITCHING ARCHITECTURE AS FUNCTIONS OF
THE NUMBER OF PORTS K_0

Switching Architecture	Cost Function
3-D	$F_1(K_0) = \beta_1 K_0^{1+\zeta}$
Multi-stage	$F_2(K_0) = \beta_2 K_0^{1+\eta} \log_2 K_0$
2-D	$F_3(K_0) = \beta_3 K_0^{2+\delta}$

coefficients associated with reliability and yield issues in the manufacturing of 3-D, multi-stage, and 2-D OXC switches, respectively.

As stated in Eq. (3), K_0 is a function of network size N , node degree Δ , and wavelengths of traffic t between node pairs. Let C_s^o denote the cost of OXC, and C_s^o is given by

$$C_s^o = NF_i[K_0(N, \Delta, t)] = NF_i[t(N-1)(H_{\min} + 1)],$$

$$i \in \{1, 2, 3\}, \quad (5)$$

where i indexes the switch type and therefore β_1 , β_2 , and β_3 are scaling factors (cost per port) for 3-D, multi-stage, and 2-D switching fabrics, respectively.

We model the cost of an OEO switch as a linear function of the number of OEO switching ports K_e :

$$C_s^e = \beta_e K_e(N, \Delta, t, r, \eta) = \beta_e \frac{t(N-1)r}{R\eta} [H_{\min}(N, \Delta) + 1],$$

$$(6)$$

where β_e is the per port cost of an OEO switch. We set β_e at \$40 k/port for a 2.5 Gb/s interface and \$80 k/port for a 10 Gb/s interface, respectively, based on the estimate in [18].

C. Network Cost

For a network equipped with an OXC, according to Eqs. (2) and (5), the total network cost C is

$$C = C_f + C_s^o = N\{\alpha\Delta + F_i[K_0(N, \Delta, t)]\}, \quad i \in \{1, 2, 3\}. \quad (7)$$

The total cost can be further normalized as cost per node—*normalized network cost*:

$$C_n = \frac{C}{N} = \{\alpha\Delta + F_i[K_0(N, \Delta, t)]\}, \quad i \in \{1, 2, 3\}, \quad (8)$$

and cost per node per unit traffic—*normalized network cost per unit traffic*

$$C_{n/t} = \frac{C_n}{t} = \frac{\{\alpha\Delta + F_i[K_0(N, \Delta, t)]\}}{t}, \quad i \in \{1, 2, 3\}. \quad (9)$$

Similarly, for a network equipped with OEO switches, the normalized network cost per unit traffic is

$$C_{n/t} = \frac{C_n}{t} = \frac{\{\alpha\Delta + \beta_e K_e(N, \Delta, t, r, \eta)\}}{t}. \quad (10)$$

IV. SCALABLE NETWORK ARCHITECTURES

A. Network Topologies That Provide a Lower Bound on Network Cost

Using the network cost models, we formulate the physical topology design problem as an optimization over the type of symmetric regular topology (denoted as “tpl.” in the formulation), the routing algorithms (denoted as “r.a.” in the formulation), and the network node degree Δ . The formulation has a general form as follows:

$$\min_{\{\text{tpl.}\}, \{\text{r.a.}\}, \Delta} C_n(N, \Delta, t)$$

$$\text{s.t. } 2 \leq \Delta \leq N-1;$$

$$\Delta \in \mathbb{Z}^+;$$

$$N \text{ and } t \text{ are given.} \quad (11)$$

As shown in [2–4], we have found that for regular networks and uniform traffic, the joint design problem of fiber connection topology, dimensioning, and routing can be solved optimally and analytically for a special class of regular graphs—Generalized Moore Graphs. That is, we proved that with minimum hop routing, Generalized Moore Graphs, whose average minimum hop distances scale favorably as $\log_{\Delta} N$, achieve the lower bound on network cost and are good reference topologies. We also showed that topologies with structures close to Generalized Moore Graphs can achieve close-to-minimum cost.

B. Network Topologies That Provide an Upper Bound (Worst Case) on Network Cost

As a natural extension of our previous work [2], we are also interested in finding a class of regular topologies that provides upper bounds (worst case) on the average hop distance and network cost. It turns out that (one-sided) Δ -nearest neighbors topologies (shown in Fig. 4), a circulant graph [6] in which each node connects to its Δ closest (one-sided) neighbors in a cyclic fashion, can achieve such bounds. To see why Δ -nearest neighbors exhibit relatively large average hop distances, we study them from the point of view of their routing spanning trees. As illustrated in Fig. 4(b), the nodes are not efficiently packed: each level is packed with only Δ nodes—the minimum requirement to maintain the connectivity. As a comparison, a Generalized Moore Graph packs $\Delta(\Delta-1)^{i-1}$ nodes at the i th level ($i \leq D-1$). By making connections between their properties of average hop distance and switching

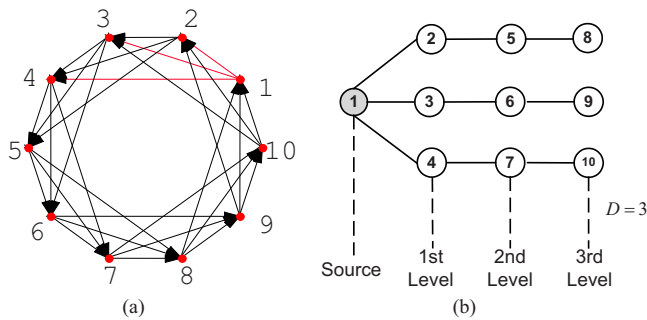


Fig. 4. (Color online) (a) (One-sided) Δ -nearest neighbors topology with $N=10$, $\Delta=3$, and $D=3$; (b) routing spanning tree from node 1.

cost model, we identified that Δ -nearest neighbors yield cost upper bounds (the proof is omitted for brevity). This result is summarized as follows:

Theorem 4.1: A Δ -nearest neighbors topology provides an upper bound on the average minimum hop distance among all regular topologies with the same node number and node degree. Moreover, Δ -nearest neighbors topologies also achieve the upper bounds on the network cost under uniform traffic.

C. Comparisons of OXC and OEO Switches

The focus of this subsection is to compare the relative cost benefits of deploying OXC or OEO switches in the network. As stated in Section III, the cost of an OEO switch depends also on the port rate R and data rate per wavelength r , while the cost of an OXC switch can be considered as rate independent. Figure 5 plots the minimal normalized network cost per unit traffic per data rate C_n^*/r as a function of data rate per wavelength for combinations of two classes of network topologies (Δ -nearest neighbors and Moore Graphs) and

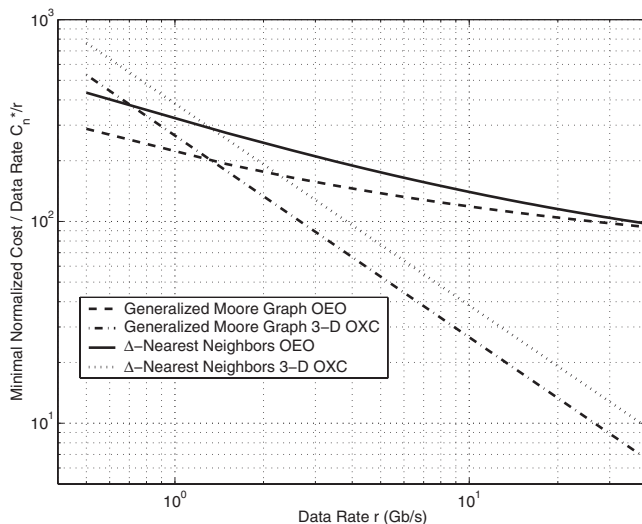


Fig. 5. Minimum normalized network cost as a function of data rate per wavelength for combinations of two classes of network topologies (Δ -nearest neighbors and Generalized Moore Graphs) and two types of switching fabrics (OEO switch and 3-D OXC). $N=50$, $\alpha=20$, $\beta_1=1$, and $\beta_e=7.5$.

two types of switching fabrics (OEO switch and 3-D OXC). The network size, fiber connection cost, OEO per port cost, and 3-D OXC per port cost are set as $N=50$, $\alpha=20$, $\beta_e=7.5$, and $\beta_1=1$, respectively. We also assume that there is one wavelength of traffic between each node pair, i.e., $t=1$. This plot demonstrates that at low data rates (e.g., <1 Gb/s per node pair) it is economical to use OEO switches. At high data rates (>10 Gb/s), networks with 3-D OXC exhibit much better scalability in terms of minimal normalized network cost per data rate, primarily due to the fact that the cost of OXC switches is intrinsically independent of data rate.

