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Novel electromagnetic radiation in a semi-infinite space filled with a double-negative metamaterial

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We have theoretically investigated the electromagnetic radiation excited by a charged particle moving along a semi-infinite space filled with a double-negative metamaterial (DNM). Cherenkov radiation in the double-negative region exhibits reversed or backward radiation behavior. The spectral density of reversed Cherenkov radiation (RCR) has a continuous distribution over the radiation frequency region. The influence of some important parameters on the Cherenkov radiation energy per unit length has been discussed. The surface wave in the vacuum region presented here also is investigated. We conclude that the amplitude of the surface wave is greatly enhanced over some normal dielectric materials cases. The enhanced surface wave may be useful for high frequency and high power vacuum electron devices with the DNM.

I. INTRODUCTION

The concept of the left-handed medium was first envisioned by the Russian scientist Veselago in 1967.¹ However, the naturally occurring material does not possess both negative permeability and negative permittivity simultaneously. Therefore, the new concept was not given much attention at that time. Since the original work,²⁻⁵ the artificially structured electromagnetic metamaterials have generated great enthusiasm among scientists and engineers.

Reversed Cherenkov radiation (RCR) was systematically studied for a charged particle in infinite isotropic and anisotropic double-negative metamaterials (DNMs), respectively.^{6,7} RCR and transition radiation generated by a charged particle through or across the DNM boundary also was theoretically investigated.⁸⁻¹¹ In addition, RCR in the waveguide partially or fully loaded a DNM was studied in details.¹²⁻¹⁵ The above results offer the basis for the experimental verification and potential applications of the RCR¹⁶⁻¹⁹ due to the reversed behavior and easy control of effective electromagnetic parameters. However, the surface wave excited by the charged particle in the vacuum region has been given few attentions.

Nowadays, Terahertz science and technology have attracted more and more attentions in the world. Many potential applications have been proved due to the unique properties of Terahertz radiation. These applications include semiconductor and high-temperature superconductor characterization, tomographic imaging, label-free genetic analysis, cellular level imaging and chemical and biological sensing.²⁰ However, there are many challenges in high power Terahertz sources, especially compact sources. Therefore, we have a new idea that the compact high power Terahertz radiation can be produced by using the DNMs. Based on the idea, we focus on the electromagnetic radiation excited by a charged particle which moves in vacuum parallel to and over a half of the volume filled with the DNM. The similar problem for normal dielectric materials was investigated before.²¹⁻²³ This particular problem presented here is of great interest for the generation of microwaves up to Terahertz waves. This is because the amplitude of the surface wave can be greatly enhanced over some normal dielectric material case when the RCR occurs in the DNM. Hence the results of this paper offer a view to develop planar microwave/Terahertz wave vacuum electron devices in the future.

II. THEORETICAL STATEMENT

We consider the general case of a charged particle moving along the boundary, as shown in Fig. 1. The charge of the particle is denoted by q , the velocity by \bar{v} , and the distance from the particle to the boundary by d . Half of the space is filled with an isotropic DNM with the rest of the space in vacuum. The effective material parameters in the DNM can be described in a Cartesian coordinate system (x, y, z)

as follows¹³

$$\varepsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma_e\omega}, \quad (1)$$

$$\mu_2(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\gamma_m\omega}, \quad (2)$$

where ω is the excitation angular frequency, ω_p the electric plasma frequency, γ_e the collision frequency representing “electronic” dissipation of the material, ω_0 the magnetic resonance frequency, γ_m the collision frequency accounting for the “magnetic” loss of the material, F the filling fraction in the SRR unit cell of the material.³ The charged particle moves along a semi-infinite space filled with the DNM in the \hat{z} direction with velocity \bar{v} . Therefore, the charge density is given by the following form

$$\rho(\bar{r}, t) = q\delta(x)\delta(y)\delta(z - vt). \quad (3)$$

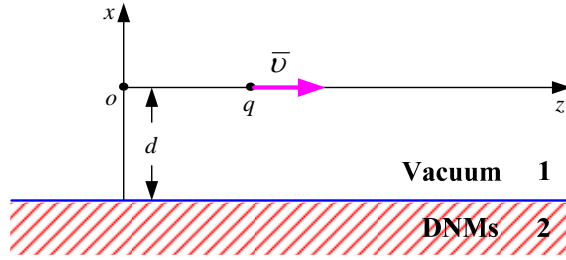


FIG. 1. A schematic diagram of a charged particle moving with speed \bar{v} along a semi-infinite space filled with a DNM.

