Nonlinear Capital Taxation Without Commitment

Nonlinear Capital Taxation Without Commitment

Emmanuel Farhi
Harvard

Christopher Sleet
Carnegie Mellon University

Iván Werning
MIT

Sevin Yeltekin
Carnegie Mellon University

February 18, 2011 (7:50pm)

Abstract

We study efficient nonlinear taxation of labor and capital in a dynamic Mirrleesian model incorporating political economy constraints. Policies are chosen sequentially over time, without commitment. Our main result is that the marginal tax on capital income is progressive, in the sense that richer agents face higher marginal tax rates.

1 Introduction

In most advanced countries, capital taxation is not only positive, but also progressive.[1] This is inconsistent with existing economic models. On the one hand, most normative theories prescribe zero capital income taxes. On the other hand, positive theories have rationalized positive tax rates on capital income, but have remained silent regarding their progressivity. The main purpose of this paper is to provide such a political economy theory that addresses both the level and the progressivity of capital taxation.

Modern optimal-tax theory is founded on the trade-off between efficiency and redistribution (Mirrlees (1971)). The losses in efficiency from taxation are determined mechanically by the economic environment—preferences, technology and information. In contrast, the desire to redistribute, often modeled by a social welfare function, may implicitly capture the outcome or demands of some political process.

However, if anything, actual policy making not only considers this trade-off, but also constantly reconsiders it: policies chosen at some point, can be reformed or replaced by

[1] We provide some evidence towards the end of this intoduction, when we discuss how our results compare to actual policies.

new ones at any later time. Due to this lack of commitment, the credibility of policies must be judged by projecting their effects into the future. The impact on future wealth inequality is of particular concern. Otherwise, large levels of inequality may create, ex post, a political demand for reform, towards policies that redistribute wealth. The purpose of this paper is to explore this idea and study optimal policy design when the credibility of policies are taken into account.

Our theory blends recent developments in optimal taxation with elements of political economy. We study a dynamic Mirrleesian model where policy is determined sequentially. We allow for the most general non-linear tax schedules for labor and capital income. Our main result shows that progressive capital taxation emerges naturally in this setting.

Our deterministic economy abstracts from aggregate or idiosyncratic uncertainty. It is populated by a continuum of agents that live for two periods and are heterogeneous in their labor productivity, which is privately observed. The latter assumption precludes the first-best outcome of full insurance. We assume that tax instruments are restricted only by this asymmetry of information and political economy considerations. Absent the latter, any incentive compatible allocation is implementable. In particular, we allow for nonlinear taxation of labor and capital income. We study two models, the first with a two-period horizon and the second with overlapping generations and an infinite horizon.

An important benchmark is the case with full commitment. If tax policy could be chosen once and for all then standard results from the optimal taxation literature apply in our economy. In particular, the seminal result by Atkinson and Stiglitz (1976) implies that the optimal tax on capital is zero. Only a nonlinear tax on labor income is required.

However, without commitment, tax policy is not set in stone at the beginning of time. In our model, it is determined sequentially over time by governments with utilitarian welfare functions that decide taxes and transfers for the current period. The utilitarian welfare function captures a concern for inequality and a desire to redistribute. This implies that, without commitment, at any point in time, the most tempting deviation is to wipe the slate clean and implement the most extreme redistribution. In particular, this involves an expropriating capital levy.

In equilibrium, this extreme outcome can be prevented if there is a cost of deviating and if no government finds the benefits of a deviation more tempting than incurring this cost. In this paper we focus on two kinds of costs: direct costs and reputational costs. Ex post, these costs may hold back governments. Central to our paper is the notion that, ex ante, policies should be designed with an eye towards their credibility, and that inequality may be a crucial determinant of the latter. We believe both features are important in modern democratic societies.

In our two-period model we introduce a direct cost of reforming tax policy. This creates an intermediate form of commitment. In our infinite horizon model with overlapping generations, we assume there are no direct costs of reforming tax policy. Instead, we focus on the concerns for the loss of reputation. Reputation there works as follows. Upon observing a deviation, the private sector's expectations may shift, anticipating future governments to behave similarly. This may lead to a bad economic outcome, in which agents do not produce to avoid expropriation.

This is formalized using a dynamic game. Equilibrium outcomes must satisfy credibility constraints that ensure that no government prefers to deviate towards full redistribution. The best outcome can then be determined as the solution to a social planning problem incorporating these credibility constraints.[2]

As a consequence of these credibility constraints, policies deviate from the normative benchmarks provided by the optimal tax literature. Our main result is that capital taxation is progressive in the sense that agents that enjoy higher consumption face higher marginal tax rates on their savings. We show that this feature can be implemented with a tax schedule on wealth that is convex. As for the level, marginal tax rates may be positive over some regions and negative over others. Indeed, in the two period version of the model, the marginal tax rate on capital is always positive at the very top and negative at the very bottom.

The intuition for these results is as follows. The sign and level of the marginal tax rate placed on any agent is determined by the net effect that an extra unit of capital held by this agent has on the credibility constraint. On the one hand, an extra unit of capital in the hands of some particular agent increases the equilibrium value of the utilitarian objective. In fact, it does so according to this agent's marginal utility. On the other hand, more capital also raises the value of a deviating policy towards full redistribution. The sign of the optimal marginal tax depends on the net of these two effects, since this determines whether it is preferable to encourage or discourage savings by any particular agent. For instance, for a very rich agent, with high consumption and low marginal utility, an extra unit of saving has a negligible effect on the equilibrium utilitarian value. However, the extra unit of capital improves non-trivially the value attached to the deviation towards full redistribution. Thus, capital may be positively taxed for rich agents. The reverse may be true for poor enough agents with low consumption and high marginal utility. Capital

[2]In the infinite horizon model, the reputational mechanism discussed above corresponds to a trigger-strategy equilibrium, where a deviation is followed by a bad continuation equilibrium. We draw on Chari and Kehoe (1990) and focus on sustainable plans or policies, a refinement that focuses on symmetric perfect Bayesian equilibria.

may be subsidized for these agents.

The same principle explains the progressivity of the marginal tax rate. The value that an extra unit of capital has on the deviation path with full redistribution is independent of who does the extra saving. The difference between this common value of one unit of capital under a deviation, and the value obtained in equilibrium from this extra capital, which equals that agent's marginal utility, is then solely a function of that agent's consumption. Thus, agents with higher consumption face a higher marginal tax on capital.

The progressivity in the taxation of capital reflects an important feature of the allocation, that individual consumption is mean reverting. Agents with higher consumption have lower average consumption growth. This requires that they face lower after-tax rates of return, explaining the progressivity in marginal taxes on capital. It is optimal to have mean reversion in consumption because this makes policies more credible. Progressive taxation of capital emerges to reduce wealth inequality by discouraging accumulation amongst the rich and encouraging it amongst the poor.

It is interesting to compare our results with actual policies. It is difficult to come up with precise estimates of the level and progressivity of capital taxes (more so than for income taxes). But progressivity of marginal tax rates is, broadly speaking, a feature of actual capital tax policy in developed economies. Factors that contribute to positive capital taxes are corporate taxes, income taxes (where income includes not only labor but also capital income, with usually some special treatment for capital gains), wealth taxes (in some countries), and estate taxes. In most countries income taxes, estate taxes, and wealth taxes when they exist, are progressive. Moreover, many tax exempted instruments to promote retirement savings suchs as IRA's and 401(k)'s have maximum annual contribution levels. The result is that at least to some extent, capital taxes are progressive.

Our theory provides the first theoretical justification, to the best of our knowledge, for this common feature of policy. However, some differences remain between our theoretical results and actual policy. First, our theory predicts that capital taxes should be negative for some households (at the bottom of the distribution). While this feature seems at odds with actual policy, the comparison is more nuanced. Consider for example the case of educational subsidies, which can be thought as negative marginal tax rates on human capital accumulation. Second, our theory does not predict that marginal tax rates on labor and capital should be related in a simple manner. This is in contrast with some features of actual policy. For example, in reality, income taxation constitutes a joint taxation of income and capital.

Related literature. In this paper we build on our previous research by Farhi and Werning (2007, 2008) and Sleet and Yeltekin (2006, 2008a,b).

Within an intergenerational setting, Farhi and Werning (2007) studies an endowment economy where altruistic agents face privately observed taste shocks. In this setting, when the welfare of the first generation is maximized, the allocation features immiseration so that there is no non-degenerate invariant distribution, as in Atkeson and Lucas (1992). Following Phelan (2006) the paper considers other efficient allocations and traces out the Pareto frontier between current and future generations by adding the constraint that the expected welfare of all future generations remain above some *exogenous* level. A key result is that these efficient allocations feature mean reversion, and as a consequence, immiseration is overturned. Farhi and Werning (2008) studies a Mirrleesian model with capital and focus on implications for taxation, especially estate taxation. The main result is that the optimal marginal estate tax is progressive and negative. That is, intergenerational transfers should be subsidized, but the marginal subsidy should be smaller for larger estates.

The current paper's setup and results build on these two papers, but with important differences. In the present model, policies are decided by successive governments, without commitment. In any given period, the current government's objective function is utiliarian over the agents currently alive. Credible policies must keep future utilitarian welfare for all successive governments above some level. Unlike the previous normative models mentioned above, this level is now *endogenous.* It corresponds to the value attached to deviating towards full redistribution, given that this then involves a direct cost (as in our two period model) or an indirect cost by triggering a bad continuation equilibrium (as in our infinite horizon model with overlapping generations). We characterize the equilibrium outcomes that maximize weighted utilitarian welfare criteria over the different generations of agents. In terms of results, our implementation using nonlinear capital taxation is similar to the one for estate taxation in Farhi and Werning (2008). Indeed, the tax schedule shares the progressivity feature in both cases. However, an important difference is that, whereas estate taxes were always negative, here we find that positive marginal taxes may be optimal.

Sleet and Yeltekin (2006) consider the implications of a lack of societal credibility in an Atkeson-Lucas economy without capital. In such settings, they show that the optimal allocation from the perspective of the initial generation solves the problem of a committed planner who attaches positive weight to later generations. In later work, Sleet and Yeltekin (2008a) integrate this analysis with an explicit model of voting over future allocations. The current paper significantly extends these results. First, the earlier papers of

Sleet and Yeltekin focus on allocations, not implementations. They do not derive implications for taxes. Second, these earlier papers do not include physical capital and assume a different demographic structure from that adopted here. The introduction of capital and the assumption of overlapping generations modifies and complicates the connection between credibility and the societal weighting of generations. The overlapping generations framework explicitly separates personal intertemporal from social intergenerational discounting. Binding credibility constraints in the future create a motive for suppressing inequality amongst the old, but have ambiguous implications for capital accumulation. In contrast, increases in a committed planner's generational discount factor raise capital accumulation, but may need not lead to a reduction in inequality amongst the old.