V. ROUTING AND WAVELENGTH ASSIGNMENT FOR GENERALIZED MOORE GRAPHS

When we addressed the OXC cost in Sections III and IV, we implicitly suppressed the wavelength continuity constraints by assuming that either there is an infinite number of wavelengths or a full wavelength conversion is available. As such, K_o ports are enough to switch K_o lightpaths, without causing wavelength blocking. Given the fact that the capacity in the metro environment is always scarce and converters currently are still expensive, we expand the scope of this work by investigating whether Moore Graphs exhibit good efficiency in wavelength dimensioning. A key measure to evaluate such an efficiency is the difference (gap) between the minimum number of wavelengths and the network congestion under uniform lightpath connections [19]. Following the approaches that are similar to those of [19], we first study the load distribution and congestion of Moore Graphs under uniform traffic. We next construct and compare the upper and lower bounds on the minimal number of wavelengths required. We show that for Moore Graphs the bounds are tight—the gap between the lower and upper bounds is at most 1 (as shown in Table II).

A. Definition and General Solving Approaches of RWA Problems

We follow the conventional definition of RWA problems: given a network fiber topology and a set of end-to-end lightpath requests, we are to determine routes and assign wavelengths that require the minimal possible number of wavelengths. If the routing is already provided, we only need to deal with the wavelength assignment (WA) problem. In solving a WA problem, *wavelength continuity constraints* [7] must be obeyed.

There are in general two approaches to solve a WA problem. The first approach involves setting up the WA problem in the form of mathematical programming and solving it by using techniques such as linear

TABLE II
RWA RESULTS FOR MOORE GRAPHS

Graph	Node Number, Node Degree, and Diameter	Min. No. of Wavelengths With Conversion	Min. No. of Wavelengths Without Conversion
Fully connected graphs	$\Delta=N-1$ $D=1$	1	1
Rings (odd no. of nodes)	N is odd $\Delta=2$	$\frac{N^2-1}{8}$	$\frac{N^2-1}{8}$
Petersen Graph	$N=10$ $\Delta=3$ $D=2$	5	5
Hoffman–Singleton Graph	$N=50$ $\Delta=7$ $D=2$	13	≤ 14
$N=3250$ $\Delta=57$ $D=2$	$N=3250$ $\Delta=57$ $D=2$	113	≤ 114

programming (LP) or nonlinear programming (NLP) [7,8]. The second approach involves first constructing either a *node conflict graph* or an *edge conflict graph* and then solving the related problems of *node coloring* or *edge coloring* [19]. We concentrate on the second approach, since this approach usually allows us to exploit good properties that are intrinsic to certain lightpath patterns (logical topologies) and fiber topologies.

The node coloring approach is widely used in solving RWA problems. For a given topology G and a set of lightpaths P , we construct a node conflict graph, denoted as G_N , as follows: each node in G_N corresponds to a lightpath in P and two nodes in G_N are connected by an (undirected) edge, if the two corresponding lightpaths in P share a common fiber. Solving the WA problem is then equivalent to finding the *node chromatic number* of the graph G_N , denoted as $\chi(G_N)$. There is a known result that provides an upper bound on the node chromatic number for a connected graph with maximal node degree [20], as summarized in the following:

Theorem 5.1: Let G_N be a connected graph with maximal degree Δ_{\max} . Suppose G_N is neither a complete graph nor an odd cycle, then $\chi(G) \leq \Delta_{\max}$.

Next we consider the edge coloring approach. Unlike the node coloring approach, which has no limitation on the length (in the number of hops) of a lightpath request, the edge coloring approach can only apply to special cases in which all lightpaths have at most two hops. The edge equivalent graph, denoted as G_L , is constructed as follows: for every edge $e \in E$ of the original fiber topology, we introduce a node v_e in G_L . For a lightpath that uses both the edges e_1 and e_2 , $e_1 \neq e_2$, we add an (undirected) edge that connects v_{e_1} and v_{e_2} . Once G_L is constructed, solving the WA problem is then equivalent to solving the edge coloring

problem of G_L . That is, we are to find the *edge chromatic number* $\Lambda_e(G_L)$ of G_L —the minimal number of colors to be assigned to the edges of G_L , such that all edges incident on a node in G_L have different colors. These colors correspond to wavelengths used in the original fiber network G .

To illustrate why the edge coloring approach can only be applied to solve RWA for lightpaths of no more than two hops, we provide a simple example, as shown in Fig. 6. We set up a lightpath of 3 hops on a line topology. It is trivial to see that 1 wavelength is enough to support this lightpath. However, if we used the edge coloring approach, the constructed edge conflict graph would be a 3-node ring, for which 3 colors (wavelengths) are required to ensure that all edges incident on a node have different colors. Obviously this is not true.

There is also a known result that provides an upper bound on the edge chromatic number for a connected graph with maximal node degree Δ_{\max} [21,22], as summarized in the following:

Theorem 5.2: For a connected graph G_L with a maximal node degree Δ_{\max} , the edge chromatic number $\Lambda_e(G_L)$ is either Δ_{\max} or $\Delta_{\max} + 1$.

B. Solving RWA Problems for Moore Graphs

In this subsection, we study whether the minimum hop routing algorithm, which minimizes the network cost, also minimizes the number of wavelengths required to establish all-to-all uniform lightpath connections for Moore Graphs. In [5], we showed that, for Moore Graphs, with minimum hop routing, the total network load generated by uniform all-to-all traffic can be evenly distributed on every fiber. In this subsection, we rely on this property to solve the RWA problem for a Moore Graph. For clarity, we again summarize the result of balanced load distribution (the proof is detailed in [5] and omitted here).

Theorem 5.3: For a Moore Graph of degree Δ and diameter D , balanced load distribution can be achieved for the static uniform all-to-all traffic, with each edge having a load of $\sum_{i=1}^D i(\Delta - 1)^{i-1}$.

When wavelength conversion is not available, RWA becomes rather complicated because of the wave-

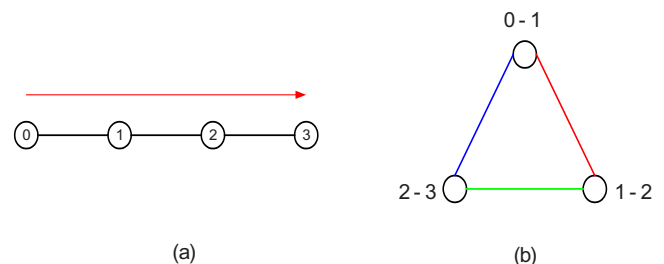


Fig. 6. (Color online) (a) A line topology and a lightpath request of 3 hops; (b) the WA result when the edge coloring approach is used.

length continuity constraints. With wavelength continuity constraint, the balanced load result (Theorem 5.3) can only be used to construct a lower bound on the number of wavelengths required. As such, we first solve the RWA problem for each instance of Moore Graphs and later extrapolate the solutions to a general result. RWA results for all the instances of Moore Graphs are summarized in Table II. Among these Moore Graphs, complete graphs, rings, and the Petersen Graph all require the same (minimal) number of wavelengths with or without wavelength conversions.

