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Lift-and-project relaxations of AC microgrid distribution system planning

Joshua A. Taylor and Franz S. Hover
Massachusetts Institute of Technology
Department of Mechanical Engineering
jatl@mit.edu, hover@mit.edu

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Abstract

We apply relaxation procedures to polynomial optimization problems that originate in transmission system planning, and obtain new convex formulations for the AC case. The approach is novel because the optimization is efficient but also addresses the true nonlinear physics directly. We illustrate the method on a test case derived from a notional shipboard distribution system.

1. INTRODUCTION

Transmission system planning (TSP) is a network design problem in which lines are selected from a candidate set to meet physical requirements while minimizing investment and operational costs [5, 7]. Linearized or 'DC' power flow is a standard simplification in the field [10], since AC flow is too complicated for most optimization scenarios. In network design problems, however, even linearized power flow creates a nonlinear, non-convex problem because the existence of a line can be a variable. Thus TSP with linearized load flow has traditionally been handled via further simplified forms, the so-called transportation and disjunctive models [7]. At the same time, in applications like a ship electrical system, line resistive characteristics are no longer negligible compared to reactances, because of the short line lengths, and the DC simplification is invalid for any purpose. An AC solution for TSP therefore has significant value, but only recently has it been approached in full [6], using an interior point method in tandem with a constructive heuristic algorithm.

Toward this end, our objective is to develop and apply new algorithms for power distribution design problems, that will improve accuracy and efficiency across several application areas. We utilize 'lift-and-project' relaxation procedures in our development, specifically that of [8]. A relaxation is an approximation to an optimization problem which always bounds the minimum below (or maximum above), and is typically easier to solve than the original problem. The phrase lift-and-project refers to lifting an optimization problem to a higher-dimensional space via the introduction of new variables, and then projecting the lifted problems solution back onto the original variables. Relaxed solutions are often suboptimal or infeasible for the original problem, but can contain a significant portion of the true optimal solution. Leveraging the maturity of commercial mixed integer linear programming solvers, our method represents the most scalable option to date for transmission system planning with full AC load flow.

We emphasize that the role of relaxation in the design process is not to create a feasible solution, but rather to construct a near-feasible solution that captures the structure of the optimal one, and therefore enables a follow-on design step that should be easier, for example a standard nonlinear programming formulation in conjunction with branch and bound. Moreover, one often finds that the relaxed solution adds key insights to the original problem, which can be studied in detail via simulation. A relaxed solution can be used to reduce or reformulate the design variables, by relabeling the solution as a fixed parameter set, and making new variables. In many cases, we have to identify a feasible initial solution in the new variable set; this is a separate problem from optimization, however.

Here we specialize our previous work [9] to a new benchmark case based on a shipboard distribution system. The all-electric ship, e.g. [3], is a prototypical micro-grid design application, for which the disjunctive and transportation models are entirely unsuited, as noted above. Note that the models given here do not address robustness, which would be of interest to the all-electric ship and some other microgrids, but via alternative cost functions such issues may be within the scope of our general approach.

2. TRANSMISSION SYSTEM PLANNING MODELS

We now derive linear models for AC transmission system planning which are similar in structure and size to the existing linear models [7]. We are given the following problem parameters: line investment vector c, a vector of real and reactive generation and demand limits $\underline{p}$, $\overline{p}$, $\underline{q}$, and $\overline{q}$, normalized flow limits $\overline{s}$, existing network ξ^0, and line construction limits $\overline{\xi}$. Let Γ denote the set of buses, Ω_0 the set of existing lines, and Ω the set of candidate lines. We follow the notational conventions that unless otherwise specified, single subscripts denote members of Γ, double subscripts members of Ω, and $i \sim j$ summation over $\Omega_0 \cup \Omega$. Let s, v, and y respectively denote complex powers, voltages and admittances. The basic

