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Time variations of Mars' gravitational field and seasonal changes in the masses of the polar ice caps

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[1] Tracking of the Mars Global Surveyor spacecraft has been used to measure changes in the long-wavelength gravity field of Mars and to estimate the seasonal mass of carbon dioxide that is deposited in the polar regions each fall and winter and sublimed back into the atmosphere every spring and summer. Observations spanning 4 Mars years have been analyzed. A clear and well-defined seasonal signal, composed of annual and semiannual periods, is seen in the lowest odd degree 3 coefficient but with less confidence in the lowest even degree 2, which is expected to be smaller and is also much more difficult to observe. Direct estimation of the seasonal mass exchange employing a simple, seasonally varying model of the size and height of each cap provides values that indicate some systematic departures from the deposition predicted by a general circulation model. Estimates are also obtained for the precession and nutation of the pole of rotation of Mars, the degree 2 tidal Love number, k_2 , and the mass of Phobos, the larger of Mars' two natural satellites.

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1. Introduction

[2] Mars has a tenuous carbon dioxide (CO₂) atmosphere that actively exchanges with the surface over the course of the planet's 687 (Earth)-day year. Variations in solar insolation associated with the planet's orbital eccentricity [Leighton and Murray, 1966], coupled with a Hadley circulation pattern that is influenced significantly [Richardson and Wilson, 2002] by the planet's south-to-north elevation difference [Smith and Zuber, 1996; Smith et al., 1999c], produce active atmospheric dynamics that drive the seasonal hemispheric transport of CO₂. Over the course of a Martian year ~18% of the total volatile mass is estimated to be exchanged between the atmosphere and surface, resulting in a redistribution of $\sim 1 \times 10^{-8}$ of the total mass of Mars. The seasonal transport of atmospheric mass affects the planetary rotation, and this effect has been addressed in theoretical studies by Defraigne et al. [2000] and observed from tracking of landers [Folkner et al., 1997a, 1997b; Yoder and Standish, 1997; Yoder et al., 2003].

[3] The seasonal CO₂ cycle was observed directly by the Viking landers via measurements of variations of atmospheric pressure at both landing sites [Hess et al., 1979, 1980; Leovy, 1985; Zurek et al., 1992], and more recently by the High-Energy Neutron Detector (HEND) and Gamma Ray Spectrometer (GRS) on the Mars Odyssey mission. A seasonal pressure change was also observed at the Pathfinder landing site over a fraction (~12%) of a Martian year [Schofield et al., 1997]. Radiative balance calculations [Paige and Ingersoll, 1985; Paige and Wood, 1992] and general circulation models (GCMs) [e.g., Haberle et al., 1993; Hourdin et al., 1995; Forget et al., 1998], constrained by Viking lander pressure data, have been used to estimate the mass of CO₂ that condenses onto the Martian surface. Simulations estimated a maximum deposition of the equivalent of about 1 m of solid CO₂ ice [Smith et al., 1999a].

[4] The Mars Global Surveyor (MGS) laser altimetry [Zuber et al., 1992; Smith et al., 1999c, 2001b] and Radio Science [Tyler et al., 1992, 2001] investigations have provided the first direct global-scale observations of the change in height of Mars' seasonal ice caps [Smith et al., 2001a; Aharonson et al., 2004] and of the seasonally exchanged CO₂ mass [Smith et al., 2001a; Yoder et al., 2003]. Preliminary observations of the seasonal mass exchange [Smith et al., 2001a] displayed general patterns consistent with GCM predictions [Smith et al., 1999b], but showed other features that were unexpected. This initial work demonstrated the feasibility of isolating small but important temporally varying geophysical signals on Mars

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from an orbiting spacecraft [Lemoine *et al.*, 2001; Yuan *et al.*, 2001; Konopliv *et al.*, 2006].

[5] In the current study we analyze MGS *X* band Doppler and range tracking observations from the mission's Radio Science experiment [Tyler *et al.*, 1992, 2001] to estimate the seasonal variations in some of the low-degree zonal coefficients of the Mars gravity field and, in conjunction with altimeter data from the Mars Orbiter Laser Altimeter (MOLA) investigation [Zuber *et al.*, 1992; Smith *et al.*, 1999c, 2001b] and bolometer observations by the Thermal Emission Spectrometer (TES) [Christensen *et al.*, 1992, 2001], both also included in the payload of MGS, make direct estimates of the north and south polar seasonal mass deposits. Simultaneously we estimate the position and direction of the pole of rotation. Our results are based upon 4 Mars years of MGS tracking observations. MGS ceased to operate in October 2006, nearly 10 years after its launch in November 1996.

2. Radio Tracking Observations

2.1. Range Rate

[6] The MGS spacecraft was in a near-polar (inclination = 92.8°), near-circular (altitude ~ 400 km) orbit with a period of 117 min. The spacecraft utilized two-way and three-way ramped Doppler tracking. Observations were at *X* band (4.2- and 3.6-cm wavelength corresponding to 7.2- and 8.4-GHz frequency) for the uplink from the ground and downlink from the spacecraft, respectively. In two-way tracking the signal is transmitted to the spacecraft, and transponded coherently back to the transmitting station on Earth. In three-way tracking the signal is transmitted and transponded from the spacecraft in the same fashion, however, different stations are used to transmit and receive. Ramping of the Doppler signal refers to a piecewise linear change in the uplink reference frequency that facilitates locking onto the downlinked signal at the receiving station.

[7] Transmission and reception of MGS radio signals utilized tracking stations in the NASA Deep Space Network (DSN) at Goldstone, California, Madrid, Spain and Canberra, Australia. Typically, MGS was tracked for one 10-h pass per day using the DSN's 34-m-diameter high-efficiency or beam waveguide antennae. The range rate observable is the Doppler shift of the tracking signal, which provided a measurement of spacecraft velocity in the line of sight. In the MGS mapping and extended missions, the Doppler tracking measurements were averaged in 10 s intervals and typically displayed an accuracy of better than 0.1 mm/s [Tyler *et al.*, 2001].

2.2. Range

[8] Except near solar conjunction where plasma noise is problematic, the MGS Radio Science investigation [Tyler *et al.*, 1992] acquired daily range measurements at ~ 3 -min intervals for an hour. These measurements provided the distance from the ground tracking station antenna to the spacecraft. The range observable is the round-trip propagation time between the ground station and spacecraft, which can be scaled to find the path length. From tracking of MGS, range was obtained simultaneously with Doppler frequency during periods of two-way tracking, and yielded

the distance to the MGS spacecraft to a few meters [Tyler *et al.*, 2001].

2.3. From Radio Tracking to Planetary Gravity

[9] Doppler range rate and range in combination provided excellent constraints on the orbit of the MGS spacecraft [Lemoine *et al.*, 1999]. Of interest in the current study are perturbations of the spacecraft orbit that are due mainly to Mars' long-wavelength gravitational field. Doppler tracking provides a measure of spacecraft velocity that is used to compute the orbits of MGS around Mars, based upon various a priori models and adjusted parameters. This orbital information is then used to produce normal equations that are solved to yield estimates of various orbital and geophysical parameters, including spherical harmonics of the gravitational potential and the masses of the seasonal ice caps.