We start with a fully connected (complete) graph, which can be treated as a (trivial) Moore Graph. In a complete graph, each node reaches every other node in exactly one hop. It is trivial that such a graph requires exactly one wavelength with or without wavelength conversion.

We next consider ring topologies (with an odd number of nodes). A known result [11] shows that it requires the same (minimal) number of wavelengths, indifferent to the wavelength conversion capabilities of the network. The minimal number of wavelengths required is $(N^2 - 1)/8$.

The rest of the instances of the existing Moore Graphs all have diameters of 2. That is, the longest lightpath has 2 hops. Using this property, we can transform a RWA problem into an edge coloring problem of a conflict graph G_L and obtain a tight upper bound on the minimal number of wavelengths. In other words, for a Moore Graph (of diameter 2), finding the minimal number of wavelengths to support a given set of lightpath requests is the same as finding the edge chromatic number of the corresponding G_L . An example of constructing the edge conflict graph of the Petersen Graph is shown in Fig. 7.

For solving a RWA problem, lightpaths using only a single fiber (edge) can always be assigned a wavelength independently from other lightpaths (using more than one fiber). Thus we only need to consider lightpaths of two hops. As shown in the proof for Theorem 5.3 (cf. [5], Section 4.5.2), for a Moore Graph under uniform all-to-all traffic, each fiber is used as a

first hop (of a two-hop path) for $\sum_{i=1}^{D-1} (\Delta - 1)^i$ times ($D=2$); each fiber is used as a second hop (of a two-hop path) for $(\Delta - 1)^{D-1}$ times ($D=2$). In other words, the conflict graph is regular with a node degree

$$\Delta(G_L) = (\Delta - 1)^{2-1} + \sum_{i=1}^{D-2} (\Delta - 1)^i = \sum_{i=1}^{D-2} i(\Delta - 1)^{i-1} - 1. \tag{12}$$

According to Theorem 5.2, the minimal number of wavelengths to support all the lightpaths of two hops is at most

$$\Delta(G_L) + 1 = \sum_{i=1}^{D-2} i(\Delta - 1)^{i-1}. \tag{13}$$

Adding one additional wavelength that is used for the lightpath of one hop, we can have an upper bound on the minimal number of wavelengths W_M as

$$W_M \leq 1 + \sum_{i=1}^{D-2} i(\Delta - 1)^{i-1}. \tag{14}$$

In summary, Theorem 5.3 provides a lower bound on the minimal number of wavelengths required for a Moore Graph. For complete graphs and rings, it requires the same (minimal) number of wavelengths with or without wavelength conversions. An upper bound on the minimal number of wavelengths is given in Eq. (14). By combining these results, we extrapolate to the following general conclusion on the minimal number of wavelengths used to support all-to-all uniform traffic:

Theorem 5.4: For a Moore Graph of degree Δ and diameter D , a minimal number of wavelengths required to support all-to-all uniform traffic with or without wavelength conversions satisfies

$$\sum_{i=1}^D i(\Delta - 1)^{i-1} \leq W_M \leq 1 + \sum_{i=1}^D i(\Delta - 1)^{i-1}. \tag{15}$$

Note that the difference between the upper and the lower bound is 1. In other words, for a Moore Graph, at most one additional wavelength is required in the absence of wavelength conversion. For the Petersen Graph, using minimum hop routing and a simple wavelength assignment heuristic, a minimum of 5 wavelengths are required to support the all-to-all uniform traffic. The heuristic is a combination of “first-fit” and “last-fit” RWA algorithm [23]. Theorem 5.4 shows that Moore Graphs are also efficient in regard to the wavelength usage, in the sense that wavelength conversions do not provide significant advantages.

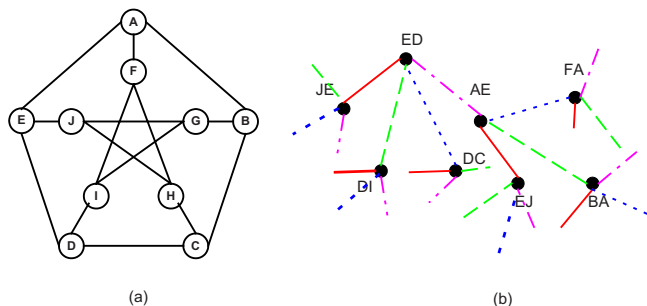


Fig. 7. (Color online) (a) Petersen Graph; (b) part of the edge conflict graph G_L and the edge coloring of the Petersen Graph.

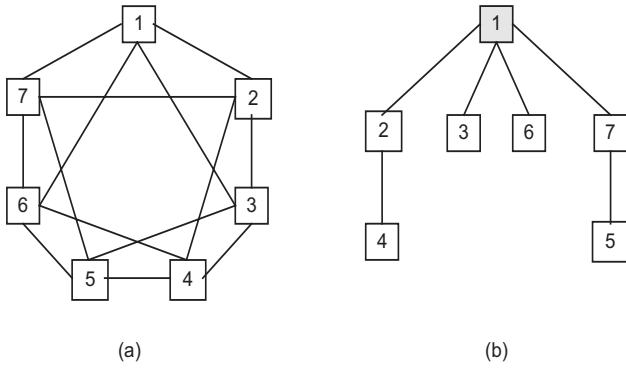


Fig. 8. (a) A Generalized Moore Graph with $N=7$ and $\Delta=4$; (b) a routing spanning tree from node 1.

C. Solving RWA Problems for Generalized Moore Graphs

The balanced load distribution property of a Moore Graph arises from its symmetric structure—each of its nodes has a fully populated routing spanning tree. For a Generalized Moore Graph, multiple minimum hop paths may exist for some source–destination pairs. As a result, the minimum hop routing may or may not balance the load or minimize the congestion even under uniform traffic. To illustrate this, we first use an example of a (undirected) Generalized Moore Graph with $N=7$ and $\Delta=3$, as shown in Fig. 8(a). In this example, with the minimum hop routing illustrated in Fig. 8(b), a load of 2 can be evenly distributed on each edge. We further show that 2 wavelengths are enough to support uniform all-to-all traffic without any wavelength conversion. We next consider another example—a Symmetric Hamilton Graph of 6 nodes and degree 3, shown in Fig. 9(a). This Symmetric Hamilton Graph can be also considered as a complete $K_{3,3}$ bipartite graph (a set of graph vertices can be decomposed into two disjoint sets, such that no two vertices within the same set are adjacent, but every pair of vertices in the two sets are adjacent). For clarity of discussion, we redraw the same Symmetric Hamilton Graph in the bipartite $K_{3,3}$ form in Fig. 9(b). For this graph, the minimum hop routing algorithm is not unique. Table III lists two different minimum hop routing algorithms. We note that neither of the algorithms can distribute the load evenly over each fiber.

