# Geographic Location and Geographic Prediction Performance Benefits for Infrastructureless Wireless Networks

Shane A. Fink
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Author .....
Department of Electrical Engineering and Computer Science
May 23, 2012

Certified by .....
Vincent W. S. Chan
Joan and Irwin Jacobs Professor of Electrical Engineering and Computer Science
Thesis Supervisor

Certified by .....
Christopher C. Yu
Group Leader - Sensors and Networks Group - Charles Stark Draper Laboratory
Thesis Supervisor

Accepted by .....
Leslie A. Kolodziejski
Chair, Department Committee on Graduate Students

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## Abstract

The field of infrastructureless wireless networks (IWNs) is a broad and varied research area with a history of different assumption sets and methods of analysis. Much of the focus in the area of IWNs has been on connectivity and throughput/energy/delay (T/E/D) tradeoffs, which are important and valuable metrics. When specific IWN routing protocols are developed, they are often difficult to characterize analytically. In this thesis we review some of the important results in IWNs, in the process providing a comparison of wideband (power-limited) versus narrowband (interference-limited) networks. We show that the use of geographic location and geographic prediction (GL/GP) can dramatically increase the performance of IWNs. We compare past results in the context of GL/GP and develop new results in this area. We also develop the idea of throughput burden and scaling for the distribution of topology and routing information in IWNs and we hope that this work provides a context in which further research can be performed.

We primarily focus our work on wideband networks while also reviewing some narrowband results. In particular, we focus on wideband networks with non-zero processing energy at the nodes, which combines with distance-dependent transmission energy as the other main source of power consumption in the network. Often the research in this area does not take into account processing energy, but there is previous work which shows that processing energy is an important consideration. The consideration of processing energy is the determining factor in whether a whisper to the nearest neighbor (WtNN) or characteristic hop distance routing scheme is optimal. Whisper to the nearest neighbor routing involves taking a large number of short hops, while characteristic hop distance routing is the scheme by which the optimal hop distance is based on the distance dependent transmission energy and the processing energy, as well as the attenuation exponent.

For a one-dimensional network, we use a uniform all-to-all traffic model to determine the total hop count and achievable throughput for three routing types: WtNN without GL/GP, WtNN with GL/GP, and characteristic hop distance with GL/GP. We assume a fixed rate system and a random and uniform node distribution. The uniform all-to-all traffic model is the model where every node communicates with every other node at a specified rate. The achievable throughput is the achievable rate at which each source can send data to each of its destinations. The results we develop show that the performance difference between WtNN with and without GL/GP is minimal for one-dimensional networks. We show the reduction in hop count of characteristic hop distance routing compared to WtNN routing is significant. Further, the achievable throughput of characteristic hop distance routing is significantly better than that of WtNN networks.

We present a method to determine the link rate scaling necessary for link state distribution to maintain topology and routing information in mobile IWNs. We developed several results, with the main

result of rate scaling for two-dimensional networks where every node is mobile. We use a random chord mobility model to represent independent node movement. Our results show that in the absence of GL/GP, there is a significant network burden for maintaining topology and routing information at the network nodes. We also derive real world scaling results using the general analytic results and these results show the poor scaling of networks without GL/GP. For networks of 100 to 1000 nodes, the rate scaling for maintaining topology in mobile wireless networks is on the order of hundreds of megabits to gigabits per second. It is infeasible to use such significant amounts of data rate for the sole purpose of maintaining topology and routing information, and thus some other method of maintaining this information will need to be utilized.

Given the growing number of devices connected to the Internet, in the future it is likely that IWNs will become more prevalent in society. Despite the significant amount of research to date, there is still much work to be done to determine the attributes of a realistic and scalable system. In order to ensure the scalability of future systems and decrease the amount of throughput necessary for network maintenance, it will be necessary for such systems to use geographic location and geographic prediction information.

Thesis Supervisor: Vincent W. S. Chan
Title: Joan and Irwin Jacobs Professor of Electrical Engineering and Computer Science

Thesis Supervisor: Christopher C. Yu
Title: Group Leader - Sensors and Networks Group - Charles Stark Draper Laboratory

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## Acronyms

| | |
|---|---|
| BGP | Border Gateway Protocol |
| bps | bits per second |
| CCDF | complementary cumulative distribution function |
| CDF | cumulative distribution function |
| DREAM | distance routing effect algorithm for mobility |
| DSR | dynamic source routing |
| GL/GP | geographic location and geographic prediction |
| GPSR | greedy perimeter stateless routing |
| IBN | infrastructure-based network |
| IID | independent and identically distributed |
| IP | Internet Protocol |
| IS-IS | intermediate system to intermediate system |
| IWN | infrastructureless wireless network |
| LAN | local area network |
| LOS | line-of-sight |
| LSRP | link-state routing protocol |
| MAN | metropolitan area network |
| MANET | mobile ad-hoc network |
| MH | Manhattan mobility model |
| MIMO | multiple-input and multiple-output |
| OSPF | open shortest path first |
| PDF | probability distribution function |
| PMLAR | predictive mobility and location-aware routing |
| RF | radio frequency |
| RIP | routing information protocol |
| RWP | random waypoint model |
| SINR | signal-to-interference-plus-noise power ratio |
| SIR | signal-to-interference power ratio |
| SNR | signal-to-noise ratio |
| T/E/D | throughput/energy/delay |
| TCP | Transmission Control Protocol |
| TDMA | time division multiple access |
| VAR | velocity-aided routing |
| WAN | wide area network |
| *whp* | with high probability |
| WP1 | with probability 1 |
| WtNN | whisper to the nearest neighbor |

## Notation

| | |
|---|---|
| $\sim$ | on the order of |
| $\xrightarrow{P}$ | converges in probability (weak law of large numbers) |
| $1_{\{H_{(i)}^{(j)}(n)\}}$ | indicator random variable for whether the first hop from node $X_{(i)}$ to $X_{(j)}$ is correct or incorrect |
| $a(n)$ | (i) parameterization term in the optimal throughput-delay tradeoff, where $\sqrt{a(n)}$ corresponds to the average distance traveled in one hop (2.11.1.2) (ii) cell size in cell routing |
| $B$ | bandwidth |
| $b$ | minimum number of bits for each node to describe its topology and routing information |
| $C$ | channel capacity |
| $\mathbb{C}_{(i)}$ | number of correct first hops from node $(i)$ |
| $C_i$ | length of the chord along which node $W_i$ travels |
| $\mathcal{C}_L^{(i)}$ | number of clusters to the left of cluster $i$ |
| $\mathcal{C}_R^{(i)}$ | number of clusters to the right of cluster $i$ |
| $D_{(i)}$ | random variable that takes the value of the direction of the nearest neighbor from node location $(i)$ |
| $D(n)$ | delay in a network of $n$ nodes |
| $d$ | distance between the transmit and receive antennas |
| $\hat{d}_{char}$ | characteristic hop distance normalized with respect to |
| $d_{char}$ | characteristic hop distance |
| $\mathbb{E}_{(i)}$ | number of error first hops from node $(i)$ |
| $E_b$ | received energy per bit |
| $\mathcal{G}(n, M)$ | Erdős–Rényi graph model where each graph with $M$ edges is equally probable |
| $G(\mathcal{X}_n; r)$ | the random graph of vertex set $\mathcal{X}_n = \{\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n\}$ |
| $\mathcal{G}\{n, Pr(edge) = p\}$ | Erdős–Rényi graph model where edges are chosen with probability $p$ |
| $\mathcal{G}^n$ | set of all Erdős–Rényi graphs with vertices $V = \{1,2, \dots, n\}$ |
| $\sqrt{G_1}$ | product of the transmit and receive antenna field radiation patterns in the LOS direction |
| $g(\Theta_{t+\tau}^i)$ | function for chord length $C_i$ |
| $g_{kj}$ | channel power gain from the transmitter of the $j$th link to the receiver of the $k$th link in the minimum SINR model |
| $\mathbb{H}_{(i)}$ | total number of hops in the path from node $X_{(i)}$ to every other node in the network |
| $H_{(i)}^{(j)}(n)$ | indicator random variable for whether the first hop from a source node location $(i)$ to a destination node location $(j)$ is correct (takes the value 0) or incorrect (takes the value 1) in a network of $n$ nodes |
| $\mathbb{H}_T^{d_{char}}$ | total number of hops in a characteristic hop distance routing network |
| $\mathbb{H}_T^{GL/GP}$ | total number of hops in a WtNN network with GL/GP |
| $\mathbb{H}_T^{non-GL/GP}$ | total number of hops in a WtNN network without GL/GP |
| $h(C_i)$ | function for traversal time $T_{traverse}^i$ |
| $\mathbb{I}_n$ | indicator random variable for record to date (Section 3.2) |

| | |
|---|---|
| $k$ | path loss exponent |
| $L$ | one dimensional network length |
| $\overline{L}$ | mean distance between two points independently and uniformly distributed on the unit area sphere (Section 2.11.1.1) |
| $\hat{L}(n)$ | maximum traffic on a single link in steady state in a network of $n$ nodes |
| $L_{(i\to j)}(t)$ | the number of traffic streams at time $t$ on the link from node $X_{(i)}$ to node $X_{(j)}$ |
| $L_{(i)}^{(j)}$ | the number of traffic streams at time $t$ on the link from node $X_{(i)}$ to node $X_{(j)}$ in steady state |
| $\hat{L}^{d_{char}}(n)$ | maximum steady state traffic on a link in a characteristic hop distance network |
| $\hat{L}^{GL/GP}(n)$ | maximum steady state traffic on a link in a WtNN network with GL/GP |
| $\hat{L}^{non-GL/GP}(n)$ | maximum steady state traffic on a link in a WtNN network without GL/GP |
| $l$ | (i) integer multiplier of $d_{char}$ such that $L = (l-1)\cdot d_{char}$ (Chapter 3)<br>(ii) size of a cell edge in cell routing (Chapter 4) |
| $\ell(n)$ | cell length for cell routing in a one-dimensional network of $n$ nodes |
| $m$ | number of cells in a two-dimensional network using cell routing |
| $N$ | ambient noise level |
| $N_o$ | noise power spectral density |
| $N(t)$ | the counting process for a Poisson process where $N(t)$ is the number of arrivals up to and including time $t$ |
| $\mathcal{N}_L^{(i)}$ | number of neighbors to the left of node $X_{(i)}$ |
| $\left\{\mathcal{N}_L^{(i)}\right\}$ | the set of neighbors to the left of node $X_{(i)}$ |
| $\mathcal{N}_R^{(i)}$ | number of neighbors to the right of node $X_{(i)}$ |
| $\left\{\mathcal{N}_R^{(i)}\right\}$ | the set of neighbors to the right of node $X_{(i)}$ |
| $n$ | number of nodes in a network or random graph |
| $n_k$ | noise power of the receiver on the $k$th link in the SINR model |
| $\mathbb{O}_n^k$ | the arbitrarily labeled $k^{th}$ ordering of the $n$ nodes in a one-dimensional network |
| $\mathbb{P}_{(i)}$ | the path from $X_{(i)}$ to all other nodes in WtNN networks without GL/GP |
| $\hat{P}_{avg}$ | per node time-average power constraint |
| $P_k$ | power of the transmitter on the $k$th link in the minimum SINR model or physical model |
| $P_r$ | received power |
| $P_t$ | transmitted power |
| $R$ | fixed transmission rate in bps (this is a random variable in Chapter 4) |
| $R_{eff}$ | effective rate of transmission |
| $R_{min}$ | minimum transmission rate for the entire network |
| $R^*$ | transmission rate corresponding to characteristic hop distance |
| $\mathbb{R}^d$ | $d$-dimensional space |
| $(R_t, \Theta_t^i)$ | stochastic starting position of node $W_i$ in the topology coherence time approach |
| $(R_{t+\tau}, \Theta_{t+\tau}^i)$ | stochastic ending position of node $W_i$ in the topology coherence time approach |
| $R_{CR}$ | minimum transmission rate for cell routing |
| $R_{min}^i$ | minimum transmission rate to provide sufficient time for transmission of topology and routing information to node $W_i$ |
| $R_{TA-SN}$ | minimum transmission rate for the topology approach with a single mobile node |
| $R_{TCTA}$ | minimum transmission rate for the distributed link coherence time approach |
| $Rx_{(k)}$ | total traffic streams received by node $(k)$ |
| $r$ | fixed transmission rate in bps |

| | |
|---|---|
| $r(n)$ | uniform throughput capacity in a network of $n$ nodes |
| $(r_n)$ | a sequence of parameters in a random graph |
| $r_{TCTA}^{min}(\phi)$ | minimum rate such that $F_{R_{min}}^{C}(r) \leq \phi$ |
| $S$ | signal level |
| $S^2$ | the surface of a three-dimensional sphere of area one square meter |
| $S_i$ | arrival epochs in a Poisson process |
| $s$ | speed of the mobile node in cell routing |
| $s_i$ | speed of node $W_i$ |
| $T(n)$ | throughput in a network of $n$ nodes |
| $T_{Rx}^{i}$ | time remaining for node $W_i$ to receive transactions after receiving the topology and routing update |
| $T_{traverse}^{i}$ | time to traverse a cell (cell routing) or the circular transmission area of a node |
| $T_{update}$ | time required to receive the topology and routing data update |
| $Tx_{(k)}$ | total traffic streams transmitted by node $(k)$ |
| $Tx_{(k)}^{(i\rightarrow j)}$ | total traffic streams transmitted by node $(k)$ for traffic from node $(i)$ to node $(j)$ |
| $t_{ss}$ | the first time instance at which the network is in steady state with respect to traffic flow |
| $V = \{1,2, \dots, n\}$ | the set of vertices in an Erdős–Rényi graph |
| $\boldsymbol{v_i}$ | velocity of node $W_i$ |
| $\boldsymbol{v_R^i}$ | radial velocity of node $W_i$ in polar coordinates |
| $\boldsymbol{v_T^i}$ | tangential velocity of node $W_i$ in polar coordinates |
| $(v_x^i, v_y^i)$ | velocity of node $W_i$ in rectangular coordinates |
| $\mathbb{W}$ | the set $\{W_1, W_2, \dots, W_n\}$ of all nodes (or node locations) in a two-dimensional network |
| $W_i$ | the $i^{th}$ arbitrarily labeled node (or node location) in a two-dimensional network |
| $\mathbb{X}$ | the set $\{X_1, \dots, X_n\}$ of unordered nodes (or node locations) $X_i$ |
| $X_{(i)}$ | the $i$th ordered node (or node location) in a one-dimensional network |
| $X_i$ | the $i$th unordered node (or node location) in a one-dimensional network |
| $\mathcal{X}_i$ | d-dimensional IID random variables representing the node locations in a random geometric graph (Section 2.1.2) |
| $\mathcal{X}_n$ | the vertex set $\{\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n\}$ of a random graph |
| $\mathbb{X}_O$ | the set $\{X_{(1)}, \dots, X_{(n)}\}$ of ordered nodes (or node locations) $X_{(i)}$ |
| $(x_t^i, y_t^i)$ | starting position of a mobile node $W_i$ in cell routing |
| $(x_{t+\tau}^i, y_{t+\tau}^i)$ | ending position of a node $W_i$ in cell routing |
| $Y_i$ | distance between nodes $X_{(i)}$ and $X_{(i+1)}$ |
| $\mathbb{Z}^+$ | the set of positive integers |
| $Z_i$ | inter-arrival times in a Poisson process |
| | |
| $\alpha$ | processing energy |
| $\beta$ | distance-dependent transmission energy |
| $\gamma_i$ | SINR for the $i$th link in the SINR model |
| $\Delta$ | guard band in the protocol model |
| $\varepsilon$ | a small value $< 1$ |
| $\zeta$ | minimum SINR for successful receptions in the physical model |
| $\Theta(\cdot)$ | asymptotic upper and lower bound (tight bound) |
| $\lambda$ | (i) signal wavelength (Section 2.11, Section 4.1.2) |

| | |
|---|---|
| | (ii) rate of a Poisson process (Appendix A.1)<br>(iii) bit rate in bits per second from source to destination |
| $\hat{\lambda}$ | achievable uniform all-to-all steady-state throughput in bits per second |
| $\lambda(n)$ | per-node transmission rate in bits per second in a network of $n$ nodes (Section 2.11.1.1) |
| $\hat{\lambda}^{d_{char}}$ | achievable throughput for a characteristic hop distance network |
| $\hat{\lambda}^{GL/GP}$ | achievable throughput for a WtNN network with GL/GP |
| $\hat{\lambda}^{non-GL/GP}$ | achievable throughput for a WtNN network without GL/GP |
| $\lambda_{\Delta}^{d_{char}}$ | the throughput performance benefit achieved by using characteristic distance hopping over a WtNN network with GL/GP |
| $\lambda_{\Delta}^{d_{char},GL/GP}$ | the throughput performance benefit achieved by using both characteristic distance hopping and GL/GP over a WtNN network without GL/GP |
| $\lambda_{\Delta}^{GL/GP}$ | the throughput performance benefit achieved by using GL/GP-enabled nodes in a network using WtNN routing |
| $\xi\left(T_{traverse}^{i}\right)$ | function for $R_{min}^{i}$ |
| $\xi(n)$ | energy per bit in a network of $n$ nodes |
| $O(\cdot)$ | asymptotic upper bound |
| $\rho$ | (i) SINR model interference reduction due to signal processing (Section 2.8.1)<br>(ii) distance between transmit and receive antennas |
| $\rho_{max}^{*}$ | characteristic hop distance |
| $\left(\rho_{max}, \Theta_{t+\tau}^{i}\right)$ | ending position of node $W_i$ in the topology coherence time approach |
| $\left(\rho_{max}, \Theta_{t}^{i}\right)$ | starting position of node $W_i$ in the topology coherence time approach |
| $\tau$ | time period of node mobility |
| $\Phi$ | Poisson point process |
| $\phi$ | probability of insufficient rate for connectivity in the topology coherence time approach |
| $\chi$ | minimum rate for the case where a single mobile node travels along the diameter of the circle formed by another node's maximum transmission range |
| $\psi$ | number of bits of topology and routing data each node must receive, for each change in topology, in order to configure the network for connectivity |
| $\Omega(\cdot)$ | asymptotic lower bound |

# Chapter 1

# Introduction

When the development of the Internet first began, it would have been difficult to imagine its use in the many tasks we now perform online every day such as checking the weather, reading the news, and executing financial transactions. It would have been even more of a stretch to predict the vast quantity of devices that would eventually interconnect with one another via the Internet or private networks. This explains modern networking problems such as the exhaustion of the address space in IPv4 [1]. The Internet evolved from connecting researchers and government organizations with the initial assumption of a fixed, wired infrastructure. Today, the network has broadened to personal computers, wireless devices such as mobile phones, and other devices as wide-ranging as ovens, cars and televisions.

Throughout all of these changes in the use of communication networks, the predominant transport and networking protocols have for some time been Transmission Control Protocol (TCP) and Internet Protocol (IP) addressing and routing. The TCP/IP protocol suite will likely continue to serve most end-user networking needs for the foreseeable future. Most successful and widely adopted uses of networking today involve a predominantly fixed wire line infrastructure. When wireless connections are used, they mostly consist of one hop wireless architectures with base station infrastructures, such as in the IEEE 802.11 protocol family. These wireless protocols are primarily designed for and work best in environments where the router is connected to the wire line infrastructure and serves as an access point to the wired Internet.

The research area of multi-hop wireless networking is broad, though recently considerable focus has been placed on so-called mobile ad-hoc networks (MANETs), or wirelessly linked networks of mobile users. This thesis uses the more general term infrastructureless wireless network (IWN) to describe both MANETs and non-mobile ad-hoc networks. While both MANET and IWN could be used to describe this area of research, IWN more accurately describes a wider range of networks, including those without mobility. We define an IWN as a network of wirelessly communicating homogeneous nodes with no fixed infrastructure, prearranged hierarchy, or deployment strategy. In many cases, such networks will be mobile. However, in other cases, such as sensor networks, the nodes may be predominantly stationary or mobile at fixed intervals of time or for periods of time. Further, the dynamic nature of links in an IWN may not necessarily be due to mobility and instead could be due to changes in the radio frequency (RF) environment, either from obstructions or interference.

The work in this thesis analyzes the performance benefits of geographic location and geographic prediction (GL/GP) information in IWNs, as compared to the traditional method of utilizing routing tables stored at networks nodes. In an obstruction-free environment, the network topology could be determined from knowledge of the location of every node in the network. We note that in an environment with obstructions, the network will likely not be able to rely completely on GL/GP information for determining the topology. We propose that the ability of IWNs to operate as functional standalone networks necessitates the use of network geographic information stored at the nodes of the network. This includes both geographic location information and geographic prediction information about the other nodes in the network. Using routing tables without geographic information[1], as in the case of the Internet, is neither bandwidth efficient nor scalable for the number of nodes expected to be operating in future IWNs, on the scale of 10 to 1000 nodes.

[1] That is, tables maintained at the nodes and updated with traditional link-state routing protocol via flood routing when the network topology changes.

## 1.1 Existing Infrastructureless Wireless Network Research

Current IWN research is primarily focused on two areas: bounds on throughput/energy/delay (T/E/D) in networks (e.g., [2], [3], [4]) and particular topology and routing algorithms (e.g., [5], [6]). Thus there exist both bounds on performance and algorithms which attempt to achieve those bounds. The work in this thesis focuses on these bounds as well as the expense of distributing topology and routing information in IWNs, particularly in terms of throughput. Thus we aim to show that in order to come close to achieving the T/E/D bounds in the analytic research, IWNs must have some way of maintaining the network topology and appropriate routing solutions without overusing the valuable network bandwidth for link-state routing protocol (LSRP).

Traditional topology-based IWN routing protocol research can be broken down into three areas (see [5]): proactive (e.g., [7] and [8]), reactive (e.g., [9] and [10]) and hybrid proactive-reactive (e.g., [11] and [12]). In traditional IWN research, nodes communicate topology and route information by sending messages to one another at regular intervals or due to trigger events. The network topology is based on a number of physical factors and Chapter 2 of [3] provides a summary of how real-world RF environments are analyzed. Most analytic IWN research focuses on easily described RF environments, as is explored in this thesis. Once the topology information is determined, routing can then be performed using a number of algorithms, including velocity-aided routing (VAR), predictive mobility and location-aware routing (PMLAR), dynamic source routing (DSR), distance routing effect algorithm for mobility (DREAM) and greedy perimeter stateless routing (GPSR) [5].

## 1.2 The Geographic Location and Prediction Assumption

At present it is difficult to compare the performance of networks with and without GL/GP for several reasons. First, much of IWN research focuses on network performance bounds. In general, when dealing

with bounds, one assumes that the network has whatever information is necessary to meet that bound. IWN performance is traditionally analyzed in the context of limitations based on interference or power (that is, energy use) and the rest of the system is not defined. Second, much of the work in this area assumes that nodes have the necessary information to perform forwarding of packets in the network to their destinations. Often no mention is made of exactly how that information is collected in the first place. Taken to its conclusion, this question of which comes first, the information to route or the routing of that information, one arrives at the cold start problem of IWNs. Further, even if the cold start problem is solved, there still exists the issue of maintaining links in a network which by its dynamic nature will not always know how to send information between every source-destination (S-D) pair in the network. In summary, the use of GL/GP at the nodes is often assumed and may be necessary to achieve performance close to the bounds in the literature.

## 1.3 Thesis Scope and Organization

The following is a brief description of the scope and organization of this thesis, by chapter:

Chapter 2 reviews some key analytical concepts related to IWNs, including graph theory, mobility and traffic models, common IWN performance evaluation criteria, and narrowband and wideband systems. Particular focus is given to the development of the wideband model as derived from the Shannon capacity. While most of the results in this thesis apply to both narrowband and wideband systems, we develop the wideband model to a greater extent as it provides some important insights into the networking problem. This chapter also reviews the prior work which motivated the work in this thesis.

Chapter 3 explores a one-dimensional IWN by way of determining the total hop count and achievable throughput for three routing types: WtNN without GL/GP, WtNN with GL/GP, and characteristic hop

distance with GL/GP. In this chapter we look at nodes communicating only with their nearest right and left neighbors and compare throughput with networks where nodes communicate over a characteristic hop distance.

Chapter 4 provides an analysis of the network capacity burden of distributing protocol information in IWNs using a simple velocity model. First we look at the burden of exchanging protocol information for common routing schemes with a single mobile node and then use a more general approach where every node in the network is mobile. We show that given some simple assumptions about node mobility, the link coherence time across the network scales poorly with the number of nodes and consequently so too does the burden of exchanging routing protocol information.

Chapter 5 summarizes the main results of the thesis and addresses the future of IWN research and applications. Here we qualitatively provide several application-side GL/GP methods for ensuring connectivity in future wireless networks.

# Chapter 2

# Modeling Infrastructureless Wireless Networks

As compared to wired networks, which have a fixed infrastructure, IWNs have a number of additional degrees of freedom which must be accounted for in modeling. Some examples include dynamic links, stochastic node distribution and node mobility. Furthermore, there are important modeling distinctions and assumptions to be made relating to power-limited versus interference-limited networks, as well as considering processing energy and transmission energy consumption. Finally, power usage is often a limiting factor in IWNs that must be optimized.

## 2.1 Modeling using Graph Theory

Graph theory represents an important aspect of IWN research, particularly as it relates to modeling the ad-hoc nature of the node distribution in such networks. We previously defined IWNs as having no fixed infrastructure, prearranged hierarchy, or deployment strategy. In other words, an IWN is a random, pseudo random or arbitrary network as compared with wired networks. Further, unlike wired networks, IWNs are not assumed to be hierarchical in nature. These two assumptions lead to stark differences between an IWN and a traditional infrastructure-based network (IBN). Thus while both IWNs and IBNs can be modeled and analyzed using graph theory, IWN modeling encounters limits of stochastic graph theory that are not an issue in IBNs, which are not generally assumed to have randomly generated

topologies. As part of our research in this area, we look at the two main areas of random graph theory[2]: random geometric graphs (see [13]) and Erdős–Rényi graphs (see [14]).

Perhaps the most important difference in modeling these two types of networks is the lack of hierarchy in an IWN as compared to an IBN, making it generally necessary to model all nodes in an IWN simultaneously. In an IBN, it is possible to break the modeling process into parts based on the hierarchy. Each level of the hierarchy can often be analyzed separately, such as the wide area network (WAN), metropolitan area network (MAN), and local area network (LAN). Whereas in an IWN certain nodes may serve the function of a backbone, not every IWN is organized in this fashion and thus for many applications it is necessary to simultaneously model the entire network

Another important difference between IWNs and IBNs is that IBNs are generally formed almost exclusively from wired links, which are relatively easier to model because their placement is deterministic, not random. In the case where there are wireless links in IBNs, such links are usually one-hop wireless links (that is, access point links). Thus the modeling of wireless links in IBNs is generally simpler because the wireless nodes do not usually communicate with one another over a series of links via other wireless nodes. While wired links and nodes may fail with some probability, this can be incorporated into the modeling. A graph theory based reliability analysis of network architectures is available in [15].

[2] Due to the wireless nature of the network, each node in an IWN is in principle connected to all other nodes with some non-zero rate. In contrast to wired networks, the wireless network is technically fully connected. This makes routing more complicated because of the richness of connectivity and the possibility of much more fluid routing.

### 2.1.1 Erdős–Rényi Graphs

An introduction to Erdős–Rényi graphs can be found in [14], and we present some of the basic concepts from that work in this section. Erdős–Rényi graphs are described by a set of vertices and a set of edges, where the edges are formed in a probabilistic manner. Unlike random geometric graphs, which are discussed in the next section, Erdős–Rényi graphs are not typically well suited for the study of IWNs in free space environments. In general, random geometric graphs more accurately model IWNs, while Erdős–Rényi graphs are well suited to the study of wired networks.

The work in [14] considers graphs where $n$ is the number of vertices and $V = \{1,2,\dots,n\}$ is the set of vertices. The set of all graphs with vertices $V = \{1,2,\dots,n\}$ is given by $\mathcal{G}^n$. The two basic models of Erdős–Rényi graphs are represented by $\mathcal{G}(n,M)$ and $\mathcal{G}\{n, Pr(edge) = p\}$. In the first model, $\mathcal{G}(n,M)$, each graph with $M$ edges is equally probable. Thus given that the maximum number of undirected edges is $N = \binom{n}{2}$, then we have $0 \leq M \leq N$ and there are $\binom{N}{M}$ equally probable graphs of type $\mathcal{G}(n,M)$, each occurring with probability $\binom{N}{M}^{-1}$. In the second model, $\mathcal{G}\{n, Pr(edge) = p\}$, edges are chosen independently and with probability $p$. Thus the number of vertices $m$ is binomially distributed with parameters $n$ and $p$. Finally, in order to relate the two types of graphs, we note as in [14] that $\mathcal{G}\left\{n, Pr(edge) = \frac{1}{2}\right\}$ is the set of all graphs $\mathcal{G}^n$ where every graph is equally probable.

### 2.1.2 Random Geometric Graphs

Random geometric graphs are formed by placing a set of points over a region of space according to some probability distribution and then forming an edge between any two points separated by a distance less than or equal to a specified value. As compared to Erdős–Rényi graphs, random geometric graphs are much better suited to the modeling of IWNs. In particular, random geometric graphs are suitable for

modeling two-dimensional IWNs in free space environments. However, similar to Erdős–Rényi graphs, the author in [13] notes that precise computation of probabilities for a random geometric graph of size $n$ is unfeasible except for small values of $n$. Thus, most of the results in this work focus on large finite graphs. The study of random geometric graphs began with [16]. An in depth analysis of random geometric graphs can be found in [13], while [17] takes a broad look at mathematical modeling of wireless networks. In this section we briefly introduce some of the major tenets of random geometric graph theory found in these works.

As in [13], let $f$ be some specified probability density function on the $d$-dimensional space $\mathbb{R}^d$. We have some sequence $\mathcal{X}_1, \mathcal{X}_2, \ldots$ of $d$-dimensional independent and identically distributed (IID) random variables, distributed according to $f$. We defined the random geometric graph $G(\mathcal{X}_n; r)$ as the graph of vertex set $\mathcal{X}_n = \{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\}$ with undirected edges connecting all nodes separated by a distance not more than $r$. This is known as Gilbert's random disk model and was first proposed in [16]. An overview of the model is given in [17]. As in [17], a random geometric graph can also be described in terms of a Poisson point process $\Phi$. A point process can be visually described as a random collection of points in space. We direct the reader to [17] for the formal definition of a point process and further exploration of random graphs modeled as Poisson point processes. The precise calculation of properties of random geometric graphs is difficult and as a result most research in this area focuses on asymptotic results and thus sequences asymptotic in $n$. That is, given some sequence of parameters $(r_n)$, one considers the properties of $G(\mathcal{X}_n; r_n)$, often as the sequence $(r_n)$ approaches zero. It is noted in [13] that questions concerning connected components of $G(\mathcal{X}; r)$ can be studied in terms of the coverage process of balls of equal radius $r/2$ centered at the points $\mathcal{X}_n$.

Unlike in Erdős–Rényi graphs, where edges are independent or nearly independent, edges in random geometric graphs are not independent. That is, an edge between nodes $X_i$ and $X_j$ implies that nodes $X_i$ and $X_j$ are close to one another (specifically, within some distance $r$). The existence of another edge between $X_j$ and $X_k$ implies that nodes $X_j$ and $X_k$ are also close to one another. Given these two edges, we can say that node $X_i$ is also close to $X_k$ (that is, node $X_i$ is located at most at a distance $2r$ from node $X_k$). The author in [13] argues that the lack of independence between node distances, given the existence of some edges, makes random geometric graphs often more realistic than Erdős–Rényi graphs. This lack of independence is termed the triangle property in [13]. That is, given three nodes $X_i$, $X_j$ and $X_k$, and given edges exist between two of them, the third edge (the hypotenuse of the triangle) is at most length $2r$ and this virtual edge forms a triangle with the other two edges.