According to the electromagnetic potentials approach, we define the electric and magnetic fields in the Gaussian system:

$$\bar{E} = -\frac{1}{c} \frac{\partial \bar{A}}{\partial t} - \nabla \varphi, \quad (4)$$

$$\bar{B} = \nabla \times \bar{A}, \quad (5)$$

where \bar{A} is the vector potential, φ is the scalar potential, and c is the light velocity in vacuum. In the theoretical analysis, we choose a set of potentials (\bar{A} , φ) to meet the Lorenz gauge. We resort to Maxwell’s equations, use the electromagnetic potentials approach described above, and arrive at the equations for potentials. Meanwhile, we assume that \bar{A} and φ have the Fourier integral representations as

follows

$$\bar{A}(\bar{r}, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \bar{A}(\bar{k}, \omega) e^{i(\bar{k} \cdot \bar{r} - \omega t)} dk_x dk_y dk_z d\omega, \quad (6)$$

$$\rho(\bar{r}, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho(\bar{k}, \omega) e^{i(\bar{k} \cdot \bar{r} - \omega t)} dk_x dk_y dk_z d\omega, \quad (7)$$

where

$$\bar{A}(\bar{k}, \omega) = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \bar{A}(\bar{r}, t) e^{-i(\bar{k} \cdot \bar{r} - \omega t)} dx dy dz dt, \quad (8)$$

$$\rho(\bar{k}, \omega) = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho(\bar{r}, t) e^{-i(\bar{k} \cdot \bar{r} - \omega t)} dx dy dz dt. \quad (9)$$

Inserting formula (3) into the equation for potential \bar{A} and using the above Fourier integral representations, we obtain the expression of the vector potential as

$$\bar{A}(\bar{r}, \omega) = \hat{z} \frac{q}{2\pi^2} \frac{1}{c} \int_{-\infty}^{+\infty} dk_y e^{ik_y y} e^{i\frac{\omega}{v}z} \int_{-\infty}^{+\infty} \frac{\mu e^{ik_x x}}{k_x^2 - k_{x1}^2} dk_x = \frac{\mu \pi}{k_{x1}} e^{ik_{x1}|x|}. \quad (10)$$

where $k_{x1} = i\sqrt{k_y^2 + \omega^2/v^2 - \varepsilon_1 \mu_1 \omega^2/c^2}$ is the x -component of the wave vector \bar{k}_1 .

Therefore, the expression (10) in the unbounded vacuum can be simplified as

$$\bar{A}_0(\bar{r}, \omega) = \hat{z} A_{0z} = \hat{z} \frac{q}{2\pi} \frac{i}{c} \int_{-\infty}^{+\infty} \frac{dk_y}{k_{x1}} e^{i(k_{x1}|x| + k_y y + \frac{\omega}{v}z)}, \quad (11)$$

For the present case, as shown in Fig. 1, $\bar{A}_1(\bar{r}, \omega)$ in the vacuum half can be written in the following form

$$\bar{A}_1(\bar{r}, \omega) = i \int_{-\infty}^{+\infty} \bar{a}_1 dk_y e^{i(2k_{x1}d + k_{x1}x + k_y y + \frac{\omega}{v}z)}, \quad (12)$$

where $\bar{a}_1 = a_{1x}\hat{x} + a_{1y}\hat{y} + a_{1z}\hat{z}$ is the unknown field coefficient in the vacuum half.

Evidently, the electromagnetic wave excited by the charged particle in the vacuum half (Fig. 1) must be the surface wave, which decays exponentially with distance far away from the interface at $x = -d$, since the Cherenkov radiation does not occur.