We also make contact with a literature on political economy incorporating limited commitment and heterogenous agents. Benhabib and Rustichini (1996) study the link between wealth and investment in a dynamic game where output in every period is split between consumption by two social groups and investment. They focus on the best subgame perfect equilirbium. The most profitable deviations involve one group extracting as much consumption as possible, leaving no resources for investment. They show in examples that this might lead to lower capital accumulation in equilibrium than in the first best. Whether these effects are more pronounced at low or high wealth levels depends on the curvature of the utility and production functions, resulting respectively in growth traps or situations with low growth at high wealth levels. Acemoglu et al. (2008) and Acemoglu et al. (2007) study a model in which policy is set by a self-interested ruler or dictator who derives utility from private consumption. They focus on the best equilibrium of the game without commitment. The ruler's preferred deviation expropriates all the economy's resources for its own private consumption; thus, higher capital increases the attractiveness of this deviation. As a result, the best equilibrium discourages accumulation, implying a positive marginal tax on capital. In contrast to our main result regarding progressivity, in their setting all agents face the same positive tax rate. In addition, unless the ruler is impatient, these distortions disappear in the long run because promised consumption transfers to the ruler are backloaded in a way that makes the credibility constraint eventually not bind.[3] Bisin and Rampini (2006) study optimal policy in an economy with unobservable endowments, when the planner has limited commitment. In their model, in contrast to ours, reforms are not associated with any cost. As a result, full redistribution always oc-

[3] Acemoglu et al. (2007) considers an extension where the ruler's objective is a weighted average of utilitarian welfare and the utility from its private consumption. This model is closer to ours, although they do not consider the ruler's weight on private consumption to be zero. We conjecture that our main result on progressive capital taxation may obtain for this extension. However, this is not addressed because they only study the aggregate distortions to capital accumulation, not individual ones.

curs in the second period. They show that in this context, it is welfare improving to give agents access to anonymous markets that the government cannot monitor.

The normative literature on capital taxation provides an important benchmark for our results. Many normative optimal taxation models prescribe zero capital taxation. On the one hand, Chamley (1986) and Judd (1985) have shown that in Ramsey models, capital taxes should not be used in steady-state to finance government expenditures. On the other hand, in a Mirrlees context, the uniform-taxation result by Atkinson and Stiglitz (1976) shows that optimal capital taxes are zero when is no uncertainty and preferences are separable. A few theoretical papers analyze non-linear capital taxes under commitment. Saez (2002) considers a model where the only source of heterogeneity is initial wealth. In this setting, an initial capital levy that fully redistributes capital is optimal. Saez assumes an exogenous upper bound on the marginal tax rate and characterizes the optimal sequence of piecewise linear capital tax schedules. Benabou (2002) constructs a model with human capital, instead of physical capital, and studies nonlinear taxation of income, within a one-dimensional parametric class. As mentioned above, Farhi and Werning (2008), study the related issue of nonlinear estate taxation.

Two branches of the political economy literature have touched upon the issue of capital taxation. Both strands of literature have rationalized positive tax rates on capital, but have largely ignored the nonlinear taxation of capital.

The first branch revolves around the idea of time inconsistency first introduced by Kydland and Prescott (1977). The typical setup is a Ramsey model with a representative agent and a government which finances a public good using linear taxes. The central idea is that once sunk, capital is inelastic, so that capital taxation is equivalent ex-post to lump sum taxation. See Fischer (1980) and Klein and Rios-Rull (2003); Klein et al. (2008) for a more recent treatment. Several papers analyze how reputation mechanisms can alleviate the time inconsistency problem and result in intermediate levels of capital taxation. See for example Kotlikoff et al. (1988), Chari and Kehoe (1990) and Phelan and Stacchetti (2001).

The second branch is closest to our paper. It studies the linkage between income distribution, redistribution and growth, but mostly abstracts from time inconsistency problems.[4] The typical setup features heterogenous agents and linear taxation combined with lump sum rebates. If the median voter is less productive than the mean voter (Persson and Tabellini (1994a)), or if the median voter derives a lesser fraction of its total income

[4]One notable exception is Persson and Tabellini (1994b), who reintroduce a time inconsistency problem in an otherwise similar model. They emphasize strategic delegation, whereby voters might elect a government that has a disproportionate stake in capital income.

from capital than the mean voter (Alesina and Rodrick (1994); Bertola (1993)), strictly positive and higher than optimal capital tax rates will be chosen in the political equilibrium.

Our model combines elements of both literatures. Time inconsistency arises in our setup because of the interaction between dynamic incentive provision and redistribution. Incentives require inequality in consumption and savings. However, because of a concern for equality, it is tempting to expropriate capital holdings and fully redistribute. The main result of the paper that capital taxes are progressive is a new insight.

2 A Two Period Economy

We begin with a simple two period version of the model that helps bring out the essential mechanism underlying our results. The economy is populated by a continuum of agents of measure one that live in periods $t = 0, 1$. Agents work only in the period $t = 0$ and consume in both periods $t = 0, 1$.

A worker with productivity θ_0 that exerts work effort e_0 delivers $n_0 = e_0 \cdot \theta_0$ effective units of labor. Productivity shocks are i.i.d. draws from a distribution F and support Θ. Utility is given by

$$u(c_0) - h\left(\frac{n_0}{\theta_0}\right) + \beta u(c_1). \tag{1}$$

We assume that u and h are twice differentiable, u is concave, h is convex, and that u and h satisfy the Inada conditions $u'(0) = \infty$, $u'(\infty) = 0$, and $h'(0) = 0$.

The technology is specified by a linear production function in labor and capital. An allocation specifies consumption and labor for each agent as a function of productivity $(c(\theta_0), c_1(\theta_0), n_0(\theta_0))$. The resource constraints are then

$$\int c_0(\theta_0)\, dF(\theta_0) + K_1 \leq \int n_0(\theta_0)\, dF(\theta_0) + RK_0, \tag{2}$$
$$\int c_1(\theta_0)\, dF(\theta_0) \leq RK_1.$$

Here K_1 denotes aggregate capital with gross rate of return $R > 0$.

Following Mirrlees (1971), we assume that an agent's productivity, θ_0, and work effort, e_0, are privately observed. Only effective labor $n_0 = e_0 \cdot \theta_0$ and consumption are publicly observable. This leads us to study efficient allocations and the tax systems that implement them. By the revelation principle, any allocation that is attainable by some mechanism or tax system must satisfy the incentive compatibility constraints

$$u(c_0(\theta_0)) - h\left(\tfrac{n_0(\theta_0)}{\theta_0}\right) + \beta u(c_1(\theta_0)) \geq u(c_0(\theta_0')) - h\left(\tfrac{n_0(\theta_0')}{\theta_0}\right) + \beta u(c_1(\theta_0')) \quad . \tag{3a}$$

Under a direct mechanism, the agent is asked to report productivity and is assigned consumption and labor as a function of this report. The constraint ensures that truth telling is optimal.

An agent can always choose not to work, which requires

$$u(c_0(\theta_0)) - h\left(\tfrac{n_0(\theta_0)}{\theta_0}\right) + \beta u(c_1(\theta_0)) \geq (1+\beta)u(0) - h(0). \tag{3b}$$

Although this constraint is often ignored, omitting it amounts to assuming that agents can be forced to choose work within the equilibrium set $\{n_0(\theta_0)\}_{\theta_0 \in \Theta}$, which requires some punishment other than the withholding of consumption. We add this restriction to the incentive constraints to capture the idea that all incentives are provided through consumption.

For any allocation we can define the labor wedge or implicit marginal tax on labor $\tau^n(\theta_0)$ for an agent with productivity θ_0 as

$$\frac{h'\left(\frac{n_0(\theta_0)}{\theta_0}\right)}{u'(c_0(\theta_0))} = \theta_0\,(1 - \tau^n(\theta_0))$$

and the intertemporal wedge or implicit marginal tax on capital $\tau^k(\theta_0)$ for an agent with productivity θ_0 as

$$u'(c_0(\theta_0)) = \beta R(1 - \tau^k(\theta_0))u'(c_1(\theta_0)).$$

Of the two wedges, in this paper we are mainly concerned with the latter.

We say that an allocation is efficient if it maximize the utilitarian objective

$$\int \left(u(c_0(\theta_0)) - h\left(\tfrac{n_0(\theta_0)}{\theta_0}\right) + \beta u(c_1(\theta_0)) \right) dF(\theta_0)$$

subject to the resource and incentive compatibility constraints. Efficient allocations solve the dual planning problem

$$\min_{\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0), K_0, K_1\}} K_0 \tag{4}$$

subject to the resource constraints (2), the incentive constraints (3a) and

$$\int \left(u(c_0(\theta_0)) - h\left(\tfrac{n_0(\theta_0)}{\theta_0}\right) + \beta u(c_1(\theta_0)) \right)\, dF(\theta_0) \geq \bar{U}. \tag{5}$$

Efficient allocations do not distort the intertemporal consumption choice. The implicit marginal tax on capital is zero.

Proposition 1. *Let $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$ be an efficient allocation of the commitment economy. Then*

$$\tau^k(\theta_0) = 0 \qquad \forall \theta_0 \in \Theta.$$

This result follows as a corollary of the celebrated uniform taxation result by Atkinson and Stiglitz (1976). They showed that when preferences for a group of goods are weakly separable from work effort, these goods should be uniformly taxed, to avoid distortions in their relative consumption. In our case, the consumption pair over both periods (c_0, c_1) is weakly separable from work effort n_0 in the first period and the result applies.

This establishes an important benchmark for the results that follow. In our economy capital taxation should be zero. Any deviation from this principle arises from the lack of commitment.

3 Lack of Commitment and Capital Taxation

We now depart from the assumption of full commitment and consider a form of limited commitment which imposes an additional restriction on allocations. We call this new restriction the credibility constraint. In this section we motivate the credibility constraint somewhat informally and study the implications of imposing it on the planning problem. In the next section we study an explicit dynamic policy game where the credibility constraint emerges as a characterization of equilibrium outcomes.

The credibility constraint is motivated as follows. When period $t = 1$ comes along the original plan calls for the consumption assignment $c_1(\cdot)$ to be carried out. Imagine, however, that this plan can be reformed in favor of an alternative assignment $\hat{c}(\cdot)$. To determine whether a reform takes place, the original assignment is compared to the reformed one using a utilitarian criterion, $\int u(c(\theta_0))dF(\theta_0)$ versus $\int u(\hat{c}(\theta_0))dF(\theta_0)$. This captures a preference for equality that is key for our results. As we review later, the utilitarian criterion can be justified by embedding our economy in a probabilistic voting game. For now, it is simpler to proceed taking this criterion as given.

