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VEHICLE-MERGING PROBLEM

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This study was conducted at the Electronic Systems Laboratory with support extended by the U. S. Department of Commerce under Contract C-85-65, Project Transport, M.I.T. DSR Project No. 76105.

Electronic Systems Laboratory
Department of Electrical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

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ABSTRACT

This paper considers the problem of deciding the best way of merging two strings of high-speed vehicles into a single guideway as well as the design of the control system required to accomplish this task. It is shown that both the optimal merging sequence and the control system can be determined by casting the problem into the framework of controlling a linear time-invariant system with quadratic performance criteria. The required control system is shown to be a linear time-invariant feedback system.

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** Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

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1. INTRODUCTION

This research was motivated by the need to establish an orderly approach to the problem of merging two or more strings of high-speed vehicles into a single guideway or lane. The position and velocity control of vehicles in a single guideway has been considered before in Refs. [1], [2], and [3]; the same approach, namely casting the problem into a linear optimal regulator one, is used here. The equations of the string as well as the performance index is identical to those used by Levine and Athans [1]. For this reason, no extensive motivation and discussion of the criterion is presented.

In both the merging and the single guideway problem a control cost functional is formulated based on the following broad specifications

1. Each pair of physically adjacent vehicles must be separated by a given desired separation distance Δ ; any (positive or negative) deviation from Δ should be penalized.
2. Each vehicle must travel with a given string velocity \bar{v} ; any (positive or negative) velocity deviation from \bar{v} should be penalized.
3. Severe accelerating or decelerating forces should be avoided except in emergency situations.

It is emphasized that emergency control is not considered here.

2. THE MERGING PROBLEM

The physical aspects of the merging problem are illustrated in Figure 1. Two strings of vehicles moving (from right to left) in two single-lane guideways A and B must merge into a single-lane guideway C. The junction point is denoted by 0. It is assumed that a command and control center is located near the junction so as to automatically control a finite number of vehicles in the guideways A, B, and C which, during some interval of time, are in the vicinity of the junction point.

The vehicles involved in the merging are denoted as follows :

Guideway A : $a_1, a_2, \dots, a_\alpha$

Guideway B : b_1, b_2, \dots, b_β

Guideway C : $c_1, c_2, \dots, c_\gamma$

For notational purposes we denote by A the set of vehicles in the guideway A.

Thus

$$A = \{a_1, a_2, \dots, a_\alpha\} \quad (2.1)$$

Similarly define

$$B = \{b_1, b_2, \dots, b_\beta\} \quad (2.2)$$

$$C = \{c_1, c_2, \dots, c_\gamma\} \quad (2.3)$$

Note that these sets are disjoint.

It is evident from Figure 1 that distances from the junction point 0 can be used to define a natural ordering of the sets A,B,C. The natural ordering is defined by the relations

$$\begin{aligned} a_1 &> a_2 > \dots > a_\alpha \\ b_1 &> b_2 > \dots > b_\beta \\ c_1 &> c_2 > \dots > c_\gamma \end{aligned} \quad (2.4)$$

and so one can define the ordered sets A^0, B^0, C^0 by

$$A^{\circ} = \{a_i \in A : a_i > a_{i+1} ; i=1,2,\dots,\alpha-1\} \quad (2.5)$$

$$B^{\circ} = \{b_i \in B : b_i > b_{i+1} ; i=1,2,\dots,\beta-1\} \quad (2.6)$$

$$C^{\circ} = \{c_i \in C : c_i > c_{i+1} ; i=1,2,\dots,\gamma-1\} \quad (2.7)$$

If one has absolute control over the vehicles, then one can merge them in many ways. For example, one can move through the junction first all of the vehicles in A and then all the vehicles in B; in this case, after all the vehicles have entered the main guideway, the order depicted in Figure 2(a) will exist. On the other hand, one may wish to interlace the vehicles as shown in Figure 2(b). At any rate, as long as the vehicles can be effectively controlled several possible merging sequences can be generated as illustrated in Figs. 2(c) and (d).

