

A Need for Speed:

A New Speedometer for Runners

by

Ravindra Vadali Sastry

Submitted to the Department of Electrical Engineering and Computer Science

in Partial Fulfillment of the Requirements for the Degrees of

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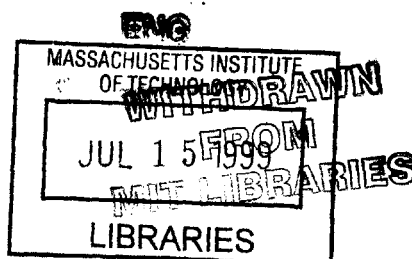
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Author _____
Department of Electrical Engineering and Computer Science
May 26, 1998

Certified by _____
Michael J. Hawley
Alex Dreyfoos Assistant Professor of Media Technology
Thesis Supervisor

Accepted by _____
Arthur C. Smith
Chairman, Department Committee on Graduate Theses



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ABSTRACT

A new method for measuring speed and energy expenditure during locomotion is presented. This method is based on physics and a simple energy balance. It is robust to the many individual variations, including running style and fitness level, that make accurate measurements with conventional devices impossible. Several methods of device calibration are discussed, including a biomechanical method that requires no trial runs.

Thesis Supervisor: Michael J. Hawley

Title: Alex Dreyfoos Assistant Professor of Media Technology, MIT Media Laboratory

1.0 Introduction

Millions of Americans run regularly; for their health, for recreation, and competitively. One of many advantages of running is that it requires no equipment – only shoes. The runner is not tied down to an exercise machine and has the freedom to enjoy the outdoors. With this freedom comes one disadvantage: runners have very little information about their workouts. Exercise equipment like ergometers and bike computers offer the user a wide range of information, both during and after the workout. People can know their speed and calorie burn at all times. Clearly, runners would like to have this information but not at the cost of having to run on a treadmill, the only real alternative.

The popularity of heart rate monitors is a testament to the widespread demand for feedback during exercise. These monitors give users a measure of how hard they're working, so that they can maintain their exertion in some desired range, and also let them track the improvement in their fitness level over time. Although heart rate is certainly a useful thing to know, it is not what runners are most interested in. It just happens to be something that is easily measured.

There are three things that runners really want to know about their runs:

- Speed
- Distance
- Energy expenditure

Experienced runners usually can make fair estimates of how fast they are going, but less experienced recreational runners lack this skill. Additionally, any intuitive estimate is only a fuzzy measure of speed – there will be a large window for error. Estimates of

distance are similar to speed estimates. Experience allows for increasingly accurate guesses of distance, but never to any real level of accuracy. At best, runners do know how far their total run is, since many running courses are measured.

Energy expenditure is the most problematic of the three. No intuitive measure is possible, and those devices that purport to measure energy expenditure all do so based on broad statistical models that may only be minimally relevant to a particular person. Some people run on treadmills, just so that they can get this information. Unfortunately, the number of calories burned that is reported by the treadmill is based only on the speed and the weight of the user. It is clear that many approximations are made to allow such an estimate, and these approximations inevitably invalidate the final results to some extent.

The needs of runners are not being satisfied, although several products are available that attempt to address these needs. Several of these are discussed, followed by a description of the basis for a new approach to measuring speed, distance, and energy expenditure.

2.0 Current Approaches

Much research has been devoted to the development of activity monitoring devices for runners and walkers. Many such products are currently available, offering varying degrees of convenience and reliability. None fully satisfy the need for precise, real-time measurements of speed, distance, and energy expenditure. Two popular types of conventional devices are discussed, followed by an evaluation of a specific, commercialized ambulatory monitoring device.

2.1 Pedometer

The dominant ambulatory monitor is the pedometer. First envisioned by Leonardo da Vinci about 500 years ago, the pedometer is intended to measure the number of steps taken by the wearer [1]. Although the pedometer depends on acceleration, it is not an accelerometer in the true sense. Steps are counted in response to vertical acceleration of the body, which causes a lever arm to move vertically and a ratchet to rotate [2]. A threshold acceleration can be set by careful adjustment of the spring, and very accurate step counts are obtained.

If step length is known, then speed and distance can be calculated with high accuracy.

$$distance = stepcount \times steplength$$

$$speed = \frac{distance}{time}$$

However, step length varies greatly by person and across the range of walking and running speeds. As expected, leg length is highly correlated with step length, but the

relationship is not deterministic: there is significant variation in the step lengths of subjects whose leg lengths are identical.

Conventional wisdom holds that step length is constant during walking and variable during running, suggesting that pedometers will perform well for walkers and poorly for runners. This conclusion turns out to be correct, although not for this reason.

Step length varies across walking speeds just as it varies across running speeds. Pedometers work well for walkers only because people do not vary their speed very much while walking. For healthy people, typical walking speed is $3 \text{ mph} \pm .5 \text{ mph}$. It is extremely uncommon for someone's preferred walking speed to be outside this range, and people can maintain their preferred speed very closely over long distances. If the step length for a particular person at their preferred walking speed is known and entered into the pedometer, a very accurate measure of distance will be obtained.

Running presents a much greater challenge. Normal running speeds range from 5 *mph* (12 minutes per mile) to 12 *mph* (5 minutes per mile). Step lengths obviously vary greatly over this speed range. However, the critical difference between running and walking is that runners are very bad at maintaining constant speed. Without constant speed, the constant step length assumption is invalid and the step count measured by the pedometer is meaningless. Large errors result when a representative step length is used for the purpose of estimating distance.

Pedometers are capable of giving acceptable distance measurements for walking and poor estimates of distance for running. They can also provide accurate measurements of step frequency which allows speed to be estimated using the following relation.

$$\text{speed} = \text{steplength} \times \frac{1}{T_s} = \text{steplength} \times f_s$$

T_s is the time taken for each stride, and f_s is the stride frequency. The fundamental problem is that stride length is a function of speed, and varies from subject to subject. The expression is more meaningful when this dependence is made explicit.

$$speed = L_s(speed) \times f_s$$

L_s is the stride length. The problems associated with speed measurement and distance measurement are identical. The calculations do have a physical basis, however. There is no such basis for estimates of the energy cost of locomotion.

The best pedometers employ empirical relations between energy expenditure and step count, producing good results only when the assumptions of the empirical model are satisfied. Since there is no theoretical basis for these models, they can not be expected to perform well in all situations. Pedometers are “designed to count steps in walking and possibly running; they should not be expected to measure other kinds of activities or total energy expenditure [2].”

2.2 Actometer

A less familiar monitoring device is the actometer. This is simply a self-winding wrist watch from which the escape mechanism has been removed [3]. The rotor is directly connected to the hand. Any accelerations that occur in the plane of the face of the watch will result in movement of the hand. An actometer records not only the fact of movement (as does the pedometer, in counting steps) but also its intensity. The rotor turns faster as the intensity of the motion increases. Since this somewhat arbitrary motion metric does not directly correspond to any gait-related parameter, purely empirical means must be used to establish a relation between the two. The drawbacks of this approach

have already been established. Without a theoretical basis, no model can account for all behaviors and regimes of operation.

2.3 Accuracy of Pedometer and Actometer

The following results, obtained by Saris, demonstrate the inadequacy of these two devices. Over the range of walking speeds, average stride length varied from 0.82 meters, at the lowest experimental speed, to 0.98 meters at the highest. Many people walk at speeds lower than those tested by Saris, resulting in average stride lengths of less than 0.70 meters. If a constant walking speed is maintained, then satisfactory results will be obtained, but the assumption that walking stride lengths are constant across the range of speeds can be seen to be somewhat erroneous: there is at least a 30% variation in stride length.

Running resulted in average stride lengths from 0.79 meters to 1.40 meters. A pedometer is obviously not robust to these changes in gait. An actometer could achieve results superior to those of the pedometer, but is still insensitive to the differences between running and walking, and it can not handle gaits that do not fall within certain bounds assumed by the empirical model. Another problem is the correlation between the pedometer and actometer results and the energy expenditure [4].

The following table and figures summarize some of the experimental findings.

Gait	Speed (km/hr)	f_s (strides/min)	Pedometer (units/min)	Actometer (units/min)
Walking	5.8	118	125	4.0
	7.2	130	137	5.4
	8.6	146	206	7.7
Running	7.2	152	213	12.2
	8.6	157	202	12.8
	10.1	158	169	13.9
	13.3	168	212	17.0
	14.8	176	76	20.4

Table 2.1: Accuracy of Pedometer and Actometer. From [3].

