FINITE DIFFERENCE MODELLING OF ACOUSTIC LOGS IN VERTICALLY HETEROGENEOUS BIOT SOLIDS

by

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ABSTRACT

This paper discusses the results of tests carried out on a finite difference formulation of Biot's equations for wave propagation in saturated porous media which vary in range and depth (Stephen, 1987). A technique for modeling acoustic logs in two-dimensionally varying Biot solids will give insight into the behavior of tube waves at permeable fractures and fissures which intersect the borehole. The code agrees well with other finite difference codes and the discrete wavenumber code for small porosity in the elastic limit of Biot's equations. For large porosity (greater than one per cent) in the elastic limit or for the acoustic limit, good agreement is not obtained with the discrete wavenumber method for vertically homogeneous media. The agreement is worst for amplitudes of the pseudo-Rayleigh wave. The amplitude of the Stoneley wave and the phase velocities of both waves could be acceptable for some applications. An example is shown of propagation across a horizontal high porosity stringer in a Berea sandstone. Reflections from the stringer are observed but given the inaccuracies of the pseudo-Rayleigh waves for vertically heterogeneous media the amplitudes for the stringer model are questionable. We propose a three stage approach for further work: 1) Use the Virieux scheme instead of the Bhasavanija scheme for the finite difference template. The Virieux scheme has been shown in other studies to be more accurate for liquid-solid interfaces. 2) Run the present code for lower frequency sources to emphasize Stoneley waves and diminish pseudo-Rayleigh waves. Stoneley waves are most sensitive to permeability variations which are the primary objective of Biot wave

studies. 3) Develop a finite difference code for Biot media with the fluid-solid boundary conditions specifically coded. This code would be suitable for studying constant radius boreholes in vertically varying Biot media.

INTRODUCTION

The objective of this study is to investigate the effects of depth dependent porous media on full waveform acoustic logs. How do tube waves behave as they pass horizontal permeable stringers? What is the coupling mechanism between body waves and tube waves at borehole discontinuities? What are the effects of depth dependent porosity and permeability?

Stephen (1987) presented a formulation to solve Biot's equations by the method of finite differences. This formulation has been implemented on the VAX 8800 at the M.I.T. Earth Resources Laboratory. In this paper we discuss tests that have been carried out to check the validity of the code and we show an example of a synthetic acoustic log in a depth dependent Biot medium.

The first step in validating the code is to demonstrate that it gives correct results in the elastic and acoustic limits of Biot theory (Biot, 1956a,b; 1962). If porosity is set to zero in Biot's equations, the equations reduce to the elastic wave equations, with independent variables of Lamé parameters (λ and μ) and density (ρ) corresponding to the solid matrix. We call this the elastic limit. If on the other hand porosity is set to 100 per cent, the Biot equations reduce to the acoustic wave equation with compressibility (k) and density (ρ) corresponding to the values for the pore fluid. We call this the acoustic limit. In either case the other Biot parameters drop out of the solution.

In our model we represent the borehole as a simple homogeneous fluid governed by an acoustic wave equation (Figure 1). This region is then merged with a transition region based on Biot's equations which allow both radial and vertical variability. At the boundary between the homogeneous fluid and Biot media sections we define Biot parameters corresponding to the fluid. Waves propagate across this numerical boundary undisturbed. Then at least two grid points into the Biot region we change the Biot parameters to represent a Biot solid. The physical effect of this boundary is computed in the finite difference formulation for heterogeneous Biot media. Specific boundary conditions are not introduced. In this way general interfaces, such as washouts or bed boundaries, can be incorporated without changing the code.

In the elastic limit (porosity of 0%) the fluid part of the Biot region (left most grid points) is obtained by setting the shear modulus (μ) to zero and choosing λ and ρ to correspond to the borehole fluid. Then at the borehole wall λ, μ and ρ are changed to correspond to elastic rock. In this limit the Biot code results can be checked against finite difference and discrete wavenumber results for vertically homogeneous elastic media.

In the acoustic limit (porosity of 100%) we choose the pore fluid parameters to be the same as the borehole fluid parameters. The resultant model is just a homogeneous fluid for which analytical results are well known.

When a finite meaningful porosity is introduced to the Biot region we postulated that the elastic limit code would correspond to a sealed boundary at the borehole wall which does not allow flow between the borehole and the permeable formation. We also postulated that the acoustic limit code would correspond to the permeable boundary case which does allow flow between the borehole and the permeable formation. Since the boundary conditions are not specifically coded in this finite difference method it is not clear in advance which if either boundary is represented in the acoustic and elastic limits. As discussed below, comparison with discrete wavenumber results shows that the elastic limit does correspond to an impermeable boundary for Stoneley waves but not for pseudo-Rayleigh waves. It is not clear what the acoustic limit code corresponds to for a sharp interface.

THE ELASTIC LIMIT OF BIOT'S EQUATIONS

Stephen (1987) reviewed Biot theory for heterogeneous media based on the presentation of Schmitt (1986) and presented the finite difference formulation. The wave equations for a heterogeneous, isotropic Biot solid are:

$$(A+N)\nabla(\nabla\cdot\vec{u}) + N\nabla^2\vec{u} + Q\nabla(\nabla\cdot\vec{U})$$

$$+\ \nabla A(\nabla\cdot\vec{u}) + \nabla N \times (\nabla\times\vec{u}) + 2(\nabla N\cdot\nabla)\vec{u} + \nabla Q(\nabla\cdot\vec{U})$$

$$= \rho_{11}\ddot{\vec{u}} + \rho_{12}\ddot{\vec{U}} + b(\dot{\vec{u}} - \dot{\vec{U}}) \tag{1}$$

$$Q\nabla(\nabla\cdot\vec{u}) + \tilde{R}\nabla(\nabla\cdot\vec{U}) + \nabla Q(\nabla\cdot\vec{u}) + \nabla\tilde{R}(\nabla\cdot\vec{U})$$

$$= \rho_{12}\ddot{\vec{u}} + \rho_{22}\ddot{\vec{U}} - b(\dot{\vec{u}} - \dot{\vec{U}})$$

where $\vec{u}$ are the displacements of the solid matrix and $\vec{U}$ are the displacements of the pore fluid.

The coefficients A, N, Q, and $\tilde{R}$ can be expressed in terms of the bulk moduli of the solid matrix (K_s), the skeleton or frame (i.e., the dry porous solid) (K_b) and the pore fluid (K_f), the shear modulus of the skeleton (μ_b) and the porosity, ($\tilde{\phi}$):

$$
\begin{aligned}
A &= \frac{(1-\tilde{\phi})\left(1-\tilde{\phi}-\frac{K_b}{K_s}\right) K_s + \tilde{\phi}\frac{K_s}{K_f}K_b}{1-\tilde{\phi}-\frac{K_b}{K_s}+\tilde{\phi}\frac{K_s}{K_f}} - \frac{2}{3}\mu_b \\
Q &= \frac{\left(1-\tilde{\phi}-\frac{K_b}{K_s}\right)\tilde{\phi}K_s}{1-\tilde{\phi}-\frac{K_b}{K_s}+\tilde{\phi}\frac{K_s}{K_f}} \\
\tilde{R} &= \frac{\tilde{\phi}^2 K_s}{1-\tilde{\phi}-\frac{K_b}{K_s}+\tilde{\phi}\frac{K_s}{K_f}} \\
N &= \mu_b .
\end{aligned}
\tag{2}
$$