The fact that there are many ways of merging the vehicles in A and B can be described mathematically in the following way. Let N denote the total number of vehicles involved, i.e.,

$$N \triangleq \alpha + \beta + \gamma \quad (2.8)$$

Let Q be the unordered set formed by the $(\alpha+\beta)$ vehicles in A and B, i.e.,

$$Q \triangleq \{q_1, q_2, \dots, q_{\alpha+\beta}\} = A \cup B \quad (2.9)$$

Define the ordering

$$q_1 > q_2 > \dots > q_{\alpha+\beta} \quad (2.10)$$

and the ordered set Q°

$$Q^{\circ} = (A \cup B)^{\circ} = \{q_j \in Q : q_j > q_{j+1} ; j = 1, 2, \dots, \alpha+\beta-1\} \quad (2.11)$$

Furthermore, define the ordered sets

$$Q_i^{\circ} = \{a_i \in A, b_i \in B : (A \cup B)^{\circ} - B = A^{\circ} \text{ and } (A \cup B)^{\circ} - A = B^{\circ}\} \quad (2.12)$$

where $i=1,2,\dots,M$. These in turn can be used to define the ordered sets

$$P_i^O = \{C^O, Q_i^O\} \quad (2.13)$$

It should be clear (see also Figure 2 for the example) that each set P_i^O has the property that

- (a) it has $N = \alpha + \beta + \gamma$ elements,
- (b) its first γ elements constitute the ordered set C^O of Eq. (2.7),
- (c) its last $\alpha + \beta$ elements involve elements of A and B such that if all the elements of B are removed, then the elements of A form the ordered set A^O and if all the elements of A are removed, then the elements of B form the ordered set B^O .

Note that the total number M of the sets Q_i^O (or P_i^O) defines the total number of possible merging sequences. It turns out that M is given by the expression

$$M = \frac{(\alpha + \beta)!}{\alpha! \beta!} \quad (2.14)$$

Since there are a total of M possible merging sequences, there are two basic problems that must be solved.

Problem 1 : Given a set of vehicles in the three guideways and given a merging sequence. How should the vehicles be controlled so that the desired merging sequence takes place in a safe and orderly manner?

Problem 2 : Is there a best (in some sense) merging sequence?

These two problems are solved in the sequel. In the next section Problem 1 will be attacked using the theory of optimal control.

3. OPTIMAL CONTROL FOR A GIVEN MERGING SEQUENCE

Denote the position at some initial time t_0 of the vehicles with respect to the junction point as follows

$$\begin{aligned}\text{Guideway A} &: z[a_1](t_0), z[a_2](t_0), \dots, z[a_\alpha](t_0) \\ \text{Guideway B} &: z[b_1](t_0), z[b_2](t_0), \dots, z[b_\beta](t_0) \\ \text{Guideway C} &: z[c_1](t_0), z[c_2](t_0), \dots, z[c_\gamma](t_0)\end{aligned}\tag{3.1}$$

It is assumed that distances along guideway C are positive, while distances along A and B are negative. Thus,

$$\begin{aligned}0 &> z[a_1](t_0) > z[a_2](t_0) > \dots > z[a_\alpha](t_0) \\ 0 &> z[b_1](t_0) > z[b_2](t_0) > \dots > z[b_\beta](t_0) \\ z[c_1](t_0) &> z[c_2](t_0) > \dots > z[c_\gamma](t_0) > 0\end{aligned}\tag{3.2}$$

The true velocities of each vehicle at t_0 will be denoted by

$$v[a_i](t_0), v[b_j](t_0), v[c_k](t_0)\tag{3.3}$$

Assume that all the vehicles are identical.* Since all vehicles must travel at the same desired velocity \bar{v} , then there is a constant force \bar{f} needed to overcome the drag at that velocity. For small deviations of the actual velocity $v(t)$ of any vehicle from the desired velocity \bar{v} one has^{1,2}

$$\frac{d}{dt} \phi(t) = -\beta \phi(t) + u(t)\tag{3.4}$$

$$\text{where :} \quad \phi(t) \triangleq v(t) - \bar{v}\tag{3.5}$$

β : drag coefficient at speed \bar{v} ($\beta > 0$)

$u(t)$: incremental acceleration

*This is not an essential assumption; the general theory can be worked out in the case of non-identical vehicles. If one assumes, however, that all the vehicles are identical one brings into focus the essential symmetry of the problem.