To avoid trivial solutions, we assume a reform costs $\kappa \geq 0$ units of goods, implying the resource constraint

$$\int \hat{c}(\theta_0)\, dF(\theta_0) \leq RK_1 - \kappa. \tag{6}$$

If a reform takes place, the criterion $\int u(\hat{c}(\theta_0))dF(\theta_0)$ is maximized by a constant consumption level:

$$\hat{c}_1(\theta_0) = RK_1 - \kappa.$$

Comparing the two alternatives, it follows that a reform can be avoided if and only if

$$\int u(c_1(\theta_0))\, dF(\theta_0) \geq u(RK_1 - \kappa). \tag{7}$$

One may interpret the fixed cost literally, perhaps as the opportunity cost of timely legislative procedures. However, its real purpose here is to allow for a simple form of limited commitment in our finite horizon setting. At one extreme, the case with $\kappa = \infty$ effectively delivers full commitment, as in the previous section. Indeed, the same outcome obtains for finite but high enough values of κ. At the other extreme, when $\kappa = 0$ there is no commitment and reform is imminent. Intermediate values of κ capture intermediate levels of commitment. Later, when we study a stationary overlapping generations economy, with an infinite horizon, we dispense with this fixed cost and study reputational equilibria, sustained by trigger strategies.

We say that allocations are credible if they satisfy inequality (7). An allocation that does not satisfy this inequality is not credible in the sense that it can be anticipated that a reform would take place in period $t = 1$. Because reforms are costly, it is best to avoid them. Thus, we consider allocations that maximize the utilitarian objective $\int \Big(u(c_0(\theta_0)) - h(\frac{n_0(\theta_0)}{\theta_0}) + \beta u(c_1(\theta_0)) \Big)\, dF(\theta_0)$ subject to the resource, incentive and credibility constraints. This leads us to study the dual planning problem

$$\min_{\{c(\theta_0), c_1(\theta_0), n_0(\theta_0), K_0, K_1\}} K_0 \tag{8}$$

subject to the resource constraints (2), the incentive constraints (3a), the credibility constraint (7)

$$\int \Big(u(c_0(\theta_0)) - h(\frac{n_0(\theta_0)}{\theta_0}) + \beta u(c_1(\theta_0)) \Big)\, dF(\theta_0) \geq \bar{U}. \tag{9}$$

Lack of commitment captures the idea that work effort has already taken place in period $t = 0$, so that incentives are no longer required, and equality is desirable in period $t = 1$.

3.1 Optimal Progressive Capital Taxation

Let $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$ be a solution to the dual planning problem. Consider the following variation around this optimum:

$$
\begin{aligned}
u(\tilde{c}_0(\theta_0)) &= u(c_0(\theta_0)) - \beta\delta_1(\theta_0) \\
u(\tilde{c}_1(\theta_0)) &= u(c_1(\theta_0)) + \delta_1(\theta_0)
\end{aligned}
$$

for any function $\delta_1(\cdot)$ and $\tilde{n}_0(\theta_0) = n_0(\theta_0)$. This perturbed allocation is incentive compatible and delivers the same utility as $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$. Thus, we can drop the promise keeping constraint (9) and the incentive constraints (3a) and minimize K_0 subject to the resource constraints (2) and the credibility constraint (7). Then the function $\delta_1(\theta) = 0$ for all $\theta \in \Theta$ is a solution to

$$\min_{\{\delta_1, K_0, K_1\}} K_0$$

subject to

$$
\begin{aligned}
\int c(u(c_0(\theta_0)) - \beta\delta_1(\theta_0))\, dF(\theta_0) + K_1 &\leq \int n_0(\theta_0)\, dF(\theta_0) + RK_0, \qquad (10)\\
\int c(u(c_1(\theta_0)) + \delta_1(\theta_0))\, dF(\theta_0) &\leq RK_1,
\end{aligned}
$$

and

$$\int u\big(c_1(\theta_0) + \delta_1(\theta_0)\big)\, dF(\theta_0) \geq u(RK_1 - \kappa). \qquad (11)$$

Let μ_0 and μ_1 be the multipliers on the resource constraints in (10) and ν be the multiplier on the credibility constraint (11). Then we have the following first order conditions:

$$
\begin{aligned}
\nu + \mu_0\beta\frac{1}{u'(c_0(\theta_0))} - \mu_1\frac{1}{u'(c_1(\theta_0))} &= 0, \\
-\nu R u'(RK_1 - \kappa) - \mu_0 + R\mu_1 &= 0,
\end{aligned}
$$

and

$$1 - R\mu_0 = 0.$$

These first order conditions can easily be rearranged to prove the following result.

Proposition 2. *Let $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$ be an efficient allocation of the no-commitment economy. Suppose that κ is low enough so that the full commitment solution is not feasible, and the* *credibility constraint is strictly binding. Then there exists $\nu > 0$ such that for all $\theta_0 \in \Theta$*

$$\tau^k(\theta_0) = \frac{R\frac{\nu}{\mu_0}(u'(RK_1 - \kappa) - u'(c_1(\theta_0)))}{1 + R\frac{\nu}{\mu_0}(u'(RK_1 - \kappa) - u'(c_1(\theta_0)))} \tag{12}$$

where $\mu_0 = \frac{1}{R} > 0$.

Several implications follow from this simple formula. First, capital taxation is progressive in the sense that the implicit marginal tax is increasing in consumption $c_1(\theta_0)$. Second, the sign of the marginal tax is determined by the sign of $u'(RK_1 - \kappa) - u'(c_1(\theta_0))$, which depends on θ_0. Indeed, for the agent consuming the most we have $RK_1 - \kappa = \int c_1(\theta_0)\, dF(\theta_0) - \kappa < \max_{\theta_0} c_1(\theta_0)$ ensuring that the marginal tax rate on capital is positive at the top. Similarly, for the credibility constraint to bind, it must be the case that consumption at the bottom is lower than consumption after a reform: $\min_{\theta_0} c_1(\theta_0) < RK_1 - \kappa$. This implies that the marginal tax rate on capital is negative at the bottom.

Corollary 1. *Suppose that κ is low enough so that the full commitment solution is not feasible, and the credibility constraint is strictly binding. Then the implicit marginal tax rate $\tau^k(\theta_0)$ is strictly increasing in θ_0, positive at the top, $\sup_{\theta_0 \in \Theta} \tau^k(\theta_0) > 0$, and negative at the bottom $\inf_{\theta_0 \in \Theta} \tau^k(\theta_0) < 0$.*

The magnitude and sign of the marginal tax rate is driven by $u'(RK_1 - \kappa) - u'(c_1(\theta_0))$ because this determines the net effect on the credibility constraint of an additional unit of capital held by an agent with productivity θ_0. This raises individual consumption, and thus raises the left hand side of the credibility constraint. At the same time, more capital raises the right hand side of the credibility constraint, the value of reforming. The former effect is smaller, the higher is θ_0 because the date 1 consumption $c_1(\theta_0)$ increases with θ_0; the latter effect is independent of θ_0. Indeed, for high enough θ_0 the net effect is that the constraint becomes tighter; the opposite is true for low enough θ_0. This explains the signs at the top and bottom.

The economic intuition is best understood by imagining an initial allocation that does not distort savings, that is, that satisfies Atkinson-Stiglitz's prescription. Suppose further that $R\beta = 1$ so that each individual's consumption is constant over time, $c_0(\theta_0) = c_1(\theta_0)$. Of course, some inequality is needed to provide incentives for work effort. The credibility of this allocation can be improved by reducing the inequality in the second period. This is accomplished while holding constant the lifetime utility of each individual by tilting the consumption of the rich towards the first period and that of the poor towards the second period. Since lifetime utility is unchanged, the same incentives are provided, but because inequality falls in the second period, credibility is improved. The cost of this allocation increases because we deviate from perfect consumption smoothing, but this effect is of second order. In essence, it is optimal to frontload the provision of incentives.

This example highlights that the progressivity in the taxation of capital reflects an important features of the allocation: individual consumption is mean reverting. Agents with higher consumption have lower consumption growth. This requires that they face lower after-tax rates of return, which requires progressivity in marginal taxes on capital.

3.2 Implementation with Taxes

We now provide a simple tax system that implements incentive compatible allocations using two separate nonlinear tax schedules, one for labor income and another for capital income. Agents face the following budget constraints:

$$\begin{aligned} c_0 + k_1 &\leq n_0 - T^n(n_0) + Rk_0, \\ c_1 &\leq Rk_1 - T^k(Rk_1). \end{aligned} \tag{13}$$

In the first period, after observing their productivity θ_0, agents make consumption, c_0, saving, k_1, and labor, n_0, choices. In the second period agents simply consume their after tax wealth. Given tax schedules T^n and T^k, a competitive equilibrium is an allocation $\{c_0(\theta_0),\ c_1(\theta_0),\ n_0(\theta_0)\}$ and $\{k_1(\theta_0)\}$ such that (i) agents optimize: each agent θ_0 maximizes their utility (1) subject to (13); and (ii) markets clear: the resource constraints (2) hold with equality. We say that tax schedules (T^n, T^k) implement an incentive compatible allocation $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$ if the latter is a competitive equilibrium for some $\{k_1(\theta_0)\}$. Our implementation result can now be simply stated.

Proposition 3. *Suppose $\{c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)\}$ is incentive compatible and nondecreasing in θ_0. Then there exist tax schedules (T^n, T^k) that implement this allocation as a competitive equilibrium.*

Incentive compatibility requires $n_0(\theta_0)$ and $u(c_0(\theta_0)) + \beta u(c_1(\theta_0))$ to be nondecreasing in θ_0. Efficient allocations with or without commitment feature nondecreasing $c_0(\theta_0)$ and $c_1(\theta_0)$. Thus, they can be implemented by separable tax schedules.

If the tax schedules are locally differentiable, then the first-order conditions for agent

θ_0 imply

$$\frac{h'\left(\frac{n_0(\theta_0)}{\theta_0}\right)}{u'(c_0(\theta_0))} = \theta_0 \left(1 - T^{n\prime}(n_0(\theta_0))\right), \tag{14}$$

$$u'(c_0(\theta_0)) = \beta R(1 - T^{k\prime}(Rk_1(\theta_0)))u'(c_1(\theta_0)). \tag{15}$$

In other words, marginal taxes equal implicit marginal taxes $T^{n\prime}(n_0(\theta_0)) = \tau^n(\theta_0)$ and $T^{k\prime}(Rk_1(\theta_0)) = \tau^k(\theta_0)$. Indeed, $T^k(Rk_1)$ is everywhere differentiable and $T^n(n_0)$ is differentiable at points where no bunching occurs, i.e. at points where $n_0(\theta_0)$ is locally strictly increasing. The corollary below spells out the implications for the capital income tax schedule and its derivative.