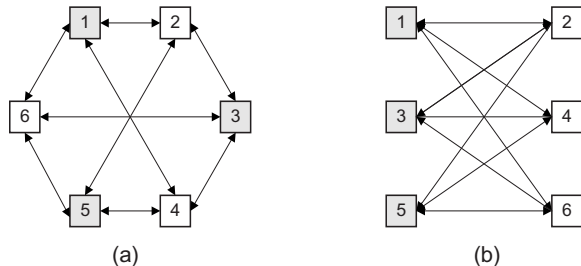


Fig. 9. (a) A Symmetric Hamilton Graph with $N=6$ and $\Delta=3$; (b) the same graph is redrawn as a $K_{3,3}$ complete bipartite graph.

TABLE III
THE MINIMUM HOP ROUTING ALGORITHMS FOR A SYMMETRIC HAMILTON GRAPH WITH $N=6$ AND $\Delta=3$

	Algorithm 1	Algorithm 2
Node i	$ i _6 \rightarrow i+1 _6,$ $ i _6 \rightarrow i+1 _6 \rightarrow i+2 _6,$ $ i _6 \rightarrow i+3 _6,$ $ i _6 \rightarrow i+3 _6 \rightarrow i+4 _6$ $ i _6 \rightarrow i+5 _6$	$ i _6 \rightarrow i+1 _6,$ $ i _6 \rightarrow i+3 _6 \rightarrow i+2 _6,$ $ i _6 \rightarrow i+3 _6,$ $ i _6 \rightarrow i+3 _6 \rightarrow i+4 _6$ $ i _6 \rightarrow i+5 _6$
Max load	4	3

TABLE IV
RWA RESULTS FOR GENERALIZED MOORE GRAPHS WITH $\Delta=3,$ 4 AND $D=2, 3$

Generalized Moore Graphs	Min. No. of Wavelengths With Conversions	Min. No. of Wavelengths Without Conversions
$N=6, \Delta=3, D=2$	3	3
$N=8, \Delta=3, D=2$	4	4
$N=14, \Delta=3, D=3$	10	10
$N=6, \Delta=4, D=2$	2	2
$N=7, \Delta=4, D=2$	2	2
$N=8, \Delta=4, D=2$	3	3
$N=9, \Delta=4, D=2$	3	3
$N=10, \Delta=4, D=2$	4	4
$N=11, \Delta=4, D=2$	4	4
$N=12, \Delta=4, D=2$	5	5
$N=13, \Delta=4, D=2$	5	5

Routing algorithm 1 incurs a maximum load of 4, while routing algorithm 2 minimizes the maximum load to 3. It is also straightforward to show that, using routing algorithm 2, a minimum of 3 wavelengths are enough to support a uniform demand ($t=1$), even in the absence of wavelength conversion. Using node coloring approaches, we investigate the wavelength assignments for Generalized Moore Graphs with $\Delta=3, 4$ and $D=2, 3$. The results are listed in Table IV. It is seen that the wavelength conversion does not reduce the minimal number of wavelengths required. We thus conclude that the wavelengths can also be efficiently provisioned for these Generalized Moore Graphs.

VI. IRREGULAR TOPOLOGIES AND NON-UNIFORM TRAFFIC

Up until now, we have been analyzing scalable network architecture by focusing on regular physical topologies and uniform traffic. In practice, network topologies are seldom regular or even regularizable. Also, the amount of demands on node pairs are rarely equal. As such, it is often difficult to directly derive the analytical expressions or solutions. Normally the

evaluation of irregular topologies under non-uniform traffic is carried out numerically. Notwithstanding, based on the framework presented so far, we can extend the results derived for symmetric regular networks under uniform traffic to evaluate the cost efficiency of irregular networks under non-uniform traffic. In particular, we can construct network cost lower and upper bounds. To this end, in Subsection VI.A, we identify conditions under which minimum hop routing is still optimal. In Subsection VI.B, we focus on irregular networks under uniform traffic. We first show that the results for Generalized Moore Graphs and Δ -nearest neighbors can be used to provide useful estimates for irregular networks. We next demonstrate how we use Generalized Moore Graphs as references to suggest possible improvements for irregular physical topologies. In Subsection VI.C we study regular networks under non-uniform traffic. We first review the concept of minimum and maximum flow trees. Based on these concepts, we provide network cost lower and upper bounds for regular networks under arbitrary (non-uniform) traffic. Finally in Subsection VI.D, by combining the results from Subsections VI.B and VI.C, we construct network cost lower and upper bounds for irregular networks under arbitrary traffic.

A. Irregular Topologies, Arbitrary Traffic, and Minimum Hop Routing

In this work, an irregular topology is characterized by the following parameters: the number of nodes N , the maximum node degree Δ_{\max} , the minimum node degree Δ_{\min} , and the average node degree $\bar{\Delta}$. $\bar{\Delta}$ is defined as

$$\bar{\Delta} = \frac{1}{N} \sum_{i=1}^N \Delta_i, \quad (16)$$

where Δ_i is the degree of node i . For convenience of discussion, we denote an irregular topology as $(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})$. The average minimum hop distance $H_{\min}(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})$ is then defined as

$$H_{\min}(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta}) = \frac{1}{N(N-1)} \sum_{i=1}^N \sum_{j=1}^{D_i} n_i(j), \quad (17)$$

where D_i denotes the network diameter from node i (the maximal hop distance from node i via minimum hop routing), and $n_i(j)$ denotes the number of nodes that are j hops away from node i .

Before analyzing the cost efficiency of an irregular network, we need to identify conditions under which minimum hop routing is still optimal. In [2], we proved that under uniform traffic, minimum hop routing is optimal for any given regular network with a non-decreasing switching cost function. For an irregu-

lar network, without regularity and nodal symmetry, the relationship between the minimum hop routing and the network cost becomes much more complicated. To maintain tractability, we restrict our analysis to linear switching cost structure (e.g., $F_1^i(K_o^i) = \beta_1 K_o^i$ for 3-D OXC, where K_o^i is the switch size at node i , cf. Table I). Under this condition we can show that minimum hop routing is still optimal. This is due to the fact that with the linearity minimizing the total switching cost is equivalent to minimizing the total network load. In other words, the minimum network load solution is also the minimum network cost solution.

B. Irregular Networks Under Uniform Traffic

1) *Lower and Upper Bounds on Network Cost:* In this subsection, we analyze the network cost of an arbitrary network under the restrictions of uniform all-to-all traffic, minimum hop routing, and linear switching cost [e.g., $F_1^i(K_o^i) = \beta_1 K_o^i$]. Our approach is to use the results of average minimum hop distances for Generalized Moore Graphs and Δ -nearest neighbors to size the average minimum hop distance of an irregular topology $(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})$. The results are presented in the following. The proofs are omitted here for brevity.

Theorem 6.1: The average minimum hop distance of an irregular topology is lower bounded by the average minimum hop distance of a Generalized Moore Graph of N nodes and node degree Δ_{\max} . That is,

$$H_{\min}(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta}) \geq H_{\min}^M(N, \Delta_{\max}). \quad (18)$$

The average minimum hop distance of an irregular topology is upper bounded by the average minimum hop distance of a Δ -nearest neighbors topology of N nodes and node degree Δ_{\min} . That is

$$H_{\min}(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta}) \leq H_{\min}^N(N, \Delta_{\min}). \quad (19)$$

Since the switching cost is proportional to the average minimum hop distance, a direct application of Theorem 6.1 provides us with a cost lower bound and a cost upper bound for an irregular network.

Theorem 6.2: For an irregular network of $(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})$, the network cost under uniform traffic has a lower bound

$$C(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta}) \geq \alpha N \bar{\Delta} + \beta_1 N(N-1) \times [H_{\min}^M(N, \Delta_{\max}) + 1], \quad (20)$$

and an upper bound

$$C(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta}) \leq \alpha N \bar{\Delta} + \beta_1 N(N-1) \times [H_{\min}^N(N, \Delta_{\min}) + 1], \quad (21)$$

The proofs are omitted here for brevity.

