



THE DISTRIBUTION OF HYDRAULIC ENERGY IN WEIR FLOW
WITH RELATION TO SPILLWAY DESIGN

by

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Cambridge, Massachusetts
May 25, 1932

Professor A. L. Merrill
Secretary of the Faculty
Mass. Institute of Technology
Cambridge, Massachusetts

Dear Professor Merrill:

I herewith submit a
thesis entitled

"The Distribution of Hydraulic
Energy in Weir Flow with Re-
lation to Spillway Design"

in partial fulfillment of the requirement
for the degree of Master of Science in
Civil Engineering.

Very truly yours,

Hunter Rouse

183077

A word of appreciation to Professor G. E. Russell
for his patience, interest, and encouragement
throughout the preparation of this thesis

S u m m a r y

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Because of the prevalent tendency among American hydraulic engineers to adhere entirely to empirical methods in the design of weirs and spillways, the author makes an effort in this thesis to clarify the actual physical circumstances accompanying flow over various spill sections.

Proceeding first of all from the purely theoretical standpoint, the basic equations of hydro-mechanics as applied to two-dimensional curving flow are developed, and the forces causing conversion of potential and pressure energy into kinetic energy are portrayed. Both the means of application of the theory of potential flow and the accompanying difficulties and limitations are briefly shown.

After a general, non-mathematical discussion of the conditions of pressure and velocity distribution occasioned by vertical curvature, the theorem of Bernoulli and the law of impulse and momentum are applied to this general case of two-dimensional water motion. These general principles are then

adapted to basic examples of weir discharge, and in the following section these applications are illustrated by the description of experimental investigations conducted by the author on five different forms of weir profile.

Finally, practical spill sections are discussed, with regard to the design of the profile and to the variation of the discharge coefficient with both head on crest and shape and dimensions of spillway profile. Model experiments made in France and at M.I.T. are described to substantiate the discussion.

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The Distribution of Hydraulic Energy in Weir Flow
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I Introduction

1. Discussion of past practice

Although recent developments in aviation have shown so clearly the practical value of studying fluid motion on a purely physical basis, hydraulic engineers are still prone to treat the flow of water from a more or less empirical standpoint. In the field of turbine design the theory of hydromechanics is gradually becoming a usable tool, in particular as it is being found that the departure of water from the ideal case of a perfect fluid may be compensated by certain approximations made in applying the theoretical laws of ideal behavior. Hence those practicing in the latter field have become rather familiar with the action of water under the conditions of curving flow and realize the existence and importance of the accelerative forces essential to the motion of water particles in curved paths.

The majority of hydraulic engineers, however, continue to show great hesitancy to venture into the complexities of this type of water motion, with the result that their nearest approach to the study of curving flow is the treatment of discharge from an orifice or sharp-crested weir in the light of a body falling freely through space. That many of the major hydraulic problems involve an acceleration of the

water particles with an accompanying departure from static pressure distribution (i.e. every case in which a change of channel cross-section occurs) is ignored; instead of seeking to comprehend the conditions at the actual transition sections, these localities are avoided by taking measurements up- and downstream where static conditions are still in effect. It is true that this procedure permits ready computation of the discharge coefficient in the case of flow at orifices, gates, and weirs, but the graphs of the coefficients over a range of heads often show variations that are difficult to explain, much less prophesy, by such methods.

Horton¹ (see Bibliography), for instance, has compiled a very extensive report on experiments with weirs of various shapes and sizes, in which he has measured only discharge and stream profile, without attempting to show why the shape of the weir may alter the conditions. Woodburn², after investigating at length the problem of the broad-crested weir, came no nearer to the actual comprehension of flow at a change of section. How much more these men could have understood had they been as deeply engrossed in the curves of internal pressure as in the curves of the water surface!

In the case of discharge over spillways, past experience has been practically the only guide; assumptions are made to enable an approximate determination of the pressure exerted upon the upstream surface and the most favorable form of the discharge face, and time has proven that the assumptions

are safe (see Creager³). But they accomplish little in improving our knowledge of the actual conditions and thus allowing a minimum of assumption in design.

It is not the writer's purpose at this time to present better methods of design or to show to any great extent where past methods are in error; instead an effort will be made to discuss the behavior of water under conditions of partial (in general, less than gravitational) acceleration due to vertical curvature of the stream lines, based upon both theoretical and experimental investigation of flow over various types of weirs, with a view toward making this sort of flow more comprehensible. The writer believes most sincerely that the study of curving flow is indispensable to further progress in applied hydraulics.

2. Recent developments in the field of curving flow

Professor Koch⁴ of the Technische Hochschule at Darmstadt, Germany, was probably the first to attempt a comprehensive study of the flow of water in a way which would bring the complexities of the subject within the grasp of the practising engineer. "We need, in place of mathematical hydrodynamics and empirical hydraulics, simple, intuitive, and practical hydrodynamics," he wrote more than a decade ago, for "knowledge of the forces exercised by flowing water is directly essential, and this knowledge is not provided

by hydraulics." Hence Koch devoted his attention in particular to such cases of water motion in which "secondary forces" arose because of the curvature of the stream paths. Besides developing in a practical way theories covering a wide range of conditions, he also tested his theories in a model laboratory at the university, with excellent results. But his experiments were confined to general cases, paving the way for other experimenters to follow in special fields.

Comparatively few have as yet followed Koch's example. Not until 1929 did Böss⁵ of Karlsruhe, Germany, publish a paper on the computation of pressure reduction in flow over a broad-crested weir, in which he developed a method for approximating the pressures over a flat or curving face on the broad assumption of linear pressure distribution over the vertical section; he further showed that the introduction of low sills at the end of broad-crested weirs in no way influenced the discharge. Yet immediately thereafter Ehrenberger⁶, of Vienna, showed incomplete knowledge of flow conditions by making false assumptions on the basis of similar experiments.

Spurred on by these investigations, the writer conducted extensive experiments for the case of ventilated discharge over a very broad weir with horizontal floor, measuring the energy distribution at numerous points in the region of transition to verify certain theoretical studies made previously⁷. In addition to this series of experiments, the

writer also investigated three other cases involving different downstream faces of the same weir, all of these investigations being conducted in the hydraulic laboratory of the Technische Hochschule at Karlsruhe, Germany. At present the writer is engaged in similar experiments with the sharp-crested weir in the M.I.T. River Hydraulic Laboratory. Each of these investigations will be discussed later in this thesis.

3. Methods of attack

Through assuming that water is an ideal fluid (frictionless, incompressible, and cohesionless) the hydromechanical theory of potential flow offers the only means of completely solving a problem of flow through a varying section. Needless to say, the neglect of the variation in energy distribution due to frictional losses involves some amount of error in the results; yet the error is surprisingly small in cases of smooth, rapid transition. The greatest drawback to this physically correct method is its extreme tediousness, for the graphical solution depends upon trial and error and repeated correction for its accuracy, which makes it of little direct use to the practical engineer.

The real value of this theory lies in the clear picture it gives of the physical action that occurs when water departs from a lineal course, an understanding that is essential

to a working knowledge of hydraulics. Once this principle of acceleration and the accompanying change in pressure is understood, more practicable hydraulic principles may be combined intelligently with experience to produce efficient and comprehensive results.

The latter principles are based largely upon Bernoulli's equation and the law of impulse and momentum with certain modifications. Contrary to the theory of potential flow, these deal with conditions over sections of the entire stream, rather than with the individual particles of water, and hence do not show as clearly the underlying characteristics of this type of fluid motion.

II Fundamental principles and their application to basic weir sections

1. General hydrodynamics of curving flow

Hydromechanics treats fluid motion from a purely physical point of view, by means of the mathematical theory of potential flow⁸. Through the assumption of an ideal fluid - that is, one which is frictionless, cohesionless, and incompressible - the motion in space of any particle may be expressed by the three equations of Euler. The solution of these equations, together with the equation of continuity, becomes possible once the boundary conditions are established; these boundary conditions are simply the profiles of the confining surfaces, or in case the water is exposed to the atmosphere at some point, the fact that the pressure there is atmospheric. The derivation of these equations follows.

Let us consider a small volume of fluid ds in length and dA in cross-sectional area (see Figure 1). The only forces acting upon this volume are gravity and the pressure of the surrounding water. That component of the total force f acting in any direction s , since the rate of pressure change in this direction is designated by $\frac{\partial p}{\partial s}$, may be expressed as follows:

From the geometry of the figure,

$$f_s = p dA - (p + \frac{\partial p}{\partial s} ds) dA + W \cos \alpha$$

Hence $f_s = -\frac{\partial p}{\partial s} ds \cdot dA - w \cdot ds \cdot dA \cdot \frac{\partial h}{\partial s}$

and $\frac{f_s}{ds \cdot dA} = -\frac{\partial}{\partial s} (p + wh)$ (1)

On the basis of this general equation we may now state: The force producing acceleration upon a unit volume of fluid in any direction is equal to the rate of decrease in the sum $(p + wh)$ in that direction.

Since $f = m \cdot a$ and $m = \frac{w}{g}$ equation 1 becomes

$$a_s = -\frac{\partial}{\partial s} \left(\frac{p}{w} + h \right) \times g \quad (2)$$

Thus what is probably the most important principle of hydromechanics becomes apparent: The acceleration of a fluid particle in any direction is equal to the product of gravitational acceleration and the rate of decrease in the sum $\left(\frac{p}{w} + h \right)$ in that direction.

In general, the velocity of any particle is a function of space and time; written in terms of the Cartesian coordinate system,

$$v_x = f_1(x, y, z, T)$$

$$v_y = f_2(x, y, z, T)$$

$$v_z = f_3(x, y, z, T)$$

If, instead of the Cartesian, we use the natural coordinate system, placed so that the particle in question lies at the center of coordinates, the general expressions for space and time acceleration become greatly simplified. In Figure 2 is shown a fluid particle traveling in a curved path. At any point o in this three-dimensional path

(the position of the particle at the given instant) natural coordinate axes may be constructed, with o as the center of coordinates. Since the velocity vector \bar{v} of the particle is tangent to the curve at o and lies along the s -axis, and the radius of curvature ρ is measured along the n -axis, the plane (s,n) is the plane of curvature at that point; the m -axis is normal to this plane.

The acceleration of a particle moving through space may be expressed

$$a = \frac{dv}{dT}$$

The component of this acceleration in the direction s then becomes

$$a_s = \frac{dv}{dT} = \frac{\partial v}{\partial s} \cdot \frac{\partial s}{\partial T} + \frac{\partial v}{\partial T} = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial T} = \frac{\partial (\frac{v^2}{2})}{\partial s} + \frac{\partial v}{\partial T}$$

in which $\frac{\partial (\frac{v^2}{2})}{\partial s}$ represents the acceleration in the s direction due to a differential movement along the curved path, and $\frac{\partial v}{\partial T}$ denotes the acceleration in the s direction at point o due to a differential change in time.

Similarly the acceleration normal to this direction along the n -axis (centripetal acceleration) will be

$$a_n = \frac{dx_n}{dT} = \frac{v^2}{\rho} + \frac{\partial v_n}{\partial T}$$

Since the curvature of the path at point o is entirely in the plane (s,n) over an infinitesimal distance, the only acceleration in the m direction is that with time:

$$a_m = \frac{\partial v_m}{\partial T}$$

These values may now be equated in turn to the expression already developed for acceleration in any direction, resulting in Euler's equations expressed in terms of natural coordinates:

$$\frac{\partial v}{\partial T} + \frac{\partial}{\partial s} \left(\frac{v^2}{2} \right) = -g \frac{\partial}{\partial s} \left(\frac{p}{w} + h \right) \quad (3a)$$

$$\frac{\partial v_n}{\partial T} + \frac{v^2}{\rho} = -g \frac{\partial}{\partial n} \left(\frac{p}{w} + h \right) \quad (3b)$$

$$\frac{\partial v_m}{\partial T} = -g \frac{\partial}{\partial m} \left(\frac{p}{w} + h \right) \quad (3c)$$

If we are dealing with only a two-dimensional case, equation 3c may be dropped. Furthermore, if the flow is a steady one (not varying with time), equations 3a and 3b may be greatly simplified. Equation 3a becomes

$$\frac{\partial}{\partial s} \left(\frac{v^2}{2g} + \frac{p}{w} + h \right) = \frac{\partial H}{\partial s} = 0$$

Since this differential expression is equal to zero,

$$\frac{v^2}{2g} + \frac{p}{w} + h = H = \text{const.} \quad (4)$$

Hence the total energy is seen to be constant along a given path or stream filament. The theory of potential flow further assumes constant energy at all points in the moving fluid (i.e. no frictional losses and no turbulence of any kind). This constant term H denotes the energy per unit weight of fluid, and hence is a linear value, commonly called total head. Equation 4 is Bernoulli's equation in its basic form; it is interesting to note that Bernoulli

published this expression about 1738, some time before Euler's equations were made known in 1755.

By subtracting the term $\frac{\partial(\frac{v^2}{2})}{\partial n}$ from both sides of equation 3b, the right side will be seen to contain the differential of H and hence is also equal to zero:

$$\frac{v^2}{\rho} - \frac{\partial(\frac{v^2}{2})}{\partial n} = -g \frac{\partial}{\partial n} \left(\frac{v^2}{2g} + \frac{p}{w} + h \right) = 0$$

The expression then resolves to

$$\frac{v^2}{\rho} = \frac{\partial(\frac{v^2}{2})}{\partial n} = v \frac{\partial v}{\partial n} \quad \text{and} \quad \frac{v}{\rho} = \frac{\partial v}{\partial n} \quad (5)$$

$$\text{or} \quad \log v = \int \frac{\partial n}{\rho} + C' \quad \text{whence} \quad v = C \cdot e^{\int \frac{\partial n}{\rho}} \quad (6)$$

Euler's equations are customarily written in their most general and least useful form - in Cartesian coordinates - which is added here for the sake of completeness:

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} + \frac{\partial v_x}{\partial T} = -g \frac{\partial}{\partial x} \left(\frac{p}{w} + h \right)$$

$$v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} + \frac{\partial v_y}{\partial T} = -g \frac{\partial}{\partial y} \left(\frac{p}{w} + h \right)$$

$$v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial T} = -g \frac{\partial}{\partial z} \left(\frac{p}{w} + h \right)$$

In addition to these equations must be considered that resulting from the law of continuity, which states that, since the fluid is assumed incompressible, the quantity of water entering the confines of a given space element must equal the quantity leaving those confines at the same time.

Considering now the two-dimensional case, in which the thickness or z dimension is unity, we see from Figure 3 that the quantity of fluid entering the element $dx \cdot dy$ through the face dx is $v_y dx$ and that leaving through the opposite face is $(v_y + \frac{\partial v_y}{\partial y} \cdot dy) dx$; similar relations are found for the other two faces. The difference between the fluid leaving and the fluid entering this element will be

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dx \cdot dy$$

which must equal zero. Hence we have the equation of continuity:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad (7)$$

We have already seen that the velocity component in any direction, for steady flow, may be expressed as a function of the coordinates. This function is called the ϕ -function, and is such that:

$$v_x = \frac{\partial \phi}{\partial x} \quad v_y = \frac{\partial \phi}{\partial y} \quad (8)$$

ϕ is commonly known as the velocity potential, and is a constant along a trajectory cutting all stream filaments at right angles.

Similarly the stream function ψ is such that its value is constant along any stream filament, and may be expressed by the following relations:

$$v_x = \frac{\partial \psi}{\partial y} \quad v_y = - \frac{\partial \psi}{\partial x} \quad (9)$$

By combining the equations 7 and 8 the familiar equa-

tion of Laplace results:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (10)$$

Potential flow may now be seen to have the following general characteristics for the case of a two-dimensional, non-rotational, steady motion: The velocity of any particle may be expressed as a potential, or a mathematical function of its coordinates; this known as the velocity potential ϕ , and has a constant value over a curve that is normal to the velocity vector or direction of motion at every point. A second function ψ , called the stream function, by means of which the velocity may also be expressed, has a constant value along the path a particle travels. Although the stream function ψ is constant along a single stream filament, its value is different for every other filament; similarly, the velocity potential ϕ , a constant over a single trajectory normal to all stream filaments, will vary from point to point along any single stream filament.

The significance of the ϕ and ψ functions becomes more apparent if equations 8 and 9 are rewritten in natural coordinates, so that the x-axis lies along s and the y-axis along n (see Figure 4). The equations then become:

$$v_s = \frac{\partial \phi}{\partial s} = \frac{\partial \psi}{\partial n} = \bar{v} ; \quad v_n = \frac{\partial \phi}{\partial n} = -\frac{\partial \psi}{\partial s} = 0 \quad (11)$$

This we see to be true, since the velocity vector lies along the s -axis at any point, so that the velocity in the n

direction must be zero in each case or equal to the derivative of the corresponding constant.

With these relationships we now have sufficient equations for the solution of problems, once the expressions are put in the proper form for the boundary conditions in question. The solution is often effected by the "method of conformal transformation", through use of complex relations involving the imaginary quantity $\sqrt{-1}$; in this way any number of solutions to equation 10 may be found, from which that sought may readily be selected. This method is of great value in determining theoretically the discharge coefficients of weirs and orifices, the best shapes of airplane sections, turbine blades, projectiles, and so forth.

However, inasmuch as these solutions involve considerable mathematical knowledge, they are of little interest to the practical hydraulician. The chief value to him of the theory of potential flow lies in the comprehensive picture it gives of the physical occurrences accompanying curvature of the stream filaments, by means of the flow net. This consists of a system of any desired number of ψ and ϕ lines drawn on the profile of a given flow, in such a way that between every pair of ψ lines or stream filaments an equal quantity of water will flow per unit of time. The velocity potential lines are so drawn that the small enclosed areas on the profile will become perfect squares as they approach the infinitesimal.

A fairly accurate net may be drawn by eye over a profile of any discharge simply by adjusting the stream and velocity potential lines so that they will divide the profile into approximate squares. This is further facilitated if the pressure distribution along the boundaries of the profile is known through measurement: from Bernoulli's theorem the velocities may be found at various points along the boundaries; then, since the velocity is inversely proportional to the side of each square (see equation 11), the accuracy of the construction may be checked.

In Figure 5 is shown such a flow net constructed for a profile of a model siphon⁸. Coloring matter introduced into the water showed the actual stream filaments, corresponding almost exactly with the theoretical lines of constant ψ .

This graphical method may also be used, independent of all measurements, to solve a discharge problem completely, provided the flow approximates that of an ideal fluid - that is, provided there is no excessive turbulence, and no three-dimensional motion⁹. If the water surface is exposed to the atmosphere, its probable curve must be assumed; if it flows between fixed surfaces, the boundaries are of course already determined. The profile is now divided into strips of approximately equal discharge through drawing by eye, as has already been explained, assumed stream lines; where the flow is linear, these stream lines will divide

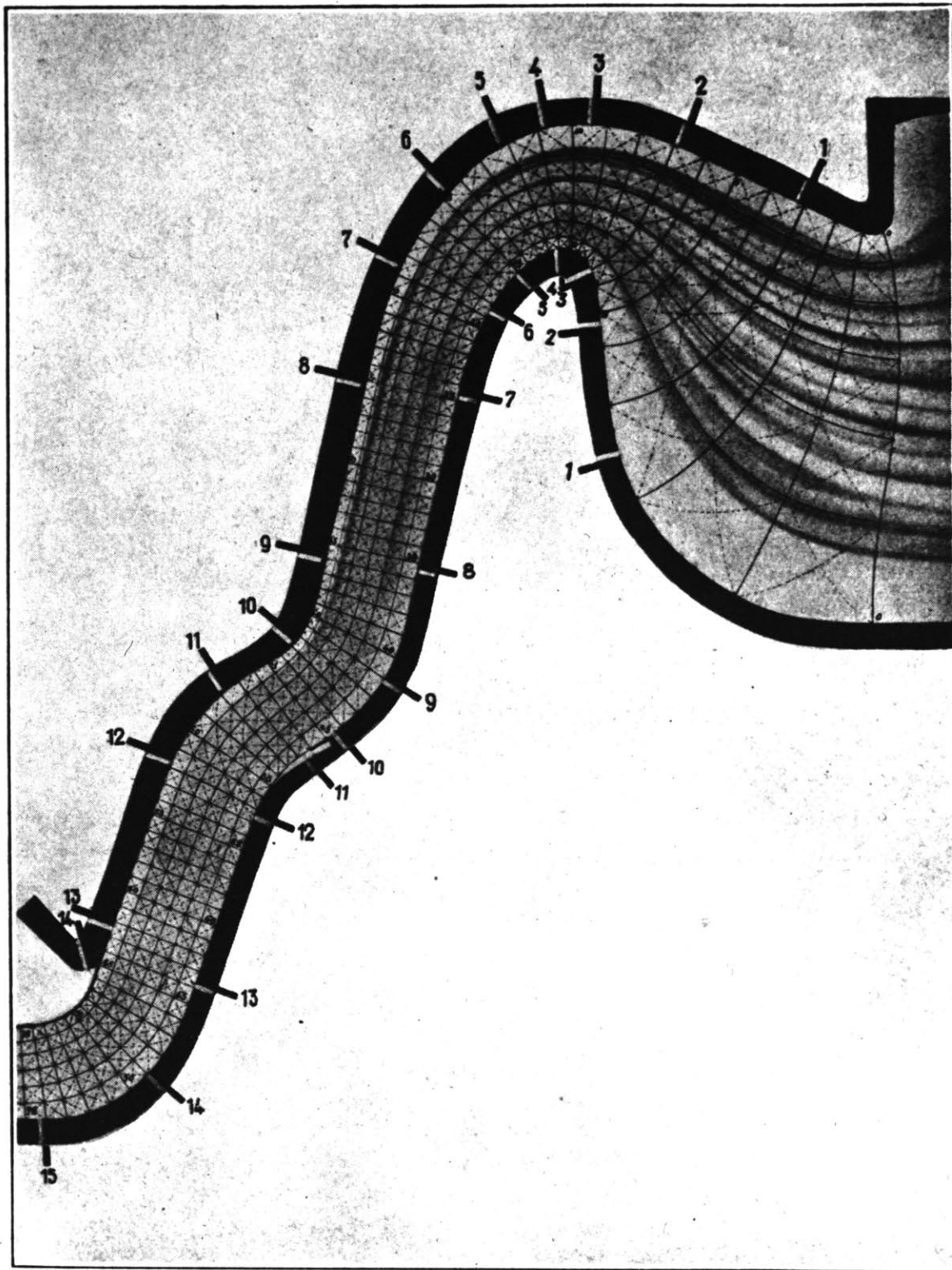


Figure 5. Comparison between theoretical and actual stream lines, from an experiment with a model of a siphon made at Hannover, Germany

the profile into equal sections (see Figure 6). The approximate accuracy of the construction may be tested by sketching in a number of velocity potential lines, to form a network of squares. It must be noted that the accuracy, as well as the quantity of work, increases with the number of stream lines that is used.

