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## Challenges in Merger Simulation Analysis

By CHRISTOPHER R. KNITTEL AND KONSTANTINOS METAXOGLOU\*

We discuss the challenges we have experienced simulating merger effects assuming a Random-Coefficient (RC) Logit on the demand side and a static Bertrand game on the supply side. In particular, we focus on the challenges in the estimation of a RC-Logit demand model, as documented in Knittel and Metaxoglou (2008). Different starting values and nonlinear search algorithms yield different demand estimates. These different demand estimates lead to economically significant variation in postmerger market outcomes.

As Aviv Nevo (2001) illustrates, a demand system combined with an oligopoly model of competition and an appropriate set of assumptions suffice to simulate merger effects. Abstracting from technical details, the model of competition offers the first-order conditions (FOCs) that equate marginal revenue to marginal costs. The demand system identifies the marginal revenue, yielding premerger marginal costs through the FOCs. The existence and uniqueness of an equilibrium are usually assumed (e.g., Steven Berry, James Levinsohn, and Ariel Pakes 1995, henceforth BLP).<sup>1</sup>

Assuming that a smaller number of competitors is the only difference between the pre- and postmerger industry structures, which can be easily handled with an adjustment to the ownership matrix entering the same FOCs, postmerger prices are solutions to a system of nonlinear equations.

We quantify the variation in postmerger market outcomes from different demand estimates due to alternative starting value/algorithm

combinations. We use the cereal data from Nevo (2000) and automobile data from Berry et al. (1995). The hypothetical merger in the cereal industry involves Kellogg's and General Mills. The hypothetical merger in the automobile industry involves GM and Chrysler.

In the case of cereals, the difference between post- and premerger prices is between  $-0.05$  cents per serving and about 19 cents. Postmerger industry profits are between \$75 million and \$5.8 billion. The change in consumer welfare also exhibits notable variation:  $-\$182$  million to \$953 million. In the case of automobiles, the average change in price is between \$28 and \$187. Profits and consumer welfare also vary widely: \$5 billion to \$35 billion, and \$219 million to \$1.5 billion, respectively, even when we exclude an outlier algorithm.

Our work should be of immediate interest to antitrust agencies that routinely review a large number of mergers under very tight time constraints and frequently employ simulation techniques to assess their potentially anticompetitive effects.<sup>2</sup>

In the next section, we discuss some details regarding the challenges in the estimation of a typical RC-Logit model. In Section II, we outline computational issues associated with the calculation of the postmerger prices in the Bertrand game. We present our findings in Section III. Conclusions are presented in Section IV.

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<sup>1</sup> For some recent theoretical work regarding the existence and uniqueness of equilibrium in pricing games among single-product firms facing RC-Logit demand curves, see Gad Allon, Awi Federgruen, and Margaret Pierson (2010).

<sup>2</sup> Since the introduction of the Hart-Scott-Rodino (HSR) Antitrust Improvements Act of 1976, all US mergers valued at more than a threshold are required to file with the Federal Trade Commission (FTC) and the Antitrust Division of the Department of Justice (DOJ). Between 1991 and 2009, approximately 45,000 mergers were filed with the two agencies. After various adjustments throughout the years, the filing threshold was set at \$65.2 million in fiscal year 2009; see Federal Trade Commission and DOJ Antitrust Division (2009). Following an HSR filing, the agencies have 30 days to conduct their preliminary review and decide whether to issue a "second request" for a more thorough investigation of a potentially anticompetitive merger.

## I. The Demand Model

The first step of the merger simulation exercise is the estimation of the underlying demand model. We refer the reader to Berry et al. (1995) and Nevo (2000) for the details of the standard BLP-type model, as well as estimation strategies. Berry et al. (1995) establish the consistency and asymptotic normality of the estimates and provide the associated variance-covariance matrix.

Following the publication of computer code by Nevo (2000), the estimation of BLP-type models has become increasingly popular. More recently, studies, including some of our work, have identified issues with the computational aspects of this class of demand models that have been dominating the industrial organization field for the last ten years or so.