Next consider any arbitrary merging sequence. As noted in Section 2 any given merging sequence defines which adjacent vehicles in guideway A or B will continue to be adjacent after they pass through the junction point. Thus, any two vehicles which will become adjacent in guideway C must be controlled in such a way so that any errors in their desired separation are to be eliminated once they go through the junction point. It, therefore, makes sense to control these vehicles prior to the junction point, because, given sufficient time, future separation errors can be corrected using smooth acceleration or deceleration forces (which are important from the passenger-comfort and energy-expenditure viewpoints).

The mathematical formulation of the system equations will now be discussed.

Given any one merging sequence indexed by i ($i = 1, 2, \dots, M$ where M is given by Eq. (2.14)). This merging sequence defines an ordered set P_i^O (see Eq. (2.13)). Let p_{k_i} denote the elements of P_i^O

$$P_i^O = \{p_{1_i}, p_{2_i}, \dots, p_{(\alpha+\beta+\gamma)_i}\} \quad (3.6)$$

This particular order of merging defines which vehicles are to be adjacent after merging has occurred. So let k_i and k_i+1 ($k_i=1, 2, \dots, \alpha+\beta+\gamma$) index adjacent vehicles under this ordering. Clearly,

$$\frac{d}{dt} z[k_i](t) = v[k_i](t) \quad (3.7)$$

$$\frac{d}{dt} z[k_i+1](t) = v[k_i+1](t) \quad (3.8)$$

Let $e_{k_i}(t)$ denote the deviation of the k_i and k_i+1 vehicles from their desired separation, i.e.,

$$e_{k_i}(t) \triangleq z[k_i](t) - z[k_i+1](t) + \Delta \quad (3.9)$$

Then, this position error satisfies the differential equation

$$\frac{d}{dt} e_{k_i}(t) = v[k_i](t) - v[k_{i+1}](t) \quad (3.10)$$

Define $\phi_{k_i}(t)$ to be the velocity deviation of the k_i -th vehicle from the desired velocity \bar{v} and let $u_{k_i}(t)$ be the acceleration applied to the same vehicle. Thus,

$$\phi_{k_i}(t) \triangleq v[k_i](t) - \bar{v} \quad (3.11)$$

$$\text{Then } v[k_i](t) - v[k_{i+1}](t) = \phi_{k_i}(t) - \phi_{k_{i+1}}(t) \quad (3.12)$$

and so Eqs. (3.10) and (3.4) yield

$$\begin{aligned} \frac{d}{dt} \phi_{k_i}(t) &= -\beta \phi_{k_i}(t) + u_{k_i}(t) \\ \frac{d}{dt} e_{k_i}(t) &= \phi_{k_i}(t) - \phi_{k_{i+1}}(t) \end{aligned} \quad (3.13)$$

For the given i -th merging sequence, define the state vector $\underline{x}_i(t)$ as the vector obtained by interlacing the velocity and position errors. Thus,

$$\underline{x}_i(t) \triangleq [\phi_{1_i}(t) \quad e_{1_i}(t) \quad \phi_{2_i}(t) \dots \phi_{(\alpha+\beta+\gamma)_i}(t)]' \quad (3.14)$$

Similarly define the control vector

$$\underline{u}_i(t) = [u_{1_i}(t) \quad u_{2_i}(t) \dots u_{(\alpha+\beta+\gamma)_i}(t)]' \quad (3.15)$$

In this case the state vector $\underline{x}_i(t)$ obeys the vector differential equation

$$\frac{d}{dt} \underline{x}_i(t) = \underline{A} \underline{x}_i(t) + \underline{B} \underline{u}_i(t) \quad (3.16)$$

where \underline{A} is the $[2(\alpha+\beta+\gamma)-1] \times [2(\alpha+\beta+\gamma)-1]$ matrix

$$\underline{A} = \begin{bmatrix} -\beta & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & -\beta \end{bmatrix} \quad (3.17)$$

and \underline{B} is the $[2(\alpha+\beta+\gamma)-1] \times (\alpha+\beta+\gamma)$ matrix

$$\underline{B} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} \quad (3.18)$$

It is important to realize that the above formulation reduces the problem of controlling the vehicles in the merging problem to the problem of controlling vehicles in a single string. The only difference is that this single string has the property that vehicles belong to the guideways A and B have the freedom of "moving over each" other as long as they are physically in different guideways prior to merging.

