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Inducing Time-Reversal-Invariant Topological Superconductivity and Fermion Parity Pumping in Quantum Wires

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We propose a setup to realize time-reversal-invariant topological superconductors in quantum wires, proximity coupled to conventional superconductors. We consider a model of quantum wire with strong spin-orbit coupling and proximity coupling to two *s*-wave superconductors. When the relative phase between the two superconductors is $\phi = \pi$ a Kramers pair of Majorana zero modes appears at each edge of the wire. We study the robustness of the phase in the presence of both time-reversal-invariant and time-reversal-breaking perturbations. In addition, we show that the system forms a natural realization of a fermion parity pump, switching the local fermion parity of both edges when the relative phase between the superconductors is changed adiabatically by 2π .

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Introduction.—Over the last few decades, it has been realized that there is a deep and unexpected relation between the properties of matter and topology. At zero temperature, there exist phases of matter that are distinguished by an underlying topological structure encoded in their ground-state wave functions. These phases are often characterized by a finite energy gap in their bulk, and protected gapless edge states with unusual properties. The bulk can either be insulating, as in the case of the recently discovered topological insulators (TIs) [1,2], or superconducting [3–5]. Phases of the latter type, known as “topological superconductors” (TSCs), support anomalous zero-energy Andreev edge states which are robust as long as the bulk quasiparticle gap remains open. These edge states have attracted much attention due to their possible future applications for topologically protected quantum information processing [6]. Recently, it has been predicted that the one-dimensional variant of a TSC can be realized by proximity-coupling a semiconducting quantum wire to a superconductor (SC) [7–11]. The resulting TSC phase has particle-hole symmetric modes at zero energy, localized at the edges of the wire, known as Majorana zero modes [12]. Signatures of such zero modes have been observed in recent experiments [13–15].

In the presence of time-reversal (TR) invariance, different types of TSCs can arise [16,17]. These phases are solid-state analogues of the B phase of superfluid ^3He [18,19]. The alloys $\text{Cu}_x\text{Bi}_2\text{Se}_3$ [20–25] and $\text{Sn}_{1-x}\text{In}_x\text{Te}$ [26] are possible candidates for these phases. In addition, it has been proposed that proximity-coupling an unconventional superconductor to a quantum wire can stabilize a one-dimensional TSC phase which supports a *Kramers pair* of Majorana zero modes at its edge [27–29], protected by TR invariance. These edge modes are characterized by an anomalous relation between the fermion parity of the edge and time-reversal symmetry [16].

In this Letter, we propose a different setup to realize time-reversal-invariant (TRI) TSCs in quantum wires. The setup is shown schematically in Fig. 1(a). A quantum wire with strong spin-orbit coupling is proximity coupled to two *s*-wave superconductors from either side [30]. We assume that the relative phase ϕ between the two superconductors can be controlled externally, e.g., by connecting the two superconducting leads and threading a flux Φ through the resulting superconducting loop. The relative phase is then $\phi = 2\pi\Phi/\Phi_0$, where $\Phi_0 = hc/2e$ is the superconducting flux quantum. We assume that the flux is applied far away from the wire, such that the magnetic field in the region of the wire is zero. The system has TR invariance for $\phi = 0$ and $\phi = \pi$. For $\phi = 0$, we expect the induced SC state in the wire to be topologically trivial. In contrast, for $\phi = \pi$ we show that a TRI topological superconductivity state is formed in the wire under a broad range of circumstances, and a Kramers pair of Majorana states appears at each edge of the wire. This follows from a general criterion for TRI topological superconductivity in centrosymmetric systems [21,31], which we extend to one-dimensional systems below.

Unlike the previous proposals [27–29], the setup we present requires only coupling to *conventional* superconductors, and may thus be easier to realize. The key for achieving a TRI TSC in our system is that the induced pairing potential in the quantum wire is odd under spatial parity, due to the π phase difference between the two external superconductors.

