

NONLINEAR REFRACTION TRAVELTIME TOMOGRAPHY

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ABSTRACT

We identify a few unique issues that are important for performing a nonlinear refraction traveltime tomography effectively. These include accuracy of traveltime and ray-path calculation for a turning ray and physical information in a refraction traveltime curve. Consequently, we develop a shortest path raytracing method with an optimized node distribution that can accurately calculate refraction traveltimes and raypaths in any velocity model. We find that minimizing misfit of refraction traveltimes with the least-squares criterion does not account for the whole physical meaning of a refraction traveltime curve. We therefore pose a different nonlinear inverse problem that explicitly minimizes misfits of both traveltimes (*integrated slownesses*) and traveltime gradients (*apparent slownesses*). As a result, we enhance the resolution of the tomographic inversion as well as the convergence speed. We regularize our inverse problem with the Tikhonov method as opposed to applying *ad hoc* smoothing to keep the inversion stable. The use of the Tikhonov regularization avoids solving an ill-posed problem and allows us to invert an infinite number of unknowns. We apply this tomographic technique to image the shallow velocity structure at a coastal site near Boston, Massachusetts. The results are consistent with a local boring survey.

INTRODUCTION

Utilizing refraction traveltime data for imaging the subsurface has long been a standard technique. It is appealing because of its low-cost field operation and easy interpretation of the data. Dobecki and Romig (1985) suggested that by 1990 seismic reflection surveying would “replace refraction surveys as the most common seismic tool for engineering and groundwater studies,” while Lankston (1989) argued that “the reflection method

offers some capabilities that the refraction method cannot offer; but the converse is also true." Indeed, from a physical point of view, refraction relies on the heterogeneity of the medium to turn seismic energy back up to receivers, providing velocity information independent of that from reflection. However, conventional refraction methods fall short of showing the true strength of this technique with oversimplified geometry and media, although they attempt to develop unique physical concepts from refraction traveltimes. On the other hand, modern tomography methods seem to ignore the values of physical concepts that have been already established for the refraction problem, but rather matching traveltimes as any other tomography (e.g., reflection and cross-hole tomography). Consequently, certain statements have often been made about the effectiveness of the refraction method without the appropriate qualifications, leading to misconceptions. In this paper we try to combine both conventional refraction concepts and recently developed tomographic techniques in an effort to establish a new perception of the refraction traveltime method.

Seismic refraction data are conventionally acquired with "forward" and "reverse" shots, and their interpretation is made using reciprocity in several ways. These include the generalized reciprocal method (Palmer, 1980), the wavefront reconstruction method (Aldridge and Oldenburg, 1992), and the wavefield extrapolation method (Clayton and McMechan, 1981; Hill, 1987). All of these methods assume that seismic velocity structures are simple, and primarily attempt to map a refractor. Tomography methods, where an approach to calculate traveltimes and raypaths in a regular grid model and an inversion technique to reconstruct seismic velocities are used, have been extensively studied and developed for crosshole and reflection geometry. However, little has been done for refraction problems. White (1989) developed a refraction traveltime tomography that applies a two-point raytracing algorithm and solves a damped least-squares problem for both velocities and refractor depths. The inverse problem is regularized with a gradient smoothing operator in a *creeping* manner (Scales *et al.*, 1990; Zhang *et al.*, 1996). Zhu and McMechan (1989) perform refraction tomography using an analytic traveltime solution and applying an inversion method that is the same for the crosshole geometry (McMechan *et al.*, 1987) except for an initial model requiring positive velocity gradients. Stefani (1995) demonstrates a turning-ray tomography which is similar to White's (1989) but inverts velocities only. Using Vidale's (1988) finite-difference approach to solve an eikonal equation without involving rays, Ammon and Vidale (1993) developed a refraction traveltime tomography regularizing the inverse problem with second-order model derivatives in a *jumping* fashion (Scales *et al.*, 1990; Zhang *et al.*, 1996). Because they explicitly construct a sensitivity matrix by repeating forward calculation for each perturbed grid, their approach is limited to a small number of model grids. In addition, Qin *et al.* (1993) show that the finite-difference approach (Vidale, 1988) is not sufficiently accurate for refraction calculation.

It appears that existing techniques for refraction traveltime tomography are not sophisticated due to either the raytracing approach used or to the ill-posed inverse problem encountered or both. Using traditional two-point raytracing algorithms limits the accu-

Nonlinear Refraction Traveltime Tomography

racy of tomographic inversion. The ray methods suffer from the problem of converging to a local minimum traveltime path, and occasionally missing the global minimum (Moser, 1991). Generally, they require the use of a continuous model parameterization and cannot be easily applied to a grid model without some form of reparameterization (Lees and Hsilev, 1992; Fischer and Lees, 1993). An ill-posed inverse problem must be effectively regularized in order to obtain a physically meaningful solution, which is independent of the model discretization (Delprat-Jannaud and Lailly, 1993; Zhang *et al.*, 1996). However, because not all regularization approaches perform well, a proper criterion for refraction traveltime tomography must be defined. More importantly, the unique properties of the refraction problem have not been utilized. As a result, existing refraction traveltime tomography techniques are not much different from those for crosshole or reflection geometry.