To illustrate this idea consider the situation shown in Figure 3. In Fig. 3(a) one has seven vehicles as they appear at the starting time t_0 . In Fig. 3(b) one sees the implications of a particular merging sequence after merger has occurred. Figure 3(c) shows the vehicles at $t = t_0$ as they appear on a fictitious "single-lane" guideway from the viewpoint of their distances from the merging point 0. It then appears that vehicle a_1 must "jump-over" b_1 , and b_2 must "jump-over" a_2 to obtain the merging sequence demanded by Fig. 3(b). This "jumping-over" phenomenon is allowed as long as it occurs prior to the reaching of the junction point of the vehicles involved.

The requirement of controlling the vehicles so that they merge according to a desired merging sequence while they are properly separated and travel near the desired velocity \bar{v} can be accomplished by defining a cost functional J_i (depending on the merging sequence) of the following quadratic type.

$$J_i = \int_{t_0}^{\infty} \left\{ q \left[\sum_{k_i=1}^{\alpha+\beta+\gamma-1} e_{k_i}^2(t) \right] + p \left[\sum_{k_i=1}^{\alpha+\beta+\gamma} \phi_{k_i}^2(t) \right] + r \left[\sum_{k_i=1}^{\alpha+\beta+\gamma} u_{k_i}^2(t) \right] \right\} dt \quad (3.19)$$

$$q > 0, \quad p \geq 0, \quad r > 0$$

The choice of this type of performance criterion has been discussed extensively previously^{1,2,3} and, for this reason, is not included here. In essence, it is the opinion of the author, that it accurately reflects any relative weighing desired in the broad specifications discussed in Section I under non-emergency operating conditions.

The form of the control-cost functional J_i of Eq. (3.19) leads to the definition of the $[2(\alpha+\beta+\gamma)-1] \times [2(\alpha+\beta+\gamma)-1]$ positive semidefinite matrix

$$\underline{Q} = \begin{bmatrix} p & 0 & 0 & \dots & 0 & 0 \\ 0 & q & 0 & \dots & 0 & 0 \\ 0 & 0 & p & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & p \end{bmatrix} \quad (3.20)$$

and of the $(\alpha+\beta+\gamma) \times (\alpha+\beta+\gamma)$ matrix \underline{R}

$$\underline{R} = r \underline{I} \quad (3.21)$$

Using these two definitions the cost functional J_i of Eq. (3.19) can be written in the form

$$J_i = \int_{t_0}^{\infty} \{ \underline{x}_i'(t) \underline{Q} \underline{x}_i(t) + \underline{u}_i'(t) \underline{R} \underline{u}_i(t) \} dt \quad (3.22)$$

It is well known⁴ that the control which minimizes the cost functional J_i is given by (under the above formulation) by

$$\underline{u}_i^0(t) = - \frac{1}{r} \underline{B}' \underline{K} \underline{x}_i(t) \triangleq - \underline{G} \underline{x}_i(t) \quad (3.23)$$

and that the minimum value J_i^0 of the cost functional J_i at $t=t_0$ is given by

$$J_i^0 = \underline{x}_i'(t_0) \underline{K} \underline{x}_i(t_0) \quad (3.24)$$

where \underline{K} is the unique positive definite solution of the matrix equation

$$-\underline{K} \underline{A} - \underline{A}' \underline{K} + \frac{1}{r} \underline{K} \underline{B} \underline{B}' \underline{K} - \underline{Q} = \underline{0} \quad (3.25)$$

The determination of the solution-matrix \underline{K} of the above equation can be accomplished in a straight forward manner using a digital computer (see, for example, Refs. [5] and [6]). Thus, given the number of vehicles, their drag coefficient β , and the weighting constants p, q , and r in the cost functional (3.19), one can determine the matrix \underline{K} ; this in turn defines the gain matrix \underline{G} (see Eq. (3.23))

$$\underline{G} \triangleq \frac{1}{r} \underline{B}' \underline{K} \quad (3.26)$$

It is important to emphasize that the matrices \underline{K} and \underline{G} are independent of the merging sequence, i.e. that these two matrices are completely independent of the index i which distinguishes one merging sequence from another.