[10] To process the tracking data, we utilized the NASA/GSFC GEODYN/SOLVE orbit determination system of programs [Rowlands *et al.*, 1993; McCarthy *et al.*, 1994; Pavlis *et al.*, 2001, 2006]. In the processing we accounted for periodic spacecraft thrusting events (aka angular momentum dumps) that served to unload momentum from reaction wheels used to maintain the MGS attitude. These events are recorded in the spacecraft SPICE kernels archived by the NASA Planetary Data System. For the planetary positions we used the DE410 planetary ephemeris (E. M. Standish *et al.*, JPL planetary and lunar ephemerides DE403/LE403, internal report, Jet Propul. Lab., Pasadena, Calif., 1995). We applied third body accelerations due to the Sun, Moon, planets, and the natural satellites of Mars (Phobos and Deimos). In addition to the direct solar radiation pressure acting on MGS, we incorporated the indirect reflected solar radiation from Mars, as well as the radiation pressure from Mars' thermal emission using spherical harmonic models derived from analysis of Viking Infrared Thermal Mapper data [Lemoine, 1992]. We used a model of the Martian atmosphere [Culp and Stewart, 1984; Stewart, 1987] as the a priori estimate of the atmospheric density at the MGS orbital altitudes and we adjusted a single drag coefficient in each orbital arc. For calculation of drag and radiation pressure we model the MGS spacecraft with a nine-plate model, consistent of six for the spacecraft bus, two for the solar arrays, and one for the high-gain antenna. We also included relativistic corrections [Moyer, 1981, 2000], including the Schwarzschild effect (i.e., the relativistic modification of the central body term in the force model) and relativistic light time effects due to the Sun, Jupiter, and Saturn. Transformations between coordinate time and atomic time are included by GEODYN in all the interplanetary measurement modeling. We corrected DSN tracking data for Earth-based tracking station coordinate effects including Earth polar motion, Earth solid tide and ocean loading effects according to International Earth Reference System conventions. We used meteorological data collected at half hour intervals at each DSN station to compute an Earth troposphere media correction for the radiometric tracking data [Hopfield, 1999]. A table of parameters and models used and adjusted in the analysis is given in Table 1. No a priori constraints were applied to any orbital parameter, including the drag, solar radiation pressure perturbations. The momentum dumps

Table 1. Summary of Models Used and/or Adjusted in Current Analysis

Model	Applied, Estimated, or Derived
Gravity field model	Applied, some parameters estimated
Lunar and planetary gravitational perturbations	Applied
Gravity perturbations by Phobos and Deimos	Applied, Phobos mass estimated
Solar radiation pressure	Estimated, 1 per arc
Mars albedo pressure	Applied
Atmospheric density (drag) at MGS altitude	Estimated, 1 per arc
Nine-plate spacecraft model	Applied
Mean atmospheric pressure at Viking lander sites	Derived
Momentum desaturation events	Estimated magnitudes
Relativistic measurement and force perturbations	Applied <i>Moyer</i> [1981, 2000]
Mars gravity tide, k_2	Estimated
DSN ground station positions and tidal and tectonic motions	Applied
Mars precession in RA & Dec	Estimated
Mars General Circulation Model (GCM)	Applied Ames GCM run 99.74 (R. M. Haberle, personal communication, 2002)
Seasonal masses of icecaps	Estimated each arc, unconstrained
Seasonal variation in atmospheric mass	Estimated each arc, constrained

were only constrained to the known time of the event, not in magnitude.

3. Gravitational Potential and Density Distribution

[11] The gravitational potential of Mars, U , can be expressed in spherical harmonics as [Kaula, 1966]

$$U(r, \theta, \lambda) = \frac{GM}{r} \left\{ 1 + \sum_{l=2}^{\infty} \left(\frac{R}{r} \right)^l \sum_{m=0}^l \cdot [(\bar{C}_{l,m} \cos m\lambda + \bar{S}_{l,m} \sin m\lambda) \bar{P}_{l,m}(\cos \theta)] \right\}, \quad (1)$$

where G is the universal constant of gravitation; M is the total planetary mass; R is the reference equatorial radius; $\bar{P}_{l,m}$ are normalized associated Legendre functions of degree l and order m ; r , λ , and θ are the body-fixed coordinates of radial distance, longitude, and colatitude; and $\bar{C}_{l,m}$ and $\bar{S}_{l,m}$ are the normalized Stokes coefficients.

[12] The Stokes coefficients of the gravitational potential can be related to the density distribution of the planet by [Chao and Gross, 1987; Chao, 1994]

$$\Gamma_{lm}(r, \theta, \lambda) = \frac{1}{(2l+1)MR^l} \int_V \rho(r, \theta, \lambda) r^l Y_{lm}(\theta, \lambda) dV, \quad (2)$$

where $\Gamma = \bar{C}_{lm} + i\bar{S}_{lm}$, $Y_{lm} = P_{lm} \sin(\theta) \exp(im\lambda)$, $\rho(r, \theta, \lambda)$ is the density distribution, and V is the planetary volume. Note that the volume integration includes the atmosphere. Using

this formalism, the seasonal redistribution of mass can be written

$$\Gamma_{lm}(r, \theta, \lambda, t) = \frac{1}{(2l+1)MR^l} \int_V \Delta\rho(r, \theta, \lambda, t) r^l Y_{lm}(\theta, \lambda) dV, \quad (3)$$

where $\Delta\rho$ represents the temporal density distribution and t is time.

3.1. Low-Degree Coefficients

[13] Of greatest importance in the detection of temporal variability in the gravity field are the low-degree terms of equation (1). The lowest degrees of the field correspond to the longest-wavelength gravity signals, which are the most sensitive to global-scale changes in the distribution of density or mass. In general, the low-degree terms are the best constrained parameters in spherical harmonic models [Balmino *et al.*, 1982; Smith *et al.*, 1993; Lemoine *et al.*, 2001; Yuan *et al.*, 2001] because potential fields have greatest power at long wavelengths [Kaula, 1966]. In addition, the long-wavelength field is sampled whenever the spacecraft is being tracked. The zonal terms, for which $m = 0$, are of key interest in the detection of CO₂ cycling, because they represent changes in the mass distribution along lines of longitude (i.e., from pole to pole). On the basis of a simulation of gravity field changes associated with the cycle of CO₂ exchange on Mars, the largest term in the time-varying geopotential is the lowest odd-degree coefficient [Smith *et al.*, 1999b; Sanchez *et al.*, 2004] followed by the lowest even-degree term (Table 2). These low-degree terms can be solved for directly from the tracking data but the signals of the coefficients have similar perturbing effects on the spacecraft orbit so separation of one coefficient from another is difficult from a single spacecraft orbit. In the first part of this paper we estimate the degree 2 and 3 zonal coefficients from perturbation of the MGS orbit.