$\bar{A}_2(\bar{r}, \omega)$ in the medium half filled with the DNM can be expressed in the following form

$$\bar{A}_2(\bar{r}, \omega) = i \int_{-\infty}^{+\infty} \bar{a}_2 dk_y e^{i[k_{x1}d + k_{x2}(x+d) + k_y y + \frac{\omega}{v}z]}. \quad (13)$$

where $\bar{a}_2 = a_{2x}\hat{x} + a_{2y}\hat{y} + a_{2z}\hat{z}$ is the unknown field coefficient in the DNM half. Let's

define radial k_{x2} following Ref. 10, where it has been done for the fast charge

incoming left-handed medium. In any passive medium, k_{x2} should be

$$k_{x2}^2 = \frac{\omega^2}{c^2} \varepsilon_2 \mu_2 - \frac{\omega^2}{v^2} - k_y^2 = \frac{\omega^2}{c^2} [\varepsilon_2' \mu_2' - \varepsilon_2'' \mu_2'' + i(\varepsilon_2'' \mu_2' + \varepsilon_2' \mu_2'')] - \frac{\omega^2}{v^2} - k_y^2, \quad (14)$$

where $\varepsilon_2' = \text{Re } \varepsilon_2$, $\varepsilon_2'' = \text{Im } \varepsilon_2$, $\mu_2' = \text{Re } \mu_2$, $\mu_2'' = \text{Im } \mu_2$, $\varepsilon_2'' > 0$, and $\mu_2'' > 0$. Thus, $\text{Im } k_{x2}^2 < 0$ in the DNM. If we consider the propagating waves in the DNMs, that is $-\pi/2 < \arg k_{x2}^2 < 0$. Therefore, we have two options: (1) $-\pi/4 < \arg k_{x2} < 0$ and (2) $3\pi/4 < \arg k_{x2} < \pi$. The first option satisfies the requirement $\text{Im } k_{x2} < 0$, and the second one does not satisfy it. Thus $-\pi/4 < \arg k_{x2} < 0$ for positive frequencies, that means $\text{Re } k_{x2} > 0$. The group velocity and the energy flux density both are directed opposite to the wave vector \bar{k}_2 , therefore these vectors have negative projection on the x -axis. This characteristic is clearly demonstrated by the dispersion diagram in the double-negative frequency region, as illustrated in Fig. 2.

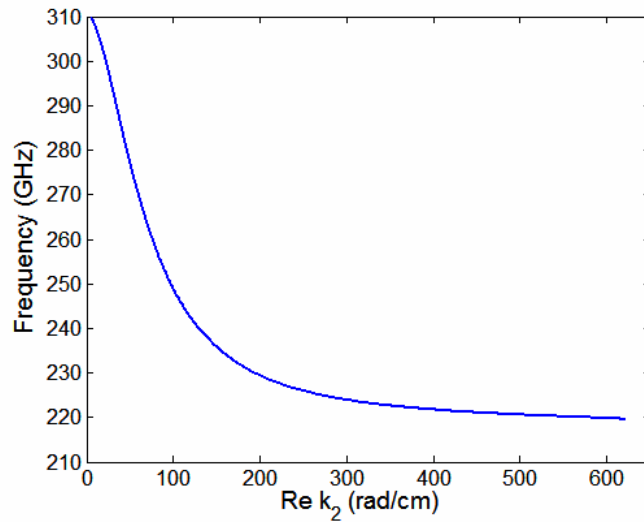


FIG. 2. A dispersion diagram for the present case.

In addition, we have the scalar potentials²³

$$\varphi_0(\bar{r}, \omega) = \frac{c}{v} A_{0z}(\bar{r}, \omega), \quad (15)$$

$$\varphi_1(\bar{r}, \omega) = 0, \quad (16)$$

$$\varphi_2(\bar{r}, \omega) = 0. \quad (17)$$

The electromagnetic field in the vacuum half is determined by the potentials φ_0 and $\bar{A}_0 + \bar{A}_1$ while the field in the medium half filled with the DNM is determined by \bar{A}_2 . Thus the field components in both the vacuum half and the DNM can be obtained by matching the fields at the boundary $x = -d$.²³

We can understand that the fields in the problem are made up of the field of a charged particle in an infinite medium and the fields due to the presence of the boundary. Thus the total radiated energy per unit length of path can be calculated as

$$\frac{dW_t}{dz} = qE_{1z} \Big|_{\substack{x=y=0 \\ z \rightarrow \infty}} = \frac{q}{c^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk_y \left[\frac{q}{2\pi k_{x1}} \left(\frac{c^2}{v^2} - 1 \right) - ca_{1z} \omega e^{i2k_{x1}d} \right] = \frac{2q}{c^2} \text{Re} \left\{ \int_0^{\infty} d\omega \int_{-\infty}^{\infty} dk_y \left[\frac{q}{2\pi k_{x1}} \left(\frac{c^2}{v^2} - 1 \right) - ca_{1z} \omega e^{i2k_{x1}d} \right] \right\}. \quad (18)$$