When $N \gg \bar{\Delta}$, $N \gg \Delta_{\max}$, and $N \gg \Delta_{\min}$, we approximate the ratio of the cost upper bound to the cost lower bound as

$$\frac{\alpha N \bar{\Delta} + \beta_1 N(N-1) H_{\min}^N(N, \Delta_{\min})}{\alpha N \bar{\Delta} + \beta_1 N(N-1) H_{\min}^M(N, \Delta_{\max})} \approx \frac{N \ln \Delta_{\max}}{\ln N \cdot 2 \Delta_{\min}}. \quad (22)$$

Equation (22) indicates that the ratio scales as $N/\ln N$.

To evaluate the gap between the lower and the upper bound, we plot in Fig. 10 the bounds in the form of normalized network cost per unit traffic $C(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})/[N(N-1)]$ as a function of network size N . In the plot, the following parameters are used: $\Delta_{\max}=6$, $\Delta_{\min}=3$, and $\bar{\Delta}=4$. Based on the estimation of a realistic cost ratio between fiber and switching in metropolitan area networks, we set $\alpha/\beta_1=40$ and $\beta_1=1$. The plot indicates that as N increases, the gap between the upper and lower bounds increases (cf. [22]). We also note that, as the size of the network increases, the lower bound of the normalized network cost per unit traffic decreases, while the upper bound first decreases and then increases. This can be explained as follows: the minimum node degree is set as a fixed value 3. This node degree (of 3) is optimal only for certain sizes of networks ($N=10 \sim 30$). As the size of the network increases, this node degree (of 3) becomes less efficient.

To have an idea on how close these two bounds match the actual cost of the irregular networks, we employ randomly generated networks. That is, for a given set of parameters (N , Δ_{\max} , Δ_{\min} , and $\bar{\Delta}$), we con-

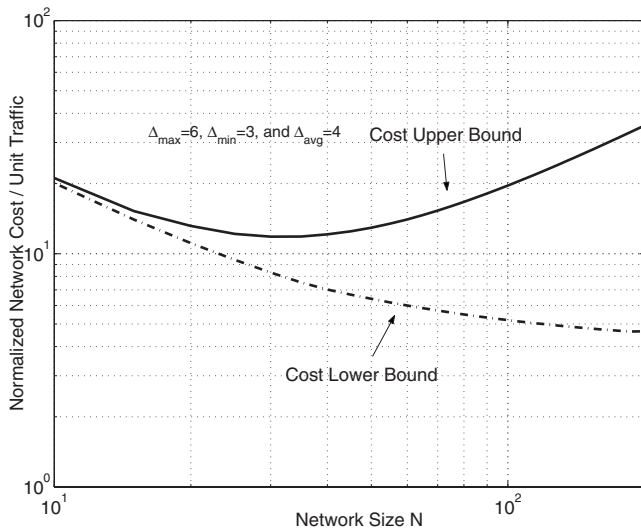


Fig. 10. Network cost upper and lower bounds (in the form of normalized network cost per unit traffic) as functions of network size N . Fiber and switching cost parameters are $\alpha/\beta_1=40$ and $\beta_1=1$.

struct around 4000 instances of networks at random, compute the cost of each of them, and compare their cost distributions with the corresponding lower and upper bounds. We plot in Figs. 11 and 12 the cost histograms for random networks of size $N=20$ and 40, respectively. The node degree parameters for each N are set as $\Delta_{\max}=6$, $\Delta_{\min}=3$, and $\bar{\Delta}=4$. These histograms demonstrate that Generalized Moore Graphs can be used for effectively sizing the cost of an irregular network, especially when N is small (e.g., $N=10 \sim 30$) and the network is densely connected (e.g., $\bar{\Delta}/N \geq 0.2$). On the contrary, the cost upper bounds generated by using Δ -nearest neighbors are loose, especially when N is large (e.g., $N \geq 40$) and the network is sparsely connected (e.g., $\bar{\Delta}/N \leq 0.1$).

To look for better estimates of network cost, we use $\bar{\Delta}$ to replace Δ_{\max} in Eq. (20) and Δ_{\min} in Eq. (21). Thus the new estimates are

$$\alpha N \bar{\Delta} + \beta_1 N(N-1) [H_{\min}^M(N, \bar{\Delta}) + 1] \quad (23)$$

and

$$\alpha N \bar{\Delta} + \beta_1 N(N-1) [H_{\min}^N(N, \bar{\Delta}) + 1]. \quad (24)$$

We also use the Symmetric Hamilton Graph [2] to provide an alternative estimate of network cost as follows:

$$\alpha N \bar{\Delta} + \beta_1 N(N-1) [H_{\min}^H(N, \Delta_{\min}) + 1], \quad (25)$$

where $H_{\min}^H(N, \Delta) = 3/4 + (N-2)/4(\Delta-1)$, as derived in [2]. Including these estimates to the corresponding Figs. 11 and 12, we find that they do give better estimates. Note that we term Eqs. (23)–(25) as “esti-

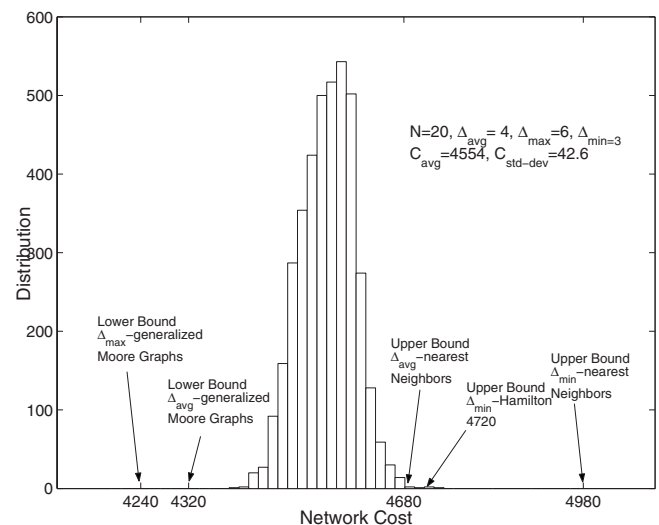


Fig. 11. Network cost histogram for randomly generated networks, with $N=20$, $\Delta_{\max}=6$, $\Delta_{\min}=3$, and $\bar{\Delta}=4$. The horizontal and the vertical axis represent the network cost and the number of instances (cases), respectively. The fiber and switching cost parameters are $\alpha/\beta_1=40$ and $\beta_1=1$.

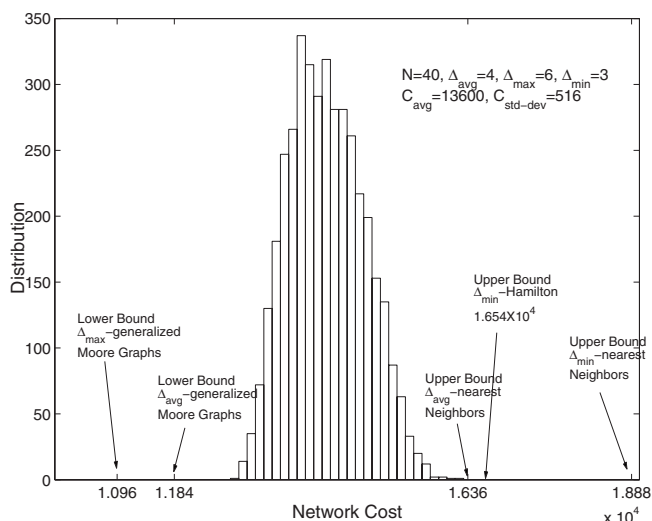


Fig. 12. Network cost histogram for randomly generated networks, with $N=40$, $\Delta_{\max}=6$, $\Delta_{\min}=3$, and $\bar{\Delta}=4$. The horizontal and the vertical axis represent the network cost and the number of instances (cases), respectively. The fiber and switching cost parameters are $\alpha/\beta_1=40$ and $\beta_1=1$.

mates,” since we have not been able to prove that they are indeed tighter bounds for every irregular network $(N, \Delta_{\max}, \Delta_{\min}, \bar{\Delta})$.