## 2.2 Mobility Models in IWNs

One of the open problems in IWN research is developing an appropriate model for mobility in a network. Given a bounded space in which the nodes operate, there are many ways to model mobility within the network and the choice of a particular model has a large impact on simulation results. The mobility model that is applicable in a given situation may be entirely unrealistic in another. Further, a routing or topology algorithm optimized for one mobility model may perform very poorly for another. Given some small assumption set about mobility, it is often possible to derive results about IWNs, particularly results related to link coherence time. Such results, while dependent on the particular mobility model chosen, nonetheless can be helpful in the analysis of network scaling.

Three popular mobility model classes in current research are: random waypoint model (RWP), Manhattan mobility model (MH), and some form of grouped mobility model such as reference point group mobility model (RPGM) [18]. Mobility model selection has a tremendous impact on simulation

results, as one would expect. Variables such as velocity, acceleration and degrees of freedom in movement are extremely important when analyzing the metrics of a given system. While we intend to address the issue of mobility models in IWNs, we aim to develop results that are as general as possible and do not rely on a particular mobility model for correctness.

## 2.3 Throughput, Energy, and Delay

One of the main differences in terms of end-user metrics for analyzing wireless networks, compared to wired networks, is the inclusion of energy as a metric. In general, wired networks are characterized in terms of throughput and delay (or latency). While energy is also an important metric for wired systems, it is not often appreciably noticed by the end user. There is literature focusing on minimizing energy consumption in the WAN and MAN, as in [19]. However, such minimization work is mostly aimed at service providers and not the end user of the system. Furthermore, as is shown in more detail in later chapters, the optimization of T/E/D involves competing objectives. For instance, utilizing mobility to increase throughput is shown in [20] to greatly increase delay. Throughout the work presented here, we explore the T/E/D tradeoff and its effect on the design of future IWNs.

## 2.4 Routing Tables in IWNs

The predominant method of internetworking in the Internet today is Border Gateway Protocol (BGP). For intranetworking, the primary methods of routing today are link-state routing protocols (for example, open shortest path first (OSPF) and intermediate system to intermediate system (IS-IS), which both use the Dijkstra algorithm) and distance vector routing protocols (for example, routing information protocol (RIP), which uses the Bellman-Ford algorithm). These methods of internetworking and intranetworking use a hierarchical structure wherein network devices maintain tables to determine next hops for routing. These tables are utilized to make routing decisions in the network, either by passing to a higher

hierarchy level any destinations not found, or routing as appropriate for those destinations that are found. Such protocols work well for wired networks that have slow dynamics in terms of link changes, but perform poorly when used in systems with fast dynamics, such as IWNs.

## 2.5 Connectivity

A connected network is the precursor for sending information, and in fact the assumption of a connected two-dimensional network lays the groundwork for most of the analysis in this thesis. Thus, it is important to note the difficulty of the issue of connectivity, as well as the contribution of GL/GP information to connectivity improvements. While connectivity in IWNs is not the focus of our work, connectivity analysis is an important area of study for IWNs and has been addressed in works such as [21], [3] and [22]. [22] provides analytic connectivity bounds while also exploring node degree in IWNs and phase transition (that is, the transition region for increasing node degree as a function of transmission distance and number of nodes). The work in [22] and [23] focuses on k-connectivity and the phase transition width of k-connectivity in IWNs. Both of these works assume a fixed transmission range and a finite number of nodes, $n$, which differentiates them from the asymptotic results often found in this area. The results in [23] are for one, two and three dimensions, while [22] focuses on two dimensions, which is our focus here.

A summary of much of the work in the area of IWN connectivity is found in Chapter 3 of [3], which looks at the main results for stationary and mobile nodes, as well as networks with and without helper nodes, among other distinctions. This work also notes those areas where results do not exist, particularly for networks with helper nodes. The impact of beam forming on IWN connectivity is explored in [21], which shows significant improvements in connectivity using beam forming at user nodes. For a fixed rate system, beam forming effectively increases the transmission range of each node based on the directivity

of the antenna array or aperture. As both beam forming and helper nodes benefit from some form of GL/GP in IWNs, it is important to address the method by which GL/GP can improve connectivity in IWNs. Thus we have seen there is previous research which shows that beam forming and helper nodes both dramatically increase the probability of connectivity of a network. In summary, there exists a wide body of literature on connectivity in IWNs for many different types of networks. Here we have provided a brief overview of some of these works and note that while we assume connectivity for many of our results, the issue of connectivity is important and non-trivial for this area of research.

## 2.6 Traffic Models

The analyses in this work focus on one of three traffic models: uniform and symmetric, uniform, and uniform all-to-all. In the uniform and symmetric traffic model, each node selects another node and both send data to one another at an average rate of $\lambda$ [bits/sec] and the routing path is assumed to be the same in both directions (see [3]). For this model there are $n/2$ S-D pairs in a network of $n$ nodes, assuming $n$ is even. In the uniform traffic model, each node arbitrarily chooses a destination and sends data to that destination at an average rate of $\lambda$ [bits/sec] (see [21] and [2]). Under this model there are $n$ S-D pairs in a network of $n$ nodes. In the uniform all-to-all model, each node sends data to every other node at an average rate of $\lambda$ [bits/sec]. Under this model there are $n(n-1)/2$ S-D pairs in a network of $n$ nodes[3]. Arguments can be made for the validity of any of these models or some other model. Ultimately, the choice of a traffic model is driven by both application and the analytic results to be obtained and thus in some cases one model is advantageous over another.

[3] We note that the scaled expectation of the uniform traffic model over all possible S-D pairings is equivalent to the uniform all-to-all model.

## 2.7 Network Types

Throughout much of the research in IWNs, there exists a well-developed method of describing the network under consideration in terms of certain characteristics. In this thesis we focus more on networks with certain characteristics over others but in this section we explain the differences in order to provide the basis for our selections. The purpose of this section is also to provide a brief overview of the main distinctive network characteristics in IWN research.

### 2.7.1 Random Versus Arbitrary

A random network consists of nodes placed according to some distribution or process (such as a PDF or a Poisson process). An arbitrary network is one where nodes are specifically placed; such networks are often used to bound performance by finding the best or worst possible topology for a given network. For example, in [3] and [21] arbitrary networks are used to find upper and lower bounds on throughput, given a specific traffic pattern. Both random and arbitrary network configurations are useful for deriving IWN results. Often random networks can be analyzed using well-developed mathematical tools, and provide general results. Aside from providing bounds, arbitrary networks can be used to analyze a network with a known a priori configuration. Thus, while useful for bounding, for more general uses each arbitrary configuration must be analyzed on a case by case basis, which is infeasible due to the dynamics from mobility and the RF environment.

### 2.7.2 Power-Limited Versus Interference-Limited

There exist two distinct areas under which most current IWN research is classified: interference-limited (or narrowband) and power-limited (or wideband). In Section 2.8 we address the common modeling methods which allow for analytic results for these two network types. Perhaps the most well-known work in narrowband IWNs is [2], from which much IWN research followed. This work utilized both a

protocol model and a physical model (see Section 2.8.1) for modeling a narrowband, fixed IWN with random node placement. The work in [3] provides a framework for power-limited IWNs; the contribution we present here is based on that framework. Such wideband systems are limited not by interference at nodes but by both transmission energy and processing energy for each hop approaching the power constraints at the nodes.

### 2.7.3 Mobile Versus Stationary Nodes

It is important in IWN research to differentiate between mobile and stationary nodes. While mobility is realistic, it increases the complexity of the analysis. Mobility can be used to increase throughput in networks, as in [24], though often at the expense of increased delay. In some cases it may not be necessary to model mobility explicitly, but instead assume some random distribution at every fixed instance of time. As noted qualitatively in [22], a uniformly random distribution does not remain uniformly random under the random waypoint model (as in [25], [26], [27], and [28]). In contrast, [22] asserts that nodes moving according to a random direction model as in [29] and [30] do maintain a uniformly random distribution, but this is also not analytically proven. Maintaining a random uniform distribution while modeling mobility is important because much of the research on the performance of IWNs, even when modeling static networks, assumes a random uniform node distribution (e.g., [2], [31], [23]).

### 2.7.4 Fixed Versus Variable Rate

Regardless of whether a network is modeled as interference-limited or power-limited, there is also the question of whether the nodes transmit at fixed rate or variable rate. That is, whether the nodes always transmit at the same rate or whether they adapt their rate, limited by the maximum rate at which they can transmit based on the Shannon limit (see [32]). In a free-space or homogenously attenuated

environment that is obstacle-free, a wireless network is always connected at some possibly arbitrarily low rate (see [3]). In contrast, a fixed rate system is either connected at the fixed rate or it is not, noting that fixed rate systems are often easier to analyze. We also note that, for analysis purposes, a fixed rate system could have a rate that is dynamically determined by the lowest rate link in the system. While we do not explore it further here, multiple-input and multiple-output (MIMO) systems and/or beamforming could in theory be used together with coding to give an approximately equal rate for all nodes. In general, fixed rate systems are more easily analyzed while variable rate systems are more general, though more difficult to analyze (see [3] for a discussion of this multi-commodity flow problem). Often both analytical approaches yield the same performance results in terms of scaling.

### 2.7.5 Networks with and without Helper Nodes

Given that maintaining connectivity in a wireless network is an important goal, there has been significant research in the area of helper nodes to maintain connectivity and improve performance. The throughput benefits of helper nodes in interference-limited networks have been studied for randomly deployed helper nodes in [2] and fixed base stations in [33]. The T/E/D tradeoff benefits of helper nodes are explored in [3] for strategically deployed helper nodes. This work also analyzes the connectivity benefits of helper nodes for different deployment strategies (e.g., random, deterministic, strategic). In this work we focus on networks with $n$ user nodes, though many of the results may also hold in or be easily modified for networks with helper nodes.

## 2.8 Narrowband and Wideband Modeling

As previously discussed, power-limited networks are networks where it is assumed that every node wishing to transmit at a given time may do so. Such networks are limited by the power necessary to transmit and, in the case of nonzero processing energy, the power necessary for processing. An

interference-limited network is one where interference between nodes is the limiting factor in the network (see [2] for one of the most well-known works on interference-limited networks). In this section we first address two ways that narrowband systems are commonly modeled. Following that, we derive the common method by which wideband systems are modeled. As this thesis focuses on wideband systems, we spend more time developing the power-limited approach to IWNs in order to provide a better foundation for the work in later chapters.

### 2.8.1 Narrowband Systems

There are a number of ways to model interference in narrowband IWNs and in this section we discuss the two models used in [2]. These two models are defined below.

> **Definition 1** - Protocol Model: a transmission from node $X_i$ to $X_j$ over the $m$th sub channel is successful if $|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|$ for all simultaneously transmitting $k$ on the same sub channel.

In Definition 1, $\Delta > 0$ comes from the specification in the protocol of a guard zone to prevent interference by a neighboring node transmitting simultaneously on the same sub-channel. Thus this protocol model definition in [2] uses the idea of a guard band around a receiving node to model interference and deals primarily with distance-dependent interference. It implicitly assumes that nodes have a transmission range (which is appropriate given a homogenous RF environment) and that the interference radius of nodes is a linear multiple of the transmission range.

**Definition 2** - Physical Model: given a set $\{X_l, l \in \mathcal{T}\}$ of nodes simultaneously transmitting at some time instant over a sub channel, and $\{P_l, l \in \mathcal{T}\}$ is the set of power levels chosen by the transmitting nodes, then the transmission from node $X_i$ to $X_j$ over the $m$th sub channel is successful if $\left(\frac{P_i}{|X_i - X_j|^k}\right) \Big/ \left(N + \sum_{\substack{l \in \mathcal{T} \\ l \neq i}} \frac{P_l}{|X_l - X_j|^k}\right) \geq \zeta$.

In Definition 2, $k$ is the path loss exponent and $\zeta$ is the minimum signal-to-interference-plus-noise power ratio (SINR) for successful receptions. The physical model assumes that signal power decays with distance from the transmitter $d$ as $d^{-k}$ where $k > 2$ and that there exists ambient noise of power level $N$. Thus the physical model is in some sense a slightly more sophisticated model that takes into account both the path loss exponent and the ambient environmental noise. We note that the physical model can be viewed as a special case of (16.1) in [34], which is the SINR model shown below:

$$\gamma_i = \frac{g_{ii} P_i}{n_i + \rho \sum_{j \neq i} g_{ij} P_j}, \forall\, i, j \in \{1, 2, \ldots, N\}. \tag{2.1}$$

In (2.1), $g_{kj} > 0$ is the channel power gain from the transmitter of the $j$th link to the receiver of the $k$th link, $P_k$ is the power of the transmitter on the $k$th link, $n_k$ is the noise power of the receiver on the $k$th link, and $\rho$ is the interference reduction due to signal processing (e.g., $\rho \approx 1/G$ for code division multiple access (CDMA) with processing gain $G$ and $\rho = 1$ for time division multiple access (TDMA)). We see that the physical model in Definition 2 is the special case where the gain is given by the distance between the sender and receiver (and the path loss exponent), and $\rho = 1$.

Both of these models are used in [2] to develop scaling results for IWNs. The specifics of these two models provide a point of comparison with power-limited networks. However, we note that there exist

other methods for modeling narrowband systems and that neither model here is necessarily better than the other.

### 2.8.2 Wideband Systems

Based on the work in [3], [34] and [35], in this section we develop the method we predominantly use for modeling transmissions in power-limited networks. We begin with the well known Shannon capacity given by

$$C = B\log_2\left(1+\frac{S}{N}\right)\left[\frac{bits}{s}\right], \tag{2.2}$$

where $C$ is the channel capacity, $B$ is bandwidth in $Hz$, and $S/N$ is the signal-to-noise ratio (SNR).

Now we let $S$ be the receiver power (that is, $S = P_r[J/s]$) and $N = N_oB[J/s]$, where $N_o$ is the noise power spectral density in $[W/Hz]$. Then we rewrite the Shannon capacity as

$$C = B\log_2\left(1+\frac{P_r}{N_oB}\right)\left[\frac{bits}{s}\right]. \tag{2.3}$$

As in [34] we define the normalized Shannon capacity as $C_b = C/B\ [bps/Hz]$ and the received energy per bit as $E_b = P_r\ /R\ [J/bit]$, where $R$ is the rate in bits per second (bps). Finally, we have $\lim_{R\to C} E_b = P_r/C\ [J/bit]$, noting that it is possible to transmit at rates arbitrarily close to the Shannon capacity but not at the Shannon capacity. Then we again rewrite the Shannon capacity and, as in [34] and [35], we solve for the minimum ratio of energy per bit to noise power spectral density in the wideband limit and, taking the limit, we have the following:

$$\lim_{C_b \to 0} \frac{E_b}{N_o} = \ln 2 \left[\frac{1}{bit}\right]. \tag{2.4}$$

Thus we have re-derived the well known minimum ratio of energy per bit to noise power spectral density in the wideband limit. We use this result and parts of the derivation in order to solve for the received power constraint for a successful transmission at a fixed rate.

### 2.8.2.1 Shannon Capacity Power Constraint

Now we assume the wideband limit and let $E_b/N_o = \ln 2 \ [1/bit]$, where $E_b = P_r \ /\mathrm{R}\,[J/bit]$. We note that we previously used $\lim_{R \to C} E_b$ and now return to the case of arbitrary $R$. Then for a given rate $R$, the received power must satisfy

$$P_r \geq RN_o \ln 2 \left[\frac{J}{sec}\right]. \tag{2.5}$$

Per (2.7) in [34], and modifying that equation to include a variable path loss coefficient, we have the following:

$$\frac{P_r}{P_t} = \left[\frac{\lambda\sqrt{G_1}}{4\pi d}\right]^k, \tag{2.6}$$

where $P_r$ is the received power, $P_t$ is the transmitted power, $\lambda$ is the signal wavelength, $\sqrt{G_1}$ is the product of the transmit and receive antenna field radiation patterns in the line-of-sight (LOS) direction, $d$ is the distance between the transmit and receive antennas, and $k$ is the variable path loss coefficient.

Then combining (2.5) with (2.6), we have

$$P_t\left[\frac{\lambda\sqrt{G_1}}{4\pi d}\right]^k \geq RN_o \ln 2 \left[\frac{J}{sec}\right]. \tag{2.7}$$

Now as in [3], we loosen the constraint on $P_t$ and let $P_t = \hat{P}_{avg}$. That is, we assume that nodes have a time-average power constraint $\hat{P}_{avg}$. We note that given a total amount of energy available to a node, and assuming the transmission system has no upper power limit, it is possible to transmit at arbitrarily high rates for arbitrarily short periods of time. However, for a realistic network this would not be a feasible method of operation. For a network which is transmitting at some average rate, we use the time-average power constraint and we have

$$\hat{P}_{avg}{G_1}^{k/2}\left[\frac{\lambda}{4\pi d}\right]^k \geq RN_o \ln 2 \left[\frac{J}{sec}\right]. \tag{2.8}$$

We let $\gamma = \lambda/(4\pi)\,[m]$ and simplify as

$$\hat{P}_{avg}{G_1}^{k/2} \geq Rd^k\left(\frac{N_o \ln 2}{\gamma^k}\right)\left[\frac{J}{sec}\right]. \tag{2.9}$$

Returning to the term $\left(N_o \ln 2/\gamma^k\right)$, we see that it has units of $\left[J/\left(bit \cdot [m]^k\right)\right]$. We note that if we let $\beta = \left(N_o \ln 2/\gamma^k\right)\left[J/\left(bit \cdot [m]^k\right)\right]$ and $G_1 = 1$, we have

$$\hat{P}_{avg} = Rd^k\beta, \tag{2.10}$$

where $\beta$ is the distance-dependent transmission energy. Given zero processing energy, this is the same as (4.9), the predominant maximum transmission range equation in [3]. It is important to note that there are two competing assumptions. We assume that $C_b \to 0$ (that is, we are operating with infinite

bandwidth). However, we also assume that we can assign a value $\gamma = \frac{\lambda}{4\pi}$. We note that for fixed $G_1$, $\gamma = f(\lambda)$, and thus the value is dependent on the transmission wavelength. In real systems the actual bandwidth used is not infinite and the rate is based on the aggregate rate for the entire frequency band used. We also note that the far field assumption, that is $d \gg \lambda$, means that $d$ would grow without bound as $\lambda$ approaches 0. However, if we assume we do not use some finite frequency band around 0 then we can effectively ignore this issue while maintaining the assumption of infinite bandwidth.

### 2.8.2.2 Shannon Capacity Maximum Hop Distance

We now look at (4.9) from [3], which is

$$\rho \leq \rho_{max} \triangleq \left[\frac{1}{\beta}\left(\frac{\hat{P}_{avg}}{R} - \alpha\right)\right]^{1/k}, \tag{2.11}$$

where $\rho$ is the transmission distance, $\rho_{max}$ is the maximum transmission distance, and $\alpha$ is the processing energy in $[J/bit]$. To determine the maximum transmission range, we let $\alpha = 0$. That is, we solve for the maximum hop distance by ignoring the processing energy and letting the transmission energy be the only factor in energy use approaching the power constraint $\hat{P}_{avg}$. Then we have

$$\rho \leq \rho_{max} = \left(\frac{\hat{P}_{avg}}{\beta R}\right)^{1/k}. \tag{2.12}$$

We note that this is (2.10) written in a different form. For the case where $\hat{P}_{avg} < \beta R$, the maximum transmission range increases with increasing $k$; we show why this condition is not possible. We rearrange (2.9) to obtain

$$d \leq \frac{\lambda}{4\pi}\left(\frac{\hat{P}_{avg}{G_1}^{k/2}}{RN_o \ln 2}\right)^{\frac{1}{k}}. \tag{2.13}$$

Then we see that the condition $\hat{P}_{avg} < \beta R$ from [3] is equivalent to $\hat{P}_{avg}{G_1}^{k/2} < RN_o \ln 2$ in (2.13). Now given that

$$\hat{P}_{avg}{G_1}^{k/2}\left[\frac{\lambda}{4\pi d}\right]^k \geq RN_o \ln 2 \left[\frac{J}{sec}\right], \tag{2.14}$$

we see that the condition $\hat{P}_{avg}{G_1}^{k/2} < RN_o \ln 2$ requires $[\lambda/(4\pi d)]^k > 1$ in order to satisfy this inequality. The far field assumption means that $d \gg \lambda$ and therefore, given that $k \geq 2$, $[\lambda/(4\pi d)]^k < 1$. Then we can state the following:

$$\hat{P}_{avg}{G_1}^{k/2} > RN_o \ln 2. \tag{2.15}$$

We note that given $[\lambda/(4\pi d)]^k < 1$, the case where $\hat{P}_{avg}{G_1}^{k/2} = RN_o \ln 2$ is not possible. This makes intuitive sense since a higher attenuation exponent requires greater power to transmit at the same rate over the same distance. Thus we see that it is not possible for the maximum transmission range to increase with increasing $k$ because we assume far field operation.

Given the per node average power constraint $\hat{P}_{avg}$, a transmission at a distance $d$ is correctly received if

$$d^k \leq \frac{\hat{P}_{avg}{G_1}^{k/2}\gamma^k}{RN_o \ln 2}\,[m^k], \tag{2.16}$$

Solving for $d$ and substituting in for $\gamma$, we have

$$d \leq \frac{\lambda}{4\pi}\left(\frac{\hat{P}_{avg}{G_1}^{k/2}}{RN_o \ln 2}\right)^{\frac{1}{k}} \triangleq \rho_{max}[m], \tag{2.17}$$

Because of the importance of ensuring $\hat{P}_{avg}{G_1}^{k/2} > RN_o \ln 2$, we express the maximum transmission range as above. We note that because $\hat{P}_{avg}{G_1}^{k/2} > RN_o \ln 2$, we can take the $k^{th}$ root of both sides of the inequality without affecting the inequality. In order to find the maximum hop distance possible in a network, we have ignored processing energy and only focused on distance-dependent transmission energy.

## 2.9 Sparse Versus Dense Networks

A key question to answer in IWN research is whether networks should be assumed to be sparse or dense[4]. The answer to this question determines whether routing solutions can be locally updated due to a change in topology (for the case of dense networks) or whether they must be globally updated (for the case of sparse networks). That is, it determines whether the network can route around link state changes or whether the entire routing solution must be recomputed. Thus for a dense network, given a topology change, the network can avoid generating topology and routing update traffic for all nodes by only generating updates for nodes in the vicinity of the change. For a sparse network, information about a topology change must be propagated to every node in the network. In some sense, one can view a dense network as divided into sub-graphs, where a local change in a particular sub-graph does not affect, or minimally affects, the optimal routing solution. A sparse network, by contrast, cannot be divided into sub-graphs for the purposes of topology change propagation.

[4] Sparse versus dense networks are further discussed in a graph theory context in [13].

## 2.10 Geographic Location and Geographic Prediction Information

In order to determine the benefits of GL/GP information in IWNs, we provide a general definition which can be applied in a number of different types of networks. First, we use GL/GP information as a single idea because of the relationship between geographic location and geographic prediction. That is, given perfect geographic information, a network would have no need for geographic prediction because the location information would never be "stale", or incorrect. Likewise, given a starting location and perfect geographic prediction, there would be no need for additional geographic information. Putting the two together, some combination of regular location updates and appropriate metrics for prediction would allow for the nodes in a network to maintain correct information about the location of all other nodes in the network. Combined with an accurate view of the network RF topology[5], the goal of geographic location and geographic prediction information is to reduce or eliminate the need for additional signaling among nodes for the purpose of maintaining an accurate network topology.

The distribution of GL/GP information in an IWN is an important challenge to which there is likely no single best solution. The GL/GP information could be distributed by the nodes in the network, using the links between the nodes or, for example, a separate low rate channel with longer transmission ranges. Another method is to use, for instance, an airborne node to keep track of and provide updates for the network topology, which reduces the load on the network for distributing this information. Regardless the method used, the important concept is to reduce the network burden of maintaining an accurate network topology view at the nodes in the network. We focus on the advantages of utilizing such information in a network while noting that the method of information distribution will likely be application-specific. What we have not yet quantified is the gain in network throughput using

[5] See [3] for a discussion of methods for calculating, modeling and estimating the RF topology.

geographic information. While significant LSRP signaling capacity via flood routing is saved, there is still a network burden for passing GL/GP information that reduces this gain.

## 2.11 Prior Motivating Work

In this section we summarize some of the previous work in the area of IWNs in order to compare performance between networks with and without GL/GP. We primarily focus on power-limited networks, as we do in other chapters, while also addressing interference-limited networks. The goal of this section is to analyze the performance gain of GL/GP in networks; we address the issue of protocol distribution in Chapter 4. Thus in this section we assume the a priori distribution of topology and routing information in the networks. In Chapter 4 we show the scaling issues posed by the distribution of this information. In this section we do not provide an exhaustive review of connectivity and T/E/D tradeoffs in IWNS, as such a review is provided in Chapter 3 and Chapter 4 of [3], respectively. However, we review some of the important results of the works in this area.

### 2.11.1 Narrowband Throughput Scaling

In this section we look at previous results in narrowband network scaling, both from the perspective of zero processing energy and non-zero processing energy. We also look at the validity of certain assumptions in order to compare the work in these areas. In this section and others, it is important to note how scaling results change based on the network assumptions. The optimal scaling results in narrowband systems are not the same as those in wideband systems, nor are the results the same for zero and non-zero processing energy, and thus it is important to differentiate among the network types.

### 2.11.1.1 Stationary Nodes and Zero Processing Energy

For stationary, randomly distributed nodes with the zero processing energy assumption, we use the results of what is considered by many to be the seminal work in IWNs. In [2], the authors explore the scaling results of fixed random and arbitrary networks, where random networks are the focus here. First, as discussed in Section 2.8.1, there are two main models of narrowband systems used in [2]: the protocol model in Definition 1 and the physical model in Definition 2. The main results of this work hold for random networks on $S^2$, which is the surface of a three-dimensional sphere of area one square meter. Note that $\bar{L}$ is the mean distance between two points independently and uniformly distributed on the unit area sphere. The following results also hold on a planar disk of unit area, where in the planar disk case $\bar{L}$ is the mean distance between two points independently and uniformly distributed on the planar disk of unit area. In the case of random networks, each node has a randomly chosen destination to which it wishes to send data at a rate of $\lambda(n)$ bits per second (that is, the uniform model discussed in Section 2.6). For both the protocol model and the physical model, the throughput capacity scales as[6]

$$\lambda(n) \in \Theta\left(\frac{R}{\sqrt{n \log n}}\right)[bits/second], \tag{2.18}$$

Thus we see that for interference-limited networks, the need to share the channel causes a reduction in per-node throughput as the number of nodes in the network increases. This means that for the interference-limited case, the throughput per node, $R$, must scale as $\Theta\left(\sqrt{n \log n}\right)$ in order to maintain constant data rate between every source-destination pair.

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[6] We define big O notation terms used in this paper and in previous works, using the conventional notation:
*Asymptotic upper bound*: $f(n) \in O(g(n)) \rightarrow \exists k > 0, n_0 : f(n) \leq g(n) \cdot k \; \forall n > n_0$
*Asymptotic lower bound*: $f(n) \in \Omega(g(n)) \rightarrow \exists k > 0, n_0 : g(n) \cdot k \leq f(n) \; \forall n > n_0$
*Asymptotic upper and lower bound (tight bound)*: $f(n) \in \Theta(g(n)) \rightarrow \exists k_1, k_2 > 0, n_0 : g(n) \cdot k_1 \leq f(n) \leq g(n) \cdot k_2 \; \forall n > n_0$
We also adopt the convention of using set notation for Big O notation, that is $f(x) \in \{\cdot\}$, instead of the notation $f(x) = \{\cdot\}$. Thus, for example, we write $T(n) \in \Theta(D(n)/n)$ instead of $T(n) = \Theta(D(n)/n)$.

We now look at the results in [31] (which was later published in [36] as Part II, or the constant-size packets model), which gives the throughput-delay scaling in static wireless networks with constant size packets. This work is based on previous work in [4] (which was later published in [20] as Part I, or the fluid model). The authors of [31] use what is referred to as the relaxed protocol model, which is the protocol model from [2], as in Definition 1. This work shows the following for a static random network:

$$T(n) \in \Theta\left(\frac{D(n)}{n}\right), \qquad \text{for } T(n) \in O\left(\frac{1}{\sqrt{n \log n}}\right). \tag{2.19}$$

This scaling holds for the entire range of achievable throughputs, which is lower bounded by $1/n$ and upper bounded by $1/\sqrt{n \log n}$. Then for the highest achievable per node throughput in a fixed network of $\Theta\left(1/\sqrt{n \log n}\right)$, as in [2], the average delay is given by $D(n) \in \Theta\left(\sqrt{n/\log n}\right)$. This also shows that in order to achieve throughput of $\Theta(1)$ as in [24] (see Section 2.11.1.2), the delay scales as $\Theta(n)$.

### 2.11.1.2 Mobile Nodes and Zero Processing Energy

In this section with look at the scaling of networks with mobile nodes and zero processing energy. We note that in the previous section on static networks, the scaling does not take into account the generally dynamic nature of infrastructureless wireless networks. Thus such scaling results are applicable in certain cases, such as sensor networks which, absent periodic node failures, behave in a static manner. However, for many applications the users are mobile and thus it is important to characterize the scaling in the case of dynamic networks.

In [24], the authors utilize the mobility of nodes to enable routing which results in better throughput scaling than in the case of fixed nodes, as in [2]. The authors show that direct communication achieves poor throughput scaling and that two-hop routes achieve the maximum throughput capacity of an

interference-limited network. In this work the authors show that it is possible to schedule $\Theta(n)$ concurrent successful transmissions per time slot with local communication between nodes. With a two-hop relay (that is, source to random relay to destination) the total throughput achieved is $\Theta(n)$ and thus the per-node throughput achieved is $\Theta(1)$. The authors note that the mobility model of IID node trajectories is idealized. However, later work in [37] shows the same $\Theta(1)$ scaling with a mobility model where each node moves randomly on a single-dimensional great circle on the unit sphere, instead of the entire two-dimensional area. As noted in [24], routing using mobility has the drawback of greatly increasing delay. We now review the results in [20] for the throughput-delay tradeoffs in mobile networks. The first range of $T(n)$ is the same as that in (2.19) and the results are also the same. Thus we have

$$T(n) \in \Theta\left(\frac{D(n)}{n}\right), \qquad \text{for } T(n) \in O\left(\frac{1}{\sqrt{n \log n}}\right). \tag{2.20}$$

For the increased range of throughput provided for my mobility, that is $T(n) \in \omega\left(1/\sqrt{n \log n}\right)$ and $T(n) \in O(1/\log n)$, the optimal throughput-delay tradeoff is

$$T(n) \in \Theta\left(\frac{1}{\sqrt{na(n) \log n}}\right), \qquad D(n) \in \Theta\left(n \log \frac{1}{a(n)}\right). \tag{2.21}$$

The optimal throughput-delay tradeoff is parameterized by $a(n)$, where $\sqrt{a(n)}$ corresponds to the average distance traveled in one hop; it is bounded as $a(n) \in \Omega(\log n/n)$ and $a(n) \leq 1$. The authors note that when mobility is used to increase throughput beyond $\Theta\left(1/\sqrt{n \log n}\right)$, that is to $\Theta\left(1/\sqrt{n}\right)$, the delay immediately jumps to $\Theta(n \log \log n)$. For the constant throughput of $\Theta(1)$, the delay is $\Theta(n \log n)$.