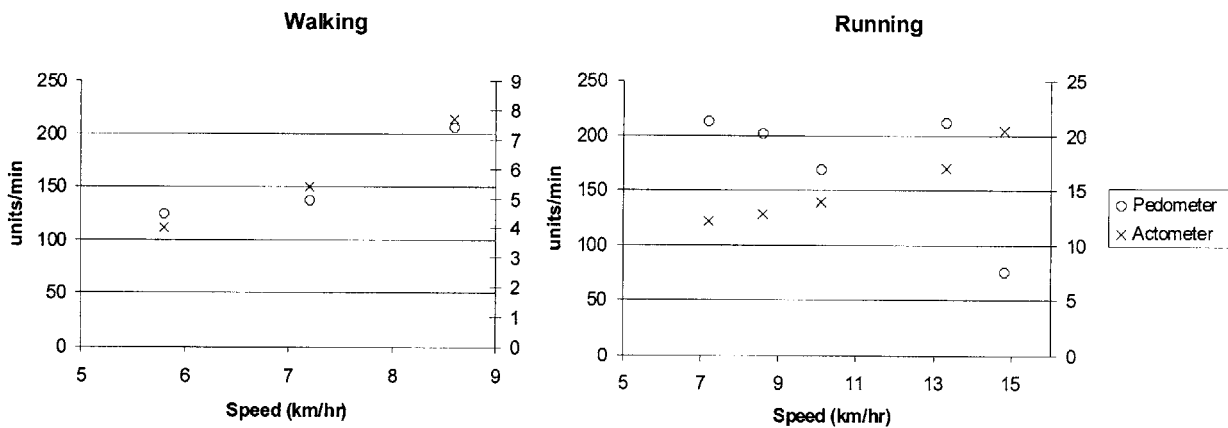


Figure 2.1: Accuracy during walking and running. From [3].

The results in figure 2.1 show that pedometers and actometers are both very good at measuring activity during walking. This is consistent with expectations. The running results are less satisfying. The pedometer performed terribly, indicating a downward trend in activity as speed increased. The actometer results were much better, showing the correct, upward trend in activity. It should be noted, however, that the arbitrariness of the actometer metric makes it incredibly hard to calibrate for an individual even though it has

acceptable performance when averaged across several subjects. Unlike the pedometer, which is calibrated with the stride length of the subject, the actometer calibration has no physical meaning. These two devices are representative of traditional activity monitors, and neither achieves satisfactory performance.

2.4 BioTrainer: A Commercialized Activity Monitor

The BioTrainer is a product marketed by Individual Monitoring Systems of Baltimore, MD. The marketing brochure makes it out to be a rather impressive piece of equipment. “This personal exercise monitor, quite different from the simple step pedometer, is a sophisticated monitoring instrument that measures and records the physical movements of your body.” It uses “advanced motion sensing technology” and the sensor in the BioTrainer is “of the same type used by NASA.” The BioTrainer is “the perfect companion for walkers and joggers” because it “precisely computes calorie burn based on body weight.”



Figure 2.3: BioTrainer. \$89.

In reality, this device is nothing more than an actometer where the mechanical components have been replaced with a solid-state accelerometer. This is certainly an improvement over the original, but does not represent any breakthrough in activity monitoring. The sensor “of the same type” that is used by NASA is the accelerometer itself. The “advanced motion sensing technology” is an algorithm that records the magnitude of the accelerometric signal; exactly analogous to the mechanically-implemented actometer algorithm. The performance of the BioTrainer is consistent with the performance of mechanical actometers.

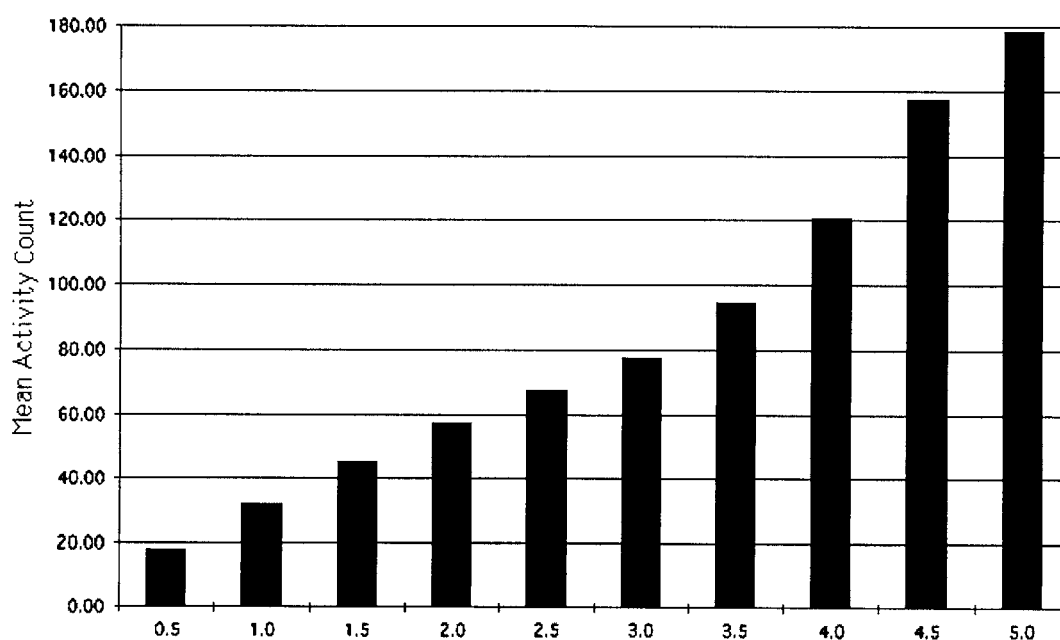


Figure 2.4: BioTrainer measurement vs. walking speed (mph)

The BioTrainer makes no attempt to calculate speed or distance, rendering its claim of being the “perfect companion for walkers and joggers” invalid. However, it can apparently compute calorie burn “precisely.” Since this device is nothing more than an actometer, its ability to compute energy expenditure is no improvement. The only

information that the BioTrainer has is the activity count which it measures, and the body weight of the subject. It must employ an empirical formula to predict calorie burn, a formula that has no way of accounting for any of the non-weight variations among subjects. This is anything but precise. Unfortunately, it accurately reflects the current state of activity monitoring.

3.0 A New Approach

The shortcomings of conventional activity monitors are not due to poor design or device malfunction. For the most part, these devices are technically sound and perform exactly as intended by the manufacturer. Their performance is unsatisfying, however. If a runner is interested in speed, distance, and energy, then the best conventional devices will provide:

- estimates of speed based on the flawed assumption of constant stride length
- estimates of distance based on the flawed speed estimate
- estimates of energy expenditure based on the speed estimate and a statistical relationship to body weight

The error in the speed estimate will be propagated into the distance and energy estimates. The energy estimate also has no physical basis: it is just a best fit based on an “average” person.

A new approach that avoids invalid assumptions and arbitrary energy cost functions would address these issues and provide a true advance in activity monitoring. Such an approach is discussed, followed by some technical considerations regarding implementation.

3.1 Theory

100 years ago, Zuntz addressed the question of whether it was more energy efficient to run on two or four legs. His were the first measurements of metabolic energy consumption in running animals. He discovered that horses use less energy than humans to move a unit body weight a unit distance. Zuntz hypothesized that there was an

energetic benefit to quadrupedal locomotion, as compared to bipedal locomotion. To test his theory, he performed measurements of oxygen consumption in running dogs and found that dogs used more energy per unit body weight than both horses and humans. After closer examination, Zuntz discovered that the energy cost of locomotion was not related to limb number but rather, to body weight itself. He found that the cost of moving a unit body weight a unit distance was related to the animal's body weight: small animals use more energy to run a given distance than do large animals [5].

More recent experiments by Taylor on a diverse group of quadrupedal mammals have established that the amount of energy required to move a unit body weight a unit distance decreases in direct proportion to body weight.

$$\frac{E_{metab}}{W_b} \propto W_b^{-0.31}$$

E_{metab} is the cost of locomotion and W_b is body weight [6]. This result was later extended to bipeds: ostriches and ponies of the same size use the same amount of energy to run [7]. Even within the same individual, the cost of running is independent of the number of limbs. Chimpanzees trained to run on either two or four legs show no difference in energy cost between the two gaits [8].

Force plate measurements of the forces exerted by the foot during running reveal another interesting point: the vertical component of the force is at least 10 times larger than the horizontal (forward) component. This finding implies what turns out to be a highly accurate first-order approximation: the energy cost of running is equal to the vertical work the leg muscles do to oppose gravity. This agrees with the finding that the cost of locomotion is independent of gait (bipedal vs. quadrupedal). All that matters is the body weight of the animal, since the total vertical work done by the animal must equal

the negative work done by gravity. If this were not the case, then the animal's center of gravity would have a net vertical displacement during running. The work done by gravity is proportional to body weight. The vertical work done by the animal must be done while its foot is in contact with the ground. This leads to the critical relation:

$$\frac{1}{W_b} \cdot \frac{\partial E_{metab}}{\partial t} = c \cdot \frac{1}{t_c}$$

where W_b is body weight in Newtons, E_{metab} is energy cost of locomotion, t_c is foot contact time, and c is a cost coefficient measured in Joules/Newton [9]. Thus, the rate of energy expenditure can be calculated if body weight and contact time are known.