Corollary 2. *(i) Consider an efficient allocation of the commitment economy. Then it can be implemented with a nonlinear income tax T^n and a zero tax on capital $T^k(Rk_1) = 0$.*
(ii) Consider an efficient allocation of the no-commitment economy and suppose that the credibility constraint (7) is strictly binding. Then it can be implemented with tax schedules (T^n, T^k). The tax on capital T^k is convex and differentiable. The marginal tax rate is positive at the top $T^{k\prime}(R\bar{k}_1) > 0$ and negative at the bottom $T^{k\prime}(R\underline{k}_1) < 0$, where $\bar{k}_1 \equiv \max_{\theta_0} k_1(\theta_0)$ and $\underline{k}_1 = \min_{\theta_0} k_1(\theta_0)$.

These results translate our previous implications for implicit marginal tax rates into implications for explicit tax systems. They also allow us to reinterpret the planning problem in terms of a choice over tax systems, instead of allocations. The no-commitment economy maximizes over tax schedules (T^n, T^k) that are credible, in the sense of avoiding a tax reform. In the second period a tax reform amounts to replacing the tax schedule for capital income T^k with a capital levy that completely expropriates capital: $\hat{T}^k(Rk_1) = Rk_1 - RK_1 + \kappa$. Note that this reformed tax schedule $\hat{T}^k$ features a marginal tax that is constant and equal to 100%. Thus, our results on the progressivity of T^k are not obtained because of a similar progressivity in $\hat{T}^k$.

3.3 Probabilistic voting

We motivated the credibility constraint by considering a utilitarian planner with no commitment. We now provide political economy underpinnings for both the lack of commitment and the utilitarian social welfare function. Indeed, efficient allocations of the no-commitment economy can be interpreted as the solution of an explicit political economy game, following the probabilistic voting model along the lines of Coughlin (1986) and Lindbeck and Weibull (1987) which is well known to lead to an outcome that maximizes a utilitarian objective. Our purpose here is to sketch how these arguments adapt to our setting.

In the political economy game policies are decided as the outcome of an election. At the beginning of each period $t = 0, 1$ two candidates $i = A, B$ face off in an election. The winner is determined by simple majority. Voting behavior is described further below. Candidates attempt to maximize the probability of winning the election. Before voting takes place, candidates present their platforms to the electorate, publicly stating the policies they will pursue if elected.

In period $t = 0$ a platform consists of taxes $(T_0^{n,i}, T_0^{k,i})$. After the election, if candidate i^* is the winner then he holds office for one period and is committed to implementing the proposed platform. In period $t = 0$ this entails enacting both taxes into law. The tax on labor T_0^{n,i^*} is then immediately implemented. The tax on capital T_0^{k,i^*} remains on the books and takes effect in period $t = 1$ if it is not reformed. At this point, agents make their consumption, saving and labor choices $(c_0^{0,i^*}(\theta_0), k_1^{0,i^*}(\theta_0), n_0^{0,i^*}(\theta_0))$.

In period $t = 1$ a new election takes place. This is the source of the lack of commitment. Candidates take the distribution of capital $k_1^{0,i^*}(\theta_0)$ and the previously enacted tax on capital T_0^{k,i^*} as given and present platforms consisting of a tax on capital $T_1^{k,i}$. Once again, if candidate j^* is the winner, then he must implement T_1^{k,j^*}. If the corresponding capital tax schedule T_1^{k,j^*} differs from the schedule enacted in the previous period, T_0^{k,i^*} then the economy incurs a resource cost κ.

In period $t = 0$ agents consider the welfare implications of platforms $j = A, B$, computing

$$v_0^j(\theta_0) = u(c_0^{0,j}(\theta_0)) + \beta u(c_1^{0,j}(\theta_0)) - h\left(\frac{n_0^{0,j}(\theta_0)}{\theta_0}\right) \qquad j = A, B,$$

where $(c_0^{0,j}(\theta_0), c_1^{0,j}(\theta_0), n_0^{0,j}(\theta_0))$ denotes the allocation that results if candidate j wins in period $t = 0$. Likewise, in period $t = 1$ agents compute

$$v_1^j(\theta_0) = u(c_1^{1,j}(\theta_0)) \qquad j = A, B,$$

where $c_1^{1,j}(\theta_0)$ denotes the allocation that results if candidate j wins in period $t = 1$.

In deciding which candidate to cast their vote for, agents care about the sum of two variables: the welfare implied by the platform and an idiosyncratic candidate-specific taste shock. In period t, an agent i with productivity θ_0 votes for A over B if and only if

$$v_t^A(\theta_0) + \varepsilon^{i,A} > v_t^B(\theta_0) + \varepsilon^{i,B}; \tag{16}$$

ties are broken by voting with equal probability for each candidate.

The ε shock captures ideological preferences, fondness based on a candidate's personality, or any other consideration that leads individuals not to vote entirely based on their self interest. It implies that for each productivity type θ_0, voters take different sides in the election. As a result, candidates choose their platform with an eye to pleasing agents across the productivity spectrum. This is in sharp contrast to the median voter setup, where there is a single type θ_0 that is the marginal voter and candidates cater their platform to this single agent.

We assume that $\Delta^i_\varepsilon = \varepsilon^{i,B} - \varepsilon^{i,A}$ is independent of θ_0. The probability that platform A wins the election is then given by

$$\int G\left(v^A_t(\theta_0) - v^B_t(\theta_0)\right)\, dF(\theta_0) \tag{17}$$

where G is the distribution for Δ_ε. The political equilibrium takes a very simple form when the distribution for Δ_ε is uniform on an interval $[-m_\varepsilon, m_\varepsilon]$ that is wide relative to the range of agent utilities. The assumption that agent utility is bounded and that m_ε is large relative to that bound ensures that $v^A_t(\theta_0) - v^B_t(\theta_0)$ lies in the interior of the support of Δ_ε.[5][6]

Since the cumulative distribution function in (17) is linear in the interior of the support, $G\left(v^A_t(\theta_0) - v^B_t(\theta_0)\right) = \left(v^A_t(\theta_0) - v^B_t(\theta_0)\right)/(2m_\varepsilon)$, each candidate positions its platform to maximize the utilitarian welfare criterion

$$\int v_t(\theta_0)\, dF(\theta_0). \tag{18}$$

Since both candidates choose the same platform, both are elected with equal probability. It follows that the subgame perfect equilibria of this political economy game coincide with efficient allocations of the no-commitment economy.

4 Overlapping Generations

We now turn to an infinite horizon overlapping generations economy. This allows us to study the credibility problem as a dynamic game. In contrast to the two-period version, we do not introduce an exogenous cost of reform. Instead, good policies are sustained by

[5] Assuming the density is uniform simplifies the analysis but is not critical. As is well known, the same results would obtain for a larger class of non-uniform distributions that ensures that the candidates' platform problem is sufficiently convex.

[6] The upper bound on effective labor is finite $\bar{n} < \infty$. There exists a bound $M > 0$ such that for all c, $n < \bar{n}$ and θ, $|u(c) - h(n/\theta)| < M$. Moreover, $2M(1+\beta) < m_\varepsilon$.

a concern for shaping private expectations regarding future policy. Formally, we consider reputational equilibria, in which the threat of reversion to a less desirable equilibrium deters deviations from a good equilibrium.

4.1 Model Setup

The time horizon is infinite with period $t = 0, 1, \ldots$ The economy is populated by overlapping generations of agents that live for two periods. Agents born in period t work only when young, n_t, and consume when young and old, c_t^y and c_{t+1}^o, in periods t and $t+1$.

An agent with productivity θ_t that exerts work effort e_t delivers $n_t = e_t \cdot \theta_t$ effective units of labor. Agents' productivities are i.i.d. draws from a distribution F with support Θ. Productivity and effort are private information; effective labor and consumption are observable.

The utility of an agent of generation t is given by

$$u(c_t^y) - h\left(\frac{n_t}{\theta_t}\right) + \beta u(c_{t+1}^o). \tag{19}$$

We assume that u and h are twice differentiable, u is concave, h is convex, and that they satisfy the Inada conditions $u'(0) = \infty$, $u'(\infty) = 0$, and $h'(0) = 0$.

The resource constraint in period t is given by:

$$C_t^y + C_t^o + K_{t+1} \leq F(K_t, N_t) \tag{20}$$

where $C_t^y = \int c_t^{y,i} di$ is aggregate consumption of young agents (indexed by $i \in [0,1]$), $C_t^o = \int c_t^{o,j} dj$ is aggregate consumption of old agents (indexed by $j \in [0,1]$), $N_t = \int n_t^i di$ is aggregate labor of the young, and K_t is aggregate capital. The production function F is assumed to be homogeneous of degree one, strictly increasing and continuously differentiable.

Policy decisions are made by a sequence of governments indexed by $t = 0, 1, \ldots$ Government t cares only about agents that are currently alive and evaluates welfare according to the utilitarian criterion

$$\tilde{\mathbb{E}}_t \left[\lambda \int u(c_t^{o,j}) dj + (1-\lambda) \int \left(u(c_t^{y,i}) - h\left(\frac{n_t^i}{\theta_t^i}\right) + \beta u(c_{t+1}^{o,i}) \right) di \right]$$

with relative weights λ and $1-\lambda$ on the old and young, respectively. Here $\tilde{\mathbb{E}}_t$ denotes the beliefs of government t.

The Policy Game. Informally, the sequence of events is as follows. At the beginning of period t the capital stock K_t and all past actions are given and publicly known. Each young agent then privately observes its own productivity θ_t^i and chooses labor n_t^i. At the end of the period output $F(K_t, \int n_t^i di)$ becomes available. The current government chooses a consumption assignment for the young and the old, as a function of their labor choice, as well as the level of capital for next period. The only constraint at this point is the resource constraint.

Note that once agents' labor decisions are sunk the government can distribute consumption equally. In this sense, the government has no commitment device to provide incentives.

We incorporate the following restrictions on strategies. We restrict attention to symmetric pure strategies for agents such that agents with the same productivity take the same action, i.e. $\theta_t^i = \theta_t^j$ with $i \neq j$ implies $n_t^i = n_t^j$. We also restrict attention to strategies for agents and governments that react to labor choices $\{n_t^i\}_{i\in[0,1]}$ only through the implied distribution of labor. In this sense, agents are treated anonymously.[7]

Formally, the game is described as follows. The relevant public history H_t at the beginning of period t consists of the sequence of past government choices and the distribution of private choices for labor:

$$H_t = \left\{H_{t-1}, \bar{c}^o_{t-1}(\cdot), \bar{c}^y_{t-1}(\cdot), K_t, G_{t-1}\right\}$$

for $t = 1, 2, \ldots$ and $H_0 = \{K_0, G_{-1}\}$. Here $\bar{c}^o_t(\cdot)$ is a function that maps the period $t-1$ effective labor n^j_{t-1} of agent j of generation $t-1$ into current consumption. Similarly, the function $\bar{c}^y_t(\cdot)$ maps the effective labor n_t^i of agent i of generation t into current consumption. Finally, G_t denotes the cumulative distribution function for $\{n_t^i\}_{i\in[0,1]}$.