Since the first term in the curly brackets of formula (18) does not make a contribution to the Cherenkov radiation,²³ the Cherenkov radiation energy per unit length of path can be calculated as

$$\frac{dW}{dz} = -\frac{2q}{c} \text{Re} \left\{ \int_0^{\infty} d\omega \int_{-\infty}^{\infty} dk_y a_{1z} \omega e^{i2k_{x1}d} \right\}. \quad (19)$$

The double integral is taken over the Cherenkov radiation frequency region and over k_y . For low absorption material, the region of integration over k_y is determined by the inequality $k_y^2 < \omega^2/c^2 (\varepsilon_2' \mu_2' - \varepsilon_2'' \mu_2'') - \omega^2/v^2$. The integrand of the formula (19) represents the spectral density.

III. NUMERICAL CALCULATION AND DISCUSSION

In order to demonstrate the physical properties of the electromagnetic radiation in the present case, we performed the numerical calculations using these parameters which are listed here: the operation frequency ω belonging to the Terahertz band, the electric plasma frequency $\omega_p = 2\pi \times 500 \times 10^9 \text{ rad/s}$, the collision frequencies $\gamma_e = \gamma_m = 1 \times 10^{10} \text{ rad/s}$, the magnetic resonance frequency $\omega_0 = 2\pi \times 219 \times 10^9 \text{ rad/s}$, the filling fraction in the SRR unit cell $F = 0.5$, the electron velocity $v = 0.2c$, and the distance $d = 0.05 \text{ mm}$. Thus, RCR can be excited by an electron over the radiation frequency range (219.10, 226.60) GHz, which falls into the double-negative region (219.05, 309.65) GHz. The total radiated energy per unit length is $1.20810035648 \times 10^6 \text{ eV/m}$ while the Cherenkov radiation energy is $1.20810035427 \times 10^6 \text{ eV/m}$. It means that the energy loss of a charge in a semi-infinite vacuum is $2.21 \times 10^{-3} \text{ eV/m}$. It is really small but in reality the loss of semi-infinite vacuum is exactly zero. Compared to the normal dielectric material case shown in Fig. 3, the results in the present case clearly show the reversed property of the Cherenkov radiation in the DNM, which is represented by solid lines with arrows, while Cherenkov radiation in the normal materials such as distilled water, which is denoted by dashed lines with arrows, is characterized by the forward behavior. The results are completely consistent with the theoretical predictions presented in Section II. Note that the radiation direction is characterized by the time-averaged Poynting vector. The Cherenkov radiation cone in unbounded isotropic DNMs is symmetrical with the z -axis.⁷ However, in the present case, the surface wave excited by an electron in vacuum is not symmetrical with the z -axis (Fig. 3) and exponentially attenuated

when far away from the interface between vacuum and the DNM.

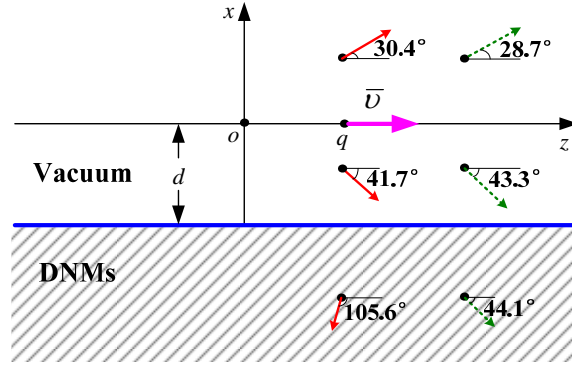


FIG. 3. The directions of the time-averaged Poynting vector in the DNM and vacuum.

We next investigated the spectral density of RCR. Fig. 4 shows that there is a continuous distribution of the spectral density for the present case, as similar to the unbounded case.⁷ Then we studied the effects on the Cherenkov radiation energy of varying several important parameters such as the electron velocity and the distance d . We find from Fig. 5 that, when the particle velocity increases, the Cherenkov radiation energy per unit length increases. This is because the RCR frequency band expands as the particle velocity increases, completely similar to the unbounded case. The Cherenkov radiation energy is much larger (10^8) than that in the unbounded case when the electron velocity is kept constant, as seen in Fig. 5.