To verify this relationship, two sets of experiments were performed. First, Kram and Taylor verified that the weight-specific cost of locomotion was proportional to running speed.

$$\frac{1}{W_b} \cdot \frac{\partial E_{metab}}{\partial t} \propto speed$$

They then showed that the rate of force application, which is proportional to the inverse of contact time, was also proportional to running speed.

$$\frac{1}{t_c} \propto speed$$

Both of these relations were verified in a diverse group of animals consisting of kangaroo rats, squirrels, hares, dogs, and ponies. The second relation, between contact time and speed, yields a method of calculating speed if contact time is known. Since pace - the amount of time required to run a fixed distance - is the inverse of speed, this relation can be rewritten as:

$$pace \propto t_c$$

Pace is a much more meaningful quantity to a runner: knowing someone ran a four minute mile means more than knowing they ran 15 *mph* for a distance of one mile. A relation between distance and contact time is implicit:

$$distance = speed \cdot time \propto \frac{1}{t_c} \cdot time$$

The shortcomings of the pedometer and actometer approximations are avoided by the use of this physics-based method. To calculate speed, distance, and energy expenditure, only body weight and contact time need to be measured.

3.2 Implementation

Body weight is a trivial parameter, but contact time proves to be a very difficult quantity to measure. The first attempt to do so was carried out by Marey over 100 years ago. Marey equipped his subjects with a clockwork recorder for making records of walking and running. Air chambers in the shoes communicated with the recorder via rubber tubes. A rubber bulb was held in the hand to start the pen recorder. The runner also carried an accelerometer on his head, although this was probably an afterthought [10].

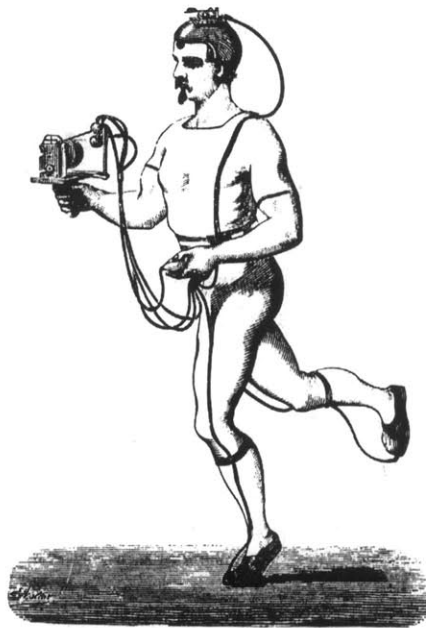


Figure 3.1: Marey's instrumented runner

The contact time measurements used by Kram and Taylor were made through the use of force plates. These are sensitive electronic scales that can accurately transduce a large range of forces while undergoing a negligible vertical displacement. Although highly accurate, force plates are very expensive and bulky. They are useful only in a research setting. A consumer device that measures contact time would have to be much cheaper and much smaller. It would have to be wearable, like conventional pedometers and actometers.

A device developed by FitSense Technology, Inc. of Wellesley Hills, MA addresses this problem. This device is compact, light, and can be worn on the shoe while walking or running. It contains a solid-state accelerometer and a microprocessor. By utilizing a painstakingly developed proprietary algorithm, the processor is able to analyze

the accelerometric waveforms, determine whether the subject is walking or running, and produce a highly accurate measure of contact time – all in real time.

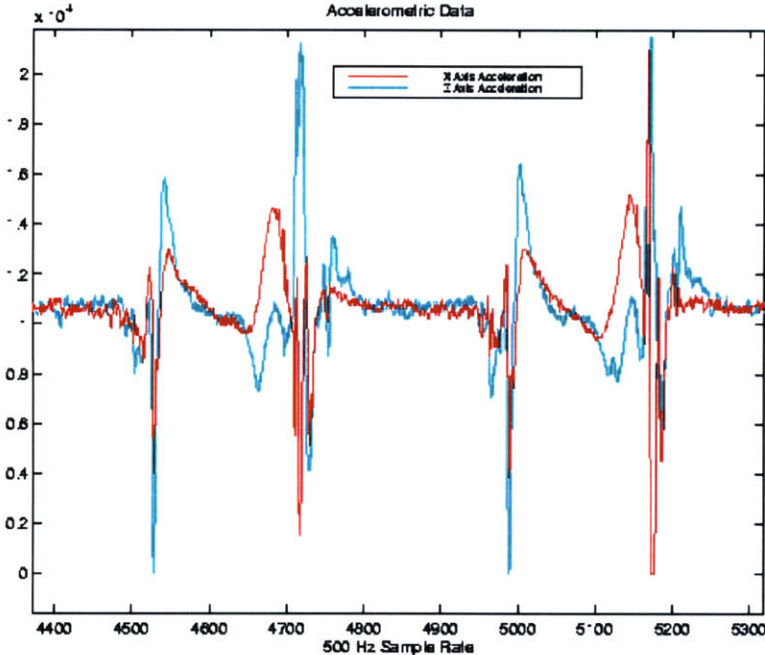


Figure 3.2: Accelerometric Data for Walking (2 strides are shown)

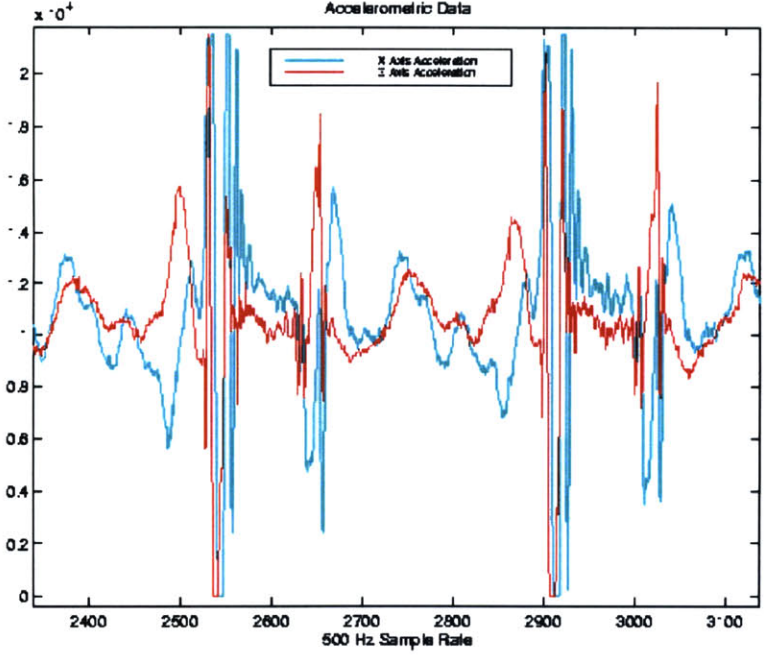


Figure 3.3: Accelerometric Data for Running (2 strides are shown)

4.0 Calibration Methods

If body weight is known and contact time can be measured accurately, all that remains unknown in the above relation is the constant of proportionality. This constant can be found by measuring the slope of a graph of contact time vs. pace.

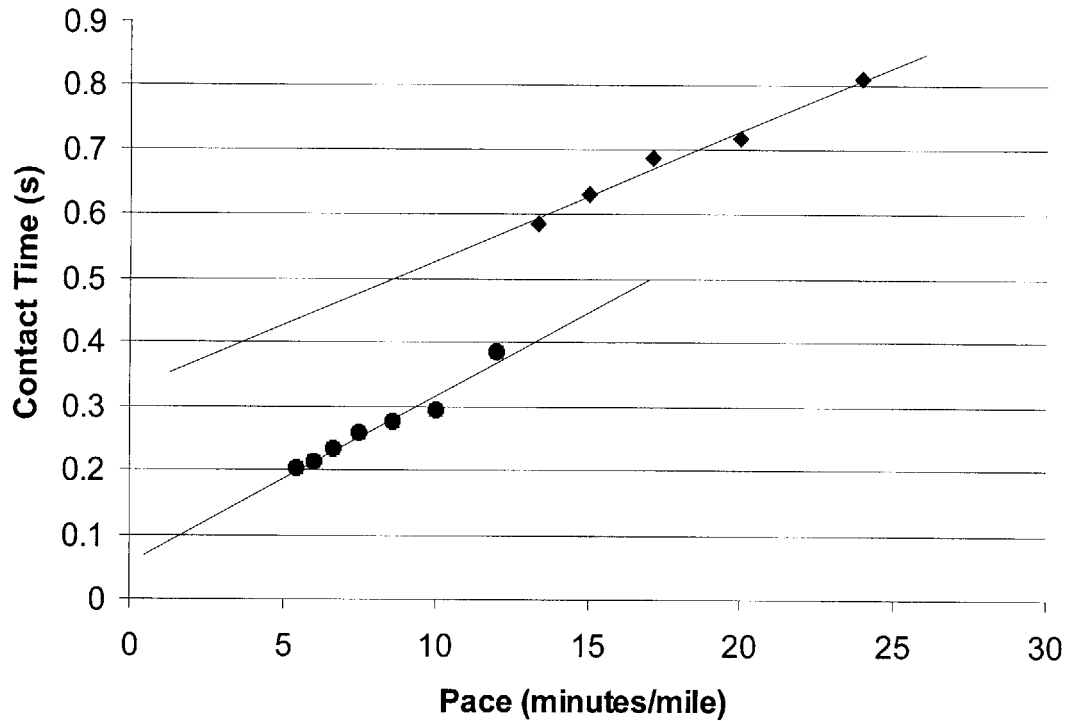


Figure 4.1: Linear t_c -Pace relation for one subject.