Before offering a brief overview of these studies, we should keep in mind that the objective function of a typical BLP model has not been shown to be globally concave (e.g., Patrick Bajari, Jeremy T. Fox, and Stephen P. Ryan 2007). A nonlinear search in the parameter space usually requires hundreds or even thousands of function evaluations, with each of them involving a call of the contraction mapping introduced in Berry et al. (1994).

Knittel and Metaxoglou (2008) illustrate that the underlying GMM problem is a nontrivial one based on data from Nevo (2000) and BLP. Using more than 10 optimization algorithms from different classes (derivative-based, direct-search, random-search) and 50 starting values, they show that economic variables of interest, such as elasticities and consumer welfare, vary widely depending on the choice of optimization routine and starting value.

Jean-Pierre Dube, Fox, and Che-Lin Su (2009) show that the temptation to implement loose stopping criteria for the contraction mapping to speed up the estimation process may cause two types of errors in parameter estimates. First, the approximation error of the inner contraction mapping propagates into the outer GMM objective function and its derivatives. Second, even when an optimization run converges, it may falsely stop at a point that is not a local minimum. The authors offer an alternative formulation of the GMM problem as a mathematical program with equilibrium constraints (MPEC), building on work of Su and Kenneth L. Judd (2008).

## II. The Bertrand Game

This section describes the second step of the merger simulation exercise assuming a Bertrand oligopoly model. Having retrieved marginal costs using the demand estimates and the FOCs of a Bertrand game, the researcher solves for the postmerger prices via simulation assuming existence and uniqueness of an equilibrium.

With demand estimates in hand, marginal costs are inferred using the FOCs of a static Bertrand model with multiproduct firms:

$$(1) \quad p = mc + \Omega(p)^{-1} s(p),$$

where  $p$  is the price vector,  $s(\cdot)$  is the vector of market shares, and  $mc$  denotes the corresponding marginal costs. The dimension of these vectors is equal to the number of the products available in the market, say  $J$ . The  $\Omega$  matrix involves the share price-derivative matrix and an ownership structure matrix. The ownership structure matrix is of dimension  $J \times J$ , with its  $(i, j)$  element equal to one if products  $i$  and  $j$  are produced by the same firm, and zero otherwise. Because prices are observed and demand estimation allows us to retrieve the elements of  $\Omega$ , estimates of marginal costs,  $\widehat{mc}$ , are directly obtained using (1).

A simple change of ones and zeros in the ownership structure matrix, along with a series of additional assumptions (Nevo 2001), allows the simulation of a change in the industry's structure, as the one implied by mergers among competitors. Simply put, a merger simulation implies the same Bertrand equilibrium with a smaller number of firms. The vector of postmerger prices  $p^{post}$  is the solution to the following system of nonlinear equations:

$$(2) \quad p^{post} = \widehat{mc} + \hat{\Omega} (p^{post})^{-1} \hat{s}(p^{post}).$$

The elements of  $\hat{\Omega} (p^{post})$  reflect changes in the ownership structure implied by the hypothetical merger. Solving for the postmerger prices is equivalent to solving a system of nonlinear equations of dimension  $J$  in the market under consideration. For example, using the cereal data, we have 94 markets with 24 products in each market. As a result, solving (2) requires the solution of 94 systems of nonlinear equations of dimension 24.