In this study, we identify a few important issues in the refraction traveltime problem and develop a new approach for rapidly and accurately conducting nonlinear refraction traveltime tomography. First, instead of using traditional two-point raytracing algorithms (Julin and Gubbins, 1977; Um and Thurber, 1987; Červený, 1987), we apply the shortest path raytracing approach (SPR) (Saito, 1989; Moser, 1991) which can expand an entire wavefront in a regular grid model from its source. To its advantage, the SPR method always finds the global minimum traveltime raypath among those lying on a preset graph of nodes and edges. With an optimized node distribution in a graph template (Zhang, 1996), the SPR method is more accurate and efficient for calculating refraction. Second, we found that minimizing traveltime data alone with a least-squares criterion does not account for the whole physical meaning of a refraction traveltime curve. The inversion produces an allowed variance for the misfit of the traveltimes (*integrated slownesses*) but without explicitly specifying a variance for the gradients of traveltime curves (*apparent slownesses*). Therefore, we pose a nonlinear inverse problem that minimizes the misfits of both traveltimes and traveltime gradients. Numerical experiments demonstrate that the inclusion of traveltime gradient data in the inversion enhances the tomographic resolution as well as the convergence speed. Third, we use the Tikhonov regularization to perform a global inversion in the sense of reconstructing the whole model as opposed to applying *ad hoc* posterior smoothing constraints to keep the inversion stable. However, we will show evidence that not all the smoothing criteria in the Tikhonov method can perform well for refraction tomography. In fact, only the second-order derivative can be stable.

In addition, we demonstrate the application of this approach to real data from a small-scale survey conducted at a coastal site near South Boston, Massachusetts. The approach has been applied to various other projects at different scales, such as characterizing a contamination site at Woburn, Massachusetts (Zhang *et al.*, 1996) and imaging the crustal structure across the Transantarctic Mountains—the Antarctic Crustal Profile Project (Zhang, *et al.*, 1996). With modification, a joint refraction and reflection traveltime tomography is developed to image the crustal structure of the California Borderland—the Los Angeles Region Seismic Experiment (Zhang, *et al.*, 1996). Other

technical developments include a constrained refraction traveltime tomography with the wavefront reconstruction (Zhang and Toksöz, 1996).

CALCULATING REFRACTION TRAVLETTIMES AND RAYPATHS

For our tomography purpose, we need a raytracing approach to calculate both traveltimes and raypaths for the refracted first-arrival or turning rays. In fact, it is very difficult to accurately and efficiently calculate these types of rays in a complex velocity model (Qin *et al.*, 1992). Raytracing is the most time-consuming step of nonlinear seismic traveltime tomography. Since usually thousands of rays must be traced for a typical tomographic inversion, saving computer time and memory is an important factor when selecting an appropriate forward modeling algorithm. On the other hand, accuracy of both traveltimes and raypath loci is also important because the accuracy of tomographic inversions depends on the errors introduced by forward calculation. Using one model parameterization to trace rays and a different one for inversion (Bishop *et al.*, 1985) introduces errors because the optimal raypaths in the model used to trace rays will not be optimal in the model used to invert. A simplified model with a few parameters allowed for raytracing and inversion (Zelt and Smith, 1992) makes it difficult to model a complex structure. For a regular grid model, traveltime calculation methods (e.g., the finite-difference method, the shortest path raytracing method) that expand the wavefront in the entire model may perform better in finding diffracted raypaths, headwaves, and raypaths to shadow zones (Vidale, 1988; Saito, 1989; Moser, 1991). Using a finite-difference method to approximate the solution of an eikonal equation (Vidale, 1988) is a fast approach for traveltime calculation. However, because it expands a "square wavefront," it is not as accurate as the SPR method (Qin *et al.*, 1992). The SPR method is flexible for any desired accuracy by adjusting the graph template size. However, for highly accurate results, this method requires vast memory and intensive calculation. Recent improvements have been made by Fischer and Lee (1993), Klimes and Kvasnicka (1993), Weber (1995), and Zhang (1996). In this study, we apply the approach developed by Zhang (1996).

One can find seismic raypaths by calculating the shortest traveltime paths through a network that represents the earth. The network consists of nodes, and the node connection is based on a graph template. The SPR method includes three steps: (1) timing nodes along an expanding wavefront from its original source or secondary source; (2) finding the minimum traveltime point along the wavefront and taking this point as a new secondary source; and (3) expanding the wavefront from this minimum time-point. These three steps are repeated until the whole model is traced. With a small number of ray legs in a graph template, SPR usually yields zig-zag raypaths in flat parts of the velocity model and produces longer raypaths (Fischer and Lees, 1993; Weber, 1995).