The prediction of a linear relationship between contact time and pace is verified, but it is important to note that both the line for walking (◆) and the line for running (●) have positive y-intercepts. Neither line passes through the origin, as predicted by the strictly proportional model of Kram and Taylor. Calibration of the device therefore requires two constants for each gait, instead of the one cost coefficient c suggested by the proportional model.

The obvious method for determining these constants would be to take several measurements, as in figure 4.1, and perform a linear fit to extrapolate the slope and y-intercept. Although this would yield the most accurate results, it is not optimal from a consumer standpoint. If a user took the device out of its box and read in the instructions that 12 trials at varying speeds were necessary to calibrate the device, it could easily be decided that the device was simply too hard to use. The perceived value of the speed, distance, and energy measurements must exceed the perceived difficulty of calibrating the device. Therefore, the goal is to determine the minimal calibration protocol that will still yield accurate measurements.

4.1 Two Point Calibration

The easiest simplification would be to exploit the high linearity of the curves. Only two points are needed to define a line. More data points are only necessary when the error in the individual measurements propagates and causes significant error in the extrapolated line fit. Since the contact time measurements made by the FitSense device are very accurate, the propagated error is not significant. If a subject's characteristic contact time is t_{c1} at pace p_1 and t_{c2} at pace p_2 , and there is a contact time measurement error of $\pm\delta$, the resulting linear fit could be anywhere in the shaded area in figure 4.2.

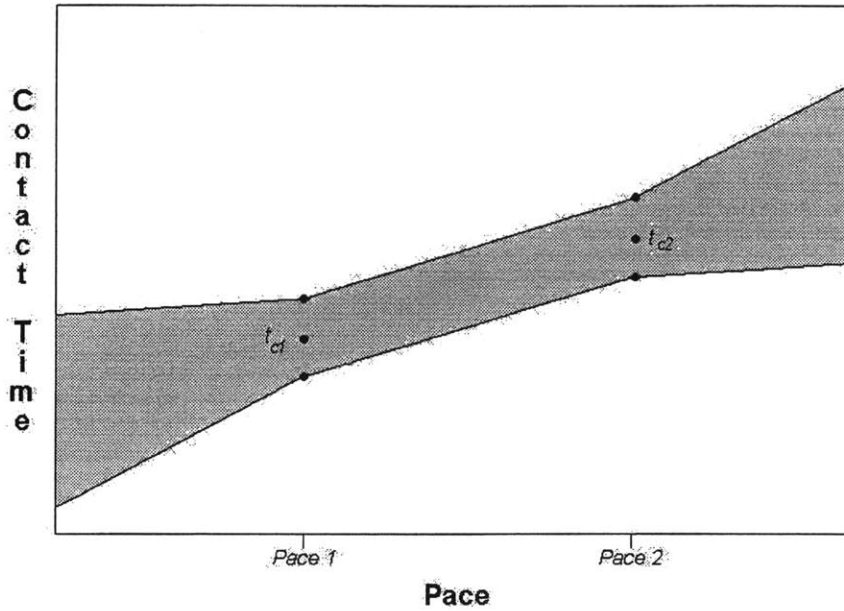


Figure 4.2: Propagation of measurement error

The relative magnitude of the measurement error δ is greatly exaggerated for the purpose of illustrating the effect of measurement error on the linear fit. The slope of the linear fit of the data in figure 4.2 is given by:

$$m_{measured} = \frac{\Delta t_c}{\Delta pace} = m_{actual} \pm \frac{2 \cdot \delta}{\Delta pace}$$

The error term can be minimized by reducing δ , which is the job of the device designer, and by maximizing $\Delta pace$. This is achieved by conducting one of the trials at a very low speed and forcing the other to a high speed.

4.2 One Point (“Pivot” Point) Calibration

To simplify calibration further, another of the trial runs must be eliminated. To define a line, however, two points are required. If only one trial is conducted, only one

point will be obtained. The other point is obtained by making a generalization based on a large set of data. The data are shown in figure 4.3.

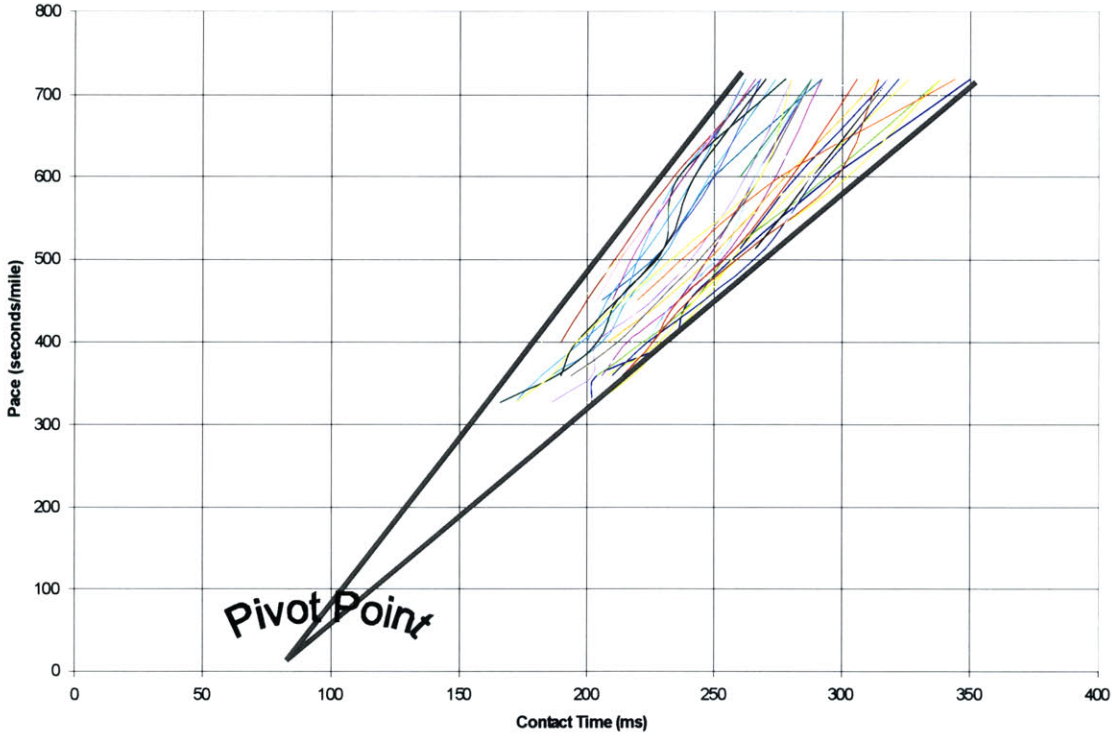


Figure 4.3: Linear t_c -Pace relations for 38 subjects

The data in figure 4.3 were obtained from 3 days of experiments, described in chapter 5. The axes in figure 4.3 are reversed, when compared to figures 4.1 and 4.2. That is why figure 4.3 implies a negative y-intercept even though the previous two figures showed a positive y-intercept.

Most notable, however, is the apparent convergence of all 38 lines at a single point: the “pivot” point. The exact coordinates of this point can be extrapolated from the data set. The physical meaning of the “pivot” point is unclear. The pace is very close to zero, implying a near infinite speed. The existence of the “pivot” point allows the calibration protocol to be reduced to one trial, to obtain the second point.

The apparent simplicity of the “pivot” point calibration comes at the cost of accuracy. The “pivot” point is calculated from a broad data set, and is not subject specific. Subjects who differ significantly from the average would get less accurate results than those who are close to the average. Despite the simplicity of the “pivot” point method, the error it introduces motivates a continued search for an optimal calibration protocol.

4.3 Biomechanical Calibration

The ideal calibration method would be one that required no test runs at all – a zero point calibration. Instead of measuring a subject’s gait while they are running, such a method would require the *a priori* prediction of the gait parameters. To do this, some relation, either statistical or theoretical, must be found that relates a subject’s gait parameters to any of their measurable biomechanical parameters. Examples are height, weight, age, gender, leg length, etc. If such a relation were discovered, calibration would be reduced to simply entering the relevant biomechanical data into the device. No test runs would be necessary.