TABLE 1—SIMULATION RESULTS: CEREALS

Algorithm	Profit	CV	Price
QNEWTON 1	3.768	−97	0.06
QNEWTON 2	3.538	953	8.38
CONJGRAD	3.798	−122	0.03
SOLVOPT	3.539	953	8.38
KNITRO 1	3.837	−63	0.12
KNITRO 2	3.837	−62	0.12
KNITRO 3	3.539	953	8.38
SIMPLEX	3.769	−87	0.08
MADS	4.024	−129	0.09
GPS	4.070	−182	−0.05
SIMANNEAL 1	5.775	817	18.96
SIMANNEAL 2	2.862	−25	7.22
GENETALG	0.075	44	1.12

TABLE 2—SIMULATION RESULTS: AUTOMOBILES

Algorithm	Profit	CV	Price
QNEWTON 1	27	711	87
QNEWTON 2	27	1,235	151
CONJGRAD	27	712	87
SOLVOPT	29	1,550	187
KNITRO 1	35	1,141	140
KNITRO 2	35	1,134	139
KNITRO 3	35	1,151	142
SIMPLEX	27	716	87
MADS	27	728	89
GPS	27	1394	172
SIMANNEAL 1	23	1,197	146
SIMANNEAL 2	5	219	28
GENETALG	187	382	45

In the results discussed below, we solve the system of nonlinear equations in (2) using a dogleg trust-region (DTR) version of Newton's method.<sup>3</sup> We implement the DTR method via the MATLAB *fsolve* function using the premerger prices as starting values.<sup>4</sup>

### III. Results

The results of our merger simulations for the cereal and the automobile industries appear in Tables 1 and 2, respectively. In both tables we report postmerger industry profits ("Profit"), total compensating variation ("CV"), and the average change in price ("Price") for 13 different algorithms we used to estimate the demand models in Knittel and Metaxoglou (2008).<sup>5</sup>

We used a set of 50 starting values for each of these 13 algorithms and solved for the postmerger prices using the set of starting values that gave rise to the minimum GMM objective function value. The demand specification for automobiles is similar, but not identical, to Berry

et al. (1995).<sup>6</sup> Our demand model for cereals is identical to Nevo (2000).

In the case of the cereal data, we report results for the market with the smallest share of the outside good.<sup>7</sup> For the automobile data, we report results for year 1990, which corresponds to the market with the largest potential size—approximately 94 million households. The postmerger industry profits are expressed in billions of dollars. The total compensating variation is measured in millions of dollars. The price changes are measured in cents per serving (cereals) and dollars (automobiles).

Depending on the algorithm used, in the case of cereals, the postmerger profits range between \$75 million and \$5.8 billion. The average change in price is between −0.05 cents and almost 19 cents per serving. The change in consumer welfare (CV) is as low as −\$182 million and as high as \$953 million.

Moving to automobiles, the postmerger profit range is between \$5 billion and \$35 billion when we exclude the outlier of \$187 billion associated with the Genetic Algorithm (GENETALG). The change in consumer welfare (CV) is between \$219 million and \$1.5 billion. Finally, the average change in price is between \$28 and \$187.

<sup>3</sup> Jorge Nocedal and Stephen Wright (1999) provide an excellent discussion of the method in their chapter 11.

<sup>4</sup> The termination tolerances for both the merit-function value and the vector of prices we are solving for are set equal to 1E-16. We also impose a maximum number of 1,000 iterations. Additionally, the Jacobian is approximated using finite differences. The remaining of the *fsolve* settings are equal to their default values.

<sup>5</sup> The total compensating variation (CV) in each market is equal to the product of the average compensating variation (ACV) with the potential market size. We calculate ACV averaging individual compensating variations calculated using equation (6) in Nevo (2001). The price changes are weighted by premerger market shares.

<sup>6</sup> For example, we don't allow for nonlinear income effects, and we use slightly different product characteristics. See Knittel and Metaxoglou (2008) for the exact specification.

<sup>7</sup> We assume a market size of 250 million as if the single market analyzed is representative of the nation. Although this assumption is somewhat arbitrary, it does not affect the variation in results discussed here.

#### IV. Conclusion

In this paper, we share our experience with merger simulations using a RC-Logit model on the demand side and assuming a static Bertrand game on the supply side. Drawing largely from our work in Knittel and Metaxoglou (2008), we show that different demand estimates obtained from different combinations of optimization algorithms and starting values lead to substantial differences in postmerger market outcomes using metrics such as industry profits and change in consumer welfare and prices.

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