### 2.11.1.3 Stationary Nodes and Non-Zero Processing Energy

Based on this throughput-delay tradeoff work, [38] explores the T/E/D tradeoff in wireless networks, using the physical model from Section 2.8.1. This work also takes into account processing energy in the narrowband scaling case. It is thus a work that bridges some of the differences between interference-limited networks, which generally do not focus on power, and power-limited networks, which generally assume unlimited bandwidth. This analysis is for fixed random networks and the key results are reviewed in [3]. For networks operating in a bounded region, taking $\Theta(1)$ number of hops achieves the optimal packet delay $\mathrm{D}(n)$ and energy per bit $\xi(n)$ scaling of $\Theta(1)$, with throughput of $\Theta(1/n)$. That is, we have the following for $T(n) \in \Theta(1/n)$:

$$D(n) \in \Theta\big(nT(n)\big), \;\; \xi(n) \in \Theta\big(D(n)\big). \tag{2.22}$$

Using whisper to the nearest neighbor (WtNN), or increasing the number of hops with $n$, achieves the maximum throughput scaling at the expense of delay and energy as follows:

$$D(n), \xi(n) \in \Theta\left(\sqrt{\frac{n}{\log n}}\right), \;\; \text{for } T(n) \in \Theta\left(\frac{1}{\sqrt{n \log n}}\right). \tag{2.23}$$

Thus, per [3], [38] shows that for interference-limited networks, taking fewer hops improves delay and energy scaling at the expense of worse pairwise throughput scaling.

## 2.11.2 Wideband Network Scaling

In this section we review the results for wideband throughput scaling. The purpose of this section is to compare the scaling results for WtNN routing and optimal routing in power-limited IWNs. Section

2.11.1.3 reviewed the scaling results for narrowband systems with non-zero processing energy. Here we see how the assumption of zero versus non-zero processing energy affects the optimal routing scheme and the resulting network scaling. While previous work has shown that WtNN is the optimal scheme in narrowband networks with zero processing energy, we review work that shows WtNN is suboptimal for wideband networks with the non-zero processing energy assumption.

## 2.11.2.1 Optimal Throughput with Non-Zero Processing Energy

In [39], the authors derive the optimal capacity scaling for power-limited wireless networks. This work shows that taking $\Theta(1)$ number of hops is throughput, energy, and delay optimal and such a scheme achieves $\Theta(1)$ uniform capacity with high probability under uniform and symmetric traffic. The definition of with high probability is as follows:

> **Definition 3** - An event is said to occur with high probability ($whp$) if the probability of the event satisfies $Pr\{\cdot\} \geq 1 - \frac{1}{n}$.

Direct transmission routing achieves $\Theta(1)$ throughput scaling with probability 1 (WP1), but it is not energy optimal. The uniform traffic model is also found in [2] and, per Section 2.6, is the model where each node arbitrarily selects a destination node and sends data to it at an average rate of $\lambda$. The uniform capacity, $\hat{\lambda}$, is the maximum $\lambda$ that every SD pair can sustain over all routing strategies. The achievable maximum $\lambda$ for a specific routing scheme is termed uniform throughput. Then we can define uniform throughput as $\hat{\lambda} \triangleq \max_{\lambda}\{Achievable\ \lambda\}$. The uniform throughput for several routing strategies is shown in [3]. The results from this work for WtNN routing are shown in Section 2.11.2.2. This can be compared to the uniform capacity (that is, the maximum uniform throughput over all routing strategies) which is $\Theta(1)$. The authors also showed that for fixed transmission rate, if signal processing energy is non-zero,

there exists a rate $R^* = \hat{P}_{avg}(k-1)/(k\alpha)$ that maximizes the upper bound. There is a corresponding characteristic hop distance defined as follows:

$$\rho^*_{max} = \left(\frac{\alpha}{\beta(k-1)}\right)^{1/k} \triangleq d_{char}. \tag{2.24}$$

Thus, for a fixed rate $R$, taking a smaller number of long hops achieves a higher throughput in general because pass-through traffic constrains the pairwise throughput (see [3]).

### 2.11.2.2 Whisper to the Nearest Neighbor Throughput

We review the results for the throughput scaling of the WtNN routing approach. While the term WtNN has been used in various works on the topic of IWNs, there is often the implicit assumption of some form of GL/GP or a complete topology map at every node. WtNN is the routing scheme where one takes a larger number of short hops in order to decrease interference and power consumption in interference-limited and power-limited networks, respectively (see [39], [2], [40]). At each hop in the network the shortest distance hop is selected and therefore, as node density increases, the number of hops increases. A comparison of WtNN with and without the zero processing energy assumption is provided in [3], noting it was shown in [40] that if signal processing energy is zero, as $n$ increases network capacity increases when taking shorter hops. In the next section, we review the results from [39] in a power-limited network with non-zero processing energy, which show that it is sub optimal. Per [39], whisper to the nearest neighbor is shown to be optimal for interference-limited networks in [2] and for power-limited networks with zero processing energy in [40].

The work in [40] provides an analysis of the throughput scaling of wideband random networks with the assumption of zero signal processing energy. This work utilizes the protocol model from Section 2.8.1,

while noting that the scaling results of the physical model are the same. Further, [40] allows for variable transmission rate based on distance, given some fixed maximum transmission power. In order to show the throughput scaling, it is assumed that each node is constrained to a maximum transmit power $P_0$ and the bandwidth is arbitrarily large. Thus we note that this work implicitly assumes that signal processing energy is negligible, as was assumed in [2], which dealt with interference-limited networks. In the limit of large bandwidth, [40] shows that the scaling for uniform throughput capacity is

$$r(n) \in O\left(\left(\sqrt{n \log n}\right)^{k-1}\right), r(n) \in \Omega\left(\frac{(\sqrt{n})^{k-1}}{\left(\sqrt{\log n}\right)^{k+1}}\right) \tag{2.25}$$

Thus it was shown in this work that the throughput in an IWN increases with increasing number of nodes, in contrast to [2].

### 2.11.2.3 WtNN Throughput with Non-Zero Processing Energy

In [3] and [39], the throughput scaling results are developed for the network in Section 2.11.2.2[7], while taking into account processing energy at the nodes. In this section we summarize those results and some of the key parts of the derivation of those results. We now begin with the following Lemma, which is Lemma 6 in Appendix B.3 of [3]:

[7] The authors in [39] also reference [44] as a point of comparison for wideband IWN throughput results without the consideration of processing energy.

**Lemma 1** - Consider a random network with n nodes independently and randomly distributed on a unit torus according to a uniform distribution. The torus is divided into square cells of area $a(n)$.

(i) If the cell size $a(n) \geq 2\frac{ln\, n}{n}$, then each cell has at least one node $whp$ [4].

(ii) If $a(n) = A$, then each cell has at least $\frac{nA}{2}$ nodes $whp$.

(iii) If $a(n) = 2\frac{ln\, n}{n}$, then each cell has at most $6\, ln\, n$ nodes $whp$.

(iv) The number of SD lines passing through any cell is $O\left(n\sqrt{a(n)}\right)$, $whp$ [4].

The derivation in [3] of the pairwise throughput of random, power-limited networks uses the scaling results of S-D lines passing through a cell from [4]. We note that in Part (i) of Lemma 1, the lower bound results from the case where routing between cells is performed optimally, in the sense that if a line is drawn between an S-D pair, traffic moves from cell to cell along that line. That is, for every cell through which the line connecting an S-D pair passes, the S-D pair traffic passes through that cell. Using this, [39] shows that the uniform throughput scaling for a random network using WtNN routing is $O\left(\sqrt{\ln n/n}\right)$ with high probability. Thus we see that when processing energy is taken into account, the throughput scales poorly with $n$ as compared to the $\Theta(1)$ throughput scaling, which occurs with high probability for any scheme with $\Theta(1)$ hops.

## 2.11.2.4 Processing Energy Considerations

As seen in the previous sections, whether or not processing energy is taken into account has a large impact on the scaling of a wireless network. For a power-limited network, we note that it is the difference between uniform throughput scaling and throughput scaling that increases with an increasing number of nodes. The question of whether or not to consider processing energy is one which can be

addressed from first principles. In [3], the author compares the fundamental limit for the energy required for a binary switch transition in electronic signal processing and the fundamental limit for transmitted energy per bit. The result is that, due to thermal noise inherent in any physical system, neither signal processing energy nor transmission energy is negligible. As the consideration is the difference between unbounded scaling and uniform scaling, as well as between WtNN routing and routing using a characteristic hop distance, it is vital to consider processing energy.

## 2.12 The Case for GL/GP Information

We saw in Section 2.11.2 that WtNN networks have poor throughput scaling when processing energy is considered, while networks with characteristic distance hopping have better throughput scaling (that is, uniform throughput scaling $whp$). This difference in throughput scaling is important for showing one of the advantages of GL/GP information in IWN. In the absence of such information, using characteristic distance hopping is not an option because nodes do not know the distances to their neighbors. Thus either WtNN or another, less optimal approach will be utilized for routing. This does not mean that WtNN is the only viable routing option in the absence of GL/GP information. However, it is among a set of available suboptimal routing schemes in the absence of such information. Thus even if another routing scheme with $\Theta(1)$ hops is used, which results in $\Theta(1)$ throughput scaling $whp$, such a scheme will not be energy and delay optimal. Therefore, we see that GL/GP allows the use of characteristic distance hopping, which is throughput optimal, while without such information the network must in general use a suboptimal routing scheme.

## 2.13 Summary

In this chapter we described some of the basic areas of IWN modeling, network performance, and network types. Section 2.1 provided a review of the major tenets of graph theory, which forms the basis

for much of IWN research, particularly as it relates to node distribution. We also introduced the concept of mobility models and the metrics of throughput, energy, and delay for measuring network performance. In Section 2.6 we looked at three traffic models for IWNs and in Section 2.7 we provided descriptions of five of the main network descriptors: random versus arbitrary node distribution, power-limited versus interference-limited modeling, mobile versus stationary nodes, fixed versus variable rate, and networks with and without helper nodes. We further developed the idea of power-limited (narrowband) versus interference-limited (wideband) networks as this is one of the most important differentiators in IWN research. Finally, we looked at some of the prior work in this area which motivates the work in this thesis, including results which can be used to compare the performance of two-dimensional networks with and without GL/GP. In summary, this chapter provides a background on IWNs and serves as the basis for the chapters to follow.

# Chapter 3

# One-Dimensional Network Throughput Scaling

Our first analytic results come from a one-dimensional, or line network. Such a network allows the development of results that are less tractable in higher dimensions. We develop throughput scaling results for three network types: WtNN without GL/GP information, WtNN with GL/GP information, and characteristic hop distance routing with GL/GP information. We use the term GL/GP to denote that every node in the network knows the location of every other node in the network. While we deal with a static network in this analysis, the results can be extended to networks with mobile nodes. This work differs from much of the work in the area of IWNs because we provide deterministic throughput scaling results.

## 3.1 Probability Distribution and Order Statistics

We define a line network to be $n$ end-user nodes randomly and uniformly distributed along a line bounded by $[0,1]$. Each node's location is selected independently of all other nodes and distributed according to a continuous uniform distribution. We denote the $i^{th}$ unordered node, and its location, as $X_i$ and we note that the locations $\{X_1, \dots, X_n\}$ are IID random variables. We call this unordered set $\mathbb{X}$, such that $\mathbb{X} = \{X_1, \dots, X_n\}$. We further define the ordered set of node locations as $\{X_{(1)}, \dots, X_{(n)}\}$ such that $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n-1)} \leq X_{(n)}$. We call this set $\mathbb{X}_O$ such that $\mathbb{X}_O = \{X_{(1)}, \dots, X_{(n)}\}$. For the

remainder of this chapter, we use $(i)$ to denote the sorted index and $i$ to denote the unsorted index. That is, when random variables are labeled with indices, we differentiate between the sorted and unsorted indices by using the two types of labels. We also use $X_{(i)}$ and $X_i$ to refer to both the node and its location. Finally, for the remainder of this work, we assume the following when referring to node location $X_{(i)}$:

$$i, n \in \mathbb{Z}^+, i \in [1, n]. \tag{3.1}$$

Next we define a set of random variables representing the distance between neighboring nodes, $\{Y_i\}_{i \epsilon B}$, where $B \in \{1, \dots, n-1\}$ and $Y_i = X_{(i+1)} - X_{(i)}$. We denote left and right to be the directions of decreasing and increasing distance from the origin, respectively, in the line network. Thus each $Y_i$ is the distance from each node to its nearest neighbor to the right. Per [21] we note that each $Y_i$ is identically distributed but that they are not independent. To see their interdependence, consider the case of one very large $Y_i$, which implies shorter spacing between other pairs on the bounded line.

We next define $X_{(n)}$ as $X_{(n)} = \max(X_1, X_2, \dots, X_n)$ and analyze its convergence. We show that $X_{(n)}$ converges in probability to 1. Noting that $X_{(i)} \leq 1$, we take the limit over $n$ of $\Pr\{|X_{(n)} - 1| > \varepsilon\}$ for any $\varepsilon > 0$. Given $(X_{(n)} - 1) \leq 0$, we can write

$$\lim_{n \to \infty} \Pr\{|X_{(n)} - 1| > \varepsilon\} = \lim_{n \to \infty} \Pr\{\max(X_1, X_2, \dots, X_n) < 1 - \varepsilon\}. \tag{3.2}$$

Then given that the positions $\{X_1, X_2, \dots, X_n\}$ are all independent, we have the following:

$$\lim_{n\to\infty} \Pr\{|X_{(n)} - 1| > \varepsilon\} = \lim_{n\to\infty} \prod_{i=1}^{n} \Pr\{X_i < 1 - \varepsilon\}. \quad (3.3)$$

Finally, due to the uniform distribution, we note that $\Pr\{X_i < 1 - \varepsilon\} = (1 - \varepsilon)$. Then we have

$$\lim_{n\to\infty} \Pr\{|X_{(n)} - 1| > \varepsilon\} = \lim_{n\to\infty} (1 - \varepsilon)^n = 0. \quad (3.4)$$

Thus we have proved that $X_{(n)} \xrightarrow{P} 1$. This effectively says that, in the limit of large number of nodes $n$, one of the $n$ nodes is arbitrarily close to the value of 1. We also note that due to the distribution being continuous, $Pr\{X_j = 1\} = 0 \; \forall \; j$ and therefore $\lim_{n\to\infty} Pr\{X_{(n)} = 1\} = 0$.

## 3.2 Maxima and Their Application to the Analysis

Having shown that $X_{(n)} \xrightarrow{P} 1$, we can further analyze this convergence by looking at how the new maxima occur (see Chapter 1 of [41]). The following analysis is undertaken because it is helpful to understand whether there are a finite number of maxima in the limit as $n$ approaches infinity or whether new maxima continue to occur. We again note that $\{X_1, X_2, \ldots, X_n\}$ are IID continuous random variables with a common PDF $f_X(x)$. Because of this, $Pr\{X_i = \alpha\} = 0 \; \forall \; \alpha \in [0,1]$ and $Pr\{X_i = X_j\} = 0 \; \forall \; i \neq j$.

We define $X_n$ as a record-to-date of the sequence $\{X_1, X_2, \ldots, X_n\}$ if $X_n > X_i \; \forall \; i < n$. We wish to find the probability that $X_n$ is a record-to-date for $n \geq 1$. We define an indicator random variable such that

$$\mathbb{I}_n = \begin{cases} 1, & X_n \; is \; a \; record - to - date, \\ 0, & otherwise \; . \end{cases}$$

Then the probability that $X_n$ is a record-to-date is given as

$$\Pr\{\mathbb{I}_n = 1\} = \Pr\{X_1 < X_n, X_2 < X_n, \dots, X_{n-1} < X_n\} = 1/n. \tag{3.5}$$

The result is obtained via an argument by symmetry. That is, each $X_j$ has an equal probability of being the record-to-date, given that the set of values $\mathbb{X} = \{X_1, X_2, \dots, X_n\}$ is a set of IID random variables.

This argument shows that the probability of any given $X_j$ in the set $\mathbb{X}$ of size $n$ being a record-to-date is equal to that of any other, for a given value of $n$. Finally, we wish to find an expression for the expected number of records-to-date that occur over the first $m$ trials (that is, the first $m$ nodes to be added to the network) for any given integer $m$. We solve for the expected number of records-to-date over the first $m$ trials:

$$E\left[\sum_{n=1}^{m} \mathbb{I}_n\right] = \sum_{n=1}^{m} E[\mathbb{I}_n] = \sum_{n=1}^{m} Pr\{\mathbb{I}_n = 1\}. \tag{3.6}$$

Taking this expectation in the limit of $m$ we have

$$\lim_{m\to\infty} \sum_{n=1}^{m} Pr\{\mathbb{I}_n = 1\} = \lim_{m\to\infty} \sum_{n=1}^{m} \frac{1}{n} = \infty. \tag{3.7}$$

We note that the harmonic series is divergent and thus there are an infinite number of maxima in the limit of $m$. We have thus shown that the probability of the $n^{th}$ node to be placed being the record-to-date is $1/n$. However, we have also shown that the expected number of times a newly added node becomes a record-to-date is infinite. This analysis tells us that in some cases, if we wish to analyze the network, we should not do so in the sense of adding nodes one at a time to the network but rather by

considering the network as a set of nodes already placed. We also note that the analysis may be simpler when viewing the network as a set of $n$ nodes rather than adding one node after another.

## 3.3 Ordering, Neighbors, and Assumptions

Given $n \geq 3, i \in [2, n-1]$, we know that the node located at $X_{(i)}$ has exactly two neighbors: one to the left and one to the right. We further note that each of the $n!$ orderings of the $n$ nodes in the network is equally probable. That is, in the case of $n = 3$:

$$\Pr\{X_1 < X_2 < X_3\} = \cdots = \Pr\{X_3 < X_2 < X_1\} = \frac{1}{6}. \tag{3.8}$$

This argument can be extended to $n$ nodes. If we let $\mathbb{O}_n^k$ be the arbitrarily labeled $k^{th}$ ordering of the $n$ nodes where $k \in \{1,2, \ldots, n!\}$, then

$$\Pr\{\mathbb{O}_n^1\} = \Pr\{\mathbb{O}_n^2\} = \cdots = \Pr\{\mathbb{O}_n^{n!}\} = \frac{1}{n!}. \tag{3.9}$$

We can also calculate the number of nodes to the right, $\mathcal{N}_R^{(i)}$, and number of nodes to the left, $\mathcal{N}_L^{(i)}$, of the node located at $X_{(i)}$. By inspection, we see that for $i \in [1, n]$,

$$\mathcal{N}_R^{(i)} = n - i, \tag{3.10}$$

$$\mathcal{N}_L^{(i)} = n - \mathcal{N}_R^{(i)} - 1 = i - 1. \tag{3.11}$$

Finally, we define the following index sets:

$$\left\{\mathcal{N}_R^{(i)}\right\} = \{i+1, \ldots, n\}, \tag{3.12}$$

$$\left\{\mathcal{N}_L^{(i)}\right\} = \{1, \ldots, i-1\}. \tag{3.13}$$

For the remainder of this chapter, we assume that $n \geq 2$. A scenario where $n = 1$ is not really a network at all and therefore this assumption does not result in any loss of generality.

### 3.3.1 Nearest Neighbors

Now that we have looked at some of the basic distribution statistics, we define the nearest neighbor as the node that is closest to a given node location $X_{(i)}$. Formally:

**Definition 4** - $D_{(i)}$ is a random variable that takes the value of the direction of the nearest neighbor from node location $(i)$ such that $D_{(i)} = \begin{cases} R, & \textit{nearest neighbor is to the right,} \\ L, & \textit{nearest neighbor is to the left.} \end{cases}$

The end nodes in a one-dimensional line network only have one nearest neighbor and the direction of that neighbor is deterministic (see Part (i) of Lemma 2). The lemma, proved in Appendix A.1, is as follows:

**Lemma 2** - Given a one-dimensional network of independent and uniformly distributed nodes where $n \geq 3$, we have the following:

(i) $D_{(1)} = R$ and $D_{(n)} = L$ occur surely.

(ii) $Pr\{D_{(i)} = L\} = Pr\{D_{(i)} = R\} = 0.5, \forall i \in [2, n-1]$.

In the sections to follow, we will use this lemma in determining the hop count and throughput for networks without GL/GP.

## 3.3.2 One-Dimensional Assumptions

In order to compare the performance of networks with and without GL/GP, we make some assumptions about the particular networks in question, most notably with regards to connectivity. The assumption of a connected graph may at first appear to be an unrealistic assumption due to the fact that establishing and maintaining connectivity in IWNs is not trivial (see [21] and [3]). However, the purpose of this work is not to analyze IWN connectivity. Rather, given a connected network, the purpose of our work is to show how GL/GP improves network performance. While it is clear that GL/GP can enable other techniques that improve connectivity, noting that it enables beamforming (as in [21]) and strategic helper node deployment (as in [3]), we aim to show that GL/GP also offers advantages beyond improvement of connectivity performance.

### 3.3.2.1 Connectivity and Topology

We assume that for all three networks types (WtNN routing without GL/GP, WtNN routing with GL/GP, and characteristic hop distance routing) the network is connected. We define connectivity using the characteristic hop distance, where this also means that WtNN networks are connected, by the definition of the WtNN routing protocol. We define connectivity as

$$Y_i \leq d_{char} \; \forall i \in \{1, \ldots, n-1\},$$

where $d_{char}$ is the characteristic hop distance defined in (2.24). In all networks we assume a fixed communication rate $R$. We further assume that for networks with characteristic hop distance routing, the nodes in the network can communicate with any of their one hop neighbors that are within a hop distance $d_{char}$. For networks with WtNN routing, we assume that all neighbor nodes are able to communicate with a rate R. However, given the WtNN routing protocol, we assume that nodes only communicate with their closest left and right neighbors and the end nodes (located at $X_{(1)}$ and $X_{(n)}$) only communicate with one other node. That is, we assume that nodes can increment their transmission range, via transmission power control, in steps of $\delta$ such that

$$\Pr\{|Y_i - Y_{i-1}| \leq \delta\} = 0 \; \forall i \in [2, n-1]. \tag{3.14}$$

Thus when incrementing the transmission range in steps of size $\delta$, the probability of finding more than one new neighbor in a given step is zero. These assumptions mean that the nodes in the network increase transmission range, starting from 0, until the topology formed is a line network and each node then transmits at rate $R$.

We note that the topology resulting from these assumptions is a unique line network where each node located at $X_{(i)}, i \in [2, n-1]$, only communicates with its left and right nearest neighbors. That is, given a set of node locations $\{X_1, \ldots, X_n\}$, the network topology created by these assumptions is deterministic. We neither assert that creating a network utilizing these assumptions is optimally efficient, nor that the resulting network is optimal for any particular metric. However, these assumptions result in a one-dimensional network topology that is useful for evaluating the whisper to the nearest neighbor routing scheme.

### 3.3.2.2 Routing in Networks without GL/GP

We assume that, for networks without GL/GP, routing is performed by sending packets to a node's nearest neighbor. That is, nodes do not have any information about the location of their destinations and there is no distributed sharing of topology information in the network. In addition, we assume that packets can be uniquely labeled and identified such that if a node receives a packet it had previously sent to its nearest neighbor, it then sends it to its other neighbor. A node only communicates with its neighbors to the left and right (per the topology creation described previously) and does not send a packet to the node from which it just received the packet, unless it is an end node. Thus if a packet reaches one end node of the network and it is not destined for that end node, then the end node sends the packet back the way it came. Intermediate nodes follow the same rules and do not send a packet back to the node from which they received it, and in this way the packet traverses the network. That is, given a first hop in a particular direction, the traffic continues to flow in that direction until it reaches the end of the network, at which point it travels back in the opposite direction if it has not yet reached the intended destination. Per Lemma 2, we note that in the one-dimensional case, the a priori probabilities that a node's nearest neighbor is to the left or right are equal, for nodes which are not end nodes.

### 3.3.2.3 Routing in Networks with GL/GP

For networks with GL/GP, we assume that each node knows the relative direction (left or right) of every other node in the network and thus can determine whether to send packets to its left or right neighbor to reach destinations with the minimum number of hops. This contrasts with the non- GL/GP case, where the nodes do not know the relative direction of the destination. For the WtNN case, networks with GL/GP route packets in the correct direction to the closest node in that direction. For the characteristic hop distance case defined in (2.24), networks with GL/GP route in the correct direction

over a hop of length approximately equal to the characteristic distance. We assume for the remainder of this chapter that networks utilizing characteristic hop distance routing also have GL/GP. For the upper bounds on characteristic hop distance routing, we do not address the issue of when there is not a node a characteristic hop distance away because we assume that the network is connected at the characteristic distance with rate $R$. For the tight bound on throughput *whp*, we show that *whp* there is a node a characteristic hop distance away. In both the WtNN and characteristic hop distance cases, the nodes know whether to send a packet to the left or right; the only difference is whether the packet is sent to the nearest neighbor or to a node a characteristic hop distance away.

#### 3.3.2.4 Communication Links and System Bandwidth

We assume that each node is capable of full duplex communication and can therefore simultaneously transmit and receive. Further, the network is operating in the wideband (or power-limited) regime such that there are as many channels as necessary for the number of nodes wishing to communicate at a given time. Thus we assume that there is no interference. These assumptions allow us to ignore issues of transmission scheduling and frequency allocation in the analysis. While these are important and non-trivial issues, they are not the main focus of this work. Our focus is on the development of performance bounds of the three network types.

## 3.4 Uniform All-to-All Traffic First Hop Analysis

To begin our analysis of one-dimensional networks, we start with an analysis of the first hop in a one-dimensional network with uniform all-to-all traffic. Let $H_{(i)}^{(j)}$ be the indicator random variable for the first hop from a source node location $(i)$ to a destination node location $(j)$, noting the use of sorted node indices (i.e., $H_{(i)}^{(j)}$ refers to source node $X_{(i)}$ and destination node $X_{(j)}$). We then define $H_{(i)}^{(j)}$ in the following way:

$$H_{(i)}^{(j)}(n) = \begin{cases} 0, & direction\ of\ first\ hop\ is\ correct\ in\ a\ network\ of\ n\ nodes, \\ 1, & direction\ of\ first\ hop\ is\ incorrect\ in\ a\ network\ of\ n\ nodes. \end{cases}$$

In subsequent sections we solve for $\Pr\left\{H_{(i)}^{(j)}(n) = 0\right\}$ and $\Pr\left\{H_{(i)}^{(j)}(n) = 1\right\}$ in networks with and without GL/GP. This first hop analysis sheds light on the differences between the network types (networks with and without GL/GP) and enables analysis of full traffic patterns, both in terms of total hop count and throughput.

### 3.4.1 WtNN Networks without GL/GP

In the non-GL/GP case, there is substantial analytic work required to show the probabilities of correct versus incorrect first hops. The probability of a node making a correct first hop is based on the probability of its nearest neighbor being to the left or the right, as well as the relative direction of the node with which it is communicating. The proof of the results to follow is found in Appendix A.2. We show that in the non-GL/GP case, for any $n$, the probability of a correct first hop is:

$$Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = 0.5 + \frac{1}{n}, n \geq 2. \tag{3.15}$$

The reason this probability is strictly greater than $1/2$ is because the first hops for the end nodes are always in the correct direction (a corollary of Lemma 2).

The probability of an incorrect first hop is given by

$$Pr\left\{H_{(i)}^{(j)}(n) = 1\right\} = 0.5 - \frac{1}{n}, n \geq 2. \tag{3.16}$$

This probability is strictly less than $1/2$ for the same reason that the probability of a correct first hop is strictly greater than $1/2$.

Combining these two results, we have the following:

$$\lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = \lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 1\right\} = 0.5. \tag{3.17}$$

Thus the probability of a correct or incorrect first hop approaches 0.5 asymptotically with increasing $n$.

### 3.4.2 Networks with GL/GP

Per our assumptions, the first hop is always in the correct direction in the GL/GP case, regardless of whether the routing type is WtNN or characteristic hop distance. Given a packet of data to send from node $(i)$ to node $(j)$, node $(i)$ sends the packet to a neighbor that is in the direction of node $(j)$, whether that node is $(j)$ or some intermediate node. Thus in the GL/GP case there are no incorrect first hops and we make the following conclusions:

$$Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = 1, \tag{3.18}$$

$$Pr\left\{H_{(i)}^{(j)}(n) = 1\right\} = 0. \tag{3.19}$$

## 3.5 Uniform All-to-All Traffic Complete Path Hop Analysis

Now that we have looked at the first hop case and compared networks without GL/GP to those with GL/GP, we continue the line network analysis by looking at the full traffic paths for uniform all-to-all traffic. We note that the analysis is heavily dependent on the assumptions about the operation of the

networks, and in particular network topology and routing control. For the results to follow, the assumptions used are described in Section 3.3.2.

### 3.5.1 WtNN Networks without GL/GP

We begin the complete path hop analysis with the non-GL/GP case, where nodes communicate with their nearest neighbors, per the assumptions. We note that, given a set of nodes $\mathbb{X}$, there is a deterministic path starting at every source and ending at every destination. We define the notation $\mathbb{H}_T^{non-GL/GP}$, which takes the value of the total number of hops in a non-GL/GP network. We have the following lemma, proven in Appendix A.3:

> **Lemma 3** - In a network of $n$ nodes without GL/GP and using WtNN routing, the total number of hops in the network for uniform all-to-all traffic is $\mathbb{H}_T^{non-GL/GP} = \frac{2}{3}n^3 - n^2 + \frac{1}{3}n$.

Thus we see that the total number of hops in such a network is independent of the node deployment for a scheme where nodes only communicate with their left and right nearest neighbors.

### 3.5.2 WtNN Networks with GL/GP

In the GL/GP case, we note that nodes always send packets in the correct direction from the source to the destination. Thus if a destination is to the right, the route from the source goes to the right. If a destination is to the left, the route from the source goes to the left. Unlike the non-GL/GP case, the paths do not have the same first hop direction. This differs from the non-GL/GP case where packets are sent to nearest neighbors and the relative direction of the destination is unknown. We define the term $\mathbb{H}_T^{GL/GP}$, which takes the value of the total number of hops in a non-GL/GP network. We have the following lemma, proven in Appendix A.4:

**Lemma 4** - In a network of $n$ nodes with GL/GP and using WtNN routing, the total number of hops in the network for uniform all-to-all traffic is $\mathbb{H}_T^{GL/GP} = \frac{1}{3}n^3 - \frac{1}{3}n$.

Thus, as in the non-GL/GP case, we see that the total number of hops is independent of the node deployment.

### 3.5.3 Characteristic Hop Distance Networks with GL/GP

In the case where nodes transmit at characteristic hop distances, we develop bounds on the total number of hops in the network, $\mathbb{H}_T^{d_{char}}$. We note that the number of hops under characteristic hop distance routing can never exceed that under WtNN network with GL/GP, given the previously defined routing rules. Characteristic hop distance is defined as follows (see [39]):

$$d_{char} = \left(\frac{\alpha}{\beta(k-1)}\right)^{1/k}. \tag{3.20}$$

In (3.20), $\alpha$ is the processing energy in $[J/bit]$, $\beta$ is the distance-dependent transmission energy in $\left[J/\left(bit \cdot [m]^k\right)\right]$, and $k$ is the variable path loss coefficient. The optimal transmission distance, $d_{char}$, is the result of taking into account both processing energy and distance-dependent transmission energy. We assume that $\alpha > 0$ and $\beta > 0$ and therefore it is never more energy efficient to hop in the wrong direction. Thus we can upper bound $\mathbb{H}_T^{d_{char}}$ as $\mathbb{H}_T^{d_{char}} \leq \mathbb{H}_T^{GL/GP}$. We develop a lower bound on the number of hops in a one-dimensional characteristic hop distance network, which is given in Lemma 5 and proven in Appendix A.5.