In addition, we consider the effect of time-reversal breaking perturbations, such as a deviation of the relative phase from π or a Zeeman field. These perturbations split the degeneracy of the edge states. Surprisingly, however, we find that the topological character of the system is not completely lost. Instead, the system forms a natural realization of a *fermion parity pump*, switching the local

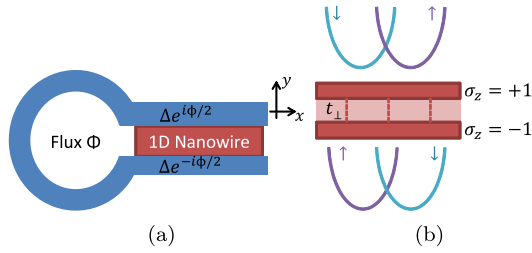


FIG. 1 (color online). (a) The general setup proposed for the realization of a TRI TSC. The TRI topological phase is obtained for $\phi = \pi$. (b) Specific model considered for the nanowire. The spin orbit coupling on the two chains comprising the wire has an opposite sign as indicated by the energy dispersion curves. A finite tunneling amplitude t_{\perp} between the two chains opens a gap at the crossing points.

fermion parity as well as flipping the local spin density at both edges when ϕ is changed adiabatically by 2π . This is a generalization of the adiabatic charge pump proposed by Thouless [32].

Conditions for TRI topological superconductivity.—We consider a system which has TR invariance and particle-hole symmetry, such that $\mathcal{T}^2 = -1$, $\mathcal{C}^2 = 1$, where \mathcal{T} , \mathcal{C} are the time-reversal and particle-hole operators, respectively (class DIII [33]). The phases in such systems are classified by a \mathbb{Z}_2 invariant in spatial dimensions $d = 1, 2$ and by a \mathbb{Z} (integer) invariant in $d = 3$. A sufficient condition for TRI topological superconductivity in centrosymmetric systems in $d = 2$ and 3 was derived in Refs. [21,31,34]. The condition states that if (1) the pairing is odd under inversion and opens a full SC gap and (2) the number of TRI momenta enclosed by the Fermi surface in the normal (non-SC) state is odd, then the system is in a TSC state. We have extended the condition to 1D systems [35], for which (2) above is replaced by the requirement that in the normal state the number of (spin-degenerate) Fermi points between $k = 0$ and $k = \pi$ is odd. In 1D centrosymmetric systems, the condition is both sufficient and necessary.

Applying this condition to the setup of Fig. 1(a), we see that for $\phi = \pi$ the induced pairing is odd under a spatial inversion $\vec{r} \rightarrow -\vec{r}$, which interchanges the two superconductors. Suppose that the wire is made of a material with a centrosymmetric crystal structure. Then, if the number of spin-degenerate bands crossing the Fermi level of the wire is odd, and if the bulk of the wire is fully gapped by the proximity effect, then the resulting state is necessarily a TRI TSC.

Note that, although our condition makes no reference to the necessity of spin-orbit coupling (SOC) in the wire, SOC is essential to realize a TSC [35]. Therefore, we expect that in the absence of SOC the bulk remains gapless for $\phi = \pi$, invalidating one of the requirements for topological superconductivity.

Model.—As an illustration, we consider a simple model of a centrosymmetric quantum wire with SOC. Our model consists of two coupled chains, with a SOC term

originating from Rashba nearest neighbor hopping and consistent with inversion symmetry. The Hamiltonian is given by

$$\mathcal{H} = \sum_k \psi_k^\dagger H_k \psi_k. \quad (1)$$

Here, $\psi_k^\dagger = (c_k^\dagger, -i s_y c_{-k})$ is a spinor in Nambu space, where $c_k^T = (c_{1,\uparrow,k}, c_{1,\downarrow,k}, c_{2,\uparrow,k}, c_{2,\downarrow,k})$, and c_{lsk}^\dagger creates an electron with momentum k and spin s at chain $l = 1, 2$. We use \vec{s} to denote Pauli matrices in spin space and $\sigma_z = \pm 1$ for the upper or lower chain, respectively; see Fig. 1(b). The Hamiltonian matrix is written as $H_k = H_{0k}\tau_z + H_{\Delta}\tau_x$, where $\vec{\tau}$ are Pauli matrices that act on the Nambu (particle-hole) space, and the matrices H_{0k} , H_{Δ} are given by

$$H_{0k} = \xi_k + \lambda_k s_z \sigma_z - t_{\perp} \sigma_x, \quad (2)$$

$$H_{\Delta} = \Delta \sigma_z, \quad (3)$$

where $\xi_k = 2t(1 - \cos k) - \mu$ and $\lambda_k = 2\lambda \sin k$. The parameters t and t_{\perp} are nearest neighbour hopping amplitudes along the chains and between the chains respectively, λ is the SOC strength, and μ is the chemical potential. H_{Δ} describes the proximity coupling to two superconductors with opposite phases. Inversion symmetry is implemented by the operator $\mathcal{P} = \sigma_x$, that interchanges the two chains, followed by $k \rightarrow -k$.