Within period t there are two stages:

1. Productivities $\{\theta_t^i\}_{i\in[0,1]}$ of young agents are realized. Agent i privately observes θ_t^i and chooses labor n_t^i. Output $F(K_t, N_t)$ is produced.

2. The current government observes H_t and the distribution G_t of current labor choices. Let $\hat{H}_t = \{H_t, G_t\}$ be the corresponding interim history. The current government then chooses current policy $\bar{c}^o_t(\cdot)$, $\bar{c}^y_t(\cdot)$ and K_{t+1} subject to the resource constraint.

This timing assumption is slightly different than that of our two-period economy. The government here decides consumption for the young after their labor choices are sunk.

[7]These two restrictions, symmetry and anonymity, are commonly imposed in the analysis of policy games. For example, they are imposed by Chari and Kehoe (1990) to define their notion of sustainable plans in the context of their Ramsey policy game.

This implies a stronger form of lack of commitment, in that there is no commitment even within a period.[8]

We now define strategies. In period t, each young agent observes the public history H_t and the private shock θ_t. We denote an agent's strategy by $\sigma_t(H_t, \theta_t)$, so that the labor choice made by agent i is given by $n_t^i = \sigma_t(H_t, \theta_t^i)$.[9] The strategy of government t is described by $\tau_t \equiv (\tau_t^{c,o}(\hat{H}_t, \cdot), \tau_t^{c,y}(\hat{H}_t, \cdot), \tau_t^K(\hat{H}_t))$, implying policies $\bar{c}_t^o(\cdot) = \tau_t^{c,o}(\hat{H}_t, \cdot)$, $\bar{c}_t^y(\cdot) = \tau_t^{c,y}(\hat{H}_t, \cdot)$, and $K_{t+1} = \tau_t^K(\hat{H}_t)$.

We denote the strategies of all generations of agents by $\sigma \equiv \{\sigma_t\}_{t=0}^\infty$ and those of governments by $\tau \equiv \{\tau_t\}_{t=0}^\infty$. We focus on Perfect Bayesian equilibria for this dynamic game.

We specialize our study of equilibria to the case where G_{-1} is the cumulative distribution function of $\{\sigma_{-1}(\theta_{-1})\}_{\theta_{-1}\in\Theta}$ for some exogenous function σ_{-1} and we include this function in H_0. Given a history H_t, the strategies (σ, τ) determine labor for agents and the government policy in period t. These actions then generate an updated history H_{t+1}. Proceeding in this way and starting from $t = 0$, we can define the entire sequence $\{H_t\}_{t=0}^\infty$ and the corresponding allocation outcome defined by $n_t(\theta_t) = \sigma_t(H_t, \theta_t)$, $c_t^y(\theta_t) = \tau_t^{c,y}(\hat{H}_t, \sigma_t(H_t, \theta_t))$, $c_t^o(\theta_{t-1}) = \tau_t^{c,o}(\hat{H}_t, \sigma_{t-1}(H_{t-1}, \theta_{t-1}))$ and $K_{t+1} = \tau_t^K(\hat{H}_t)$. We rank outcomes according to a utilitarian welfare measure placing weight λ_t on the welfare of generation t:

$$U = \lambda_{-1} \int u\left(c_0^o(\theta_{-1})\right) dF(\theta_{-1}) + \sum_{t=0}^{\infty} \lambda_t \int \left(u\left(c_t^y(\theta_t)\right) - h\left(\frac{n_t(\theta_t)}{\theta_t}\right) + \beta u(c_{t+1}^o(\theta_t)) \right) dF(\theta_t).$$

We are interested in characterizing the best equilibrium from the perspective of this objective. We work with the dual problem: K_0, such that there exists an equilibrium with associated utility equal to $\underline{U}$.

Following Chari and Kehoe (1990), we next develop a set of necessary and sufficient conditions for an allocation to be the outcome of an equilibrium. We term these allocation outcomes sustainable. Studying the best equilibrium is then reduced to a constrained programming problem.

[8]An alternative timing assumption is to allow the government to commit within a period to a rule for consumption for the young (or to some tax schedules). One difficulty is that, in order to satisfy the resource constraint, one must consider commitment to rules that adjust with the realized distributions of labor G_t. For this reason, this model is technically harder to work with but has similar implications.

[9]Note that the strategy does not depend on i, capturing our restriction to symmetric strategies.

4.2 Sustainable Allocations and the Credibility Constraint

An *allocation* $\left(\{c^o_t, c^y_t, n_t\}_{t\geq 0}, \{K_t\}_{t\geq 0}\right)$ consists of sequences of consumption functions for the old $c^o_t : \Theta \to \mathbb{R}^+$, consumption functions for the young $c^y_t : \Theta \to \mathbb{R}^+$, labor supply functions $n_t : \Theta^t \to \mathbb{R}^+$, and capital stocks $K_t \in \mathbb{R}^+$. We denote by $C^y_t = \int c^y_t dF(\theta_t)$ the aggregate consumption of young agents, $C^o_t = \int c^o_t(\theta_{t-1})$ the aggregate consumption of old agents, and $N_t = \int n_t(\theta_t) dF(\theta_t)$ the aggregate labor of the young. We denote by G_t is the cumulative distribution function for labor in period t of $\{n_t(\theta_t)\}_{\theta_t \in \Theta}$.

Feasible allocations. The allocation is *incentive compatible* if for all t

$$u(c^y_t(\theta_t)) - h\left(\tfrac{n_t(\theta_t)}{\theta_t}\right) + \beta u(c^o_{t+1}(\theta_t)) \geq u(c^y_t(\theta'_t)) - h\left(\tfrac{n_t(\theta'_t)}{\theta_t}\right) + \beta u(c^o_{t+1}(\theta'_t)) \quad \forall \theta_t, \theta_t{}' \in \Theta,$$
$$u(c^y_t(\theta_t)) - h\left(\tfrac{n_t(\theta_t)}{\theta_t}\right) + \beta u(c^o_{t+1}(\theta_t)) \geq (1+\beta)u(0) - h(0). \quad (21)$$

We say that an allocation $\left(\{c^o_t, c^y_t, n_t\}, K_t\right)$ is *feasible* if: (i) it is incentive compatible; (ii) it satisfies the following resource constraint for every t:

$$\int c^0_t(\theta_{t-1}) dF(\theta_{t-1}) + \int c^y_t(\theta_t) dF(\theta_t) \leq F\big(K_t, \int n_t(\theta_t) dF(\theta_t)\big) - K_{t+1}. \quad (22)$$

Sustainable allocations. We say that an allocation $\left(\{c^o_t, c^y_t, n_t\}_{t\geq 0}, \{K_t\}_{t\geq 0}\right)$ is *sustainable* if it is the outcome of an equilibrium. Following Chari and Kehoe (1990), we can derive a simple set of necessary and sufficient conditions for a feasible allocation to be the outcome of an equilibrium.

Consider a function $\tilde{W}(K, G_-)$. In any period t we define the credibility constraint as the following inequality:

$$\lambda \int u(c^0_t(\theta_{t-1})) dF(\theta_{t-1}) + (1-\lambda) \int \left(u(c^y_t(\theta_t)) + \beta u(c^o_{t+1}(\theta_t))\right) dF(\theta_t)$$
$$\geq \hat{W}\left(K_t, G_t; \tilde{W}\right), \quad (23)$$

where

$$\hat{W}(K, G; \tilde{W}) = \max_{c^o, c^y, K'} \left\{\lambda u(c^o) + (1-\lambda)\left(u(c^y) + \beta \tilde{W}(K', G)\right)\right\} \quad (24)$$

subject to the resource constraint $c^0 + c^y + K' \leq F(K, \int n dG(n))$.

We say that an allocation is sustainable given $\tilde{W}$ if: (i) it is feasible; and (ii) it satisfies the sequence of credibility constraints (23).

Given a pair of equilibrium strategies (σ_R, τ_R), we denote by $W^{(\sigma_R, \tau_R)}(K, G_-)$ the associated equilibrium value for the expected payoff of the initial old $\int u\left(\hat{c}_0^o(n_{-1})\right) dG_{-1}(n_{-1})$ when the initial history is given by $K_0 = K$ and $G_{-1} = G_-$. We can then construct trigger strategies that revert to (σ_R, τ_R) upon a deviation. The following proposition characterizes the allocations that are outcomes of such equilibria.

Proposition 4. *Let (σ_R, τ_R) be a pair of equilibrium strategies. An allocation is sustainable given $W^{(\sigma_R,\tau_R)}$ if and only if it is the outcome of an equilibrium with trigger strategies reverting to (σ_R, τ_R) upon a deviation.*

Let (σ_R, τ_R) be a pair of equilibrium strategies and consider a trigger strategy equilibrium in which a deviation is followed by a reversion to (σ_R, τ_R). The credibility constraint ensures that the period t government prefers the equilibrium outcome to deviating. For a period t government the benefits of a deviation are threefold. First, the government can equalize the consumption of the old. Just as in the two period model, work effort for generation $t-1$ has already taken place in period $t-1$, so that incentives are no longer required, and equality in consumption for the old in period t is desirable for the utilitarian objective. Second, the government can equalize consumption of the young: work effort for generation t has already taken place in the first stage of period t, so that incentives are no longer required, and equality in consumption for young agents in period t is desirable for the utilitarian objective. Finally, the government can achieve its optimal balance (depending on the relative Pareto weight λ) between the consumption of the old, the consumption of the young, and is also free to choose the level of capital for the next period. These benefits are reflected in the definition of $\hat{W}$, which incorporates that there is no consumption inequality within a generation and that the level of consumption for the young, consumption for the old and capital are chosen freely. The period t government must weigh these benefits against the fact that the deviation triggers a reversion to (σ_R, τ_R) in period $t+1$, so that the average utility for its old will be $W^{(\sigma_R,\tau_R)}$, as reflected in the definition of $\hat{W}$.[10]

The greatest deterrent is achieved by the lowest possible value for $W^{(\sigma_R,\tau_R)}$, which we call the worst and denote by $W(K, G_-) = \inf W^{(\sigma_R,\tau_R)}(K, G_-)$ where the infimum is over pairs of equilibrium strategies (σ_R, τ_R). The next proposition shows that the worst can be used to characterize the entire set of equilibrium outcomes.

Proposition 5. *If an allocation is the outcome of an equilibrium, then it is sustainable given the worst W. Conversely, suppose that there exists a pair of equilibrium strategies (σ_R^W, τ_R^W) leading*

[10]Note that we do not include the disutility of labor on the left-hand side of equation (23) and in the definition of $\hat{W}$. This simplification arises from the fact that the period t government chooses policies after generation t agents have made their labor supply decisions.

to the worst $W^{(\sigma_R^W,\tau_R^W)}(K,G_-) = W(K,G_-)$. Take any allocation that is sustainable given W. Then this allocation is the outcome of an equilibrium with trigger strategies reverting to (σ_R^W,τ_R^W) upon a deviation.