Further it is convenient to express the coefficients K_b, N and K_f in terms of the compressional and shear velocities of the dry but still porous rock, α_m, β_m, the density of the matrix, ρ_s, and the pore fluid velocity and density, α_f and ρ_f:

$$
\begin{aligned}
K_b &= (1-\tilde{\phi})\rho_s(\alpha_m^2 - \tfrac{4}{3}\beta_m^2) \\
N &= (1-\tilde{\phi})\rho_s\beta_m^2 \\
K_f &= \rho_f\alpha_f^2 .
\end{aligned}
\tag{3}
$$

We assume that b, ρ_{11}, ρ_{12}, and ρ_{22} are frequency independent which is acceptable over the narrow band of frequencies used to represent the source pulse (see Appendix E of Stephen et al., 1985). The viscous coupling coefficient at low frequencies is:

$$
b = \frac{\eta\tilde{\phi}^2}{\tilde{k}}
\tag{4}
$$

where η is the dynamic viscosity of the fluid and $\tilde{k}$ is the intrinsic permeability. The mass coupling coefficient is :

$$
\rho_{22} = \frac{4}{3}\tilde{\phi}\rho_f
\tag{5}
$$

and the other coupling coefficients are:

$$
\begin{aligned}
\rho_{12} &= \rho_2 - \rho_{22} \\
\rho_{11} &= \rho_1 - \rho_{12} ,
\end{aligned}
\tag{6}
$$

where ρ_1 and ρ_2 are the liquid and solid phase densities per unit volume:

$$\begin{aligned} \rho_1 &= (1-\tilde{\phi})\rho_s \\ \rho_2 &= \tilde{\phi}\rho_f \,. \end{aligned} \tag{7}$$

If the medium is homogeneous the wave equations for heterogeneous media reduce to equations (47) in Schmitt (1986).

In the elastic limit, porosity goes to zero and the coefficients in Biot's equations (2) become:

$$\begin{aligned} A &= K_b - \frac{2}{3}\mu_b \; = \lambda_b \\ Q &= 0 \\ \tilde{R} &= 0 \\ N &= \mu_b \end{aligned} \tag{8}$$

where K_s equals K_b if porosity is zero and λ_b and μ_b are Lamé constants for the solid. Furthermore,

$$\rho_{12} = \rho_{22} = \rho_2 = b = 0 \,. \tag{9}$$

The second of Biot's equations (1) is redundant and the first becomes the elastic wave equation for heterogeneous media:

$$\rho_s \ddot{\vec{u}} = (\lambda+\mu)\nabla(\nabla\cdot\vec{u}) + \mu\nabla^2\vec{u} + \nabla\lambda(\nabla\cdot\vec{u}) + \nabla\mu\times(\nabla\times\vec{u}) + 2(\nabla\mu\cdot\nabla)\vec{u} \,. \tag{10}$$

If the shear modulus, μ, vanishes then this reduces to the acoustic wave equation in terms of displacements. Equations (1) can be used to compute wave propagation in the fluid filled borehole by setting porosity and shear modulus to zero. Also by setting porosity to zero and keeping the shear modulus finite the same equations can be used for an elastic formation.

If we use this elastic limit (with shear modulus equal to zero) for the borehole fluid then at the borehole wall the fluid motion will couple to the matrix rather than the pore fluid. We postulated that this would correspond to an impermeable boundary case which could be caused by mud cake or casing.

THE ACOUSTIC LIMIT OF BIOT'S EQUATIONS

In the acoustic limit, porosity goes to unity and the coefficients in Biot's equations (2) become:

$$\begin{aligned} A &= 0 \\ Q &= 0 \\ \tilde{R} &= K_f = \lambda_f \\ N &= 0\,. \end{aligned} \tag{11}$$

Now the densities and the viscous coupling coefficient become:

$$\begin{aligned} \rho_1 &= 0 \\ \rho_2 &= \rho_f \\ \rho_{11} &= \tfrac{1}{3}\rho_f \\ \rho_{12} &= -\tfrac{1}{3}\rho_f \\ \rho_{22} &= \tfrac{4}{3}\rho_f \\ b &= \tfrac{\eta}{k}\,. \end{aligned} \tag{12}$$

In this case the first of Biot's equations (1) is redundant and the second becomes the acoustic wave equation for heterogeneous media:

$$\rho_f \ddot{\vec{U}} = \lambda_f \nabla(\nabla \cdot \vec{U}) + \nabla\lambda_f(\nabla \cdot \vec{U})\,. \tag{13}$$

This gives a second independent way to compute wave propagation in a fluid filled borehole from Biot's equations (1): set porosity to one hundred per cent.

If we use this acoustic limit for the borehole fluid then at the borehole wall the fluid motion will couple with the pore fluid rather than the matrix. We postulated that this would correspond to a permeable boundary case (i.e., open pores).

TESTS OF THE BIOT CODE IN HOMOGENEOUS FLUIDS

In both the elastic and the acoustic limits the finite difference Biot codes should give results corresponding to a point source in a homogeneous fluid. This is a physically trivial example but is non-trivial numerically and is a good zeroth order test of the code. The microseismograms for both cases are shown in Figure 2. It is reassuring that the codes are stable and accurate for this model. The results also confirm that the source strength in the two cases is equivalent. So the amplitude differences shown below between acoustic and elastic limits are due to either propagation in the formation or coupling to the formation.

TESTS OF THE BIOT CODE IN THE ELASTIC LIMIT

We show two sets of results for the finite difference Biot code in the elastic limit. In the first set all microseismograms are computed using the finite difference Biot code and models correspond to media with 0, 0.1, 1.0 and 19 % porosity (Figure 3). (Table 1 shows the model parameters for all models in the paper.) The microseismograms for very small porosity are similar to the ones for zero porosity and the microseismogram features change slowly with slow changes in porosity. This supports the notion that the code is working correctly in this limit.

In the second set of microseismograms, results for the elastic and acoustic limit finite difference Biot codes with small (0.1%) porosity are compared to results from the discrete wavenumber method, the Stephen finite difference formulation (with a specific boundary condition, Stephen et al., 1985) and the Bhasavanija finite difference formulation (without a specific boundary condition, Bhasavanija, 1983). (The Stephen finite difference code is documented in Hunt and Stephen, 1987. The Bhasavanija formulation is based on developments by Nicoletis, 1981.) The latter three solutions were discussed by Stephen et al. (1983; 1985) and Stephen and Cheng (1990). Good agreement is obtained between the discrete wavenumber, Stephen, Bhasavanija, and the elastic Biot results. The acoustic Biot result is quite a bit smaller in amplitude, and also does not show the P wave arrival clearly. (Figure 4).

A more quantitative comparison can be made by applying Prony's method and looking at amplitude and phase velocity curves versus frequency for individual phases (Figure 5). The Biot solution for the elastic limit is very similar to the Bhasavanija result which differs slightly from the Stephen elastic finite difference solution. A consistent feature of the Bhasavanija and Biot elastic limit result is that the pseudo-Rayleigh wave amplitudes are overestimated. In the time series this is observed in a larger amplitude Airy phase (the tail end) of the pseudo-Rayleigh wave packet. In the amplitude plots (Figure 5) the pseudo-Rayleigh wave is up to 16 per cent too large. The Stoneley wave amplitudes are underestimated by up to 20%. The pseudo-Rayleigh wave velocities are quite good but the Stoneley wave velocities show considerable scatter, probably due to the low amplitude of the Stoneley wave. For the Biot finite difference in the acoustic limit, the psuedo-Rayleigh wave amplitudes and velocities are seriously underestimated and the Stoneley waves are non-existent. It appears for now that the acoustic limit results are unphysical. It is not clear to us what this means.