3.2. Masses of Seasonal Ice Caps

[14] The changes in the gravity field are almost exclusively due to movement of carbon dioxide between the atmosphere and the ice caps on a seasonal cycle. A relationship exists between the simplest ice cap model, a point mass (M_{NP} , M_{SP}) at each pole, and the coefficients of the gravity field [Karatekin *et al.*, 2005; M. T. Zuber and D. E. Smith, Estimation of temporal changes in the mean atmospheric pressure of Mars from MGS Doppler tracking, paper presented at Mars Atmosphere Modelling and Observations Workshop, Granada, Spain, 2003] as

$$M_{NP} = \frac{1}{2} (C_{2,0} + C_{3,0}) \cdot M_{Mars} \quad (4)$$

$$M_{SP} = \frac{1}{2} (C_{2,0} - C_{3,0}) \cdot M_{Mars}. \quad (5)$$

Table 2. Predicted Gravity Coefficient Variations

Coefficient	Predicted Value ^a
Data span	~4 Mars years
$C_{2,0}$	3×10^{-9}
$C_{3,0}$	5.5×10^{-9}

^aExpected peak-to-peak variation for seasonal exchange of CO₂ [from Smith *et al.*, 1999b].

Table 3. Data Summary

Data	Description
Data span	~4 Mars years
Data span, L_s	~1450°
Number Doppler observations	~5 × 10 ⁶
Rate of Doppler observation	0.1 Hz
Quality of Doppler observations	0.1 m s ⁻¹
Number range observations	~5 × 10 ⁴
Rate of range observations	~1 per 3 min, 1 hr per day
Quality of range observations	~2 m after bias removal
Total number of 5-day arcs	475
Number of arcs used in analysis	434

This simple representation is discussed later in conjunction with our results for $C_{2,0}$ and $C_{3,0}$.

[15] Naturally, the observable changes in gravity due to the seasonal changes are not restricted to the lowest degrees and orders but affect almost all of the coefficients to some extent. Thus, in order to quantify more fully the changes in the masses of the seasonal caps it is arguably preferable to solve directly for these masses rather than through a very limited number of gravity coefficients. This can be accomplished by introducing a more complex model of the ice cap involving both the physical dimensions of the seasonal caps and their elevations. In the second part of the paper we estimate the masses of the seasonal caps by combining the tracking data with imaging, altimetry, and atmospheric pressure variations.

3.3. Orbital Data Sets

[16] The MGS tracking data were analyzed in approximately 5-day orbital arcs covering the period February 1999 to August 2006. Out of a total of nearly 475 orbital arcs we accepted 434 for analysis. It is customary to analyze seasonal effects on Mars with respect to the solar longitude, L_s . This parameter is defined from 0°–360° over a Martian year and $L_s = 0°$ corresponds to the vernal equinox in the northern hemisphere. The data span covers over 1440 degrees of L_s , 4 complete Mars years of 687 days each. A summary of the data is given in Table 3.

4. Estimation of Seasonal Zonal Gravity Coefficients

[17] The solution for the lowest-degree zonal harmonics were obtained simultaneously with the 6 orbital parameters,

Figure 1. Variation in zonal gravity coefficient $C_{3,0}$ as a function of season when estimated in combination with the orbit and supporting parameters. A four-frequency fit (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) is shown in black. The variation is of order 5×10^{-4} of the nominal value of $C_{3,0}$.

Figure 2. Variation in zonal gravity coefficient $C_{5,0}$ as a function of season when estimated in combination with the orbit and supporting parameters. A four-frequency fit (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) is shown in black. The variation is of order 10^{-4} of the nominal value of $C_{5,0}$.

a drag coefficient, a solar radiation pressure coefficient, along with estimates of the effective acceleration for each of the momentum dumps performed by the spacecraft during the approximate 5-day period covered by the data. All 434 arcs were analyzed simultaneously to provide 434 independent solutions for each of the coefficients $C_{2,0}$; $C_{3,0}$; $C_{3,0}$ and $C_{5,0}$; and $C_{2,0}$ and $C_{4,0}$ adjusted simultaneously. In some solutions we also adjusted GM and the Love number k_2 as parameters common to all orbital arcs.

[18] The largest perturbation of the MGS orbit by the seasonal transportation of CO₂ is expected from the $C_{3,0}$ term [Smith *et al.*, 1999b] followed by $C_{2,0}$ (see Table 2). Our individual normalized solutions for $C_{3,0}$ and $C_{5,0}$ for the 4 Mars years are shown in Figures 1 and 2 where the values are plotted as a function of L_s . For computational convenience the coefficients are normalized according to Kaula [1966] where

$$\text{Normalization Factor}(l, m) = \left[\frac{(l-m)!(2l+1)(2-\delta)}{(l+m)!} \right]^{1/2} \quad (6)$$

and $\delta = 1$, $m = 0$; and 0 for $m \neq 0$. For geophysical interpretation of gravity coefficients it is necessary to unnormalize the coefficients.

[19] Figure 3 shows the unnormalized $C_{3,0}$ and $C_{5,0}$ values on the same graph and indicates that the individual solutions are identical, therefore representing the same

Figure 3. Solution for the unnormalized gravity coefficient $C_{5,0}$ superimposed upon the unnormalized solution for $C_{3,0}$. The two solutions are statistically identical. A four-frequency fit (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) is shown in blue/red.

Table 4. Four-Frequency Solutions to Normalized Values of Low-Degree Zonal Gravity Field

Value	Amplitude $\times 10^{-9}$	Sigma $\times 10^{-9}$	Phase Degrees	Sigma Degrees
$C_{3,0}$				
L_s	3.082	0.068	38.92	1.25
$2L_s$	-0.546	0.068	-29.07	7.16
$3L_s$	0.280	0.067	-53.17	13.99
$4L_s$	-0.086	0.068	-8.5	44.27
$C_{5,0}$				
L_s	2.479	0.055	39.32	1.26
$2L_s$	-0.441	0.055	-26.48	7.15
$3L_s$	0.246	0.054	-50.94	12.85
$4L_s$	-0.052	0.055	-7.23	58.60
$C_{2,0}$				
L_s	-17.32	1.22	-327.2	3.9
$2L_s$	6.95	1.23	23.9	9.8
$3L_s$	-8.71	1.22	343.8	7.9
$4L_s$	2.87	1.20	-69.7	24.2
$C_{4,0}$				
L_s	10.25	0.60	-129.7	3.3
$2L_s$	5.27	0.60	-14.4	6.5
$3L_s$	-1.01	0.59	-74.1	34.4
$4L_s$	0.53	0.60	-22.3	63.7
$C_{3,0}$ With $C_{2,0}$				
L_s	3.070	0.068	39.07	1.06
$2L_s$	-0.555	0.068	-32.35	7.08
$3L_s$	0.262	0.068	-47.80	14.97
$4L_s$	-0.086	0.068	-3.95	44.67
$C_{2,0}$ With $C_{3,0}$				
L_s	-1.95	0.71	24.3	20.6
$2L_s$	-2.54	0.70	-69.8	16.2
$3L_s$	2.22	0.70	97.0	18.4
$4L_s$	-0.55	0.70	-62.6	73.1

physical perturbation of the MGS orbit. We are therefore unable to separate these 2 coefficients in the Mars gravity field. The $C_{5,0}$ perturbation includes the same orbital frequency perturbations as $C_{3,0}$. Extending this to the next odd-degree zonal, $C_{7,0}$, does not provide the same perturbation of the orbit. Any solution for an odd-degree zonal coefficient absorbs some or all of the signal from other odd-degree zonals. Separability of these coefficients requires more than one orbit with different characteristics.