Selecting now any ϕ trajectory, the radius of curvature ρ of the stream lines is measured at each intersection with the chosen ϕ line. These values are plotted against those of n , measured along the ϕ line from outer to inner border of the profile (see Figure 6). Since the velocity is inversely proportional to the distance between each pair of ψ lines, the curve of v against n may also be plotted; this will of course be the curve $v = e^{\int_0^n \frac{\partial n}{\rho}}$ (equation 6).

The slope of the tangent of the latter curve will equal at every point the value $\frac{\partial v}{\partial n}$, from which, together with the plotted values of v and ρ , the corresponding ratio $\frac{v}{\rho}$ is checked (equation 5). If this is found to be in error, the ψ lines must be changed until the relation is satisfied. Then it is an easy matter to find the curve of pressure distribution from Bernoulli's equation. This process is repeated for various ϕ trajectories. In the case of a free surface, the surface pressures must be found to be zero - otherwise the assumed surface curve is in error, and must be revised.

The foregoing pages have given briefly the general methods by which certain hydraulic problems may be solved through use of Euler's equations and the theory of potential flow; obviously such a solution would be impossible with only the usual equations of hydraulics. Needless to say, this method is tedious and dependent upon the limits of graphical accuracy; furthermore, it ignores whatever losses may occur through friction. However, the assumption of constant energy at every point in reality causes no great error in the final results for the majority of cases of smooth transition, so that the ideal conditions represented by the theory of potential flow may usually be taken as basically indicative of actual occurrences.

For cases of ideal, straight-line flow, the pressure is a direct function of the depth, so that the pressure head at any point is equal to the vertical distance of that point below the free water surface, and the velocity is then the same at any point over the vertical section. Actually the velocity distribution is influenced by floor friction; however, the pressure will retain its static distribution so long as the flow is linear, and the total head of every filament must then change according to the variation in velocity head of that filament.

Thus in open channels the total energy will vary from surface to bottom directly with the variation in velocity head, so that the value H_m for an entire vertical section

must necessarily be an average. But since we shall treat in this thesis comparatively short distances without excessive turbulence, the loss of energy from section to section of a given stream filament will often be so small that it may be neglected. Hence we may for our purposes assume that the total energy in each filament remains nearly constant, but that it may vary from filament to filament.

While Bernoulli's equation, with correction for energy loss, is commonly applied to pipe flow and lineal flow in open channels, we have seen that it is equally valid in cases of departure from conditions of static pressure distribution. Correct as this may be, however, it is of little assistance in determining pressure and velocity at any point, when both are unknown. For this we must depend upon equation 5.

The development of equations 1 and 2 is of primary importance in that it shows clearly the mechanics of curving flow. Water cannot follow a curved course unless forces exist tending to make the particles deviate from linear paths. Curvature upwards requires in general an external force such as that provided by a change in floor level or by a sill or weir; this external force is transmitted to the particles within the stream by an increase above the normal static pressure. Curvature downwards requires a reduction in the floor reaction to the weight of the water - that is, the weight of the water no longer produces normal

static pressure over a vertical section, but instead a reduced pressure; the portion of the weight not supported by the floor causes the downward acceleration.

From equations 1 and 2 it will be seen that the velocity, and hence the pressure, is a complex function of the radius of curvature ρ . Since this function is dependent upon a number of factors, the resulting pressure distribution is in general non-linear, so that only in case of moderate curvature may the distribution curve be approximated by a straight line; this approximation does not, however, correspond to static conditions, for the pressure head varies directly with, but is no longer equal to, the depth below the surface.

From the foregoing discussion it is not difficult to visualize the part gravity plays in the vertical acceleration of water. So long as there is a force great enough to resist the gravitational effect upon the water particles - the resistance of the channel bed, for instance - there will be no vertical acceleration; hence the particles continue to move in a straight line. Should this resistance decrease or increase, the particles will begin to undergo a vertical acceleration. What is difficult to visualize is the fact that acceleration in any direction is the result of not only one but two forces - gravity and pressure drop. We have already seen that the force tending to accelerate a unit volume of water in a given direction is equal to the

drop per unit of distance in the expression $(p + wh)$ in that direction. As h decreases downwards, let us consider D (depth below the free surface) as increasing in that direction, so that we may write for the vertical force acting downwards upon a unit volume

$$f_D = -\frac{\partial}{\partial D} (p + wh) = w \frac{\partial D}{\partial D} - \frac{\partial p}{\partial D} = w - \frac{\partial p}{\partial D} \quad (12)$$

Hence the downward force upon a unit volume is its weight minus the rate of increase of pressure intensity in that direction. Under static conditions the rate of increase of pressure exactly counterbalances the weight, so that there is no acceleration in the vertical. But in the case of curving flow, the pressure distribution is often such that the particles in certain regions of the flow are being accelerated by forces much greater than gravity alone; in a contracting jet just emerging from a sharp-crested orifice, for instance, particles below the centerline of the orifice are being driven downwards not only by gravity, but also by a pressure drop; the same is true for any weir discharging into the atmosphere.

Since gravity has only a vertical component, it can cause no acceleration horizontally. Yet if equation 12 is rewritten for the horizontal direction x ,

$$f_x = -\frac{\partial p}{\partial x} \quad (13)$$

it will be seen that a horizontal acceleration may still result because of pressure drop in the horizontal direction.

This horizontal acceleration results in an increase in velocity to equal the decrease in pressure intensity, for the potential head does not change and the total energy must remain constant. Similarly a vertical acceleration requires an increase in vertical velocity and a decrease in pressure and potential heads, the total head still remaining unchanged.

With these considerations in mind, we may now proceed to discuss several hydraulic principles which will be of assistance in computations of the simpler cases of curving flow.

2. The use of Bernoulli's equation

A graphical representation of Bernoulli's equation is commonly used in cases of pipe and open channel flow. For pipe lines the total head is given on a longitudinal plot of the system by a line lying the distance H above the assumed geodetic base; hence this energy line is independent of the distance of the pipe below it. Below the energy line a distance equal to the velocity head is plotted the pressure gradient, so that the vertical distance between the pressure gradient and the pipe axis always equals the average pressure head at the section in question. The elevation of the pipe axis above the base is of course the average potential head.

This representation for open channel flow is much

simpler, since the pressure gradient lies at the water surface in all cases of static pressure distribution - i.e. when the flow is linear. Hence the energy line lies above the water surface an amount equal to the velocity head. Since the velocity is seldom constant over a vertical section, a factor α_u must, strictly speaking, be used to correct the value $\frac{V_m^2}{2g}$, so that the true average velocity head k_m becomes

$$k_m = \frac{1}{D} \int_0^D \frac{v}{V_m} \cdot \frac{v^2}{2g} \cdot dD = \alpha_u \cdot \frac{V_m^2}{2g} ; \quad \alpha_u = \frac{1}{Q V_m^2} \int_0^D v^3 dD \quad (14)$$

The factor α_u may vary from 1.0 in the ideal case of equal distribution to perhaps 1.31 for abnormal parabolic distribution in which the surface velocity is some five times the bed velocity. Under normal conditions of fairly uniform channel lining, it should not exceed 1.1, and is often so small as to be negligible.

In the ideal case the energy gradient is horizontal. Actually, however, frictional losses cause the energy line to drop at a corresponding rate. Excessive turbulence results in a pronounced decrease in the value H , but for our problems of smooth transition over a short distance, the slope is always very slight and may often be neglected entirely. It must also be realized that the energy line generally represents an average value for the entire vertical section, for were a line to be plotted for every stream filament in the section, the individual values of H would

vary through a considerable range. Hence only for ideal conditions will H_m give the correct total head for more than one or two points in the vertical section.

So long as the pressure distribution remains static, the above conditions hold true. But once the stream filaments commence to curve, the pressure gradient may no longer be plotted as a single line. To take an average value, as is done for the velocity head, is still advantageous for pipe flow, but has no purpose whatsoever where the upper water surface is exposed to the atmosphere. For purposes of computation, however, pressure, velocity, and potential (now designated by z) heads may be written as average values as follows:

$$H_m = \frac{1}{D} \int_0^D \left(\frac{p}{w} + \frac{v^2}{2g} + z \right) dD = \frac{P}{wD} + \alpha_v \frac{V_m^2}{2g} + \frac{D}{2}$$

These values are shown graphically in Figure 7. Since the velocity with which the velocity head is computed is the actual velocity at every point (that is, a vector quantity whose direction varies over the section) and the depth is measured most easily in the vertical, in order to express the velocity in terms of depth and discharge, the cosine of the average angle of inclination of the stream lines with the horizontal must be introduced:

$$V_m = \frac{Q}{n} = \frac{Q}{D \cos \alpha}$$

Hence the pressure "area" $\frac{P}{w}$ at any vertical section may

be expressed:

$$\frac{P}{w} = DH_m - \frac{D^2}{2} - \frac{Q^2}{D \cos^2 \alpha} \quad (15)$$

In regions where the curvature is not extreme, the pressure area is still approximately a triangle, so that the pressure intensity at the bottom of the stream (the pressure exerted on the floor) may be computed from the relation

$$p_f = \frac{2P}{D}$$

The term "critical depth" is one often used in linear flow to designate that depth at which, for a given value of H , the maximum discharge will occur - or, conversely, that depth at which, for a given discharge, the total energy will be a minimum. This may be derived mathematically as follows: The expression

$$Q = vD = DV\sqrt{2g(H-D)} \quad (17)$$

is differentiated with respect to D and the result placed equal to zero, H being treated as the constant term.

This gives the relation

$$H = \frac{3}{2} D \quad (18)$$

for the maximum Q at that value of H . As the velocity head is $H - D$, the critical velocity is found to be \sqrt{gD} .

In turn, the discharge at critical depth becomes:

$$Q = D \sqrt{gD}$$

and

$$H = \frac{3}{2} D = \frac{3}{2} \sqrt[3]{\frac{Q^2}{g}} \quad (19)$$

If we designate as "streaming flow" that at which the average velocity is lower than wave velocity ($v_w = \sqrt{gD}$), and that at which the velocity is greater as "shooting flow", then we may say that the critical velocity is that which borders on both shooting and streaming flow. Hence the critical depth is that at which the critical velocity occurs.

However, this value is a mean of the velocities over the entire section; when the velocity is unevenly distributed, both streaming and shooting flow may occur at different points over a single section. Hence the familiar test for determining the type of flow by means of making surface waves and noticing whether they travel up- or downstream is often subject to error.

In certain cases, which will be discussed later in this thesis, the critical depth is also the border depth between linear and curved flow. Yet this is not true for sudden transitions, such as the sharp-crested weir, for here the surface begins to show curvature long before either the crest or a depth approaching the critical has been reached. Often the error is made of computing the critical depth for a given discharge, and locating this depth on the profile of such a transition at a point where the stream lines are already curving. But curvature denotes non-static pressure distribution, which entails an increase above normal velocity for that depth. Hence, though the velocity may apparently

still be at the border between streaming and shooting flow, this will be seen to be completely erroneous when one recalls that this critical depth denotes maximum discharge only in case of static pressure distribution.

It will be remembered that equation 19 was derived on the basis that the sum $(\frac{P}{W} + z)$ remained a constant - i.e. that the pressure head equaled the depth below the water surface - from which the critical velocity was found to equal wave velocity. Yet one must not forget that the velocity of waves is a function of pressure head, since in a body of water in which the pressure is reduced to zero by any cause (water freely falling through space, for instance) wave motion will cease. Hence, under conditions of curving flow it is very inadvisable to compare the terms "critical velocity" and "wave velocity" as derived for linear flow, or even to consider either value in such cases until our knowledge of water movement in non-linear paths is more complete.

Professor Böss, in the paper already mentioned⁵, has given an approximate method for expressing discharge in terms of water depth and an "underpressure" factor, proving conclusively that, for a given depth, the discharge will increase above that computed by equation 19 as the pressure decreases below static conditions. This becomes obvious after considering Bernoulli's equation, for a drop in pressure demands an increase in velocity, thus

resulting in greater discharge past a section of a given depth. The theoretical limit of this variation is that at which the pressure is reduced to the vapor pressure of water, beyond which no increase in velocity may result; this is, of course, an imaginary condition to illustrate the case in question, for a stream with the upper surface exposed to the atmosphere would disrupt before such a low pressure could be reached. The application of these points will be shown in a later section, in connection with discharge over spill sections.

3. Law of impulse and momentum

In frequent use at the present time is a principle of mechanics which enables the computation of the theoretical height of water attained in the hydraulic jump. It must be noted that, despite the common assumptions involving "alternate stages" as given indirectly by Bernoulli's theorem through equation 17, transitions may, in general, be solved satisfactorily only by use of the law of impulse and momentum. That this principle is applicable to curving flow as well as linear may be seen from the following derivation of the general equations.

Upon an isolated portion of a stream bounded by the channel walls and floor and two vertical sections at right-angles to the walls (only the two-dimensional case will be

considered) as shown in Figure 8a, the force causing acceleration is the vector sum of the floor pressure, the pressures exerted upon the limiting vertical sections, and the weight of the water. This results in a change of momentum between the two sections according to the relation $f = m \cdot a$. Resolving force and momentum into horizontal and vertical components, the following equations are obtained:

Horizontal

$$\int_0^y p_f \cos \beta dy + \int_0^{D_a} p_a dD - \int_0^{D_b} p_b dD = \int_0^{D_b} m v_b \cos^2 \alpha_b dD - \int_0^{D_a} m v_a \cos^2 \alpha_a dD$$

$$P_f \cos \beta + P_a - P_b = 2w \frac{v_b^2}{2g} \cos^2 \alpha_b D_b - 2w \frac{v_a^2}{2g} \cos^2 \alpha_a D_a$$

$$P_a - P_b + P_f \cos \beta = \frac{w Q^2}{g D_b} - \frac{w Q^2}{g D_a} \quad (20)$$

Vertical

$$W - \int_0^x p_f \sin \beta dx = \int_0^{D_b} m v_b \sin \alpha_b \cos \alpha_b dD - \int_0^{D_a} m v_a \sin \alpha_a \cos \alpha_a dD$$

$$W - P_f \sin \beta = 2w \frac{v_b^2}{2g} \sin \alpha_b \cos \alpha_b D_b - 2w \frac{v_a^2}{2g} \sin \alpha_a \cos \alpha_a D_a$$

$$W - P_f \sin \beta = \frac{w Q \tan \alpha_b}{g D_b} - \frac{w Q \tan \alpha_a}{g D_a} \quad (21)$$

Two approximations have been made in this derivation: first, the transition is assumed smooth and rapid, so that the effect of floor friction in retarding the acceleration may be neglected; second, the term α_u already mentioned, to correct the error involved by using the square of the average velocity, has been omitted, since it is a troublesome factor and yet will not cause more error than the

graphical approximation of the average angle α .

When the flow is linear and horizontal at each of the two sections and taking place on a level floor, the first equation may be rewritten:

$$\frac{D_a^2}{2} + 2 D_a \frac{V_a^2}{2g} = \frac{D_b^2}{2} + 2 D_b \frac{V_b^2}{2g} \quad (22)$$

since the floor no longer affects the acceleration; similarly the second equation here becomes zero. In this form the relation is used for the case of the hydraulic jump¹¹. (See Figure 8b). That this differs from the Bernoulli equation for constant energy

$$D_a + \frac{V_a^2}{2g} = D_b + \frac{V_b^2}{2g}$$

and hence depends upon energy loss to preserve equilibrium may be seen by changing equation 22 into the following form:

$$\frac{D_a}{4} + \frac{V_a^2}{2g} = \frac{D_b}{4} + \frac{V_b^2}{2g}$$

Should the transition be such that the flow actually passes through the true critical depth, for purposes of computation the first vertical section may be taken at this point. Since

$$\frac{D_a^2}{2} + \frac{Q^2}{gD_a} = \left(\frac{2}{3} H_c\right)^2 + \frac{H_c}{3} \cdot \frac{2}{3} H_c = \frac{2}{3} H_c^2$$

equation 20 then becomes for any other section downstream:

$$\frac{P}{W} = \frac{2}{3} H_c^2 + \frac{Q^2}{gD} \quad \text{and} \quad \frac{P}{W} = \frac{2P}{WD} \quad (23)$$

in which H_c is the height of the energy line above the floor at the critical section. This equation will give results practically equal to those given by equation 15.

Equation 21 is often of use in the computation of the floor pressure over an irregular section, since the difference between the weight of the water and the vertical component of floor pressure is equal to the vertical component of the change in momentum. By proceeding in short horizontal intervals, the curve of floor pressure may thus be closely approximated.

4. Application of above considerations
to elementary spill sections
a. The sharp-crested weir

The discharge equation for the suppressed, sharp-crested weir is obtained by treating it as a rectangular orifice of infinite width operating under a head such that the water surface is at the same level as the upper edge of the orifice. If the water is considered to approach the weir with a certain velocity, the expression for the discharge per unit of channel width will then be:

$$Q = C \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot (H^{3/2} - k^{3/2}) \quad (2A)$$

where H is the height of the energy line above the weir crest and k the velocity head of the approaching water. Designating the head on the weir crest by h , which is equal to $H - k$, the above expression may be rewritten

in the form

$$Q = C \cdot K \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot h^{3/2} \quad (25)$$

wherein

$$K = \left[\left(1 + \frac{k}{h}\right)^{3/2} - \left(\frac{k}{h}\right)^{3/2} \right]$$

The coefficient C denotes the contraction of the jet, depending not upon the velocity of approach but upon the direction of approach. For a weir of infinite height, the water will approach radially until near the vicinity of the crest (see Figure 9), where each stream filament will curve by an amount depending upon the interaction of all filaments upon one-another. This coefficient may be shown theoretically¹² to equal the value $\frac{\pi}{\pi+2}$ or 0.6110. For a weir of finite height (Figure 10) the water will not approach radially, but in an average direction tending more and more toward the horizontal as the weir becomes lower; hence the contracting effect becomes less, so that the value C will increase as the ratio $\frac{h}{D}$ varies from 0 to 1. The coefficient K for the weir of great height approaches unity, also increasing as the weir becomes lower.

Hence, since the coefficients C and K are dependent upon the physical proportions of water depth and weir height, the principle of geometrical similitude shows that for a given ratio of $\frac{h}{D}$ the two coefficients will always be the same, regardless of the discharge. In reality, however, the effect of viscosity causes a departure varying inversely

with h , so that under very low heads the actual discharge coefficient μ is greater than $C \cdot K$ for the value of $\frac{h}{D}$ in question.

