

# Levy Flight as a Robotic Search Pattern

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by

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Submitted to the  
Department of Mechanical Engineering  
in Partial Fulfillment of the Requirements for the Degree of

***Bachelor of Science in Mechanical Engineering***

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## **Abstract**

Levy flights have been recently found to approximate the trajectories of animal foragers in their search for resources and food. Levy flights have proved to be effective in searching tasks because of their characteristic combination of long trajectories followed by bursts of short trajectories. In addition, some developments in robotics are currently geared toward robot miniaturization and mimicry of living creatures. For example, robots based on the features of hummingbirds and bees have already been developed though they are not yet autonomous. Incorporating Levy flight searching strategies into these miniaturized robots could yield more efficient and effective searching machines.

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## 1. Introduction



**Figure 1:** Quadrotor with motion sensors and onboard computer (Simonite).

Our future relies on robots and computer intelligence. Technologies that would one day become in place making our lives easier and more enjoyable. Robots are now used in industrial applications typically to handle repetitive and hazardous task like a forging shop, welding shop and elsewhere in assembly. The other applications of robots are in exploring, inspection, and maintenance in remote and hazardous environments including the outer planet and the deep sea. The trend in robotics is towards miniaturization; we want to build robots the size of little ants and flying bees. We want to build robots the size of cockroaches. Particularly when handling hazardous material, very tiny robots are needed. Energy efficiency is another issue because if you want a robot to be autonomous, it should carry its own power pack so it should be energy efficient. It cannot afford to draw a lot of power.

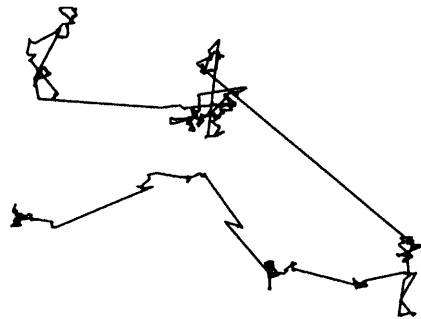
Scientists and engineers have begun to chase nature and are trying to reverse engineer insects and birds. How can such birds fly the way they do? Much work has been done to build small robots that are able to fly, but how will they be put to work? With these new minute robots, search and rescue missions can be completed more rapidly. In the future, we might be able to send a swarm of quadrotors, pictured in figure 1, to a disaster zone and search for missing people or inspect dangerous locations. Robots are now being designed to be more autonomous and efficient.

“The exploitation of active radar techniques using resonant antennas has permitted the accurate reconstruction of the flight paths of moths and honey bees at the field scale” (Reynolds). Studies have shown that animals and insects move in a manner approximated by Levy flights when they are searching for food or resources. In the same manner as the anatomy of animals and insects are being reversed engineered, there might be a way of implementing flight paths that are followed by foragers to conduct search and rescue missions by autonomous flying robots. Robots can become more useful and autonomous if we are able to incorporate and mimic the way animals move which is very closely approximated by a Levy flight.

## 2. Levy Flight

### 2.1 Introduction to Levy Flight

Animals move for many different reasons including: searching for food, looking for a possible mate, water, and avoiding being caught by predators. “If the environment is unchanging or wholly predictable, animals may develop knowledge of where to locate resources and they will exploit that knowledge” (Reynolds). Unfortunately, this is not always the case. Animals have to conduct non-oriented searches with little or prior knowledge of where resources will be found. In foraging theory, it is often assumed that a forager has a movement described as a Gaussian, Rayleigh, and other classical distributions with well-defined variances (Viswanathan). In recent years, there has been a lot of scientific research that shows many animals use an exploration strategy estimated by Levy Flights when they are scavenging for food or resources. Levy flights model activities that involve a lot of small steps, interspersed with occasional very large excursions.



**Figure 2:** Trajectory of a Levy flight with 100 steps; characterized by the small steps and occasional large steps. (Yang).