Figure 4. Variation in zonal gravity coefficient $C_{2,0}$ as a function of season when estimated singly in combination with the orbit and supporting parameters. A four-frequency fit (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) is shown in black. The variation is of order 5×10^{-5} of the nominal value of $C_{2,0}$. **Figure 5.** Variation in zonal gravity coefficient $C_{4,0}$ as a function of season when estimated singly in combination with the orbit and supporting parameters. A four-frequency fit (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) is shown in black. The variation is of order 5×10^{-3} of the nominal value of $C_{4,0}$.

[20] However, the odd-degree variation is very clear in Figures 1, 2, and 3 and its departure from a perfect sinusoid is well defined. The pattern can be well represented by the frequencies L_s , $2L_s$, $3L_s$, and $4L_s$ and the amplitudes and phases are shown in Table 4. The uncertainty in the amplitude at the L_s period (1 Mars year) is $\sim 2\%$, the $2L_s$ period (semiannual) is about 14% and the $3L_s$ period is about 25%. We assume that at some level there are changes from year to year within the 4-year data span.

[21] The individual solutions for the even zonal coefficients $C_{2,0}$ and $C_{4,0}$ are shown in Figures 4 and 5, but unlike $C_{3,0}$ and $C_{5,0}$ there is no common pattern and the variations in Table 4 are an order of magnitude larger than expected. Figure 6 shows that when adjusted together the variation in $C_{2,0}$ is reduced significantly but $C_{4,0}$ is statistically unchanged, from which we conclude that $C_{2,0}$ absorbs significant power from $C_{4,0}$ when estimated on its own, but not in reverse.

[22] The difficulty in separating coefficients of odd degree and even degree was not surprising since coefficients of even or odd degrees have similar orbital affects. However, we also were interested in whether odd degrees were separable from even degrees, and in particular if $C_{3,0}$ was independent of $C_{2,0}$ and vice versa.

[23] Figure 7 shows the unnormalized individual $C_{3,0}$ values and the $C_{3,0}$ values obtained when the $C_{2,0}$ is adjusted simultaneously with $C_{3,0}$. The two solutions for $C_{3,0}$ are essentially identical and indicate that $C_{2,0}$ is having no effect on the estimation of $C_{3,0}$. The coefficient is

Figure 6. Simultaneous adjustment of $C_{2,0}$ and $C_{4,0}$. The parameter $C_{4,0}$ is almost unaffected by the presence of $C_{2,0}$. The reverse is not true. Four-frequency fits (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) to the recoveries are shown in black.

Figure 7. Variation in zonal gravity coefficient $C_{3,0}$ (unnormalized) when estimated simultaneously with $C_{2,0}$ and in combination with the orbit and supporting parameters and superimposed upon $C_{3,0}$ adjusted alone. Four-frequency fits (periods of L_s , $2L_s$, $3L_s$, and $4L_s$) to the recoveries are shown in blue/red. There is no discernable impact on $C_{3,0}$ of solving simultaneously for $C_{2,0}$.

therefore completely independent of $C_{2,0}$ but not necessarily of higher zonal coefficients. The four-frequency simultaneous solutions for $C_{2,0}$ and $C_{4,0}$ are shown in Table 4. The presence of $C_{2,0}$ in a simultaneous solution involving $C_{3,0}$ or $C_{4,0}$ does not affect either of these coefficients.

[24] Figure 8 shows results for $C_{2,0}$ in the simultaneous adjustment of $C_{2,0}$ and $C_{3,0}$. The $C_{2,0}$ signal has decreased considerably in Figure 8 compared to Figure 4, in contrast to $C_{3,0}$ which is unchanged (Figure 7). These results lead us to the conclusion that the observed variation of $C_{3,0}$ representing the lumped effect of several odd zonals, is a robust solution for the seasonal effects of the movement of CO_2 on the gravity field of Mars but that $C_{2,0}$ is a relatively weak result and dependent on other coefficients being adjusted at the same time.

[25] A rationalization of the weakness of the $C_{2,0}$ result can be found in the size of the orbital perturbation by $C_{2,0}$. The largest perturbation of the MGS orbit by $C_{2,0}$ is in the right ascension of the node. In order to maintain its sun synchronous orbit the node precesses at about 0.5° per day, or 30 km at the equator. The seasonal perturbation of $C_{2,0}$ has a half amplitude of about 1×10^{-6} of $C_{2,0}$, so the daily perturbation of the node of MGS is about $30 \text{ km} \times 10^{-6}$, which is about 3 cm at the equator. In our orbital arcs of

Figure 8. Variation in zonal gravity coefficient $C_{2,0}$ (unnormalized) when estimated simultaneously with $C_{3,0}$ and in combination with the orbit and supporting parameters. A significant change in $C_{2,0}$ (see Figure 4) when $C_{3,0}$ is adjusted simultaneously. The range of values is reduced by at least a factor of 5.

5 days in length this only amounts to about 15 cm. The primary perturbation is quasi semiannual; each hemisphere contributes to the seasonal change with an annual period but the two hemispheres are out of phase owing to the obliquity of the Martian spin axis that causes northern and hemisphere seasons as on Earth. The perturbation accumulates to a peak-to-peak displacement of the orbital plane of less than 10 m at the equator. Although it is probably measurable we believe it is a major factor in not obtaining a strong solution for the seasonal change in $C_{2,0}$ and that the signal is absorbed into other parameters that can vary with a semi-annual period.

[26] In order to try to assess the geophysical value of the seasonal $C_{2,0}$ and $C_{3,0}$ coefficients we have derived the polar masses from equations (4) and (5) from the combined solution. As a check on the calculations, since the quality of $C_{2,0}$ and $C_{3,0}$ solutions are so different, we produced a solution in which we solved directly for $(C_{2,0} + C_{3,0})$ and $(C_{2,0} - C_{3,0})$ simultaneously. The results were almost identical and the latter are shown in Figure 9 compared with GCM values. The weakness of the solution for the seasonal masses is indicated by the large scatter of the data in Figure 9 which is mainly due to the solution for $C_{2,0}$. Both the north and south show seasonal variations, although the northern mass also shows a strong semiannual component. Although both the north and south show seasonal variations they do not agree with the GCM and are inconsistent with present understanding about the lack of ice at the summer poles. This may in part be due to trying to represent the mass variation as the combination of two sinusoids. In trying to interpret Figure 9 it is important to remember that the model is simple and that the estimated

Figure 9. (top) Point mass estimate for seasonal cap masses using $C_{2,0}$ and $C_{3,0}$ coefficients using equations (4) and (5). Solid lines are the best fit to annual and semiannual periods (L_s , $2L_s$). (bottom) Best fit curves of Figure 9 (top) shown with the computed variation by the Ames GCM [Haberle et al., 2002]. Gravity curves show seasonal variations for each cap, but the semiannual period is much larger than the GCM result. This is probably due in large measure to the gravity recovery but also in part to the simplicity of the point mass seasonal cap model.