Zhang (1996) describes two improvements. The traveltime errors in SPR are due to space discretization and angle discretization. These two errors are independent, i.e., decreasing grid size does not reduce error due to the limited angle coverage (Moser, 1991).

Nonlinear Refraction Traveltime Tomography

One simple approach that can reduce error due to angle discretization is to optimize node distribution in the graph template prior to the raytracing. Because the angle error is associated with the largest angle that can be sampled in a graph template, the angle differences for a given number of rays in a graph template should be minimized. Figure 1 shows a graph template that contains two nodes on each grid boundary. The equal node distribution shown by the dashed line was used previously (Nakanishi and Yamaguchi, 1986; Moser, 1991; Fischer and Lees, 1993; Weber, 1995). The open circles and solid lines in Figure 1 give the optimized node distribution in which all the propagation angles are equal (22.5°). If the number of nodes on each grid boundary is more than two, the optimized propagation angles cannot be exactly equal, but have an overall minimum difference. Another node optimization is made by analyzing the velocity model prior to the raytracing. Instead of timing all the network nodes with the same graph template, the areas in the model that are smooth with velocity variations less than an allowed interval are determined, and the nodes in these areas are eliminated. This avoids the zig-zag raypath problem, and also enhances calculation efficiency. Thus, our approach includes two steps to optimize node distribution.

Figure 2 shows a comparison of traveltime errors due to different raytracing approaches for a simple two-layer model. For this example, we use only two nodes on each cell boundary. In modeling real data, we usually use five nodes for better accuracy in a complex model. Due to uneven angle sampling, using regular nodes produces a large error in some areas including refraction in this case. Without additional computation effort, simply adjusting the node distribution according to Figure 1 reduces some amount of error. Further optimizing the nodes by eliminating them in “smooth areas” produces results with negligible errors.

MINIMIZING A PHYSICALLY MEANINGFUL OBJECTIVE FUNCTION

Here, we solve a nonlinear regularized inverse problem. Starting from an initial model constructed using a conventional analytical approach for refraction, we iteratively update traveltimes and raypaths without assuming any interfaces or velocity functionals in the model. Our inversion approach is unique in two ways. First, the input data include not only refraction traveltimes (*integrated slownesses*) but also the gradients of the traveltimes (*apparent slownesses*). As we mentioned previously, combining two types of data will compromise the drawback of the least-squares criterion for the refraction problem. Second, we apply the Tikhonov regularization (Tikhonov and Arsenin, 1977) to explicitly constrain the model roughness as opposed to applying *ad hoc* posterior smoothing to keep the inversion stable. Specifically, we minimize the following objective function,

$$\Phi(m) = (1 - \omega)\|d - G(m)\|^2 + \omega\|\hat{d} - \hat{G}(m)\|^2 + \tau\|Dm\|^2 \quad (1)$$

$$= (1 - \omega)S_1 + \omega S_2 + \tau S_3 \quad (2)$$

where d is the travelttime data; $G(m)$ is the calculated travelttime data for current model m ; \hat{d} is the travelttime gradient data; $\hat{G}(m)$ is the calculated travelttime gradient data; D is a regularization operator; τ is a smoothing trade-off parameter; and ω is a weighting factor between the data misfit norm and the data gradient misfit norm.

Inversion Algorithm

To iteratively minimize the objective function (1) without explicitly dealing with the large sensitivity matrices, one can first use the Gauss-Newton (GN) method to linearize the stationarity equation associated with minimizing the objective function (1) and then use a Conjugate Gradient (CG) approach to solve inversion for each iteration (Scales, 1987). One can also directly minimize the objective function using CG methods (Matarese, 1993). The following algorithm takes the first approach:

$$\begin{aligned} & ((1 - \omega)A_k^T A_k + \omega B_k^T B_k + \tau L^T L + \epsilon_k I) \Delta m_k = \\ & (1 - \omega)A_k^T (d - G(m_k)) + \omega B_k^T (D_x d - D_x G(m_k)) - \tau L^T L m_k, \end{aligned} \quad (3)$$

$$m_{k+1} = m_k + \Delta m_k, \quad k = 1, 2, 3, \dots, N, \quad (4)$$

where A_k is the sensitivity matrix for the travelttime data (for the slowness and reflector derivative calculation, see Bishop *et al.*, 1985). B_k contains the sensitivity for the travelttime gradient, in which the derivative term is the difference of the raypath lengths in each model grid associated with the travelttime picks at two adjacent receivers. However, we do not construct matrices A_k and B_k in our inversion. Because we used the CG approach to solve equation (3), we only need to provide the results of A_k or A_k^T multiplying an arbitrary vector, B_k , or B_k^T multiplying an arbitrary vector in each CG iteration (Scales, 1987). Since raypaths are stored in compact vectors, these operations only require simple vector mapping. Thus, we are able to deal with large inverse problem efficiently.