5.0 Experimental Results

In the fall of 1998, a battery of experiments were performed in conjunction with FitSense Technology and the Biomechanics Laboratory of the Sargent College of Health and Rehabilitation Sciences, Boston University. 38 Subjects were recruited to perform a battery of walking and running trials on a specially instrumented treadmill equipped with a force plate. Their speeds and contact times were recorded by a computer that was linked to the treadmill. A variety of biomechanical data were also recorded for each subject. These data consisted of: gender, height, weight, age, shoe size, self-reported inseam, measured inseam, pivot length (distance from hip pivot to ground), hip height (distance from iliac crest to ground), calf circumference, thigh circumference, and vertical leap. The hope was that a link between biomechanical parameters and gait parameters would be found.

5.1 Equipment

A Gaitway treadmill fitted with Kistler force plates was used for data collection. The treadmill's A/D converter connected directly to a data logging computer. In order to alleviate the inherent awkwardness some subjects experience when running at high speeds on a treadmill, a video camera was aimed at the runners' feet, feeding a television monitor at eye level. It was hoped that this feedback would assure the subjects that they were not in danger of falling off the back of the treadmill so that they could remain relaxed during the entire battery of trials.

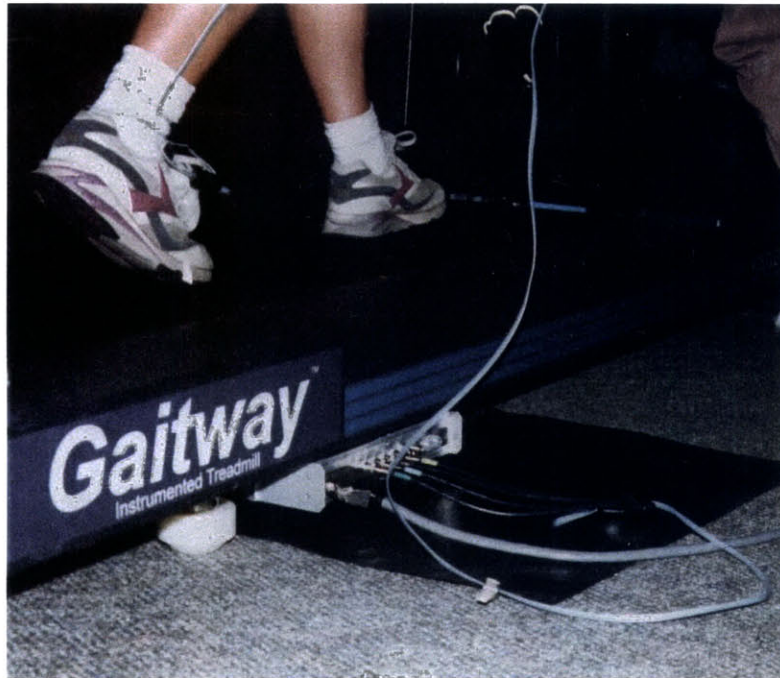


Figure 5.1: A/D converter output of Gaitway treadmill.



Figure 5.2: Data logging computer and operator.

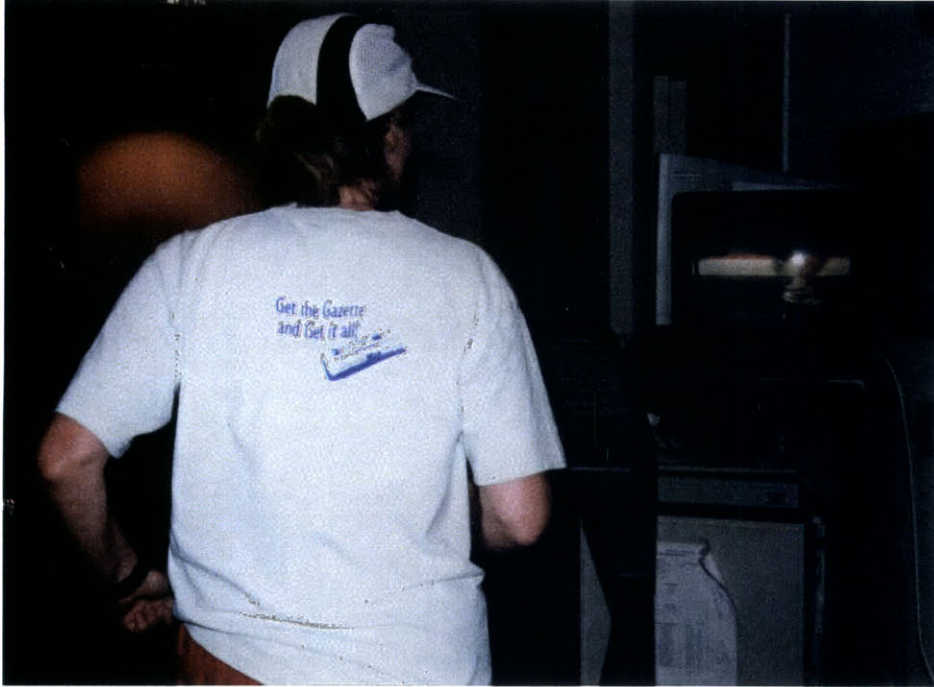


Figure 5.3: Television monitor showing subject's feet.

5.2 Protocol

Upon arrival, each subject filled out a survey reporting their gender, height, weight, age, shoe size, and self-reported inseam. Measurements of inseam, pivot length, hip height, calf circumference, and thigh circumference were then made by the author and other FitSense Technology personnel. Treadmill trials followed, after which vertical leap was measured.

Each treadmill trial consisted of 20-30 strides at a fixed speed. Walking trials came first, in *.5 mph* increments from *2.5 mph* to *4.5 mph*. Running trials followed, in *1 mph* increments from *5 mph* to the subject's maximum comfortable speed, typically between *9* and *12 mph*. During each trial, contact times were measured and averaged across the trial, resulting in one measured contact time for each trial. These average contact times were plotted against pace to yield linear fits for walking and running. Each

gait is characterized by two parameters, slope and y-intercept, resulting in four total gait parameters measured for each subject.



Figure 5.4: Walking trial in progress. Author is standing.



Figure 5.5: Running trial in progress.

5.3 Analysis

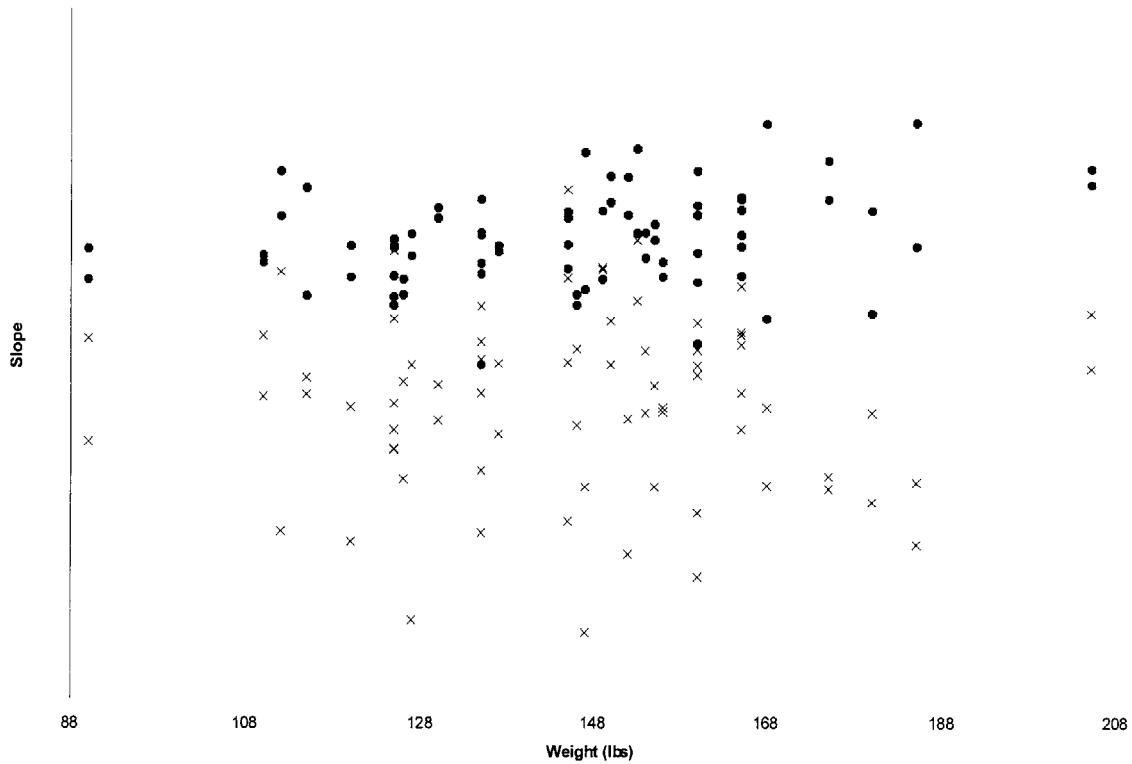


Figure 5.6: Slope parameter vs. subject weight.