Figure 10. Seasonal polar cap size model derived from TES bolometer observations [Kieffer *et al.*, 2000; Kieffer and Titus, 2001].

mass from gravity includes the atmosphere whereas the GCM data only show the CO₂ surface ice.

5. Estimation of the Masses of Seasonal Ice Caps and Interchange With Atmosphere

[27] We have also used the tracking data to make direct estimates of the masses of the seasonal caps. We have accomplished this by constructing a model of each seasonal ice cap in size and height, deriving a gravity field for that cap, and solving for a scale factor for the model that best fits the tracking data. For this approach to be effective we must have a model that represents the seasonal ice cap as a function of time. In addition, it is necessary to include the effect of the mass of the atmosphere on the orbit and apply appropriate constraints in terms of the conservation of total mass. Thus our model includes both seasonal CO₂ ice caps and the CO₂ atmosphere, and thus represents all the components of the volatile material that participates in the mass exchange. The residual cap, and the remainder of the atmospheric mass are considered part of the static mass of the planet.

[28] We represent each seasonal ice cap as a cone of material that is symmetric about the pole and draped over the topography. The size and height of each cap varies with L_s . We have used the TES thermal observations as a measure of the cap size [Kieffer *et al.*, 2000; Kieffer and Titus, 2001]. Figure 10 shows the sizes of the caps as a function of L_s and our quantization of the cap size at 5° of latitude. It is noted that the TES data indicate that the seasonal cap in the south reaches a maximum of about latitude 49°S and that the maximum in the north is about 52°N. Our digitization assigns both latitudes 50°, thus making the seasonal caps of equal size. This approximation may be an important limit on our model because the surface area between latitudes 49° and 52° is large. This is a correction we will make in future work. Note that the cap never actually goes to zero in our model but is constrained to be of radius 1°. The reason for this will be discussed in great detail later but a nonzero value enables us to estimate a mass

for the cap even if other data suggest it should be zero. The height of the cap is based on altimetry [Smith *et al.*, 1999c, 2001b] and we have adopted a maximum height of 1 m at both poles at lat 90°. The height of the material was assumed to decrease linearly to zero at the cap edge at latitudes 50°N and 50°S as suggested by the MOLA altimetry results [Smith *et al.*, 2001a]. In numerical experiments we found the size of the cap to be more important than the height of the cap. Finally, we assume a density for the material of 1000 kg m⁻³. Together with the shape and volume as a function of L_s we are able to derive a time-variable gravity field for each cap.

[29] The atmospheric mass component of the volatile cycle is modeled as a surface mass layer draped over the topography. We assume a constant atmospheric pressure on the areoid from which the surface mass is derived, taking into account the topography. For this calculation we assume a density scale height of 8 km between the areoid and the surface. Our volatile model thus has three components: two seasonally varying polar caps and an atmospheric surface layer over the whole planet. Figure 11 shows a schematic of this model.

5.1. Gravity Field for Volatile Model Components

[30] We computed the gravity field for each polar volatile component from the model shape, size, and density, as a function of L_s . Each ice cap was draped over a $1 \times 1^\circ$ model of the topography and the geopotential coefficients computed by numerically integrating over the surface layer to degree and order 90. Although we computed all the coefficients we only used the zonal harmonics since our model of each cap was symmetric about the rotation pole with the exception of topographic variations in longitude. The expression for the zonal gravity potential of the north seasonal cap, V_{NP} , is of the form

$$V_{NP} = K_1 \left(\frac{GM_{NP}}{r} \right) \cdot \left[\left(\frac{a}{r} \right)^2 P_2(\cos \theta) C_{2,0} + \left(\frac{a}{r} \right)^3 P_3(\cos \theta) \cdot C_{3,0} + \dots \right], \quad (7)$$

where K_1 is a scale factor, M_{NP} is the computed mass of the seasonal cap, a is the mean equatorial radius of Mars,

Figure 11. Schematic of the seasonal covering of a residual Martian ice cap. Our seasonal cap model overlays the topography of the cap at 1° of latitude and longitude resolution. The mass of the atmosphere is modeled as a global layer that varies with L_s and is uniform over the entire planet.

Figure 12. Input atmospheric mass model, which represents the variation about the mean atmospheric mass of about 3×10^{16} kg. An uncertainty (sigma) of 4×10^{15} kg was assigned to the mass in the adjustment.

$P_l(\cos\theta)$ is the Legendre polynomial of degree l , r is the distance from the center of mass of Mars and θ is the colatitude. Similarly, there is a corresponding potential, V_{SP} for the south polar seasonal cap

$$V_{SP} = K_2 \left(\frac{GM_{SP}}{r} \right) \cdot \left[\left(\frac{a}{r} \right)^2 P_2(\cos\theta) C_{2,0} + \left(\frac{a}{r} \right)^3 P_3(\cos\theta) \cdot C_{3,0} + \dots \right] \quad (8)$$

and for the atmospheric surface layer

$$V_{atm} = K_3 \left(\frac{GM_{atm}}{r} \right) \cdot \left[\left(\frac{a}{r} \right)^2 P_2(\cos\theta) C_{2,0} + \left(\frac{a}{r} \right)^3 P_3(\cos\theta) \cdot C_{3,0} + \dots \right]. \quad (9)$$

In the above expressions the geopotential coefficients $C_{2,0}$, $C_{3,0}$, $C_{4,0}$, $\dots$, $C_{90,0}$ are determined from the volatile models of the caps and atmosphere, and K_1 , K_2 , K_3 are the constants to be estimated from the data simultaneously with the orbit and other parameters.

[31] The masses of the seasonal caps are therefore $K_1 M_{NP}$, $K_2 M_{SP}$ and the atmosphere is $K_3 M_{atm}$. These masses are constrained to sum to zero so that there is no change in the total mass of volatile material.

[32] The expressions for potential of the seasonal caps and atmosphere are added to the potential of the solid planet to form the total gravity potential. Thus,

$$V(\text{Mars}) = V(\text{solid planet}) + V(\text{seasonal caps} + \text{atmosphere}). \quad (10)$$

[33] This potential is used to compute the orbit of MGS and associated parameters, and also the scale parameters for the masses of the volatile components. In the analysis, the scale factors have been estimated for the three masses from the tracking data in each orbital arc of approximately five days.