TESTS OF THE BIOT CODE FOR PERMEABLE MEDIA

In this suite of examples we show results for an acoustic log in Berea sandstone with 19% porosity. Microseismograms are shown for the elastic limit finite difference case, for the acoustic limit finite difference case, and for the discrete wavenumber method for permeable amd impermeable boundaries (Figure 6). The time series for the elastic limit Biot code for 19% sandstone show a large amplitude Airy phase of the pseudo-Rayleigh wave packet. The acoustic limit Biot code for the same model has overall absolute signal levels about a half smaller for all phases. This could be attributed to the larger attenuation caused by fluid flow across the boundary.

The large amplitude Airy phase is also observed in both discrete wavenumber cases. However neither case shows as large an Airy phase as the acoustic limit finite difference code and the greatly reduced amplitudes for the permeable case are not observed.

The Prony's method results (Figure 7) for discrete wavenumber show nearly identical phase velocities for the permeable and impermeable cases for each of the Stoneley and pseudo-Rayleigh waves. Pseudo-Rayleigh wave amplitudes are also comparable but the Stoneley wave amplitudes are slightly higher for the impermeable boundary.

On comparing the elastic limit Biot theory with the impermeable boundary discrete wavenumber result one sees that Stoneley wave amplitudes are comparable but that Stoneley wave phase velocities are lower and have more scatter in the finite difference case. For the pseudo-Rayleigh wave, phase velocities are comparable but amplitudes are over estimated by a factor of two in the finite difference results.

The acoustic limit Biot theory underestimates the pseudo-Rayleigh wave phase velocities by about 10% and overestimates their amplitude by about 50%. The Stoneley waves are essentially nonexistent. Although the acoustic limit is a valid solution of Biot's equations in heterogeneous media it is not clear which boundary condition it corresponds to or which physics it represents.

A HIGH POROSITY STRINGER

The objective of this work is to develop a code for predicting wave propagation in two dimensional Biot media. Notwithstanding the inaccuracies described above, we show here results for a 38% porosity stringer imbedded in a 19% porosity sandstone (Figure 8). Difference logs (the logs for a uniform formation are subtracted from the logs for the stringer) are shown for both acoustic and elastic limits for the same model. For the elastic limit case, reflections of PL and pseudo-Rayleigh waves can be observed from the top of the stringer which was at 1.99 m depth, and reflections of pseudo-Rayleigh waves can also be seen from the bottom of the stringer at 2.39 m depth. The large amplitude Airy phase is not evident in the reflections. Pseudo-Rayleigh wave amplitude and/or velocity anomalies are also observed within the stringer (from 1.99 to 2.39m depth). For the acoustic limit, clear pseudo-Rayleigh wave reflections are observed from the top and bottom of the stringer. PL wave reflections are not detectable. This is not surprising since the PL waves are undetectable in the downgoing wavefield.

So we have a code, the elastic limit version, which generate reasonable answers for Biot media which varies in two dimensions. The acoustic limit case works for the 2-D media but we are uncertain what the results mean even in the 1-D case.

CONCLUSIONS

The finite difference method does provide a means for obtaining solutions to wave propagation in two-dimensional Biot media. All of the physics of acoustic logs are present in the finite difference results. However the agreement between finite difference and discrete wavenumber results is not quantitatively good. Pseudo-Rayleigh wave amplitudes in particular, which were inaccurate by about 15% for elastic media are inaccurate by up to 100% (a factor of 2) for Biot media. We are assuming that the discrete wavenumber which treats a sharp boundary between the fluid and the biot solid is correct.

Part of our study was to investigate whether the elastic or acoustic limits of the Biot equations for heterogeneous media correspond to the permeable and impermeable boundaries used in the discrete wavenumber approach. If so, solutions for boundaries of arbitrary shape could be obtained easily by the finite difference method. The elastic limit Biot code came closest to the discrete wavenumber results, but for the pseudo-Rayleigh wave particularly, the finite difference results differed more from either discrete wavenumber results than the discrete wavenumber results did from each other. The acoustic limit Biot code gives such low amplitude results without significant PL or Stoneley waves and it must be describing an entirely different physical process (for attenuation) than either discrete wavenumber result.

The mathematical represention of the boundary is different between the finite difference method and the discrete wavenumber method and the results are different. However, which method gives the best agreement with either laboratory or field data? The acoustic limit Biot results are a bonafide solution to Biot's equations and may very well represent field results in some situations, especially in the cases with large porositeis.

On a positive note the Stoneley wave results agreed much better. Since Stoneley waves are most sensitive to the permeability issues of interest (Burns, 1988) we should concentrate further study on lower frequency sources which enhance Stoneley wave effects.

The Bhasavanija approach may not be the best finite difference method for liquid-solid boundary problems. The Virieux approach has a stability criteria which is independent of shear wave velocity and has tested well in other studies (e.g., Dougherty and Stephen, 1988). We would like to apply the Virieux code to the acoustic logging problem for both elastic and Biot media. Results should be better.

Finally, we obtained good results in finite differences for elastic media when we specifically coded the fluid-solid boundary condition (Stephen et al., 1985). We should take a similar approach for Biot media. By specifically coding the interface we lose the ability to simply introduce a rough borehole wall into the code. However we could still have a depth dependent media behind the wall and the results would still be quite interesting.

ACKNOWLEDGEMENTS

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FIG	CFRQ	KS	ROS	VPM	VSM	PORO	VPF	ROF	VISC	PERM	VP1	RO1	RAD	METHOD
2A	10.6	2.25	1.0	1.5	0.01	0.0	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ELASTIC
2B	10.6	19.12	2.336	3.735	2.08	1.0	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ACOUSTIC
3A	15.0	19.05	2.65	3.67	2.17	0.0	1.5	1.0	0.01	0.2	1.5	1.0	8	FD BIOT ELASTIC
3B	15.0	19.05	2.65	3.67	2.17	0.001	1.5	1.0	0.01	0.2	1.5	1.0	8	FD BIOT ELASTIC
3C	15.0	19.05	2.65	3.67	2.17	0.01	1.5	1.0	0.01	0.2	1.5	1.0	8	FD BIOT ELASTIC
3D	15.0	19.05	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	8	FD.BIOT ELASTIC
4A	10.6	N/A	2.3	4.0	2.3	N/A	N/A	N/A	N/A	N/A	1.8	1.2	9.5	FD ELASTIC STEP
4B	10.6	N/A	2.3	4.0	2.3	N/A	N/A	N/A	N/A	N/A	1.8	1.2	9.5	DW ELASTIC
4C	10.6	N/A	2.3	4.0	2.3	N/A	N/A	N/A	N/A	N/A	1.8	1.2	9.5	FD ELASTIC BHASA
4D	10.6	20.58	2.3	4.0	2.3	0.001	1.8	1.2	0.01	0.2	1.8	1.2	9.5	FD BIOT ELASTIC
4E	10.6	20.58	2.3	4.0	2.3	0.001	1.8	1.2	0.01	0.0001	1.8	1.2	9.5	FD BIOT ACOUSTIC
6A	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	DW BIOT IMPERM
6B	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	DW BIOT PERM
6C	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ELASTIC
6D	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ACOUSTIC
8A	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ELASTIC
8B	10.6	37.9	2.65	3.67	2.17	0.19	1.5	1.0	0.01	0.2	1.5	1.0	9.5	FD BIOT ACOUSTIC