**Figure 3:** Distribution of the 100 steps shown in Figure 2 (Yang).

Levy flights can be seen almost everywhere. Deer and albatross, as well as sharks, use Levy flights when searching for food. To avoid spending too much time in unproductive areas, animals need to develop search strategies that generate a fractal distribution of stopping points to succeed in the location of food. Children also perform a form of Levy flight when playing hide and seek. The seeker runs across the yard to a place with several possible hiding spots and searches around that area. If no one is found, the seeker runs across the yard to another spot and continues the search until people are found.

## 2.2 Levy Stable Distribution

Levy Flights are a type of Markovian stochastic process. The individual jumps of Levy flight have lengths are distributed with the probability density function  $\lambda(x)$  decaying at large  $x$  as  $\lambda(x) \approx |x|^{-1-\alpha}$  with  $0 < \alpha < 2$  (Chechkin). A Levy flight is defined by the characteristic function of the Levy probability density function, also known as its Fourier Transform:

$$p_{\alpha,\beta}(k; \mu, \sigma) = F\{p_{\alpha,\beta}(x; \mu, \sigma)\} \quad (1)$$

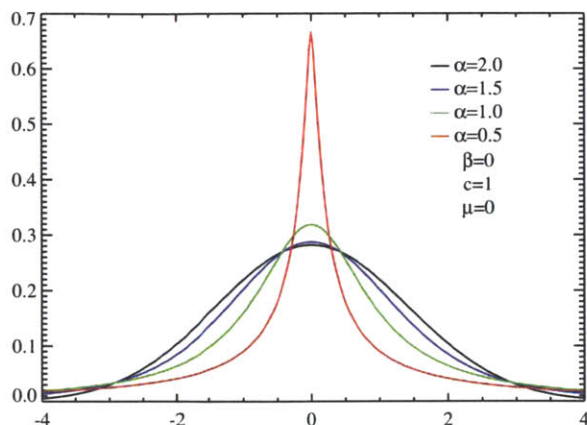
which is defined as:

$$\int_{-\infty}^{\infty} dx e^{ikx} p(x; \mu, \sigma) = \exp\left[i \mu k - \sigma^\alpha |k|^\alpha \left(1 - i\beta \frac{k}{|k|} \bar{\omega}(k, \alpha)\right)\right] \quad (2)$$

where

$$\bar{\omega}(k, \alpha) = \begin{cases} \tan \frac{(\pi\alpha)}{2}, & \text{if } \alpha \neq 1, 0 < \alpha < 2, \\ -\frac{2}{\pi} \ln|k|, & \text{if } \alpha = 1. \end{cases} \quad (3)$$

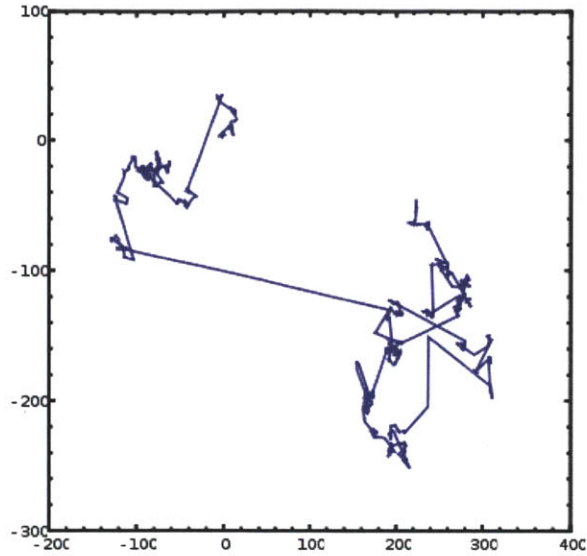
It is noticeable that the characteristic function is governed by four parameters:  $\alpha$ ,  $\beta$ ,  $\mu$ , and  $\sigma$ . The exponent  $\alpha$  must lie between 0 and 2, inclusively, and is known as the index of stability or the Levy index. It defines the asymmetry of the distribution. The skewness parameter,  $\beta$ , lies between -1 and 1, inclusively, and defines the asymptotic decay of the probability density function.  $\mu$  is a shift parameter and  $\sigma > 0$  is a scale parameter. "The shift and scale parameters play a lesser role" and a Levy stable probability density function can set  $\mu = 0$  and  $\sigma = 1$  (Chechkin). The stable probability density function can be expressed in terms of elementary functions in three instances: a Gaussian distribution where  $\alpha = 2$ , Cauchy distribution where  $\alpha = 1$  and  $\beta = 0$ , and a Levy-Smirnov distribution where  $\alpha = 1/2$  and  $\beta = 1$ .