We note that we previously dealt with a one-dimensional network with a normalized length of 1. Now we let $L$ be the network length and we assume $L$ is some integer multiple of $d_{char}$ such that[8] $L = (l-1) \cdot d_{char}$ where $(l-1) \in \mathbb{Z}^+$ and we further assume that $n/l \in \mathbb{Z}^+$. Now we define the characteristic hop distance normalized with respect to $L$ as $\hat{d}_{char} = d_{char}/L = (l-1)^{-1}$. The optimal network for minimum hop count is where nodes are clustered at integer multiples of $\hat{d}_{char}$, that is at the locations $\{0, (l-1)^{-1}, 2(l-1)^{-1}, \ldots, 1\}$, with an equal number of nodes at each cluster. Thus we can view the network as essentially subdivided into $l$ clusters with $n/l$ nodes at each cluster. Finally, we view the entire network as $n/l$ sub-networks, where each sub-network consists of one node at each cluster and thus consists of $l$ nodes. We have the following lemma for the bounds on $\mathbb{H}_T^{d_{char}}$:

**Lemma 5** - In a line network of $n$ nodes with GL/GP and using characteristic hop distance routing, the total number of hops in the network for uniform all-to-all traffic is bounded as $\frac{1}{6l}(2n - l^2 + l)(2n + nl^2 - 3l) \leq \mathbb{H}_T^{d_{char}} \leq \frac{n}{3}(n^2 - 1)$.

## 3.5.4 Network Comparison

In this section we compare the total hop count results among the three network types we analyzed. As hop count is a proxy for both energy use and network delay, this comparison is valuable for determining the benefits of GL/GP and characteristic distance hopping. We have the following comparing $\mathbb{H}_T^{GL/GP}$ to $\mathbb{H}_T^{non-GL/GP}$:

$$2 \geq \frac{\mathbb{H}_T^{non-GL/GP}}{\mathbb{H}_T^{GL/GP}} = 2 - \frac{3}{n+1} \geq 1. \tag{3.21}$$

[8] We use $l-1$ in order to use $l$ for the number of network clusters.

In the limit, we can see that

$$\frac{\mathbb{H}_T^{non-GL/GP}}{\mathbb{H}_T^{GL/GP}} = 2 \in \Theta(1). \tag{3.22}$$

While it is not surprising that the GL/GP case has a deterministic result, it is interesting that the non-GL/GP case also does. Although the first hop is asymptotically correct with probability $1/2$, this correctness condition does affect the total number of hops taken in the network under uniform all-to-all traffic. For this traffic pattern, the network layout is arbitrary with respect to calculating the total number of hops. It should be noted that this is only true for this traffic pattern and for these routing rules. If the topology creation was such that a node could communicate with more than its two closest neighbors, then our analysis would need to be modified to reflect this.

Now we determine the benefits of characteristic distance routing in one-dimensional networks as compared to WtNN routing with and without GL/GP. For the case of WtNN routing without GL/GP, we have the following:

$$1 \leq 2 - \frac{3}{n+1} \leq \frac{\mathbb{H}_T^{non-GL/GP}}{\mathbb{H}_T^{d_{char}}} \leq \frac{2ln(n-1)(2n-1)}{(2n-l^2+l)(2n+nl^2-3l)}. \tag{3.23}$$

The lower bound is the ratio $\mathbb{H}_T^{non-GL/GP}/\mathbb{H}_T^{GL/GP}$ from (3.21). For the upper bound, given some $l$, we have the following:

$$\frac{\mathbb{H}_T^{non-GL/GP}}{\mathbb{H}_T^{d_{char}}} \in O(n). \tag{3.24}$$

We see that the upper bound on the improvement of characteristic distance hopping for the total hop count in a network is of the order $n$. For the case of WtNN routing with GL/GP, we have the following:

$$1 \leq \frac{\mathbb{H}_T^{GL/GP}}{\mathbb{H}_T^{d_{char}}} \leq \frac{2ln(n+1)(n-1)}{(2n-l^2+l)(2n+nl^2-3l)}. \tag{3.25}$$

We see that because $\mathbb{H}_T^{d_{char}}$ is upper bounded by $\mathbb{H}_T^{GL/GP}$, the lower bound on the ratio of $\mathbb{H}_T^{GL/GP}$ to $\mathbb{H}_T^{d_{char}}$ is 1. For the upper bound we have the following:

$$\frac{\mathbb{H}_T^{GL/GP}}{\mathbb{H}_T^{d_{char}}} \in O(n). \tag{3.26}$$

We see that the upper bound on the improvement of characteristic distance hopping for the total hop count in a network is again of the order $n$. Given $\mathbb{H}_T^{non-GL/GP}/\mathbb{H}_T^{GL/GP} \in \Theta(1)$, we expect the upper bound on the performance improvement of characteristic distance hopping to be of the same order for the GL/GP WtNN case as for the non-GL/GP WtNN case.

## 3.6 Uniform All-to-All Traffic Throughput

Having analyzed the hop count in one-dimensional networks with uniform all-to-all traffic, we now determine the throughput of such networks. In the uniform all-to-all traffic model each node sends data to every other node at an average rate of $\lambda\ [bits/sec]$. We use arbitrary network configurations to develop bounds on the throughput for the cases of WtNN routing without GL/GP and characteristic

distance routing with GL/GP. We define a new term called the achievable uniform all-to-all steady-state throughput, $\hat{\lambda}$ $[bits/sec]$, in a manner similar to [39], [3] and [21]. We express $\hat{\lambda}$ as

$$\hat{\lambda} \triangleq \max_{\lambda}\{Achievable\ \lambda\}, \forall t \geq t_{ss}, \tag{3.27}$$

where achievable means it can be sustained between every S-D pair and $t_{ss}$ is the first time instance at which the network is in steady state with respect to traffic flow. We note that we are considering the maximization over all achievable throughputs because, regardless of the network type, it is always possible to obtain worse performance given a poor enough scheduling or power control scheme.

Now we let $L_{(i\rightarrow j)}(t)$ be the number of traffic streams at time $t$ on the link from node $X_{(i)}$ to node $X_{(j)}$. We note that due to the full duplex communication assumption there exists another, not necessarily identical term $L_{(j\rightarrow i)}(t)$. We can quantify the link traffic in terms of the number of data streams because each data stream has the same rate $\lambda$ under our traffic model. Now we formally define $t_{ss}$ as follows[9]:

$$t_{ss} \triangleq \arg\min_{t}\{L_{(i\rightarrow j)}(t) = L_{(i\rightarrow j)}(t + t_0), \forall t_0 > 0\}. \tag{3.28}$$

There could exist a transient $\hat{\lambda}$ when $t < t_{ss}$, which may not be achievable in the steady state and therefore the condition on $t$ in (3.28) is required. Because we are dealing with steady state throughput, we define the following:

[9] We define $t_{ss}$ as the time after which the number of streams on any given link is a constant. At network startup, there may be some transient traffic load that could be sustained for a period of time before all of the pass-through traffic is accounted for on every link.

$$L_{(i)}^{(j)} \triangleq L_{(i\to j)}(t), \forall t \geq t_{ss}. \tag{3.29}$$

$L_{(i)}^{(j)}$ is the steady state throughput in terms of number of streams on the link from node $X_{(i)}$ to node $X_{(j)}$. Finally, we note that in the case where only nearest neighbors communicate, the following is true:

$$L_{(i)}^{(j)} = 0 \; \forall \; i \in [2, n-1], j \notin \{i+1, i-1\}. \tag{3.30}$$

Thus we are only concerned with the non-zero terms $L_{(i)}^{(i+1)}$ and $L_{(i)}^{(i-1)}$. We again note that due to the full duplex communication assumption there are also non-zero terms $L_{(i+1)}^{(i)}$ and $L_{(i-1)}^{(i)}$ (i.e., the receiving links for node $(i)$ in the full duplex system). Because the traffic pattern in question is uniform all-to-all, each node is a sink (receiver) for the same number of streams as for which it is a source (transmitter). Further, given that a node relays a pass through traffic stream destined for some other node, it both sends and receives that stream the same number of times. Whether a given stream is sent once or twice depends on whether that stream took a correct or incorrect first hop, respectively. Therefore, it makes no difference whether we analyze the set of transmitting or receiving links for a given node; the results are the same. We arbitrarily choose to analyze the steady state throughput in terms of the sending links of node $X_{(i)}$, those being $L_{(i)}^{(i+1)}$ and $L_{(i)}^{(i-1)}$. With a fixed rate system, we know that each link[10] is rate limited such that

$$\lambda\left(L_{(i)}^{(i+1)} + L_{(i)}^{(i-1)}\right) \leq R. \tag{3.31}$$

We are concerned with the bound on transmitted traffic data rate, noting that the bound is the same for receiving. Given that some links may be more highly loaded than others, we define the following:

[10] Noting that $L_{(n)}^{(n+1)} = 0$ and $L_{(1)}^{(0)} = 0$.

$$\hat{L}(n) \triangleq \max_i \left\{ L_{(i)}^{(i+1)} + L_{(i)}^{(i-1)} \right\}. \tag{3.32}$$

Finally, we see that

$$\hat{\lambda} = \frac{R}{\hat{L}(n)} \tag{3.33}$$

In order to develop bounds or solve for $\hat{\lambda}$ (depending on the network case), we determine the maximum number of traffic streams $L_{(i)}^{(j)}$ on any link in steady state. By bounding $\hat{L}(n)$ we are then able to bound $\hat{\lambda}$. In the analysis to follow, we differentiate between networks using WtNN routing with and without GL/GP by using the terms $\hat{L}^{GL/GP}(n)$ or $\hat{L}^{non-GL/GP}(n)$ and $\hat{\lambda}^{GL/GP}$ or $\hat{\lambda}^{non-GL/GP}$, respectively. For GL/GP networks using characteristic distance hopping, we denote these quantities as $\hat{L}^{d_{char}}(n)$ and $\hat{\lambda}^{d_{char}}$.

## 3.6.1 WtNN Networks without GL/GP

In WtNN networks without GL/GP, we find upper and lower bounds on the achievable throughput, $\hat{\lambda}^{non-GL/GP}$, in the network. Thus we find the best and worst case results for arbitrary network deployments. We continue to assume connectivity and therefore we are not concerned with the absolute distances between neighboring nodes. Rather, we are concerned with the relative distances between nodes, since this determines whether a node's nearest neighbor is to its right or its left.

Given the ordered set of nodes $\mathbb{X}_O = \{X_{(1)}, \ldots, X_{(n)}\}$, there are a total of $n$ nodes in the network and thus a total of $n(n-1)$ S-D pairs under uniform all-to-all traffic. We solve for upper and lower bounds

on $\hat{\lambda}^{non-GL/GP}$. The lower bound on throughput is proven in Appendix A.6 and the upper bound is proven in Appendix A.7. Summarizing these results, we have the following lemma:

**Lemma 6** - In a network of $n$ nodes using WtNN routing without GL/GP, the maximum achievable throughput per S-D pair under uniform all-to-all traffic is bounded as $\frac{R}{n^2-n-1} \leq \hat{\lambda}^{non-GL/GP} \leq \frac{2R}{n^2+n-2}$.

### 3.6.2 WtNN Networks with GL/GP

In networks with GL/GP, while the throughput is again determined by the most highly loaded link, the load on that link is deterministic. Thus it does not depend on the values of $D_{(k)}$, due to the deterministic nature of a one-dimensional network using GL/GP. Specifically, the first hop is not dependent on the nearest neighbors of the nodes in the network. Thus, there are no upper and lower bounds because the traffic only moves in a deterministic manner.

We prove Lemma 7 in Appendix A.8 and we note that other methods exist to solve for the throughput given the network assumptions. In particular, the GL/GP case can be solved using a min cut/max flow analysis, which would take a vastly different approach and require a shorter derivation[11]. However, because such an analysis does not easily extend to the non-GL/GP case, we do not use it here. By following the same approach as in the non-GL/GP case, it is easier to identify where and how the two cases differ. We have the following result:

[11] The max cut/min flow approach is used in Appendix A.9 to derive the upper bound on $\hat{\lambda}^{d_{char}}$ in Section 3.6.3.

**Lemma 7** - In a of network of $n$ nodes using WtNN routing with GL/GP, the maximum achievable throughput per S-D pair under uniform all-to-all traffic is bounded as $2R/(n^2-1) \leq \hat{\lambda}^{GL/GP} \leq 2R/(n^2-2)$.

### 3.6.3 Characteristic Hop Distance Networks

In the case where nodes transmit at characteristic hop distances, we bound the maximum number of traffic streams on a link in a similar manner to the hop count bounding in Section 3.5.3. As noted in Section 3.3.2.3, we assume that such networks utilize GL/GP. The lower bound on $\hat{\lambda}^{d_{char}}$ is the same as the result for the WtNN network with GL/GP. We note that given the routing rules for WtNN networks with GL/GP, every node handles all pass through traffic intersecting it in the network and thus its throughput represents a lower bound for GL/GP networks. The upper bound on $\hat{\lambda}^{d_{char}}$ is derived from a max-flow min-cut argument (see Appendix A.9). We have the following lemma, using the notation from Section 3.5.3:

**Lemma 8** - In a network of $n$ nodes using characteristic hop distance routing with GL/GP, the maximum achievable throughput per S-D pair under uniform all-to-all traffic is bounded as $2R/(n^2-1) \leq \hat{\lambda}^{d_{char}} \leq 2R/(nl-2)$.

Thus we see that $\hat{\lambda}^{d_{char}} \in \Omega(1/n^2)$ and $\hat{\lambda}^{d_{char}} \in O(1/n)$.

### 3.6.4 Network Comparison

In this section we compare the achievable throughput results for one-dimensional networks among the three network types in order to determine the benefits of GL/GP and characteristic hop distance

routing. First, we define $\lambda_{\Delta}^{GL/GP}$, which represents the performance benefit achieved by using GL/GP-enabled nodes in a network using WtNN routing:

$$\lambda_{\Delta}^{GL/GP} \triangleq \frac{\hat{\lambda}^{GL/GP}}{\hat{\lambda}^{non-GL/GP}}. \tag{3.34}$$

Next we define $\lambda_{\Delta}^{d_{char},GL/GP}$, which represents the performance benefit achieved by using both characteristic distance hopping and GL/GP over a WtNN network without GL/GP:

$$\lambda_{\Delta}^{d_{char},GL/GP} \triangleq \frac{\hat{\lambda}^{d_{char}}}{\hat{\lambda}^{non-GL/GP}}. \tag{3.35}$$

Finally, we define $\lambda_{\Delta}^{d_{char}}$, which represents the performance benefit achieved by using characteristic distance hopping over a WtNN network with GL/GP:

$$\lambda_{\Delta}^{d_{char}} \triangleq \frac{\hat{\lambda}^{d_{char}}}{\hat{\lambda}^{GL/GP}}. \tag{3.36}$$

### 3.6.4.1 Comparison of WtNN with and without GL/GP

We first consider the benefit of a network using WtNN and GL/GP over a network using WtNN routing and no GL/GP, and we see the following for the one-dimensional case:

$$1 < \lambda_{\Delta}^{GL/GP} < 2. \tag{3.37}$$

Thus in the case of uniform all-to-all traffic, the performance benefit achieved by a network with GL/GP is $\Omega(1)$ and $O(1)$ and therefore $\lambda_{\Delta}^{GL/GP} \in \Theta(1)$. In Section 3.5.4 we showed that $\mathbb{H}_T^{non-GL/GP}/\mathbb{H}_T^{GL/GP} \in$

$\Theta(1)$ and thus it is not surprising that the throughput benefit of GL/GP in WtNN networks is also $\Theta(1)$. We also note here that what we mean by WtNN without GL/GP is a network where there is no routing information available to the nodes; nodes simply forward to their nearest neighbor until a packet reaches the destination. For one-dimensional networks, throughput scaling for a "random walk" routing method (that is, whispering to the nearest neighbor with no destination information) scales the same as when nodes have perfect knowledge of their destinations. This is an interesting result both because the result does not depend on the network configuration and because, as we show in Chapter 4, the distribution of topology and routing information in IWNs requires significant throughput per node.

### 3.6.4.2 Benefits of Characteristic Distance Hopping

Next we compare the throughput benefits of characteristic distance hopping to networks using WtNN routing with and without GL/GP. We begin with the comparison to networks without GL/GP by looking at $\lambda_{\Delta}^{d_{char},GL/GP}$. We have the following:

$$1 < \lambda_{\Delta}^{d_{char},GL/GP} \leq \frac{2(n^2-n-1)}{nl-2}. \tag{3.38}$$

We note that $\lambda_{\Delta}^{d_{char},GL/GP} \in O(n)$ and thus the achievable throughput gain is proportional to the reduction in hop count calculated in Section 3.5.4.

Finally, we calculate the throughput benefits of characteristic distance hopping over WtNN networks with GL/GP and obtain the following[12]:

[12] We note that the inequality in Lemma 7 is due to the odd/even cases for $n$. Thus the lower bound in (3.39) can be satisfied with equality because we assume that $n$ is the same for the networks we are comparing.

$$1 \leq \lambda_{\Delta}^{d_{char}} \leq \frac{n^2 - 1}{nl - 2}. \tag{3.39}$$

We note that $\lambda_{\Delta}^{d_{char}} \in O(n)$, where again the results are as expected given the hop count comparison results in Section 3.5.4. Given that the achievable throughput of WtNN networks with GL/GP is $\Theta(1)$ compared to WtNN networks without GL/GP, we expect that $\lambda_{\Delta}^{d_{char}}$ and $\lambda_{\Delta}^{d_{char},GL/GP}$ are of the same order. That is, because the achievable throughput of WtNN networks with and without GL/GP is of the same order, we expect the performance improvement of characteristic hop distance networks over either WtNN network to be the same.

## 3.7 A Tight Bound on Throughput Scaling

Given the bounds in Section 3.6.4 for throughput scaling for characteristic hop distance routing, we develop a tight upper bound on $\hat{\lambda}^{d_{char}}$ to show that the upper bounds on $\lambda_{\Delta}^{d_{char},GL/GP}$ and $\lambda_{\Delta}^{d_{char}}$ are tight. We following a similar method as used in [3] to derive Lemma 1. In Appendix A.10, we show that the throughput scaling upper bound for characteristic distance hopping is tight with high probability (see Definition 3), that is $\hat{\lambda}^{d_{char}} \in \Theta(1/n)\ whp$. Given this, the upper bounds on the benefits of characteristic distance hopping are tight $whp$. Thus we have the following Lemma:

> **Lemma 9** - Consider a random one-dimensional network with $n$ nodes independently and randomly distributed on a line of unit length according to a uniform distribution. Under uniform all-to-all traffic, the maximum achievable throughput of a network using characteristic hop distance routing over a network using WtNN routing is $\Theta(n)\ whp$.

Lemma 9 summarizes the performance improvement of characteristic hop distance routing over the WtNN protocol for one-dimensional networks. Per Section 2.12, GL/GP enables the use of characteristic

distance hopping by allowing nodes to determine the locations of neighbors in the network. Thus GL/GP enables this throughput performance improvement. While direct transmissions between sources and destinations would also allow for $\Theta(1/n)$ throughput (noting that every node needs to transmit to every other node under our traffic model), such a scheme is not energy efficient.

## 3.8 Summary

In this chapter we have compared three different types of one-dimensional networks: WtNN routing without GL/GP, WtNN routing with GL/GP, and characteristic hop distance routing with GL/GP. In particular, we looked at a total hop count comparison and a throughput comparison, where the total hop count can be seen as a proxy for energy and delay in the network. We showed that, compared to a network using characteristic hop distance routing, the total number of hops in a one-dimensional network using WtNN routing, either with or without GL/GP, scales as $\Omega(1)$ and $O(n)$. We also showed that the achievable throughput of a network using characteristic hop distance routing is $\Omega(1)$ and $O(n)$ greater than that of a network using WtNN with or without GL/GP. Finally, we showed that, with high probability, the achievable throughput of a network using characteristic hop distance routing is $\Theta(n)$ greater than that under WtNN routing. Thus we have shown the performance benefits of characteristic hop distance routing combined with GL/GP, compared to WtNN routing. Characteristic distance hopping is one way to ensure that the hop distance is constant as the number of nodes increases; these results could be extended for other constant hop distance routing schemes. GL/GP enables significant performance improvements in terms of throughput for one-dimensional networks. While often the research in IWNs focuses on two-dimensional networks, the one-dimensional results in this chapter do have real world applications, such as a network of vehicles traveling on a road.

# Chapter 4

# Rate Scaling in Two-Dimensional Power-Limited Networks

In this chapter we continue our analysis of two-dimensional IWNs by looking at the rate scaling necessary for the distribution of topology and routing information. While considerable work has been done to analyze throughput, energy consumption, and delay scaling and tradeoffs in IWNs, there is less attention dedicated to affect the distribution of topology and routing information in these networks. In Section 4.1, we define and differentiate between network topology and network routing, terms which are sometimes used interchangeably in the literature. Further, assumptions are often made concerning what information is available at nodes for routing decisions without addressing the equally important question of the requirements and costs of collecting, distributing, and maintaining that information. Related work was performed in [3] for the period until a local topology change, both for new links connecting and old links disconnecting, in terms of "on the order of" results[13]. These results in [3] used a mobility model where nodes move in a random direction with a velocity $v$ for a random duration of time before changing direction. In this chapter we use a specific mobility model to develop lower bounds on the rate scaling due to link state distribution for maintaining topology and routing information in IWNs.

[13] That is, results using the "~" notation.

## 4.1 Assumptions and Definitions

The following is a list of important assumptions and definitions for analysis of rate scaling due to the distribution of topology and routing information in two-dimensional IWNs.

### 4.1.1 Connectivity

We assume for the purposes of this chapter that we are dealing with a network that is connected. That is, we assume that a path exists from every node to every other node, where a path could be a single hop from the source to the destination or a series of hops through intermediate nodes. In addition to this, to ensure packet delivery given a connected network, it is necessary for the network to distribute updated topology and routing information in order to route a packet from any source node to any destination node.

### 4.1.2 Transmission Rate

We assume that when nodes transmit, they do so at a fixed rate $R$ $[bits/s]$. In this chapter $R$ represents the transmission rate as a random variable and the lowercase $r$ represents the value taken by $R$. Given the per node average power constraint $\hat{P}_{avg}$, per Section 2.8.2.2, a transmission over a hop distance of $\rho$ is correctly received if

$$\rho \leq \rho_{max} = \frac{\lambda}{4\pi}\left(\frac{\hat{P}_{avg}{G_1}^{k/2}}{RN_o \ln 2}\right)^{\frac{1}{k}} [units], \tag{4.1}$$

where $\lambda$ is the signal wavelength, $\hat{P}_{avg}$ is time-average power constraint, $\sqrt{G_1}$ is the product of the transmit and receive antenna field radiation patterns in the LOS direction, $R$ is the transmission rate, $N_o$ is the noise power spectral density in $[W/Hz]$, and $k$ is the path loss exponent. We set the units to be

arbitrary for a more general result. We note that we assume $\alpha = 0$, as in Section 2.8.2.2. That is, we assume zero processing energy because we wish to find an upper bound on the maximum possible transmission distance.

For the remainder of this chapter, we will use $\rho_{max}$ as it is defined in (4.1), and we note that it is a function of both $R$ and $\hat{P}_{avg}$. We also note that the connectivity of a fixed-rate network is dependent on $\rho_{max}$. The purpose of the work in this chapter is not to address connectivity, but rather to address the required transmission rate for the distribution of protocol information. In the sections to follow, we develop bounds on the rate $R$ necessary for the propagation of protocol (topology and routing) information. One may naturally ask how $\rho_{max}$ can appear in such a bound if it is also a function of $R$. We can view $\rho_{max}$ as a function of the ratio $\hat{P}_{avg}/\mathrm{R}$. Then if we assume there exists a $\rho_{max}$ necessary to ensure connectivity (noting that connectivity is a condition for the work in this chapter) then $\hat{P}_{avg}$ must scale as $R$ in order to maintain connectivity and ensure enough rate exists to propagate protocol information. Given some $\rho_{max}$ necessary for connectivity and some $\hat{P}_{avg}$, the bound on $R$ necessary for the distribution of protocol information may be unachievable and thus the network will not be able to ensure the distribution of the protocol information.

### 4.1.3 Network Topology

We define network topology as a description of the network from which it is possible to make routing decisions. We assume for the purposes of this chapter that, given a connected network, the network topology can be used to make routing decisions for the communication of information from any node in the network to any other node in the network. Some examples of network topology include: a square matrix of 0's (disconnected) and 1's (connected), a list of neighbor nodes, and a square matrix of link

connection rates. We adopt the topology of a square matrix of 0's and 1's. We define a topology change event for a fixed rate system as follows:

> **Definition 5** - A topology change event in a fixed rate system is defined as the action of a node entering or leaving the transmission range $\rho_{max}$ of another node in the network.

For cell routing, as in Section 4.2, we define a topology change event as follows:

> **Definition 6** - A topology change event for cell routing with a fixed transmission rate is defined as the action of a node leaving (or entering) a cell.

We note that in order to leave a cell, a node must enter a new cell. Thus the definition can be written for a node leaving or entering a cell.

As in the one-dimensional case, we again assume a network of $n$ nodes while initially not specifying the area in which they are distributed (such as a plane). We denote the $i^{th}$ arbitrarily labeled node and its location, as $W_i$ and we denote the set of nodes and locations as

$$\mathbb{W} = \{W_1, W_2, \dots, W_n\} \equiv \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}. \tag{4.2}$$

### 4.1.4 Network Routing

We define routing as the use of the network topology information to forward information from one node in a network to another node in a network. There are many examples of network routing protocols. For packet-switched networks, as mentioned in Section 2.4, the two primary routing types are link-state routing protocols and distance vector routing protocols. In Section 4.2 we analyze cell routing,

where the analysis is a particular case of the more general and routing-agnostic topology approach of Section 4.3. Likewise, Section 4.4 is also a routing-agnostic topology approach. In general, the aim of this chapter is to develop results that do not depend on the particular routing scheme selected.

### 4.1.5 Protocol Burden

In order to maintain a connected network and ensure packet delivery, the following are necessary: the entire network must be able to have current topology and routing information and then be able to forward a packet from any source node to any destination node. We also assume that the network is always connected. That is, it should always be possible for any node to communicate with any other node, either directly or through one or more intermediate nodes. For now, we assume that given some network topology, each node must receive $\psi$ bits of topology and routing data in order to configure the network for connectivity. In Section 4.5 we make assumptions about the value of $\psi$ and use those assumptions to develop real world bounds on network scaling as a function of $\psi$. Based on this protocol burden, we provide the following definition of the probability of successful protocol distribution:

> **Definition 7** - Given an IWN with fixed rate $r$, a change in topology at time $t_1$, and a subsequent change in topology at time $t_2$ (where $t_2 - t_1 > 0$ WP1), the probability of successful protocol distribution is defined as the probability that the rate $r$ is sufficient for every node in the network to receive $\psi$ bits of data to in a time less than or equal to $t_2 - t_1$.

In the sections to follow, this definition will allow us to solve for bounds on $R$ for protocol distribution, as well as the distribution of $R$ for the successful distribution of protocol information.

## 4.2 Rate Scaling for Cell Routing

In this section we look at a particular routing scheme in two-dimensional networks called cell routing. We consider cell routing as it appears in [39], where the cell geometry lends itself to tractable analytic bounds. Cell routing divides the network into a fixed number of cells and then routes traffic from source to destination by traveling through cells (see Fig. 1, derived from [3], [21], and [39]). A line is drawn from source to destination and the traffic moves through the cells that the line intersects. Traffic is divided among the nodes in a cell, which forward it to the next cell in the path. We note that cell routing is not necessarily implementable but it helps provide bounds on the achievable performance of IWNs. Cell routing is used in various ways to derive results in [2], where Voronoi regions are used as cells on the surface of a sphere. The surface of a torus is considered for the cell routing analysis in [4] and in later work by the same authors in [20]. The work in [39] builds on some of the analysis in [4]. In [39], the authors show that taking $\theta(1)$ hops achieves the pair-wise throughput scaling of $\theta(1)$ and the optimal delay and energy per bit scaling. The work in [39] did not assume that the dominant energy use in IWNs is from transmission energy.

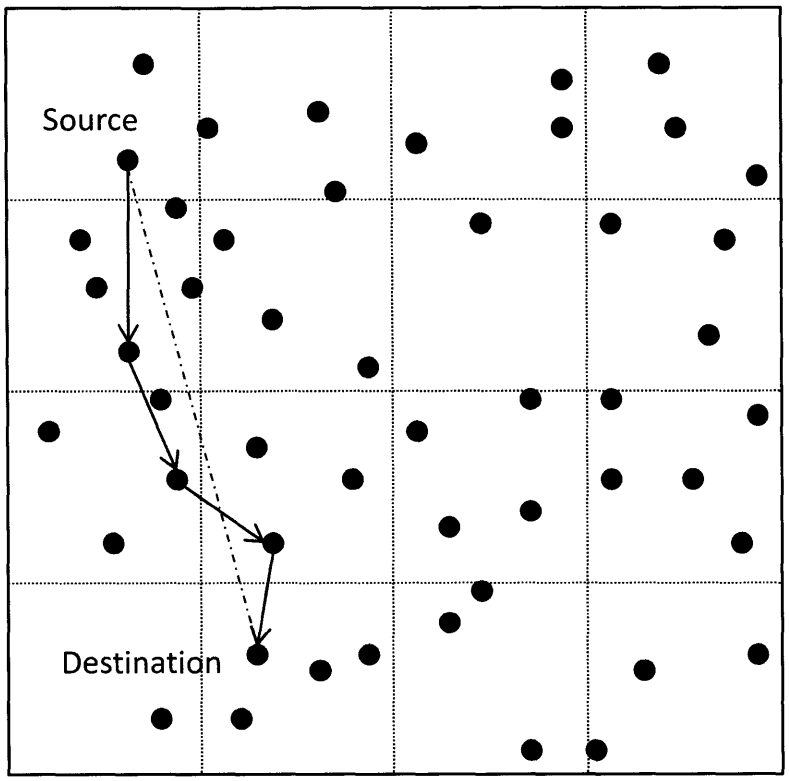


Fig. 1. Diagram of Cell Routing

## 4.2.1 Network Structure and Constraints

As in [3], we assume a unit torus operating region, where we can view the network as a unit area square grid. For a simpler representation, we view the unit torus as a unit square grid of cells with size $l$ x $l$. We have the following bound on the grid size:

$$\sqrt{\frac{2}{n}} \leq l \leq \frac{\rho_{max}}{2\sqrt{2}}. \tag{4.3}$$

The lower bound is due to the requirement that there is at least one node in every cell with high probability, and the upper bound ensures that nodes in adjacent cells can communicate with one

another, including diagonal connectivity. See Section 2.11.2.3 for a review of these results. Examining (4.3), we note that this also bounds the number of cells, $m = (l^2)^{-1}$, as follows:

$$\frac{8}{{\rho_{max}}^2} \leq m \leq \frac{n}{2}. \tag{4.4}$$

We briefly look at the conditions under which (4.3) cannot be met (that is, conditions when the upper bound is less than the lower bound). In this case it is not possible for the cells to be simultaneously large enough to ensure there is one node in each cell with high probability and small enough to ensure connectivity amongst nodes in adjacent cells. Rearranging the upper and lower bounds in (4.3), we see that this occurs for

$$n < \frac{16}{{\rho_{max}}^2}. \tag{4.5}$$

Thus in order to meet the bounds in (4.3), we must simultaneously satisfy the following bound:

$$n \geq \frac{16}{{\rho_{max}}^2}. \tag{4.6}$$

## 4.2.2 Node Mobility and Protocol Burden

We now look at node mobility in a random network of $n$ nodes and its effect on transmission rate scaling in networks without GL/GP information. We assume a fixed rate system for the purposes of this analysis. We assume that for every time period $\tau$, each node $i$ has a starting position given as $(x_t^i, y_t^i)$ and an ending location $(x_{t+\tau}^i, y_{t+\tau}^i)$, and we assume that the nodes travel in a straight line at a constant velocity. Then the nodes have velocity and speed given by:

$$\boldsymbol{v_i} = \left(\frac{|x_{t+\tau}^i - x_t^i|}{\tau}, \frac{|y_{t+\tau}^i - y_t^i|}{\tau}\right) = (v_x^i, v_y^i) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)\left[\frac{units}{s}\right], \tag{4.7}$$

$$s_i = \|\boldsymbol{v}_i\|\left[\frac{units}{s}\right]. \tag{4.8}$$

Given the straight line movement assumption, we note that this means all higher derivatives are zero. For the analysis to follow, we assume that only one node is mobile and that connectivity is to be maintained with every other node in the network, in terms of distributing topology and routing information. Any given node can be the one mobile node, where the speed is the same and thus we let $s_i = s$ and $|\boldsymbol{v}_i| = |\boldsymbol{v}| \; \forall \; i \in [1, n]$.