LEGEND:

FIG	Figure number in paper
CFRQ	Center freuquency of source (kHz)
KS	Bulk modulus of solid matrix (GPa)
ROS	Density of solid matrix (gm/cm^3)
VPM	Compressional wave velocity of saturated rock (km/s)
VSM	Shear wave velocity of saturated rock (km/s))
PORO	Porosity of the formation (%)
VPF	Compressional wave velocity of the pore fluid (km/s)
ROF	Density of pore fluid (gm/cm^3)
VISC	Viscosity of the pore fluid (poise)
PERM	Permeability of the formation (darcy)
VPB	Compressional wave velocity of the borehole fluid (km/s)
ROB	Density of the borehole fluid (gm/cm^3)
RAD	Radius of the borehole (CM)
METHOD	FD-Finite differences, DW-Discrete wavenumber

Table 1: Parameters used in this paper

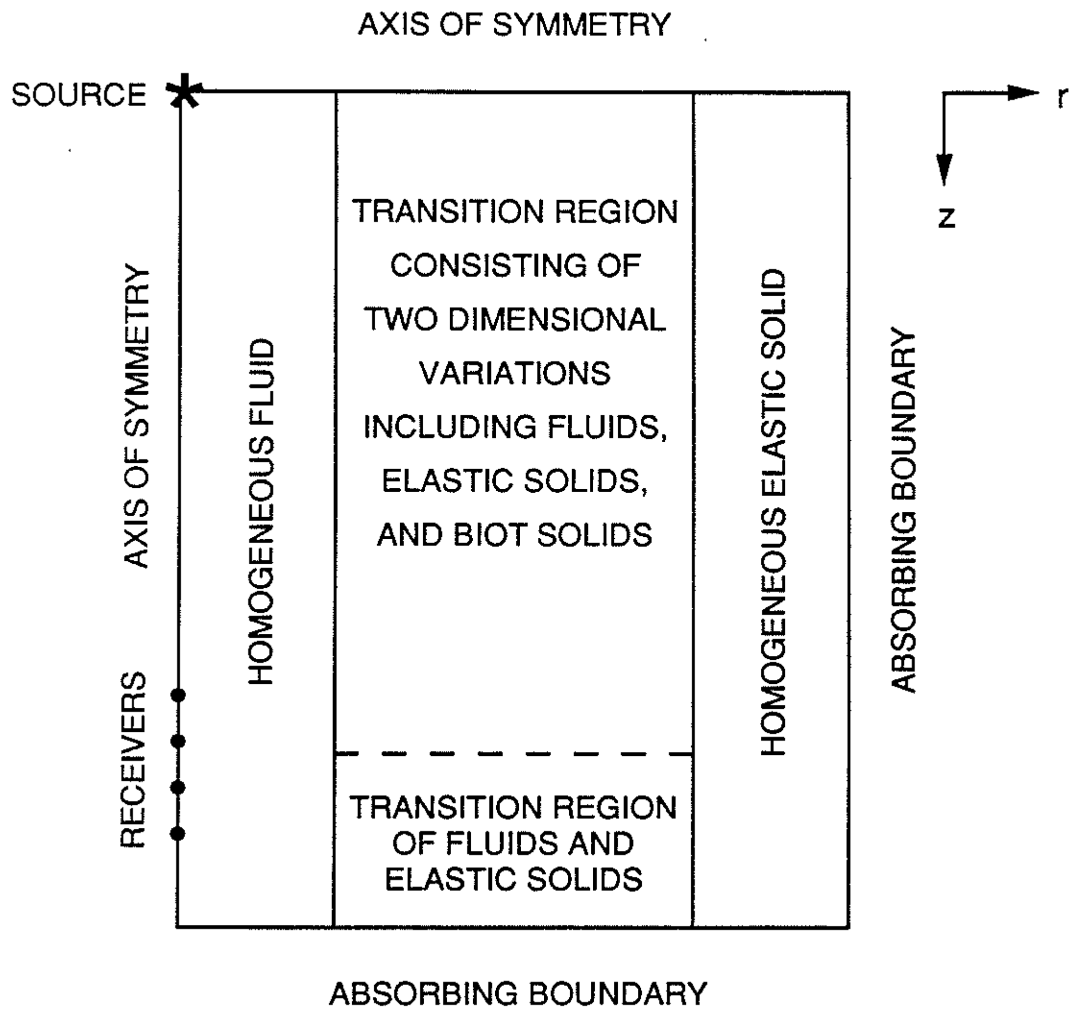


Figure 1: The grid configuration used for finite difference synthetic acoustic logs in Biot solids.

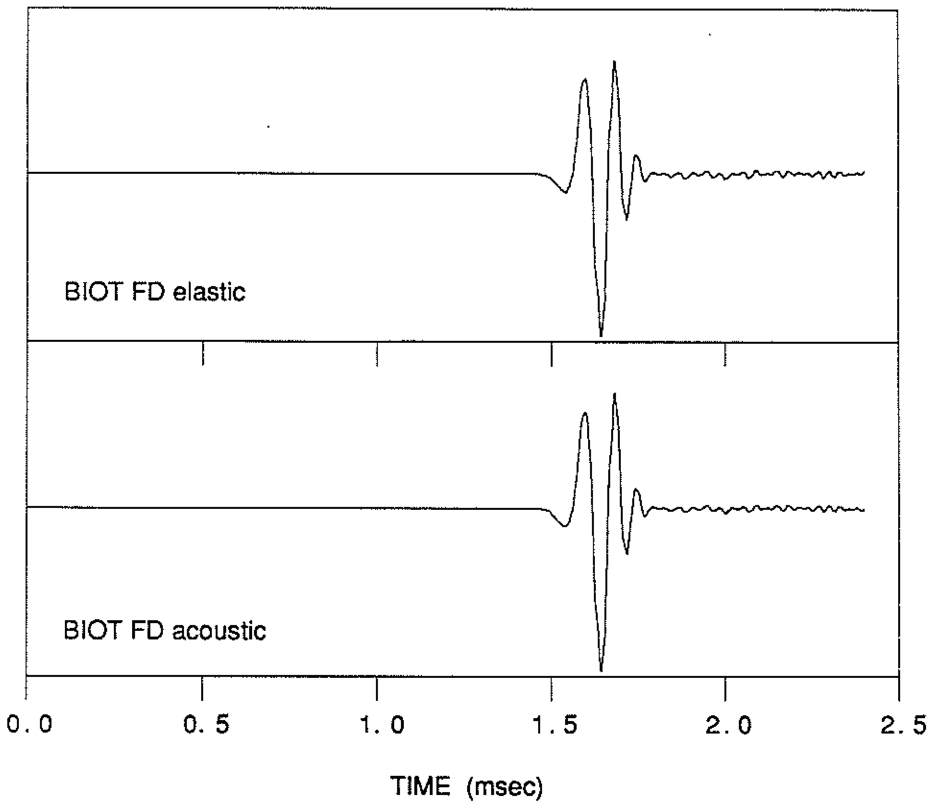


Figure 2: Microseismograms for homogeneous media are a useful check on the codes and confirm the source strength in each case. a) Biot finite difference code for the elastic limit for homogeneous water. b) Biot finite difference code for the acoustic limit for homogeneous water. Wave forms and amplitudes are idenetical.