Because of the difficulty of expressing the variation of these coefficients theoretically, the best weir formulae at present are empirical, based upon the results of innumerable experiments. That of Rehbock (1929)¹³, in metric units, is typical of these:

$$Q = \left[1.782 + \frac{0.24 h_e}{p} \right] l \cdot h_e^{3/2}$$

in which $h_e = h_0 + 0.0011$ m, p is the weir height, and Q the discharge in cubic meters per second. For the development of these formulae, a plot of h against Q does not show to good advantage either the systematic variation of or the errors it may cause in the computed discharge. These difficulties may be overcome by graphing finite values of h as ordinates against the abscissa μ . For a single height of weir this will of course give a single curve, which may be drawn to a large scale because of the small variation in μ . On this plot, however, only a small range of the value $\frac{h}{D}$ may be shown; furthermore, the empirical formula applies accurately only to values of this ratio $\frac{h}{D}$ not greater than about 0.6.

If, on the other hand, μ as ordinate is plotted against $\frac{h}{D}$ between the limits 0.0 and 1.0, it will be seen that will ascend to infinity before the limit $\frac{h}{D} = 1.0$ is reached.

In Figure 12 such a graph is shown; it is interesting to note that the theoretical curve of von Mises¹² and the empirical curve of Rehbock¹³ give practically the same results over a large range, yet both reaching infinity at the limiting value $\frac{h}{D} = 1.0$. That this is erroneous will be shown in the following section.

We have just seen that both C and K will increase with decreasing height of weir, hence tending to increase the discharge for a given head - yet it must be realized that the two coefficients represent entirely different occurrences. An increase in K denotes greater velocity and hence greater discharge past a section of given depth, but this term has nothing whatsoever to do directly with the shape of the profile at the crest; on the other hand, C denotes a change in shape of the nappe, but does not depend upon the magnitude of the velocity (except of course when the distribution is exaggerated). This holds true despite the fact that both coefficients are functions of $\frac{h}{D}$.

Let us now consider the action which causes contraction of the nappe. If the weir is very high, all particles will approach radially, so that no two stream filaments will have the same direction. In the vicinity of the crest the action is two-fold: first, gravitation causes a downward acceleration, resulting in a downward curve in the upper surface of the water; second, the stream filaments, having opposing components of momentum, so interact that the direc-

tion of each is changed to conform more nearly to the average direction of all filaments. According to the theorem of Bernoulli, the loss in potential head due to gravitational action requires an equal increase in velocity head. Just past the crest no external forces other than gravity act upon the particles - hence the vertical acceleration of the nappe as a whole is that of gravity.

Furthermore, the fact that each filament is curved by the interaction of all filaments in the crest vicinity shows that some internal force must exist causing this curvature. That this internal force must be due in part to a drop of pressure normal to each filament in the direction of curvature will be seen from equation 3b. If we consider a vertical section at the crest, we will see that the pressure is atmospheric at both upper and lower limits of the section; yet here the curvature is at its maximum value, so that there must exist considerable pressure within the stream - that is, independent of gravity, there is a drop in pressure causing acceleration upwards in the upper portion of the nappe and downwards in the lower portion. But as gravity is the sole outer force acting upon the section, it must be concluded that the total acceleration of the whole section is that of a freely falling body, although the upper particles accelerate more slowly and the lower ones more rapidly than this average value.

As this interaction of the stream filaments continues,

the filaments become more nearly parallel and the pressure accordingly decreases as the section moves farther from the crest. As this action occurs over a comparatively short distance, not far from the crest the internal pressure will approach the atmospheric. It has been shown that gravitation will produce only vertical acceleration in the nappe, yet equation 13 also shows that a reduction in pressure may cause an increase in the horizontal velocity component. Hence the horizontal component will increase until the internal pressure within the falling sheet is reduced practically to zero; from that point on, the vertical thickness of the nappe will remain constant (see Figure 9), denoting a constant horizontal component of velocity. Only after this point has been reached may the profile of the nappe be calculated correctly by the customary equations for freely falling bodies.

This entire discussion applies fully as well to weirs of small height, except that the contraction coefficient increases (i.e. the contraction decreases) as the weir becomes lower. As will be shown in the following section, for the case of maximum contraction coefficient - a weir of zero height - pressure still occurs within the nappe at the crest, and the conditions of acceleration are similar to those of the foregoing case. Although contraction still exists as the weir becomes lower, the rise of the lower surface becomes less; for the limiting case, the lower sur-

face curve is tangent to the horizontal at the crest.

Professor Koch was probably the first to investigate through the use of piezometers the pressure distribution over the upstream face of the weir in a thorough manner. He found⁴, in the case of a high weir, that the pressure head at the floor was equal to the total depth of water in the channel - that is, the depth beyond the limit of the drop-down curve. In the vicinity of the floor the pressure retained this static distribution, but departed from the static triangle as it approached the crest, at which point it reached zero. Similar conclusions were reached by Professor Harris¹⁴ of the University of Washington, who attempted to express the coefficient of contraction as a function of this pressure reduction.

Since the theoretical derivation of the contraction coefficient is based upon the fact that the weir face is normal to the approach channel both vertically and horizontally, it is to be expected that any departure from this condition will affect the discharge. For instance, if the weir is tilted downstream, somewhat the same result will be obtained as in the case of a very shallow weir; that is, the average direction of approach will become more nearly horizontal, thus causing less contraction of the jet and consequently greater discharge. The opposite is true if the weir is inclined upstream. It must be noted that this variation in discharge is due directly to only the contrac-

tion coefficient; the variation in velocity of approach with the change in discharge is a secondary matter, which is in full accord with our earlier statement that the velocity coefficient is a function of velocity of approach and the contraction coefficient a function of direction of approach.

b. Horizontal floor with abrupt, ventilated fall

A weir may be considered as variable in height between the limits of infinity and zero. While the infinite height is never attained in nature, the dimensions of a weir of finite height are often so great in comparison to the head on the crest that it may be considered in this category; the infinitesimally low weir is of course the same as an abrupt, vertical drop in the channel floor, it being assumed in all cases that the tailwater is sufficiently far below the crest that it does not influence the discharge.

Let us assume for the moment a horizontal, rectangular channel of great length, leading from a reservoir of constant water level and ending in such an abrupt drop, and consider first the ideal frictionless case. Once the gate at the channel entrance is removed, water will begin to flow into the channel, because of the drop in pressure in that direction; this flow will increase until it reaches its maximum value for the difference in level between reservoir surface

and channel floor. If we designate this difference by H , the discharge will equal, per foot of width,

$$Q = q^{1/2} D_c^{3/2} = \left(\frac{2}{3} H_c \right)^{3/2} q^{1/2} \quad (26)$$

according to equation 19. In the vicinity of the crest, however, a distinct drop in the surface will occur, similar to that at the sharp-crested weir. Theoretically this curve will extend infinitely far upstream, but this departure from the horizontal is in reality noticeable over only a short distance. Hence the water will flow at approximately the critical depth until comparatively near the crest. Since the surface near the crest shows a noticeable downward curve, the pressure within the stream must decrease with the increasing velocity, in accordance with our previous discussion of the principles of curving flow. Yet due to the length of the channel, the surface curve is practically horizontal some distance downstream from the reservoir, so that the actual critical depth is reached before the surface begins to drop; thus the discharge is a function of an actual critical depth occurring in straight-line flow.

Let us now consider the case involving friction. Since the energy line must slope downward by an amount equal to the rate of energy loss, and since the floor is horizontal, the maximum discharge for a section some distance downstream could not be computed from the difference

between reservoir and channel-floor levels, for this no longer represents the total head at that section. Hence the discharge will decrease by an amount depending upon the lost head between reservoir and crest, because the section of least energy determines the maximum discharge for the entire channel.

Since the critical section has moved nearer the crest, because of frictional loss the water surface has now a gradual slope between the channel entrance and the critical section, and the drop in surface level due to curvature of flow may be considered to begin at the latter section. Hence the discharge will depend in all actual cases upon the length of channel and the frictional factor, but the critical depth will always be reached at some point before the crest. It is obvious that the critical section for a given channel is by no means fixed in position, moving farther from the crest with increasing elevation of the reservoir surface.

Two transition sections occur in the case just discussed: that at entrance, when the surface drops because of partial transformation of potential and pressure heads into velocity head; and that at the abrupt fall. Both transitions involve curvature of the stream filaments, but the practical range of influence is comparatively small.

However, should these two sections lie very near together (as in the case of a small broad-crested weir), the two surface curves will run together, so that at no point

between the transition sections may straight-line flow be considered to exist; hence it would be futile to attempt to locate the critical section. That this mistake is often made is shown by the numerous descriptions of efforts to develop a short, submerged weir for which the "critical depth" will always occur at a fixed section; (see Woodburn and accompanying discussion²).

Our previous discussion of contraction at a weir crest does not apply to the abrupt fall, because the water is no longer deflected upwards by any obstruction in its path. Yet a certain contraction still occurs at the crest because of the downward curvature of the stream filaments. This contraction coefficient may be computed by changing equation 26 into the typical weir formula as follows:

$$Q = \sqrt{g} \cdot D^{3/2} = C \cdot K \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot D_c^{3/2} \quad (27)$$

in which $\mu = \frac{3\sqrt{2}}{4} = 1.06$ and $K = \left[\left(1 + \frac{1}{2}\right)^{3/2} - \left(\frac{1}{2}\right)^{3/2} \right] = 1.485$

$$\text{hence } C = \frac{\mu}{K} = 0.715$$

The coefficient C has been found by the writer to equal the ratio between the depth at the crest and the critical depth (the corresponding relationship for other weirs will be mentioned later); unfortunately, however, the physical explanation and significance of the fact are not yet clear, and the matter is the basis of present experimental and theoretical research which the writer is now conducting. Although the coefficient may be computed theoretically through

a wide range of weir heights, this method is based upon conditions totally different from those actually existing - i.e. the force of gravity is neglected.¹² Hence the actual limiting value for μ of 1.06 (see curve of $\mu : \frac{h}{D}$ in Figure 12) must really represent the final point on the curve of the coefficient.

In general, the abrupt fall or weir of zero height compares very closely with those in our foregoing discussion. Although the sheet is fully ventilated so that after the crest is passed the pressure is atmospheric both above and below, there still exists appreciable pressure within the nappe until the filaments approach parallelism. Until that point the fall curve may not be treated as following the trajectory of a free body; any vertical section completely through the nappe is freely accelerated downward, but its horizontal component of velocity increases as the pressure within the nappe approaches the atmospheric.

A drop of pressure on the floor similar to that on the face of a sharp-crested weir also occurs, varying from the static head at the critical section to zero at the crest. Once the discharge and stream profile are known, this variation in floor pressure, as well as the pressure "areas" through the entire transition, may be computed by either equation 15 or 23. In the German dissertation already mentioned⁷, the writer developed the following formula, on the basis of equal velocity distribution at the critical section,

to determine the pressure distribution at any section in the entire profile:

$$\frac{p}{w} = H - y - \frac{1}{2g} \left[v_1 - (v_1 - v_2) \left(\frac{y}{D} \right)^{\frac{v_m - v_2}{v_1 - v_m}} \right]^2$$

Here y represents the elevation of a point above the floor or lower border of the falling sheet, D the thickness of the sheet, and H the height of the energy line above this lower border; v_1 , v_2 , and v_m are the lower, upper, and mean velocities respectively, as computed from the height of the energy line, depth, discharge, and floor pressure.

That this formula is somewhat tedious to apply is quite apparent. Its real value, however, lies in the fact that it shows that the pressure is only slightly influenced by unequal velocity distribution due to floor friction, for computed results check very closely with measured values for a case involving considerable frictional loss.

A noteworthy comparison may easily be made of the variation of discharge with constant total head over a crest of constant elevation, for weir heights of infinity and zero, and an intermediate stage at which the head on the crest equals the height of the weir crest above the channel floor (see Figures 9, 10, 11). Let us take as tangible values a height of energy gradient above the crest of 3 feet and a unit width of weir crest of 1 foot. Assuming that viscosity has no appreciable influence at this depth, the coefficient for a weir of infinite height will

be 0.611 (see page 31). Substituting the proper values in equation 25, we find the following discharge:

$$Q_{p=\infty} = \mu \cdot \frac{2}{3} \cdot \sqrt{2g} \ell h^{3/2} = 0.611 \times \frac{2}{3} \times 8.025 \times 1 \times 3^{3/2} = 16.95 \text{ c.f.s.}$$

Similarly a weir of zero height at the end of a long, horizontal floor will discharge under critical conditions for linear flow; since the depth is two-thirds and the velocity head one-third of 3 feet, the corresponding discharge will be:

$$Q_{p=0} = \sqrt{g} \ell h^{3/2} = \sqrt{32.2} \times 1 \times 2^{3/2} = 16.05 \text{ c.f.s.}$$

Both Rehbock and von Mises give approximately equal coefficients for a weir of height equal to the head, or about 0.687 (Figure 12). Hence the discharge for a head of 2.85 feet and a weir height of 2.85 feet will satisfy the requirements:

$$Q_{h=p} = 0.687 \times \frac{2}{3} \times 8.025 \times 1 \times 2.85^{3/2} = 17.65 \text{ c.f.s.}$$

It is quite apparent that the maximum discharge for a given total energy will not be the maximum discharge for linear flow - nor will it be that at which the head on the weir is greatest; instead it occurs at some point between the two limits. However, the variation of about 10% between the extreme discharges is relatively small. Significant is the fact that while the acting head on the crest may vary according to the height of weir, a very low weir causes a

a comparatively large velocity head to exist; hence, for equal heads on weirs of greatly different heights, the lower weir will require a much higher reservoir level (at the elevation of the energy gradient) than that with a deeper channel of approach. This point must not be ignored in weir and spillway design.

c. Effect of non-ventilation on weir discharge

Both cases just discussed involve complete ventilation of the nappe, so that atmospheric pressure will exist at the upper and lower surfaces of the falling sheet. Let us now assume that the ventilation is not complete, and the space underneath the nappe is at a pressure below atmospheric. This portion of the flow is now under the influence of another external force - air pressure from above - so that both gravity and this outer pressure produce downward acceleration. The air pressure, furthermore, acts normal to the upper surface, so that it retards acceleration in the horizontal direction as well.

Two results are at once obvious: the nappe is forced in the direction of the weir face, and the reduction of pressure below the nappe requires an increase in velocity in the lower portion of the stream. In this way the entire profile is affected: if the supply does not vary, this means that the head on the weir must decrease; if on the other

hand the supply head is constant, then the discharge must increase. That the discharge over an unventilated, broad-crested weir will not change, is obvious from the fact that non-ventilation may affect the position of the critical section, but the critical depth will still have the same magnitude.

Were it possible to evacuate the air entirely from under the nappe, the sheet would cling to the downstream face of the weir as long as the pressure did not approach the vapor pressure of water. Except for very low heads at an abrupt fall, or under higher heads for sharp-crested weirs, this is a very difficult condition to attain in nature, because small irregularities in flow at any point will allow air to enter the sheet so that it tears itself away from the weir face. This difficulty is further increased by the fact that the sheet carries air with it as it plunges into the tail water, thus constantly replenishing the supply under the nappe; however, this is never sufficient to ventilate the nappe completely, as air is also removed in the same way.

Such reduction of pressure below the falling sheet will obviously change not only the stream profile but also the pressure distribution at every point at which the stream filaments change in direction, degree of curvature, and position. Since it is impossible to maintain a constant magnitude of underpressure below a nappe plunging into the open

tailwater, it is equally impossible to foretell the exact conditions of a partially ventilated discharge. For this reason all measuring weirs must be completely ventilated before measurements may be depended upon.

A more definite type of non-ventilated weir is one in which the falling sheet is guided by a straight or curved downstream face. If, for a given discharge and height of weir, this downstream face is so constructed that it fits exactly the curve of the lower surface of the nappe when the weir is discharging at that head, the stream profile will not be changed; that is, the falling sheet will behave as though it were fully ventilated, and the pressure at all points of contact with this curved face will be atmospheric. For any other discharge these conditions will not hold; similarly, any other form of curved face for the original discharge will also cause a change in conditions.

Let us suppose, for instance, that there is a greater head on the weir than that for which the curved face was designed. Were the particles to follow their normal trajectories, unless air were supplied under the nappe, a pressure equal to that of the vapor pressure of water would exist at the spill face; this is obviously impossible. Hence the greater pressure exerted by the atmosphere upon the outer surface of the sheet forces it back against the spill face, at which surface negative pressure exists, depending in magnitude upon the amount the stream filaments are deviated

from their normal trajectories. Similarly, the pressure will be raised above the atmospheric if the curved spill face is designed for a greater discharge than that occurring.

Should the spill face be straight - that is, making an abrupt angle with the original weir crest - different conditions would prevail. It will be seen from equations 5 and 6 that an infinitely small radius of curvature - i.e. such an abrupt angle - demands an infinitely great velocity. From Bernoulli's theorem we see at once that this is a physical impossibility, inasmuch as the limit of pressure reduction is the vapor pressure of water. Hence the stream filaments will not curve abruptly, but will take such a form as is consistent with the possible reduction of pressure, and the space between the lowermost filament and the spill face will be filled with a ground-roller acting as a permanent cushion for the sheet above. At all vertical sections passing through such a roller it is exceedingly difficult to compute either pressure or velocity distribution, for without careful revision the equations already developed will not apply.

From this discussion it will be seen why all transition sections should be made gradual, with well rounded corners, except at points of discharge into free air. In many cases rounding the corners will prevent the formation of a ground roller, even though it causes considerable negative pressure in the vicinity of the curvature; this is due to the distribution of the curvature over a finite distance, rather than

expecting it to occur at a single point. Similarly, it will be found that an easy curve at the top of a steep spill face will practically insure a clinging sheet; since the curvature is gradual rather than abrupt, and takes place over a finite distance, it is often impossible to ventilate such a sheet even by forcing air under the water at the spill face, for unless a roller tends to form, there will be no fixed space into which air can force its way and cause the sheet to jump free.

III Experimentation

1. Description of three series

In order to illustrate in a definite way the principles discussed in the preceding pages, the author wishes to describe three series of experiments which he has conducted with basic weir forms. Of these investigations, that involving the ventilated, sharp-crested weir of variable height is a portion of the research in which the author is now engaged at the M.I.T. River Hydraulic Laboratory; that involving the simple, ventilated fall formed the basis of a Doctor's dissertation which he presented at the Technische Hochschule of Karlsruhe in Baden, Germany; following the completion of the latter investigation, the writer conducted similar studies of the same model, introducing straight, downstream faces of two different slopes and one circular crest. In the last section of this thesis experiments by other investigators of models of actual spillways will also be described.

2. Laboratory apparatus and procedure

The author's research in the Karlsruhe River Hydraulic Laboratory was conducted in a glass-walled experimental flume 50 cm wide, 70 cm deep, and 5 meters long (Photo 1). In an additional section at the upstream end of the flume was lo-

cated a sharp-crested measuring weir equipped with a piezometer well and vernier hook gage reading to 0.01 cm. The flume was provided with horizontal steel rails on which rode a gage carriage, so that the gage was adjustable in all three directions (Photo 2).

To provide sufficient depth of water on the model crest and still not cause undue turbulence at the measuring weir, and also to enable the introduction of piezometer inlets in one wall of the channel, a concrete wall was built into the glass flume, thus reducing the channel width to 25 cm. The channel floor was also built of smooth concrete some 35 cm above the floor of the experimental canal, terminating in a section of brass angle carefully set into the concrete and finished flush with both floor and vertical downstream face. The latter face was ventilated by means of a $1\frac{1}{8}$ -inch pipe leading into the atmosphere below the flume. The temporary channel wall in the vicinity of the crest was formed by an iron plate 70 cm x 140 cm in size, set flush with the concrete wall and enameled; this plate contained the piezometer borings.

The profile of the flowing water was measured with point and hook gages mounted on the movable gage carriage above the flume. Longitudinal distances from the crest were read from a millimeter scale fastened to one of the rails, and a vernier scale permitted vertical readings to 0.01 cm. This gage carriage was also used to support several Pitotubes

used during the experiments.

Pressure readings on the channel floor and downstream face of the weir were accomplished by means of 22 piezometers, consisting of 2-mm brass tubing soldered in holes drilled at intervals in strips of finished brass, the strips then being set into the concrete floor flush with the surface. Similar tubes, of copper, were inserted in holes drilled in numerous vertical rows in the iron wall near the crest, a total of 75 of these wall inlets being provided.

A special apparatus allowing the simultaneous observation of ten piezometer standpipes was utilized for the actual reading of pressure head (Photo 4). This consisted of ten $\frac{1}{8}$ -inch glass tubes mounted in front of a mirror, the tubes connecting with the inlets by means of rubber tubing. Moving vertically on gears at either side of the apparatus was a rigid carriage with a horizontal wire to be adjusted to the meniscus of each water column; at the side of the carriage was secured a millimeter scale with vernier, reading to 0.01 cm. The entire apparatus could be adjusted according to a spirit level on the carriage, by means of a thumbscrew at the lower corner of the mirror frame.