AC power flow model is given by

$$\begin{aligned}
\text{NLAC} \quad & \min_{\xi,s,v} \quad \sum_{i\sim j} c_{ij}\xi_{ij} \\
\text{s.t.} \quad & s_{ij} = \left(\xi^0_{ij}+\xi_{ij}\right)\left(v_i v_i^* y_{ij}^* - v_i v_j^* y_{ij}^*\right) \\
& \underline{p}_i \le \operatorname{Re}\sum_j s_{ij} \le \overline{p}_i \\
& \underline{q}_i \le \operatorname{Im}\sum_j s_{ij} \le \overline{q}_i \\
& \underline{v}_i \le |v_i| \le \overline{v}_i \\
& \left|s_{ij}\right| \le \left(\xi^0_{ij}+\xi_{ij}\right)\overline{s}_{ij} \quad (i,j)\in\Omega_0\cup\Omega \\
& 0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij}\in\mathbb{N}
\end{aligned}$$

Note that although line variables and parameters are non-directional, i.e. $\xi_{ij}=\xi_{ji}$, $\overline{s}_{ij}=\overline{s}_{ji}$ and so on, sending and receiving power flows s_{ij} and s_{ji} are not.

2.1. Linear AC models

We first must rewrite NLAC in terms of real, polynomial constraints so we may begin to build a relaxation. Let $y=g+jb$, $v=w+jx$, $s=p+jq$, and let $b^s=b+b^{sh}$, where b^{sh} is the line shunt susceptance. Applying the relaxation procedure of [8], NLACS is then is given by

$$\begin{aligned}
\text{NLACS} \quad & \min_{\xi,p,q,w,x} \quad \sum_{i\sim j} c_{ij}\xi_{ij} \\
\text{s.t.} \quad & p_{ij} = \left(\xi^0_{ij}+\xi_{ij}\right)\left(b_{ij}(w_j x_i - w_i x_j)\right. \\
& \quad \left. -g_{ij}(x_i x_j + w_i w_j) + g_{ij}(w_i^2+x_i^2)\right) \\
& q_{ij} = \left(\xi^0_{ij}+\xi_{ij}\right)\left(g_{ij}(w_j x_i - w_i x_j)\right. \\
& \quad \left. +b_{ij}(x_i x_j + w_i w_j) - b^s_{ij}(w_i^2+x_i^2)\right) \\
& \underline{p}_i \le \sum_j p_{ij} \le \overline{p}_i \\
& \underline{q}_i \le \sum_j q_{ij} \le \overline{q}_i \\
& \underline{v}_i^2 \le w_i^2 + x_i^2 \le \overline{v}_i^2 \\
& \sqrt{p_{ij}^2+q_{ij}^2} \le \left(\xi^0_{ij}+\xi_{ij}\right)\overline{s}_{ij} \\
& \quad (i,j)\in\Omega_0\cup\Omega \\
& 0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij}\in\mathbb{N}
\end{aligned}$$

The line capacity constraint involving the square root represents a slight obstacle: although it can be expressed polynomially, fourth order products of voltage variables would have to be included, rendering the size of the resulting relaxation impractically large. We instead approximate them so that p and q are involved linearly; a few options are apparent. To keep the formulation general, we introduce the constants τ^1 and τ^2 and replace (1) with

$$\tau^1_{ij}\left|p_{ij}\right| + \tau^2_{ij}\left|q_{ij}\right| \le \left(\xi^0_{ij}+\xi_{ij}\right)\overline{s}_{ij}$$

Although we have only used a single constraint in this approximation, any set of piecewise linear constraints can also be used.