5.2. Seasonal Mass Estimates

[34] We analyzed the same 434 orbital arcs of MGS that we used in the spherical harmonic estimations in section 5.1. The primary difference between the solutions is that instead of solving for 1 or 2 spherical harmonic

coefficients of the gravity field we solved for the masses of the seasonal CO_2 ice caps and the mass of the atmospheric surface layer. As in section 5.1, we solved for the orbital momentum desaturation events simultaneously with the other parameters, all of which were freely adjusted with the exception of the atmosphere which was loosely constrained to an a priori model. This a priori model was based upon a simulation by the Ames General Circulation Model (GCM) [Haberle *et al.*, 1993; Pollack *et al.*, 1990, 1993; Haberle *et al.*, 2002] and assigned a sigma of 4×10^{15} kg, equivalent to approximately 50% of the mass of the atmospheric variability. There were no a priori values for the masses of the ice caps. Figure 12 shows the a priori atmospheric mass model. Positive and negative values represent the variation about the mean.

[35] Seasonal CO_2 masses of the north pole, south pole and atmosphere, recovered every five days, are shown in Figure 13. The x axis plots continuous L_s and shows almost 4 complete Mars years. The solid line though the data is a best fit to four frequencies L_s , $2 L_s$, $3 L_s$, and $4 L_s$ and the values of amplitude and phase are shown in Table 5. The blue dashed line shows a simulation of the expected mass exchange from the Ames GCM [Haberle *et al.*, 2002]. The results for the northern hemisphere show distinct asymmetry with a slow accumulation and a more rapid decline which appears to repeat from year to year. Dry ice begins to accumulate on the surface earlier than predicted by the GCM. The pattern of deposition and sublimation in the southern hemisphere is more symmetric but suggests a slight increase in mass beginning very soon after the sublimation phase has been completed, at $L_s \sim 300$.

[36] The atmosphere shows clearly the predominantly annual (L_s) and semiannual ($2 L_s$) frequencies. The scatter of the data about the four-frequency fit suggests the constraint applied to the atmospheric data through the a priori model is not controlling the atmospheric results since the estimated sigmas are approximately one tenth of the a priori constraint. The negative values for the atmosphere are a result of the constraint that the total mass of the planet remains constant. The variable component of the atmospheric mass is $\sim 18\%$ – 20% of the total mass and thus there is a large constant reservoir of atmosphere to be added to the results for the total atmospheric mass. This reservoir of mass is included in the mass of the planet as part of the central body.

[37] Figure 14 shows all the data plotted against one cycle of L_s with the Ames GCM values superimposed. The general agreement with the GCM is evident but in Figures 13 and 14 it is also apparent that the northern mass never appears to reach zero and the southern mass drops below the zero in the last few months of the year. It is not clear if these features are physically real and if the data are identifying a process of which we are unaware and not included in our modeling, or if it is a result of the process we have followed in estimating the volatile masses. But we note the inherent challenge in separating north and south polar signals. Figure 15 shows the residuals to the four-frequency fit to the poles and atmosphere, and Figure 16 shows a histogram of the residuals and indicates the south pole residual data set is biased low while the atmosphere is biased slightly high.

[38] In Figure 15 the three residual curves show little or no systematic large departure from noise but we can use

Figure 13. Estimated mass values of (top) the north seasonal cap, (middle) the south seasonal cap, and (bottom) the variable component of the atmosphere. The value of the a priori sigma for the atmosphere was chosen to be comparable to the recovered sigmas for the seasonal caps. The blue dashed line shows the prediction from the Ames GCM. Note that the accumulation of surface ice in the northern hemisphere occurs sooner than predicted.

these results to check for longer-term changes in the mass distribution by checking for any slopes in the residual patterns. In Figure 15 the slopes through the data (kg/degree of L_s) are

$$NP : +(1.4 \pm 0.5) \times 10^{11}; \quad (11)$$

$$SP : -(0.8 \pm 0.6) \times 10^{11}; \quad (12)$$

$$ATM : -(0.6 \pm 0.6) \times 10^{11}. \quad (13)$$

The slopes sum to zero because of the conservation of volatile mass but these slopes provide an estimate of the ability to detect long-term changes. For example, the NP slope is equivalent to an increase in mass of 2.8×10^{13} kg/Earth year or about 200 Earth years to form the equivalent of the seasonal cap, or 50,000 Earth years to form the equivalent of the north polar residual (water ice) cap [Zuber *et al.*, 1998; Smith *et al.*, 2001b]. Considering the uncertainties of the slopes it is not clear if there is a migration of material from the south pole to the north pole, or why that should be the case, but that possibility cannot be ruled out.

[39] Malin *et al.* [2001] suggested that observed changes in the CO_2 patterns around the south pole from year to year indicate a possible long-term loss of material to the atmosphere, or possibly elsewhere, including the north polar cap although there is no evidence for this. Malin *et al.*'s estimate of the decrease in mass at the south pole is 2×10^{13} kg/Earth year of CO_2 , a value comparable to our increase in mass at the north.

[40] Thus, for the estimate of Malin *et al.* [2001] it would take 30 to 150 Martian years to remove a 3-m-thick layer of CO_2 , of similar order to our estimate of 200 Earth years to create an equivalent mass at the north pole. Further, since there is no evidence to support a long-term increase in the CO_2 north polar seasonal cap, any mass increase must be in the residual water ice cap. Malin *et al.* [2001] suggest their observations would sublime a south pole residual cap of CO_2 in a few thousand Martian years, in general agreement with our 50,000 Earth years to form the north residual cap, considering the uncertainties in both estimates.

[41] In Figure 17 we show the differences between the seasonal masses of the poles and atmosphere, and the GCM.

Table 5. Four-Frequency Solutions for Masses of Seasonal Caps and Variation of Atmospheric Mass

Value	Amplitude $\times 10^{15}$ kg	Sigma $\times 10^{15}$ kg	Phase Degrees	Sigma Degrees
<i>North Pole</i>				
L_s	1.534	0.028	43.64	1.07
$2L_s$	0.486	0.029	47.88	3.34
$3L_s$	0.192	0.029	29.10	8.42
$4L_s$	-0.086	0.068	-8.5	44.27
<i>South Pole</i>				
L_s	3.058	0.032	223.84	0.61
$2L_s$	0.917	0.033	52.85	2.02
$3L_s$	-0.125	0.033	25.20	14.64
$4L_s$	0.018	0.033	-4.45	102.69
<i>Atmosphere</i>				
L_s	1.524	0.034	44.035	1.28
$2L_s$	-1.42	0.034	51.132	1.38
$3L_s$	-0.067	0.034	36.158	28.65
$4L_s$	0.0537	0.034	-40.26	35.86

Figure 14. Estimated masses shown by season and compared with the Ames GCM. Note that the GCM was the a priori for the atmosphere. The consistency of the values over 4 Mars years is evident from the dispersion of the individual values. There was no a priori for the polar masses.

This comparison highlights the major periodic differences between the observed and GCM values and it is evident in all three charts that it is the annual terms which differ most. From the distribution of residuals about the GCM it is evident that real interannual changes in the magnitudes of the seasonal depositions could easily be lost in the scatter of the data, but there is no evidence of any systematic trends with time.