The maximum possible discharge consistent with smooth flow - 125 liters per second per meter of crest - was selected, and the elevation of the upper surface of the water was measured carefully at intervals extending to a point over $1\frac{1}{2}$ meters upstream from the crest; below the crest

both upper and lower surfaces were measured over a distance of 25 centimeters downstream. Following the determination of the profile, both floor and wall piezometer readings were made. Then by means of three different Pitotubes the velocity distribution was found at pertinent cross-sections, and the pressure inlet of the tube further used to check the wall measurements.

Much the same procedure was followed with three smaller discharge quantities - 81.19, 44.19, and 8.74 liters per second per meter - the first two being chosen to give respectively three-quarters and one-half as great linear dimensions as the original discharge; these quantities were computed from the geometrical relation

$$\frac{Q_a}{Q_b} = \left(\frac{L_a}{L_b} \right)^{3/2}$$

in which Q represents discharge per unit length of crest, and L represents any linear measurement.

On the completion of these experiments, the downstream end of the dam was given a straight slope of 1 horizontal to 2 vertical (Photo 7), and similar runs were made with identical discharge quantities. The slope was then changed to $1\frac{1}{2}$ horizontal to 1 vertical (Photo 6), and the measurements repeated, and finally a fourth group of runs was conducted with a downstream face in the form of a quarter circle with a radius of 20 centimeters (Photo 5). Inlets for wall piezometers had previously been arranged so as to

provide measurements in seven different vertical sections for each model form and discharge. Two more sections of piezometers just above and below the crest would have been very desirable, but it was thought unwise to risk interference with the flow caused by too many openings in the wall at this strategic point.

Studies of the energy distribution in the case of the sharp-crested weir now being conducted by the writer in the M.I.T. River Hydraulic Laboratory follow much the same procedure; except for a few minor differences in arrangement and method, both the apparatus and the routine are fundamentally identical. To date nine different heads have been measured for both elevation and pressure distribution, with the weir crest 40 cm above the channel floor. It is planned to investigate the same discharges for crest heights of 20, 10, 5, $2\frac{1}{2}$, and 0 cm. Piezometer inlets corresponding to those in the floor of the former cases have been provided in the upstream face of the weir, and the wall measurements have been replaced by those made with a small flat plate moved parallel to the plane of flow, in whose center is a small opening connecting to a glass standpipe and hook-gage; the general arrangement of this apparatus will be seen from Photo 3.

3. Results of the experiments

Figures 14 - 17 show in comprehensive form the principle measurements made on each of the four model set-ups in the Karlsruhe laboratory, all under a discharge of 125 liters per second per meter of crest; in addition in Figure 13 is plotted a discharge profile for a 40-centimeter sharp-crested weir (developed by means of geometrical similitude from a profile measured at M.I.T. for a slightly smaller rate of flow) for the same discharge quantity as that of the other models. On these plots are shown the following: the profile of each weir in the vicinity of the crest; the profile of the water surfaces; the distribution of pressure head on the wetted portions of the weirs; the distribution of pressure head over various vertical sections throughout the transition regions; the position of the critical section (for all except the sharp-crested weir) as computed from the assumption of uniform velocity distribution according to equation 19; the elevation of the energy gradient; and in the case of the two straight slopes, the approximate magnitude of the ground-rollers.

Let us first consider the sharp-crested weir. It was found that the curve of the upper surface extends upstream practically 90 cm or about $1\frac{1}{2}$ times the total depth of water in the flume. The gradual drop as the crest is approached indicates a gradual conversion of pressure head into velocity

head, since this downward curvature represents reduction below the static head. This reduction is best seen from the distribution curves at the crest section; here there is atmospheric pressure at both upper and lower limits of the section, where both curves must reach zero. The curve of pressure head on the weir face appears to be tangent to a line intersecting the vertical axis at the same level as the horizontal surface of the approaching water - in reality it is somewhat above this, by an amount approaching the velocity head. This is explained by the fact that there is a stagnation point at the corner between weir and floor, so that the total head H occurs here as pressure head; since the floor friction somewhat reduces the velocity in the lowest regions, this would also reduce the pressure head at the point of stagnation.

Obviously there is considerable pressure within the stream just over the crest caused by the interaction of the curving stream filaments; the reduction of this pressure, with the accompanying approach of the filaments to parallelism, causes an increase in the horizontal component of velocity until the internal pressure is practically zero. Experiments show that this section of zero pressure occurs approximately at that point where the mid-point of the section is at the level of the crest; here the pressure readings were almost exactly zero, and furthermore the sheet shows a constant vertical dimension from this section on, showing that the

horizontal component of velocity no longer increases.

It is noteworthy that the vertical thickness of the of the stream at the section where the lower surface has reached its maximum elevation, when divided by the head h , gives approximately the coefficient of contraction C . This also applies to a weir of zero height (i.e. the simple fall herein discussed).

According to the theory of potential flow, the stream filaments (or Ψ lines) fill every part of the channel profile, and there will be no turbulence. Actually, however, for weirs of finite height there is an unsettled region near the foot of the upstream weir face, where eddies and whirls are constantly forming. This not only causes intermittent "furling" of the lower nappe surface, but undoubtedly influences somewhat the general form of the nappe by decreasing the average convergence angle of the stream filaments. It is, of course, impossible to determine the magnitude of this change, as it is very likely small. It has often been suggested that this turbulent section might be obviated by inclining the upstream face of the weir; obviously this would result in a greatly changed nappe.

Let us now turn to the weir of zero height, or the simple fall at the end of a long, horizontal floor (see Figure 14). It will be seen that the energy gradient has a slight slope due to frictional loss, so that the critical section occurs in the vicinity of the crest; the curve of

floor pressures proves, however, that the reduction of pressure below the static due to curvature of the filaments does not occur until the critical section has been passed. So long as this curvature is slight, the reduction in floor pressure is gradual, and the pressure distribution in the vertical is approximately linear. Just before the crest the floor pressure begins to decrease more rapidly as the curvature becomes greater, and the pressure distribution in the vertical becomes a pronounced curve. The bending of the filaments reaches its maximum at the crest, where the floor pressure is reduced to zero (atmospheric). The free sheet finally approaches a normal fall trajectory as the internal pressure decreases to zero some distance from the crest.

It has already been mentioned that the depth at the crest divided by the critical depth equals the coefficient of contraction C . Furthermore, the pressure "area" at any section, as well as the vertical thickness of the sheet at the section of zero pressure, may be computed very closely by the law of impulse and momentum as expressed in equations 20 and 21, for weirs of any height.

Turning our attention to the three non-ventilated cases in which the water is guided by a downstream face, we at once note one feature in common with that of the simple, ventilated fall: the discharge is governed by the conditions of maximum straight-line flow, and not by the shape of the downstream face, for the computed critical depth will be found in each

case in approximately the same vicinity above the crest. It will be seen, however, that the crest depth is different in all four cases, showing that the depth is a function of the pressure reduction caused by the rate of curvature of the stream filaments.

It is to be understood that these conditions are true only if the water approaches over a comparatively long horizontal floor, for were the same downstream faces to be added to a plate weir of finite height, the conditions of maximum discharge for linear flow would not apply; hence a reduction of pressure at the crest section for such a weir would result in either increased discharge or decreased head, depending on whether the head on the crest or the discharge, respectively, were to remain constant. This point is of great importance.

From the photographs of the two sloping faces (see Photos 6 and 7) it will be seen that a ground roller is developed in each model just below the crest, because the filaments cannot make an abrupt angle without excessive pressure reduction. It is the formation of such a roller, together with considerable negative pressure, which permits the entrance of air, for unless such a roller can form at some point, there will be no place at which the sheet will tend to detach itself from the face. Photo 8 shows the discharge down the steeper slope just in the process of aeration; a comparison of Photos 7 and 8 will show that the aerated

space has exactly the same form as is shown by the coloring matter in the roller. The effect of easy curvature is shown by the face rounded on the fairly large radius of 20 cm. Here the curvature of the face finally causes a negative pressure head greater than the depth of water above at that point; yet because of the steady curving of the face there is no tendency for the sheet to spring loose. This tendency was very great in the case of the steeper slope with the sudden angle at the crest.

However, it must be noted that the terms "abrupt" and "gradual" are relative to velocity and head. Obviously a curve of small radius will have much the same effect as a sharp angle, and even a smooth, easy curve could not lead water around a complete vertical circle - it would tear itself loose when the negative pressure reached an exaggerated magnitude. Such extreme cases are not considered in this discussion. Hence we may safely say that a clinging sheet may be obtained by the avoidance of abrupt change in weir profile, even though the steady curve of the face causes gradual development of considerable negative pressure.

In the light of equations 2 and 5, close study of the pressure distribution curves in all five cases will prove highly enlightening. Since the aim of this thesis, however, is to show the relation of external influences upon the sheet as a whole, the internal pressures will not be discussed further.

IV. Application to practical spill sections

In spillway design there are three general factors which govern the shape of the profile: stability, discharge, and economy. The stability of a structure depends upon its resistance to sliding and to the overturning moment caused by the pressure of the water upon all wetted portions of the section; hence an accurate knowledge of the total pressure is essential. Under discharge come several secondary factors: the ability of the spillway to discharge the greatest possible quantity of water per second under a given head; the opportunity to estimate this discharge within reasonable limits of accuracy; and the certainty of even flow of water to eliminate both an overturning force due to negative pressure and the possibility of vibration caused by intermittent partial ventilation of the nappe. Economy depends not only upon a minimum of materials, but also upon a minimum of labor in ^building forms and placing the concrete or masonry.

Under our discussion of weirs, we have noted the fact that the pressure head upon the upstream face has as a maximum value the total head of the flow, or the sum of pressure and velocity head. While we are not dealing with uplift in this thesis, it is only logical to assume that such uplift is then a function of the total head rather than just the depth of water behind the weir. From the curves of measured pressures on models, the curve of pressure reduction

from bed to crest may be approximated. One must remember, however, that the curve will not reach zero at the crest of the spillway, but at that point which represents the crest of the sharp-crested weir, if the spillway profile is patterned after the ventilated nappe: furthermore, this will be true only if the profile has been designed properly for the existing discharge. At any rate, a short distance below the crest the upstream pressure will be practically equal to the distance below the energy gradient, because the rapid reduction in pressure occurs just before the crest is reached.

It has long been the custom (see Creager³, Hanna and Kennedy¹⁵) to design the lower spill face according to the trajectory of free fall based upon the assumed velocity at the crest section. While it has already been pointed out that this is not a trajectory of free fall (since the horizontal component of velocity is not constant), the error involved in this method will not be serious under normal circumstances. However, once this curve has been ascertained, one is no longer at liberty to modify the shape of the upper crest, for only so long as the **total** curve follows the lower surface of the ventilated sheet of a sharp-crested weir will conditions be similar. The crest may not be widened, nor may a rounded sill be added at the upstream face, without changing the flow decidedly.

An upstream projection of small radius or the failure to round the upstream edge of the normal crest will cause

a roller to form as cushion between the sheet and the top of the spillway. This not only results in underpressure, but also in reduction of the discharge section; if the roller is of some magnitude, it may cause a pronounced break in the coefficient curve at a certain point in the range of head variation.

If the crest is flat for a short distance, constant (atmospheric) pressure will no longer exist at the spill face. Furthermore, a sudden change from this flat portion to a downward slope will produce another cushioning roller, this time in a more serious position. The existence of a roller here indicates pressure below the atmospheric, and where both negative pressure and a roller occur simultaneously, the danger of air entering the sheet is very grave. It is seldom that such a case will ventilate to any considerable extent if the sheet touches the face again at a point lower down, for the air is partially swept away again by the rush of the water; but this process will recur constantly, due to minor disturbances in the flow, and if of sufficient magnitude may cause periodic vibration of the entire structure.

Hence we see that any departure from the profile of a ventilated weir sheet will at once alter conditions of discharge. Obviously this profile must be that of the highest expected discharge, and furthermore must be for a weir of equivalent height above the channel floor. It is well to

bear in mind that although approximation of the profile by tangents and circular curves may cause some error, the principal thing to be avoided is an abrupt change of slope; despite the occurrence of some amount of negative pressure, the sheet is not likely to ventilate itself unless a roller is allowed to form between the sheet and the spill face.

In Figures 18 and 19 are shown two spill profiles investigated in the hydraulic laboratory of Professor Camichel, at Toulouse, France¹⁶. In the first case only a slight roller exists at the flat portion of the crest; at the next curve the transition is fairly steady, and once again only a small roller results - since the sheet returns almost immediately to the straight face, there is very little tendency to ventilate. In the second case, however, both changes of slope are so abrupt and unfavorable as to cause the formation of large rollers, the second of which occurs at a point of abnormal negative pressure (see Photograph of stream filaments in Figure 20); this is a very dangerous condition.

This possibility of vibration due to intermittent partial ventilation is especially serious for cases of flow over metal gates sometimes used to provide additional head above the crest level of the spillway; should these be permanent fixtures, severe oscillation may result in considerable damage to the structure. Similar conditions exist

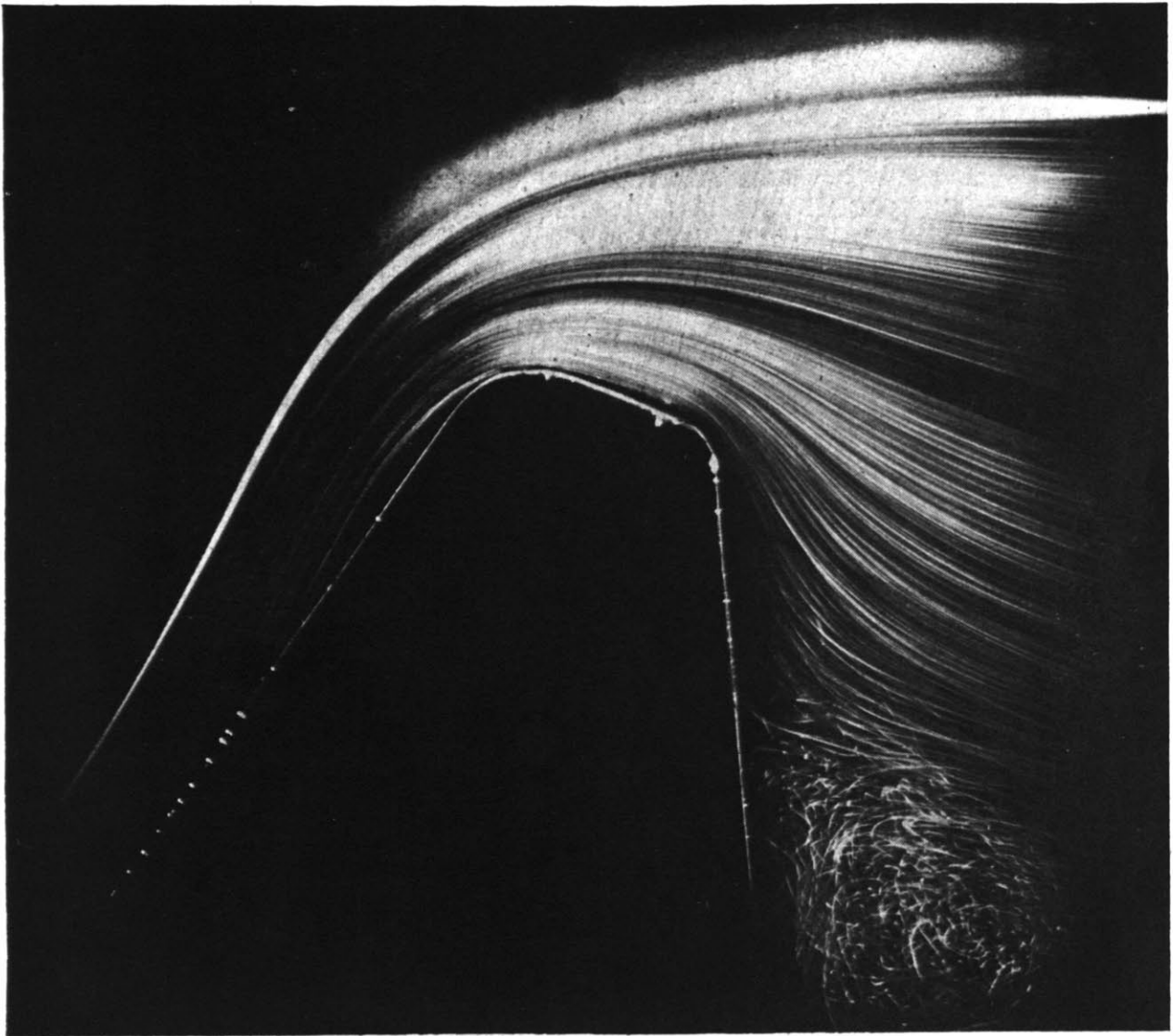


Figure 20. Study of stream filaments on a model of the Puechabon spillway at the Toulouse laboratory, France, under the direction of Professor Camichel. The model is constructed of brass at very small scale, and the stream paths photographed by means of a suspension of aluminum particles in the flowing water under strong overhead illumination. The photograph shows not only the converging filaments at the crest section, but also the turbulent region just above the spillway, and the great possibility for the sheet to spring loose at the point of formation of the ground roller at the beginning of the abrupt slope.

in the case of thin reinforced concrete weirs and spillways, whose nature precludes the possibility of designing to the natural curve of the free nappe; as these must usually discharge into the tailwater at reduced pressure, the unventilated sheet may easily set the whole structure in vibration; if the periods of oscillation of the water and the structure bear the proper harmonic relationship to each other, the magnitude of the vibration may be sufficient to loosen the entire section.

Unless the spillway profile follows absolutely the profile of a ventilated nappe for the same discharge and depth of approach, the discharge coefficient cannot be predetermined with any degree of accuracy; even in the latter case it may be foretold for only that one predetermined discharge, and for other discharges will vary according to the dimensions of the section in a way that cannot be prophesied except through experience with equivalent structures.

Should rollers form above the crest at a certain critical head, the curve of the coefficient will not show a steady transition; instead the curve will appear to follow different paths above and below this point; this is especially true of low weirs operating under a wide range of head. Hence a greater number of actual measurements on the structure during discharge will be necessary to determine the proper flow curve; obviously the avoidance of such

secondary variation through careful design of the profile is to be preferred.

Under our discussion of weir discharge, we saw that from a constant-level reservoir approximately equal quantities would discharge over a crest of fixed elevation, regardless of the depth of water behind the crest - that is, regardless of weir height. For each weir height the corresponding depth of water would vary, due to the velocity of approach, but the accompanying change in coefficients would be sufficient to hold the discharge to a variation of only about 10%. However, the crest of a spillway patterned after the weir nappe has its maximum elevation some distance above the weir crest, or at the maximum elevation of the lower surface of the ventilated sheet (about $h/10$ above the crest for a weir of great height). Hence a horizontal floor would conform to this higher level rather than to that of the imaginary sharp crest of the weir as before. This will further reduce the discharge over a horizontal floor, assuming the energy gradient to lie at the same elevation as for the spillway of finite height. Thus we see that flattening the crest of the spillway will tend to decrease the discharge by an amount increasing with the width of the horizontal section. This has as its maximum value a decrease in discharge of about 27% when the critical section actually occurs on the crest.

On the other hand, giving the crest a sharper curvature

than that of the free sheet will increase the discharge for a given head by reducing the internal pressure at the crest section. Needless to say, this is of dubious value beyond a certain limit, for it increases the chance of formation of a roller at the point of too abrupt transition, with the consequent danger of vibration.

A portion of the experimental results obtained by Lord and Fearnside¹⁷ in their investigation of model spillway discharge in the M.I.T. River Hydraulic Laboratory pertains to our discussion. The floor of their approach channel was movable, so that the pressure distribution and surface profile could be studied at different discharges and different depths of approach, with the final level of the floor at the crest elevation. In the final curves of the discharge coefficient, the effect of depth of approach was readily apparent, for the curves showed consistent progression from one floor level to the next, varying in value from 2.80 for the smallest value of h/D up to 5.67 for all heads (critical discharge) on the horizontal floor.

Piezometers in the approach floor and spill face gave very satisfactory curves of pressure distribution, showing the departure of the profile of the spillway from the ideal curve of zero pressure distribution. For a given head the negative pressure on the spillway face increased with decreasing depth of approach, due to the increasing

momentum of the water particles as h/D approached unity.