Efficient allocations. For any allocation, we can compute the associated utility

$$U\big(\{c_t^o,c_t^y,n_t\},K_t\big) = \lambda_{-1}\int u\left(c_0^o(\theta_{-1})\right)dF(\theta_{-1}) + \sum_{t=0}^{\infty}\lambda_t\int\left(u(c_t^y(\theta_t)) - h\left(\frac{n_t(\theta_t)}{\theta_t}\right) + \beta u(c_{t+1}^o(\theta_t))\right)dF(\theta_t).$$

For a given $\underline{U}$, we define an allocation to be *efficient* given $\tilde{W}$ if it solves the following planning problem:

$$\min_{\{c_t^0,c_t^y,n_t\}_{t\geq 0},\{K_t\}_{t\geq 0}} K_0 \tag{25}$$

subject to $\big(\{c_t^0,c_t^y,n_t\}_{t\geq 0},\{K_t\}_{t\geq 0}\big)$ being a sustainable allocation given $\tilde{W}$ and the constraint that $U\big(\{c_t^o,c_t^y,n_t\},K_t\big)\geq \underline{U}$.

There are similarities and differences in the determinants of credibility constraints in the overlapping generations planning problem (25) and in the two period planning problem (8) of Section 3. In both models, there is a benefit from deviating in order to equalize the consumption of the old, who have exerted work effort in the previous period. In the overlapping generations setting, there are two additional benefits from deviating: equalizing consumption for the young who have exerted effort earlier in the period, and achieving an optimal balance between consumption of the old and consumption of the young. These benefits are absent in the two period model in which there is only one generation: there are only old agents and no young agents at the time when a deviation is considered. The costs of a deviation are also slightly different. In the overlapping generations model, the cost of a deviation comes in the future in the form of a bad continuation equilibrium for the old in the next period. In the two period model, the cost of a deviation comes in the same period in the form of a waste of resources.

A roadmap. Suppose that there exists a pair of equilibrium strategies (σ_R^W,τ_R^W) leading to the worst. Then efficient allocations given W solve the problem that we set out to answer: these allocations are the ouctomes of equilibria with associated utility equal to $\underline{U}$ which use the least initial resources K_0. More generally, efficient allocations given $W^{(\sigma_R,\tau_R)}$ are the outcomes of the trigger equilibria which use the least amount of initial resources K_0 among the equilibria which revert to (σ_R,τ_R) upon a deviation and have associated utility equal to $\underline{U}$.

In Sections 4.3 and 4.4, we provide a characterization of efficient allocations given any function $\tilde{W}$ such that $\hat{W}(K, G; \tilde{W})$ is increasing and differentiable in K. We can then apply these results to the case where $\tilde{W} = W^{(\sigma_R, \tau_R)}$ or $\tilde{W} = W$ in order to obtain characterizations and implementations of the corresponding equilibrium outcomes of the policy game. In Section 4.5, we provide sufficient conditions for $\hat{W}(K, G; W)$ to be increasing and differentiable in K.

4.3 Optimal Progressive Capital Taxation

For any allocation, period t and productivity $\theta_t \in \Theta$, we can define the labor wedge or implicit marginal tax on labor $\tau_t^n(\theta_t)$ as

$$\frac{h'\left(\frac{n_t(\theta_t)}{\theta_t}\right)}{u'(c_t^y(\theta_t))} = \theta_t F_N\left(K_t, N_t\right)\left(1 - \tau_t^n(\theta_t)\right)$$

and the intertemporal wedge or implicit marginal tax on capital $\tau_t^k(\theta_t)$ as

$$u'(c_t(\theta_t)) = \beta F_K\left(K_{t+1}, N_{t+1}\right)(1 - \tau_{t+1}^k(\theta_t))u'(c_{t+1}^o(\theta_t)).$$

We can derive necessary conditions for optimality in the planning problem (25) exactly as in Section 3. Putting multipliers $\{\mu_t\}_{t\geq 0}$ and $\{\nu_t\}_{t\geq 0}$ on the resource constraints (22) and the credibility constraints (23), we can derive two key necessary first order conditions:

$$\lambda \nu_{t+1} + \mu_t \beta \frac{1}{u'(c_t^y(\theta_t))} - \mu_{t+1} \frac{1}{u'(c_{t+1}^o(\theta_t))} = 0$$

and

$$-\nu_{t+1}\hat{W}_K(K_{t+1}, N_{t+1}) - \mu_t + F_K(K_{t+1}, N_{t+1})\mu_{t+1} = 0.$$

These first order conditions can easily be rearranged to prove the following result.

Proposition 6. *Consider a function $\tilde{W}(K, G_-)$ and assume that $\hat{W}(K, G; \tilde{W})$ is increasing and differentiable in K. Let $\left(\{c_t^o, c_t^y, n_t\}_{t\geq 0}, \{K_t\}_{t\geq 0}\right)$ be an efficient allocation given $\tilde{W}$. Then there exist positive multipliers $\{\mu_t\}_{t\geq 0}$ and $\{\nu_t\}_{t\geq 0}$ such that for all t and $\theta_t \in \Theta$,*

$$\tau_{t+1}^k(\theta_t) = \frac{\frac{\nu_{t+1}}{\mu_t} F_K(K_{t+1}, N_{t+1})\left(\frac{\hat{W}_K(K_{t+1}, G_{t+1}; \tilde{W})}{F_K(K_{t+1}, N_{t+1}; \tilde{W})} - \lambda u'(c_{t+1}^o(\theta_t))\right)}{1 + \frac{\nu_{t+1}}{\mu_t} F_K(K_{t+1}, N_{t+1})\left(\frac{\hat{W}_K(K_{t+1}, G_{t+1}; \tilde{W})}{F_K(K_{t+1}, N_{t+1}); \tilde{W})} - \lambda u'(c_{t+1}^o(\theta_t))\right)}. \tag{26}$$

This proposition is the exact analogue for the dynamic overlapping generations setting of Proposition 2 proved in context of a two period economy.

Let $\left(\hat{c}^o(K, G; \tilde{W}), \hat{c}^y(K, G; \tilde{W}), K'(K, G; \tilde{W})\right)$ be the allocation following a deviation—the solution of program (24). Applying the Envelope theorem, we find that $\hat{W}_K(K, G; \tilde{W}) = \lambda F_K(K, N; \tilde{W}) u'(\hat{c}^o(K, G; \tilde{W}))$. Hence we can rewrite the formula for the implicit marginal tax rate on capital (26) as follows:

$$\tau^k_{t+1}(\theta_t) = \frac{\frac{\nu_{t+1}}{\mu_t} \lambda F_K(K_{t+1}, N_{t+1}) (u'(\hat{c}^o(K_{t+1}, G_{t+1}; \tilde{W})) - u'(c^o_{t+1}(\theta_t)))}{1 + \frac{\nu_{t+1}}{\mu_t} \lambda F_K(K_{t+1}, N_{t+1}) (u'(\hat{c}^o(K_{t+1}, G_{t+1}; \tilde{W})) - u'(c^o_{t+1}(\theta_t)))}. \tag{27}$$

Exactly as in the two period model, the magnitude and sign of the marginal tax rate $\tau^k_{t+1}(\theta_t)$ is driven by $u'(\hat{c}^o(K_{t+1}, G_{t+1}; \tilde{W})) - u'(c^o_{t+1}(\theta_t))$ because this determines how much an additional unit of capital saved in the hands of an agent with productivity θ_t tightens the credibility constraint. An extra unit of capital at date $t+1$ raises individual consumption, and thus raises the left hand side of the date $t+1$ credibility constraint. At the same time, more capital raises the right hand side of the date $t+1$ credibility constraint, the value of deviating. The former effect is smaller, the higher is θ_t because the equilibrium marginal utility of old age consumption $u'(c^o_{t+1}(\theta_t))$ decreases with θ_t. The latter effect is independent of θ_t because the marginal uitlity of old age consumption $u'(\hat{c}^o(K_{t+1}, G_{t+1}; \tilde{W}))$ after a deviation is independent of θ_t.

4.4 Tax Implementation

Proceeding as in Section 3 we provide a simple tax system that implements the efficient allocation. Let W_t be the wage and $R_t - 1$ be the rental rate of capital. Firms rent labor and capital and seek to maximize profits $F(k_t, n_t) - W_t n_t - R_t k_t$. Agents are subject to the following budget constraint

$$\begin{aligned} c^y_t + k_{t+1} &\leq W_t n_t - T^n_t(W_t n_t), \\ c^o_{t+1} &\leq R_{t+1} k_{t+1} - T^k_{t+1}(R_{t+1} k_{t+1}). \end{aligned} \tag{28}$$

After observing their productivity θ_t agents make consumption, saving and labor choices. Given prices $\{W_t, R_t\}_{t \geq 0}$ and tax schedules $\{T^n_t, T^k_t\}$, a competitive equilibrium is an allocation $\left(\{c^0_t, c^y_t, n_t\}_{t \geq 0}, \{K_t\}_{t \geq 0}\right)$ such that (i) agents optimize: each agent maximizes their utility (1) subject to (28); (ii) firms maximize profits; and (iii) markets clear. We then say that the tax schedules $\{T^n_t, T^k_t\}$ implement the allocation under consideration. As in Section 3 this requires that $F_N(K_t, N_t) = W_t$, $F_K(K_t, N_t) = R_t$, and that the marginal tax rates

$T_t^{n\prime}(n_t(\theta_t))$ and $T_t^{k\prime}(R_{t+1}k_{t+1}(\theta_t))$ correspond to the implicit marginal tax rates on labor $\tau_t^n(\theta_t)$ and capital $\tau_{t+1}^k(\theta_t)$ of the underlying allocation. The following corollary is a direct consequence of Propositions 3 and 6.

Corollary 3. *Consider a function $\tilde{W}(K, G_-)$ and assume that $\hat{W}(K, G; \tilde{W})$ is increasing and differentiable in K. For a given $\underline{U}$, consider an efficient allocation given $\tilde{W}$. Then it can be implemented with a nonlinear labor income tax T_t^n and a nonlinear tax on capital T_t^k. If the credibility constraint (23) is binding in period t, then tax on capital T_t^k is convex, with the marginal tax rate $T_t^{k\prime}$ strictly increasing in capital income $R_t k_t$. If the credibility constraint is not binding in period t, then the tax on capital T_t^k is zero.*

4.5 Characterizing the worst

The function $W(K, G_-)$ does not depend on G_-. Slightly abusing notation, we write $W(K)$. We fully characterize it in the case where the utility function u is bounded below and $F(K, 0) = 0$. We show that in this case, the deviation payoff $\hat{W}(K, G; W)$ is increasing and differentiable in K. The fact that $\hat{W}(K, G; W)$ is increasing in K is a direct consequence of its definition.