We found that refraction travelttime inversion behaves similarly to many other nonlinear geophysical inversion problems in the sense that large nonlinearity occurs in the early inversion stage due to a poor starting model, and it becomes approximately linearized when the model is close to the true solution. Therefore, we include a variable damping term $\epsilon_k I$ in the left-hand side of equation (3), and define $\epsilon_k = \alpha \times rhs$, where α is an empirical parameter (about 0.01) and rhs is the rms misfit norm of the right-hand side of equation (3). If the objective function is not minimized well and is nonlinear, then rhs is large, and a large damping ϵ_k is applied and small model updates are allowed. With the inversion proceeding further and rhs decreasing, a smaller ϵ_k drives the convergence speed faster in the latter stage. Such a variable damping term is essential for successfully linearizing a nonlinear inverse problem because it ensures that we will not miss the “global” solution.

Nonlinear Refraction Traveltime Tomography

Inclusion of Traveltime Gradient Data

Refraction data are conventionally utilized in the forms of absolute traveltime at each receiver and traveltime derivative (or traveltime gradient) across receivers. For instance, the latter is defined as a velocity-analysis function in the GRM method (Palmer, 1980). Both types of data provide independent information about the subsurface, and alternatively provide more constraints on the raypaths. The absolute traveltime represents *integrated slownesses* along the raypath, while the traveltime gradient gives a local *apparent slowness* of a refractor. On the other hand, because the ray lengths in the refraction problem may range over two orders of magnitude, and short rays constrain the velocity more tightly than do long rays, structure ambiguity increases with ray length if only traveltimes are considered. However, accounting for traveltime gradients in one way or another in interpretation can constrain this ambiguity; this is also the essence of the conventional approaches. Therefore, modern refraction traveltime tomography should utilize both types of data in a more robust way.

Figure 3 shows synthetic tomography results for fitting traveltimes alone and fitting both traveltimes and their gradients. A second-order smoothing operator is applied for regularization (see next section). As one may expect, the paths of deep refracted rays can hardly be well-constrained; consequently, there may be many “ray solutions.” However, precisely accounting for the shapes of traveltime curves, i.e., explicitly minimizing the misfits of both traveltimes and their gradients, can better constrain velocities as well as the refractor geometry. Physically, traveltime gradients contain subsurface information with the “wave” resolution. On the other hand, minimizing the misfit of traveltimes alone with the least-squares criterion does not quantify the misfit of traveltime gradients. The inclusion of traveltime gradient data is therefore necessary in refraction problems.

Tikhonov Regularization

When an inverse problem is ill-posed, no matter how sophisticated the optimization approach, there can be no definitive “solution” based on the data alone. In other words, minimizing the data misfit only (even including traveltime gradient data) does not have one “global solution.” Of course, one can try to increase the grid size to the point where the number of grids is equal to the rank of the least-squares matrix. Such a coarse model will not be sufficient to produce accurate forward solutions. For refraction traveltime tomography, most methods attempt to apply *ad hoc* constraints to keep the inversion stable. These include filling with nonzero sensitivity between rays (Hole, 1992; Cai and Qin, 1994), applying model gradient derivatives to damp model stepsize in a *creeping* manner (White, 1989; Stefani, 1995), or using a posterior low-pass filter to smooth the model after each iteration (Zhu and McMechan, 1989). We chose to solve an inverse problem that explicitly minimizes data misfit as well as model roughness using Tikhonov regularization (Tikhonov and Arsenin, 1977). This leads to a different inverse problem: for given traveltime and traveltime gradient variances, we can determine one unique trade-off parameter [τ in equation (1)] that yields an optimal

regularized solution with regard to the entire model.

Indeed, the use of the Tikhonov regularization may produce a “smooth” solution. However, when we say “model smoothness” or “model roughness,” we must answer, “by what criterion?” In fact, we found that not all smoothness criteria of the Tikhonov method are stable for the refraction traveltime tomography. We look into the following criteria:

$$\text{first - order smoothing : } D = \nabla, S_3 = \int (\nabla m(\mathbf{x}))^2 d\mathbf{x}, \quad (5)$$

$$\text{second - order smoothing : } D = \nabla^2, S_3 = \int (\nabla^2 m(\mathbf{x}))^2 d\mathbf{x}, \quad (6)$$

$$\text{third - order smoothing : } D = \nabla^3, S_3 = \int (\nabla^3 m(\mathbf{x}))^2 d\mathbf{x}. \quad (7)$$

Each of these derivatives gives one criterion of smoothness. One can also apply a higher-order derivative operator and define a different smoothness criterion for inversion. Figure 4 shows a comparison of the inversion results for smoothness at different criteria. We select proper parameters so that all these inversions converge to the same data misfit variance. It appears that the first-order smoothness criterion produces a nonphysical solution. The second- and third-order smoothness criteria allow physical details to be resolved. However, our experiences with the third-order smoothness criterion for the refraction problem are not encouraging, because it often over-resolves the details even though a large smoothing trade-off parameter is selected. The second-order smoothness criterion seems appropriate for the refraction problem. See Delprat-Jannaud and Lailly (1993) for a discussion about appropriate smoothness criteria for reflection traveltime tomography, and Zhang *et al.* (1996) for electrical tomography.