Given the grid structure for cell routing, let us assume that the mobile node begins and ends on the edge of a given cell, traveling in a straight line. Thus we assume that a mobile node completely traverses a given cell. We can define the time taken by node $W_i$ to traverse a given cell as the random variable $T_{traverse}^i \; [seconds]$ and we bound it as follows:

$$0 < T_{traverse}^i \leq \frac{\sqrt{2}l}{|\boldsymbol{v}|}. \tag{4.9}$$

Note that in the above bound, we define traversing a cell as entering and then exiting a cell, and thus the time necessary to do so must be non-zero. Given our assumptions about node velocity, the longest distance a node can traverse in a straight line is the diagonal of a cell. We note that if a node traverses the diagonal of a cell in a straight line, then $v_x^i = v_y^i$.

In order to maintain a connected network and ensure packet delivery, it is necessary for every node in the network to have current topology and routing information and then be able to forward a packet

from any source node to any destination node. Between the time a destination node enters and exits a given cell, the topology and routing information of the network must be updated and the source information forwarded to the destination node. Both of these must occur in a time bounded by $T^i_{traverse}$. Once a node enters a new cell, we assume that the topology and routing information must be re-created.

For a node to receive a given packet in cell routing, the packet must arrive at the cell where the node is located. We assume that there exists an algorithm in the network that allows nodes within a cell to determine the optimal cell routing paths for all S-D pairs. We also assume that the network is always connected. That is, it is always possible for any node to communicate with any other node, assuming that the topology and routing information is current. Finally, we assume that the network routing does not take advantage of geographic prediction information. In other words, if a node moves from one cell to another, it is necessary to run the routing algorithm again to determine routes to the node.

As previously stated, each node must receive a total of $\psi$ bits to update topology and routing information, which for a fixed rate system of rate $R$ requires a total time of $T_{update} = \psi/\mathrm{R}\ [seconds]$. We require that the network remain connected, and therefore the mobile node must be able to communicate with every other node in the network. Thus it is necessary that the mobile node receive the routing update information from every other node. For a single mobile node traversing the diagonal of a cell, the time remaining to receive transaction data (as opposed to topology and routing update information) is given as

$$T^i_{Rx} = T^i_{traverse} - T_{update} \leq \frac{\sqrt{2}l}{|\boldsymbol{v}|} - \frac{\psi}{R}, \tag{4.10}$$

where $T_{Rx}^{i}$ is the time available for receiving transactions for node $W_i$. During this time traversing a cell, the rate available for receiving transaction data, or the effective rate, $R_{eff}$, is given by the following:

$$R_{eff} = R\frac{T_{Rx}^{i}}{T_{traverse}^{i}}. \tag{4.11}$$

That is, given a network with a fixed transmission rate $R$, some of that rate is consumed by receiving topology and routing update information in a mobile network, leaving only some fraction of the rate available for receiving transaction data from other nodes. Before moving on, we look at the boundary cases in order to provide some insight. The boundary cases are as follows:

$$R_{eff} = \begin{cases} 0, & T_{update} = T_{traverse}^{i}, \\ R, & T_{update} = 0. \end{cases} \tag{4.12}$$

Thus we see that if $T_{update}$ is equal to $T_{traverse}^{i}$, then all of the receiving rate is used to provide the topology and routing update to the mobile node. If $T_{update}$ is 0, then the effective receiving rate is $R$ because no information is sent to update the network topology and routing information at the mobile node.

### 4.2.3 Rate Scaling Bound

In order to achieve non-zero effective transmission rate, $R_{eff} \geq 0$, we must satisfy $T_{Rx}^{i} \geq 0$, which is equivalent to $T_{traverse}^{i} \geq T_{update}$. Given that $T_{traverse}^{i} \leq \sqrt{2}l/|\boldsymbol{v}|$ and $T_{update} = \psi/\mathrm{R}$, we have the following inequality:

$$\frac{\sqrt{2}l}{|\boldsymbol{v}|} \geq \frac{\psi}{R}. \tag{4.13}$$

We can rearrange (4.13) to get

$$R \geq l^{-1}\left(\frac{\psi|\boldsymbol{v}|}{\sqrt{2}}\right), \tag{4.14}$$

where, from (4.3) we know that $l^{-1}$ must satisfy

$$\frac{2\sqrt{2}}{\rho_{max}} \leq l^{-1} \leq \sqrt{\frac{n}{2}}.$$

We use the lower bound on $l^{-1}$ to find the lower bound on $R$. Substituting the lower bound on $l^{-1}$ into (4.14), we then have the following:

$$R \geq \frac{2\psi|\boldsymbol{v}|}{\rho_{max}}. \tag{4.15}$$

Per [39], cell routing, which uses $\theta(1)$ hops, achieves the pair-wise throughput scaling of $\theta(1)$ and the optimal delay and energy per bit scaling in a static network. The rate scaling in this case is a function of the amount of information each node must receive due to a change in topology, which is $\psi$ bits. In Section 4.5.2 we examine the value of $\psi$ and the scaling for each of the approaches considered in this chapter. We note that, per (4.15), networks without GL/GP information require $R$ to scale linearly with velocity. Finally, we note that smaller cell sizes (that is, smaller $\rho_{max}$) require higher transmission rates in order to achieve non-zero throughput.

## 4.3 Fixed Rate Scaling - Topology Approach

Having analyzed the rate scaling in cell routing, we now take a routing-agnostic look at rate scaling in power-limited, fixed transmission rate networks. Both [3] and [21] looked at fixed as well as variable rate systems. We analyze fixed rate systems, which provide a more simple analysis framework. For the purposes of the rate scaling analyses in this chapter, cell routing is a specific case of the topology approach. The upper bound on cell size is a function of maximum transmission range and the lower bound is a function of the number of nodes. Both bounds give a deterministic number of cells in the network. In this section we view a node's transmission area as the area to be traversed by a mobile node and, like the cell routing upper bound, this area is a function of the transmission range.

### 4.3.1 Network Structure and Constraints

In this section, we also continue to assume a unit torus[14] operating region, as in Section 4.2, to avoid edge effects in the network. We label the analysis to follow as the topology approach, noting that we are not concerned with the routing method used in the network. We are only concerned with changes in topology. Given Definition 5 and our assumptions regarding the protocol burden for updating topology information at all the nodes in the network, it is necessary for every node to receive $\psi$ bits for every change in topology.

### 4.3.2 Node Mobility and Protocol Burden

We begin by looking at a simple mobility model that is very similar to that of Section 4.2. We use the same notation as in Section 4.2.2 to describe a node's starting location, ending location, velocity, and speed. Due to the straight line movement assumption, we note that this means all higher derivatives are

[14] We use a unit torus operating region in order to simplify the math for the case where part of a node's transmission area (that is, the circle swept by the maximum transmission range) is located outside of the network.

zero. Given our assumption of a fixed rate system, nodes can communicate at a rate $R$ up to a maximum distance of $\rho_{max}$. We similarly define the time node $W_i$ spends within the communication range of another node as $T^i_{traverse}$. Since the maximum distance that a node can travel within the communication range of another node is the diameter of the circle with radius $\rho_{max}$, we have the following bounds on the random variable $T^i_{traverse}$:

$$0 < T^i_{traverse} \leq \frac{2\rho_{max}}{|\boldsymbol{v}|}. \tag{4.16}$$

Between the time a destination node enters the transmission range of another node and before it leaves that transmission range, the topology and routing information at each node in the network must be updated and then the source information transmitted to the destination node. Both of these must occur in a time bounded by $T^i_{traverse}$.

As previously stated, each node must receive a total of $\psi$ bits to update its topology and routing information, which for a fixed rate system of rate $R$ requires a total time of $T_{update} \geq \psi/\mathrm{R}$. It is necessary that every node receive the topology and routing update information from every other node within the time limit $T^i_{traverse}$. For any given node $W_i$ traversing the transmission range of another node, the time remaining to receive transaction data, after completing the topology and routing information update, is given as

$$T^i_{Rx} = T^i_{traverse} - T_{update} \leq \frac{2\rho_{max}}{|\boldsymbol{v}|} - \frac{\psi}{R}. \tag{4.17}$$

The effective rate, $R_{eff}$, is again given by the following:

$$R_{eff} = R\frac{T_{Rx}^{i}}{T_{traverse}^{i}}. \tag{4.18}$$

### 4.3.3 Rate Scaling Bound - Single Mobile Node

We now look at the situation where only a single node in the network is mobile and we bound the transmission rate of the system. In order to achieve non-zero effective transmission rate, $T_{Rx}^{i} \geq 0$ must be satisfied. This is equivalent to requiring $T_{traverse}^{i} \geq T_{update}$. Given (4.17), we can substitute and rearrange the expression to get

$$R \geq \frac{\psi|v|}{2\rho_{max}}. \tag{4.19}$$

Thus we see that for the case of a single mobile node in the routing-agnostic topology approach, the rate scaling is of the same order as in the cell routing case. This results from the fact that we are simply analyzing the geometry of a particular region determined by some multiple of $\rho_{max}$. The only difference between the topology approach and the cell routing approach is the coefficient in front of $\rho_{max}$.

## 4.4 Fixed Rate Scaling - Topology Coherence Time Approach

In the previous two sections, we developed a transmission rate scaling bound for a single mobile node that moves at a constant velocity over some geometric area. Now we look at a more realistic model of topology coherence time by incorporating a random mobility model. We begin by considering the link coherence time distribution for a single mobile node using the topology approach. Then, we allow all nodes to be mobile, together with some assumptions that allow the derivation of tractable analytical results.

### 4.4.1 Network Structure and Constraints

As in Section 4.2 and Section 4.3, we assume a unit torus operating region. One necessary but not sufficient condition for a connected network is that each node is within a distance $\rho_{max}$ of at least one other node in the network. Thus given that we assume the network is connected, we can assume that each node is within the maximum transmission distance of at least one other node in the network.

### 4.4.2 Single Node Mobility Model

As before, we continue to assume constant node velocity. For the analysis in this section, we consider the starting position of a given node $W_i$, that is $(\rho_{max}, \Theta_t^i)$, in polar coordinate notation. This location is distributed randomly and uniformly on the circumference of the circle formed by the transmission range of some other node with which $W_i$ is connected. Similarly, the ending location of node $W_i$, that is $(\rho_{max}, \Theta_{t+\tau}^i)$, is also distributed randomly and uniformly on the circumference of the circle formed by the maximum transmission range circle of this node. Thus the path of the mobile node in this mobility model is defined as a randomly distributed chord on the circle of radius $\rho_{max}$ (see Fig. 2). We assume that all nodes travel at a constant speed $s_i = |\boldsymbol{v_i}| = |\boldsymbol{v}|$ where $\boldsymbol{v_i} = \boldsymbol{v}_T^i + \boldsymbol{v}_R^i$ (that is, the velocity is the sum of its tangential velocity, $\boldsymbol{v}_T^i$, and its radial velocity, $\boldsymbol{v}_R^i$). Thus the amount of time necessary for the given node $W_i$ to traverse the transmission range of a node with which it is connected is dependent on the length of the randomly distributed chord.

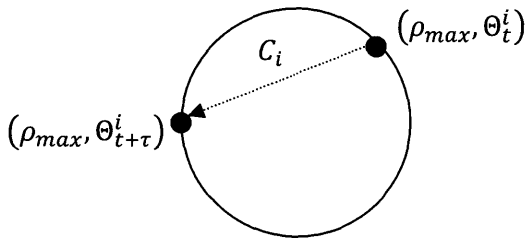


Fig. 2. Diagram of the random chord mobility model

We again define the time a node spends within the communication range of another node as $T^i_{traverse}$. If we let $C_i$ be the length of the chord along which node $W_i$ travels, then we can define the traversal time for node $W_i$ as follows:

$$T^i_{traverse} = \frac{C_i}{|v|}. \tag{4.20}$$

## 4.4.2.1 Chord Length Probability Distribution

We now determine the probability distribution of $C_i$. We consider one half of the circle in our analysis, noting that by symmetry the distribution on the other half of the circle is the same. We assume that $(R_t, \Theta^i_t) = (\rho_{max}, 0)$, where again by symmetry any other starting angle has the same distribution. We see that, per the previously stated assumptions, we begin on the circumference of the circle. Given that we assume that the path ends on the circumference of the circle, we let $(R_{t+\tau}, \Theta^i_{t+\tau}) = (\rho_{max}, \theta)$ where $\theta$ is the stochastic ending angle of mobile node $W_i$. The problem of determining a definition for a random chord on a circle is known as Bertrand's Paradox. The derivation to follow for the distribution of a random chord on a circle was confirmed by but developed independently from [42]. The authors in [42] also look at a number of other probabilistic models for the random chord problem. See Fig. 3 for a diagram of the mobility model.

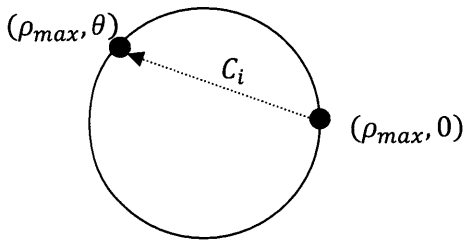


Fig. 3. Diagram of the random chord mobility model

We let $\Theta_{t+\tau}^{i}$ be uniformly distributed over $[0,\pi]$ and we have the following function for the chord length:

$$C_i = 2\rho_{max}\sin\frac{\Theta_{t+\tau}^{i}}{2} \triangleq g\left(\Theta_{t+\tau}^{i}\right). \tag{4.21}$$

That is, $C_i$ is a function of the random variable $\Theta_{t+\tau}^{i}$ and thus is itself a random variable. For its cumulative distribution function (CDF) we have the following (see Appendix B.1):

$$F_{C_i}(c) = \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{c}{2\rho_{max}}\right), & 0 \le c \le 2\rho_{max}, \\ 1, & 2\rho_{max} < c, \\ 0, & otherwise. \end{cases} \tag{4.22}$$

## 4.4.2.2 Traversal Time Probability Distribution

We now derive the CDF for the time to traverse another node's transmission range by noting that

$$T_{traverse}^{i} = \frac{C_i}{|\boldsymbol{v}|} = h(C_i). \tag{4.23}$$

Thus $T_{traverse}^{i}$ is a function of the random variable $C_i$ and is itself a random variable. We have the following CDF (see Appendix B.2):

$$F_{T_{traverse}^{i}}(t) = \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{t\cdot|\boldsymbol{v}|}{2\rho_{max}}\right), & 0 \le t \le \frac{2\rho_{max}}{|\boldsymbol{v}|}, \\ 1, & \frac{2\rho_{max}}{|\boldsymbol{v}|} < t, \\ 0, & otherwise. \end{cases} \tag{4.24}$$

Thus we have derived the distribution for the traversal time of a single mobile node moving along a random chord on the circle formed by the transmission range of another node.

### 4.4.2.3 Rate Scaling Bound

In this section we derive the rate scaling bound for a single mobile node in a manner similar to Section 4.3. We have an update time of $T_{update} \geq \psi/\mathrm{R}$ and for any given node $W_i$ we have the following for the time remaining to receive data:

$$T_{Rx}^{i} = T_{traverse}^{i} - T_{update}. \tag{4.25}$$

In order to have a non-zero effective transmission rate, $R_{eff}$, it is necessary that $T_{Rx}^{i} \geq 0$. Thus we note that $R \geq \psi/T_{traverse}^{i}$. Now, we state the following definition:

$$R_{min}^{i} \triangleq \frac{\psi}{T_{traverse}^{i}}. \tag{4.26}$$

That is, for a given node $W_i$, we define the minimum rate, $R_{min}^{i}$, to provide sufficient time for transmission of topology and routing information to the node. For a single mobile node we require that $R \geq R_{min}^{i}$. We note that we have defined this new quantity $R_{min}^{i}$ as a random variable. We derive the CDF for the minimum rate necessary to ensure that the topology and routing update information can be sent in an amount of time equal to the $T_{traverse}^{i}$. We have the following (see Appendix B.3):

$$F_{R_{min}^i}(r) = \begin{cases} 0, & r < \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ 1 - \dfrac{2}{\pi}\sin^{-1}\left(\dfrac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right), & 0 \leq \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}} \leq r, \\ 1, & otherwise. \end{cases} \tag{4.27}$$

Equation (4.27) provides an expression for the CDF of the of the minimum transmission rate to ensure that topology information is communicated within the link coherence time of a single mobile node. That is, for some rate $r$, $F_{R_{min}^i}(r)$ is the probability that this rate will be sufficient to maintain connectivity in the network, given the random chord mobility model. For example, if $r = \psi|\boldsymbol{v}|/(2\rho_{max})$, then $F_{R_{min}^i}(r) = 0$. This value of $r$ is the rate necessary if the randomly selected chord is of length equal to the diameter of the circle of radius $\rho_{max}$. Given the continuous distribution of chord length, WP1 a chord of length other than the diameter is selected.

The range of the inverse sine function is $[0, \pi/2]$ and it is decreasing in $r$; then given the expression in (4.27), $F_{R_{min}^i}(r)$ is increasing in $r$. This makes intuitive sense because one would expect that a higher rate gives a higher probability of sufficient rate for the distribution of topology and routing information. By the same reasoning, $F_{R_{min}^i}(r)$ is increasing in $\rho_{max}$ and thus a larger value of $\rho_{max}$ results in a higher probability of a rate $r$ being sufficient for the distribution of topology and routing information. Equivalently, given some fixed $r$ and assuming we maintain $r \geq \psi|\boldsymbol{v}|/(2\rho_{max})$, then $F_{R_{min}^i}(r)$ is decreasing in $\psi$ and $|\boldsymbol{v}|$. Thus given an increase in the amount of topology and routing information distributed, or an increase in velocity, the probability of a $r$ being sufficient for the network decreases.

### 4.4.3 Multiple Mobile Nodes

In Section 4.4.2 we developed an expression for the rate scaling for the single node case. In this section we build on the single node mobility model and incorporate the mobility of all nodes in the network. We

begin with the complementary CDF (CCDF) of the minimum transmission rate for a single node, where the CDF is given in (4.27):

$$F^{C}_{R^{i}_{min}}(r) = \begin{cases} 1, & r < \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ \dfrac{2}{\pi}\sin^{-1}\left(\dfrac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right), & 0 \leq \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}} \leq r, \\ 0, & otherwise. \end{cases} \tag{4.28}$$

We apply this CCDF to multiple mobile nodes. We assume that each node moves independently of every other node. Further, we simplify our analysis by ignoring the fact that the relationship between one node's movement and another node's movement can effectively increase or decrease $T^{i}_{traverse}$. We note that in a network with highly correlated movement (that is, nodes in general move in the same direction), the minimum transmission rate necessary decreases due to a corresponding increase in $T^{i}_{traverse}$. We assume that nodes do not move in cooperation, which would increase $T^{i}_{traverse}$, in order to obtain a more general bound for networks where node locations and velocities are not controlled. Further, if the network topology is centrally controlled, then distributing network topology and routing information becomes less of an issue. This is because the nodes would have direct information (from the controller) as well as indirect information (the fact that the future topology will be similar to the current topology). In the case where one node's movement decreases $T^{i}_{traverse}$ of another node, the transmission rate would have to increase to accommodate more frequent topology changes. Our assumption of independent movement thus provides a lower bound that would still hold in the case of node movement that decreases $T^{i}_{traverse}$, and this bound is analytically tractable.

### 4.4.3.1 Distribution of Minimum Rate

Given a fixed transmission rate network, we find the distribution on the maximum of the minimum per-node rate of transmission. We define the minimum rate for the network as $R_{min} \triangleq \max_i\{R_{min}^i\}$. We require that the entire network maintain connectivity in the sense of nodes being able to communicate with one another, which requires the distribution of updated topology and routing information. We begin by solving for the probability that $R_{min}$ is greater than some value $r$. We ignore the otherwise case in the CDF $F_{R_{min}^i}(r)$, because the chord length cannot take other values, and we have the following (see Appendix B.4):

$$F_{R_{min}}^C(r) = \begin{cases} 0, & r < \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ 1 - \left[1 - \dfrac{2}{\pi}\sin^{-1}\left(\dfrac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right)\right]^n, & 0 \leq \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}} \leq r. \end{cases} \tag{4.29}$$

Equation (4.29) is the CCDF for the rate necessary for successful protocol (topology and routing information) distribution. Thus $F_{R_{min}}^C(r)$ is the probability that a rate $r$ will not be sufficient for successful protocol distribution, given the random chord mobility model. In the next section we solve for the rate necessary given some specified probability of insufficient rate for successful protocol distribution.

### 4.4.3.2 Rate Scaling for Successful Protocol Distribution

We now use $F_{R_{min}}^C(r)$ to consider the rate scaling for the successful distribution of protocol information (see Definition 7). From (4.19) in Section 4.3.3, we have the following bound on $R$:

$$R \geq \frac{\psi|\boldsymbol{v}|}{2\rho_{max}}. \tag{4.30}$$

We let $\chi = \psi|\boldsymbol{v}|/(2\rho_{max})$ to aid in the tractability of the following results. We normalize $F^{C}_{R_{min}}(r)$ with respect to this lower bound on $R$. Substitution of $\chi$ results in the following:

$$F^{C}_{R_{min}}(r) = \begin{cases} 0, & r < \chi, \\ 1 - \left[1 - \frac{2}{\pi}\sin^{-1}\left(\frac{\chi}{r}\right)\right]^{n}, & 0 \leq \chi \leq r. \end{cases} \tag{4.31}$$

We solve for the rate such that the probability of insufficient rate for connectivity is less than or equal to some specified value $\phi$. That is, $Pr\{R_{min} > r\} \leq \phi$ or $F^{C}_{R_{min}}(r) \leq \phi$. We solve this for two ranges of $\phi$: $0 \leq \phi \leq 1$ and $0 \leq \phi \leq 1/2$, where the more restrictive range allows for a tighter bound. These results are proved in Appendix B.5. For the first range of $0 \leq \phi \leq 1$, we have

$$\arg\min_{r}\left\{F^{C}_{R_{min}}(r) \leq \phi\right\} \geq \frac{n\psi|\boldsymbol{v}|(1-\phi)}{\pi\rho_{max}}, \forall\, 0 \leq \phi \leq 1, \tag{4.32}$$

The result in (4.32) is the minimum $r$ such that the probability of sufficient rate for successful protocol distribution is greater than or equal to $1 - \phi$.

For the case where $0 \leq \phi \leq 1/2$, we have

$$\arg\min_{r}\left\{F^{C}_{R_{min}}(r) \leq \phi\right\} \geq \frac{6n\psi|\boldsymbol{v}|}{\phi\pi^{3}\rho_{max}}, \forall\, 0 \leq \phi \leq \frac{1}{2}, \tag{4.33}$$

We primarily focus on (4.33) because probabilities less than $1/2$ for successful probability of protocol distribution are less interesting due to the low probability. We see that the minimum rate scaling is

increasing with $n$, $\psi$ and $|v|$ and decreasing in $\phi$ and $\rho_{max}$. Thus higher values of $\phi$ (that is, lower probabilities of successful protocol distribution) require lower rates. Likewise, a larger transmission range causes a linear decrease in the minimum rate scaling. Of particular note, the rate scaling is linearly dependent on the velocity, which means that systems with higher velocities require higher rates. In the sections to follow we develop a bound on $\psi$ and use that to develop real world results for rate scaling for a given packet size.

## 4.5 Rate Scaling for All-to-All Topology Updates

In the previous sections, we have used the variable $\psi$ to denote the number of bits each node must receive per topology update. We now make assumptions about how much information each node needs to describe its topology and routing information and use that to develop a bound on the amount of new topology and routing information every node in the network must receive. We note that we are operating under the assumption that nodes do not use any predictive elements in their protocol communication. Thus each time the network topology changes, we assume that every node must be notified of the entire network topology. That is, in order for routes to exist between every S-D node pairing in a connected network, it is necessary for the entire network to update its topology and routing. If we assume that each node requires no less than $b$ bits to describe its topology and routing information, then every node must receive at least $(n-1)b$ bits for every change in topology. Then we have the bound $\psi \geq (n-1)b$.

We note that to uniquely identify every node in the network, we must have $b \geq \lceil \log_2 n \rceil$. However, here we focus on the bound of $(n-1)b$. We make no assumption about the process by which routing is performed, which may require the transmission of additional information. We only assume that in order

ensure communication is possible between all nodes, each node must send $b$ bits of information to every other node in the network.

### 4.5.1 Rate Scaling for Lower Bounded Update Size

In this section we present each of the rate scaling bounds from this chapter under the assumption of the bound $\psi \geq (n-1)b$. We let $R_{CR}$ represent the minimum rate for cell routing, $R_{TA-SN}$ represent the minimum rate for the topology approach with a single mobile node, and $R_{TCTA}$ represent the minimum rate for the topology coherence time approach. Then we have the following:

$$R_{CR} \in \Omega(n),$$

$$R_{TA-SN} \in \Omega(n),$$

$$\arg\min_{r_{TCTA}} \{F^{C}_{R_{min}}(r) \leq \phi\} \in \Omega(n^2).$$

Thus we see that the minimum rate for the single mobile node cases scale with $n$ and for the multiple mobile node case the minimum rate scales with $n^2$. Given the independence assumption for the multiple mobile node case, it makes intuitive sense that the scaling would add another factor of $n$. Thus for the single mobile node cases, the scaling is due to the bound $\psi \geq (n-1)b$. For multiple mobile nodes, the additional factor of $n$ comes from the independent movement of the nodes.

### 4.5.2 Distributed Link Coherence Time Rate Scaling for IP Packets

We now use an actual value for $b$ and present plots for the distributed link coherence time rate scaling for various values of $\phi$. We let $b = 12000\ [bits]$ (noting that standard IP Ethernet packets are 1.5 kB, or

12000 bits). Thus we make the assumption here that each node describes its local topology and routing information using one IP packet. We select values for $\phi$ of $0.05$, $0.01$ and $0.001$, corresponding to probabilities of successful protocol distribution of 95%, 99% and 99.1%, respectively. The lower bound on rate is plotted for specific ratios of transmission range to velocity, that is $\rho_{max}/|\boldsymbol{v}|$. For ratios other than those plotted, the results can be scaled by a factor of $|\boldsymbol{v}|/\rho_{max}$, noting that the figures are log-log plots. We define $r_{TCTA}^{min}(\phi) \triangleq \arg\min_{r_{DLCT}}\{F_{R_{min}}^{C}(r) \leq \phi\}$ and we have the following bounds, which we plot as a function of $n$ for $10 \leq n \leq 1000$:

$$r_{TCTA}^{min}(0.05) \geq \frac{1.44n(n-1)|\boldsymbol{v}|}{\pi^3\rho_{max}}\ [Mbps]$$

$$r_{TCTA}^{min}(0.01) \geq \frac{7.2n(n-1)|\boldsymbol{v}|}{\pi^3\rho_{max}}\ [Mbps]$$

$$r_{TCTA}^{min}(0.001) \geq \frac{72n(n-1)|\boldsymbol{v}|}{\pi^3\rho_{max}}\ [Mbps]$$

We plot the minimum rates for $\rho_{max}/|\boldsymbol{v}|$ ratios of 1, 10 and 100 (see Figs. 4-6) and a scaling comparison for $\rho_{max}/|\boldsymbol{v}| = 100$ (see Fig. 7).