As described in [9], define the new variables:

$$\begin{aligned}
\alpha_i &= w_i^2 + x_i^2 \\
\delta_{ij} &= \xi_{ij}\left(w_i^2+x_i^2\right) \\
\mu_{ij} &= b_{ij}(w_j x_i - w_i x_j) - g_{ij}(x_i x_j + w_i w_j) \\
&\quad + g_{ij}(w_i^2+x_i^2) \\
\nu_{ij} &= g_{ij}(w_j x_i - w_i x_j) + b_{ij}(x_i x_j + w_i w_j) \\
&\quad - b^s_{ij}(w_i^2+x_i^2) \\
\phi_{ij} &= \xi_{ij}\left(b_{ij}(w_j x_i - w_i x_j) - g_{ij}(x_i x_j + w_i w_j)\right. \\
&\quad \left. + g_{ij}(w_i^2+x_i^2)\right) \\
\psi_{ij} &= \xi_{ij}\left(g_{ij}(w_j x_i - w_i x_j) + b_{ij}(x_i x_j + w_i w_j)\right. \\
&\quad \left. - b^s_{ij}(w_i^2+x_i^2)\right)
\end{aligned}$$

These new variables have implicit constraints given by

$$\begin{aligned}
& g_{ij}(\mu_{ij}-\mu_{ji}) - b_{ij}(\nu_{ij}-\nu_{ji}) \\
& \quad = \left(g_{ij}^2 + b_{ij}b^s_{ij}\right)(\alpha_i - \alpha_j) \\
& b_{ij}(\mu_{ij}+\mu_{ji}) + g_{ij}(\nu_{ij}+\nu_{ji}) \\
& \quad = \left(g_{ij}b_{ij} - g_{ij}b^s_{ij}\right)(\alpha_i + \alpha_j) \\
& g_{ij}(\phi_{ij}-\phi_{ji}) - b_{ij}(\psi_{ij}-\psi_{ji}) \\
& \quad = \left(g_{ij}^2 + b_{ij}b^s_{ij}\right)(\delta_{ij} - \delta_{ji}) \\
& b_{ij}(\phi_{ij}+\phi_{ji}) + g_{ij}(\psi_{ij}+\psi_{ji}) \\
& \quad = \left(g_{ij}b_{ij} - g_{ij}b^s_{ij}\right)(\delta_{ij} - \delta_{ji})
\end{aligned}$$

Let Φ denote the set on which the variables μ, ν, ϕ, ψ, α, and δ satisfy these equalities. Forming constraints containing up to second-order terms and substituting the new variables, we have

$$
\begin{aligned}
\text{LAC} \quad & \min_{\xi,\mu,\nu,\phi,\psi,\alpha,\delta} \quad \sum_{i\sim j} c_{ij}\xi_{ij} \\
\text{s.t.} \quad & \{\mu,\nu,\phi,\psi,\alpha,\delta\} \in \Phi \\
& \underline{p}_i \leq \sum_j \xi^0_{ij}\mu_{ij} + \phi_{ij} \leq \overline{p}_i \\
& \underline{q}_i \leq \sum_j \xi^0_{ij}\nu_{ij} + \psi_{ij} \leq \overline{q}_i \\
& \underline{v}_i^2 \leq \alpha_i \leq \overline{v}_i^2 \\
& \underline{v}_i^2\xi_{ij} \leq \delta_{ij} \leq \overline{v}_i^2\xi_{ij} \\
& \underline{v}_i^2\left(\overline{\xi}_{ij} - \xi_{ij}\right) \leq \overline{\xi}_{ij}\alpha_i - \delta_{ij} \leq \overline{v}_i^2\left(\overline{\xi}_{ij} - \xi_{ij}\right) \\
& \tau^1_{ij}\left|\mu_{ij}\right| + \tau^2_{ij}\left|\nu_{ij}\right| \leq \overline{s}_{ij} \quad (i,j) \in \Omega_0 \\
& \tau^1_{ij}\left|\phi_{ij}\right| + \tau^2_{ij}\left|\psi_{ij}\right| \leq \overline{s}_{ij}\xi_{ij} \\
& \tau^1_{ij}\left|\overline{\xi}_{ij}\mu_{ij} - \phi_{ij}\right| + \tau^2_{ij}\left|\overline{\xi}_{ij}\nu_{ij} - \psi_{ij}\right| \\
& \quad \leq \overline{s}_{ij}\left(\overline{\xi}_{ij} - \xi_{ij}\right) \quad (i,j) \in \Omega_0 \\
& 0 \leq \xi_{ij} \leq \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}
\end{aligned}
$$

Through the the relaxation procedure, the AC transmission system planning problem has thus been posed as a mixed integer linear program.