Proposition 7. *(i) For any function $\tilde{W}(K, G_-)$, the deviation payoff $\hat{W}(K, G; \tilde{W})$ is increasing in K.*
(ii) Suppose that the utility function u is bounded below and that $F(K, 0) = 0$. Then $W(K) = u(0)$ and $\hat{W}(K, G; W)$ is increasing and differentiable in K.

5 Numerical Illustration

In this section we provide an illustrative numerical example. Agents are assumed to have preferences of the form:

$$\sum_{\Theta} \left(\frac{c^y(\theta)^{1-\sigma}}{1-\sigma} - \psi \frac{(n(\theta)/\theta)^{1+\gamma}}{1+\gamma} + \beta \frac{c^o(\theta)^{1-\sigma}}{1-\sigma} \right) f(\theta).$$

The utilitarian welfare weights are assumed to be geometric with $\lambda_t = \delta^t$ and, in the benchmark case, the production function is Cobb-Douglas with $F(K, N) = K^\alpha N^{1-\alpha}$. The latter assumption implies that Proposition 7 is applicable.[11]

[11] We have also computed examples with partial depreciation of capital in which Proposition 7 is not applicable. This extension greatly complicates the numerical procedure without significantly altering the results. Our benchmark calibration approximates depreciation to be 100% over 30 years.

The complete list of parameters for this economy is: $\{\sigma, \psi, \gamma, \beta, f, \Theta, \lambda, \delta, \alpha\}$. σ is chosen to be 0.9 which is both consistent with the boundedness of agent utilities and close to the log specification widely used in macroeconomics. γ is set to 1 implying a constant Frisch elasticity of 1 which is consistent with values suggested by Kimball and Shapiro (2008) and Erosa et al. (2010). ψ is set so that at the steady state of a calibrated OLG market equilibrium, agents work on average about 40% of their time when young. In the benchmark economy, β is set to 0.2. Interpreting the model period as 30 years this corresponds to an annual discount factor of about 0.95. We briefly comment on the implications of other β choices below. The labor productivity distribution is calibrated using CEX hourly wage data for the year 2000.[12] We suppose that wages are generated by competitive markets and that the wage of individual i is given by $w_i = w\theta_i$ where w is the average wage and θ_i the individual's relative labor productivity. The distribution of θ_i in the data is interpreted as the productivity distribution in the calibrated model. We assume that labor productivities in the highest percentile of the distribution are generated according to a conditional Pareto distribution and extend the distribution accordingly. In the benchmark calibration, the societal discount δ is set to 0.5 and the political weighting parameter λ is set to 0.68. We consider other possible values for δ and λ and briefly describe their implications below.

We numerically solve for the steady state of the social planning problem (25) with the parameter values given above. This steady state is described by consumption and labor functions $\{c^{y*}, c^{o*}, n^*\}$ and a capital stock K^*. An explicit statement of the problem is given in the appendix.

The optimal per annum marginal tax rate is defined to be:

$$1 - \tau(\theta) = \frac{\left(\frac{u'(c^{y*}(\theta))}{\beta u'(c^{o*}(\theta))}\right)^{\frac{1}{T}} - 1}{R^{\frac{1}{T}} - 1},$$

where $T = 30$, the length of the model period, and R is the marginal product of capital in the model. Figure (1) illustrates this tax for our benchmark case. The calibrated productivity distribution is also shown. The marginal capital tax rate is increasing and concave in productivity: unproductive agents face large marginal subsidies and highly productive agents face positive marginal taxes. This shape of the benchmark marginal tax profile is common to many other cases that we have computed.

The values of the agent's discount factor β, the political weight λ and the utilitarian

[12] The data are from Heathcote et al. (2010) Sample A. Detailed description of the sample is available in that paper.

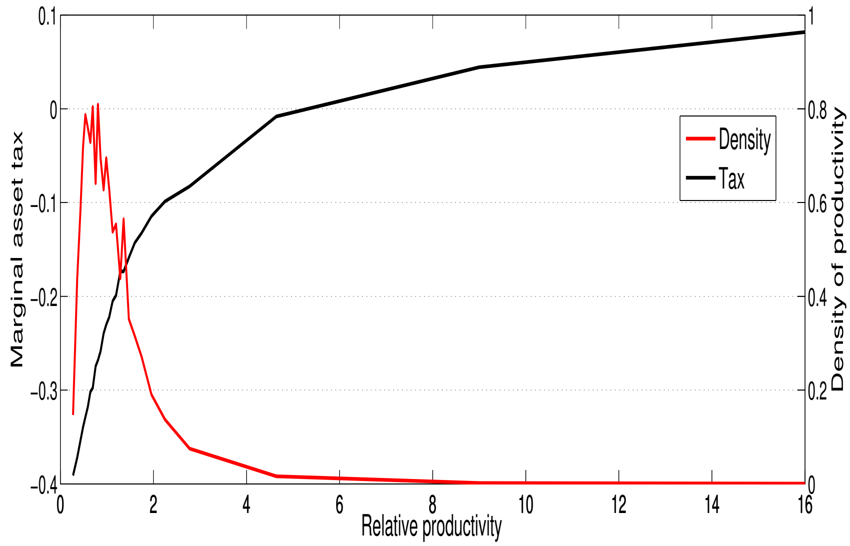

Figure 1: Marginal capital tax.

welfare weight δ are important in determining whether the credibility constraint binds or not in steady state. Smaller values of β and larger values of λ cause the government to attach less weight to the future and to the severe outcome that ultimately follows a defection. This makes the credibility constraint more likely to bind. Higher values of δ create greater mismatch between the utilitarian and the political objective, again making the credibility constraint more likely to bind. Changes to these parameters cause the relative optimal and defection marginal utilities of the old to change which can alter the sign of the marginal capital taxes at a given productivity. They also change the credibility multiplier which scales these relative marginal utilities and, hence, the tax function, see equation (27). These issues are illustrated in Figure (2) for different λ values. For values below 0.65, the credibility constraint does not bind and marginal capital taxes are 0. As λ rises, the predominant effect is the pivoting of the tax function caused by the rise in the credibility multiplier. In economic terms, as the weight of the old in the political system rises, governments are more tempted to defect. The optimal steady state tax function then becomes more progressive, suppressing ex post inequality amongst the old and diluting this temptation.

In the preceding examples, reversion to the worst equilibrium occurred following a deviation. Figure (3) illustrates a numerical experiment in which political deviations have milder consequences. Now there is reversion to the worst equilibrium with probability 0.15 and to a Markov equilibrium with probability 0.85. Consequently, defection payoffs are increased and the credibility constraint binds over a much larger set of (λ, δ) values.

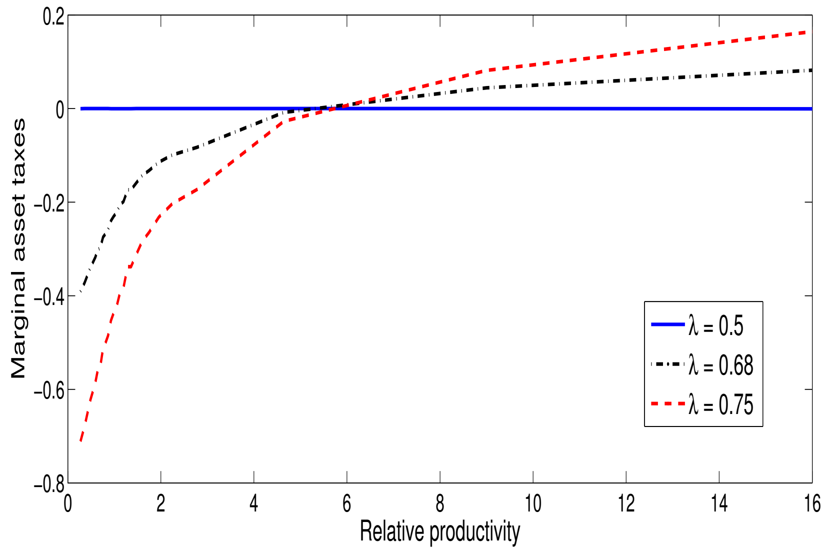

Figure 2: Marginal capital taxes for various political share, λ, values.

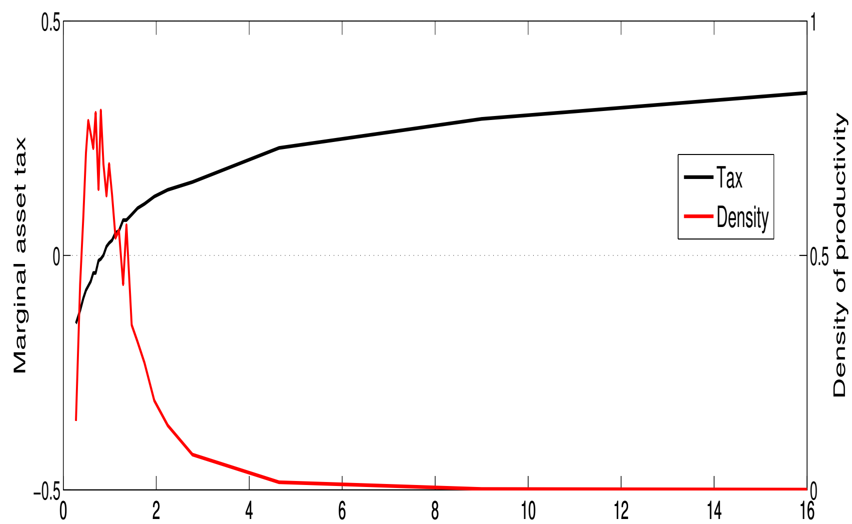

Figure 3: Marginal asset taxes: Milder punishment equilibrium, $\lambda = 0.4$, $\delta = 0.4$.

Figure (3) shows the the optimal tax function for such an equilibrium with λ and δ equal to 0.4. The monotonicity and approximate concavity of the tax function is preserved, but taxes are now positive over a much larger range of productivities. The lower value of λ coupled with reversion to the milder equilibrium causes a defecting political planner to allocate relatively more resources to the consumption of the young and to capital accumulation and less to consumption of the old. The lower value of δ contributes to a relatively greater allocation of consumption to the old at the optimum. Consistent with

(27), this combination of effects alters relative marginal utilities at the optimum and after a defection, inducing higher marginal asset taxes.

6 Conclusion

The basic idea behind our result can be stated as follows: in settings where: (a) the credibility of future policies is of concern, and (b) credibility depends on keeping inequality in check, policies will be put into place to avoid the accumulation of inequality. A progressive tax on capital is one such policy.

Our simple model delivers this sharp result in a transparent way. The main mechanism, however, appears robust, so we conjecture that the progressivity of capital taxation is likely to survive a number of extensions. Moreover, perhaps there are similar implications for other dimensions of policy that may also reduce inequality and promote credibility, such as educational policies.

References

Acemoglu, Daron, Michael Golosov, and Aleh Tsyvinski, "Dynamic Mirrlees Taxation under Political Economy Constraints," 2007. mimeo.