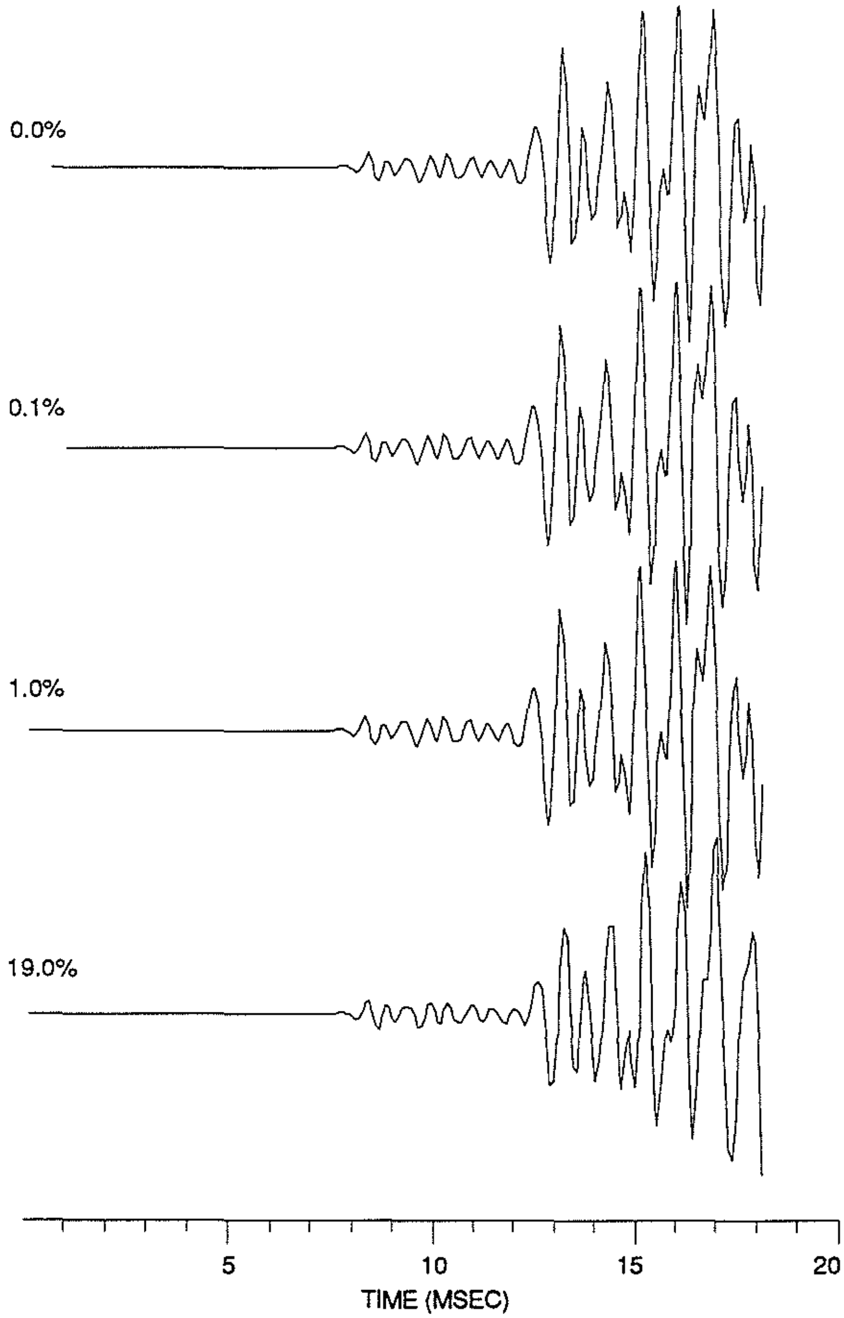


Figure 3: A comparison of traces at 2.20m below the source using the Biot code with a) 0.0%, b) 0.1%, c) 1.0%, and d) 19.0% porosity. The model parameters are shown in Table 1. Small changes in porosity result in small changes to the traces. This is a simple test of the Biot code.

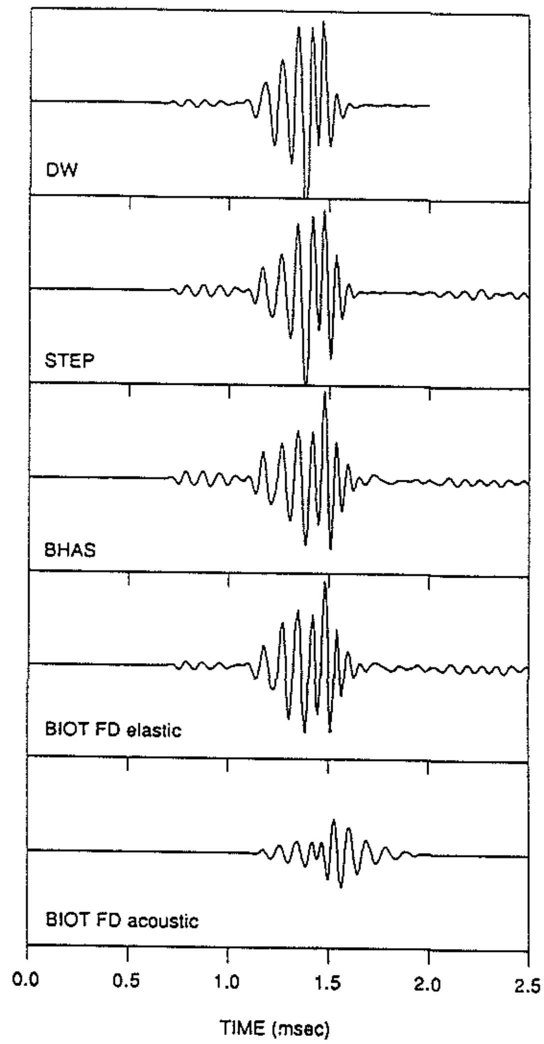


Figure 4: Five methods are compared for an acoustic log in elastic media: a) the Stephen elastic finite difference method with specifically coded boundary conditions; b) the discrete wavenumber method for elastic media; c) the Bhasavanija elastic finite difference method without specifically coded boundary conditions; d) the Biot finite difference method for the elastic limit with 0.1% porosity; and e) the Biot finite difference method for the acoustic limit with 0.1% porosity. All traces are at 2.19m depth. The Stephen finite difference and discrete wavenumber results compare well. The Airy phase is larger for the Bhasavanija elastic scheme. The Biot finite difference scheme for the elastic limit is very similar to the Bhasavanija elastic result. The Biot finite difference scheme for the acoustic limit is very different with undetectable PL waves and lower overall amplitudes than the other methods.