As far as the matter of economy of design is concerned, the author will make no attempt to settle so complex a problem abstractly, depending as it does not only upon general discharge and stability factors, but also upon the actual conditions at the site in question. It is his earnest conviction, however, that a clear picture of the physical characteristics of curving flow will enable the engineer to adapt his design of the spillway profile to suit both the demands of the flowing water and the requirements of structural stability, and thus attain economy in the broadest sense of the word. To this end the author has discussed the hydromechanics and hydraulics of spill sections to the best of his present knowledge and ability.

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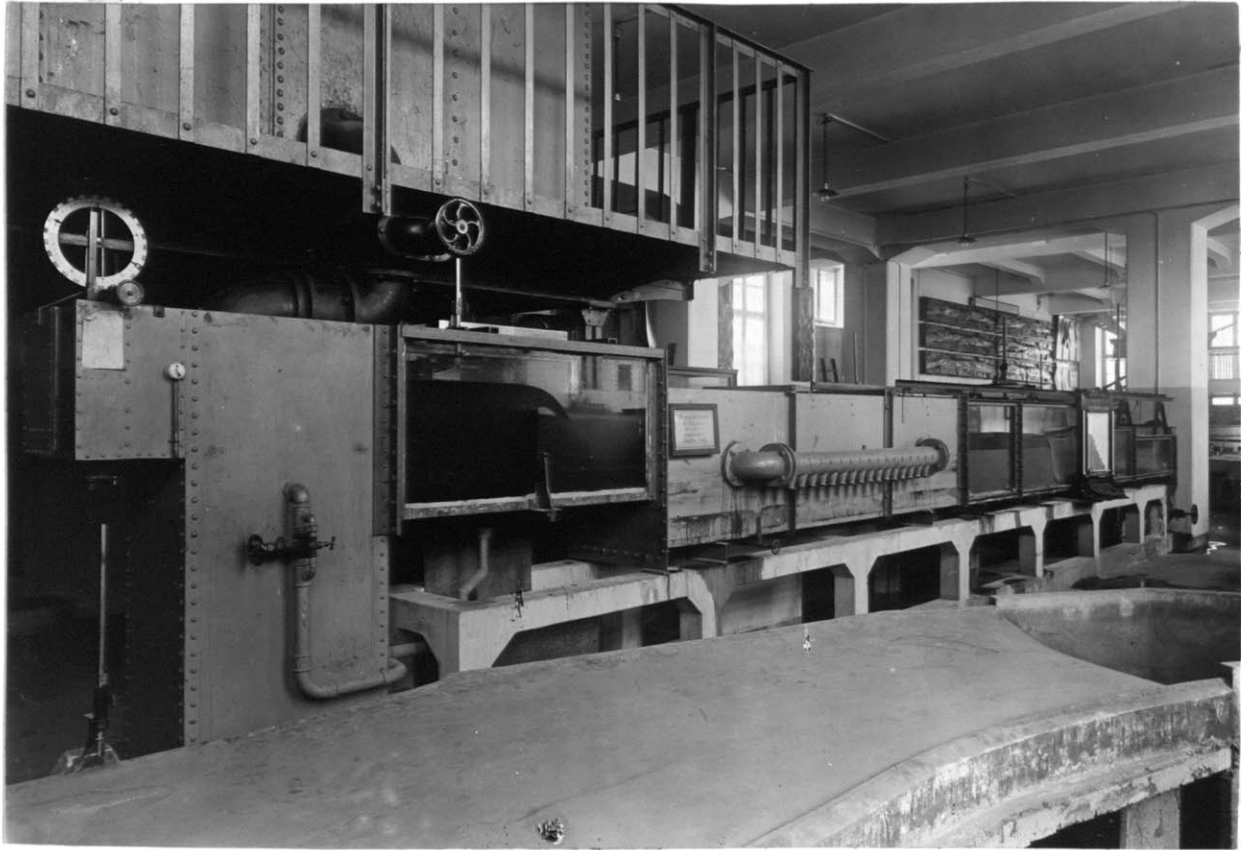


Photo 1. Side view of glass-walled experimental flume in Karlsruhe laboratory looking downstream from measuring weir. At extreme left is gage well for weir, and at lower end of flume may be seen the stream profile at a discharge of 125 l/s/m over circular crest; standpipe apparatus for piezometer readings is standing next to model.

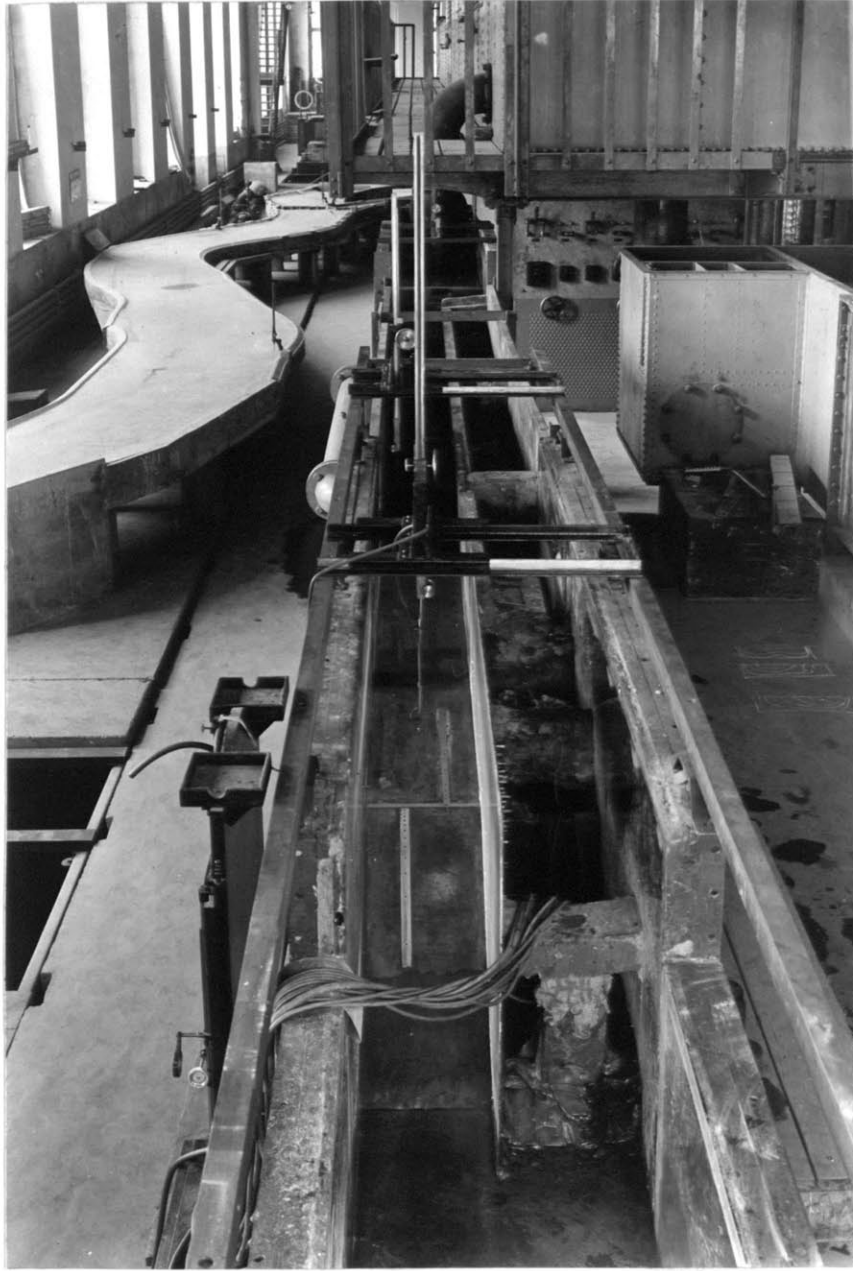


Photo 2. View of experimental flume from above, looking over spill crest upstream toward measuring weir. Brass strips holding floor piezometers, plate-iron wall with wall piezometers, gage carriage with Pitot tube, and general layout are clearly visible.

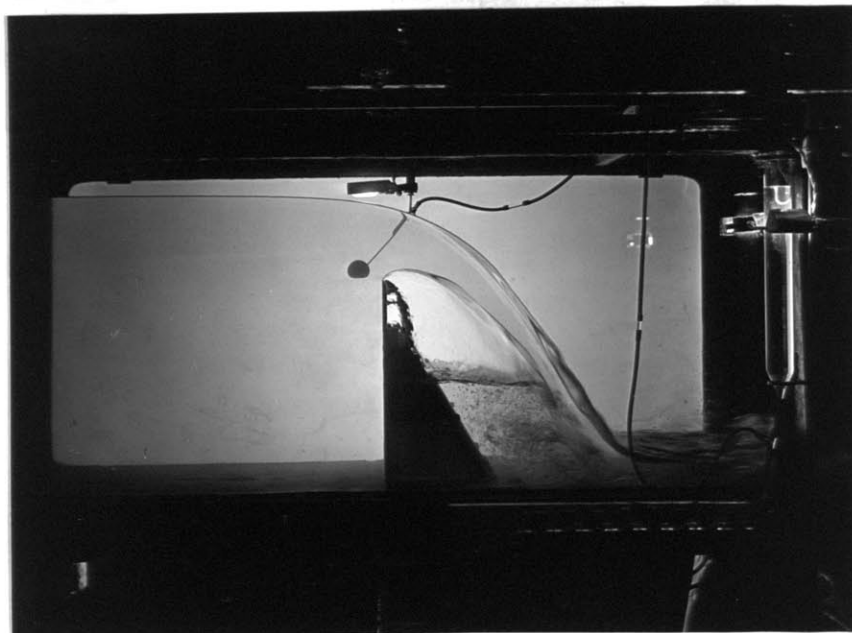


Photo 3. Sharp-crested weir (M.I.T. laboratory) discharging 125 l/s/m. Movable plate piezometer and standpipe with hook gage for pressure readings may be readily seen.

Photo 4. Ventilated fall of 125 l/s/m. Standpipes show all measured floor pressures. Wall inlets visible through nappe.

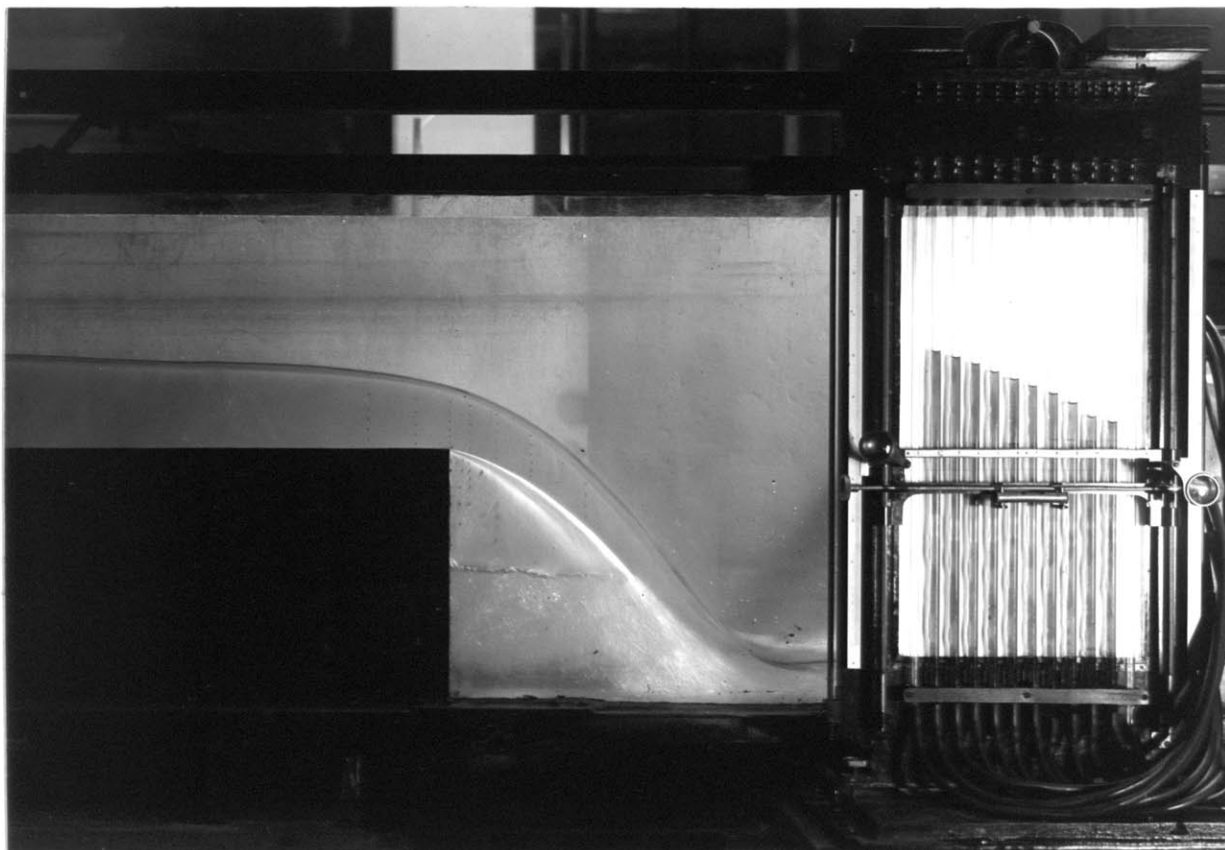
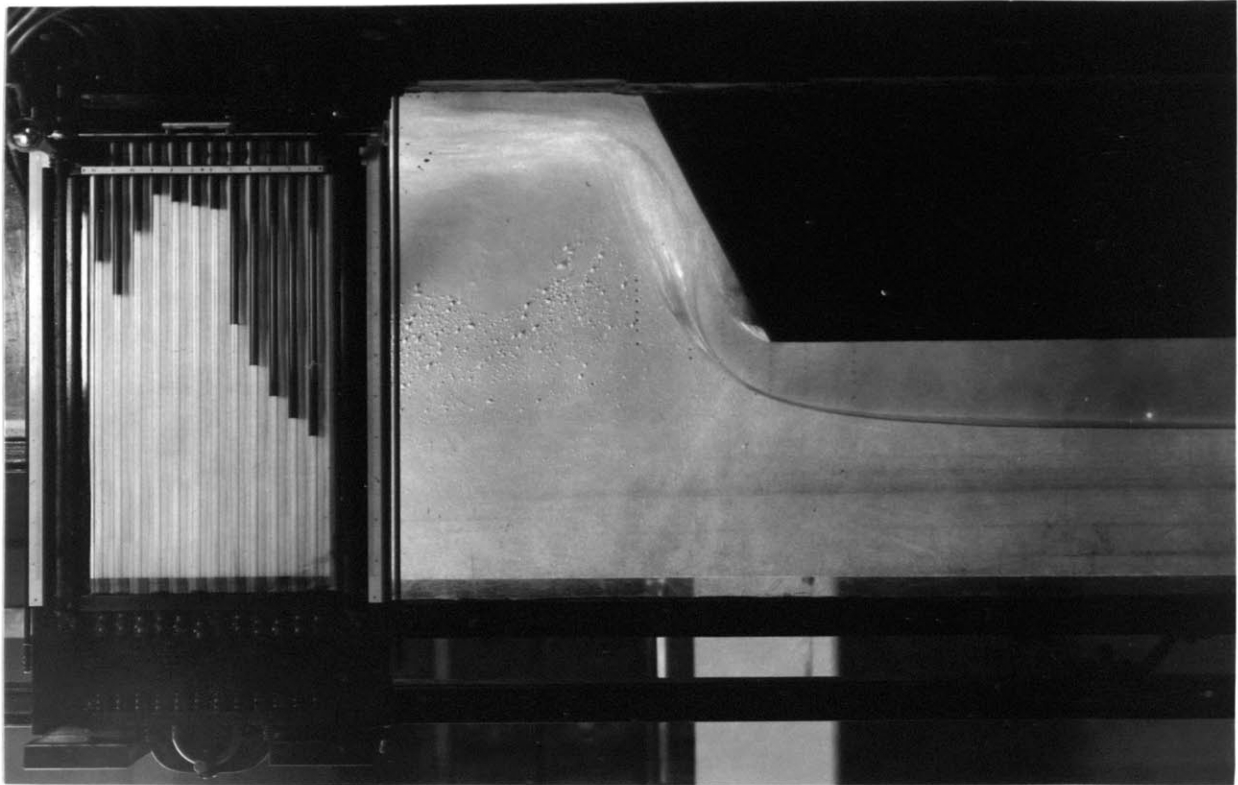




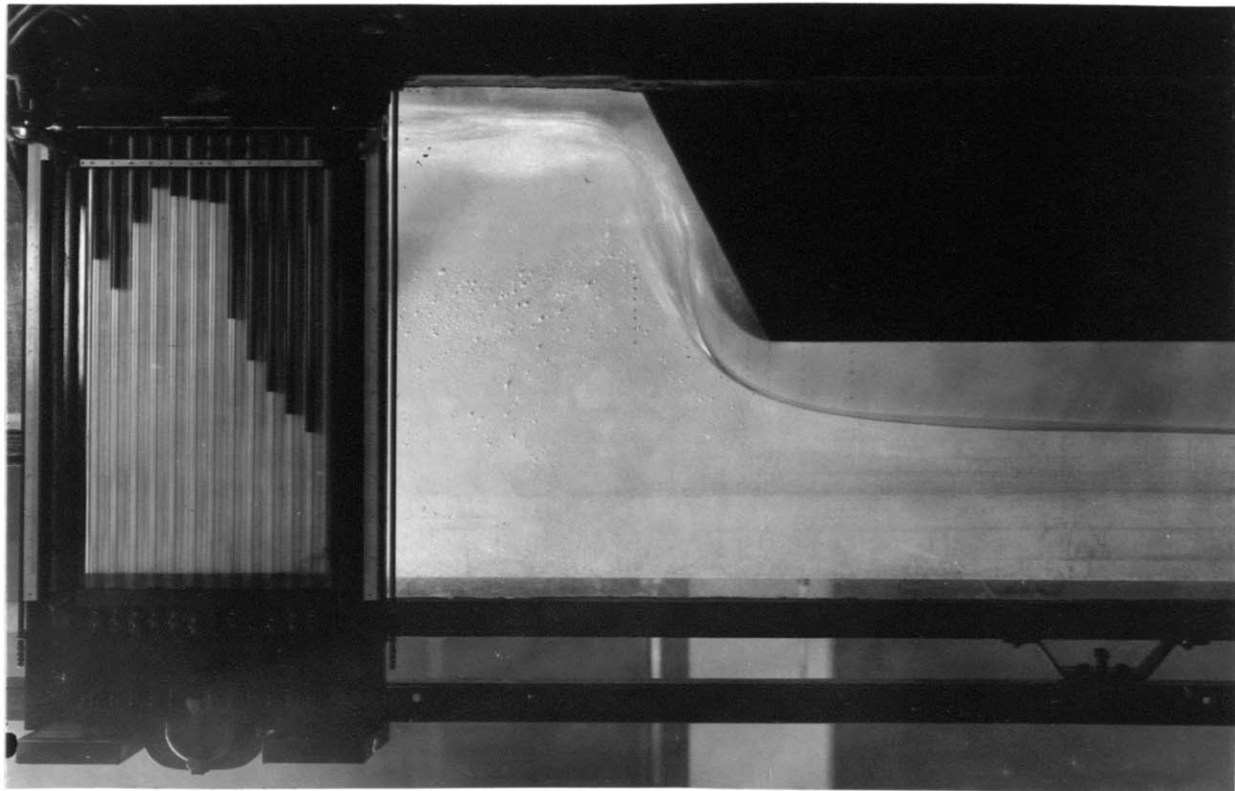
Photo 5. Discharge of 125 l/s/m, radius of curvature of downstream face 20 cm. Standpipes show pressures on weir face through entire transition.

Photo 6. Discharge of 125 l/s/m, slope of weir face 1:1½; standpipes show pressures on weir floor and slope through entire transition; color shows limits of roller.





Photos 7 and 8. Discharge of 125 l/s/m, slope of weir face 2:1. Standpipes show pressures over floor and downstream face. Roller in above photo is colored with dye; in lower picture air has entered, displacing water in roller.