3. APPLICATIONS

3.1. Computational example

In this section we apply LAC to a to a 19-bus, 46-candidate-line example which was abstracted from a notional shipboard power system [2]. The outcome of this example calculation is not considered to be directly applicable to the electric ship specifically, but rather as a result representative of a microgrid design, having geographic content and highly variable loads. Tables 1 and 2 contain all relevant problem data.

Reactive power limits are assumed to be one tenth of real power limits, reactance and resistance are both set to length over line capacity, and $\overline{\eta}$ to two for all lines. The base layout η^0 was chosen to be a ring connecting the power converter modules. Voltage limits are not enforced in this example. Line costs are proportional to length times capacity, where length is assumed to be the 'Manhattan distance' (x-distance plus y-distance plus z-distance) between two buses. We use a slightly more complicated flow limit approximation than (2.1.), which is a proper relaxation:

$$\left|p_{ij}\right| \leq \left(\eta_{ij} + \eta^0_{ij}\right)\overline{s}_{ij}, \quad \left|q_{ij}\right| \leq \left(\eta_{ij} + \eta^0_{ij}\right)\overline{s}_{ij}$$

$$\left|p_{ij}\right| + \left|q_{ij}\right| \leq \sqrt{2}\left(\eta_{ij} + \eta^0_{ij}\right)\overline{s}_{ij}.$$

The mixed integer linear program was solved using the modeling language AMPL [4] and solver CPLEX [1], in 1.0 seconds on a laptop representative of current standards. Table 3 and Fig. 1 show which lines were selected (including the base design). Inspection shows that large loads are connected directly to large generators, as expected. Non-intuitive connections, or lack of connection, elsewhere are generally due to the fact that the z-distances are not evident in the figure, and that certain connections were simply not part of the allowable set in Table 1.

Table 2. Bus real power limits and locations

Bus	$\underline{p}$ (kW)	$\overline{p}$ (kW)	x (m)	y (m)	z (m)
1: fwd. mn. gen.	0	35000	18.0	2.1	5.8
2: fwd. aux. gen.	0	3500	23.5	-2.1	5.8
3: PCM 2a	-500	-500	22.1	0	5.6
4: PCM 1a	-500	-500	33.8	-2.7	3
5: PCM 1a	-500	-500	10.4	7.0	8.2
6: aft mn. gen.	0	35000	-10.4	0.6	5.8
7: aft aux. gen.	0	3500	-43.0	-0.6	5.8
8: PCM 2a	-500	-500	-4.7	0	5.6
9: PCM 1a	-500	-500	8.5	-5.2	3
10: PCM 1a	-500	-500	-18	7	8.2
11: PCM 2a	-100	-100	-40.8	0	5.6
12: port rud.	-50	-50	-68	4.1	3.0
13: stbd. rud.	-50	-50	-68	-4.1	3
14: PCM 1a	-100	-100	-30.8	-1.8	3
15: PCM 1a	-100	-100	-50.9	6.4	8.2
16: port motor	-37000	-37000	-14.9	4	3
17: stbd. motor	-37000	-37000	13.4	-4	3
18: port rdr.	-2300	-2300	30	4.2	20
19: stbd. rdr.	-2300	-2300	30	-4.2	20

Table 3. LAC solution

Line	Additions
1 - 5	1
1 - 17	1
2 - 5	1
3 - 4	2
6 - 16	1
7 - 15	1
8 - 9	1
9 - 14	2
11 - 14	1
12 - 15	1
13 - 15	1
17 - 18	1
17 - 19	1