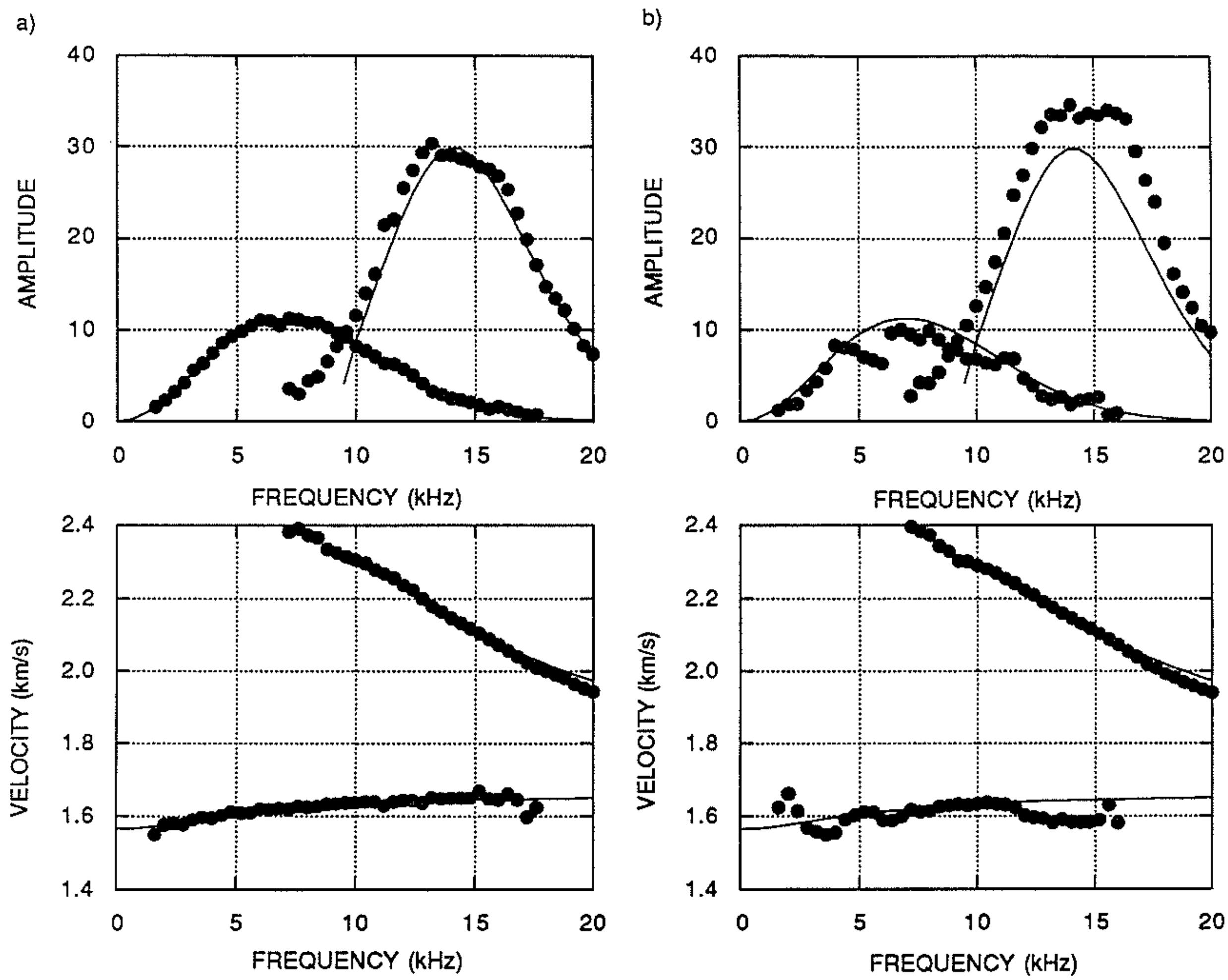
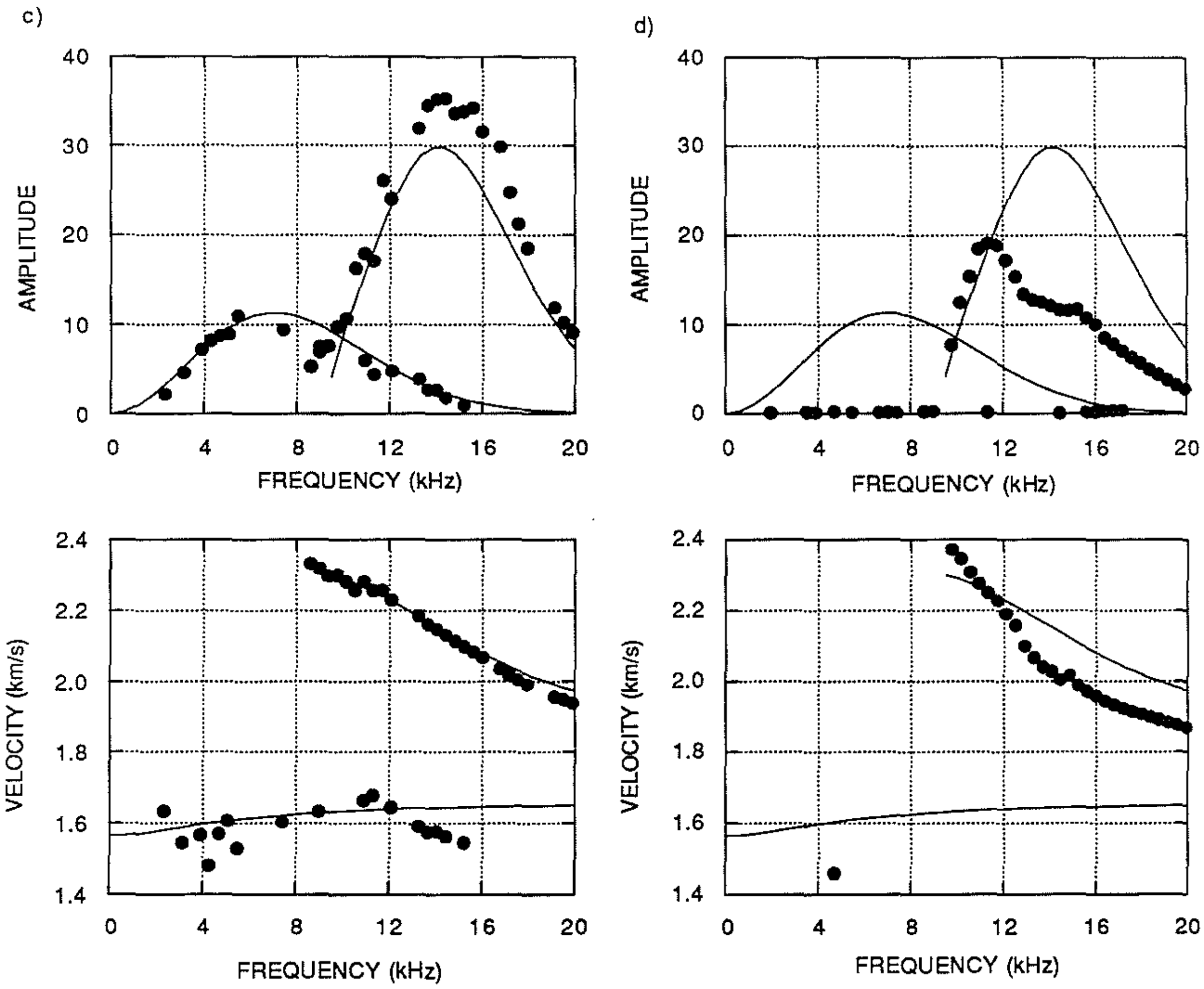


Figure 5: The Prony's method results for the examples in Figure 4 give a more quantitative comparison. The solid lines for the amplitudes in each case are the residue theory results. a) Results for the Stephen elastic finite difference code. b) Results for the Bhasavanija elastic finite difference scheme. The pseudo-Rayleigh wave amplitudes are overestimated by about 15% and the Stoneley wave amplitudes are slightly underestimated. c) Results for the Biot finite difference method for the elastic limit for 0.1% porosity. Results are similar to the Bhasavanija elastic scheme. d) Results for the Biot finite difference method for the acoustic limit for 0.1% porosity. The pseudo-Rayleigh wave amplitudes are underestimated and the Stoneley waves are non-existent.



c)
d)
AMPLITUDE
FREQUENCY (kHz)
VELOCITY (km/s)
0
4
8
12
16
20
10
30
40
1.4
1.6
1.8
2.0
2.2
2.4