The Hamiltonian (1) can be diagonalized by a Bogoliubov transformation. The spectrum is given by

$$E(k) = \pm \left[\xi_k^2 + \lambda_k^2 + t_{\perp}^2 + \Delta^2 \pm 2\sqrt{\xi_k^2 t_{\perp}^2 + \xi_k^2 \lambda_k^2 + t_{\perp}^2 \Delta^2} \right]^{1/2}. \quad (4)$$

Each band is doubly degenerate, as expected from the symmetry of the system under time reversal and inversion. For $|t_{\perp}| > |\mu|$ and $\Delta = 0$, there is a single spin-degenerate band crossing the Fermi level. For $0 < |\Delta| \ll t_{\perp}$, the spectrum becomes fully gapped with a minimum gap $\Delta_{\min} \approx |\Delta| |\lambda_{k_F}| / \sqrt{t_{\perp}^2 + \lambda_{k_F}^2}$ at the Fermi points. (k_F is the Fermi momentum.) In this case, the condition above is satisfied, and the system is in the TRI TSC phase. The gap remains open as long as $t_{\perp}^2 > \mu^2 + \Delta^2$. For $\lambda = 0$ the system remains gapless.

At the edge of a system in the TRI TSC phase, we expect to find a single Kramers pair of Majorana zero modes. To see that this is indeed the case, we note that the model (1) can be thought of as two copies of the model considered in Refs. [8,10],

$$\tilde{H}_k = (\xi_k + \tilde{\lambda}_k \sigma_z) \tau_z - B \sigma_x + \Delta \tau_x, \quad (5)$$

describing a semiconducting wire with Rashba spin-orbit coupling in an external magnetic field given by $B = t_{\perp}$. Note that in (1), s_z is conserved, and can be replaced by its eigenvalue ± 1 . Then, the unitary transformation $U = e^{i(\pi/4)(1-\sigma_z)(1-\tau_z)}$ maps H_k to \tilde{H}_k with $\tilde{\lambda}_k = s_z \lambda_k$. The

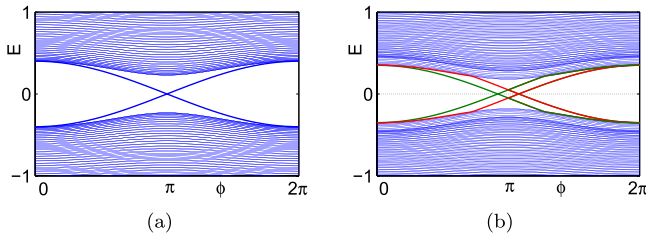


FIG. 2 (color online). Energy spectrum of a finite nanowire as a function of the relative phase between the two superconductors for (a) zero and (b) nonzero magnetic field. The length of the nanowire is $N_x = 200$. The lattice model parameters are $t_\perp = 2.5$, $t = 1$, $\lambda = 1$. The chemical potential is set to $\mu = 0$ and the magnitude of the SC pairing is $\Delta = 0.4$. (a) For $\phi = \pi$ the system is TRI and in the topological phase as can be seen from the presence of zero-energy states. (b) Here $B = 0.05$ in the z direction. Energy levels crossing the bulk gap correspond to edge states at the opposite edges of the wire (red and green online). At $\phi = \pi$ the two Majorana modes on each edge split. The crossing at zero energy does not disappear, but is shifted away from $\phi = \pi$.

model (5) has been shown [8,10] to support a single Majorana zero mode at the edge for $t_\perp^2 > \mu^2 + \Delta^2$. Hence, the two-chain model (1) has a *pair* of zero modes at the edge, one for each value of s_z . These zero modes form a single Kramers pair. We have verified this explicitly (see Fig. 2(a)).