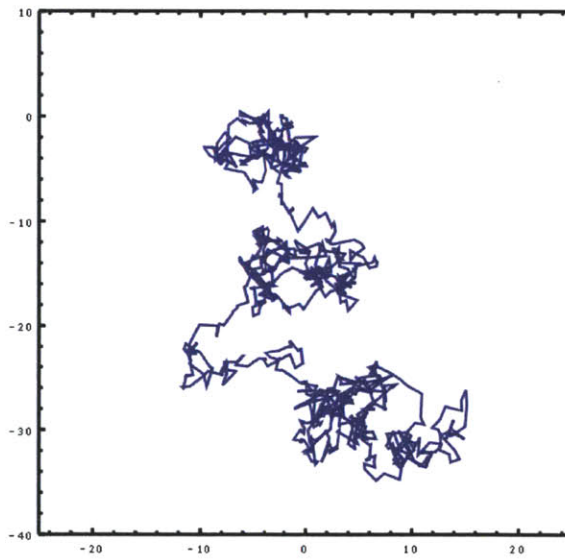
**Figure 4:** Probability density functions. Gaussian distribution –  $\alpha=2$ .  
Cauchy distribution –  $\alpha=1$ . Levy distribution –  $\alpha=0.5$  .

“A Levy distribution is advantageous when target sites are sparsely and randomly distributed, irrespective of the value of  $\mu$  chosen, because the probability of returning to a previously visited site is smaller than for a Gaussian distribution” (Viswanathan).

As can be noticed from the figure below, a larger area is visited by a Levy flight rather than a Gaussian walk. A Gaussian walk is more effective when trying to search a small known location since it will visit nearby places more often with smaller steps. In the Gaussian walk, each step contributes equally to the average movement while in the Levy flight, long steps are more frequent and make more dominant contributions.



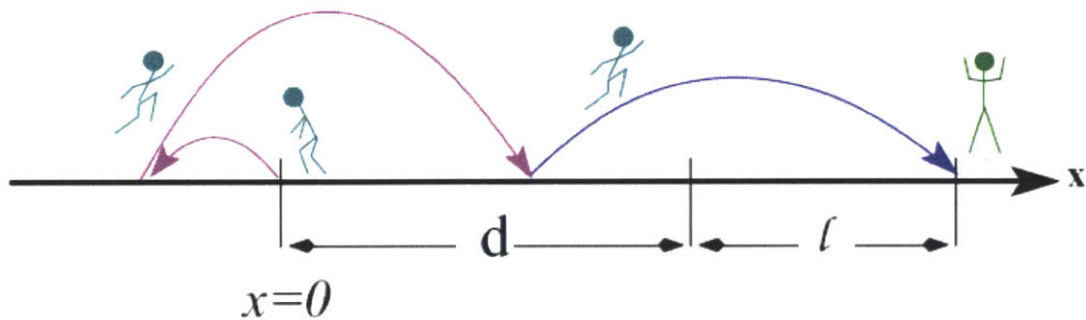
**Figure 5:** Trajectory of a Levy Flight with 1000 steps (Wikipedia).