Table 1. Line data

Line	Length (m)	s (MW)	η^0	Line	Length (m)	s (MW)	η^0
1 - 4	23.4	20.3	0	6 - 9	27.5	20.3	0
1 - 5	14.9	20.3	0	6 - 10	16.4	20.3	0
1 - 9	19.6	20.3	0	6 - 14	25.6	20.06	1
1 - 10	43.3	20.3	0	6 - 15	48.7	20.06	0
1 - 14	55.5	20.06	0	6 - 16	10.7	40	0
1 - 15	75.6	20.06	0	6 - 17	31.2	40	0
1 - 16	37.6	40	0	7 - 9	58.9	3.3	0
1 - 17	13.5	40	0	7 - 10	35	3.3	0
2 - 4	13.7	3.3	0	7 - 14	16.2	3.06	0
2 - 5	24.6	3.3	0	7 - 15	17.3	3.06	0
2 - 9	20.9	3.3	0	7 - 16	35.5	23	0
2 - 10	53	3.3	0	7 - 17	62.6	23	0
2 - 14	57.4	3.06	0	8 - 9	21	0.6	0
2 - 15	85.3	3.06	0	8 - 10	22.9	0.6	0
2 - 16	47.3	23	0	9 - 14	42.7	0.36	1
2 - 17	14.8	23	0	11 - 14	14.4	0.12	0
3 - 4	17	0.6	0	11 - 15	19.1	0.12	0
3 - 5	21.3	0.6	0	12 - 14	43.1	0.12	0
4 - 5	38.3	0.6	1	12 - 15	24.6	0.12	0
4 - 6	50.3	20.3	0	13 - 14	39.5	0.12	0
4 - 7	81.7	3.3	0	13 - 15	32.8	0.12	0
4 - 9	27.8	0.6	1	14 - 15	33.5	0.12	1
5 - 6	29.6	20.3	0	16 - 18	62.1	21.5	0
5 - 7	63.4	3.3	0	16 - 19	70.1	21.5	0
5 - 10	28.4	0.6	1	17 - 18	41.8	21.5	0
5 - 15	61.9	0.36	1	17 - 19	33.8	21.5	0

3.2. Multiple scenarios

Returning to the case of a shipboard electrical system as a prototype microgrid, we note that a ship may require multiple sets of loads, for example the operating conditions of traveling at high speed, versus active combat. Because these are such different load scenarios, there is no analogue in terrestrial system design.

A conservative approach would be to simply optimize with each load bus consuming the maximum power over all scenarios; this however, will lead to highly conservative designs. We can instead produce designs that are not overly conservative by creating constraints and variables for each scenario, and optimizing the same objective as usual. Suppose we are given n_S scenarios, and for each scenario $k = 1, \ldots, n_S$ we have a set of minimum and maximum bus power levels at each bus i, $\underline{p}_i^k$, $\overline{p}_i^k$, $\underline{q}_i^k$, and $\overline{q}_i^k$. Using a separate set of variables for each set of power levels, we have the following multiple scenario transmission planning problem:

$$
\begin{array}{lll}
\text{MSAC} & \min\limits_{\xi,s,v} & \sum\limits_{i\sim j} c_{ij}\xi_{ij} \\
\text{s.t.} & & s_{ij}^k = \left(\xi_{ij}^0 + \xi_{ij}\right)\left(v_i^k v_i^{k*} y_{ij}^* - v_i^k v_j^{k*} y_{ij}^*\right) \quad \forall k \\
& & \underline{p}_i^k \le \operatorname{Re}\sum\limits_j s_{ij}^k \le \overline{p}_i^k \quad \forall k \\
& & \underline{q}_i^k \le \operatorname{Im}\sum\limits_j s_{ij}^k \le \overline{q}_i^k \quad \forall k \\
& & \underline{v}_i \le \left|v_i^k\right| \le \overline{v}_i \quad \forall k \\
& & \left|s_{ij}^k\right| \le \left(\xi_{ij}^0 + \xi_{ij}\right)\overline{s}_{ij} \quad (i,j) \in \Omega_0 \cup \Omega, \forall k \\
& & 0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}
\end{array}
$$

From here, relaxations may be developed by identically applying the procedures of the previous section.