Noncentrosymmetric perturbations.—If inversion symmetry of the quantum wire is broken, one expects the TSC phase to be robust over a finite range of parameters as long as the system remains TRI. As an example for such a perturbation we consider Rashba-type spin orbit coupling in a direction perpendicular to the plane of the wire, given by $\delta H = \lambda_R s_y \tau_z$. It turns out that when λ_R reaches a critical value of the order of Δ the superconducting gap closes. As we show [35] this is due to the fact that the pairing potential does not couple time-reversed states in this case, but states related by inversion symmetry. Therefore a critical value of the pairing potential is required to open a gap at the Fermi energy once inversion symmetry is broken.

We conclude that in order to obtain the TSC phase, it is necessary that the crystalline structure of the wire material is centrosymmetric. In addition, the setup has to have an approximate inversion center, such that the Rashba-type spin-orbit coupling is small.

Another question we address is what happens if the magnitude of the pairing potential on the two sides of the wire is not exactly equal. The SC pairing can then be decomposed into odd (Δ_o) and even (Δ_e) spatial components, such that the pairing amplitudes on the upper and lower chains are $\Delta_{1,2} = \Delta_e \pm \Delta_o$, respectively. Once Δ_e is nonzero, the condition for topological superconductivity formulated above is no longer satisfied. However, for small enough Δ_e we expect the gap to remain finite and therefore the system remains in the topological phase. Using the

lattice Hamiltonian (1) and computing the phase diagram explicitly [35] we find that as we increase Δ_e the gap remains open up to values of the order of a half of Δ_o . We thus conclude that our setup does not rely on the strength of the proximity coupling to the two superconductors being exactly equal in magnitude.

Effect of TR breaking.—Once time-reversal symmetry is broken the two Majorana modes on each edge are no longer protected, and we expect them to split from zero energy. In the suggested setup, TR invariance can be broken either by changing the phase difference ϕ away from π , or by applying a magnetic (Zeeman) field \vec{B} in the quantum wire. The Zeeman field is modelled by adding a term $-\vec{B} \cdot \vec{s}$ to the Hamiltonian H_k . The low energy effective Hamiltonian on the edge can be written in terms of the local Majorana operators γ_1, γ_2 . Denoting by $\lambda(\vec{B}, \phi)$ the coupling between the two Majoranas due to broken TR invariance, and demanding the Hamiltonian to be Hermitian we conclude the coupling term must be of the form $\lambda i \gamma_1 \gamma_2$. Note that under time reversal $T i \gamma_1 \gamma_2 T^{-1} = -i \gamma_1 \gamma_2$. Expanding λ around $\vec{B}=0$, $\phi = \pi$ to lowest order in both parameters, we see that the coupling must be of the form $\lambda(\vec{B}, \phi) \propto \vec{B} \cdot \hat{n} + \alpha(\phi - \pi)$, where \hat{n} is some unit vector (note that all even orders in the expansion must vanish due to TR invariance). This suggests that only a single component of the magnetic field (parallel to \hat{n}) couples between the Majorana modes. Moreover, for a given magnetic field we can vary the flux and bring the coupling back to zero.

Using the lattice model (1) and calculating numerically the BdG spectrum of a finite system, we have confirmed these results. We find that to linear order in the magnetic field, only the z component of the field leads to shifting of the edge energy levels away from zero at $\phi = \pi$. For nonzero B_z we vary the relative phase ϕ between the SC pairings on the two opposite sides of the nanowire and plot the energy spectrum obtained (see Fig. 2(b)). It is clearly seen that when $B_z \neq 0$ the zero crossings are shifted away from $\phi = \pi$.

Adiabatic pumping.—One can now consider an adiabatic cycle in parameter space where the phase ϕ changes by 2π . From the arguments above, we expect a single level crossing to occur at each edge for some value of ϕ which depends on B_z [36]. The two states that cross differ by their local fermion parity, and therefore they cannot mix. We argue that such a cycle falls into the nontrivial class of adiabatic cycles in 1D particle-hole symmetric Hamiltonians (class D), discussed in Refs. [37,38], and serves as a *fermion parity pump*. Indeed, one can define the parity of the right (left) edge as $P_{R,L} = \prod_i (-1)^{n_i}$, where $n_i = c_i^\dagger c_i$ is the occupation of site i , letting i run over all the sites in the right (left) half of the wire. An adiabatic sweep through the cycle changes the fermion parity at each edge, i.e., takes the expectation value $\langle P_{R,L} \rangle$ to $-\langle P_{R,L} \rangle$, respectively. One can construct an