2) *Generalized Moore Graphs as References for Possible Improvements for an Irregular Physical Topology:* In this subsection we use examples to demonstrate how we can use Generalized Moore Graphs as references to suggest possible improvement for irregular physical topologies. In particular, we consider two representative networks, which are labeled as Network 1 and Network 2. Their physical topologies are illustrated in Figs. 13(a) and 14(a), respectively. The key network design parameters are summarized in Table V. In our study, we assume that all fiber connections have the same cost and the switching cost is linear. For clarity of discussion, we redraw the topologies of Network 1 and Network 2 in the form of chordal rings in Figs. 13(b) and 14(b), respectively. For each of the networks, we look for the minimum number of switching ports required numerically. The results show that to support all-to-all uniform traffic, a total

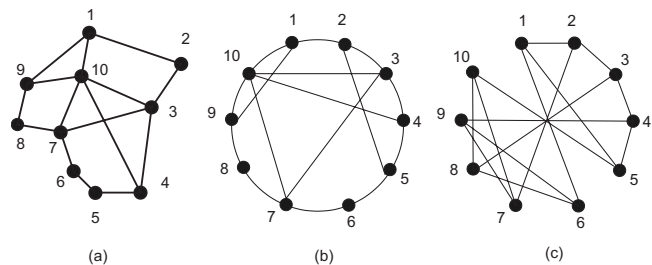


Fig. 13. (a) Network 1 with $N=10$, $\Delta_{\max}=5$, $\Delta_{\min}=2$, and $\bar{\Delta}=3$; (b) Network 1 redrawn as a chordal ring; (c) the Petersen Graph (redrawn) with $N=10$ and $\Delta=3$.

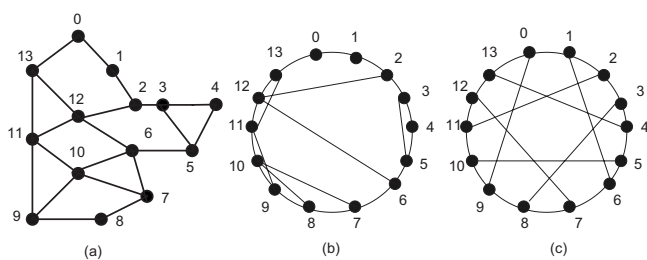


Fig. 14. (a) Network 2 with $N=14$, $\Delta_{\max}=4$, $\Delta_{\min}=2$, and $\bar{\Delta}=3$; (b) Network 2 redrawn as a chordal ring; (c) the Heawood Graph with $N=14$ and $\Delta=3$.

of 261 and 636 switching ports are required for Network 1 and Network 2, respectively. We then connect the same set of nodes in the form of Generalized Moore Graphs. In particular, we suggest that the nodes in Network 1 are connected via the Petersen Graph and the nodes in Network 2 are connected via the Heawood Graph (also a chordal ring), as shown in Figs. 13(c) and 14(c), respectively. For a fair comparison, here we let each suggested network use the same number of fiber connections as the original one does. The parameters and results of the suggested networks are also listed in Table V. It is seen that each of these suggested networks requires fewer ports. We believe that the savings are likely to be more pronounced for larger networks. An interesting trend is observed in these figures. From the perspective of a chordal ring, the original topologies of Network 1 and Network 2 tend to have more “local” connections—most of the nodes connect to their neighbors. In comparison, the improved topologies tend to have more “diagonal” connections—more nodes are linked across the ring. The trends observed in these examples, combined with the concepts and the bounds to be developed in Subsections VI.C and VI.D, provide us guidelines for designing an algorithm for constructing cost-optimal topology.

C. Regular Networks Under Non-Uniform Traffic

In this subsection, we focus on evaluating cost efficiencies of regular networks under non-uniform traffic. In particular, we derive network cost lower and upper bounds for any regular networks of node num-

TABLE V
PARAMETERS FOR NETWORK 1 AND NETWORK 2

	Network 1	Petersen Graph	Network 2	Heawood Graph
N	10	10	14	14
Δ_{\max}	5	3	4	3
Δ_{\min}	2	3	2	3
$\bar{\Delta}$	3	3	3	3
No. of fibers	30	30	52	52
No. of ports	261	240	636	560

ber N , node degree Δ , and traffic matrix $T=[t_{i,j}]$. We still assume that the switching cost at each node is linear with the number of ports. As such, minimum hop routing requires the least number of switching ports. For simplicity and clarity, we also make the assumption that all the fiber connections have the same cost.

1) *Minimum and Maximum Flow Trees*: The derivation of cost lower and upper bounds hinges on the concepts of minimum and maximum flow trees. The concept of a minimum flow tree is developed as a tool to analyze network congestion [24]. Following the rationale (of constructing a minimum flow tree), we introduce the concept of a maximum flow tree for our work.

The construction of a minimum flow tree is based on the routing spanning tree of Generalized Moore Graphs. That is, in a regular topology of node degree Δ , for each node there can be at most Δ destinations one hop away, Δ^2 destinations two hops away, etc. Moreover, for each source the Δ destinations with the largest traffic are connected by one-hop paths, the next Δ^2 destinations in descending order of traffic are connected by two-hop paths, and so on. The intuition of a minimum flow tree is to minimize the propagation of larger traffic values. As such, the larger traffic $t_{i,j}$ is closer to source node i . As an example, for the traffic matrix shown in Fig. 15(a), the construction of a minimum flow tree from node A is illustrated Fig. 15(b).

The construction of a maximum flow tree is based on the routing spanning tree of Δ -nearest neighbors. That is, in a regular topology of node degree Δ , for each node there are Δ destinations one hop away, Δ destinations two hops away, etc. That is, each (except probably the last) level is packed with only Δ nodes. Moreover, for each source the Δ destinations with the

smallest traffic are connected by one-hop paths, the next Δ destinations in ascending order of traffic are connected by two-hop paths, and so on. The intuition of a maximum flow tree is to maximize the propagation of larger traffic values. As such, the smaller traffic $t_{i,j}$ is closer to source node i . As an example, for the traffic matrix shown in Fig. 15(a), the construction of a maximum flow tree from node A is illustrated in Fig. 15(c). Note that minimum and maximum flow trees represent the best or the worst cases of traffic flow in a degree Δ topology, respectively. Thus, they are mostly not realizable within a given arbitrary traffic demand and a topology.