**Figure 6:** Trajectory of a Gaussian walk with 1000 steps (Wikipedia).

In the Levy flight trajectory, an area more than twice as big as the one in the Gaussian walk is visited. Levy flights with  $\mu = 2$  are optimal for the location of stationary targets that are randomly and sparsely distributed, and once visited are not depleted but instead remain targets for future searches (Viswanathan).

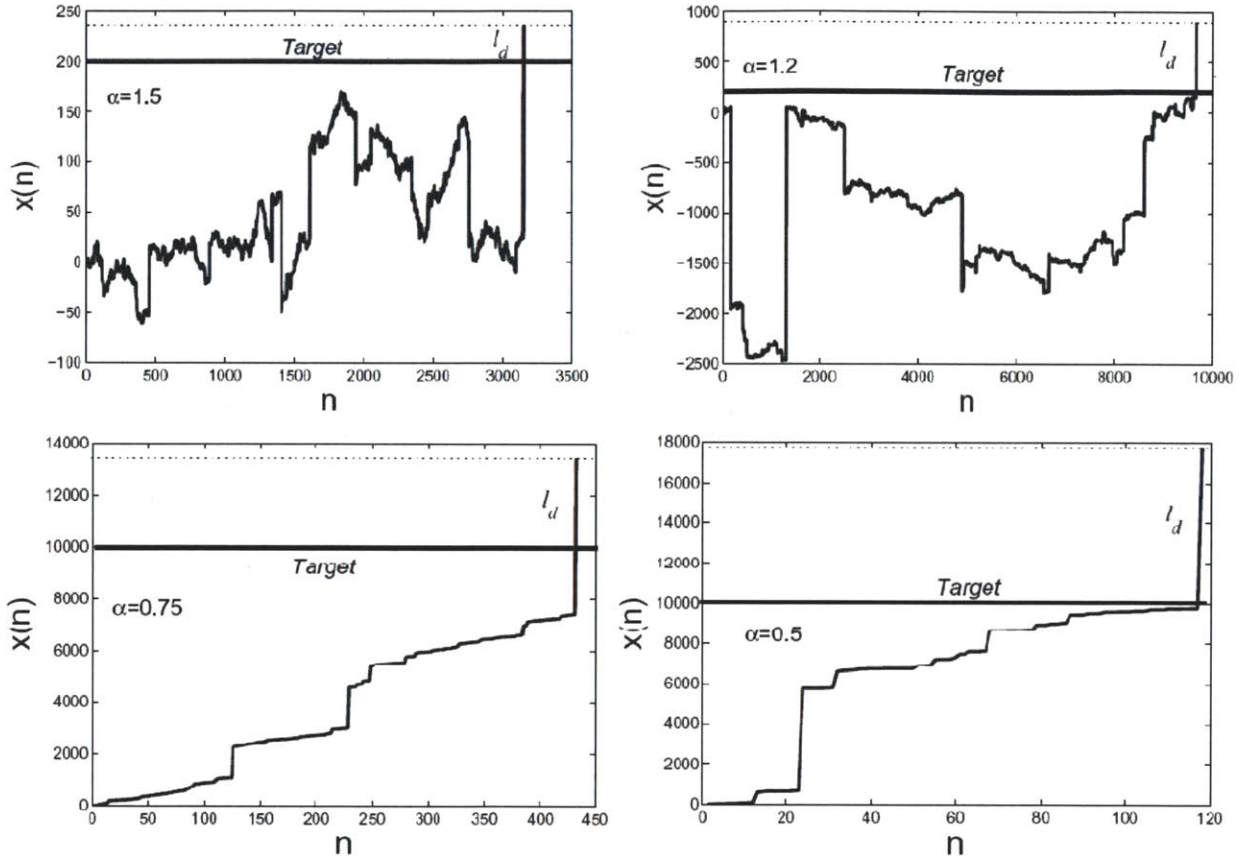
In nature, it has been shown that if an animal is searching for resources, a search strategy based on a Levy flight is superior, where the step-lengths are drawn from a statistical distribution with a power-law tail, because it maximizes the chance of random encounter (Rhodes). Two properties of a Levy flight that are of great importance are the first passage time and the first passage leapover (Checkkin). The first passage time measures how long it takes for an animal to reach or cross a certain target spot. The first passage leapover measures the distance the forager overshoot a specific target spot.