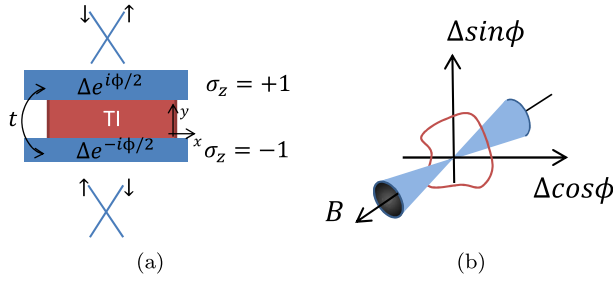


FIG. 3 (color online). (a) 2D QSH model with finite tunneling probability between the edges. (b) A closed path in parameter space encircling the gapless region (cones) corresponding to a fermion parity pump. The path is not contractible due to the persistence of the gapless region for $B \neq 0$.

explicit \mathbb{Z}_2 topological invariant characterizing the pumping process and show that it is nonzero for the cycle considered [35].

In situations in which one component of the total spin is conserved, e.g., S_z , the cycle also pumps a quantum of spin angular momentum $S_z = 1/2$ between the two edges. Repeating this cycle twice is equivalent to the \mathbb{Z}_2 spin pump discussed in Refs. [39,40]. The spin pumping property can be used as an experimental signature of the anomalous edge states. At $\phi = \pi$ each edge supports two degenerate (many-body) states with an opposite expectation value of S_z . Since the two states differ by adding a single electron or hole, they must have $\langle S_z \rangle = \pm 1/4$ [35]. When ϕ is changed adiabatically by 2π , the local spin of the edge switches. If S_z is not conserved, the unit of spin transferred between the edges during the adiabatic cycle is not quantized; however, we still expect $\langle S_z \rangle$ of each edge to flip its sign over one cycle.

The pumping property becomes particularly transparent if one considers an alternative model, illustrated in Fig. 3(a). Consider a strip of a 2D quantum spin Hall (QSH) material with 1D helical edge states. If the width of the strip is finite, the tunneling amplitude t_\perp between the edge states is nonzero. The opposite sides of the strip are proximity coupled to two s -wave SCs with a phase difference of ϕ .

In absence of a magnetic field and in the $t_\perp \rightarrow 0$ limit, a cycle in which ϕ changes by 2π can be realized by passing a superconducting vortex through the QSH strip (between the two SCs), along the x direction. Such a vortex induces a voltage along the y direction, which in turn will lead to a spin current along the x direction. The total spin transferred between the ends of the QSH strip in this process is $1/2$, corresponding to a single fermion. Hence, such a cycle exactly serves as a fermion parity pump. Note that the use of a QSH is not essential for the pumping phenomena. In the QSH model, however, the origin of the pumping is evident.

Denoting the two edges of the QSH state by $\sigma_z = \pm 1$, we can write the following low energy effective Hamiltonian:

$$H = (vks_z\sigma_z - t_\perp\sigma_x - \mu)\tau_z + Bs_z + \Delta \cos \frac{\phi}{2} \tau_x + \Delta \sin \frac{\phi}{2} \sigma_z \tau_y. \quad (6)$$

Here, v is the velocity of the edge modes, μ is their chemical potential, B is an applied Zeeman field, and Δ is the induced pairing potential. We examine the phase diagram of the system in the parameter space spanned by Δ , ϕ and B , Fig. 3(b). For $B = 0$ and $\phi = 0, \pi$, the system is TRI. For $\mu > t_\perp$, the gapless point $\Delta = 0$ separates between the trivial and the topological phases. When a magnetic field is turned on, the gapless point does not disappear but turns into a finite region $|\Delta| \leq |B|$. As we change ϕ by 2π , the path in parameter space encircles a gapless region and cannot be contracted to a point without crossing it. This is a consequence of the fermion parity pumping property of this cycle [41].

Discussion.—We have presented a general setup to realize a time-reversal-invariant TSC by proximity coupling a quantum wire with strong SOC to conventional superconductors. The TSC phase can be identified by the presence of a pair of zero-energy Majorana bound states at each edge, protected by time-reversal symmetry. Thus, we expect a zero-bias peak to appear in the tunneling conductance into the edge of the system when the phase difference between the two superconductors is $\phi = \pi$. Intriguingly, varying ϕ adiabatically by 2π pumps both fermion parity and spin between the edges.

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