MODELING FIELD DATA

We applied nonlinear traveltime tomography to a small-scale refraction survey at a coastal site near Boston, Massachusetts. The goal of the survey was to locate those areas where bedrock is deep so that construction of a new storm-drainage system may proceed without costly blasting. The environment at the working site was quite unusual because the survey area was covered by sea water during the high-tide period, and exposed for only one or two hours during the low tide each day. We show the results along two survey lines, each consisting of 24 geophones and 12 shots. A geophone spacing of 10 feet was used in all these surveys.

Figure 5a shows the seismic waveforms recorded from a forward shot and a reverse shot on line 1. Data from both shots show relatively delayed first arrivals between receivers 7 and 14, although the topography along line 1 is flat. Moreover, the amplitudes of these delayed first arrivals are relatively small. The evidence suggests that a low-velocity zone with strong seismic attenuation exists beneath receivers 7 to 14. It becomes more obvious when we placed sources (using a hammer and air gun) at locations between receiver 7 and 14; such effects were then observed at all of the traces.

Nonlinear Refraction Traveltime Tomography

Figure 6a shows traveltime data from 12 shots along line 1. Corrections for trigger time were made for a few shots based on reciprocity. These traveltime curves do not suggest a simple velocity structure, instead indicate complexity in the shallow seismic media. In another case, Figure 5b shows waveform data recorded from survey line 2, and Figure 6b shows traveltime data from 12 shots. In contrast to line 1, this is a case that demonstrates influences due to high-velocity anomalies in the shallow structure, *i.e.*, an intermediate velocity zone sits on the bedrock and outcrops the surface in the central area. Using these traveltime data, we performed tomography studies with models consisting of 250×100 grids (grid spacing of 1.0 ft). Our results are presented in Figure 7. The calculated traveltime data corresponding to our final solutions are plotted in Figure 6 (grey dots). As one can see from these cases, although the recorded data show complexities, the resolved bedrock topography is quite simple. The complexities are mostly due to the shallow velocity structures. Shortly after the seismic tomography was completed, a field boring survey was conducted at the same site. The results are consistent with the tomographic solutions (Kutrubes *et al.*, 1996).

SUMMARY

We described an improved shortest path raytracing method for calculating traveltimes and raypaths in any velocity model. We introduced a nonlinear traveltime tomography method that uses first arrivals and resolves velocity on a regular grid. We found that utilizing both traveltimes and their gradients results in physically meaningful solutions. Our inverse problem is regularized by the Tikhonov method with a second-order smoothing criterion which allows us to deal with an infinite number of unknowns. Finally, the validity of our approach was proven by applications to real data.

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Nonlinear Refraction Traveltime Tomography

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Regular vs. Optimized Node Distribution

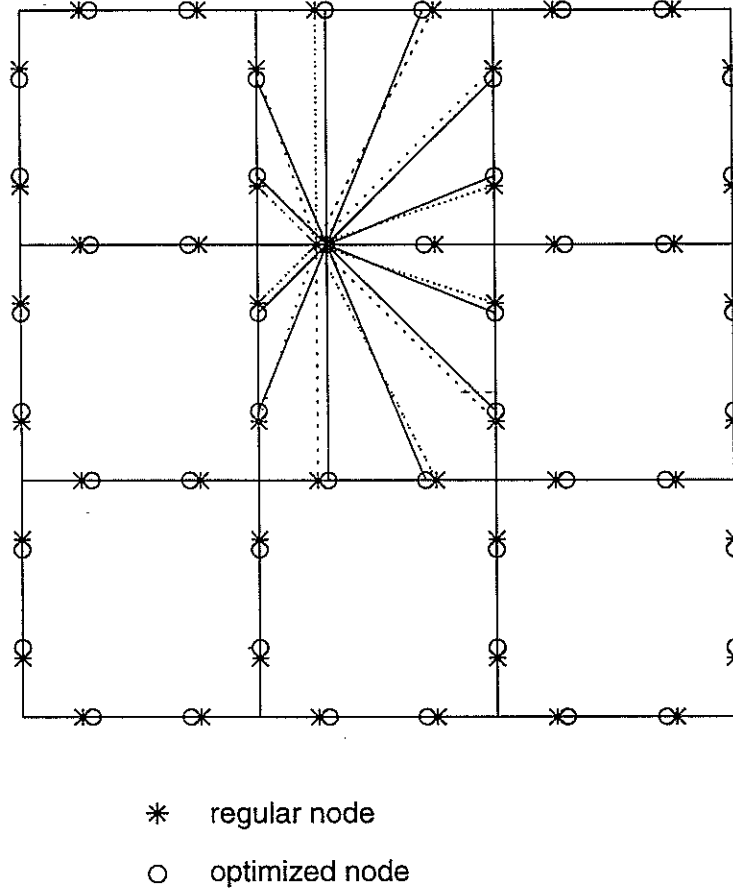


Figure 1: A graph template for the shortest path raytracing method. The star denotes a regular node that samples a model cell interface equally; the open circle represents a one-step optimized node. Slightly shifting nodes along the cell boundary, so that all the angles are equal or close, can reduce raytracing error due to angle discretization.