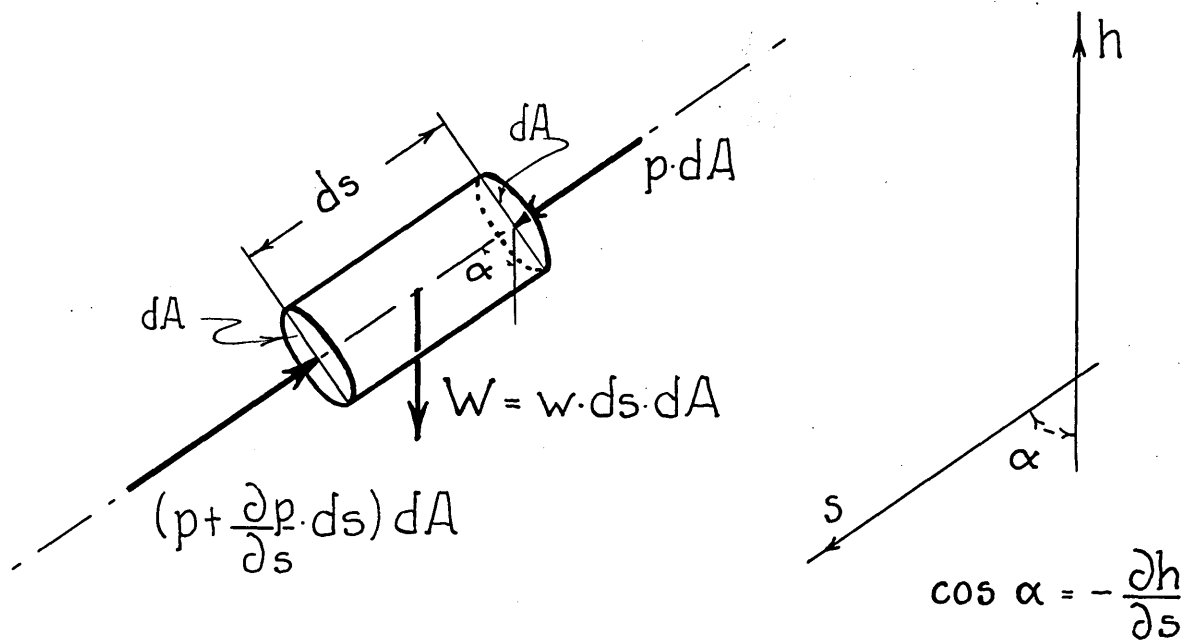


Fig. 1

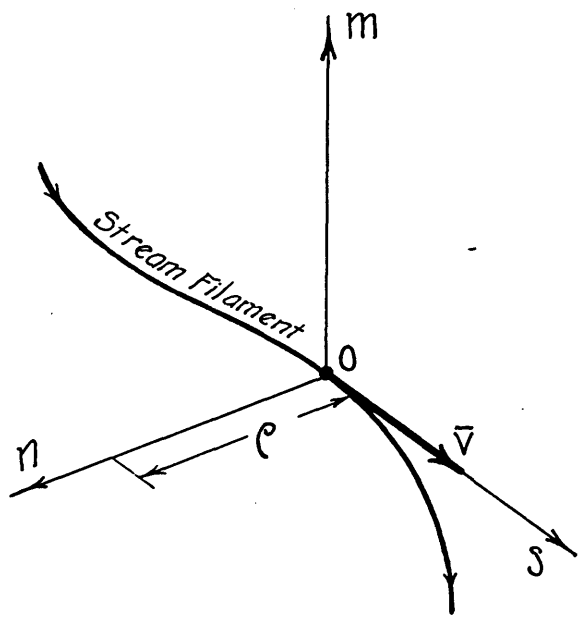


Fig. 2.

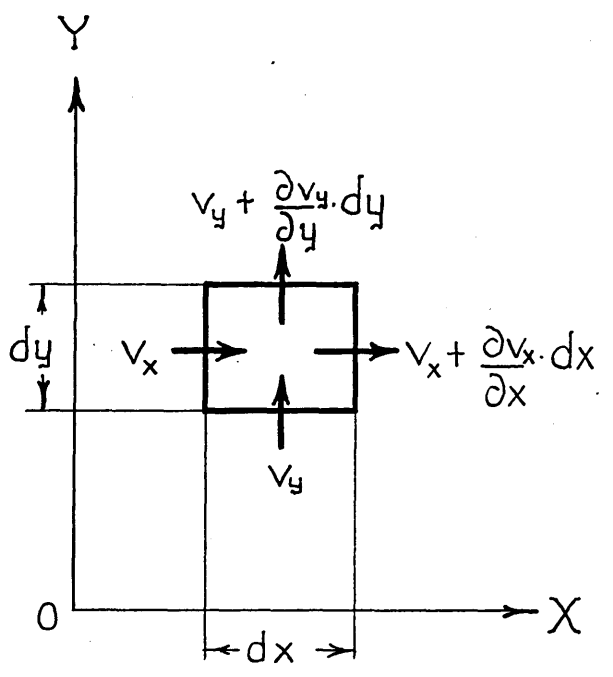


Fig. 3.

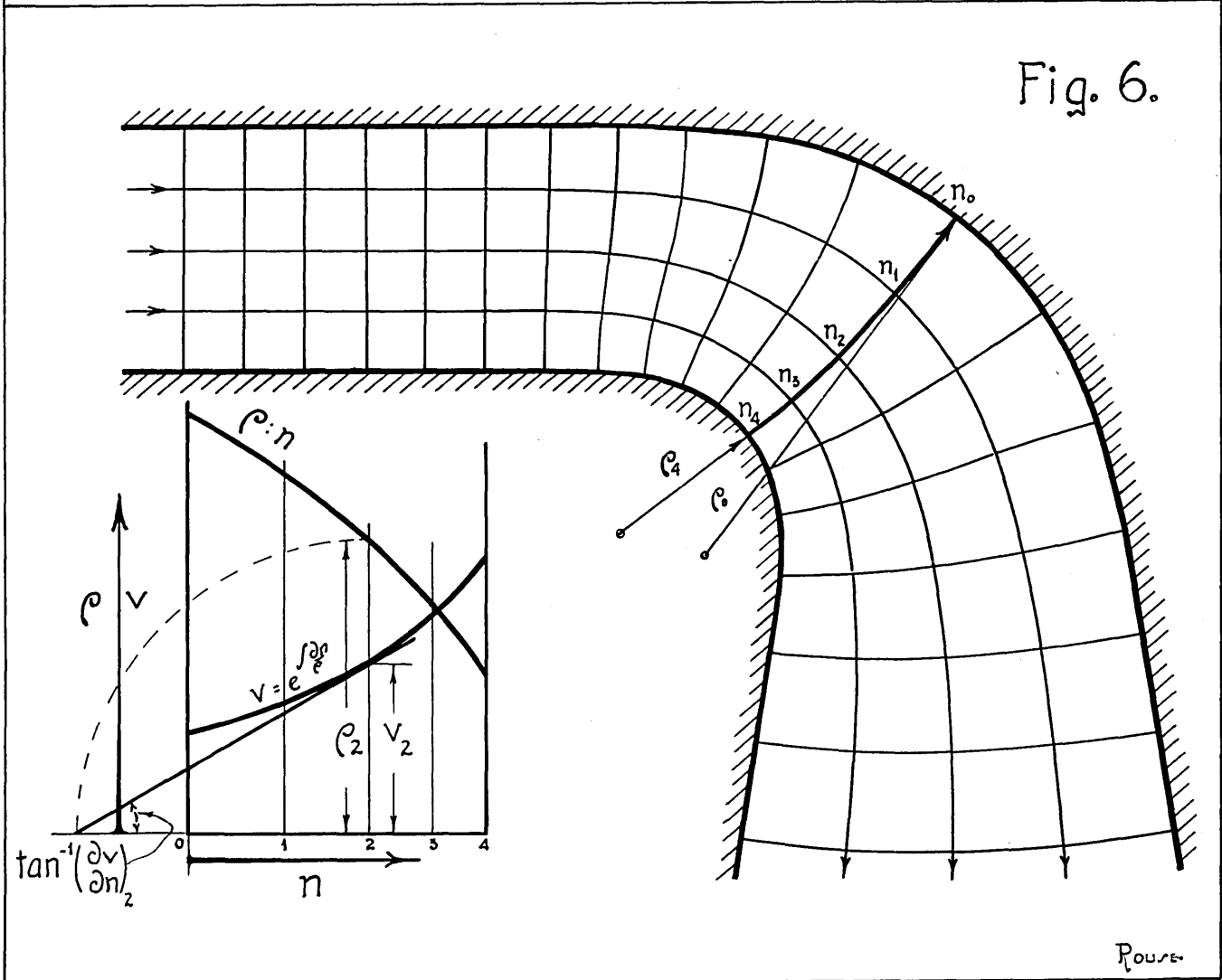
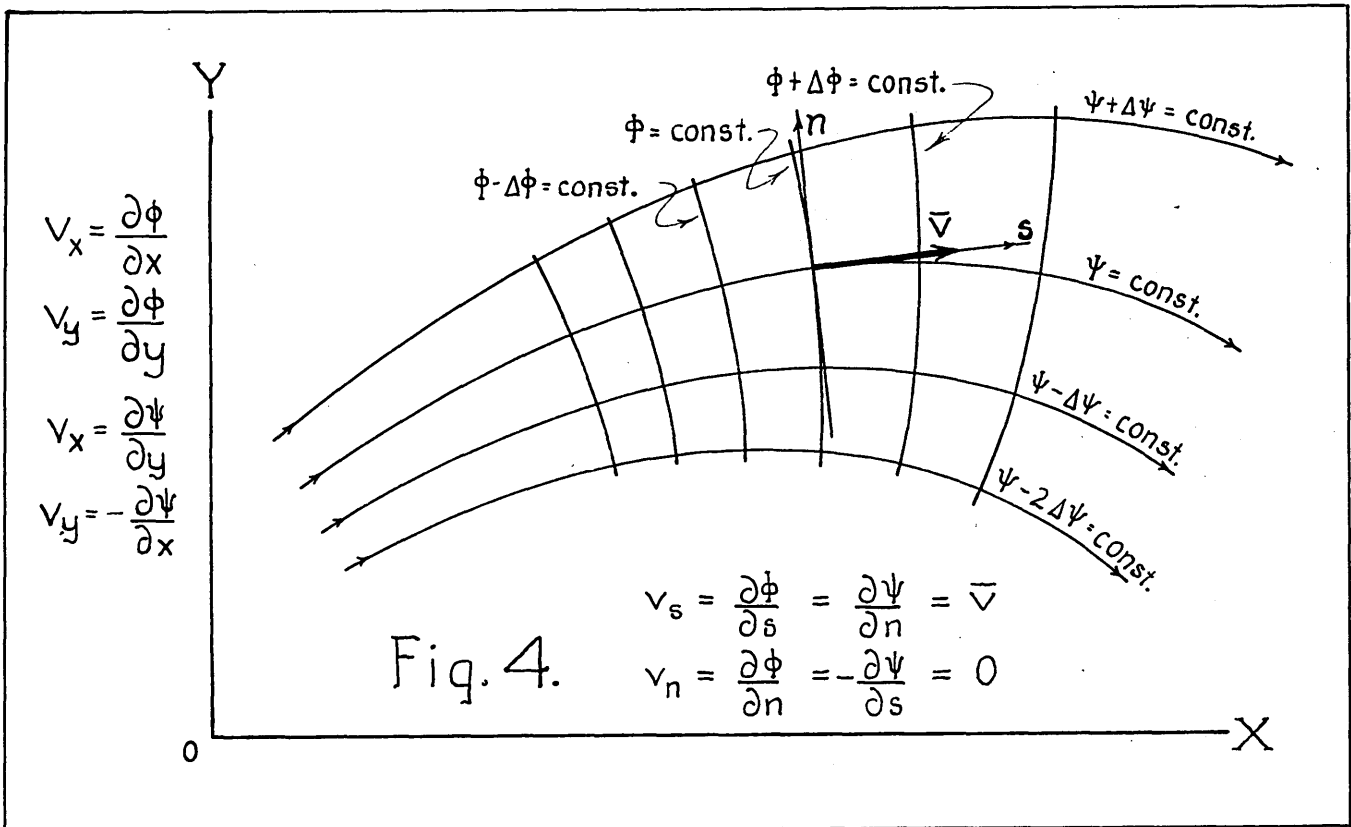


Fig. 7a

Energy distribution assuming constant H at all points (Ideal flow)

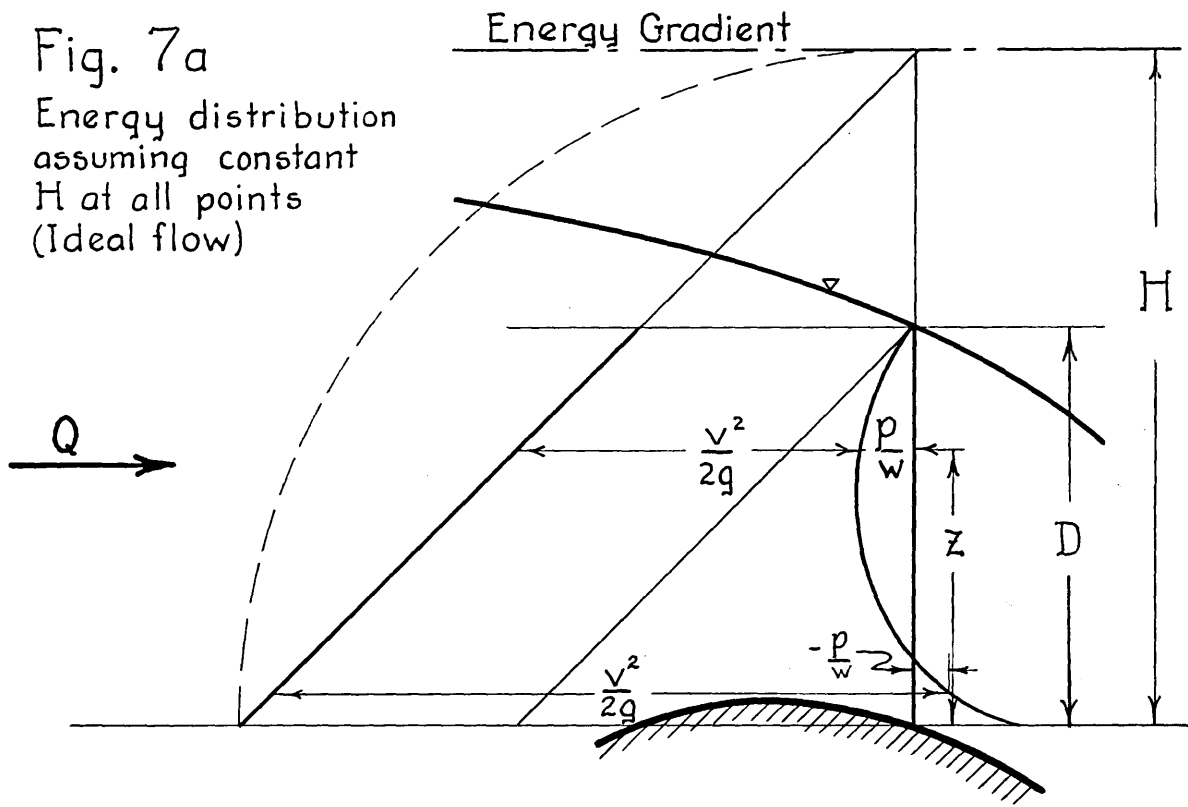
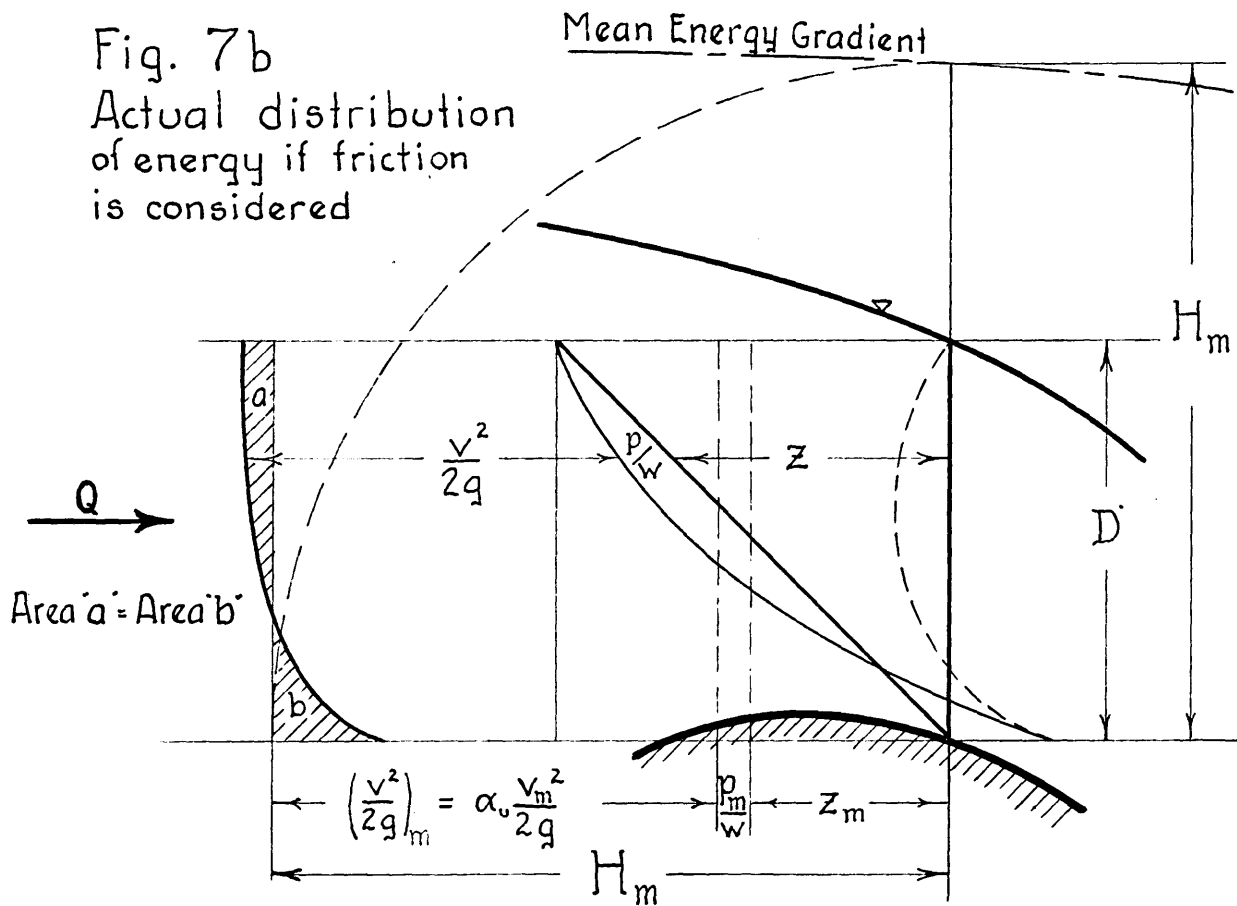