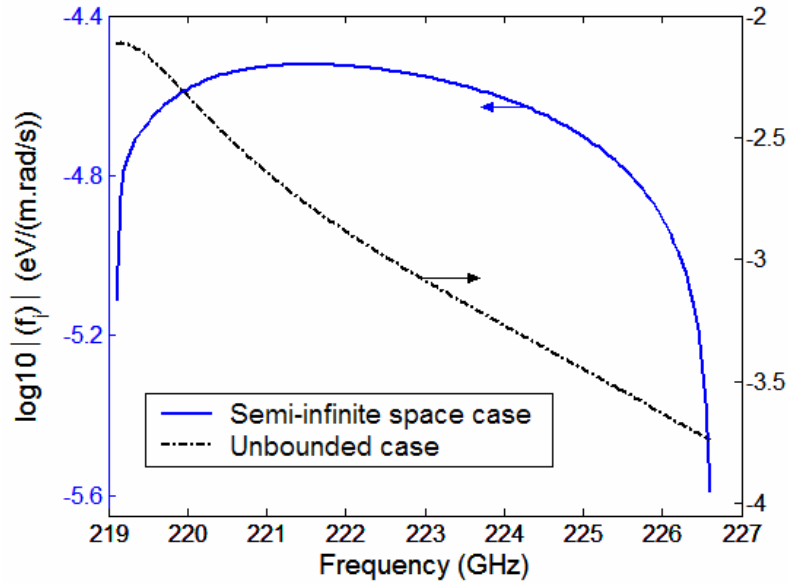


FIG. 4. Comparison of the spectral density over the radiation frequency band between the present and unbounded cases.

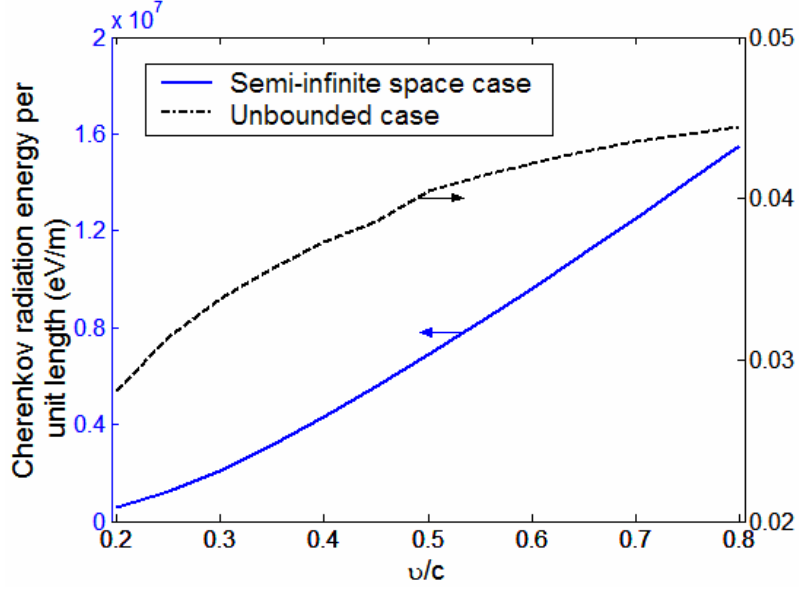


FIG. 5. Comparison of the Cherenkov radiation energy as a function of the electron velocity between the semi-infinite space and unbounded cases.

Meanwhile, we focused on the effect of the distance d on the Cherenkov radiation energy. The numerical results are shown in Fig. 6. It can be seen that the total radiated energy increases as the distance d decreases. This is due to the property of the surface wave. In other words, the field is exponentially attenuated when the distance d increases. The fact presented here can offer an important method to enhance the Cherenkov radiation energy, *i.e.*, increasing the charged particle's speed and/or letting the charged particle moves as close to the interface as possible.