Figure 5.6 shows the relationship between the slope of the t_c – pace relationship and body weight. The filled circles (●) represent running and the crosses (×) represent walking. It is clear that no linear trend exists between slope and weight. Further analysis reveals that the scatter of the data is too large to fit any significant nonlinear model as well. This analysis was repeated for all of the biomechanical data, with equally unsuccessful results. No linear or nonlinear models could be matched to the data.

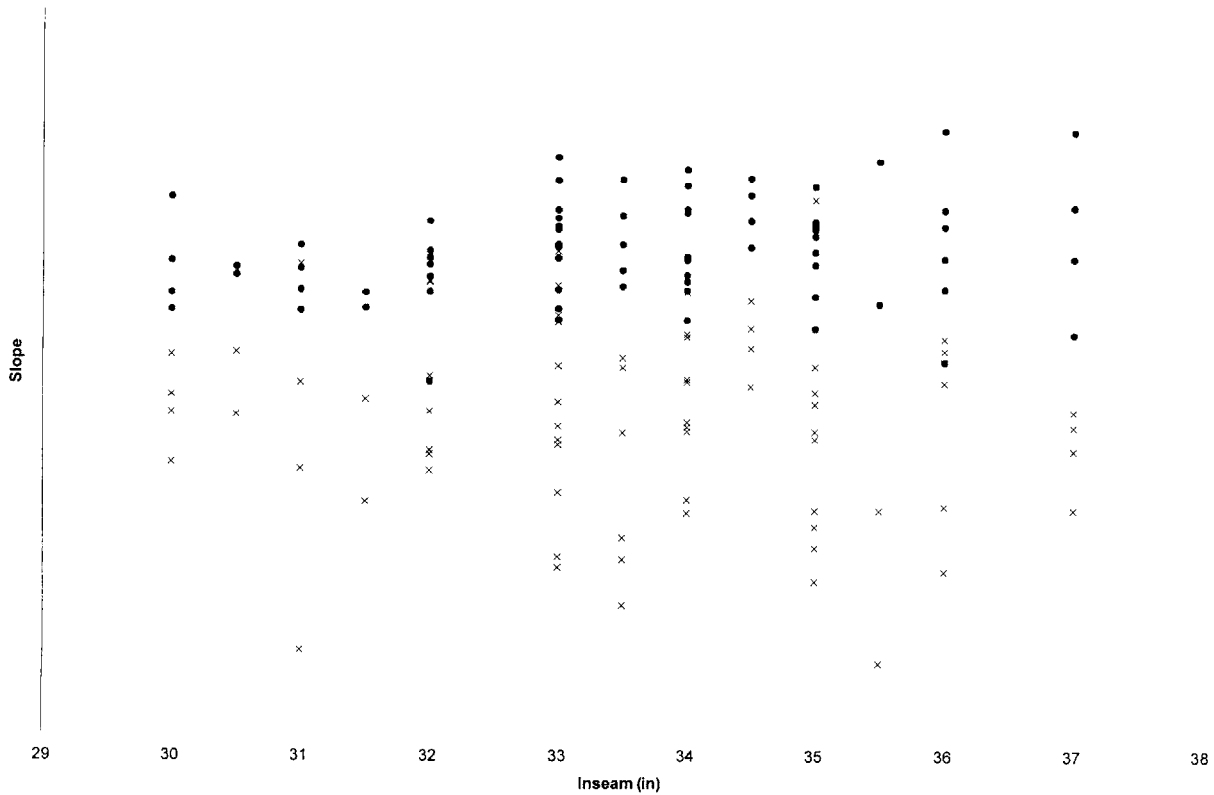


Figure 5.7: Slope vs. Inseam.

A number of biomechanical parameters (height, inseam, pivot length, hip height) were measured because they relate closely to stride length, which it was hoped would be a primary determinant of gait. All four of these parameters had very similar behavior when plotted against slope so only one is shown here. The filled circles (●) represent running and the crosses (×) represent walking. Surprisingly, no relation was found here, either.

In the end, no relation was discovered between any of the biomechanical parameters that were measured and the slopes and intercepts of the t_c – pace lines. All of the plots were similar to figures 5.6 and 5.7. While walking and running were easily differentiated, no significant trends were exposed. The optimal biomechanical calibration proved to be incredibly elusive.

6.0 Theoretical Model

Although the experiments failed to demonstrate a physiological basis for gait parameters, it should not be assumed that no basis exists. Even if a statistical correlation were indicated by the experiments, the exact mechanism of gait determination would be unknown. If gait *is* determined by physiology, then what are the relevant physiological parameters? What is the mechanism?

6.1 Linear model of leg mechanics

A simple model for leg mechanics has been proposed by McMahon and Greene. It is shown in figure 6.1.

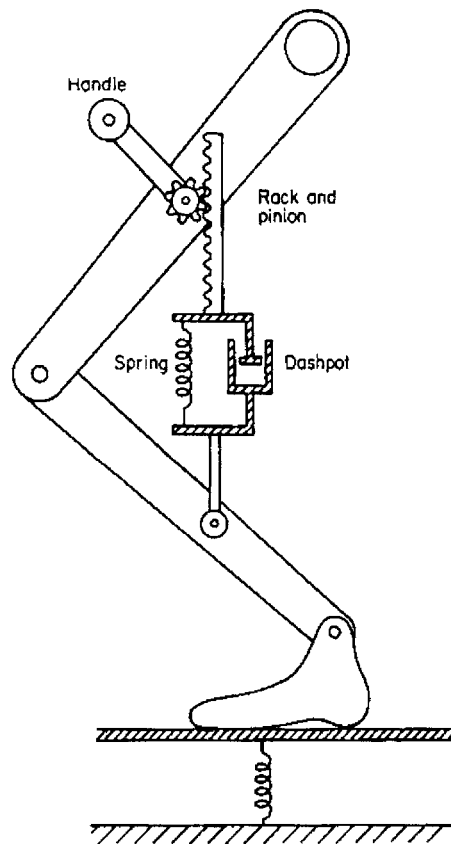


Figure 6.1: Linear leg model. From [11].

This model is based on the intrinsic features of isolated muscle and the global features of muscle controlled by reflex networks and movement commands. Contact time t_c is approximated as half the period of resonant vibration of the system.

$$t_c \cong \frac{\Pi}{\omega_r}$$

The resonant frequency ω_r is determined by body mass m , ground stiffness k_i , leg muscle stiffness k_m , and leg damping b . For tracks and paved surfaces, k_i is effectively infinite and does not affect the resonant frequency. Body mass is easily measured, so the problem reduces to the determination of the mechanical properties of the leg muscles.

The dashpot in the leg model corresponds to the damping element in the model of the muscle spindle organ, shown in figure 6.2.

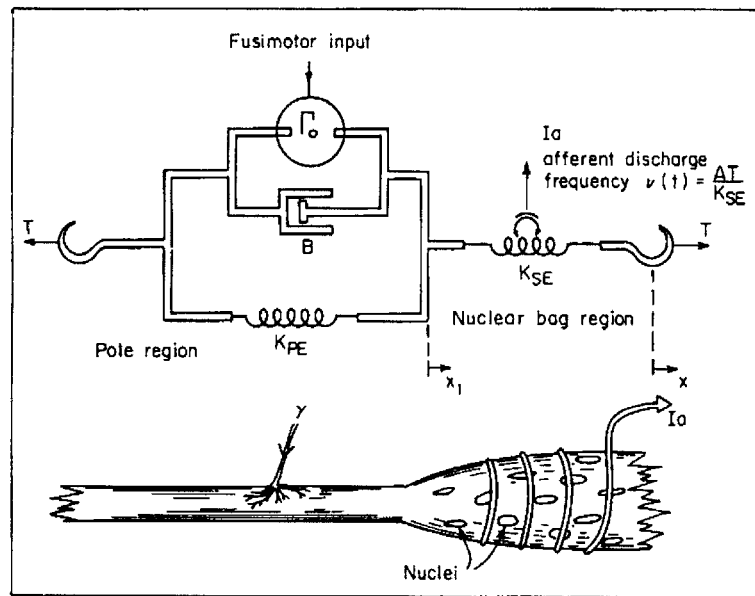


Figure 6.2: Muscle spindle model. From [12].

The spindle gives information about its length and its rate of change of length. The feedback control mechanism for muscle can therefore be considered a standard PD

controller, resulting in increased stability but a low-pass filtered step response. The spindle response has been termed a sluggish version of the length plus the rate of change of length of the organ. It is this sluggishness which is represented by damping element B in figure 6.2 and damping element b in figure 6.1.