Fig. 7b

Actual distribution of energy if friction is considered



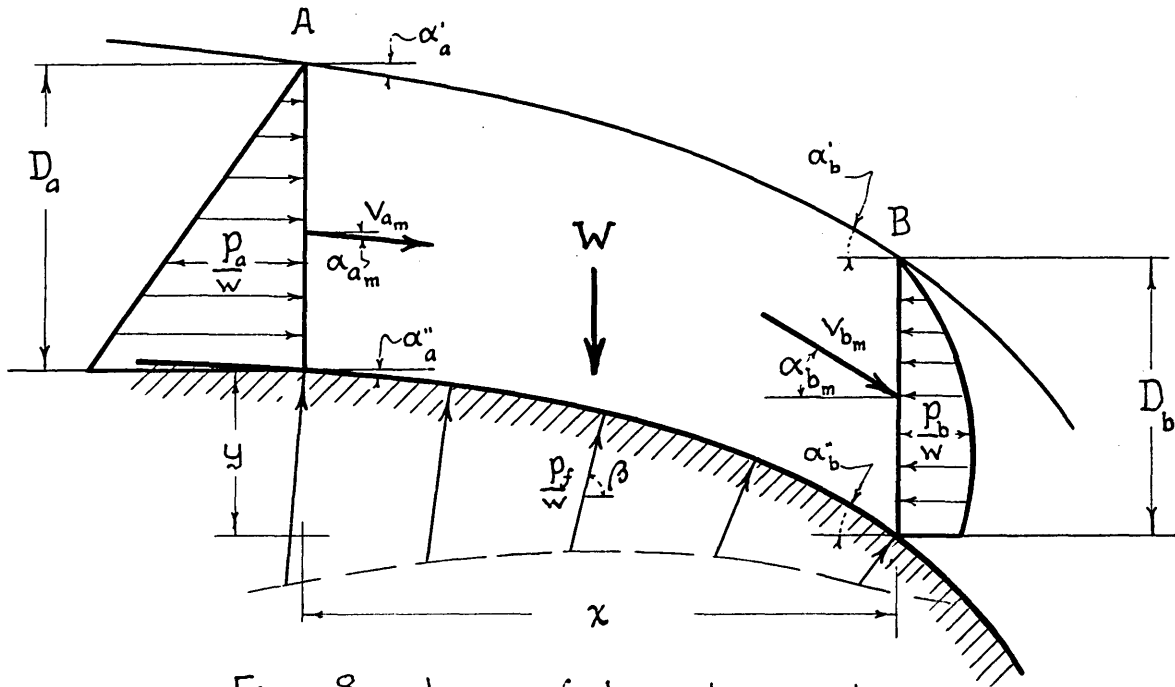
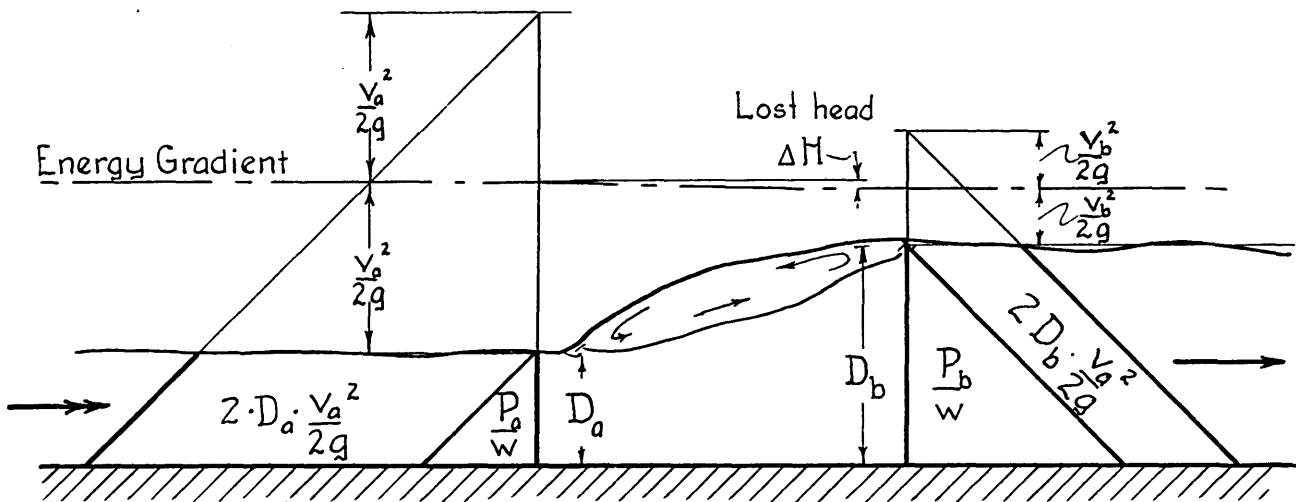
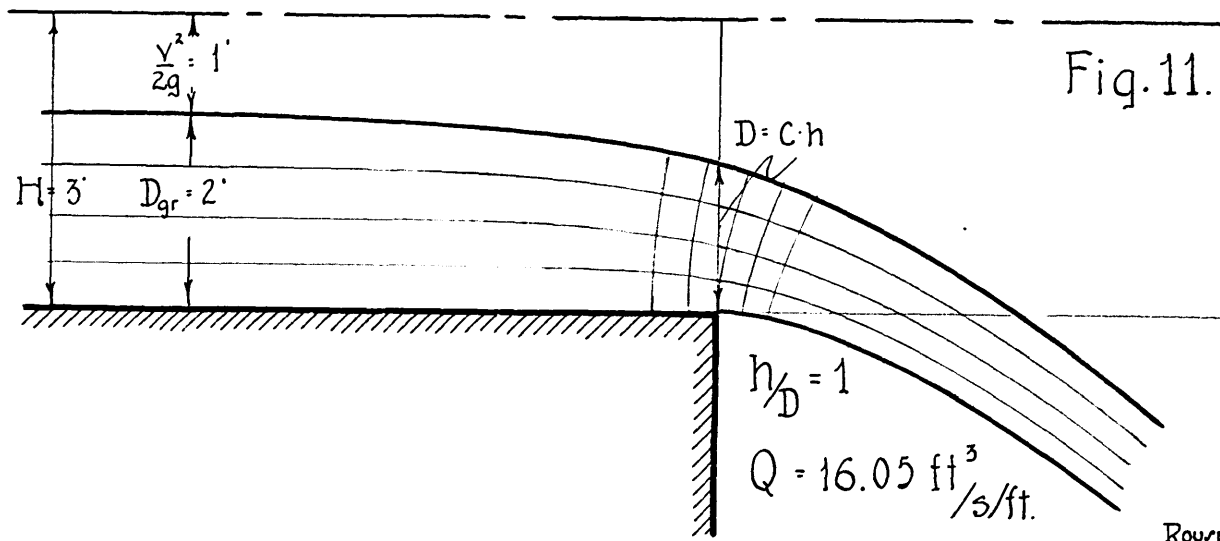
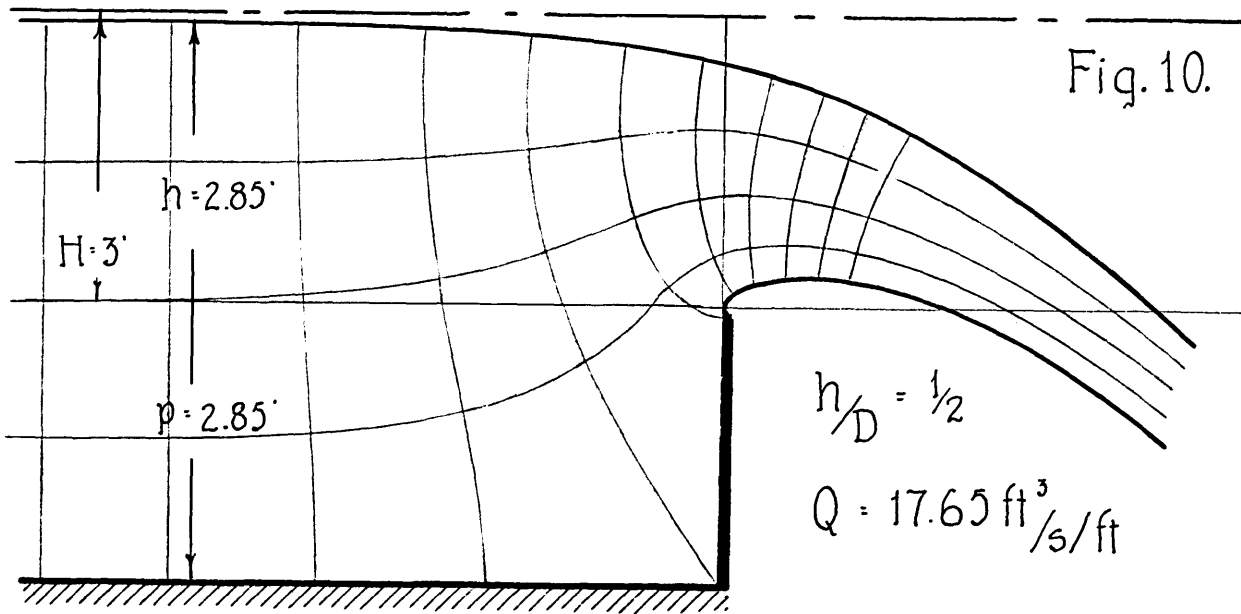
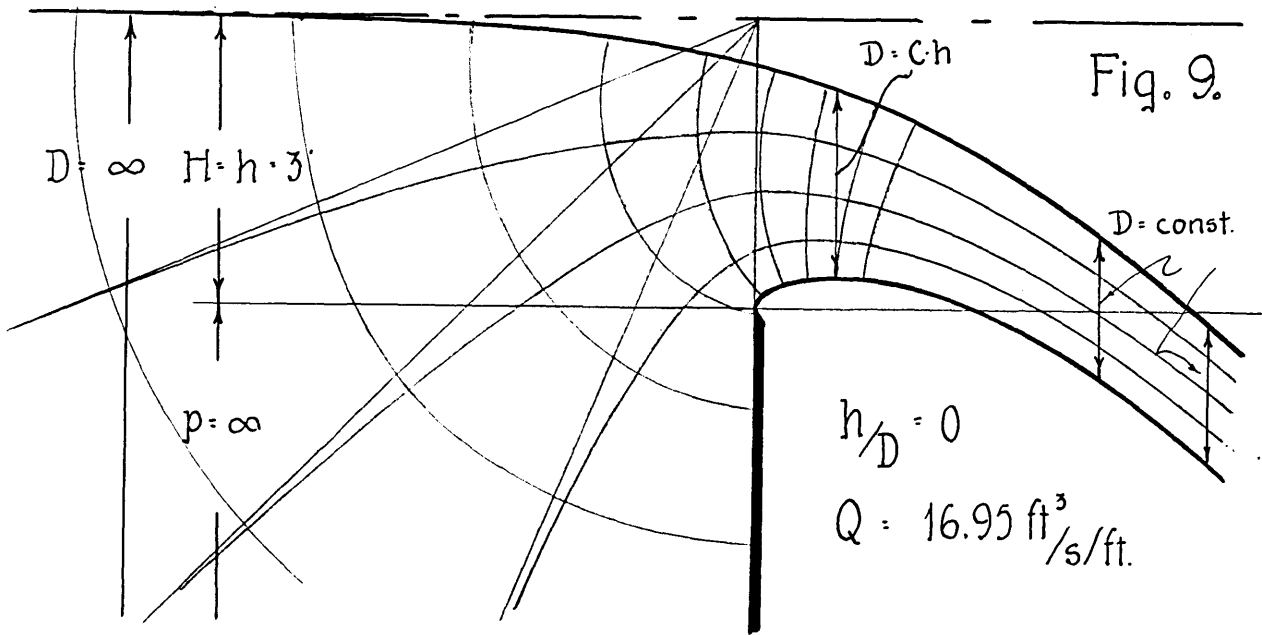


Fig. 8a. Law of Impulse and Momentum; forces acting to accelerate an isolated portion of the stream



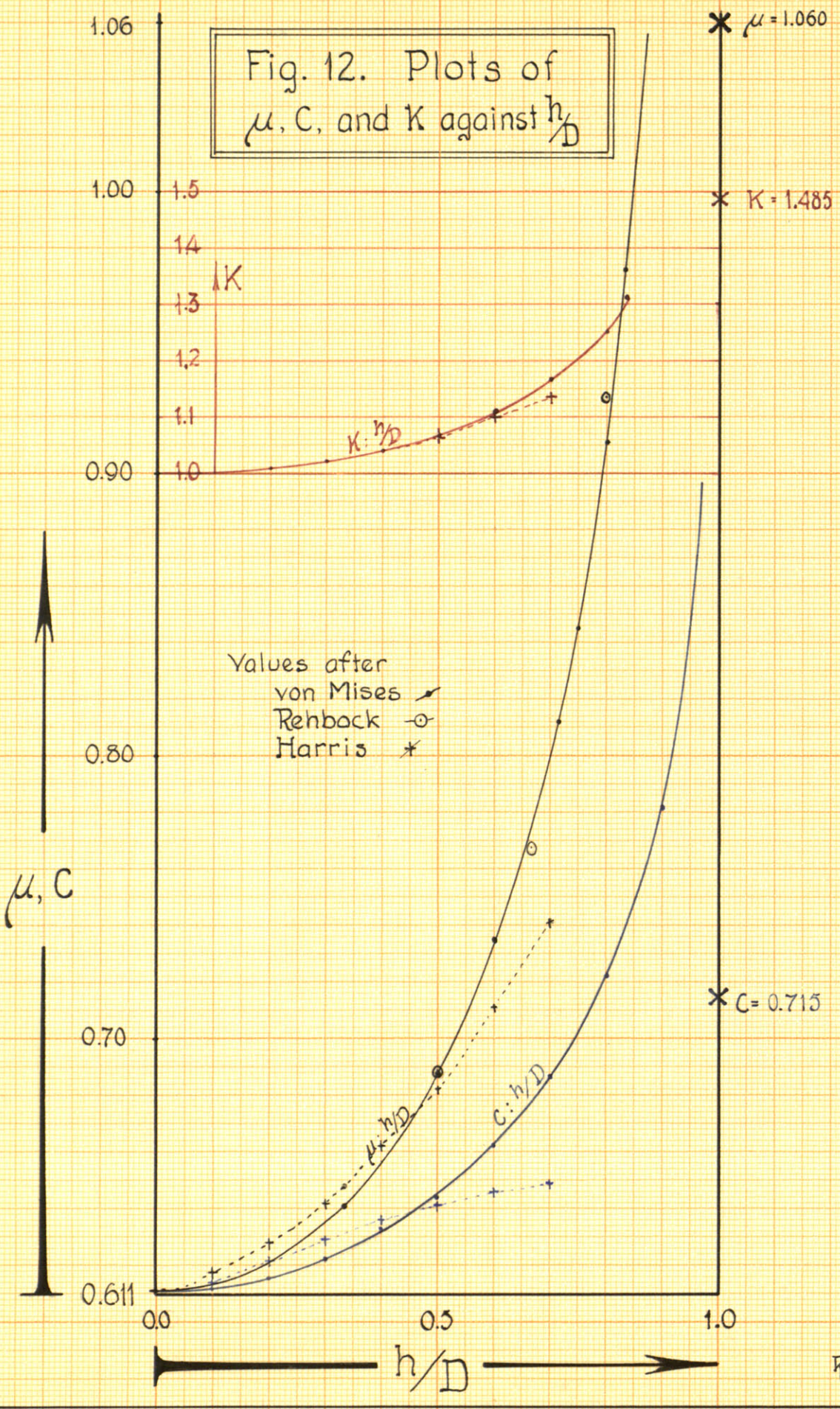
$$\frac{P_a}{w} + 2D_a \frac{V_a^2}{2g} = \frac{P_b}{w} + 2D_b \frac{V_b^2}{2g} \quad \Delta H = \frac{V_a^2 - V_b^2}{2g} + D_a - D_b$$

Fig. 8b. Law of Momentum as applied to the Hydraulic Jump; graphical representation of "Supporting Forces" after Rehbock



Route

Fig. 12. Plots of μ , C , and K against h/D



Rouse

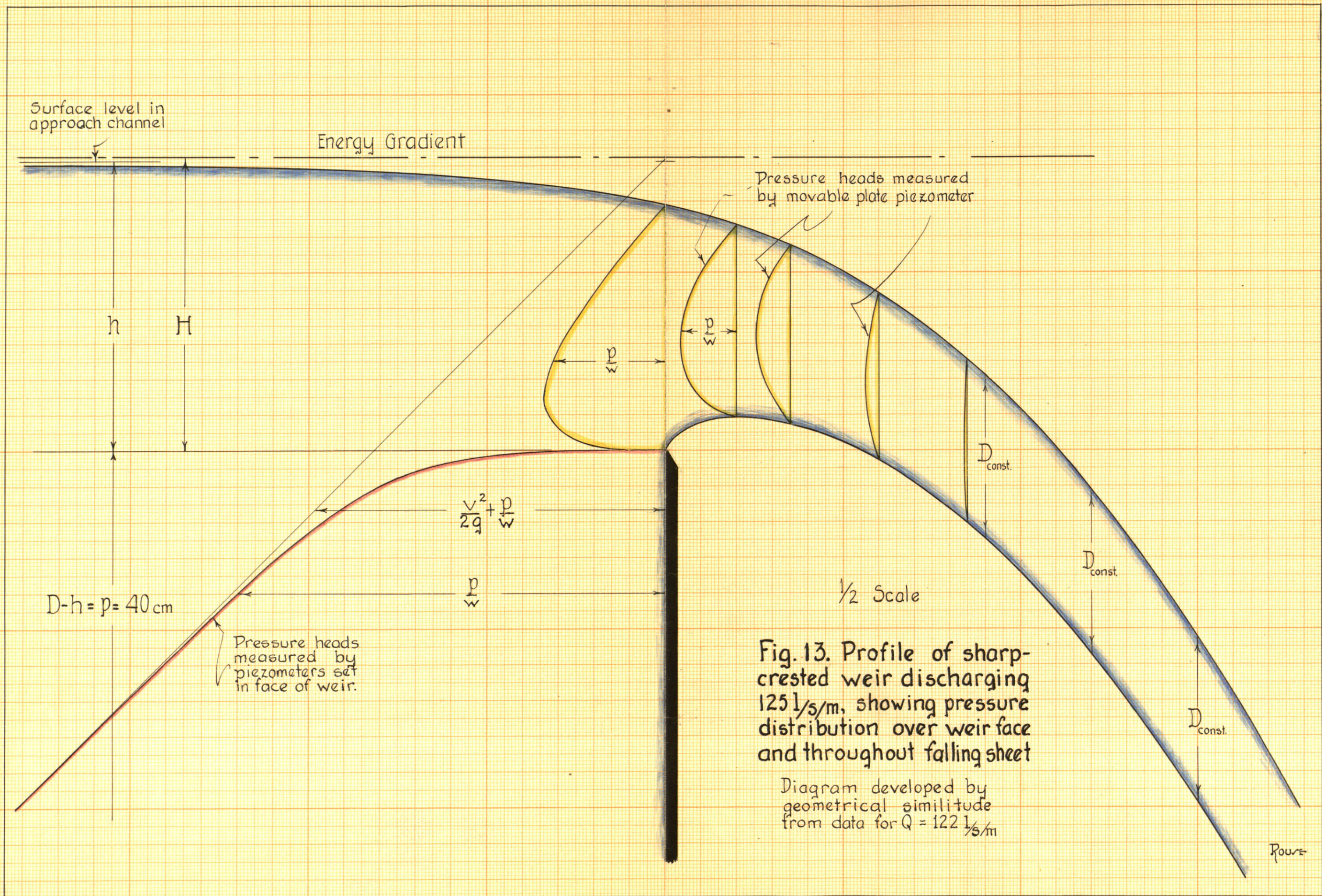


Fig. 13. Profile of sharp-crested weir discharging 125 l/s/m , showing pressure distribution over weir face and throughout falling sheet

Diagram developed by geometrical similitude from data for $Q = 122 \text{ l/s/m}$

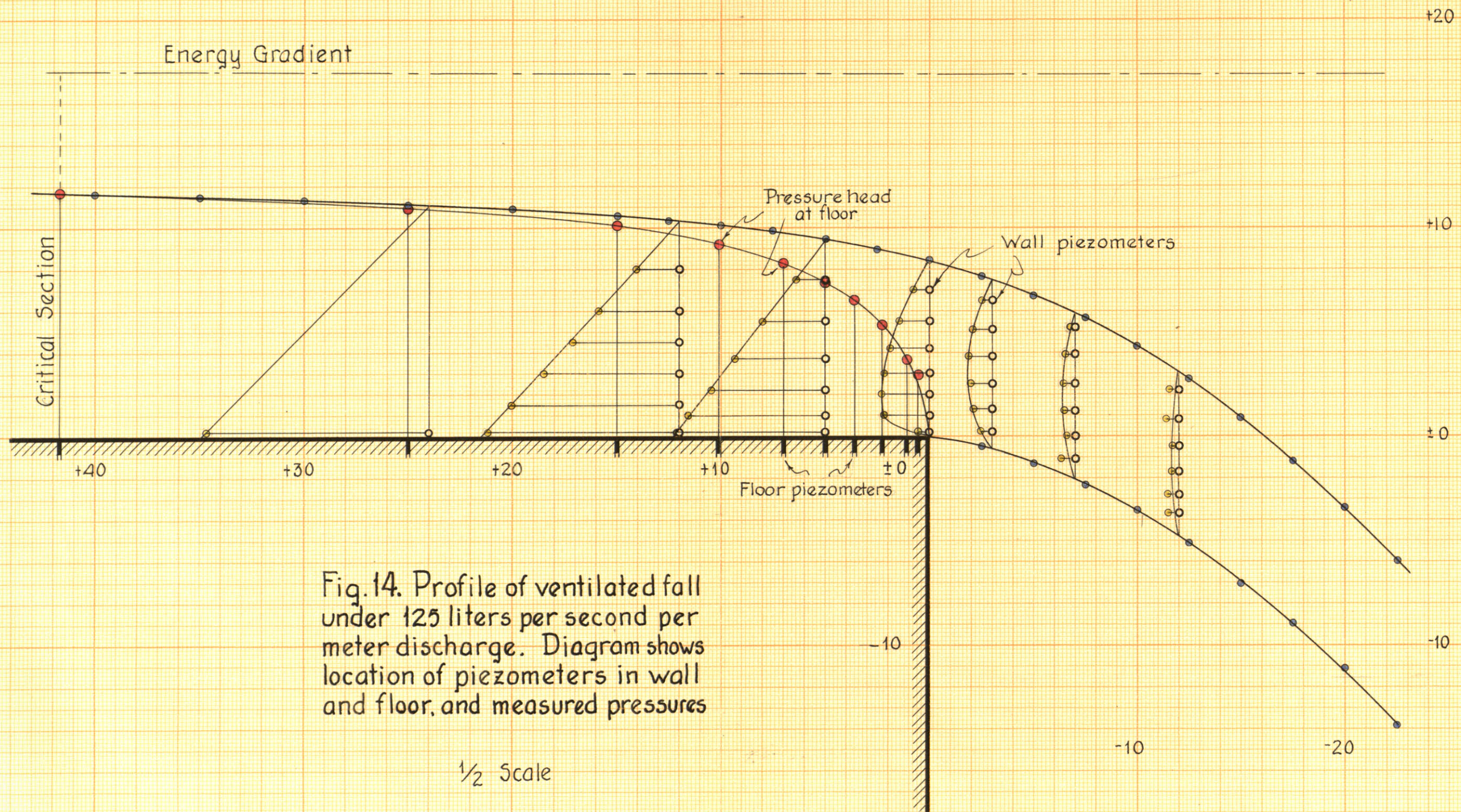


Fig.14. Profile of ventilated fall under 125 liters per second per meter discharge. Diagram shows location of piezometers in wall and floor, and measured pressures