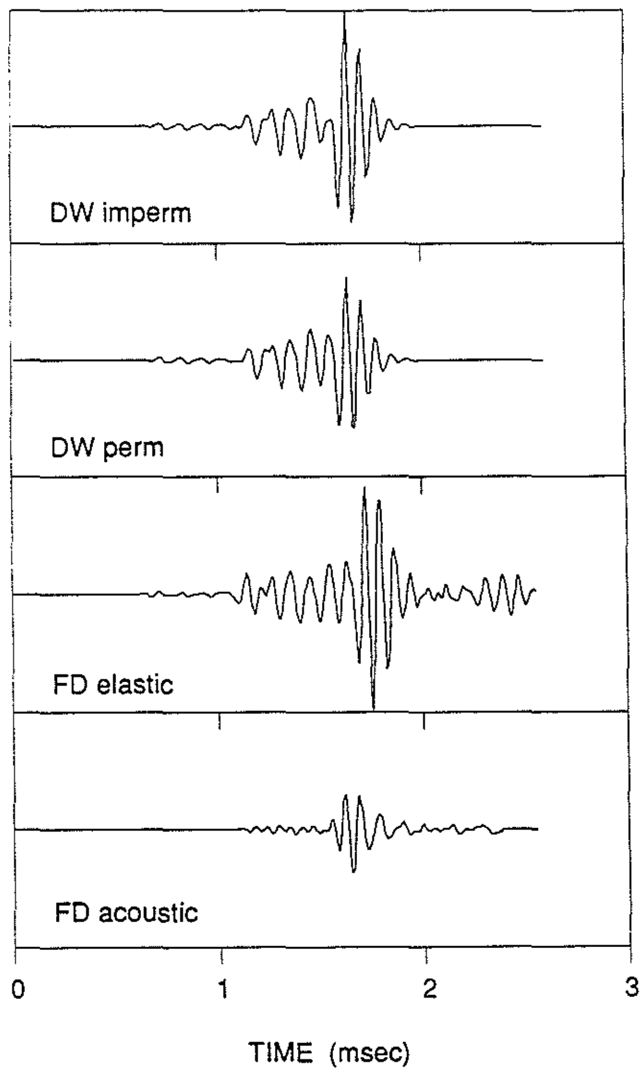


Figure 6: Four methods are compared for sonic logging in a 19% porosity Berea sandstone (All traces are at 2.19m depth.): a) the Biot discrete wavenumber result for an impermeable borehole wall; b) the Biot discrete wavenumber result for a permeable borehole wall; c) the Biot finite difference code in the elastic limit; and d) the Biot finite difference code in the acoustic limit. The finite difference result in the elastic limit has a larger Airy phase than the discrete wavenumber results. The finite difference result in the acoustic limit has much less overall amplitude.

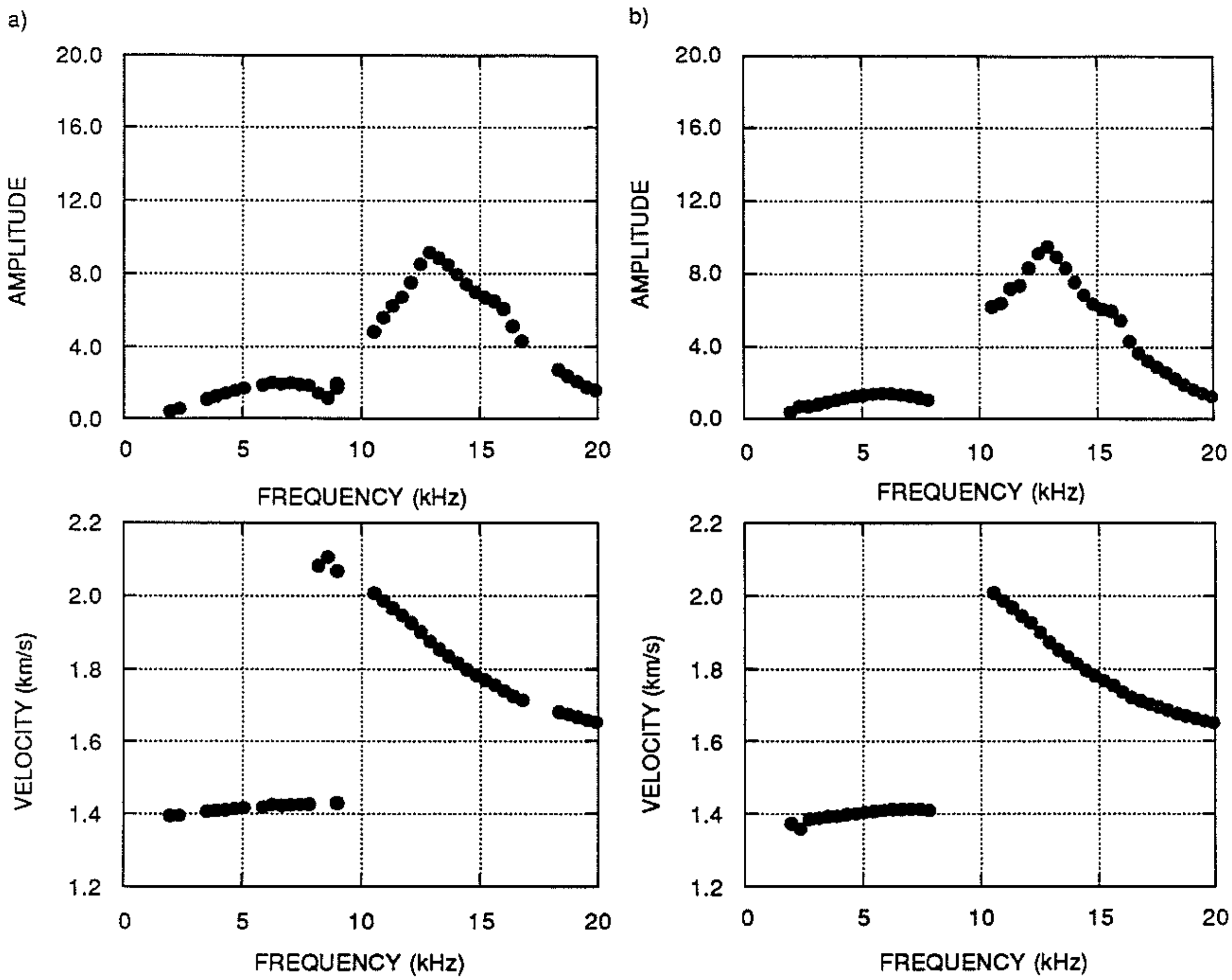
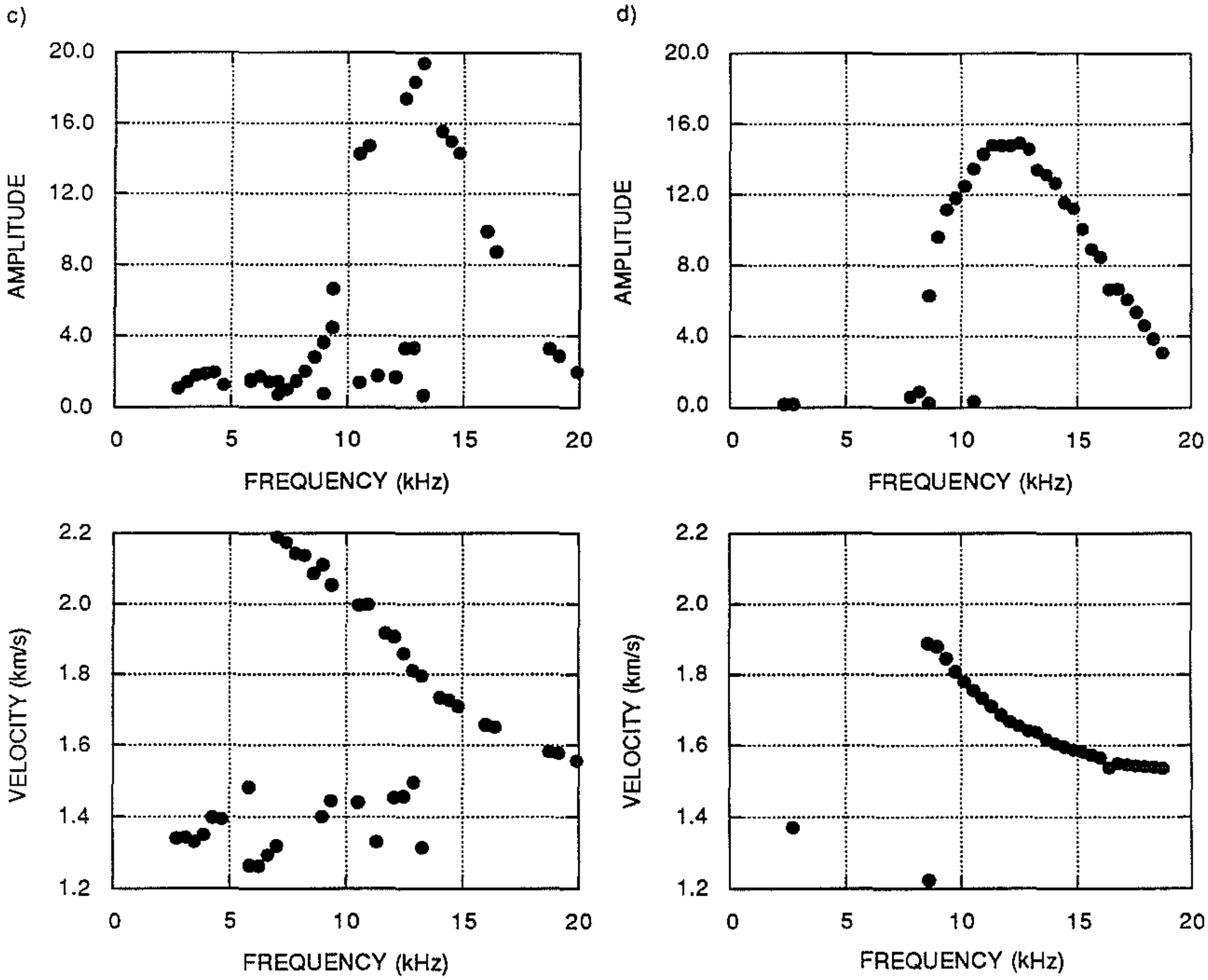