Nonlinear Refraction Traveltime Tomography

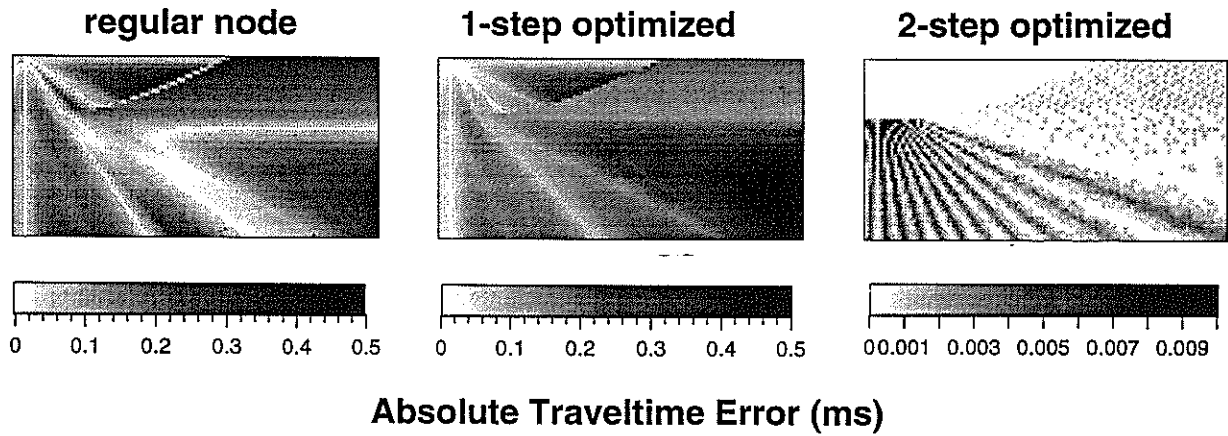


Figure 2: Comparison of raytracing accuracy due to different graph templates. A two-layer model (200 m by 50 m) is discretized with grid size of 1.0 ft. The upper-layer velocity is 2500 m/s and the lower-layer velocity is 4500 m/s. Using regular nodes produces the largest refraction traveltime error (0.4 ms); shifting nodes for minimizing angle differences reduces the refraction traveltime error down to 0.2 ms; further eliminating nodes in homogeneous areas can greatly reduce error to be negligible.

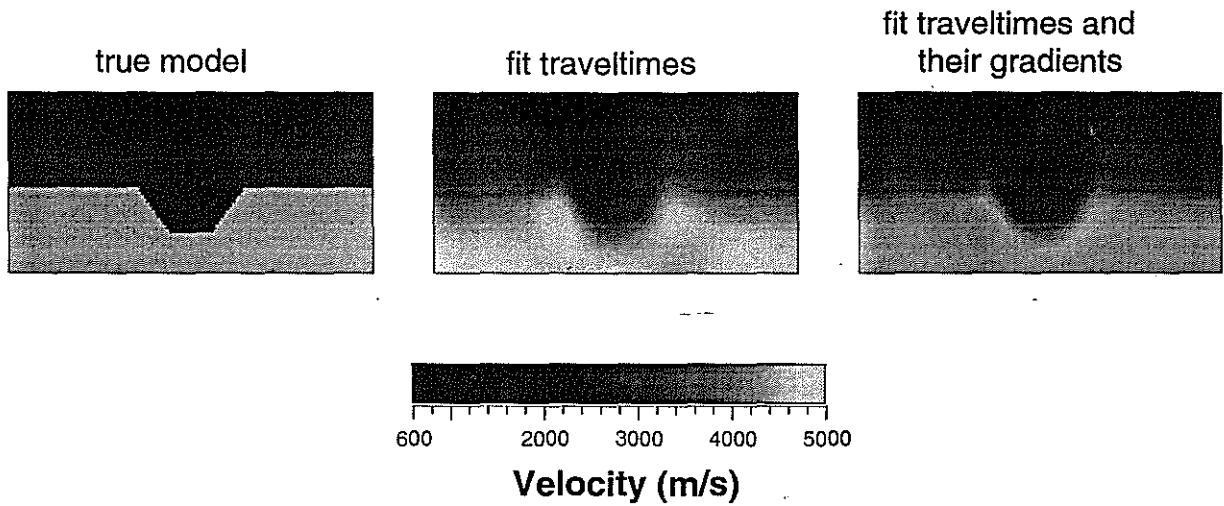


Figure 3: Numerical experiment for resolving a graben model containing a near-surface low-velocity zone. Using traveltime data alone may bias deep velocities and refractor geometry, while fitting both traveltimes and their gradients can better constrain the solution.