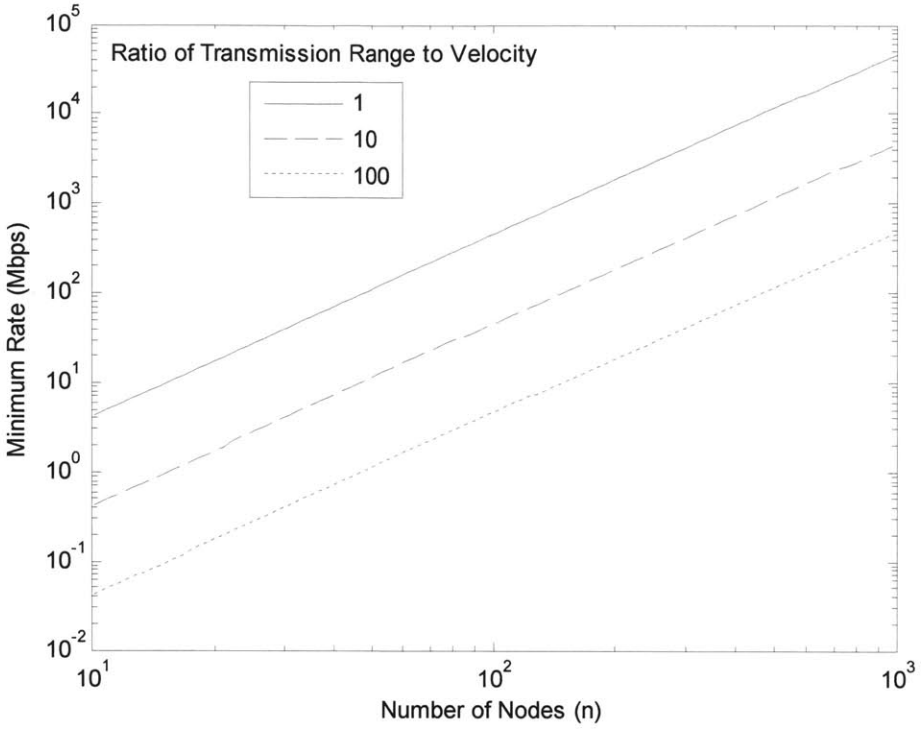


Fig. 4. Bit Rate Scaling for 95% Probability of Protocol Distribution

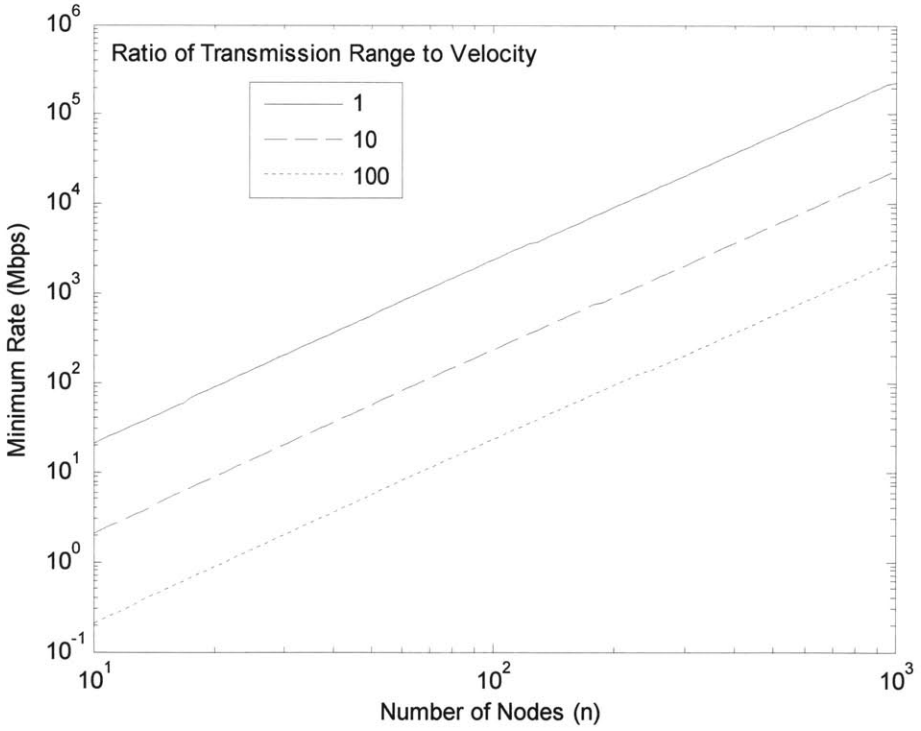


Fig. 5. Bit Rate Scaling for 99% Probability of Protocol Distribution

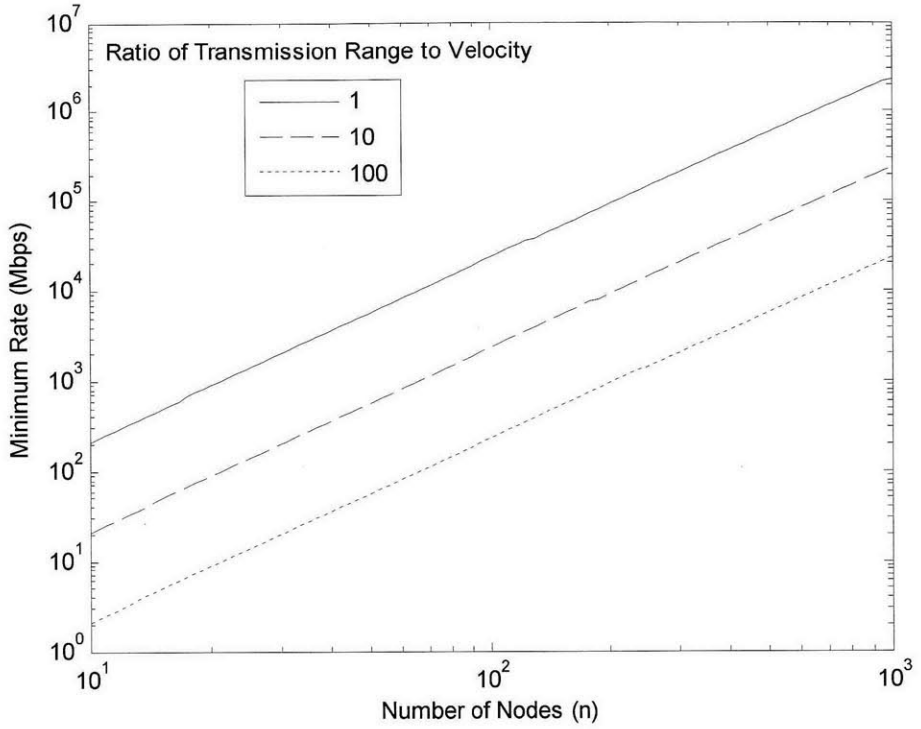


Fig. 6. Bit Rate Scaling for 99.9% Probability of Protocol Distribution

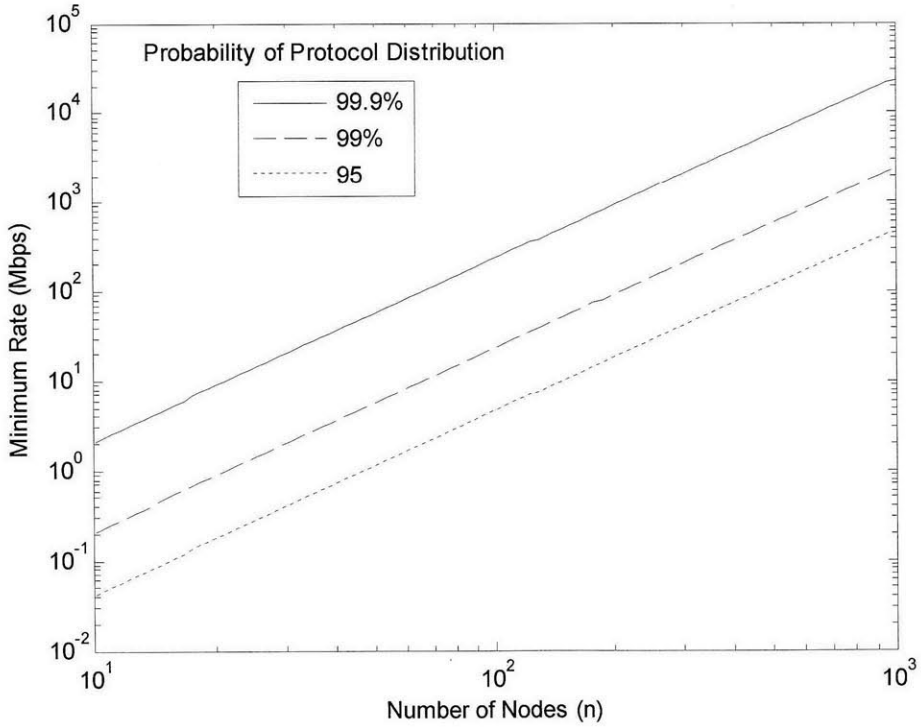


Fig. 7. Bit Rate Scaling Comparison for $\rho_{max}/|\boldsymbol{v}| = 100$

## 4.6 Summary

In this chapter we have looked at the rate scaling required for the distribution of protocol information in IWNs after topology changes resulting from node mobility. We have not addressed the issue of new links in the topology, which was discussed in [3] in terms of the order of time until a new connected link. In this chapter we presented lower bounds on rate scaling for simple mobility models and two different network views: the topology approach for a single mobile node (of which cell routing is a special case) and the topology coherence time approach. We showed that under our mobility model, and assuming every node must transmit some minimum amount of information for each topology change, the transmission rate for a fixed rate network must scale as $\Omega(n^2)$. Thus we have shown that significant node transmission rates are necessary for the distribution of topology and routing information under the assumptions considered in this chapter. Given the large amount of throughput necessary for distributing this information, we propose the use of GL/GP information for maintaining topology and routing information at the nodes in the network. GL/GP information can be used to lessen this burden by reducing the amount of signaling necessary to maintain this information at the nodes in the network. It is clear from this chapter that, absent a predictive element in the maintenance of protocol information, the throughput scaling necessary in IWNs is significant and poses a problem for network scalability. Note that we have not addressed the network burden of disseminating GL/GP information. It may seem that the network burden is equivalent to that of LSRP without GL, but with GP the need for updating GL is greatly reduced and thus less burden will be imposed on the network.

# Chapter 5

# Conclusion

In the time since the publishing of [2], which is considered by many to be the seminal work in IWNs, significant research has been directed at the characterization of such networks. While certain assumptions differ among the works in this area, the research primarily focuses on connectivity, throughput, energy, and delay. These are very important metrics to characterize in IWNs, whether narrowband or wideband. Such analyses are extremely useful in determining the types of protocols that are optimal for network design. For instance, it is important to determine the optimality of a concept such as WtNN before setting out to design an implementable protocol for topology distribution and routing in IWNs. Further, the analytic work in this area is useful for determining how well a design performs compared to the theoretical bounds on performance.

We began this work with an overview of the field of IWNs and, in particular, the characteristics which differentiate IWNs. We also reviewed and compared the scaling results for two-dimensional IWNs, as well as the assumptions for these results, in order to build a picture of the current landscape in this area. In Chapter 2 we reviewed some of the important results in connectivity and T/E/D tradeoffs and scaling. Further, in Chapter 3 we looked at the scaling of one-dimensional networks using a slightly different approach than the current body of research. We developed results for comparing three types of one-dimensional networks and, in particular, for comparing networks with and without GL/GP information.

Finally, we looked at the burden of distributing topology and routing information in networks in order show how GL/GP information can improve network performance.

In addition to connectivity and T/E/D scaling and tradeoffs, it is important to determine other characteristics of an implementable system, such as the burden of distributing topology and routing information in the network as explored in Chapter 4. The impact of this burden is useful both in terms of ensuring a network is implementable and adjusting the bounds on performance to take into account this additional requirement. Further, quantizing this burden is also useful as it relates to implementable networks. For instance, we showed in Chapter 3 that one-dimensional WtNN networks experience a minimal benefit from GL/GP information. Such results are helpful in evaluating the features of a system for implementation.

The use of GL/GP is one method we propose for maintaining topology and routing information in IWNs in order to enable forwarding of packets between nodes in the network. As compared to the node-to-node distribution of topology and routing information, the use GL/GP offers the ability to reduce the network burden of maintaining this information. One method of distributing GL/GP information is via node-to-node communication, such as how topology and routing information is distributed. Another method is to use, for instance, an airborne node to keep track of the nodes in the network and provide updates of the network topology via signaling. Regardless of the method used to reduce the protocol burden, GL/GP can improve throughput via directional antennas, user helper nodes and characteristic distance hopping, in addition to reducing the protocol burden. The use of such information will help to improve the performance of networks as IWNs become more prevalent. Moreover, with GP the need for GL updates is greatly reduced, further decreasing the need for capacity to distribute network protocol information.

We propose that future IWN research take into account the burden of maintaining topology information at every node, particularly when simulating or emulating networking protocols. In addition, we have laid the groundwork for analytic results on the protocol burden of IWNs. Some possible next steps in this area of research are to determine the rate scaling for different mobility schemes or to develop a general approach to protocol scaling in IWNs. Another area of research is to develop methods to quantify the costs and benefits of GL/GP information. For example, this could involve determining the performance tradeoffs of better information versus the cost of distributing that information. Finally, an important area of research is to determine the effect of error in geographic prediction and the relationship between that error and how often geographic location updates are required.

The analysis of IWNs presents a number of challenges, from the well-researched area of connectivity to the impact of protocol distribution on network throughput. As IWNs move from concept to reality, this work helps to lay the foundation for understanding the burden of maintaining topology and routing information at the nodes in the network. Given the use of GL/GP information, much research remains to determine both the amount of information and the accuracy of the information to be distributed. We hope that this work motivates continued GL/GP research in IWNs and lays the foundation for future results. Despite the differences in the assumptions of the various works in this field, a clearer vision continues to emerge regarding what future IWNs will look like. We hope that the results we have presented here will make that vision a little sharper.

# Appendix A

# Derivations for Equations in Chapter 3

## A.1 Proof for Results in Section 3.3

In Section 3.3 we defined the nearest neighbor as the node that is closest to a given node location $X_{(i)}$. That is, we let $D_{(i)}$ be a random variable that takes the value of the direction of the nearest neighbor from node $(i)$ such that

$$D_{(i)} = \begin{cases} R, & \textit{nearest neighbor is to the right,} \\ L, & \textit{nearest neighbor is to the left.} \end{cases}$$

We noted that the following occur surely:

$$D_{(1)} = R,$$

$$D_{(n)} = L.$$

That is, the end nodes in a one-dimensional line network only have one nearest neighbor and the location of that neighbor is deterministic. Now $D_{(i)}, i \in [2, n-1]$, can be expressed as follows:

$$\{D_{(i)} = R\} = \{|Y_{i-1}| \geq |Y_i|\},$$

$$\{D_{(i)} = L\} = \{|Y_{i-1}| < |Y_i|\}.$$

We have used the following:

$$Pr\{|X_{(i)} - X_{(i-1)}| = |X_{(i)} - X_{(i+1)}|\} = 0.$$

Next we prove that

$$Pr\{D_{(i)} = L\} = Pr\{D_{(i)} = R\} = 0.5 \; \forall \; i \in [2, n-1].$$

We start by looking at the ordered set $\mathbb{X}_O$ and we consider any $X_{(i)}$ as the arrival epoch in a Poisson process conditioned on $n$ arrivals by time 1 (per [43]). We use Chapter 2 [41] to define arrival processes and Poisson processes. An arrival process is a sequence of increasing random variables, $0 < S_1 < S_2 < \cdots$, where the random variables $S_i$ are arrival epochs. These arrival epochs can also be specified using the sequence of inter-arrival times $Z_1, Z_2, \ldots$, where these are positive random variables defined such that $Z_1 = S_1$ and $Z_i = S_i - S_{i-1}$. Further, these inter-arrival times are usually IID. Given the set of $Z_i$, the arrival epochs can be specified as

$$S_n = \sum_{i=1}^{n} Z_i.$$

A second alternative for specifying the arrival process is by the counting process $N(t)$, where for each $t > 0$, the random variable $N(t)$ is the number of arrivals up to and including time $t$ (that is, the number of arrivals on the interval $(0, t]$). We note that the set of all $N(t)$ is uncountably infinite. Then the following events are equivalent (see (2.3) in [41]):

$$\{S_n > t\} = \{N(t) < n\}.$$

Then the arrival process can be specified by the joint distributions of either the arrival epochs $S_i$, the inter-arrival intervals $Z_i$, or the counting random variables $N(t)$. A Poisson process is an arrival process that is most conveniently described by its inter-arrival times, where each $Z_i$ has the PDF given by

$$f_Z(z) = \lambda e^{-\lambda z}, z \geq 0,$$

where $\lambda$ is the rate of the process.

Per [43], we can view the node locations $\mathbb{X}_0$ as the arrival epochs in a Poisson process conditioned on $n$ arrivals by time 1, noting that an arrival process is a sequence of increasing random variables. As previously derived, $X_{(i)} < X_{(i+1)}$ implies that $X_{(i+1)} - X_{(i)}$ is a positive random variable $Y_i$. The process is considered to start at time 0 and multiple arrivals cannot occur simultaneously. We look at any $X_{(i)}$ conditioned on $n$ and all the other arrival epochs, that is $\{X_{(i)} | \{X_{(j)}, \forall j \neq i\}, X_{(n)} < 1\}$. Given the uniform distribution, $X_{(i)}$ conditioned on $n$ and all the other node locations is then uniformly distributed between $X_{(i+1)}$ and $X_{(i-1)}$. In the case of $X_{(1)}$, it is uniformly distributed between 0 and $X_{(2)}$ and, in the case of $X_{(n)}$, it is uniformly distributed between $X_{(n-1)}$ and 1. Since the distribution is uniform, we see that the events $\{D_{(i)} = R\}$ and $\{D_{(i)} = L\}$ are equally probable. Given this, we state

$$Pr\{|X_{(i)} - X_{(i-1)}| \geq |X_{(i)} - X_{(i+1)}|\} = Pr\{|X_{(i)} - X_{(i-1)}| < |X_{(i)} - X_{(i+1)}|\} = 0.5,$$

which allows us to make the following equivalent statement:

$$Pr\{D_{(i)} = L\} = Pr\{D_{(i)} = R\} = 0.5, \forall i \in [2, n-1].$$

Thus we have proved that for $i \in [2, n-1]$, the probability of having a nearest neighbor to the left is equal to the probability of having a nearest neighbor to the right.

## A.2 Proof for Results in Section 3.4.1

In the non- GL/GP case, we show that, for any $n$,

$$\lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = \lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = 0.5.$$

In addition, we obtain results which can be used to calculate these probabilities for any value of $n$. We define two additional variables for the number of correct and "error" first hops, respectively, from node $(i)$ as $\mathbb{C}_{(i)}$ and $\mathbb{E}_{(i)}$. We define $1_{\left\{H_{(i)}^{(j)}(n)\right\}}$ as the indicator random variable for whether the first hop from node $X_{(i)}$ to $X_{(j)}$ is correct or incorrect. Then we can write $\mathbb{C}_{(i)}$ and $\mathbb{E}_{(i)}$ in terms of $H_{(i)}^{(j)}$ as follows:

$$\mathbb{C}_{(i)} = \sum_{j\neq i} 1_{\left\{H_{(i)}^{(j)}(n)=0\right\}},$$

$$\mathbb{E}_{(i)} = \sum_{j\neq i} 1_{\left\{H_{(i)}^{(j)}(n)=1\right\}}.$$

In this case we are using the uniform all-to-all traffic model and thus there are $n(n-1)$ S-D traffic pairs. We assume that the network is fully connected (i.e., a path exists from each node to every other node, possibly through some number of intermediate nodes).

We first look at the two extreme cases for the non-GL/GP network, those being $H_{(1)}^{(j)}(n)$ and $H_{(n)}^{(j)}(n)$. In both of these cases, the first hop is always in the correct direction because all nodes lie to the right and left, respectively, of node locations $X_{(1)}$ and $X_{(n)}$. Thus, by this argument we have:

$$\Pr\left\{H_{(1)}^{(j)}(n) = 0\right\} = 1 \; \forall j \in [2, n],$$

$$\Pr\left\{H_{(n)}^{(j)}(n) = 1\right\} = 1 \; \forall j \in [1, n-1].$$

We now prove the following for the non- GL/GP case:

$$\lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = \lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = 0.5.$$

To do this we return to the ordered set $\mathbb{X}_O$. We have shown that the probabilities of a nearest neighbor being left or right of a given node are equally likely events. We have also calculated the number of right and left neighbors for a given node $X_{(i)}$ as

$$\mathcal{N}_R^{(i)} = n - i,$$

$$\mathcal{N}_L^{(i)} = n - \mathcal{N}_R^{(i)} - 1 = i - 1.$$

We begin the proof by first looking at the case of the middle node in a network where $n$ is odd and $n \geq 3$, that is the node of index $\left(\left\lceil\frac{n}{2}\right\rceil\right) = \left(\frac{n+1}{2}\right)$. We can see that for such a node $\mathcal{N}_R^{(i)} = \mathcal{N}_L^{(i)} = (n-1)/2$.

We see that for the middle node, $X_{\left(\frac{n+1}{2}\right)}$,

$$\mathbb{C}_{\left(\frac{n+1}{2}\right)} = \begin{cases} \mathcal{N}_R^{\left(\frac{n+1}{2}\right)}, & if D_{\left(\frac{n+1}{2}\right)} = R, \\ \mathcal{N}_L^{\left(\frac{n+1}{2}\right)}, & if D_{\left(\frac{n+1}{2}\right)} = L, \end{cases}$$

$$\mathbb{E}_{\left(\frac{n+1}{2}\right)} = \begin{cases} \mathcal{N}_L^{\left(\frac{n+1}{2}\right)}, & if D_{\left(\frac{n+1}{2}\right)} = R, \\ \mathcal{N}_R^{\left(\frac{n+1}{2}\right)}, & if D_{\left(\frac{n+1}{2}\right)} = L. \end{cases}$$

Then we can plug in the values previously calculated and we obtain

$$\mathbb{C}_{\left(\frac{n+1}{2}\right)} = \begin{cases} \frac{n-1}{2}, & if D_{\left(\frac{n+1}{2}\right)} = R, \\ \frac{n-1}{2}, & if D_{\left(\frac{n+1}{2}\right)} = L, \end{cases}$$

$$\mathbb{E}_{\left(\frac{n+1}{2}\right)} = \begin{cases} \frac{n-1}{2}, & if D_{\left(\frac{n+1}{2}\right)} = R, \\ \frac{n-1}{2}, & if D_{\left(\frac{n+1}{2}\right)} = L. \end{cases}$$

Finally, we make use of $\mathbb{C}_{(i)}$ and $\mathbb{E}_{(i)}$ and use these values to calculate the probability of selecting a path with a correct versus an error first hop using the law of total probability as follows:

$$Pr\left\{H^{(j)}_{\left(\frac{n+1}{2}\right)}(n) = 0\right\} = \sum_{d=\{L,R\}} Pr\left\{H^{(j)}_{\left(\frac{n+1}{2}\right)}(n) = 0 \middle| D_{\left(\frac{n+1}{2}\right)} = d\right\} Pr\left\{D_{\left(\frac{n+1}{2}\right)} = d\right\}.$$

We assume that the probability of selecting any particular path is equal to that of any other path. Then substituting in we get

$$Pr\left\{H^{(j)}_{\left(\frac{n+1}{2}\right)}(n)=0 \,\middle|\, D_{\left(\frac{n+1}{2}\right)}=d\right\}=\begin{cases}\dfrac{\mathbb{C}_{\left(\frac{n+1}{2}\right)}}{\mathbb{C}_{\left(\frac{n+1}{2}\right)}+\mathbb{E}_{\left(\frac{n+1}{2}\right)}}, & D_{\left(\frac{n+1}{2}\right)}=R,\\ \dfrac{\mathbb{C}_{\left(\frac{n+1}{2}\right)}}{\mathbb{C}_{\left(\frac{n+1}{2}\right)}+\mathbb{E}_{\left(\frac{n+1}{2}\right)}}, & D_{\left(\frac{n+1}{2}\right)}=L,\end{cases}$$

$$=\begin{cases}0.5, & D_{\left(\frac{n+1}{2}\right)}=R,\\ 0.5, & D_{\left(\frac{n+1}{2}\right)}=L.\end{cases}$$

This simplifies to

$$Pr\left\{H^{(j)}_{\left(\frac{n+1}{2}\right)}(n)=0\right\}=\sum_{d=\{L,R\}}\frac{1}{2}Pr\left\{D_{\left(\frac{n+1}{2}\right)}=d\right\}=\frac{1}{2}.$$

Then likewise we see that

$$Pr\left\{D_{\left(\frac{n+1}{2}\right)}(n)=1\right\}=\frac{1}{2}.$$

Thus we proved that the probabilities of the first hop being correct or incorrect are equal in a randomly selected path from the middle node $X_{\left(\frac{n+1}{2}\right)}$ in a network where $n$ is odd. Now we expand the proof to take into account all nodes $X_{(i)}$ in the network for $i \in [1,n]$, where $n$ is either even or odd. First, we again develop expressions for correct and error first hops for $i \in [1,n]$ as:

$$\mathbb{C}_{(i)} = \begin{cases} \mathcal{N}_R^{(i)}, & if D_{(i)} = R, \\ \mathcal{N}_L^{(i)}, & if D_{(i)} = L, \end{cases}$$

$$\mathbb{E}_{(i)} = \begin{cases} \mathcal{N}_L^{(i)}, & if D_{(i)} = R, \\ \mathcal{N}_R^{(i)}, & if D_{(i)} = L. \end{cases}$$

We then obtain the following:

$$\mathbb{C}_{(i)} = \begin{cases} n - i, & if D_{(i)} = R, \\ i - 1, & if D_{(i)} = L, \end{cases}$$

$$\mathbb{E}_{(i)} = \begin{cases} i - 1, & if D_{(i)} = R, \\ n - i, & if D_{(i)} = L. \end{cases}$$

Now we wish to calculate the probability that a randomly selected path takes a correct first hop versus an error first hop. We again use the law of total probability and, assuming each path is equally likely to be selected, for a given $i \in [1, n]$:

$$Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = \sum_{d \in \{L,R\}} Pr\left\{H_{(i)}^{(j)}(n) = 0 \middle| D_{(i)} = d\right\} Pr\{D_{(i)} = d\}.$$

Now substituting we obtain the following:

$$Pr\left\{H_{(i)}^{(j)}(n) = 0 \middle| D_{(i)} = d\right\} = \begin{cases} \dfrac{\mathbb{C}_{(i)}}{\mathbb{C}_{(i)} + \mathbb{E}_{(i)}} = \dfrac{n - i}{n - 1}, & D_{(i)} = R, \\ \dfrac{\mathbb{C}_{(i)}}{\mathbb{C}_{(i)} + \mathbb{E}_{(i)}} = \dfrac{i - 1}{n - 1}, & D_{(i)} = L. \end{cases}$$

Then plugging back in we solve as follows:

$$Pr\left\{H_{(i)}^{(j)}(n)=0\right\}=\frac{n-i}{n-1}Pr\{D_k=R\}+\frac{i-1}{n-1}Pr\{D_k=L\}$$

$$=\frac{n-i}{n-1}\cdot\frac{1}{2}+\frac{i-1}{n-1}\cdot\frac{1}{2}$$

$$=\frac{1}{2}.$$

By extension, we note that

$$Pr\left\{H_{(i)}^{(j)}(n)=1\right\}=\frac{1}{2}.$$

Now we must take into account the end nodes (which have no incorrect first hops), as well as the probability of selecting a particular path. We calculate the probability of any randomly selected path taking a correct first hop as follows:

$$Pr\left\{H_{(i)}^{(j)}(n)=0\right\}=\sum_{i=1}^{n}\sum_{j\neq i}Pr\left\{H_{(i)}^{(j)}(n)=0|k,l\right\}Pr\{k,l\}$$

$$=\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j\neq i}Pr\left\{H_{(i)}^{(j)}(n)=0|k,l\right\}$$

$$=\frac{1}{n(n-1)}\left(2(n-1)+\sum_{i=2}^{n-1}\sum_{j\neq i}Pr\left\{H_{(i)}^{(j)}(n)=0|k,l\right\}\right)$$

$$=\frac{2(n-1)+\frac{1}{2}(n-2)(n-1)}{n(n-1)}$$

$$= 0.5 + \frac{1}{n}, n \geq 2.$$

Thus the probability of selecting a path with a correct first hop approaches $1/2$ asymptotically with increasing $n$. Using the same method, we can calculate the probability of an error first hop as

$$Pr\left\{H_{(i)}^{(j)}(n) = 1\right\} = \sum_{i=1}^{n}\sum_{j\neq i} Pr\left\{H_{(i)}^{(j)}(n) = 1|k,l\right\} Pr\{k,l\}$$

$$= \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j\neq i} Pr\left\{H_{(i)}^{(j)}(n) = 1|k,l\right\}$$

$$= \frac{1}{n(n-1)}\sum_{i=2}^{n-1}\sum_{j\neq i} Pr\left\{H_{(i)}^{(j)}(n) = 1|k,l\right\}$$

$$= \frac{\frac{1}{2}(n-2)(n-1)}{n(n-1)}$$

$$= 0.5 - \frac{1}{n}, n \geq 2.$$

Thus the probability of selecting a path with an error first hop approaches $1/2$ asymptotically with increasing $n$. We have therefore also shown that in the non-GL/GP case, for any $n$,

$$\lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = \lim_{n\to\infty} Pr\left\{H_{(i)}^{(j)}(n) = 0\right\} = 0.5.$$

## A.3 Proof for Results in Section 3.5.1

We recall the following:

$$D_{(i)} = \begin{cases} R, & nearest\ neighbor\ is\ to\ the\ right, \\ L, & nearest\ neighbor\ is\ to\ the\ left, \end{cases}$$

$$H_{(i)}^{(j)}(n) = \begin{cases} 0, & first\ hop\ correct\ in\ a\ network\ of\ n\ nodes, \\ 1, & first\ hop\ incorrect\ in\ a\ network\ of\ n\ nodes, \end{cases}$$

$$\mathcal{N}_R^{(i)} = n - i,$$

$$\mathcal{N}_L^{(i)} = n - \mathcal{N}_R^{(i)} - 1 = i - 1.$$

In Section 3.5.1 we defined the term $\mathbb{H}_T^{non-GL/GP}$, which takes the value of the total number of hops in a non-GL/GP network. Here we also define the term $\mathbb{H}_{(i)}$, which takes the value of the total number of hops in the path from node $X_{(i)}$ to every other node in the network. We finally define the term $\mathbb{H}_{(i)}^{(j)}$, which takes the value of the total number of hops on the path from node $X_{(i)}$ to some node $X_{(j)}$. Then we have the following:

$$\mathbb{H}_T^{non-GL/GP} = \sum_i \mathbb{H}_{(i)} = \sum_i \sum_{j \neq i} \mathbb{H}_{(i)}^{(j)}.$$

We start with a network where $n = 3$. Then we note the following by inspection:

$$\mathbb{H}_{(i)} = \sum_{j \neq i} \mathbb{H}_{(i)}^{(j)} = \begin{cases} 3, & i = 1,3, \\ 4, & i = 2. \end{cases}$$

We see that if $D_{(2)} = R$ then $\mathbb{H}_{(2)}^{(1)} = 3$ and $\mathbb{H}_{(2)}^{(3)} = 1$. If $D_{(2)} = L$ then $\mathbb{H}_{(2)}^{(1)} = 1$ and $\mathbb{H}_{(2)}^{(3)} = 3$. We note that if the nearest neighbor is to the right, the path to the left node is increased by 2 hops because the path first hops right then back to the starting node, then to the destination to the left. A symmetric argument is true for the case when the nearest neighbor is to the left. We then see that in a network of $n = 3$,

$$\mathbb{H}_T^{non-GL/GP} = \sum_i \mathbb{H}_{(i)} = 10.$$

Now we extend our analysis to a network of $n$ nodes. We note that paths are once again deterministic based on our traffic routing scheme. We prove this by noting the following, where $\mathbb{P}_{(i)}$ is the path from $X_{(i)}$ to all other nodes:

$$\{D_{(i)} = R\} \leftrightarrow \{|X_{(i)} - X_{(i+1)}| \leq |X_{(i)} - X_{(i-1)}|\}, \forall i \in [2, n-1],$$

$$\{D_{(i)} = R\} \leftrightarrow \mathbb{P}_{(i)} = \{X_{(i+1)}, X_{(i+2)}, \dots, X_{(n)}, X_{(n-1)}, \dots, X_{(2)}, X_{(1)}\}.$$

We see the following from this proof:

$$\{D_{(k)} = R\} \leftrightarrow \mathbb{P}_{(i)} = \{X_{(i+1)}, X_{(i+2)}, \dots\}.$$

We note that $\{D_{(k)} = R\} \leftrightarrow \mathbb{P}_{(i)} = \{X_{(i+1)}, X_{(i)}, \dots\}$ is not possible. This is a result of our assumption of how packets are routed in the network. We assume that a node only communicates with its two nearest neighbors and does not send a packet to the node from which the packet was just received (except for the end nodes). This means that once a packet is routed in one direction it continues to travel in that

direction until it reaches its destination or the end of the network. If a packet reaches one of the ends of the network and that particular end node is not the destination, then the packet is routed back toward the other end of the network.