Figure 7: Prony's method results for the examples in Figure 6. a) Results for the Biot discrete wavenumber method for an impermeable boundary. b) Results for the Biot discrete wavenumber method for a permeable boundary. The Stoneley wave amplitudes are less than for the impermeable case. c) Results for the Biot finite difference method in the elastic limit. The pseudo-Rayleigh wave amplitudes are overestimated by 100% compared to discrete wavenumber, but the phase velocities are similar. d) Results for the Biot finite difference method in the acoustic limit. The pseudo-Rayleigh wave amplitudes are over estimated by 50% and the Stoneley waves are essentially non-existent.


c)
d)
AMPLITUDE
VELOCITY (km/s)
FREQUENCY (kHz)
20.0
16.0
12.0
8.0
4.0
0.0
2.2
2.0
1.8
1.6
1.4
1.2
0
5
10
15
20

a)

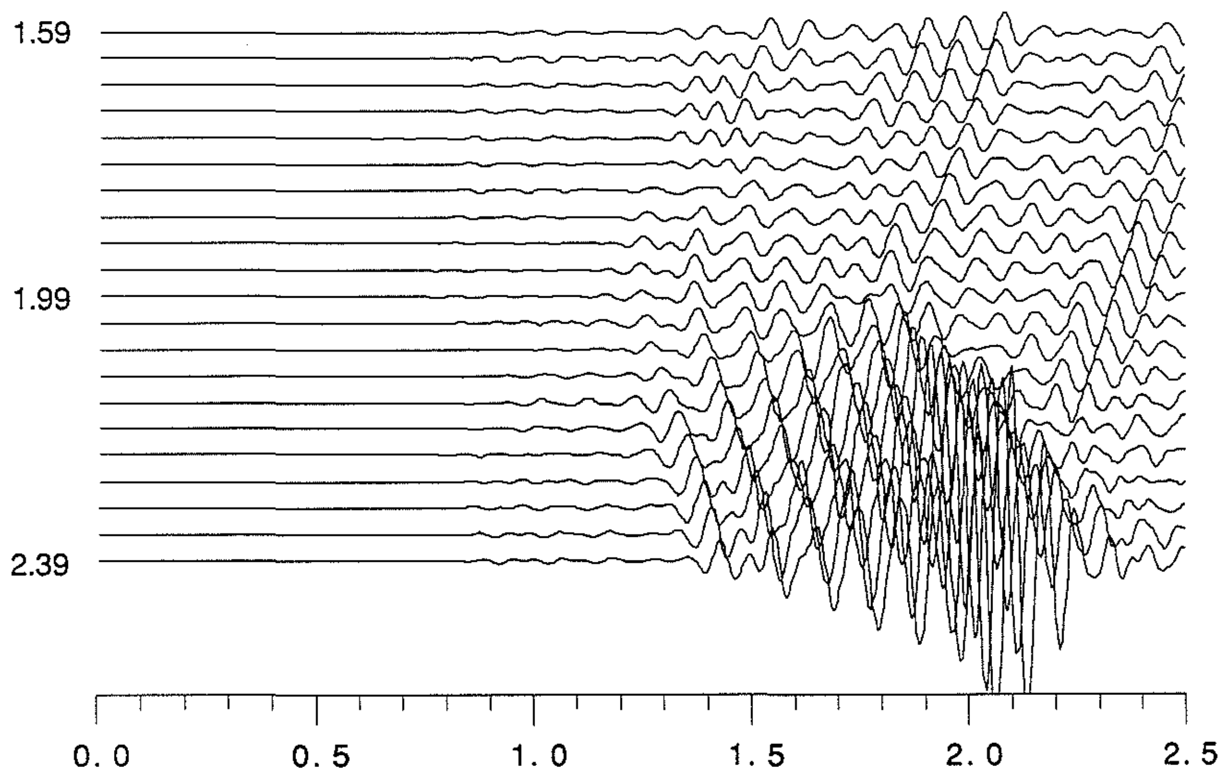


Figure 8: Examples of synthetic sonic logs in a 19% Berea sandstone with a 38% porosity stringer between 1.99 and 2.39m below the source. a) Difference microseismograms for the Biot finite difference method for the elastic limit. Pseudo-Rayleigh wave reflections are observed from the top and bottom of the stringer. PL wave reflections are observed from the top of the stringer. b) Difference microseismograms for the Biot finite difference method for the acoustic limit. The amplitudes are amplified by a factor of two with respect to those in a). Psuedo-Rayleigh wave reflections are also observed from the top and bottom of the stringer. PL reflections are undetectable.

b)