Nonlinear Refraction Traveltime Tomography

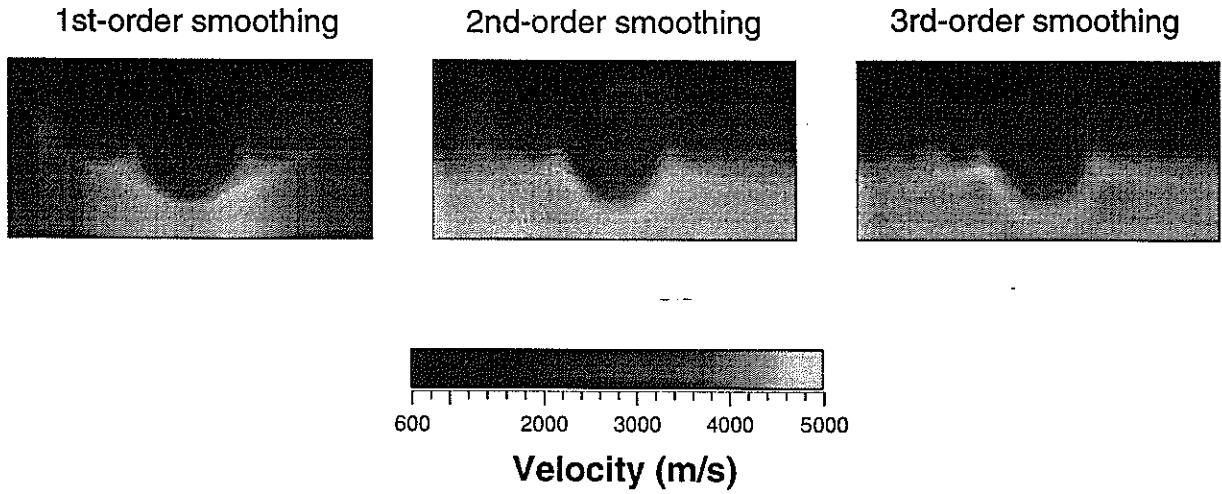
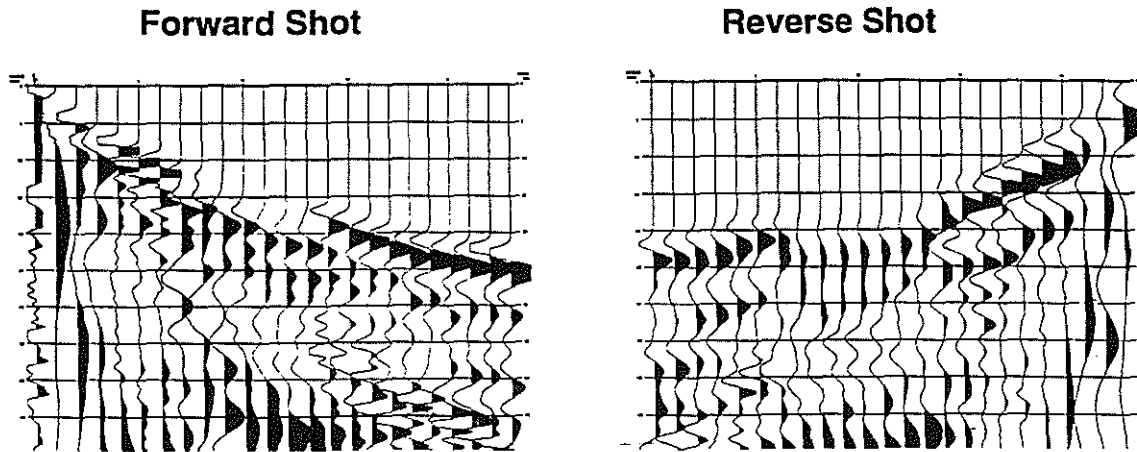


Figure 4: For the model shown in Figure 3, inversions are conducted with different smoothing criteria. The second-order smoothing criterion has the best performance.