**Figure 7:** “Schematic representation of the leapover problem: the random walker starts at  $x=0$  and after a number of jumps crosses the point  $x=d$ , overshooting it by distance  $l$ . For narrow jump length distributions, each jump is so small that crossing the point  $d$  is equal to arriving at this point” (Checkkin).

Information on the leapover behavior is important to the understanding of how far a forager searches for food (Reynolds). Below are sample trajectories of symmetric and

one-sided Levy motion. It can be noticed that jumps get larger as  $\alpha$  decreases and as jumps get larger so does the size of the leapover.

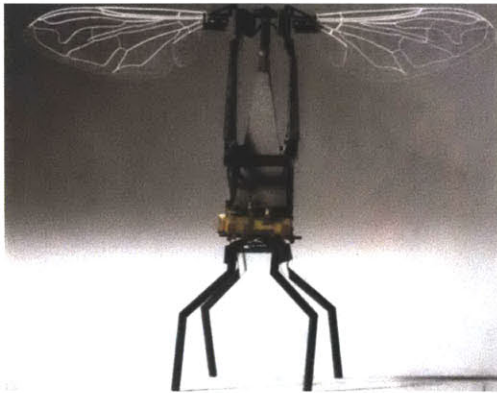


**Figure 8:** Trajectories of levy flights. Target is at  $d = 200$  in the case of the top two simulations and at 10,000 in the lower simulations (Checkkin).

In addition, it is noticeable that the smaller steps took longer to reach the target even though the target was set at 200. The trajectory with  $\alpha = \frac{1}{2}$  was able to reach the target site faster than the other iterations, approximately at 117 steps. A Levy flight proved to be more efficient to cross a target even though the overshoot was greater.

### 3. Possible Applications

#### 3.1 RoboBees



**Figure 9:** RoboBee (Simonite). **Figure 10:** RoboBee next to a cracker (Simonite).

As engineers continue to design and prototype small aircraft for tasks such as military use and search & rescue missions, they are beginning to look at nature for inspiration. “RoboBees, shown on figure 9 and 10, could be equipped with sensors and sent out after a natural disaster to collect environmental data or search for survivors, says their creator, Robert Wood, a professor of electrical engineering at Harvard University” (Technology Review). These RoboBees are tiny robots that have a huge potential due to their size which is a lot smaller than a round cracker as can be noticed in figure 2. The size of RoboBees makes them vulnerable to being lost or destroyed but also to be cheaply manufactured. As of now, RoboBees are still in prototype mode and can only be flown using wires connected to an off board power supply. In the future, it would be beneficial to have this RoboBees autonomously search areas using a Levy flight pattern while providing communicating information amongst each other and a master computer.

### 3.2 Hummingbird



**Figure 11:** “A U.S. Air Force airman directs the pilot of an F-16 Fighting Falcon aircraft into position on the end of the runway”. Department of Defense photo by Airman 1st Class Andrew Oquendo, U.S. Air Force.

The role of the navy has been very large aircrafts like the F-16 pictured above which is twice as tall as the airman. Lately, the Navy has also started to move towards biomimicry and are pursuing to reverse engineer birds, especially a hummingbird.