The only effect that a particular merging sequence has is in the construction of the state vector \underline{x}_i and of the control vector \underline{u}_i . The precomputation of the gain matrix \underline{G} and the definition of the state vector \underline{x}_i for a particular merging sequence immediately define the best possible control $\underline{u}^0(t) = -\underline{G}\underline{x}_i(t)$ for that particular merging sequence. From a physical point of view, this means that in two distinct merging sequences, the measured state variables (position and velocity errors) are weighted by different combinations of the same feedback gains to produce the appropriate control accelerations or decelerations.

This completes the solution of the first posed problem, namely, how to control for any given merging sequence. The question of determining the best possible merging sequence is treated in the following section.

4. DETERMINATION OF THE BEST MERGING SEQUENCE

The fact that one can use quadratic performance criteria for controlling the vehicles for any given sequence implies that this method of control is satisfactory with respect to the broad specifications stated in section 1. Indeed, the simulation results reported in Refs. [1], [2] and [3] reinforce the validity of this type of criterion from the transient response point of view. Therefore, it is reasonable to use the same criterion to determine the best possible merging sequence.

As before, let i denote a particular merging sequence. Let $\underline{x}_i(t_o)$ denote the state vector induced by this sequence. If one uses the optimal control scheme described above, then (see Eq. (3.24)) the cost

$$J_i^o = \frac{1}{2} \underline{x}_i'(t_o) \underline{K} \underline{x}_i(t_o) \quad (4.1)$$

is the minimum possible control cost associated with the i -th merging sequence. Use of this scalar control cost can thus be used to decide which of two merging sequences is "better." Thus, consider another merging sequence with associated state vector $\underline{x}_k(t_o)$ at t_o , with $k \neq i$; the minimum control cost associated with this sequence is

$$J_k^o = \frac{1}{2} \underline{x}_k'(t_o) \underline{K} \underline{x}_k(t_o) \quad (4.2)$$

$$\text{If} \quad J_k^o < J_i^o \quad (4.3)$$

then one can deduce that the k -th merging sequence is superior to the i -th merging sequence, because the latter will have more errors and control efforts than the former.

Once one has accepted the ordering of the merging sequences on the basis of their associated minimum control costs, then it becomes a trivial matter to define the optimal merging sequence. This can be done as follows :

suppose that a merging sequence indexed by i^* has the property that

$$J_{i^*}^0 \leq J_i^0 \quad \text{for all } i = 1, 2, \dots, M \quad (4.4)$$

Then i^* indexes the optimal merging sequence.

To determine the optimal merging sequence one must merely compute the M scalars $J_1^0, J_2^0, \dots, J_M^0$ by

$$\begin{aligned} J_1^0 &= \frac{1}{2} \underline{x}_1^T(t_0) \underline{K} \underline{x}_1(t_0) \\ J_2^0 &= \frac{1}{2} \underline{x}_2^T(t_0) \underline{K} \underline{x}_2(t_0) \\ &\dots \dots \dots \\ J_M^0 &= \frac{1}{2} \underline{x}_M^T(t_0) \underline{K} \underline{x}_M(t_0) \end{aligned} \quad (4.5)$$

and select the smallest one. This procedure defines the optimal merging sequence indexed by i^* , its corresponding state vector \underline{x}_{i^*} , and the associated optimal feedback control scheme given by

$$\underline{u}_{i^*}(t) = -\underline{R}^{-1} \underline{B}^T \underline{K} \underline{x}_{i^*}(t) = -\underline{G} \underline{x}_{i^*}(t) \quad (4.6)$$

In this manner, one has a systematic computerized solution to the selection of the best possible control sequence and of the control system which will insure that it takes place in a smooth and efficient manner.

5. DISCUSSION

The solution to the posed problem involves a certain amount of on-line computation that must be performed at the command and control center. The computational aspects of the problem will be discussed below.

One of the major simplifying factors inherent in this problem is that the \underline{K} matrix and the gain matrix \underline{G} are independent of the actual merging sequence. Thus, knowledge of

- (a) the total number of trains $N = \alpha + \beta + \gamma$ in the three guideways,
- (b) the mass m and drag coefficient β of each train, and
- (c) the weighting factors p , q , and r in the cost functional (3.19)

is sufficient to generate \underline{K} and \underline{G} . Thus, these two matrices can be pre-computed and stored in the computer memory.