2) *Network Cost Lower and Upper Bounds for Regular Networks Under Non-Uniform Traffic*: With the concepts of minimum and maximum flow trees, we are ready to provide cost lower and upper bounds. To derive a lower bound, we first perform a permutation for traffic matrix $T=[t_{i,j}]$, such that the elements of each row are in a descending order. That is, for $1 \leq i \leq N$ and $1 \leq j \leq N$, let π_i^{low} be a permutation of $(1, 2, \dots, N)$ and $\pi_i^{\text{low}}(j)$ be the j th element of π_i , such that

$$t_{i,\pi_i^{\text{low}}(j)} \geq t_{i,\pi_i^{\text{low}}(j')} \text{ for } j \leq j'. \quad (26)$$

We also define D as the network diameter and m_k^{low} as the number of nodes packed in the 1st to k th levels of the minimum flow tree. That is,

$$m_k^{\text{low}} = \begin{cases} \sum_{i=1}^k \Delta^i, & \text{if } 1 \leq k \leq D-1, \\ N-1, & \text{if } k = D. \end{cases} \quad (27)$$

As stated in [5], setting up a lightpath of k hops requires $k+1$ ports. A direct application of this fact gives us the lower bound on the total switching cost for any regular network of node number N and node degree Δ , with a traffic matrix $T=[t_{i,j}]$. That is,

$$C_s \geq \beta_1 \sum_{i=1}^N \sum_{k=1}^D \sum_{j=m_k^{\text{low}}}^{N-1} t_{i,\pi_i^{\text{low}}(j)}(k+1). \quad (28)$$

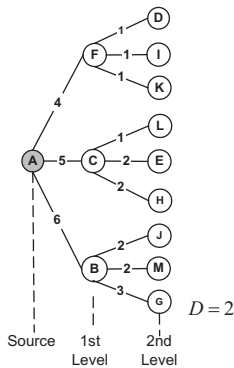
Since the fiber cost is $C_f = \alpha N \bar{\Delta}$, we have a lower bound of network cost

$$C \geq \alpha N \bar{\Delta} + \beta_1 \sum_{i=1}^N \sum_{k=1}^D \sum_{j=m_k^{\text{low}}}^{N-1} t_{i,\pi_i^{\text{low}}(j)}(k+1). \quad (29)$$

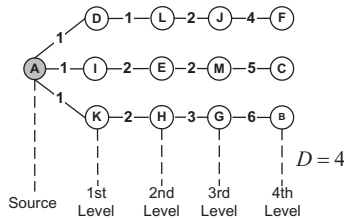
To derive an upper bound, we first perform a permutation for traffic matrix $T=[t_{i,j}]$, such that the elements of each row are in an ascending order. That is, for $1 \leq i \leq N$ and $1 \leq j \leq N$, let π_i^{up} be a permutation of $(1, 2, \dots, N)$ and $\pi_i^{\text{up}}(j)$ be the j th element of π_i^{up} , such that

	A	B	C	D	E	F	G	H	I	J	K	L	M
A	0	6	5	1	2	4	3	2	1	2	1	1	2

(a)



(b)



(c)

Fig. 15. (a) Traffic demand from node A to other nodes; (b) minimum flow tree from node A; (c) maximum flow tree from node A.

$$t_{i\pi_i^{\text{up}}(j)} \leq t_{i\pi_i^{\text{up}}(j')} \text{ for } j \leq j'. \quad (30)$$

We also define m_k^{up} as the number of nodes packed in the 1st to k th levels of the maximum flow tree. That is,

$$m_k^{\text{up}} = \begin{cases} k\Delta, & \text{if } 1 \leq k \leq D-1, \\ N-1, & \text{if } k = D. \end{cases} \quad (31)$$

A direct application of the fact that a k -hop lightpath requires $k+1$ ports gives us the upper bound on total switching cost for any regular network of node number N and node degree Δ , with a traffic matrix $T = [t_{i,j}]$. That is,

$$C_s \leq \beta_1 \sum_{i=1}^N \sum_{k=1}^D \sum_{j=m_k^{\text{up}}}^{N-1} t_{i\pi_i^{\text{up}}(j)}(k+1). \quad (32)$$

Since the fiber cost is $C_f = \alpha N \bar{\Delta}$, we have an upper bound of network cost

$$C \leq \alpha N \bar{\Delta} + \beta_1 \sum_{i=1}^N \sum_{k=1}^D \sum_{j=m_k^{\text{up}}}^{N-1} t_{i\pi_i^{\text{up}}(j)}(k+1). \quad (33)$$

D. Irregular Networks Under Arbitrary Traffic

The derivation provided in Subsection VI.C can be extended to provide network cost lower and upper bounds for irregular networks under non-uniform traffic. As in the previous subsection, we assume minimum hop routing and linear switching cost at every node.

The permutation of the traffic matrix $T = [t_{i,j}]$ for the lower bound is the same as in Eq. (26). However, m_k^{low} is defined differently in comparison with Eq. (27):

$$m_k^{\text{low}} = \begin{cases} \sum_{i=1}^k \Delta_{\text{max}}^i, & \text{if } 1 \leq k \leq D-1, \\ N-1, & \text{if } k = D. \end{cases} \quad (34)$$

The rest of the derivation follows that leads to c.f. Eq. (28). As such, the cost lower bound has the same form as Eq. (29).

Similarly, the permutation for traffic matrix $T = [t_{i,j}]$ for the upper bound is the same as in Eq. (30). m_k^{up} is defined differently in comparison with Eq. (31):

$$m_k^{\text{up}} = \begin{cases} k\Delta_{\text{min}}, & \text{if } 1 \leq k \leq D-1, \\ N-1, & \text{if } k = D. \end{cases} \quad (35)$$

The rest of the derivation follows that leads to Eq. (32). As such, the cost upper bound has the same form as Eq. (33).

We note that the performance of these bounds depends on the variance of the Δ_i and $t_{i,j}$. However, the insights gained in the process of constructing such bounds help us to understand the structure and con-

figuration imposed by optimality. In particular, the study of the properties of Generalized Moore Graphs and Minimum Flow Tree yields an important topology design guideline: a cost-effective physical topology should minimize the propagation of large traffic volume. Using this guideline together with the characterization of the tradeoff between fiber and switching resources, we propose a topology design algorithm in the following subsection.

E. A Cost-Efficient Topology Design Algorithm

In this subsection, we propose an algorithm for topology construction under realistic design scenarios (arbitrary traffic, non-symmetric topology, and distance-dependent fiber connection cost), based on the following two design guidelines:

- Guideline 1. The cost-efficient topology is the result of the tradeoff between the fiber and the switching cost.
- Guideline 2. A good network topology minimizes the propagation of large traffic flow.

The algorithm, with its pseudocode detailed in Algorithm 1, consists of two components. The first component is based on Guideline 1. That is, providing a set of budgets allows us to trade off fiber against switch cost. In particular, this component involves assigning a set of n_B monotonically increasing budgets B_i on total fiber length used in the network. That is, $B_i < B_j$ for $i < j$ and $i, j \in \{1, 2, \dots, n_B\}$. Note that the minimal fiber length budget equals the length of a minimum spanning tree (MST), with distance $d_{i,j}$ as the weight ($d_{i,j}$ is set to 1 if the fiber cost is not distance dependent). The maximal fiber length budget equals the total length of a fully connected (complete) network. The quality of the solution is affected by the number of budgets assigned—cardinality of the set n_B . In the worst case, we start with a MST and add one fiber connection (edge) for each subsequent budget B_i ($i \in \{2, 3, \dots, N(N-1)\}$), until we reach the fully connected network. In other words, the maximal cardinality of the set is $(N-1)^2 + 1$. For large network size N , analytical results on the fiber and switching costs [2–4] can inform us the solution regions for good physical topology, thus reducing the solution space.