1) waveform data from line 1



2) waveform data from line 2

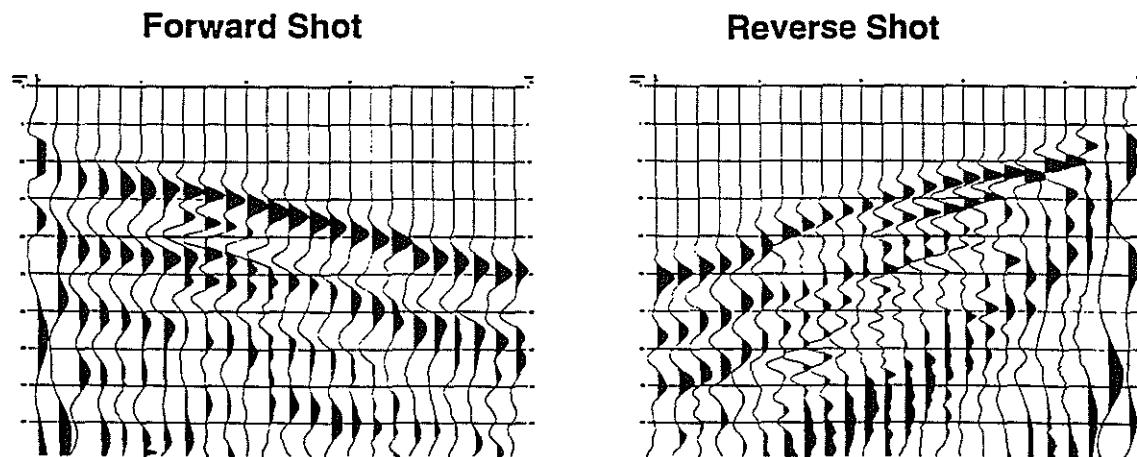


Figure 5: Waveform data from forward and reverse shots on line 1 and line 2. Note the influences due to a shallow low-velocity zone between trace 7 and 14 along line 1, and the influences due to a shallow high-velocity zone along line 2. Time interval is 5 ms.

Nonlinear Refraction Traveltime Tomography

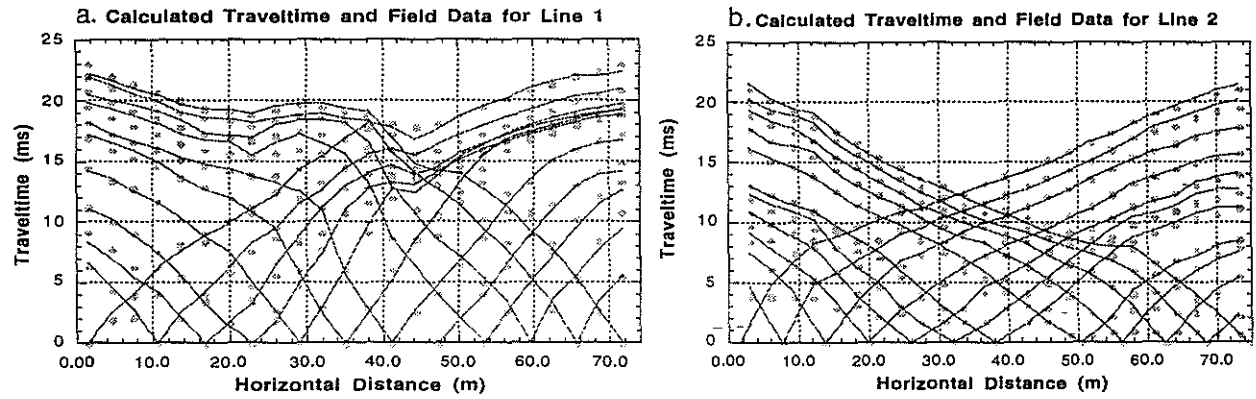


Figure 6: Field traveltime data (curves) from line 1 (a) and line 2 (b) and the calculated traveltimes (grey dots) for the resolved models shown in Figure 7.

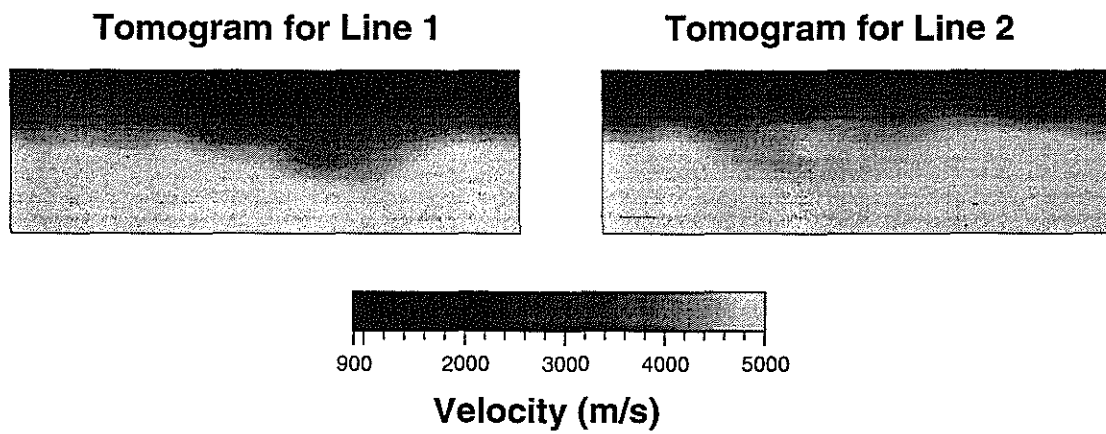


Figure 7: Tomographic imaging results for refraction survey line 1 and line 2.