The determination of the best merging sequence, based on the comparison of the control costs (4.5), requires the formation of the state vectors $\underline{x}_i(t_0)$, $i = 1, 2, \dots, M$ where $M = (\alpha+\beta)!/\alpha!\beta!$. In essence, each merging sequence assigns an index to each vehicle and defines which vehicles will be adjacent after the merging has been completed. The state vector $\underline{x}_i(t_0)$ is then formed by first measuring the position and velocity errors using Eqs. (3.9) and (3.10) and then interlacing them as indicated by Eq. (3.14). Thus the formation of the state vector for each merging sequence requires a total of $2N-1$ additions. The determination of each control cost J_i^0 requires a total of $(2N-1)^2 + 2N - 1$ multiplications and $(2N-1)^2 + 2N-2$ additions. Thus to evaluate the control cost for each merging sequence one needs to perform

$$\begin{array}{ll} 4N^2 - 2 & \text{additions} \\ 4N^2 - 2N & \text{multiplications} \end{array}$$

Since there are M possible merging sequences then it follows that the

maximum number of operations required to determine the best merging sequence is

$$M(4N^2 - 2) \text{ additions}$$

$$M(4N^2 - 2N) \text{ multiplications}$$

As an example, suppose that one deals with a total of 12 vehicles ($N=12$) five in guideway A five in guideway B ($\alpha=\beta=5$) and two in the main guideway C. Then, the total number of the possible merging sequences is

$$M = \frac{(5+5)!}{5!5!} = 252$$

and so to determine the best merging sequence one needs

$$\text{total number of additions} = 252 \times 574 = 144,648$$

$$\text{total number of multiplications} = 252 \times 552 = 139,104$$

Assuming* that each floating point addition operation requires, on the average, 10 microseconds and each floating point multiplication requires 5 microseconds, it is easy to deduce that in this example one can determine the best merging sequence in less than 3 seconds of computer time by comparing all possible 252 merging sequences. Of course, any setting of priorities and/or additional statistical information regarding the position and velocity error distribution can be used to drastically reduce the merging sequences that should be considered and, hence, the required computation time. It therefore appears that the computational requirements for optimal merging are not unrealistic with respect to current computer technology.

*These figures are typical for the IBM 7094 digital computer.

This problem has been formulated and solved under the assumption of continuous-time measurements and control. The identical procedure can also be used in the sampled-data version of this problem,⁸ i.e., when the state variables are measured in discrete instants of time and the optimal accelerations and decelerations applied are constant throughout the sampling interval. The reason is that in the sampled-data problem one still has a quadratic form which determines the minimum control cost.

6. CONCLUSIONS

An orderly and systematic procedure has been proposed for the control of high-speed vehicles during the merging phase through the use of optimal control theory. It has been demonstrated that the computations required to determine the best merging sequence are not unrealistic.

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FIGURE CAPTIONS

Figure 1 Visualization of the merging problem and the indexing of the vehicles.

Figure 2 Four distinct possible merging sequences of six vehicles, initially equally distributed in guideways A and B, and their associated ordered sets. In this case, there are 20 possible merging sequences.

Figure 3 (a) Initial vehicle distribution; (b) desired merging sequence; (c) equivalent "single-guideway" control problem.