The second component, based on Guideline 2, involves iteratively connecting node pairs based on an updated criterion. For a given budget, we first connect the nodes via a minimum spanning tree. Next, as long as the fiber length is within the budget, we add one fiber connection at a time to increase the network connectivity. Let $t_{i,j}$, $h_{i,j}$, and $d_{i,j}$ denote the demand, number of hops, and Euclidian distance between node i and node j , respectively. We use $M_{i,j} = t_{i,j}h_{i,j}/d_{i,j}$ (if fiber connection cost is not distance dependent, $M_{i,j} = t_{i,j}h_{i,j}$) as a criterion. Each time we add a new fiber span, we select the node pair with the largest $M_{i,j}$. We

repeat the same process until the total fiber length reaches (or surpasses) the budget. These steps, based on Guideline 2, not only allow large traffic to travel as few hops as possible [illustrated in Fig. 16(a)], but also to make effective use of fiber resources [illustrated in Fig. 16(b)]. Note that the maximum number of iterations is $N(N-1)$, when the fiber length budget equals the total length of a fully connected (complete) network.

Algorithm 1 Cost-Efficient Topology Design

Assign a set of n_B monotonically increasing budgets B_i on fiber length.

That is, $B_i < B_j$ for $i < j$ and $i, j \in \{1, 2, \dots, n_B\}$.

for $i = 1 : n_B$ **do**

Connect all the nodes via a minimum hop spanning tree (MST), with weight $w_{i,j} = d_{i,j}$.
 $length_F = length_{MST}$
while $length_F \leq B_i$ **do**

Construct (update) a $N \times N$ matrix M , with $M_{i,j} = \frac{t_{i,j} h_{i,j}}{d_{i,j}}$.
 Connect nodes i and j , $M_{i,j} = \max_{i,j} \{M_{i,j}\}$.
 $length_F = length_F + d_{i,j}$
end while

Routing and Wavelength Assignment (RWA) for the network.
 Calculate the network cost C_i .

end for

Choose the topology with $\min_i \{C_i\}$.

As a preliminary testing of the performance of the algorithms, we apply Algorithm 1 to small networks with 7 nodes ($N=7$). For each test case, nodes are uniformly distributed in an area of $20 \times 20 \text{ km}^2$. A demand $t_{i,j}$, in the unit of number of lightpaths, is chosen from a set of uniformly distributed integers $\{1, 2, \dots, 6\}$. We assume that the fiber cost is linear with distance. The switch cost is linear, with switch-to-fiber cost ratio β_e/α set to 2. We test a total of 500 cases, for each of which we apply both Algorithm 1 and exhaust-

ive search and then compare the costs (generated by each approach). In Fig. 17 we plot the histogram of the performance offset (the difference between the costs obtained by heuristics and exhaustive search). The figure demonstrates that in 25% of cases Algorithm 1 produces the optimal solutions. The maximum offset is 5%.

VII. CONCLUSION

Using the tool of graph theory and optimization, we established a theoretical foundation of designing networks that not only require a low initial capital investment, but also have good scalability—a decreasing cost-per-node-per-unit-traffic as user number and transaction size increase. By analyzing the tradeoffs among important network resources, we found that for regular networks and uniform traffic, the joint design problems of fiber connection topology, dimensioning, and routing can be solved optimally and analytically. We prove that with minimum hop routing, Generalized Moore Graphs, whose average minimum hop distances scale favorably as $\log_{\Delta} N$, achieve the lower bound on network cost and require the minimum (or close to minimum) number of wavelengths to support a given uniform traffic demand. In addition, Generalized Moore Graphs and their close relatives have good scalability: the minimal normalized cost per unit traffic decreases with increasing network size. In summary, our works identify Generalized Moore Graphs as fundamental architectures in the context of cost efficiency. These architectures represent a drastic departure from currently used ones in MANs, such as rings or interconnected rings.

Our results demonstrate that switching technologies have a tremendous impact on the final topological architectures. The optimal topologies connecting the

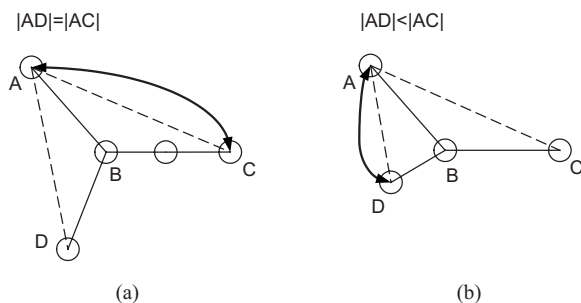


Fig. 16. A topology design algorithm: each time a fiber span is added, the node pair with the largest $t_{i,j}h_{i,j}/d_{i,j}$ is connected, where $t_{i,j}$, $h_{i,j}$, and $d_{i,j}$ denote the demand, number of hops, and Euclidian distance between node i and node j , respectively. In (a) we have $d_{A,D} = d_{A,C}$ and $t_{A,D} = t_{A,C}$. Since $h_{A,C} > h_{A,D}$, a fiber span is added between nodes A and C. In (b) we have $h_{A,D} = h_{A,C}$ and $t_{A,D} = t_{A,C}$. Since $d_{A,D} < d_{A,C}$, a fiber span is added between nodes A and D. Using $t_{i,j}h_{i,j}/d_{i,j}$ as a criterion limits the travel of large traffic to as few hops as possible and makes effective use of fiber at the same time.

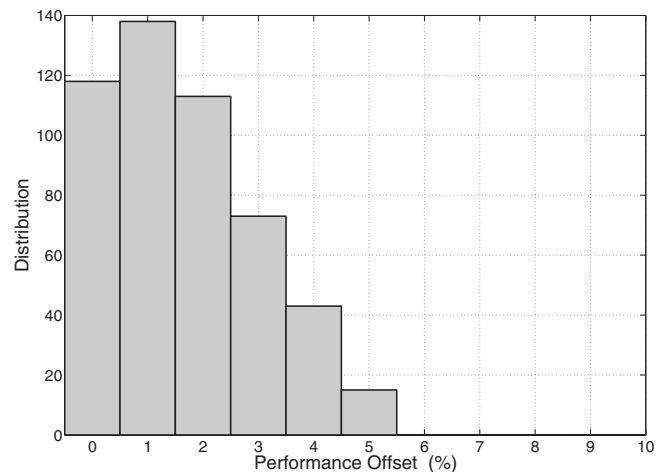


Fig. 17. The histogram of the performance offset (the difference between the costs obtained by heuristics and exhaustive search), with $N=7$ and $\beta_e/\alpha=2$. 500 cases are tested.

same set of nodes can differ significantly when different switching fabrics are used, even when these topologies are designed to serve the same traffic demand. A comparison of the cost benefit between OXC and OEO switches shows that at low data rates it is economical to use OEO switches; at high data rates, it is more cost-advantageous to use OXC switches.

We also addressed more realistic design scenarios—irregular network topologies and (static) non-uniform traffic. We showed that if the switching cost is linear with port counts, minimum hop routing is still optimal. The results of Generalized Moore networks can be used to provide useful estimates for the cost of irregular networks. Also the unique structure of a Generalized Moore Graph—each of its nodes has a full (or almost full) Δ -ary routing spanning tree—can be exploited to suggest improvements for irregular physical topologies. Moreover, the study of the constructions of Generalized Moore Graphs yields a general yet crucial design guideline for irregular topology and arbitrary traffic: a cost-effective physical topology should minimize the propagation of large traffic flows. This principle guides us to propose a network design heuristic for arbitrary topologies and traffic. The preliminary tests show that the networks generated by the heuristic have minimum or close to minimum costs.

Last but not the least, the analytical framework employed in our research thus far is general enough to be applied in our pursuit of the following future research directions:

- A comparison of cost advantages and disadvantages between flat and hierarchical architectures.
- Scalable network architecture design under demand uncertainty.
- Optimized evolution of a physical network topology over a multi-period planning horizon.

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