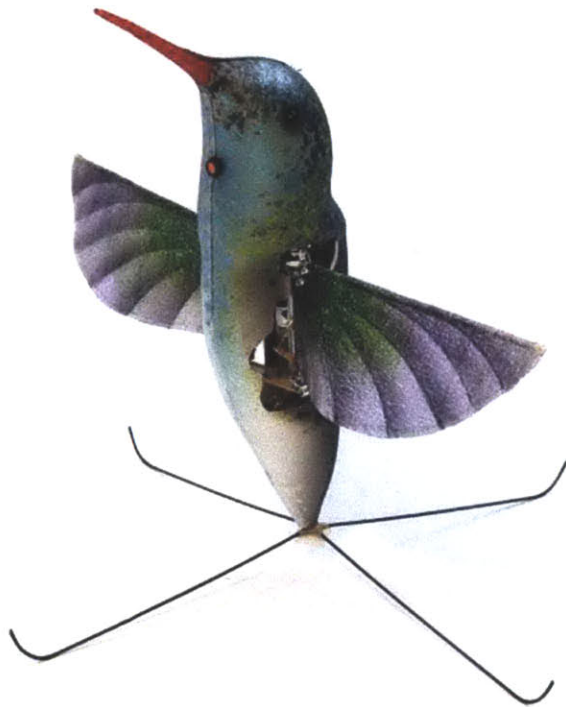
“Conventional aircraft designs can be scaled down only so far, but birds and bugs are a fruitful source of alternative blueprints for cheap, agile miniature flying vehicles”

(Technology Review). The hummingbird is of great importance because of its ability to dart from one location to another and avoid obstacles when traveling at fast speeds.

With smaller aircrafts it would be easier to go to places where it wouldn't be possible



with an F-16. A robot bird similar to the one shown in figure 5 would be able to sit at a tree or light pole and watch a doorway as people walk in and out of a building. This new surveillance aircrafts will be harder to see and nobody will notice it. This aircraft will make it easier to gather intelligence without being exposed to danger.