4. CONCLUSION

We have applied a general relaxation for polynomial optimization problems to AC transmission system planning. By forming the problem as a mixed integer linear program, we are taking advantage of mature tools in convex optimization to achieve efficient and reliable solutions to this difficult problem.

The newly obtained model has been applied here to an AC microgrid example based on an abstracted shipboard distribution system, for which the traditional simplifying assumptions in TSP are not valid. The algorithm found a solution in modest computation time, that forms a cogent basis for further refinement.

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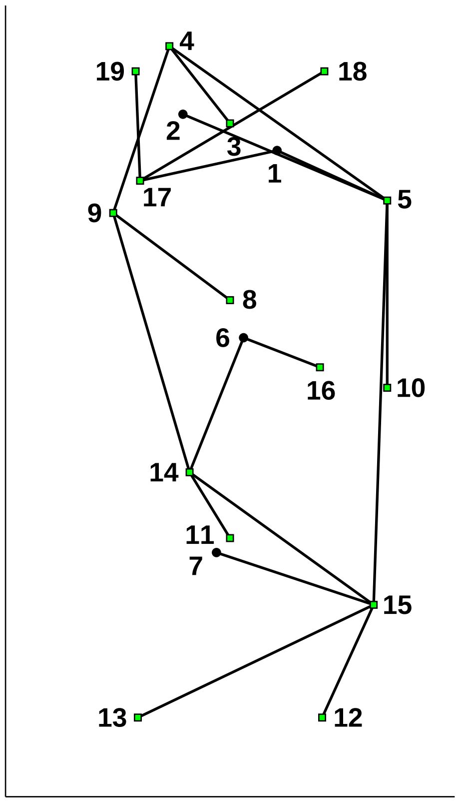

Figure 1. Solution with buses arranged geographically according to x and y coordinates given in Table 1 (the aspect ratio has been modified to aid viewing). Squares denote loads and circles generation. Note that Buses 17 and 18 are not connected to Bus 3.

REFERENCES

[1] IBM ILOG CPLEX.

[2] BMT Syntek. IPS electric plant one-line diagram for a notional destroyer, 2003.

[3] Norbert Doerry, Henry Robey, John Amy, and Chester Petry. Powering the future with the integrated power system. *Naval Engineers Journal*, 108(3):267–282, 1996.

[4] Robert Fourer, David M. Gay, and Brian W. Kernighan. *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press, November 2002.

[5] G. Latorre, R.D. Cruz, J.M. Areiza, and A. Villegas. Classification of publications and models on transmission expansion planning. *Power Systems, IEEE Transactions on*, 18(2):938 – 946, May 2003.

[6] M.J. Rider, A.V. Garcia, and R. Romero. Power system transmission network expansion planning using AC model. *Generation, Transmission Distribution, IET*, 1(5):731 –742, September 2007.

[7] R. Romero, A. Monticelli, A. Garcia, and S. Haffner. Test systems and mathematical models for transmission network expansion planning. *IEE Proceedings - Generation, Transmission and Distribution*, 149(1):27–36, 2002.

[8] H. D. Sherali and C. H. Tuncbilek. A global optimization algorithm for polynomial programming problems using a reformulation-linearization technique. *Journal of Global Optimization*, 2:101–112, 1992.

[9] J. A. Taylor and F. S. Hover. Linear relaxations for transmission system planning. *Submitted to IEEE Trans. on Power Systems*, 2010.

[10] A. J. Wood and B. F. Wollenberg. *Power generation, operation, and control*. Knovel, second edition, 1996.