5.3. Global Mean Atmospheric Pressure

[42] The atmospheric mass can be used to calculate the global mean atmospheric pressure by dividing the mass by the surface area of Mars. It is not the same as the observed pressure at an individual location but it does become a potential method for attempting to detect global changes over a long time period. Figure 18 shows the mean global pressure calculated for the altitudes of the Viking 1 and Viking 2 lander sites from both the atmospheric mass estimated here and also the Ames GCM, using the same

Figure 16. Histogram of residuals (see Figure 16) to the seasonal polar masses and atmosphere.

method, and for comparison the observed pressures at the landing sites by the landers [Hess *et al.*, 1979, 1980]. In order to make the calculation it is necessary to know the atmospheric pressure scale height at each of the lander sites below the areoid. We have estimated the scale height to be on average 9 km at V1 and 8 km at V2 for their altitudes of -3.6 km and -4.5 km, respectively. The difference in amplitudes of the annual and semiannual variations at the V1 site is evident between the observed pressure and the GCM but at the V2 site it is less obvious and the most noticeable difference is in phase. Full agreement between the global mean pressure and the observed pressure is not expected since local effects are included in the global estimates and the global values are equally valid at any other location at the same altitude.

5.4. Rotation Pole

[43] Part of the overall solution for the seasonal masses included an estimation of the direction of the spin pole from each of the 5-day orbital arcs. This adjustment was included in case our model of the rotation of Mars was incomplete (or in error) and because the seasonal deposition at the two poles has a direct effect on the precession and nutation of Mars, which therefore could be modified when the seasonal polar masses are estimated. Figures 19a and 19b show the recovered values for the right ascension (RA) and declination (DEC) and the IAU2000 a priori values. Figure 19c shows the formal error estimates of the RA and DEC, which are largest at identical times. It is estimated that the formal

Figure 15. Residuals to four-frequency fits to north and south polar masses and atmosphere. Although there are periods when all residuals depart from around zero, the overall pattern suggests random noise (see Figure 16).

Figure 17. Residuals between observations and Ames GCM.

errors of the weighted solutions are too large by factors of about 3.6 for the RA and 3.3 for the DEC.

[44] The largest departures from the a priori, particularly in declination, occur when the standard deviations of the recovered values are largest, which occur when the MGS orbit is edge-on as viewed from Earth. This geometry occurs twice every synodic period with Earth and at these times the node of MGS orbit and the inclination are poorly determined. In these configurations the longitude and inclination are almost insensitive to the observations of range and range rate. The RA and DEC results (Figures 19a and 19b) show clear linear trends and possible annual and semiannual variations. The results for the linear terms and the annual and semiannual periodic terms are listed in Table 6. The linear variations are observations of the precession of Mars pole which, when combined, lead to a precession rate of 7369 ± 53 mas/a. Table 6 also shows the estimates for the precession by *Smith and Zuber* [2008] and *Konopliv et al.* [2006]. Our value is about 2.5% less than the value obtained from Viking and Pathfinder lander data and from a combination of MGS and Odyssey orbit data [*Konopliv et al.*, 2006], the latter being based upon a 30-year data set compared to the 7.5 years of orbiter-only data used in this study. Our model of precession and nutation based upon the values in Table 6 is shown in Figure 20. The annual and semiannual periodic terms show a full range variation of order 35 m about the ~ 300 -m linear precession.

5.5. Other Parameters

[45] In addition to the parameters already discussed, the solution included estimates for the product of the gravitational constant and the masses of Mars and Phobos, and the second-degree zonal Love number, k_2 . These results are shown in Table 7 in comparison to those of *Konopliv et al.* [2006] and *Lemoine et al.* [2001]. Estimates of these parameters were not the goal of this study but their

simultaneous estimation with other parameters of interest is believed to have improved the overall quality of the solution by removing extraneous signals. We note that k_2 is larger than previous estimates, suggesting the deep interior

Figure 18. Mean global pressure variation at the (a) Viking 1 and (b) Viking 2 landing sites derived from the atmospheric mass from gravity observations and the GCM. Also shown are the measured pressures at each site by the lander.

Figure 19. (top) Right ascension and (middle) declination of the pole of rotation of Mars; estimated in red and a priori in blue dash. (bottom) The estimated standard deviations.

of Mars is less dissipative than indicated by previous studies [cf. Yoder *et al.*, 2003].

6. Interpretation of Results

6.1. Spherical Harmonic Solutions

[46] We have made preliminary estimates of the variations in the zonal terms (degrees 2–5) of the Martian gravity field due to the seasonal exchange of CO_2 between the planet’s atmosphere and surface. The planetary flattening term, $C_{2,0}$, is found to be very dependent on whether it is adjusted simultaneously with other zonal terms, and it was very difficult to obtain a reliable value. We only obtained values for $C_{2,0}$ within a plausible range when adjusted in combination with $C_{3,0}$ or $C_{4,0}$. However, the

two patterns of variation for $C_{2,0}$ with L_s were significantly different. Estimates of $C_{3,0}$ and $C_{5,0}$, when obtained individually, are indistinguishable, indicating that these two coefficients represent the same gravity signal and are the accumulated effects of many odd-degree zonal coefficients. We note that a solution for $C_{7,0}$ does not represent the same perturbation as $C_{3,0}$ and $C_{5,0}$. Our results also indicate that $C_{3,0}$ is unchanged when adjusted simultaneously with $C_{2,0}$, which also results in a more reasonable estimation for $C_{2,0}$ when compared with a prediction from a GCM. The $C_{4,0}$ term, whether adjusted singly or in combination with $C_{2,0}$, is statistically unchanged but $C_{2,0}$ changes significantly, and to a more realistic value when adjusted with $C_{4,0}$. We generally conclude that isolation of the perturbation of any single low-degree coefficient is extremely difficult from a single spacecraft orbit or from several spacecraft orbits with similar orbital parameters. Thus, the geophysical interpretation of any individual coefficient (as distinct from a full gravity model) is inherently unreliable.

[47] We attempted to use our simultaneous estimates of $C_{3,0}$ and $C_{2,0}$ with the point mass model for the ice caps and derived seasonal variations which had similarities to the GCM but with significantly different amplitudes. In addition to the poor recoverability of $C_{2,0}$, the simplicity of the point mass model was probably a significant contributor to the disagreement with the GCM or with our subsequent results obtained by directly estimating the masses.

6.2. Direct Estimation of Seasonal Mass

[48] We made direct estimates of the seasonal masses of the ice caps at ~ 5 -day intervals based upon an ice cap model derived from TES bolometer data [Kieffer *et al.*, 2000; Kieffer and Titus, 2001] for the size of the cap and MOLA altimetry results [Smith *et al.*, 2001a] for the thickness of the seasonal cap. The variation in the Martian atmospheric mass (Figure 12) was obtained from an atmospheric GCM [Haberle *et al.*, 2008]. This variation was used as a priori in our computations but only weakly constrained in the least squares adjustments of the orbits and masses in order to permit departures from the GCM should our data require it. The total CO_2 mass of the seasonal caps and the atmosphere was conserved. The model was driven by a weakly constrained atmospheric GCM model [Haberle *et al.*, 2002] and the results for the total CO_2 mass of the poles and the atmosphere constrained to conserve mass. This model produced seasonal variations in ice cap masses similar to those of the GCM, but with some systematic differences. Results indicated that CO_2 mass was deposited more slowly in the northern hemisphere but sublimation was faster and similar to the GCM.