The spring in the leg model corresponds to the stiffness of the leg. Experiments on cat soleus muscles showed that the stretch reflex acts to maintain the stiffness of the leg approximately constant in the range of medium to high forces. This is depicted in figure 6.3.

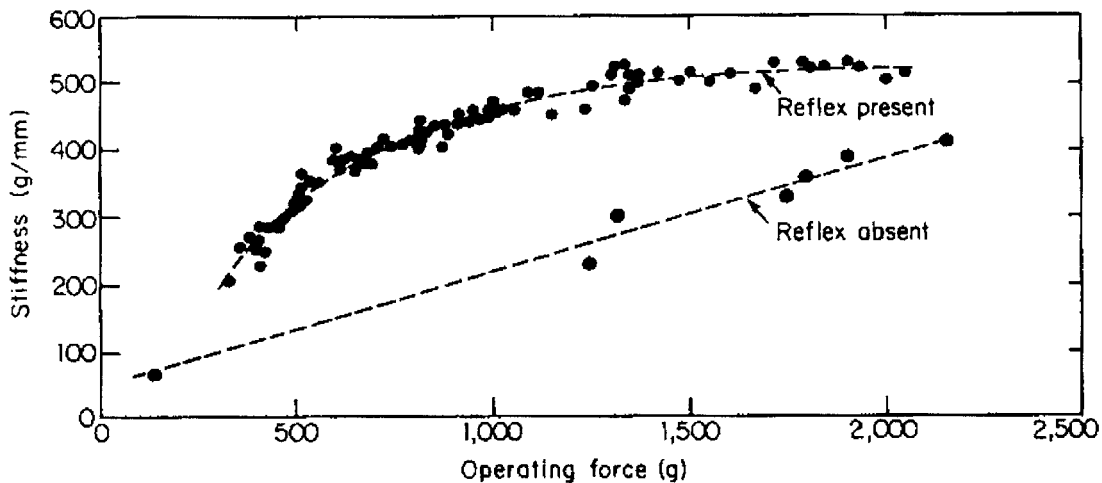


Figure 6.3: Effect of Stretch Reflex on Muscle Stiffness. From [13].

A second study by Greene and McMahon confirmed that the overall leg stiffness in humans behaves the same way as the isolated cat soleus muscle [14]. They concluded that the muscles of the legs behave like a linear spring of approximately constant stiffness over a wide range of moderate and high forces. It is this constant stiffness which is represented by the spring element k_m in figure 6.1.

Although simple, McMahon's linear leg model has a solid physiological basis. To test the model, subjects were asked to run at full speed on an experimental track. Their foot contact times were measured by high-speed photography and by a force plate under one of the board track panels [15]. Their results support the linear leg model. Across a range of body weights, contact times were fairly accurately estimated by a half period of the resonant vibration of the leg, which can be independently measured with the method described in [14].

6.2 Nonlinear model of leg mechanics

Although the linear leg model successfully predicted contact times for full-speed running, it makes no predictions for contact times at other speeds. This is due to the dependence of the of the model parameters on the applied load – they are not constant, linear elements. Fig 6.3 clearly demonstrates the dependence of leg stiffness on the operating force.

McMahon's model exploits the nearly constant stiffness exhibited at high forces and approximates the leg's stiffness as a simple spring element. At high speeds, this is a valid approximation. A device that only works at high speeds is of little interest to the recreational runner. Furthermore, the technical difficulties associated with measuring contact times at very high speeds are non-trivial. A device that was specific to high speeds would be extremely difficult to design, and would not appeal to very many consumers.

McMahon's model must be expanded to account for the whole range of forces and the whole range of running speeds. The method employed by Greene and McMahon

could be used to map out the whole stiffness-force curve for any subject. This is an extremely time-consuming procedure however, and must be performed in a specially equipped facility. To be a viable solution to the problem of calibration, any method must be of comparable complexity to the two point calibration method or the simpler, “pivot” point method.

Given the accuracy of the linear leg model at high speeds and forces, it is reasonable to expect the expanded, nonlinear model to be equally accurate across all speeds and forces. Until a simple method of determining and parameterizing the leg stiffness and damping becomes available, the existence of such a model, however accurate, remains of purely academic interest.

7.0 Additional Considerations

The entire analysis so far has focused on level, treadmill running. Even if a basis for biomechanical calibration is never determined, there are several issues that still need to be addressed. How does treadmill running relate to normal, overground running? How does running up or down hills affect the contact time relations described in chapter 3.1? Finally, how does trail running differ from running on hard, regular surfaces? All of these questions must be answered before the perfect speed-measuring device can be designed.

7.1 Treadmill vs. Overground Running

Many researchers in the field of gait analysis assume that treadmill and overground locomotion are identical. This assumption allows them to conduct all of their experiments on a treadmill and to generalize all their results to all running. Unfortunately, this assumption is invalid. Many people know this intuitively: running on a treadmill feels different than overground running.

Some researchers have acknowledged the difference and have tried to quantify it. One investigation, involving 12 adults and 14 children, showed that step frequency f_s during treadmill locomotion increased 7% in adults and 10% in children [16]. This resulted in a decreased contact time at every speed. For the adults, walking trials were conducted at 3.4 *mph*. Average contact time changed from .630 seconds to .587 seconds, a 7% reduction. For the children, walking trials were conducted at 2.5 *mph*. Average contact time changed from .606 seconds to .531 seconds, a 12% reduction.

The resulting t_c -pace relation is still linear, but the slope and intercept are different than for the treadmill case. The lines in figure 4.1 would be rotated and

translated. The discrepancy between treadmill and overground data is a fundamental issue in the field of gait analysis. It potentially invalidates the bulk of the work that has been performed. A thorough investigation into the differences and their causes is imperative to ensure the validity of gait analysis research.

7.2 Gradient Running

Another major consideration is the effect of hills on energy consumption and the t_c -pace relation [17, 18]. Figure 7.1 shows the weight-specific rate of energy consumption as a function of speed.

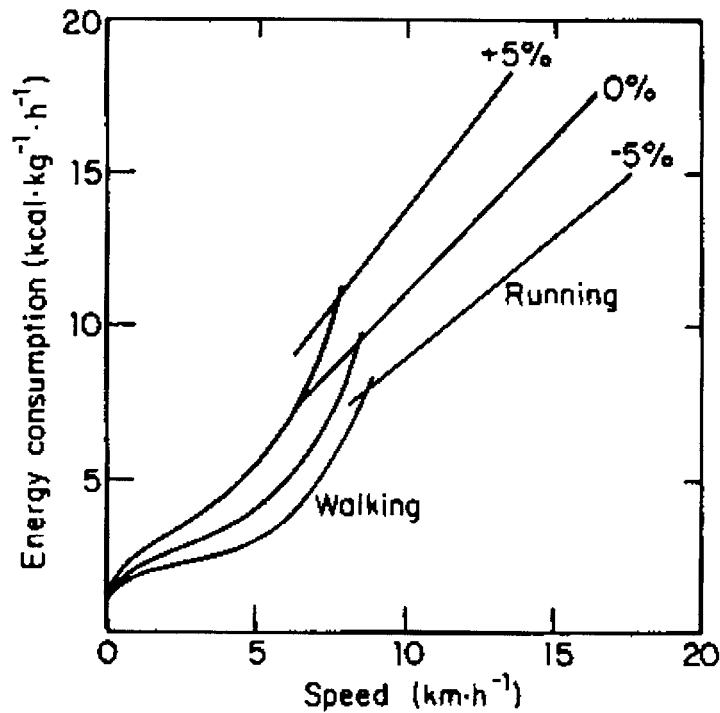


Figure 7.1: Effect of grade on energy consumption rate. From [12].

As expected, the positive grade increased energy cost and the negative grade lowered the cost of locomotion. The intersection of the curves for walking and running represent the optimal transition between walking and running. This is the critical speed for which running is more efficient than walking and becomes the preferred gait. At 0% grade, this transition is shown to be at approximately 5 *mph*.

The energetic gain associated with negative gradients do not continue indefinitely, however. This is easily experienced when trying to run down a very steep hill. A great deal of energy must be expended in slowing down, in order to prevent falling over. The optimum negative grade would be the one that does not require any braking effort. This is illustrated in figure 7.2. The most efficient negative grade is -10%, after which energy cost begins to rise again. The energetic cost of running on a -25% grade is comparable to the cost of running on level ground. Positive grades quickly increase the cost of running. At high positive gradients, walking becomes like climbing a ladder: the muscles are doing only positive work against gravity. The minimum of each curve corresponds to the most efficient speed for each gradient.