From before we know that the total number of hops for uniform all-to-all traffic for path $\mathbb{P}_{(i)}$ from node $X_{(i)}$ is given by

$$\mathbb{H}_{(i)} = \sum_{j \neq i} \mathbb{H}_{(i)}^{(j)} = \sum_{j \in \{\mathcal{N}_R^{(i)}\}} \mathbb{H}_{(i)}^{(j)} + \sum_{j \in \{\mathcal{N}_L^{(i)}\}} \mathbb{H}_{(i)}^{(j)}.$$

We see that we have separated the sums over left and right neighbor nodes, noting one set is on the direct path, and the other set of nodes requires the path to reach the end of the network before returning in the correct direction. Then we solve:

$$\mathbb{H}_{(i)} = \begin{cases} \sum_{j=i+1}^{n} (j-i) + \sum_{j=1}^{i-1} \big(2(n-i) + (i-j)\big), & \{D_{(i)} = R\}, \\ \sum_{j=1}^{i-1} (i-j) + \sum_{j=i+1}^{n} \big(2(i-1) + (j-i)\big), & \{D_{(i)} = L\}, \end{cases}$$

$$= \begin{cases} \frac{1}{2}(i-n-1)(i-n) + \frac{1}{2}(4n-3i)(i-1), & \{D_{(k)} = R\}, \\ \frac{1}{2}(i^2-i) + \frac{1}{2}(n-i)(3i+n-3), & \{D_{(k)} = L\}, \end{cases}$$

$$= \begin{cases} \frac{1}{2}n^2 - \frac{3}{2}n + ni - i^2 + i, & \{D_{(k)} = R\}, \\ \frac{1}{2}n^2 - \frac{3}{2}n + ni - i^2 + i, & \{D_{(k)} = L\}, \end{cases}$$

$$= \frac{1}{2}n^2 - \frac{3}{2}n + ni - i^2 + i.$$

Thus while the total number of hops in a path is dependent on the ordered node index $(i)$ and the direction of its first hop, the total number of hops for uniform all-to-all traffic is only dependent on the ordered node index $(i)$. Then given a node $X_{(i)}$, regardless of whether $D_{(i)} = R$ or $D_{(i)} = L$, the number of hops for uniform all-to-all traffic is the same.

Now we finally write

$$\mathbb{H}_T^{non-GL/GP} = \sum_i \mathbb{H}_{(i)}$$

$$= \sum_{i=1}^{n} \left( \frac{1}{2}n^2 + \frac{3}{2}n + kn - k^2 + k + 1 \right)$$

$$= \frac{2}{3}n^3 - n^2 + \frac{1}{3}n.$$

## A.4 Proof for Results in Section 3.5.2

In Section 3.5.2 we defined the term $\mathbb{H}_T^{GL/GP}$, which takes the value of the total number of hops in a GL/GP network. We also use the term $\mathbb{H}_{(i)}$, defined in Appendix A.3. Now we can calculate $\mathbb{H}_T^{GL/GP}$ as follows:

$$\mathbb{H}_T^{GL/GP} = \sum_i \mathbb{H}_{(i)}$$

$$= \sum_i \sum_{j \neq i} \mathbb{H}_{(i)}^{(j)}$$

$$= \sum_i \left( \sum_{j \in \{\mathcal{N}_R^{(i)}\}} \mathbb{H}_{(i)}^{(j)} + \sum_{j \in \{\mathcal{N}_L^{(i)}\}} \mathbb{H}_{(i)}^{(j)} \right).$$

We note that the above is a sum over all nodes $i \in [1, n]$. For each $i$ there is a sum of the hops to the right neighbors and a sum of the hops to the left neighbors of node $X_{(i)}$, noting that each path hops left or right through all neighbors between the source and destination. Now we simplify as

$$\mathbb{H}_T^{GL/GP} = \sum_{i=1}^{n} \left( \sum_{j=i+1}^{n} (j - i) + \sum_{j=1}^{i-1} (i - j) \right)$$

$$= \sum_{i=1}^{n} \left( \frac{1}{2}(i - n - 1)(i - n) + \frac{1}{2}(i^2 - i) \right)$$

$$= \frac{1}{3}n^3 - \frac{1}{3}n.$$

## A.5 Proof for Results in Section 3.5.3

We again note that we are dealing with a normalized one-dimensional network length of 1 and, as in Section 3.5.3, describe the optimal case for the lower bound on total hop count. We let $L$ be the network length and we assume $L$ is some integer multiple of $d_{char}$ such that $L = (l - 1) \cdot d_{char}$ where $(l - 1) \in \mathbb{Z}^+$ and we further assume that $n/l \in \mathbb{Z}^+$. Now we define the characteristic hop distance normalized with respect to $L$ as $\hat{d}_{char} = d_{char}/L = (l - 1)^{-1}$. The optimal network for minimum total hop count is where nodes are clustered at integer multiples of $\hat{d}_{char}$, that is at the locations

$\{0, (l-1)^{-1}, 2(l-1)^{-1}, \ldots, 1\}$. Thus we can view the network as essentially subdivided into $l$ clusters with $n/l$ nodes at each cluster. Finally, we view the entire network as $n/l$ sub-networks, where each sub-network consists of one node at each cluster and thus consists of $l$ nodes.

We note that we cannot view each sub-network as an independent unit because the sub-networks need to be able to communicate with one another. To calculate the total hop count, we sum over each cluster and each node within each cluster. Once we have specified a cluster and a node in the sum, we then must calculate all the hops for uniform all-to-all traffic. We now define the number of clusters to the right and left of cluster $(i)$, where $i \in [1, l]$ as follows:

$$\mathcal{C}_R^{(i)} = l - i,$$

$$\mathcal{C}_L^{(i)} = l - \mathcal{C}_R^{(i)} - 1 = i - 1.$$

Then we lower bound $\mathbb{H}_T^{d_{char}}$, including the term $(n/l - 1)$ for hops within a cell, as follows:

$$\mathbb{H}_T^{d_{char}} \geq \sum_{a=1}^{l} \sum_{b=1}^{n/l} \left( \left(\frac{n}{l} - 1\right) + \frac{n}{l} \sum_{c \in \left\{\mathcal{C}_R^{(b)}\right\}} \mathbb{H}_{(b)}^{(c)} + \frac{n}{l} \sum_{d \in \left\{\mathcal{C}_L^{(b)}\right\}} \mathbb{H}_{(b)}^{(d)} \right)$$

$$= \sum_{a=1}^{l} \sum_{b=1}^{n/l} \left( \left(\frac{n}{l} - 1\right) + \frac{n}{l} \sum_{c=a+1}^{l} (c - a) + \frac{n}{l} \sum_{d=1}^{a-1} (a - d) \right)$$

$$= \sum_{a=1}^{l} \sum_{b=1}^{n/l} \left( \left(\frac{n}{l} - 1\right) + \frac{n}{2l}(l - a)(l - a + 1) + \frac{n}{2l}(a^2 - a) \right)$$

$$= \frac{1}{6l}(2n - l^2 + l)(2n + nl^2 - 3l).$$

We combine this with the upper bound $\mathbb{H}_T^{d_{char}} \leq \mathbb{H}_T^{GL/GP}$ from Section 3.5.3 and have the following bounds:

$$\frac{1}{6l}(2n - l^2 + l)(2n + nl^2 - 3l) \leq \mathbb{H}_T^{d_{char}} \leq \frac{n}{3}(n^2 - 1).$$

Then given that $l = 1 + L/d_{char}$, we have the following:

$$\frac{\left(2d_{char}^2 n + Ld_{char} - L^2\right)\left(2d_{char}^2 n + nL^2 - 3Ld_{char}\right)}{6Ld_{char}^3} \leq \mathbb{H}_T^{d_{char}} \leq \frac{n}{3}(n^2 - 1).$$

## A.6 Proof for Results in Section 3.6.1 (Throughput Lower Bound)

In the uniform all-to-all traffic case, given the set of nodes $\mathbb{X}_O = \{X_{(1)}, \ldots, X_{(n)}\}$, there are a total of $n$ nodes in the network and thus a total of $n(n-1)$ S-D pairs. We solve for a lower bound on $\hat{\lambda}^{non-GL/GP}$, which means an upper bound on $\hat{L}^{non-GL/GP}(n)$. We look at maximizing $L_{(k)}^{(k+1)} + L_{(k)}^{(k-1)}$. Before continuing, we define the variable $Tx_{(k)}^{(i\to j)}$ $[streams]$ as the transmission traffic in steady state for node $X_{(k)}$ as generated by S-D pair $X_{(i)}$ and $X_{(j)}$. We also equivalently define a received traffic variable $Rx_{(k)}^{(i\to j)}$. We let $Tx_{(k)}$ be the total transmitted traffic in steady state for node $X_{(k)}$ and we define an equivalent variable $Rx_{(k)}$. We write the following:

$$\hat{L}^{non-GL/GP}(n) \leq \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{Tx_{(k)}\}\right\} = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \left\{\sum_{i,j\in[1,n], i\neq j} Tx_{(k)}^{(i\to j)}\right\}\right\}.$$

Given the symmetry of the network, and given the max-max argument, we see that

$$\max_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\{Tx_{(k)}\}\right\}=\max_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\{Rx_{(k)}\}\right\},$$

where $Rx_{(k)}$ is the total traffic received by node $(k)$. In other words, by symmetry

$$\max_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\left\{\sum_{i,j\in[1,n],i\neq j}Tx_{(k)}^{(i\to j)}\right\}\right\}=\max_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\left\{\sum_{i,j\in[1,n],i\neq j}Rx_{(k)}^{(i\to j)}\right\}\right\}.$$

We know from previous arguments that the number of right and left neighbors, including the end nodes, for node $(k)$ are given, respectively, as

$$\mathcal{N}_R^{(k)}=n-k,$$

$$\mathcal{N}_L^{(k)}=n-\mathcal{N}_R^{(k)}-1=k-1.$$

Then given some node $(k)$, we can calculate $\sum_{i,j\in[1,n],i\neq j}Tx_{(k)}^{(i\to j)}$ as follows:

$$\sum_{i,j\in[1,n],i\neq j}Tx_{(k)}^{(i\to j)}=\sum_{i=1}^{n}\sum_{j\neq i}Tx_{(k)}^{(i\to j)}$$

$$=\sum_{j\neq k}Tx_{(k)}^{(k\to j)}+\sum_{i\neq k}Tx_{(k)}^{(i\to k)}+\sum_{1<i<k}\sum_{1\leq j<i}Tx_{(k)}^{(i\to j)}$$

$$+\sum_{1\leq i<k}\sum_{i<j<k}Tx_{(k)}^{(i\to j)}+\sum_{1\leq i<k}\sum_{k<j\leq n}Tx_{(k)}^{(i\to j)}+\sum_{k<i<n}\sum_{i<j\leq n}Tx_{(k)}^{(i\to j)}$$

$$+\sum_{k<i\leq n}\sum_{k<j<i} Tx_{(k)}^{(i\to j)}+\sum_{k<i\leq n}\sum_{1\leq j<k} Tx_{(k)}^{(i\to j)}.$$

Now before moving on we check that all traffic pairs are accounted for in these summations. We have

$$\sum_{j\neq k} 1 = n-1,$$

$$\sum_{i\neq k} 1 = n-1,$$

$$\sum_{1<i<k}\sum_{1\leq j<i} 1 = \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 1 = \frac{1}{2}(k-1)(k-2),$$

$$\sum_{1\leq i<k}\sum_{i<j<k} 1 = \sum_{i=1}^{k-1}\sum_{j=i+1}^{k-1} 1 = \frac{1}{2}(k-1)(k-2),$$

$$\sum_{1\leq i<k}\sum_{k<j\leq n} 1 = \sum_{i=1}^{k-1}\sum_{j=k+1}^{n} 1 = (k-1)(n-k),$$

$$\sum_{k<i<n}\sum_{i<j\leq n} 1 = \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 1 = \frac{1}{2}(n-k)(n-k-1),$$

$$\sum_{k<i\leq n}\sum_{k<j<i} 1 = \sum_{i=k+1}^{n}\sum_{j=k+1}^{i-1} 1 = \frac{1}{2}(n-k)(n-k-1),$$

$$\sum_{k<i\leq n}\sum_{1\leq j<k} 1 = \sum_{i=k+1}^{n}\sum_{j=1}^{k-1} 1 = (k-1)(n-k).$$

Taking the sum of all of the above terms, we get $n(n-1)$ and therefore we have accounted for all of the traffic pairs in the network. We explain the terms as follows:

$$\sum_{j\neq k} Tx_{(k)}^{(k\to j)} \equiv traffic\ for\ which\ node\ k\ is\ a\ source,$$

$$\sum_{i\neq k} Tx_{(k)}^{(i\to k)} \equiv traffic\ for\ which\ node\ k\ is\ a\ destination,$$

$$\sum_{1<i<k}\sum_{1\leq j<i} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ left\ of\ k\ the\ left\ of\ i,$$

$$\sum_{1\leq i<k}\sum_{i<j<k} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ left\ of\ k\ to\ the\ right\ of\ i,$$

$$\sum_{1\leq i<k}\sum_{k<j\leq n} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ left\ of\ k\ to\ the\ right\ of\ k,$$

$$\sum_{k<i<n}\sum_{i<j\leq n} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ right\ of\ k\ to\ the\ right\ of\ i,$$

$$\sum_{k<i\leq n}\sum_{k<j<i} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ right\ of\ k\ to\ the\ left\ of\ i,$$

$$\sum_{k<i\leq n}\sum_{1\leq j<k} Tx_{(k)}^{(i\to j)} \equiv traffic\ from\ the\ right\ of\ k\ to\ the\ left\ of\ k.$$

We recall the following:

$$H_{(i)}^{(j)}(n) = \begin{cases} 0, & first\ hop\ correct\ in\ a\ network\ of\ n\ nodes, \\ 1, & first\ hop\ incorrect\ in\ a\ network\ of\ n\ nodes. \end{cases}$$

Now we look at the previous eight summations term by term. In the case where node $X_{(k)}$ is a traffic source, the amount of transmitted traffic depends on whether the first hop is correct from node $X_{(k)}$ to the destination node $X_{(j)}$. Thus we have:

$$\sum_{j \neq k} Tx_{(k)}^{(k \to j)} = (n-1) + \sum_{j \neq k} H_{(k)}^{(j)}(n) = (n-1) + \mathbb{E}_{(k)}.$$

We see that there is one deterministic transmission for each of the $(n-1)$ destinations of node $X_{(k)}$, as well as one additional stochastic transmission for each incorrect first hop transmission from node $X_{(k)}$.

When $X_{(k)}$ is the destination of a given traffic stream, it never re-transmits the received traffic and thus we have the following:

$$\sum_{i \neq k} Tx_{(k)}^{(i \to k)} = 0.$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the left of $X_{(i)}$, whether or not transmission traffic is generated for node $X_{(k)}$ depends on if the first hop is in the correct direction. If this is the case (i.e., $H_{(i)}^{(j)}(n) = 0$) then there is no transmission traffic generated for node $X_{(k)}$. If the first hop is in the incorrect direction, then there are two units of transmission traffic generated for node $X_{(k)}$. When traffic is sent in the incorrect direction, it has to be transmitted by node $X_{(k)}$ twice: once toward one

end of the network (the incorrect end) and again toward the other end of the network (the correct end). This gives us the following term:

$$\sum_{1<i<k}\sum_{1\le j<i} Tx_{(k)}^{(i\to j)} = \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n).$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the right of $X_{(i)}$ and $j$ is less than $k$, no transmission traffic is generated, regardless of the value of $H_{(i)}^{(j)}(n)$. We therefore simplify as follows:

$$\sum_{1\le i<k}\sum_{i<j<k} Tx_{(k)}^{(i\to j)} = \sum_{i=1}^{k-1}\sum_{j=i+1}^{k-1} 0 = 0.$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the right of $X_{(k)}$, regardless of the value of $H_{(i)}^{(j)}(n)$ there is one unit of transmission traffic generated for $X_{(k)}$ and we obtain:

$$\sum_{1\le i<k}\sum_{k<j\le n} Tx_{(k)}^{(i\to j)} = \sum_{i=1}^{k-1}\sum_{j=k+1}^{n} 1 = (k-1)(n-k).$$

The following three terms have symmetric arguments to the previous three terms and are as follows:

$$\sum_{k<i<n}\sum_{i<j\le n} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n),$$

$$\sum_{k<i\le n}\sum_{k<j<i} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n}\sum_{j=k}^{i-1} 0 = 0,$$

$$\sum_{k<i\leq n}\sum_{1\leq j<k} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n}\sum_{j=1}^{k-1} 1 = (k-1)(n-k).$$

Then taking only the non-zero terms, we obtain:

$$\sum_{i=1}^{n}\sum_{j\neq i} Tx_{(k)}^{(i\to j)} = (n-1) + \mathbb{E}_{(k)} + \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) + 2(k-1)(n-k).$$

From previous sections we know that, for any node $X_{(l)}$,

$$\mathbb{E}_{(l)} = \begin{cases} \mathcal{N}_L^{(l)}, & if D_{(l)} = R, \\ \mathcal{N}_R^{(l)}, & if D_{(lk)} = L. \end{cases}$$

Now we have the following:

$$Tx_{(k)} = \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) + \mathbb{E}_{(k)} + (n-1) + 2(k-1)(n-k).$$

Now given

$$\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) = \begin{cases} 2\mathbb{E}_{(i)}, & D_{(i)} = R, \\ 0, & D_{(i)} = L, \end{cases}$$

$$\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) = \begin{cases} 0, & D_{(i)} = R, \\ 2\mathbb{E}_{(i)}, & D_{(i)} = L, \end{cases}$$

we see that the term $\max_{\{D_{(i)}, i\in[1,n]\}}\{\max_k\{Tx_{(k)}\}\}$ is maximized for the following condition:

$$D_{(i)} = \begin{cases} R, & 2 \le i \le k-1, \\ L, & k+1 \le i \le n-1. \end{cases}$$

Now looking only at the two summations, we can simplify as follows:

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{ \sum_{i=2}^{k-1} \sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1} \sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) \right\} = \sum_{i=2}^{k-1} 2\mathcal{N}_L^{(i)} + \sum_{i=k+1}^{n-1} 2\mathcal{N}_R^{(i)}$$

$$= \sum_{i=2}^{k-1} 2(i-1) + \sum_{i=k+1}^{n-1} 2(n-i)$$

$$= 2k^2 - 2kn - 2k + n^2 - n + 2.$$

Combining with the previous work, we have the following:

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_k \{Tx_{(k)}\} \right\} = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_k \left\{ \mathbb{E}_{(k)} + (n-1)^2 \right\} \right\}$$

$$= \max_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_k \left\{ \begin{pmatrix} \mathcal{N}_L^{(k)} + (n-1)^2, \{D_{(k)} = R\} \\ \mathcal{N}_R^{(k)} + (n-1)^2, \{D_{(k)} = L\} \end{pmatrix} \right\} \right\}$$

$$= (n-1)^2 + \max_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_k \left\{ \begin{pmatrix} \mathcal{N}_L^{(k)}, \{D_{(k)} = R\} \\ \mathcal{N}_R^{(k)}, \{D_{(k)} = L\} \end{pmatrix} \right\} \right\}$$

$$= (n-1)^2 + \max_k \left\{ \mathcal{N}_L^{(k)}, \mathcal{N}_R^{(k)} \right\}.$$

We note that the nearest neighbors to nodes $X_{(1)}$ and $X_{(n)}$ are deterministic, thus we have

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{Tx_{(k)}\}\right\} = (n-1)^2 + \max_{k\neq 1,n}\{k-1, n-k\}$$

$$= (n-1)^2 + \max_{k\in[2,n-1]}\{k-1, n-k\}$$

$$= n^2 - n - 1.$$

We can now lower bound the throughput in the non-GL/GP case as:

$$\hat{\lambda}^{non-GL/GP} \geq \frac{R}{\max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \left\{L_{(k)}^{(k+1)}, L_{(k)}^{(k-1)}\right\}\right\}}$$

$$= \frac{R}{n^2 - n - 1}.$$

# A.7 Proof for Results in Section 3.6.1 (Throughput Upper Bound)

For the upper bound on throughput, we look at minimizing over all $\{D_{(i)}, i \in [1,n]\}$ such that we maximize $\hat{\lambda}^{non-GL/GP}$. That is, we solve for a lower bound on $\hat{L}^{non-GL/GP}(n)$ by looking at the most loaded node (in terms of transmissions) minimized over all possible nearest neighbor arrangements. We use the variables $Tx_{(k)}^{(i\to j)}$ and $Tx_{(k)}$ as defined in Appendix A.6. We have the following:

$$\hat{L}^{non-GL/GP}(n) \geq \min_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{Tx_{(k)}\}\right\} = \min_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \left\{\sum_{i,j\in[1,n], i\neq j} Tx_{(k)}^{(i\to j)}\right\}\right\}.$$

We have the following from Section 3.8.1.1:

$$Tx_{(k)} = \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) + \mathbb{E}_{(k)} + (n-1) + 2(k-1)(n-k).$$

Now we have

$$\hat{L}^{non-GL/GP}(n) \geq \min_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_{k} \{Tx_{(k)}\} \right\}$$

$$= \min_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_{k} \left\{ \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) + \mathbb{E}_{(k)} + (n-1) + 2(k-1)(n-k) \right\} \right\}$$

$$= (n-1) + \min_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_{k} \left\{ \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) + \mathbb{E}_{(k)} + 2(k-1)(n-k) \right\} \right\}.$$

At this point we note that each of these terms is non-negative. We optimize over the terms individually. We can do this because, as we show, the optimizations all result in the same selection for the set of nearest neighbors $\{D_{(i)}, i \in [1,n]\}$ and the node $(k)$. We thus have the following terms:

$$(n-1),$$

$$\min_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_{k} \{2(k-1)(n-k)\} \right\},$$

$$\min_{\{D_{(i)}, i\in[1,n]\}} \left\{ \max_{k} \left\{ \sum_{i=2}^{k-1}\sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) \right\} \right\},$$

$$\min_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{\mathbb{E}_{(k)}\}\right\}.$$

The first term, $(n-1)$, is deterministic.

Now we look at the second term, $\min_{\{D_{(i)}, i\in[1,n]\}}\{\max_k\{2(k-1)(n-k)\}\}$, and we have the following:

$$\min_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{2(k-1)(n-k)\}\right\} = \max_k\{2(k-1)(n-k)\}.$$

Relaxing the integer constraint, we determine the maximum:

$$\max_k\{2(k-1)(n-k)\} = 2\max_k\{(k-1)(n-k)\}$$

$$= 2\max_k\{-k^2 + kn + k - n\}$$

$$= 2\max_k\left\{-\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2}{4} - \frac{n}{2} + \frac{1}{4}\right\}$$

$$= 2(k-1)(n-k)|_{k=\frac{n+1}{2}}.$$

Returning to the integer constraint, we see that this term is maximized for

$$k = \begin{cases} \frac{n+1}{2}, & n\ odd, \\ \frac{n}{2}, & n\ even. \end{cases}$$

Now solving for the maximum, we have:

$$\max_k\{2(k-1)(n-k)\} = \begin{cases} \frac{1}{2}(n-1)^2, & n\ odd, \\ \frac{1}{2}n(n-2), & n\ even. \end{cases}$$

We see that the third term, $\min_{\{D_{(i)}, i\in[1,n]\}}\left\{\max_k\left\{\sum_{i=2}^{k-1}\sum_{j=1}^{i-1}2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n}2H_{(i)}^{(j)}(n)\right\}\right\}$, is minimized for the following condition:

$$D_{(i)} = \begin{cases} L, & 2 \le i \le k-1, \\ R, & k+1 \le i \le n-1. \end{cases}$$

As in the previous section, we have the following:

$$\sum_{j=1}^{i-1}2H_{(i)}^{(j)}(n) = \begin{cases} 2\mathbb{E}_{(i)}, & D_{(i)} = R, \\ 0, & D_{(i)} = L, \end{cases}$$

$$\sum_{j=i+1}^{n}2H_{(i)}^{(j)}(n) = \begin{cases} 0, & D_{(i)} = R, \\ 2\mathbb{E}_{(i)}, & D_{(i)} = L, \end{cases}$$

$$\mathbb{E}_{(i)} = \begin{cases} \mathcal{N}_L^{(i)}, & if D_{(i)} = R, \\ \mathcal{N}_R^{(i)}, & if D_{(i)} = L. \end{cases}$$

Then we have

$$\max_k\left\{\sum_{i=2}^{k-1}\sum_{j=1}^{i-1}2H_{(i)}^{(j)}(n) + \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n}2H_{(i)}^{(j)}(n)\right\} = \max_k\left\{\sum_{i=2}^{k-1}0 + \sum_{i=k+1}^{n-1}0\right\}$$

$$= 0.$$

Finally, we look at the fourth term:

$$\min_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\{\mathbb{E}_{(k)}\}\right\}=\min_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\left\{\mathcal{N}_L^{(k)},\mathcal{N}_R^{(k)}\right\}\right\}$$

$$=\begin{cases}\max\left\{\mathcal{N}_L^{\left(\frac{n+1}{2}\right)},\mathcal{N}_R^{\left(\frac{n+1}{2}\right)}\right\}, & n\ odd,\\ \max\left\{\mathcal{N}_L^{\left(\frac{n}{2}\right)},\mathcal{N}_R^{\left(\frac{n}{2}\right)}\right\}, & n\ even,\end{cases}$$

$$=\begin{cases}\frac{n-1}{2}, & n\ odd,\\ \max\left\{\frac{n-1}{2},\frac{n}{2}\right\}, & n\ even,\end{cases}$$

$$=\begin{cases}\frac{n-1}{2}, & n\ odd,\\ \frac{n}{2}, & n\ even.\end{cases}$$

Now putting the results together, we have the following:

$$\min_{\{D_{(i)},i\in[1,n]\}}\left\{\max_{k}\{Tx_{(k)}\}\right\}=\begin{cases}(n-1)+\frac{1}{2}(n-1)^2+\frac{n-1}{2}, & n\ odd,\\ (n-1)+\frac{1}{2}n(n-2)+\frac{n}{2}, & n\ even,\end{cases}$$

$$=\frac{1}{2}(n^2+n-2).$$

Now we arrive at the upper bound as:

$$\hat{\lambda}^{non-GL/GP} \leq \frac{R}{\min\limits_{\{D_{(i)}, i\in[1,n]\}} \left\{\max\limits_{k}\left\{L_{(k)}^{(k+1)}, L_{(k)}^{(k-1)}\right\}\right\}}$$

$$= \frac{2R}{n^2+n-2}.$$

## A.8 Proof for Results in Section 3.6.2

We again note that given the set of nodes $\mathbb{X}_O = \{X_{(1)}, \ldots, X_{(n)}\}$, there are a total of $n$ nodes in the network and thus a total of $n(n-1)$ traffic links. We solve for $\hat{\lambda}^{GL/GP}$ by determining $\hat{L}^{GL/GP^{GAN}}(n)$, noting that unlike the GL/GP case, there is a deterministic result. As in the non-GL/GP case, we look at maximizing $L_{(k)}^{(k+1)} + L_{(k)}^{(k-1)}$ and we write the following (using notation as defined in Appendix A.6):

$$\hat{L}^{GL/GP}(n) = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\{Tx_{(k)}\}\right\} = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\left\{\sum_{i,j\in[1,n], i\neq j} Tx_{(k)}^{(i\to j)}\right\}\right\}.$$

Given the symmetry of the network, and given the max-max argument, we see that

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\{Tx_{(k)}\}\right\} = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\{Rx_{(k)}\}\right\}.$$

In other words, by symmetry we know that

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\left\{\sum_{i,j\in[1,n], i\neq j} Tx_{(k)}^{(i\to j)}\right\}\right\} = \max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_{k}\left\{\sum_{i,j\in[1,n], i\neq j} Rx_{(k)}^{(i\to j)}\right\}\right\}.$$

We again note that the number of right and left neighbors, including the end nodes, for node $(k)$ are given, respectively, as

$$\mathcal{N}_R^{(k)} = n - k,$$

$$\mathcal{N}_L^{(k)} = n - \mathcal{N}_R^{(k)} - 1 = k - 1.$$

Then given some node $(k)$, we can calculate $\sum_{i,j\in[1,n],i\neq j} Tx_{(k)}^{(i\to j)}$ as

$$\sum_{i,j\in[1,n],i\neq j} Tx_{(k)}^{(i\to j)} = \sum_{i=1}^{n}\sum_{j\neq i} Tx_{(k)}^{(i\to j)}$$

$$= \sum_{j\neq k} Tx_{(k)}^{(k\to j)} + \sum_{i\neq k} Tx_{(k)}^{(i\to k)} + \sum_{1<i<k}\sum_{1\leq j<i} Tx_{(k)}^{(i\to j)}$$

$$+ \sum_{1\leq i<k}\sum_{i<j<k} Tx_{(k)}^{(i\to j)} + \sum_{1\leq i<k}\sum_{k<j\leq n} Tx_{(k)}^{(i\to j)} + \sum_{k<i<n}\sum_{i<j\leq n} Tx_{(k)}^{(i\to j)}$$

$$+ \sum_{k<i\leq n}\sum_{k<j<i} Tx_{(k)}^{(i\to j)} + \sum_{k<i\leq n}\sum_{1\leq j<k} Tx_{(k)}^{(i\to j)}.$$

These terms are the same as those for the non-GL/GP case and Appendix A.6 provides an explanation of the terms. We now look at each term individually, as in the non-GL/GP case. In the GL/GP case, the value of $H_{(k)}^{(l)}(n)$ plays no role because every hop is in the correct direction, in contrast to the non-GL/GP case. When node $X_{(k)}$ is a traffic source, the amount of transmitted traffic is given as:

$$\sum_{j\neq k} Tx_{(k)}^{(k\to j)} = (n-1) + \sum_{j\neq k} H_{(k)}^{(j)}(n) = (n-1).$$

We see that there is one deterministic transmission for each of the $(n-1)$ destinations of node $X_{(k)}$ and zero additional stochastic transmissions for each incorrect first hop transmission from node $X_{(k)}$, noting that there are no incorrect first hops from node $X_{(k)}$.

When $X_{(k)}$ is the destination of a given traffic stream, it never re-transmits the received traffic and thus we have the following:

$$\sum_{i\neq k} Tx_{(k)}^{(i\to k)} = 0.$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the left of $X_{(i)}$, whether or not transmission traffic is generated for node $X_{(k)}$ is dependent on whether or not the first hop is in the correct direction. In the case of GL/GP, there are never incorrect first hops. This results in the following:

$$\sum_{1<i<k} \sum_{1\leq j<i} Tx_{(k)}^{(i\to j)} = \sum_{i=2}^{k-1} \sum_{j=1}^{i-1} 2H_{(i)}^{(j)}(n) = 0.$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the right of $X_{(i)}$ and $j$ is less than $k$, as in the non-GL/GP case no transmission traffic is generated, regardless of the value of $H_{(i)}^{(j)}(n)$. We therefore simplify as follows:

$$\sum_{1\leq i<k} \sum_{i<j<k} Tx_{(k)}^{(i\to j)} = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k-1} 0 = 0.$$

When $X_{(i)}$ is to the left of $X_{(k)}$ and $X_{(j)}$ is to the right of $X_{(k)}$, as in the non-GL/GP case there is one unit of transmission traffic generated for $X_{(k)}$ and we obtain:

$$\sum_{1\le i<k}\sum_{k<j\le n} Tx_{(k)}^{(i\to j)} = \sum_{i=1}^{k-1}\sum_{j=k+1}^{n} 1 = (k-1)(n-k).$$

The following three terms have symmetric arguments to the previous three terms and the results are as follows:

$$\sum_{k<i<n}\sum_{i<j\le n} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n-1}\sum_{j=i+1}^{n} 2H_{(i)}^{(j)}(n) = 0,$$

$$\sum_{k<i\le n}\sum_{k<j<i} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n}\sum_{j=k}^{i-1} 0 = 0,$$

$$\sum_{k<i\le n}\sum_{1\le j<k} Tx_{(k)}^{(i\to j)} = \sum_{i=k+1}^{n}\sum_{j=1}^{k-1} 1 = (k-1)(n-k).$$

Then taking only the non-zero terms, we write

$$\sum_{i=1}^{n}\sum_{j\neq i} Tx_{(k)}^{(i\to j)} = (n-1) + 2(k-1)(n-k).$$

Now we maximize:

$$\max_{\{D_{(i)}, i\in[1,n]\}}\left\{\max_{k}\{Tx_{(k)}\}\right\} = \max_{\{D_{(i)}, i\in[1,n]\}}\left\{\max_{k}\{(n-1)+2(k-1)(n-k)\}\right\}$$

$$= \max_k\{(n-1) + 2(k-1)(n-k)\}$$

$$= (n-1) + \max_k\{2(k-1)(n-k)\}.$$

Now we look at the term $\max_k\{2(k-1)(n-k)\}$ and, relaxing the integer constraint, we determine the maximum as follows:

$$\max_k\{2(k-1)(n-k)\} = 2\max_k\{(k-1)(n-k)\}$$

$$= 2\max_k\{-k^2 + kn + k - n\}$$

$$= 2\max_k\left\{-\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2}{4} - \frac{n}{2} + \frac{1}{4}\right\}$$

$$= 2(k-1)(n-k)|_{k=\frac{n+1}{2}}.$$

Returning to the integer constraint, we see that this term is maximized for

$$k = \begin{cases} \frac{n+1}{2}, & n\ odd, \\ \frac{n}{2}, & n\ even. \end{cases}$$

Solving for the maximum, we have:

$$\max_k\{2(k-1)(n-k)\} = \begin{cases} \frac{1}{2}(n-1)^2, & n\ odd, \\ \frac{1}{2}n(n-2), & n\ even. \end{cases}$$

Solving for the entire term, we have

$$\max_{\{D_{(i)}, i\in[1,n]\}} \left\{\max_k \{Tx_{(k)}\}\right\} = \begin{cases} \frac{1}{2}(n^2-1), & n\ odd, \\ \frac{1}{2}(n^2-2), & n\ even. \end{cases}$$

Finally, we can bound the throughput as

$$\hat{\lambda}^{GL/GP} = \frac{R}{\hat{L}^{GL/GP}}$$

$$= \begin{cases} \frac{2R}{n^2-1}, & n\ odd, \\ \frac{2R}{n^2-2}, & n\ even. \end{cases}$$

We note that this can also be written as $2R/(n^2-1) \leq \hat{\lambda}^{GL/GP} \leq 2R/(n^2-2)$.