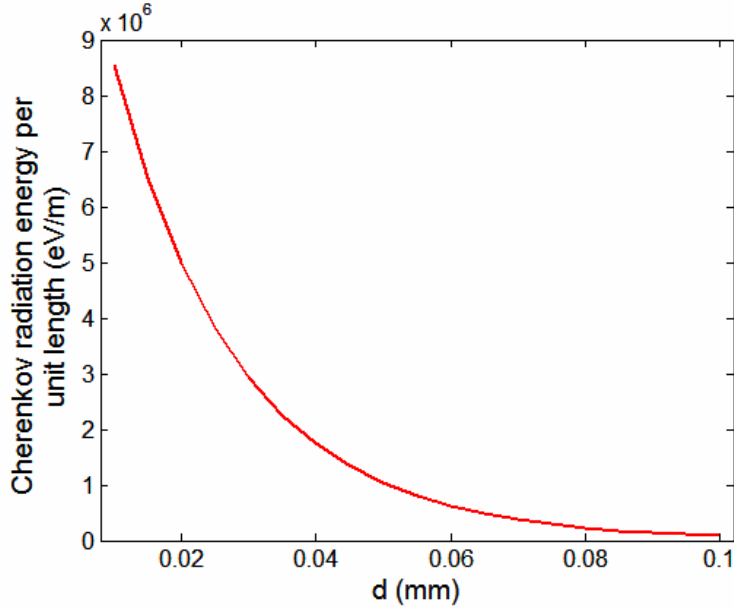


FIG. 6. Total radiated energy as a function of the distance d for the present case.

Finally, we studied the amplitude of the surface wave in the vacuum half. Here, we chose the parameters presented above and the operating frequency 220 GHz. For this surface wave, the amplitude of the time-averaged Poynting vector at $x = d/2$ for the present case has been enhanced ~ 3 to $\sim 5 \times 10^3$ times as compared with that for the normal dielectric material case, in which the DNM is replaced by the normal dielectric materials. The results are clearly illustrated in Fig. 7. We note that $|\langle \bar{S}_{vd} \rangle|$ and $|\langle \bar{S}_{vn} \rangle|$ denote the amplitudes of the time-averaged Poynting vector at $x = d/2$ in the vacuum half for the present and the normal dielectric material cases, respectively. When the relative permittivity value for the normal materials is less than ~ 25 , the Cherenkov radiation can not occur for the present case since the Cherenkov radiation condition is not satisfied. It is an important finding that the amplitude of the surface wave has been greatly enhanced using the DNM compared to some normal dielectric materials (such as BaO, its relative permittivity is 34 ± 1) cases. The significant advantage benefits the wave generation, especially Terahertz wave generation because Terahertz wave is more difficult both to produce and to detect.

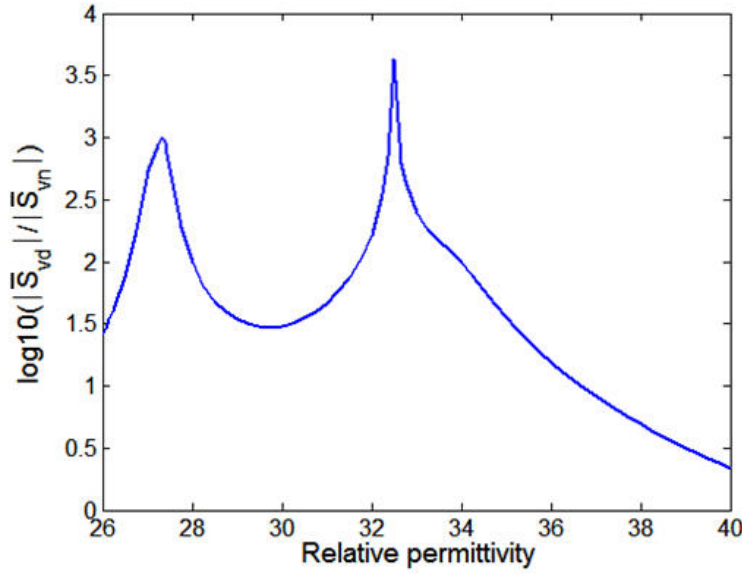


FIG. 7. Comparison of the amplitude of the surface wave between the present and the normal dielectric material cases.

IV. CONCLUSION

In this paper, we have developed a charged particle model to theoretically study the electromagnetic radiation in the space consisted of two different media. Here, half of the space is in vacuum and the other half is filled with a type of metamaterials exhibiting the double-negative behavior in a narrow frequency band. The present study shows that there exists the reversed characteristic for the Cherenkov radiation in the semi-infinite DNM. An important way to enhance the Cherenkov radiation energy is to decrease the distance d and/or increase the charged particle velocity. The

amplitude of the surface wave is greatly enhanced when DNMs are adopted over some normal dielectric materials cases. The results can offer a view to develop planar microwave devices, especially Terahertz vacuum electron sources.

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