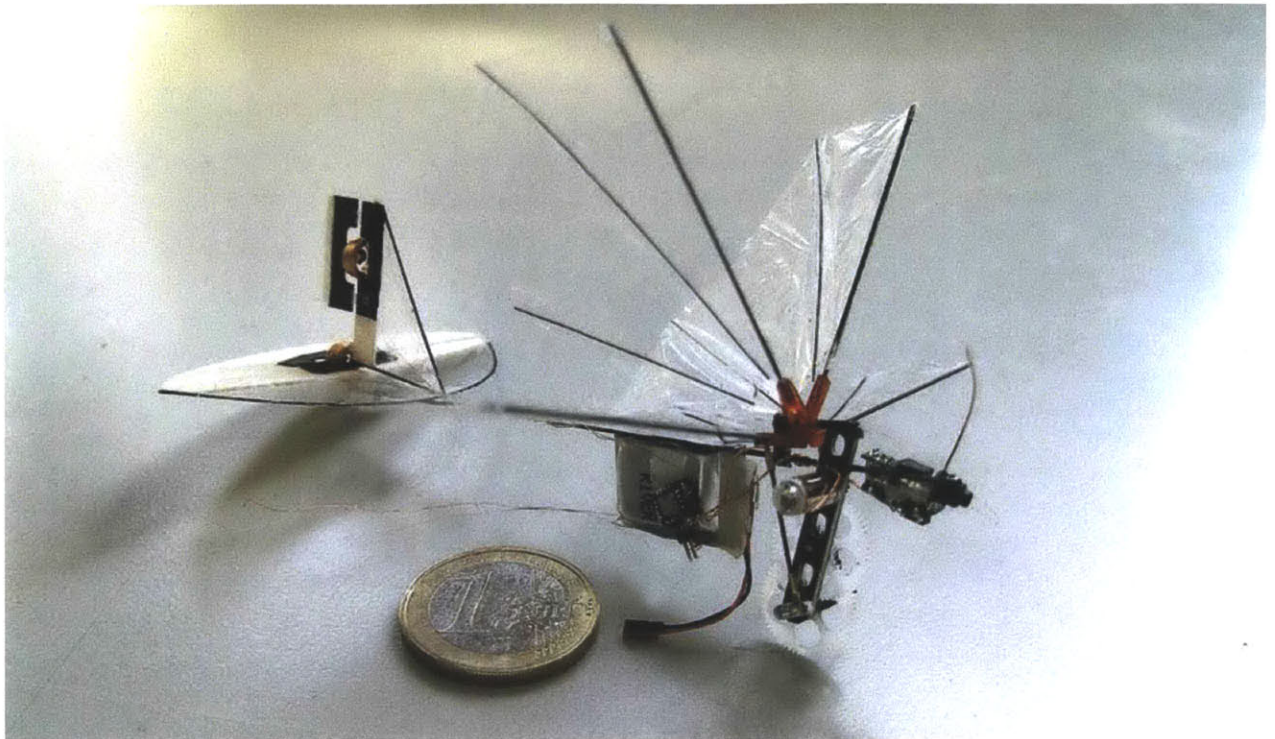


**Figure 12:** Hummingbird Robot developed by AeroVironment (Simonite).

Engineers are also cooperating with biologists to study and reengineer the hummingbird. In nature, we can see the hummingbird fly, stop, hover and feed from a plant while hovering. “This hummingbird aircraft can mimic darting in and out of open doorways in an urban environment, flip in midair, and hold its position against winds of up to eight kilometers per hour” (Technology Review). Unfortunately, it can

only fly for approximately 11 minutes while being controlled by a pilot using a live video stream from the camera seen on the hummingbird's chest.

### 3.3 DelFly Micro



**Figure 13:** The DelFly Micro pictured next to a one-euro coin (Simonite).

The DelFly Micro, pictured in figure 13 next to a one-euro coin, is the smallest flapping-wing craft capable of free flight while carrying a camera and a wireless transmitter. It has a wingspan of 10 centimeters and weighs three grams. The team responsible for the DelFly, at Delft University of Technology, in the Netherlands, has achieved superior control by combining flapping wings with a conventional airplane-style tail (Technology Review). A dozen of these robots using a Levy flight search strategy could be able to search a disaster zone in minimal time.

## 4. Identified Problems

The levy flight imposes several assumptions:

The first being that the animal is searching for a stationary resource; (2) the animal is determining its own trajectory; (3) the animal is only searching for the resource; (4) the search domain is open and is not restricted by the presence of boundaries; (5) the resource is randomly and “sparsely” distributed, i.e., the animal has no prior knowledge of target locations; and (6) the searcher has perfect perceptual and prey-capture capabilities so that targets within the searcher’s perceptual range are never missed and are always captured (Reynolds).

As stated by Reynolds, the advantage an animal has over a robot is that it can still make its own decision on which direction to head by inspecting its environment. A robot running a Levy flight simulation will be unable to decipher between key elements in its environment. It will continue to move randomly until a target is found or reached.

The task of searching for a target becomes more difficult when the target is moving. In a search and rescue mission, a person injured will be trying to find help while someone or in this case a robot tries to find the person. Instead of being able to

plot a map of which locations it has visited before, it will need to revisit sites to not miss a relocating target.

Another possible problem will be recognizing dangers while searching for a target. In a military scenario, the robot must be able to escape from enemy and avoid detection. Even though the robot might be disguised as an insect, the danger of being swatted or stepped on still exists.

If scientist and engineers are able to successfully create a robot that an insect, there will be much debate on whether these robots will be used only in search and rescue mission or to spy on citizens and possible terrorists. Individuals will have to pay more attention to their surroundings due to fear of espionage since these robots are so tiny and undetectable.

## 5. Conclusion

Nature has a great deal of science that is yet to be discovered and understood. Engineers have begun to reverse engineer cheetahs, birds, and insects while scientists have been doing research on the way organisms interact.

Today most of the robots are not completely autonomous and cannot think for themselves. In the same way that engineers are mimicking the gait of cheetahs when they are running, it is important to learn and understand animals as a whole. Even though Levy flights are fairly new to science, plenty of work has been put in to show that it best approximates the search pattern of animals. If we can couple this search pattern into robots along with decision making, a more effective robot could be possibly engineered. The platforms in which a Levy flight can be introduced are out there being improved as were seen in the section: Possible Applications.

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