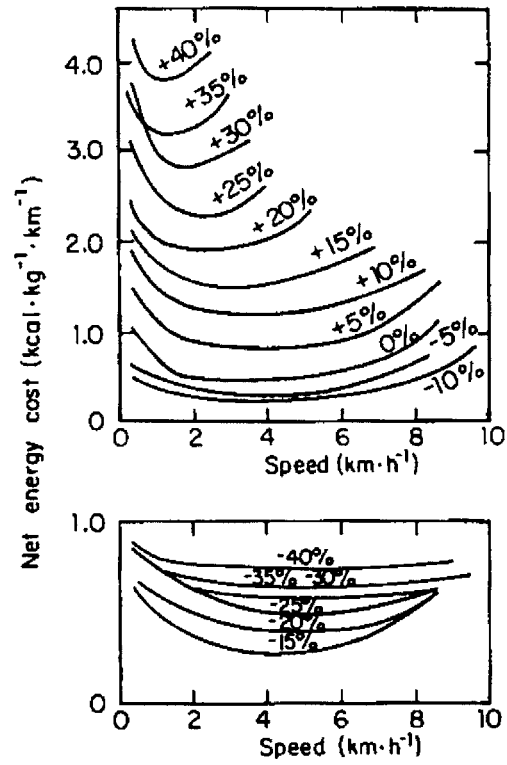


Figure 7.2: Energy cost per unit distance. From [12].

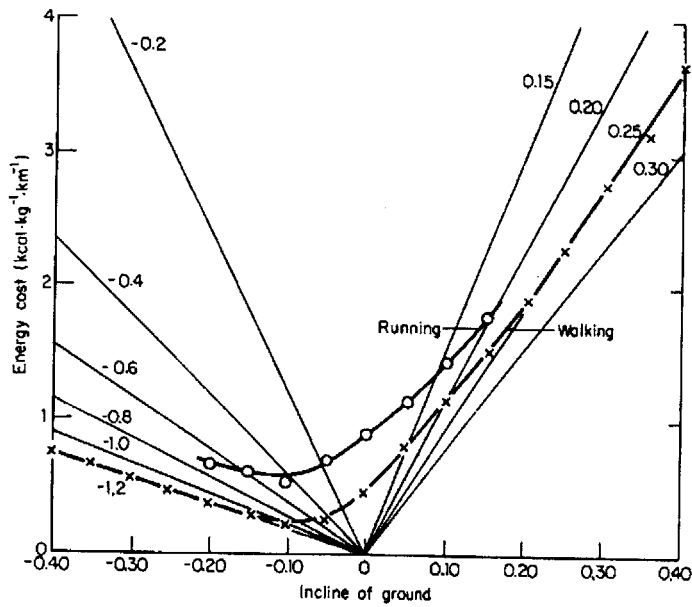


Figure 7.3: Efficiency of locomotion. From [12].

If the minimum of each curve in figure 7.2 is plotted against gradient, the standard efficiency curves are produced, shown in figure 7.3. The radial lines from 0 show constant efficiencies.

The effect of gradient running is intuitive and easily observed. Although correction factors could be incorporated that adjust the t_c -pace relation depending on grade, there is still great difficulty in determining what the grade is. How can a small, wearable device in constant motion make an accurate assessment of grade? Until this technical issue is solved, it will remain difficult to account for the energy cost of gradient running in a small device. For now, it is necessary to use the t_c -pace relation for 0% grade at all grades. Since most running takes place at level, or near-level, gradients, this is a valid approximation. Additionally, most runners cover circuit courses when they run. To some extent, the extra cost of running up hills will be offset by the increased efficiency of running down the same hills. Although figure 7.3 clearly indicates that the increased efficiency at negative grades is less than the decreased efficiency at positive grades, the offset will still tend to mitigate the measurement errors due to the level ground assumption.

7.3 Off-Road Running

It has been established that running on natural “off-road” terrain elicits a higher energy demand than road running [19]. One possible cause for this difference is the irregularity of the terrain, resulting in a less relaxed running rhythm. Runners often feel that when running, “breaking” their stride, or having to adjust their running for some reason results in an increased energy cost. Thus, regularity of stride is necessary for

maximum running comfort and efficiency. This irregularity is qualitative and hard to incorporate into a model of the cost of locomotion.

Another possible cause of the increased energy demand can be understood in light of the linear leg model described in chapter 6.1. Figure 6.1 shows the leg model, which includes an element representing the compliance of the ground. For the analysis presented in chapter 6.1, this ground compliance k_t was assumed to be infinite, so it did not affect the resonant frequency of the system or the contact time. This assumption was valid since running on very hard surfaces like roads or tracks was being considered.

Off-road running presents a very different situation. The terrain is potentially very soft, having a finite and measurable k_t . Springs do not dissipate energy: all the force applied by the runner will be returned. This is a good model of roads and tracks. Soft terrain, however, absorbs energy from the runner's foot and does not return it. A dashpot must be also included to account for this behavior.

This obviously results in decreased efficiency. More importantly, however, the equations in chapter 3.1 are no longer valid. A correction term must be included, to account for the energy dissipated by the ground. A new expression might look like

$$\frac{\partial E_{metab}}{\partial t} = c \cdot \frac{W_b}{t_c} + \frac{\partial E_{dissipated}}{\partial t}$$

The dissipation term would depend on the ground compliance k_t , ground damping b_t , body weight W_b , and contact time t_c .

$$\frac{\partial E_{dissipated}}{\partial t} = f(k_t, b_t, W_b, t_c)$$

8.0 References

- [1] Groves, David. "Beyond the Pedometer: New Tools for Activity Monitoring and Energy Expenditure." *The Physician and Sports Medicine* 16 (June 1988) 160-166.
- [2] Montoye, Henry J. *Measuring Physical Activity and Energy Expenditure*. Champaign, IL: Human Kinetics, 1996.
- [3] Saris, W.H.M. and Binkhorst, R.A. "The Use of Pedometer and Actometer in Studying Daily Physical Activity in Man. Part I: Reliability of Pedometer and Actometer." *European Journal of Applied Physiology* 37 (1977) 219-228.
- [4] Bassett, David R., Barbara E. Ainsworth, Sue R. Leggett, Clara A. Mathien, James A. Main, David C. Hunter, and Glen E. Duncan. "Accuracy of five electronic pedometers for measuring distance walked." *Medicine and Science in Sports and Exercise* 28 (1996) 1071-1077.
- [5] Zuntz, N. "Uber den Stoffverbrauch des Hundes bei Muskelarbeit." *Arch. ges. Physiol.* 68 (1897) 191-211.
- [6] Taylor, C.R., N.C. Heglund, and G.M.O. Maloiy. "Energetics and mechanics of terrestrial locotion. I. Metabolic energy consumption as a function of speed and body size in birds and mammals." *Journal of Experimental Biology* 97 (1982) 1-21.
- [7] Fedak, M.A., and Seeherman, H.J. "Reappraisal of energetics of locomotion shows identical cost in bipeds and quadrupeds including ostrich and horse." *Nature* 282 (1979) 713-716.
- [8] Taylor, C.R., and Rowntree, V.J. "Running on two or on four legs: which consumes more energy?" *Science* 179 (1973) 186-187.
- [9] Kram, Rodger and Taylor, C. Richard. "Energetics of running: a new perspective." *Nature* 346 (19 July 1990) 265-267.
- [10] Marey, E.J. *Animal Mechanism: A Treatise on Terrestrial and Aerial Locomotion*. New York: Appleton, 1874.
- [11] McMahon, T.A., and Greene, P.R. "The influence of track compliance on running." *Journal of Biomechanics* 12 (1979) 893-904.
- [12] McMahon, T.A. *Muscles, Reflexes, and Locomotion*. Princeton, NJ: Princeton University Press, 1984.
- [13] Hoffer, J.A., and Andreassen, S. "Factors affecting the gain of the stretch reflex and soleus muscle stiffness in premammillary cats." *Soc. Neurosci. Abstr.* 4 (1978) 935.
- [14] Greene, P.R., and McMahon, T.A. "Reflex stiffness of man's anti-gravity muscles during kneebends while carrying extra weights." *Journal of Biomechanics* 12 (1979) 881-891.
- [15] McMahon, T.A., and Greene, P.R. "Fast Running Tracks." *Scientific American* 239 (1978) 148-163.
- [16] Stolze, H. et al. "Gait analysis during treadmill and overground locomotion in children and adults." *Electroencephalography and Clinical Neurophysiology* 105 (1997): 490- 497.

- [17] Minetti, A.E., L.P. Ardigo, and F. Saibene. "Mechanical Determinants of the Minimum Energy Cost of Gradient Running in Humans." *Journal of Experimental Biology* 195 (1994) 211-225.
- [18] Eaton, Michael D., David L. Evans, David R. Hodgson, and Reuben J. Rose. "Effect of Treadmill Incline and Speed on Metabolic-Rate During Exercise in Thoroughbred Horses." *Journal of Applied Physiology* 79 (1995) 951-957.
- [19] Creagh, U., T. Reilly, and A. Lees. "Kinematics of running on 'off-road' terrain." *Ergonomics* 41 (1998) 1029-1033.