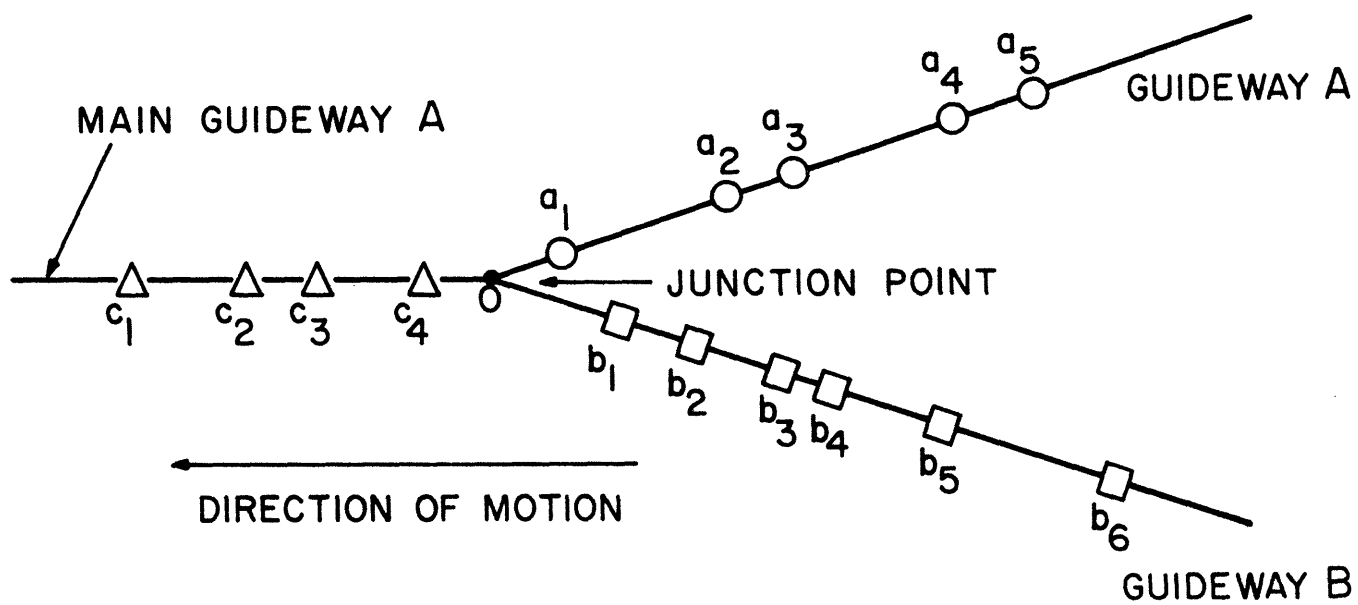
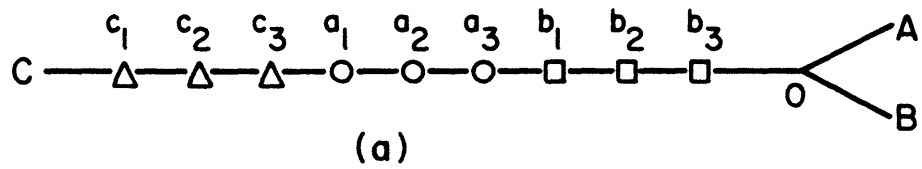
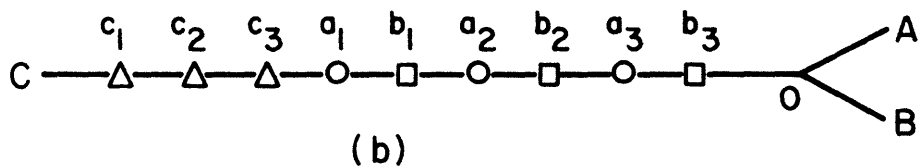


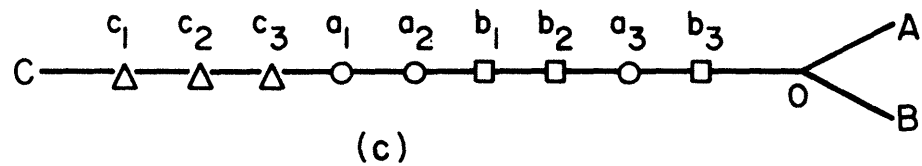
Figure 1



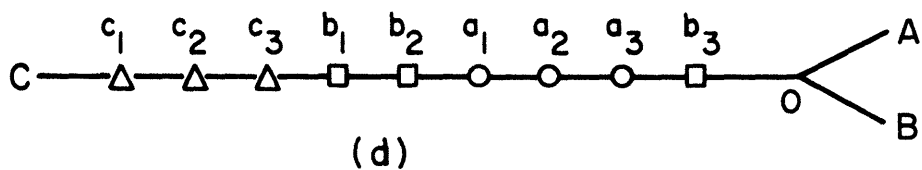
$$P_a^0 = \{c_1, c_2, c_3, a_1, a_2, a_3, b_1, b_2, b_3\}$$