## A.9 Proof for Results in Section 3.6.3 (Throughput Upper Bound)

We use the same network assumptions and notation as in Section 3.5.3 and Appendix A.5. We further assume that there are $l$ total clusters and each node in a cluster forwards the same amount of traffic, which is throughput optimal. We use a max-flow min-cut argument for the derivation to follow, where we develop a lower bound on $\hat{L}^{d_{char}}(n)$ to obtain an upper bound on $\hat{\lambda}^{d_{char}}$. We use the optimal network configuration, where we additionally assume the optimal condition that the traffic through any cluster of nodes is equally divided among the nodes in that cluster. Using notation as defined in Appendix A.6, the lower bound on streams for a given link is given by

$$\hat{L}^{d_{char}}(n) \geq \max_i \{Tx_{(i)}\},$$

where $i \in [1, l]$ and the lower bound on $\hat{L}^{d_{char}}(n)$ gives an upper bound on $\hat{\lambda}^{d_{char}}$. For a given cluster $(i)$, the traffic served per node, that is $Tx_{(i)}$, is given by

$$Tx_{(i)} = (\mathrm{n}-1) + \frac{l}{\mathrm{n}}\left[\left(\frac{n}{l}c_L^{(i)}\right)\left(\frac{n}{l}c_R^{(i)}\right) + \left(\frac{n}{l}c_R^{(i)}\right)\left(\frac{n}{l}c_L^{(i)}\right)\right].$$

We see that the traffic served per node is composed of three terms. This is very similar to the three terms in the GL/GP throughput case of Section 3.6.2 and Appendix A.8. The main difference is that the traffic passing through a given cluster is equally divided among the nodes in that cluster. We now solve for the maximum number of streams per node in the best case scenario. We have the following:

$$\hat{L}^{d_{char}}(n) \geq \max_i\left\{(\mathrm{n}-1) + \frac{2n}{l}c_R^{(i)}c_L^{(i)}\right\}$$

$$= (\mathrm{n}-1) + \max_i\left\{\frac{2n}{l}(l-i)(i-1)\right\}.$$

Where we have

$$\max_i\left\{\frac{2n}{l}(l-i)(i-1)\right\} = \frac{2n}{l}\max_i\{(l-i)(i-1)\}$$

$$= \frac{2n}{l}\max_i\{-i^2 + i\cdot l + i - l\}.$$

This is maximized for the same value of $i$ as for the equivalent terms in Appendix A.7 and Appendix A.8, noting that the expression to maximize is the same if we let $i = k$ and $l = n$. This is expected because,

ignoring the scaling term of $n/l$, we are looking at an identical network with clusters instead of nodes.

Then we have the following for the non-integer constrained case:

$$\max_i \left\{ \frac{2n}{l}(l-i)(i-1) \right\} = (l-i)(i-1)|_{i=\frac{l+1}{2}}.$$

Returning to the integer constraint, this is maximized for

$$i = \begin{cases} \frac{l+1}{2}, & l\ odd, \\ \frac{l}{2}, & l\ even. \end{cases}$$

Now solving for the maximum, we have

$$\max_i \left\{ \frac{2n}{l}(l-i)(i-1) \right\} = \begin{cases} \frac{n}{2l}(l-1)^2, & l\ odd, \\ \frac{n}{2}(l-2), & l\ even. \end{cases}$$

Now solving for the entire term, we have

$$\max_i \{Tx_{(i)}\} = \begin{cases} \frac{ln}{2} + \frac{n}{2l} - 1, & l\ odd, \\ \frac{ln}{2} - 1, & l\ even. \end{cases}$$

Then we can bound the throughput as

$$\hat{\lambda}^{d_{char}} \leq \frac{R}{\max_i \{Tx_{(i)}\}}$$

$$= \begin{cases} \dfrac{2lR}{l(ln-2)+n}, & l\ odd, \\ \dfrac{2R}{ln-2}, & l\ even. \end{cases}$$

Then noting the odd case is strictly upper bounded by the even case, we have

$$\hat{\lambda}^{d_{char}} \leq \frac{2R}{nl-2}.$$

Substituting for $l = 1 + L/d_{char}$, we have the following:

$$\hat{\lambda}^{d_{char}} \leq \frac{2dR}{n(d_{char}+L)-2d_{char}}.$$

## A.10 Proof for Results in Section 3.7

The following Lemma, based on Lemma 1 (which is itself based on Lemma 6 in Appendix B.3 of [3]), is used to drive the $\Theta(1/n)$ throughput scaling for uniform all-to-all traffic in a one-dimensional network with characteristic distance hopping:

**Lemma 10** - Consider a random one-dimensional network with $n$ nodes independently and randomly distributed on a line of unit length according to a uniform distribution. The line is divided into cells of length $\ell(n)$.

(i) If the cell size $\ell(n) \geq 2\frac{ln\,n}{n}$, then each cell has at least one node $whp$.

(ii) If $\ell(n) = L$, then each cell has at least $\frac{nL}{2}$ nodes $whp$.

(iii) If $\ell(n) = 2\frac{ln\,n}{n}$, then each cell has at most $6\,ln\,n$ nodes $whp$.

(iv) The number of transmitted traffic streams for any cell is $O(n^2)$.

The proof of (i) is derived from [4], which was also used to prove Part (i) of Lemma 1 in Section 2.11.2.3. Given that the one-dimensional network is a special case of the two-dimensional network, we can directly use this result from [4]. Parts (ii) and (iii) are derived in a similar manner from the two-dimensional case of Lemma 1. Part (iv) is the only part of this Lemma that is not derived from previous works. We determine the maximum number of outgoing (transmitted) or incoming (received) streams of traffic. Per Appendix A.6, the number of transmitted or received streams for any cell is the same, given that all pass through traffic is received and transmitted once, and the amount of source traffic and destination traffic for a cell is the same. Then we note that the number of SD pairs in the network is $n(n-1)$ and thus the maximum number of streams transmitted by any cell in network is $O(n^2)$. As in [3], we combine this with Part (ii) and see that each node needs to be able to transmit the following $whp$:

$$O\left(\frac{n^2}{\left(\frac{nL}{2}\right)}\right) = O(n)\ [streams].$$

Then we note that $\hat{\lambda} = \max_{\lambda}\{Achievable\ \lambda\}$ and $\hat{\lambda} = R/\hat{L}(n)$, where

$$\hat{L}(n) = \max_{k}\left\{L_{(k)}^{(k+1)} + L_{(k)}^{(k-1)}\right\}$$

$$= \max_{k}\{0(n)\}\ whp$$

$$= \Theta(n)\ whp.$$

Then we finally have the result $\hat{\lambda} = \Theta\left(\frac{1}{n}\right)\ whp.$

# Appendix B

# Derivations for Equations in Chapter 4

## B.1 Proof for Results in Section 4.4.2.1

Given $\Theta_{t+\tau}^{i}$ is uniformly distributed over $[0,\pi]$, we have the following CDF:

$$F_{\Theta_{t+\tau}^{i}}(\theta)=\begin{cases}\dfrac{\theta}{\pi}, & 0\le\theta\le\pi,\\ 1, & \pi<\theta,\\ 0, & otherwise.\end{cases}$$

Given the function for the chord length

$$C_i=2\rho_{max}\sin\frac{\Theta_{t+\tau}^{i}}{2}\triangleq g\left(\Theta_{t+\tau}^{i}\right),$$

we note that $C_i$ is a function of the random variable $\Theta_{t+\tau}^{i}$ and thus is itself a random variable. Then we have the following CDF:

$$F_{C_i}(c)=\Pr\left\{2\rho_{max}\sin\frac{\Theta_{t+\tau}^{i}}{2}\le c\right\}.$$

Given $g(\Theta_{t+\tau}^i) \triangleq 2\rho_{max} \sin\frac{\Theta_{t+\tau}^i}{2}$, we see that $g^{-1}$ exists[15] and we simplify the CDF of $C_i$ as:

$$F_{C_i}(c) = Pr\left\{\Theta_{t+\tau}^i \leq 2\sin^{-1}\left(\frac{c}{2\rho_{max}}\right)\right\}$$

$$= F_{\Theta_{t+\tau}^i}\left(2\sin^{-1}\left(\frac{c}{2\rho_{max}}\right)\right)$$

$$= F_{\Theta_{t+\tau}^i}(g^{-1}(c)).$$

Finally, we have the CDF of $C_i$:

$$F_{C_i}(c) = \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{c}{2\rho_{max}}\right), & 0 \leq 2\sin^{-1}\left(\frac{c}{2\rho_{max}}\right) \leq \pi, \\ 1, & \pi < 2\sin^{-1}\left(\frac{c}{2\rho_{max}}\right), \\ 0, & otherwise, \end{cases}$$

$$= \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{c}{2\rho_{max}}\right), & 0 \leq c \leq 2\rho_{max}, \\ 1, & 2\rho_{max} < c, \\ 0, & otherwise. \end{cases}$$

## B.2 Proof for Results in Section 4.4.2.2

We note that time to traverse another node's transmission range, $T_{traverse}^i$, is given by

$$T_{traverse}^i = \frac{C_i}{|\boldsymbol{v}|} = h(C_i).$$

[15] It is important to note that the reason $g^{-1}$ exists is because the equation for the chord length includes the term $\Theta_{t+\tau}^i/2$. Thus given the range of $\Theta_{t+\tau}^i$ is $[0,\pi]$, then the range of $\Theta_{t+\tau}^i/2$ is $[0,\pi/2]$, a range for which the sin function is invertible.

Thus $T^i_{traverse}$ is a function of the random variable $C_i$ and is itself a random variable. Now we have the following for its CDF:

$$F_{T^i_{traverse}}(t) = Pr\left\{\frac{C_i}{|\boldsymbol{v}|} \leq t\right\}.$$

We see that this is a constant linear operation, noting that while $|\boldsymbol{v}|$ is a variable, it is not a random variable. We further note that $h^{-1}$ exists and we simplify the CDF of $T^i_{traverse}$ as:

$$F_{T^i_{traverse}}(t) = Pr\{C_i \leq t|\boldsymbol{v}|\}$$

$$= F_{C_i}(t \cdot |\boldsymbol{v}|)$$

$$= F_{C_i}\left(h^{-1}(t)\right).$$

The CDF of $T^i_{traverse}$ is then:

$$F_{T^i_{traverse}}(t) = \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{t \cdot |\boldsymbol{v}|}{2\rho_{max}}\right), & 0 \leq t \cdot |\boldsymbol{v}| \leq 2\rho_{max}, \\ 1, & 2\rho_{max} < t \cdot |\boldsymbol{v}|, \\ 0, & otherwise, \end{cases}$$

$$= \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{t \cdot |\boldsymbol{v}|}{2\rho_{max}}\right), & 0 \leq t \leq \frac{2\rho_{max}}{|\boldsymbol{v}|}, \\ 1, & \frac{2\rho_{max}}{|\boldsymbol{v}|} < t, \\ 0, & otherwise. \end{cases}$$

## B.3 Proof for Results in Section 4.4.2.3

We now derive the CDF for the minimum rate necessary to ensure that the topology update information can be sent in an amount of time equal to the $T^i_{traverse}$. We have the following:

$$R^i_{min} = \xi\left(T^i_{traverse}\right).$$

The traversal time for a given node $W_i$ be is given by the random variable $T^i_{traverse}$. Now we have the following CDF:

$$F_{R^i_{min}}(r) = Pr\{R^i_{min} \leq r\}$$

$$= Pr\left\{\frac{\psi}{T^i_{traverse}} \leq r\right\}.$$

Because $\xi^{-1}$ exists, we can simplify as follows:

$$F_{R^i_{min}}(r) = Pr\left\{\frac{\psi}{r} \leq T^i_{traverse}\right\}$$

$$= 1 - Pr\left\{T^i_{traverse} \leq \frac{\psi}{r}\right\}$$

$$= 1 - F_{T^i_{traverse}}\left(\frac{\psi}{r}\right)$$

$$= 1 - F_{T^i_{traverse}}\left(\xi^{-1}(r)\right).$$

The CDF of $R_{min}^{i}$ is then:

$$F_{R_{min}^{i}}(r) = 1 - F_{T_{traverse}^{i}}\left(\frac{\psi}{r}\right)$$

$$= 1 - \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\frac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right), & 0 \le \frac{\psi|\boldsymbol{v}|}{r} \le 2\rho_{max}, \\ 1, & 2\rho_{max} < \frac{\psi|\boldsymbol{v}|}{r}, \\ 0, & otherwise, \end{cases}$$

$$= \begin{cases} 1 - \frac{2}{\pi}\sin^{-1}\left(\frac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right), & 0 \le \frac{\psi|\boldsymbol{v}|}{2\rho_{max}} \le r, \\ 0, & r < \frac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ 1, & otherwise, \end{cases}$$

$$= \begin{cases} 0, & r < \frac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ 1 - \frac{2}{\pi}\sin^{-1}\left(\frac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right), & 0 \le \frac{\psi|\boldsymbol{v}|}{2\rho_{max}} \le r, \\ 1, & otherwise. \end{cases}$$

We have now defined the CDF of the minimum transmission rate necessary for a single mobile node.

## B.4 Proof for Results in Section 4.4.3.1

We start with the following:

$$F_{R_{min}}^{C}(r) = Pr\{R_{min} > r\}$$

$$= Pr\left\{\max_{i}\{R_{min}^{i}\} > r\right\}$$

$$= \bigcup_i Pr\{R_{min}^i > r\}$$

$$= 1 - \bigcap_i Pr\{R_{min}^i \leq r\}.$$

Now we ignore the otherwise case of the CDF $F_{R_{min}^i}(r)$ in (4.27). By the independence of each value of $T_{traverse}^i$, and thus each value of $R_{min}^i$, we get the following:

$$F_{R_{min}}^C(r) = \begin{cases} 0, & r < \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}}, \\ 1 - \left[1 - \dfrac{2}{\pi}\sin^{-1}\left(\dfrac{\psi|\boldsymbol{v}|}{2r\rho_{max}}\right)\right]^n, & 0 \leq \dfrac{\psi|\boldsymbol{v}|}{2\rho_{max}} \leq r. \end{cases}$$

## B.5 Proof for Results in Section 4.4.3.2

Given the expression for $F_{R_{min}}^C(r) \leq \phi$, we have the following, for $r \geq \chi$:

$$F_{R_{min}}^C(r) \leq \phi$$

$$1 - \left[1 - \frac{2}{\pi}\sin^{-1}\left(\frac{\chi}{r}\right)\right]^n \leq \phi$$

$$\frac{\chi}{\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right]} \leq r.$$

We have the following Taylor series expansion:

$$\sin x = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2n+1)!} x^{2i+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots,$$

we can state

$$\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right] = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2n+1)!}\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right]^{2i+1}.$$

We note that $\pi/2 - (\pi/2)(1-\phi)^{\frac{1}{n}} < 1$. Thus we see that the term $\left[\pi/2 - (\pi/2)(1-\phi)^{\frac{1}{n}}\right]^i$ is monotonically decreasing in $i$. For $x = \left[\pi/2 - (\pi/2)(1-\phi)^{\frac{1}{n}}\right]$, we have

$$\left(\frac{(-1)^i}{(2n+1)!} x^{2i+1} + \frac{(-1)^{(i+1)}}{(2n+1)!} x^{2(i+1)+1}\right) < 0, \forall i \in \mathbb{Z}^+.$$

We see that, after the first term, the sum of every following pair of terms is less than zero. The upper bound for $\sin\left[\pi/2 - (\pi/2)(1-\phi)^{\frac{1}{n}}\right]$ is then given by

$$\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right] \leq \frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}.$$

Now we have

$$r \geq \frac{\chi}{\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right]}$$

$$\geq \frac{\chi}{\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}}$$

$$= \frac{2\chi}{\pi}\left[\frac{1}{1-(1-\phi)^{\frac{1}{n}}}\right].$$

We first note the following:

$$1-(1-\phi)^{\frac{1}{n}} = \begin{cases} 1, & \phi = 1, \\ 0, & \phi = 0. \end{cases}$$

Thus we have

$$\lim_{\varepsilon \to 0}\left(\frac{1}{1-(1-\phi)^{\frac{1}{n}}}\right) = \infty,$$

$$\lim_{\varepsilon \to 1}\left(\frac{1}{1-(1-\phi)^{\frac{1}{n}}}\right) = 0.$$

We also note that the expression $1/\left(1-(1-\phi)^{\frac{1}{n}}\right)$ for $\phi = 1$ is equal to 1. We see that when $\phi$ approaches 1, the rate is trivially lower bounded by 0. In the work to follow, we develop a lower bound on $1/\left(1-(1-\phi)^{\frac{1}{n}}\right)$ for all $\phi$ and a tighter lower bound for $0 \leq \phi \leq 1/2$. We begin with a bound based on the Maclaurin series and applicable for all $0 \leq \varepsilon \leq 1$. We have the following Maclaurin series:

$$\frac{1}{1-(1-\phi)^{\frac{1}{n}}} = \sum_{i=0}^{\infty}(1-\phi)^{\frac{i}{n}}$$

$$\geq \sum_{i=0}^{n} (1-\phi)^{\frac{i}{n}}.$$

For $0 \leq \phi \leq 1$ and $0 \leq i \leq n$, we have the inequality

$$(1-\phi)^{i} \geq (1-\phi)^{j}, \forall i \leq j.$$

Thus we note that $(1-\phi)^{\frac{i}{n}} \geq (1-\phi)$ and we have:

$$\sum_{i=0}^{n} (1-\phi)^{\frac{i}{n}} \geq \sum_{i=0}^{n} (1-\phi)$$

$$= n(1-\phi).$$

We can bound $r$ as follows:

$$\arg\min_{r} \{F_{R_{min}}^{C}(r) \leq \phi\} \geq \frac{\chi}{\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\phi)^{\frac{1}{n}}\right]}$$

$$\geq \frac{2\chi n(1-\phi)}{\pi}.$$

Now replacing with $\chi = \frac{\psi|v|}{2\rho_{max}}$, we have

$$\arg\min_{r} \{F_{R_{min}}^{C}(r) \leq \phi\} \geq \frac{n\psi|v|(1-\phi)}{\pi\rho_{max}}, \forall\, 0 \leq \phi \leq 1,$$

We proceed to develop a tighter bound on $r$ for the case where $0 \leq \phi \leq 1/2$. We note that it is possible to develop a bound for any desired range. This particular range of $\phi$ is selected for both its applicability to real networks and for the method by which the bound can be developed. We lower bound $r$, which means we want the following bounds, where $a$ is a constant to be determined:

$$(1-\phi)^{\frac{1}{n}} \geq a \rightarrow$$

$$1-(1-\phi)^{\frac{1}{n}} \leq 1-a \rightarrow$$

$$\frac{1}{1-(1-\phi)^{\frac{1}{n}}} \geq \frac{1}{1-a}.$$

We begin by writing $(1-\phi)^{\frac{1}{n}}$ in series form as:

$$(1-\phi)^{\frac{1}{n}} = \sum_{k=0}^{\infty} \frac{(\ln(1-\phi))^k}{n^k(k!)}$$

$$= 1 + \frac{\log(1-\phi)}{n} + \sum_{k=2}^{\infty} \frac{(\ln(1-\phi))^k}{n^k(k!)}.$$

We note that $\ln(1-\phi) < 0$ and thus $(\ln(1-\phi))^k$ is positive for $k$ even and negative for $k$ odd. Thus we have

$$\left[\frac{(\ln(1-\phi))^k}{n^k(k!)} + \frac{(\ln(1-\phi))^{k+1}}{n^{k+1}\big((k+1)!\big)}\right] > 0, \forall k \geq 2.$$

We have the following:

$$(1-\phi)^{\frac{1}{n}} \geq 1 + \frac{\ln(1-\phi)}{n}.$$

Alternatively, we note

$$1-(1-\phi)^{\frac{1}{n}} \leq -\frac{\ln(1-\phi)}{n}.$$

The Taylor series for $-\ln(1-\varepsilon)$ is as follows:

$$-\ln(1-\phi) = -\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j}(-\phi)^j$$

$$= -\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j}(-1)^j \phi^j$$

$$= -\sum_{j=1}^{\infty} \frac{(-1)^{2j+1}}{j}\phi^j$$

$$= \sum_{j=1}^{\infty} \frac{\phi^j}{j}.$$

We see that the Taylor series representation of $-\ln(1-\phi)$ converges from below. That is, we cannot upper bound directly using the Taylor series (e.g., by truncation of terms). Now we write the following:

$$-\ln(1-\phi) = \phi + \phi\sum_{j=2}^{\infty}\frac{\varepsilon^{j-1}}{j}.$$

Given the restriction on $\phi$ to $0 \leq \phi \leq 1/2$, we then state the following:

$$\phi^{j-1} \leq \left(\frac{1}{2}\right)^{j-1} \leq \frac{1}{j}, \forall j \geq 2.$$

Therefore, we can state:

$$-\ln(1-\phi) = \phi + \phi\sum_{j=2}^{\infty}\frac{\varepsilon^{j-1}}{j}$$

$$\leq \phi + \phi\sum_{j=2}^{\infty}\frac{1}{j^2}$$

$$= \frac{\phi\pi^2}{6}.$$

Thus we have the following bound:

$$1-(1-\varepsilon)^{\frac{1}{n}} \leq \frac{\varepsilon\pi^2}{6n}.$$

We thus bound $r$ as follows:

$$\arg\min_{r}\{F^{C}_{R_{min}}(r) \leq \phi\} \geq \frac{\chi}{\sin\left[\frac{\pi}{2} - \frac{\pi}{2}(1-\varepsilon)^{\frac{1}{n}}\right]}$$

$$\geq \frac{2\chi}{\pi}\left[\frac{1}{1-(1-\varepsilon)^{\frac{1}{n}}}\right]$$

$$= \frac{12\chi n}{\pi^3 \varepsilon}.$$

Now replacing with $\chi = \frac{\psi|v|}{2\rho_{max}}$, we have

$$\arg\min_r\{F^C_{R_{min}}(r) \leq \phi\} \geq \frac{6n\psi|v|}{\varepsilon\pi^3\rho_{max}}, \forall\, 0 \leq \varepsilon \leq \frac{1}{2}.$$

# Bibliography


[1] The Internet Corporation for Assigned Names and Numbers. (2011, Feb.) ICANN.org. [Online]. http://www.icann.org/en/news/releases/release-03feb11-en.pdf

[2] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388-404, Mar. 2000.

[3] L. L. Dai, "Proactive mobile wireless networks: an infrastructureless wireless network architecture for delay-sensitive applications," PhD thesis, EECS Department, MIT, Cambridge, MA, 2008.

[4] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *Proceedings of the 23rd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Hong Kong, 2004, pp. 464-475.

[5] K.-T. Feng, C.-H. Hsu, and T.-E. Lu, "Velocity-assisted predictive mobility and location-aware routing protocols for mobile ad hoc networks," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 1, pp. 448-464, Jan. 2008.

[6] L. Hu, "Topology control for multihop packet radio networks," *IEEE Transactions On Communications*, vol. 41, no. 10, pp. 1474-1481, Oct. 1993.

[7] C. E. Perkins and P. Bhagwat, "Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers," in *Proceedings of the AMC Conference on Communications Architectures, Protocols and Applications (SIGCOMM)*, London, U.K., 1994, pp. 234-244.

[8] S. Murthy and J. J. Garcia-Luna-Aceves, "An efficient routing protocol for wireless networks," *ACM Mobile Networks and Applications*, vol. 1, no. 2, pp. 183-197, Oct. 1996.

[9] C. E. Perkins and E. M. Royer, "Ad-hoc on-demand distance vector routing," in *Proceedings of the Second IEEE Workshop on Mobile Computing Systems and Applications (WMCSA)*, New Orleans, LA, Feb. 1999, pp. 90-100.

[10] V. D. Park and M. S. Corson, "A highly adaptive distributed routing algorithm for mobile wireless networks," in *Proceedings of the 16th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Kobe, Japan, 1997, pp. 1405-1413 vol. 3.

[11] A. Iwata, C.-C. Chiang, G. Pei, M. Gerla, and T.-W. Chen, "Scalable routing strategies for ad hoc wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 8, pp. 1369-1379, Aug. 1999.

[12] R. Sivakumar, P. Sinha, and V. Bharghavan, "CEDAR: a core-extraction distributed ad hoc routing algorithm," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 8, pp. 1454-1465, Aug. 1999.

[13] M. Penrose, *Random Geometric Graphs*. Oxford, U.K.: Oxford University Press, 2006.

[14] B. Bollobás, *Random Graphs*, 2nd ed. Cambridge, U.K.: Cambridge University Press, 2001.

[15] G. E. Weichenberg, "High-reliability architectures for networks under stress," SM thesis, EECS Department, MIT, Cambridge, MA, 2003.

[16] E. N. Gilbert, "Random plane networks," *Journal of the Society for Industrial and Applied Mathematics*, vol. 9, no. 4, pp. 533-543, Dec. 1961.

[17] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029-1046, Sept. 2009.

[18] D. Son, A. Helmy, and B. Krishnamachari, "The effect of mobility-induced location errors on geographic routing in ad hoc networks: analysis and improvement using mobility prediction," in *Wireless Communications and Networking Conference*, vol. 1, New Orleans, LA, March 2004, pp. 189-194 vol. 1.

[19] K. X. Lin, "Green optical network design: power optimization of wide area and metropolitan area networks," M.Eng thesis, EECS Department, MIT, Cambridge, MA, 2011.

[20] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal throughput-delay scaling in wireless networks-part I: the fluid model," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2568-2592, June 2006.

[21] M. F. Carey, "Connectivity, throughput, and end-to-end latency in infrastructureless wireless networks with beamforming-enabled devices," SM thesis, EECS Department, MIT, Cambridge, MA, 2011.

[22] C. Bettstetter, "On the minimum node degree and connectivity of a wireless multihop network," in *Proceedings of the 3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing*, Lausanne, Switzerland, 2002, pp. 80-91.

[23] X. Ta, G. Mao, and B. D. O. Anderson, "On the phase transition width of k-connectivity in wireless multihop networks," *IEEE Transactions on Mobile Computing*, vol. 8, no. 7, pp. 936-949, July 2009.

[24] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477-486, Aug. 2002.

[25] J. Broch, D. A. Maltz, D. B. Johnson, Y.-C. Hu, and J. Jetcheva, "A performance comparison of multi-hop wireless ad hoc network routing protocols," in *Proceedings of the 4th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom)*, Dallas, TX, 1998, pp. 85-97.

[26] E. M. Royer and C. E. Perkins, "Multicast operation of the ad-hoc on-demand distance vector routing protocol," in *Proceedings of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom)*, Seattle, WA, 1999, pp. 207-218.

[27] S. R. Das, C. E. Perkins, and E. M. Royer, "Performance comparison of two on-demand routing protocols for ad hoc networks," in *Proceedings of the 19th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Tel-Aviv, Israel, 2000, pp. 3-12.

[28] G. Holland and N. Vaidya, "Analysis of TCP performance over mobile ad hoc networks," in *Proceedings of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom)*, Seattle, WA, 1999, pp. 219-230.

[29] C. Bettstetter, "Mobility modeling in wireless networks: categorization, smooth movement, and border effects," *SIGMOBILE Mobile Computing and Communications Review*, vol. 5, no. 3, pp. 55-67, July 2001.

[30] R. A. Guérin, "Channel occupancy time distribution in a cellular radio system," *IEEE Transactions on Vehicular Technology*, vol. 36, no. 3, pp. 89-99, Aug. 1987.

[31] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay scaling in wireless networks with constant-size packets," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, Adelaide, Australia, 2005, pp. 1329-1333.

[32] C. E. Shannon, "A mathematical theory of communication," Reprinted with corrections from *The Bell System Technical Journal*, vol. 27, pp. 379-423, 623-656, July, Oct. 1948. [Online]. http://cm.bell-labs.com/cm/ms/what/ shannonday/shannon1948.pdf

[33] B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks," in *Proceedings of the 22nd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, San Francisco, CA, 2003, pp. 1543-1552 vol. 2.

[34] A. Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge University Press, 2005 [Online]. http://www.knovel.com/web/portal/browse/display?_EXT_KNOVEL_DISPLAY_bookid=2338&VerticalID=0

[35] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1319-1343, June 2002.

[36] A. El Gamal, J. Mammen, and B. Prabhakar, "Optimal throughput-delay scaling in wireless networks—part II: constant-size packets," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5111-5116, Nov. 2006.

[37] S. N. Diggavi, M. Grossglauser, and D. N. C. Tse, "Even one-dimensional mobility increases ad hoc wireless capacity," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT)*, Lusanne, Switzerland, 2002, p. 352.

[38] A. El Gamal and J. Mammen, "Optimal hopping in ad hoc wireless networks," in *Proceedings of the 25th IEEE International Conference on Computer Communications (INFOCOM)*, Barcelona, Spain, 2006, pp. 1-10.

[39] L. L. Dai and V. W. S. Chan, "Throughput of power-limited wireless networks with processing energy considerations," in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM)*, San Francisco, CA, 2006, pp. 1-6.

[40] R. Negi and A. Rajeswaran, "Capacity of power constrained ad-hoc networks," in *Proceedings of the 23rd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Hong Kong, 2004, pp. 443-453.

[41] R. G. Gallager, *Discrete Stochastic Processes*, 2nd ed., Jan. 2011, unpublished draft.

[42] S. S. Chiu and R. C. Larson, "Bertrand's paradox revisited: more lessons about that ambiguous word, random," *Journal of Industrial and Systems Engineering*, vol. 3, no. 1, pp. 1-26, Spring 2009.

[43] R. G. Gallager, private communication, Aug. 2011.

[44] H. Zhang and J. C. Hou, "Capacity of wireless ad-hoc networks under ultra wide band with power constraint," in *Proceedings of the 24th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Miami, FL, 2005 , pp. 455- 465 vol. 1.