Table 6. Results for Motion of Spin Axis

Parameter	This Paper	Folkner <i>et al.</i> [1997b]	Konopliv <i>et al.</i> [2006]
$d(\text{RA})/dt$ (10^{-3} deg/year)	-1.025 ± 0.004	-1.061 ± 0.007	
$d(\text{DEC})/dt$ (10^{-3} deg/year)	-0.614 ± 0.007	-0.609 ± 0.004	
RA annual amp (10^{-5} deg); phase (deg)	-8.7 ± 0.8 ; 38.2 ± 4.8	14 ± 1.6	11.1 ± 0.9
RA semi-annual amp; phase (deg)	10.6 ± 0.8 ; -61.6 ± 4.0	-3.0 ± 1.5	-3.1 ± 0.9
DEC annual amp (10^{-5} deg); phase (deg)	-9.4 ± 1.1 ; 274.8 ± 7.2	-4.7 ± 2.3	-6.2 ± 1.1
DEC semi-annual amp (10^{-5} deg); phase (deg)	-17.0 ± 1.0 ; 65.4 ± 3.3	-2.2 ± 1.6	-3.6 ± 0.9
Precession rate (mas/year)	-7369 ± 53	-7576 ± 35	-7568 ± 21

Figure 20. Precession model derived from MGS spacecraft with linear, annual, and semiannual components. The equivalent surface motion is shown in meters. The oscillation amounts to a full range of approximately 35 m.

Deposition to form the seasonal southern cap was found to be much more symmetric, similar to the GCM, except that the accumulation appears to start as soon as the sublimation was complete and, in our model, the mass went slightly negative in the summer although the total mass increase was in good agreement with the GCM. We do not put any significance on the slightly negative mass values but consider the trends in the accumulation of mass to be more important, indicating the possibility of other mass accumulation processes and distribution being involved. The repeatability of the seasonal patterns is clear in Figure 13; even the slightly negative overshoot of sublimation is repeated during three of the four southern hemisphere summers, and also the early rise in the northern hemisphere summer is repeated in each year. The consistency of the patterns, if not actually due to early accumulation, is likely to be the result of an inadequate seasonal polar cap model during at least the early northern and southern summers. We note that during the summer seasons our model does not force the mass to be zero by making the area and precipitation zero. However, there is nothing in the cap model that forces the masses to be nonzero.

[49] The results for the atmosphere follow closely the a priori of the GCM in Figures 13 and 14; even though they are only weakly constrained at 50% of the seasonal variation in atmospheric mass. The lack of departure from the a priori is believed to be an indication that the GCM is a good representation of the seasonal variation within the atmosphere and that the gravity data are in agreement. The error estimates shown in Figure 13 suggest that the atmosphere is not overly constrained and the quality of the atmospheric results are comparable to those for the polar masses. The major difference between a priori atmosphere and the estimated atmosphere is shown in Figure 17 to be largely in the annual component. We believe this difference is real but cannot rule out the possibility that it arises as a result of our approach or from systematics in our data or models.

[50] We used the atmospheric mass results to derive the global mean atmospheric pressure on the areoid, a measure that is probably only obtainable from gravity measurements, yet an important parameter that describes the Mars atmosphere, and inferred the pressure variation at the altitudes of the Viking 1 and 2 landing sites (Figure 18). These were compared to the GCM and to the observed pressures made by the two landers. In general, the mean pressure provided a prediction of the actual pressure at the landing site to about 10% to 15%.

[51] We also looked for the presence of any long-term systematic changes in masses of the ice caps and the atmosphere by attempting to identify slopes and amplitude changes in the data sets. The observed slopes represent the sublimation of the equivalent mass of the south polar cap in about 10^5 Earth years and the north polar cap in about 50,000 Earth years.

[52] The solutions for the direction of the rotation pole provided estimates for the precession in right ascension and declination of the pole as a function of time (Figure 19), and for the amplitudes of the annual and semiannual nutations (Table 6). The declination of the pole position was particularly sensitive to the Earth-Mars-MGS orbit geometry but weighted solutions let to an estimate of the precession rate of $(5.608 \pm 0.040) \times 10^{-6}$ degrees day $^{-1}$, which can be compared with the [Konopliv *et al.*, 2006] value of $(5.756 \pm 0.017) \times 10^{-6}$ degrees day $^{-1}$.

[53] The annual and semiannual amplitudes of the nutations given in Table 6 are equivalent to 5- to 10-m amplitudes on the planet's surface with standard deviations of about 1 m for both the RA and DEC for both annual and semiannual periods. A reconstruction of the motion of the rotation pole from Table 6 is shown in Figure 20.

7. Conclusions

[54] Part of our intention in this study was to develop an approach to monitor the seasonal mass accumulation at the poles and the average atmospheric pressure of the Martian atmosphere from the tracking of Mars orbiting spacecraft. We anticipate spacecraft tracking to be available at Mars more generally than instruments able to measure seasonal CO $_2$ mass directly on the Martian poles or in the Martian atmosphere. An extended time series of measurements of the seasonal mass cycle could lead to the detection of interannual variations or even longer-term climate change, as suggested by the linear trends in the masses obtained in this paper. These observations can be used, along with other data types, to "tune" general circulation models, increasing their value as exploratory tools to simulate the present and past atmosphere of Mars.

[55] Of our two approaches, studying the low-degree gravity and the direct estimation of the mass exchange,

Table 7. Results for GM and Tides and Comparison With Other Recent Estimates

Parameter	This Study	Konopliv <i>et al.</i> [2006]
Mars GM ($\times 10^5$)	42828.3727 ± 0.0004	42828.37440 ± 0.00028
Phobos GM ($\times 10^5$)	7.06 ± 0.18	7.162 ± 0.005
Mars k_2	0.236 ± 0.058	0.152 ± 0.009

the latter was clearly superior and led to a multiyear pattern of measurements that could be used to infer the level of interannual variation in CO₂ mass distribution. We believe that it is highly likely that a better model of the seasonal polar caps that does not assume longitudinal symmetry and has greater latitudinal resolution would produce a more accurate recovery of the seasonal mass exchange. Although a repetitive seasonal model of the polar caps may be overly simplistic, any significant change in magnitude or phasing is likely to be revealed in the gravity signals seen in spacecraft tracking. In future analyses, increasing the temporal baseline and adding observations from other spacecraft will greatly improve the determinations.

[56] However, even with the current limited data set, we have demonstrated that the detection of small changes in the Martian gravity field is clearly possible, and can be related to fundamental atmospheric and planetary dynamical phenomena. The results underscore the possibility of using tracking of an orbital spacecraft to monitor the annual and interannual variability of the global-scale cycling of CO₂ on Mars.

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