1/2 Scale

ROUSE

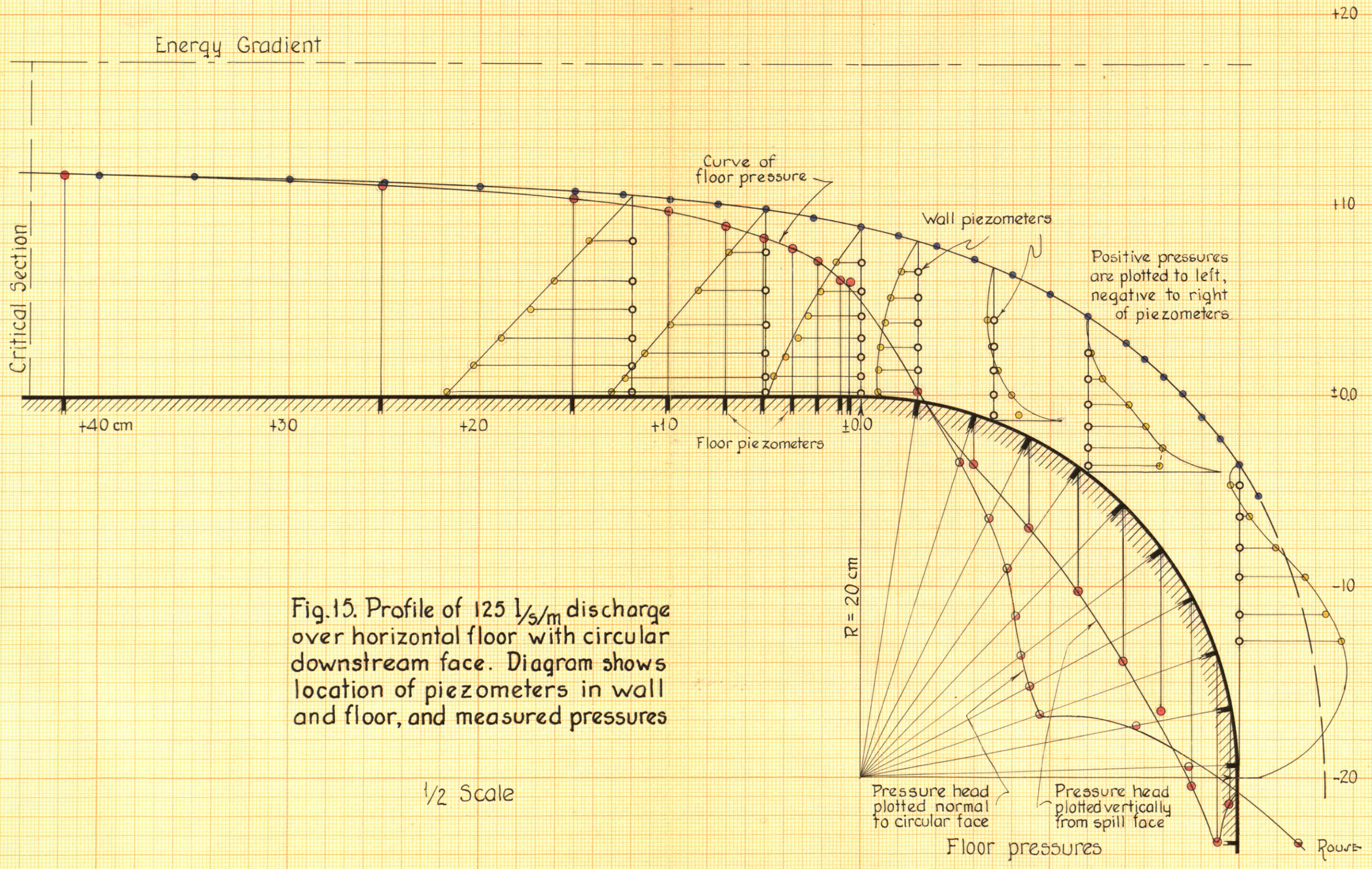


Fig.15. Profile of 125 l/s/m discharge over horizontal floor with circular downstream face. Diagram shows location of piezometers in wall and floor, and measured pressures

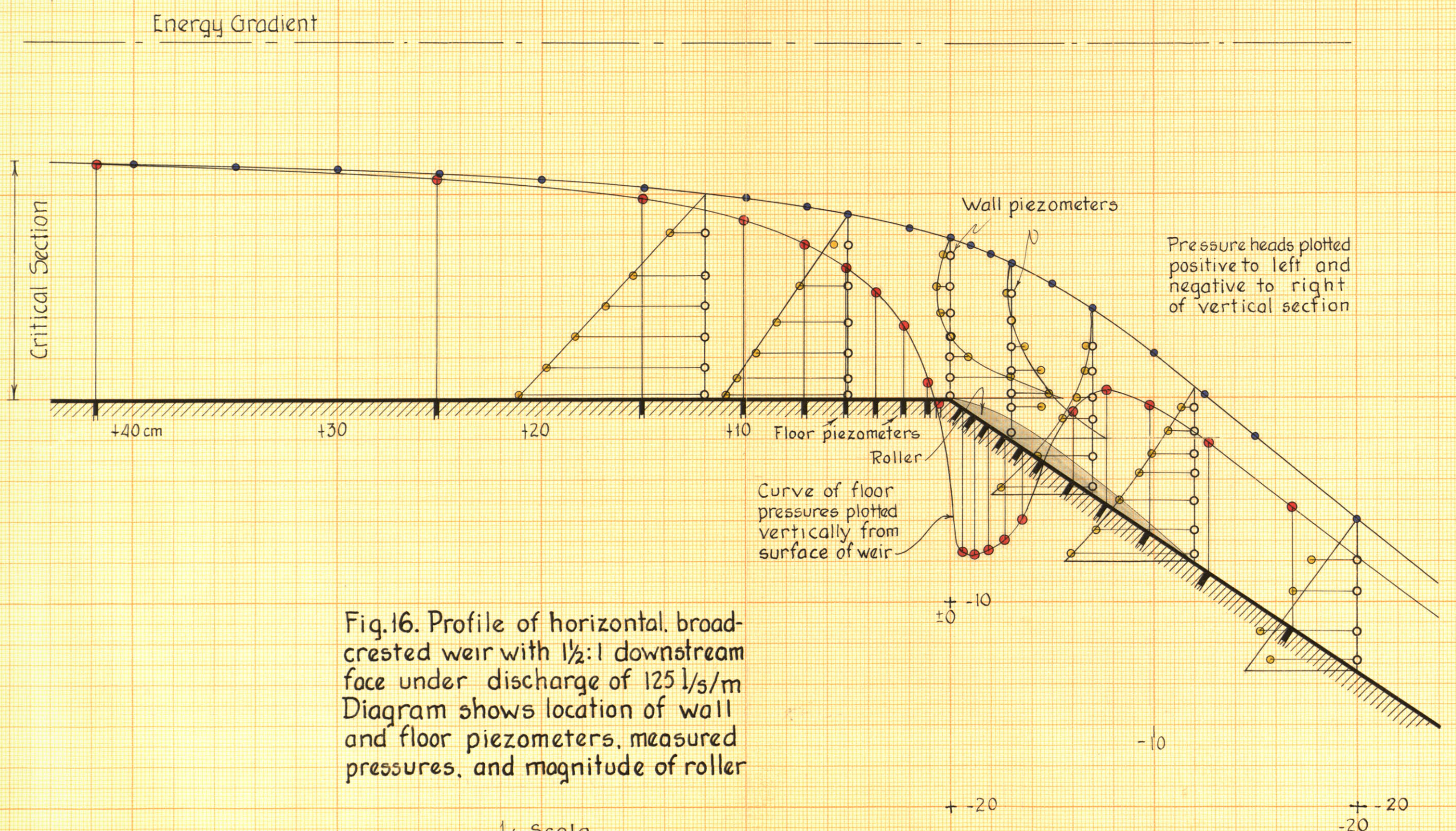


Fig.16. Profile of horizontal, broad-crested weir with $1/2:1$ downstream face under discharge of 125 l/s/m . Diagram shows location of wall and floor piezometers, measured pressures, and magnitude of roller

$1/2$ Scale

ROUSE

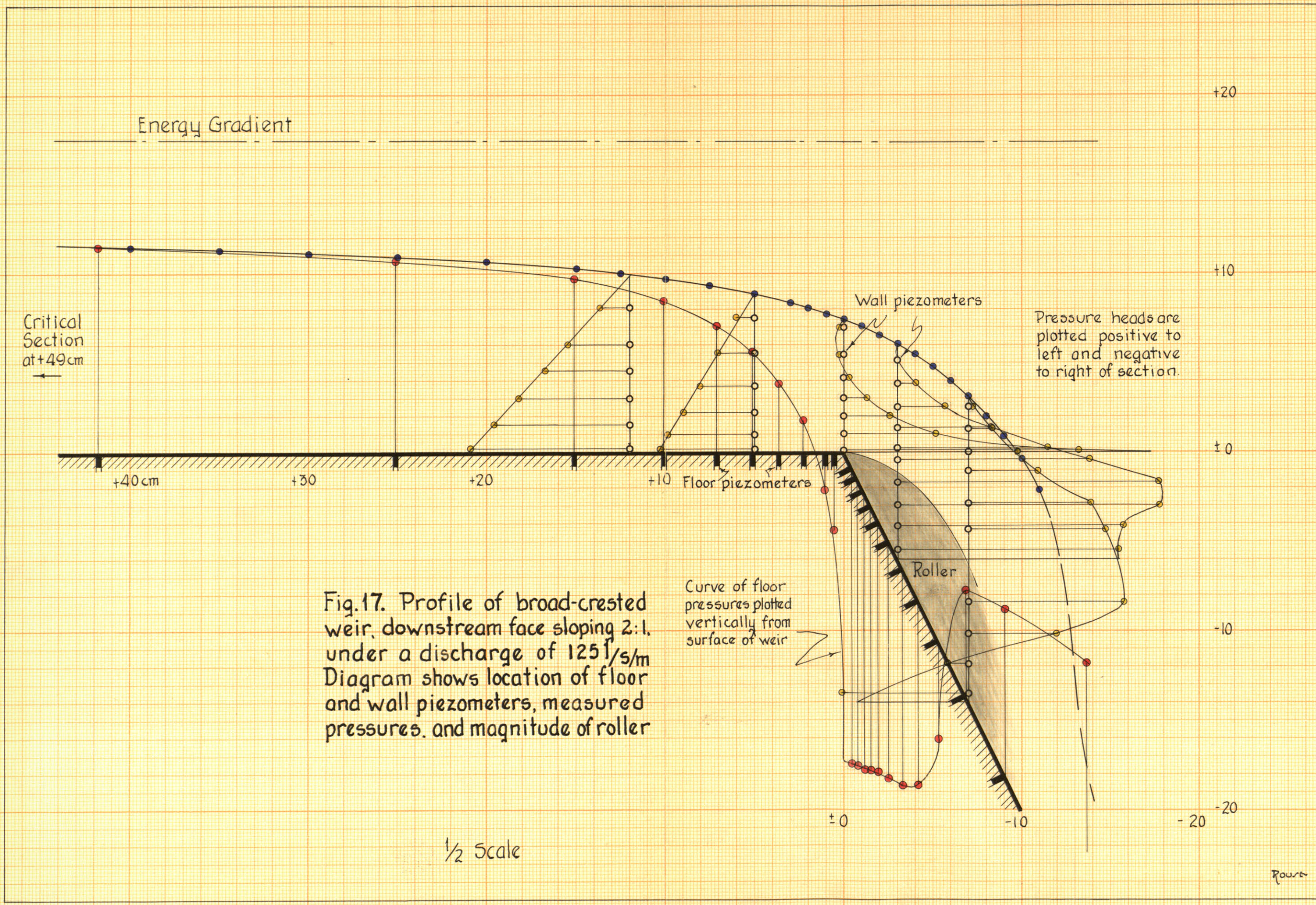


Fig.17. Profile of broad-crested weir, downstream face sloping 2:1, under a discharge of 125 l/s/m . Diagram shows location of floor and wall piezometers, measured pressures, and magnitude of roller

1/2 Scale

Power

Surface level in approach channel

Energy Gradient

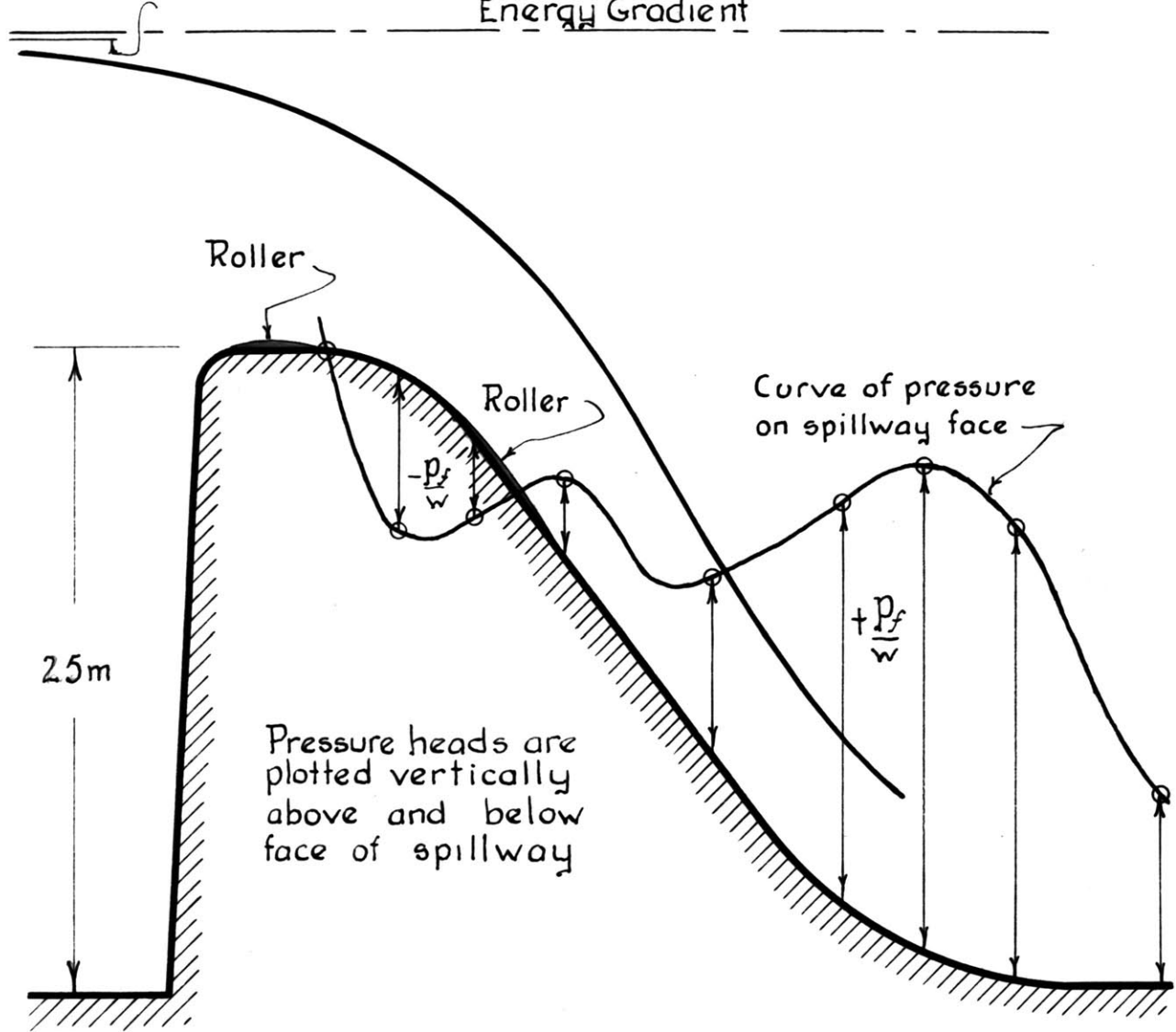


Fig. 18. Model study of Pinet spillway, France, at scales of 1:300 and 1:48.5, after Camichel, Toulouse

Surface level in approach channel

Energy Gradient

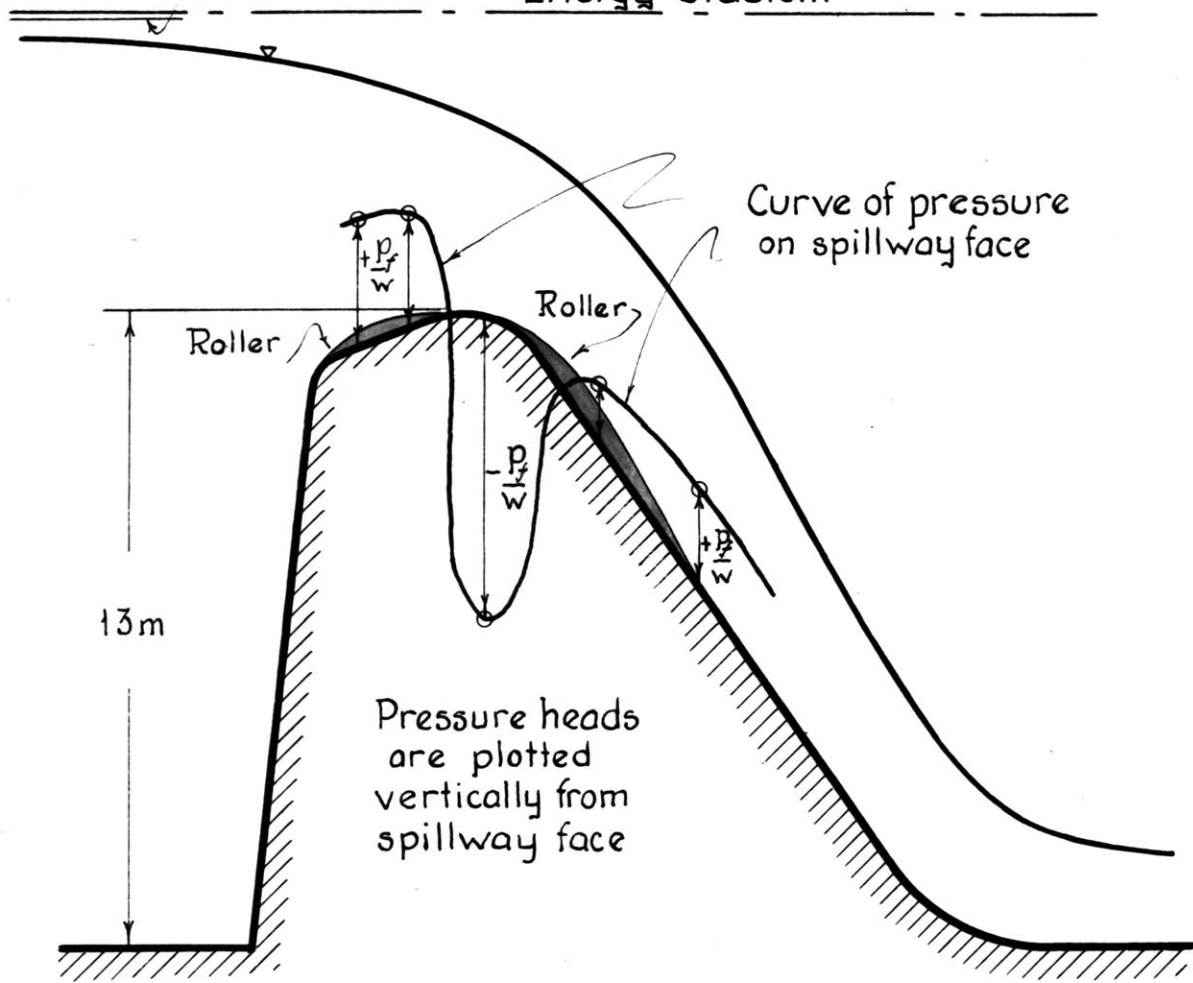


Fig.19. Results of model study of Puechabon spillway at scales of 1:100 and 1:19.5, after Camichel, Toulouse