$$P_b^0 = \{c_1, c_2, c_3, a_1, b_1, a_2, b_2, a_3, b_3\}$$



$$P_c^0 = \{c_1, c_2, c_3, a_1, a_2, b_1, b_2, a_3, b_3\}$$



$$P_d^0 = \{c_1, c_2, c_3, b_1, b_2, a_1, a_2, a_3, b_3\}$$

Figure 2

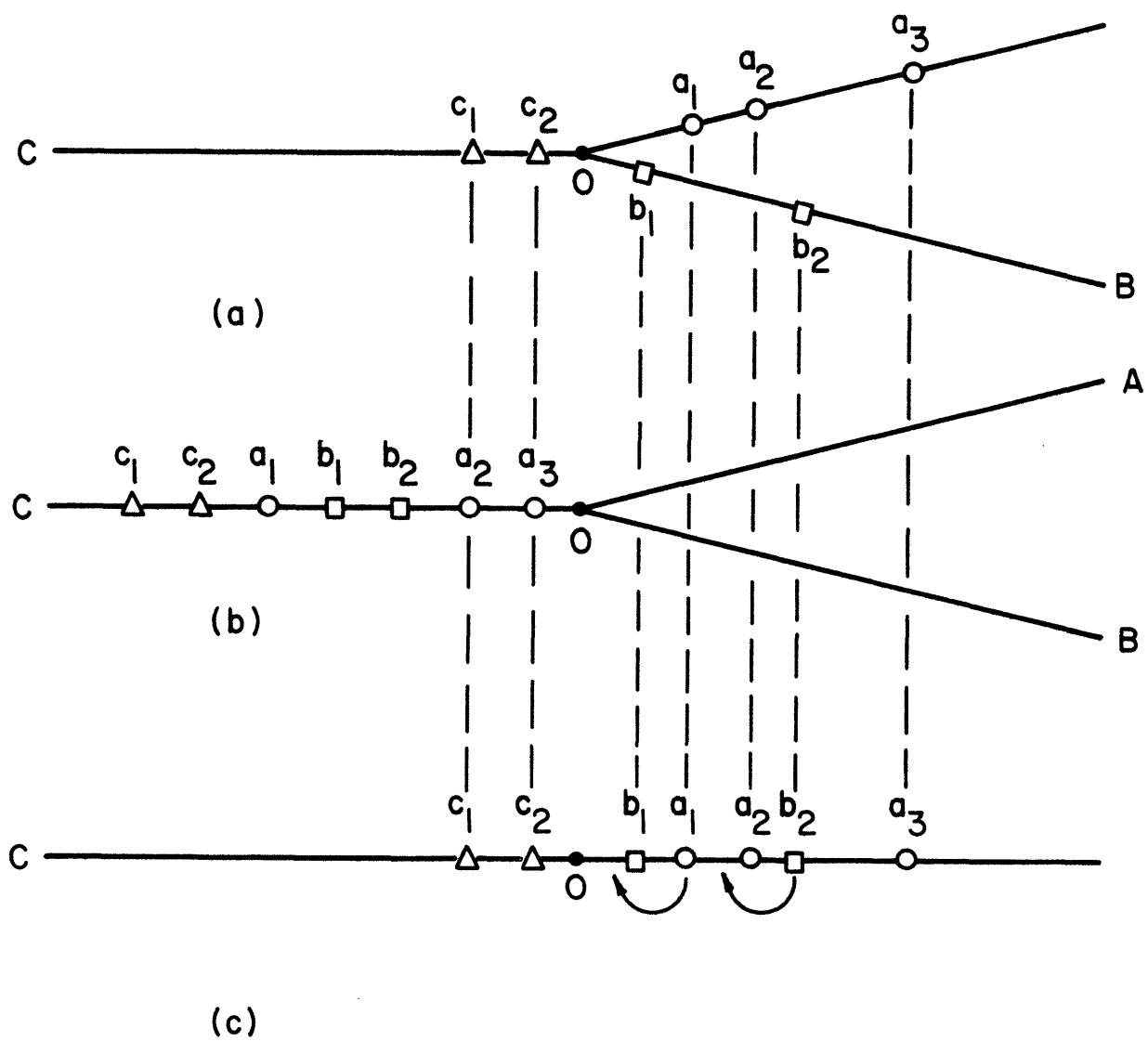


Figure 3