_ , _ , and _ , "Political Economy of Mechanisms," *Econometrica*, 05 2008, *76* (3), 619–641.

Alesina, Alberto and Danny Rodrick, "Distributive Politics and Economic Growth," *Quarterly Journal of Economics*, 1994, *109*, 465–490.

Atkeson, Andrew and Robert E., Jr. Lucas, "On Efficient Distribution with Private Information," *Review of Economic Studies*, 1992, *v59* (3), 427–453.

Atkinson, Anthony B. and Joseph E. Stiglitz, "The Design of Tax Structure: Direct vs. Indirect Taxation," *Journal of Public Economics*, 1976, *6*, 55–75.

Benabou, Roland, "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, March 2002, *70* (2), 481–517.

Benhabib, Jess and Aldo Rustichini, "Social Conflict and Growth," *Journal of Economic Growth*, 1996, *1*, 125–139.

Bertola, Guiseppe, "Factor Shares and Savings in Endogenous Growth," *American Economic Review*, 1993, *83*, 1184–1199.

Bisin, Alberto and Adriano A. Rampini, "Markets as beneficial constraints on the government," *Journal of Public Economics*, May 2006, *90* (4-5), 601–629.

Chamley, Christophe, "Optimal Taxation of Capital in General Equilibrium," *Econometrica*, 1986, *54*, 607–22.

Chari, V.V. and Patrick J. Kehoe, "Sustainable Plans," *Journal of Political Economy*, 1990, *98*, 616–636.

Coughlin, Peter J., "Elections and income redistribution," *Public Choice*, 1986, *50* (1–2), 27–91.

Erosa, Andres, Luisa Fuster, and Kambourov Gueorgi, "The Heterogeneity and Dynamics of Individual Labor Supply over the Life Cycle: Facts and Theory," 2010. mimeo.

Farhi, Emmanuel and Iván Werning, "Inequality and Social Discounting," *Journal of Political Economy*, 2007, *115(3)*, 365–402.

_ and _ , "Progressive Estate Taxation," 2008. mimeo.

Fischer, Stanley, "Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government," *Journal of Economic Dynamics and Control*, 1980, *2*, 93–107.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante, "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*, January 2010, *13* (1), 15–51.

Judd, Kenneth L., "Redistributive Taxation in a Perfect Foresight Model," *Journal of Public Economics*, 1985, *28*, 59–83.

Kimball, Miles S. and Matthew D. Shapiro, "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?," NBER Working Papers, National Bureau of Economic Research, Inc July 2008.

Klein, Paul and Jose-Victor Rios-Rull, "Time-Consistent Optimal Fiscal Policy," *International Economi Review*, 2003, pp. 1217–1245.

_ , Per Krusell, and Jose Victor Rios-Rull, "Time-Consistent Public Policy," *Review of Economic Studies*, 07 2008, *75* (3), 789–808.

Kotlikoff, Larry, Torsten Persson, and Lars Svensson, "Social Contracts as Assets: a Possible Solution to the Time-Inconsistency Problem," *American Economic Review*, 1988, *78*, 662–677.

Kydland, Finn and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85*, 473–491.

Lindbeck, Assar and Joerg Weibull, "Balanced-budget Redistribution as the Outcome of Political Competition," *Public Choice*, 1987, pp. 273–97.

Mirrlees, James A., "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 1971, *38* (2), 175–208.

Persson, Torsten and Guido Tabellini, "Is Inequality Harmful for Growth," *American Economic Review*, 1994, *84*, 600–621.

_ **and** _ , "Representative Democracy and Capital Taxation," *Journal of Public Economics*, 1994, *55*, 53–70.

Phelan, Christopher, "Opportunity and Social Mobility," *Review of Economic Studies*, 2006, *73* (2), 487–505.

_ **and Ennio Stacchetti**, "Sequential Equilibria in a Ramsey Tax Model," *Econometrica*, November 2001, *69* (6), 1491–1518.

Saez, Emmanuel., "Optimal Progressive Capital Income Taxes in the Infinite Horizon," 2002. NBER WP9046.

Sleet, Christopher and Sevin Yeltekin, "Credibility and Endogenous Social Discounting," *Review of Economic Dynamics*, 2006, *9*, 410–437.

_ **and** _ , "Optimal Growth and Inequality in Uncommitted Societies," 2008. mimeo.

_ **and** _ , "Politically Credible Social Insurance," *Journal of Monetary Economics*, 2008, *55*, 129–151.

7 Appendix

Proof of Proposition 3 Consider an allocation $c_0(\theta_0), c_1(\theta_0), n_0(\theta_0)$ that satisfies the assumptions of Proposition 3. The implicit capital tax for agent θ_0 is given by

$$\tau(\theta_0) = 1 - \frac{u'(c_0(\theta_0))}{\beta R u'(c_1(\theta_0))}.$$

We define the capital tax schedule T^k as a solution of the following ODE:

$$T^{k\prime}(Rk) = \tau\big(c_1^{-1}(Rk - T^k(Rk))\big)$$

where c_1^{-1} is the inverse of the function c_1.[13] This defines a concave function T^k over $c_1(\Theta)$. We can extend this function to a concave function T^k over $\mathbb{R}^+$. Note that since $\tau(\theta_0) < 1$, $Rk - T^k(Rk)$ is strictly increasing in k. Hence we can define the function $k(\theta_0)$, increasing in θ_0, as follows:

$$Rk(\theta_0) - T^k(Rk(\theta_0)) = c_1(\theta_0).$$

Define the labor income tax T^n so that

$$n_0(\theta_0) - c_0(\theta_0) - T^n(n_0(\theta_0)) = k(\theta_0)$$

and for $n \notin \{n(\theta_0)\}_{\theta_0 \in \Theta}$, let $T^n(n) = n$. Let $y(\theta_0) = n_0(\theta_0) - T^n(n_0(\theta_0))$.

Consider, for a given level of income y net of labor income tax, the following problem. Maximize

$$U(k, c_1; y) \equiv u(y - k) + \beta u(c_1) \tag{29}$$

subject to

$$c_1 = Rk - T^k(Rk).$$

By construction, $\big(k(\theta_0), c_1(\theta_0)\big)$ satisfies the first order conditions in (29) when $y = y(\theta_0)$.

Note that we have the following single crossing property:

$$\frac{\partial\big(-\frac{U_{c_1}}{U_k}\big)}{\partial y} > 0$$

Together with the fact that $y(\theta_0)$ and $c_1(\theta_0)$ are increasing in θ_0, this is enough to en-

[13] Possible flat portions of $c_1(\theta_0)$ define discontinuous jumps in the inverse function c_1^{-1}.

sure that for $y = y(\theta_0)$, $\big(k(\theta_0), c_1(\theta_0)\big)$ attains the maximum in (29). Hence an agent of type θ_0 who supplies $n(\theta_0')$ units of effective labor will optimally choose to consume $\big(c_0(\theta_0'), c_1(\theta_0')\big)$ when confronted with the taxes T^n and T^k. Since the original allocation is incentive compatible, working $n(\theta_0)$ and consuming $\big(c_0(\theta_0), c_1(\theta_0)\big)$ is the optimal choice for an agent of type θ_0. Moreover, and agent of type θ_0 is always better off working $n(\theta_0)$ and consuming $\big(c_0(\theta_0), c_1(\theta_0)\big)$ than choosing any $n \notin \{n(\theta_0)\}_{\theta_0 \in \Theta}$ and then being forced to consume 0 in both periods. Therefore, the taxes T^n and T^k implement the allocation.

Proof of Proposition 7 Part (i) follows directly from the definition of $\hat{W}(K, G; W)$. Part (ii) can be proved as follows. Suppose that u is bounded below. Then we necessarily have $W(K) \geq u(0)$. When $F(K, 0) = 0$ and the utility u is bounded below, it is easy to construct an equilibrium that achieves this expected payoff for the initial old. We now explain how to construct such an equilibrium.

Agents strategies are defined as follows. For every public history H_t, and shock θ_t, we specify $\sigma_t(H_t, \theta_t) = 0$. For every interim history $\hat{H}_t$, let $(c^o(\int n_t dG_t(n_t)), c^y(\int n_t dG_t(n_t)))$ be the solution of $\max_{c^o, c^y} \lambda u(c^o) + (1-\lambda)u(c^y)$ subject to $c^o + c^y = F(K_t, \int n_t dG_t(n_t))$. After every interim history $\hat{H}_t$, the strategy of government t, and for every past labor choice n_{t-1} of an old and current labor choice n_t of a young, is described by $\tau_t^{c,o}(\hat{H}_t, n_{t-1}) = c^o(\int n_t dG_t(n_t))$, $\tau_t^{c,y}(\hat{H}_t, n_t) = c^y(\int n_t dG_t(n_t)_t)$, and $\tau_t^K(\hat{H}_t) = 0$. It is easy to see that these strategies form an equilibrium, and that the corresponding equilibrium payoff for the initial old is given by $u(0)$.

Numerical procedure for the numerical illustration We numerically solve for the steady state of the social planning problem (25). Specifically, we extract the problem of a single generation from (25) and obtain multipliers $\left\{\frac{\mu^*}{\chi^*}, \frac{\nu^*}{\chi^*}\right\}$ and an allocation $\{c^{y*}, c^{o*}, n^*, K^*\}$

such that (i) the allocation maximizes:

$$\chi^* \int \left(u(c^y(\theta)) - h\left(\frac{n(\theta)}{\theta}\right) + \beta u(c^o(\theta)) \right) dF(\theta) \tag{30}$$

$$- \mu^* \left(\int c^y(\theta) dF(\theta) - F\left(K, \int n(\theta) dF(\theta)\right) \right) - \delta\mu^* \int c^o(\theta) dF(\theta) - \delta^{-1}\mu^* K$$

$$+ \nu^* \left[(1-\lambda) \int \{u(c^y(\theta)) + \beta u(c^o(\theta))\} dF(\theta) - \widehat{W}(K, G; W) \right]$$

$$+ \delta\nu^*\lambda \sum_{\Theta} u(c^o(\theta))\pi(\theta)$$

where G is the labor distribution induced by F and n, subject to the incentive constraints, and (ii) the multiplier-allocation pair satisfies the complementary slackness conditions:

$$0 = \mu^* \left[F(K^*, \int n^*(\theta) dF(\theta)) - \int c^{y*}(\theta) dF(\theta) - \int c^{o*}(\theta) dF(\theta) - K^* \right],$$

$$0 = \nu^* \left[\lambda \int u(c^{o*}(\theta) dF(\theta) + (1-\lambda) \int [u(c^{y*}(\theta) + \beta u(c^{o*}(\theta)] dF(\theta) - \widehat{W}(K^*, G^*; W) \right]$$

with both multipliers and net constraint terms non-negative. Since in the benchmark case with full depreciation, Proposition 7 applies, $W = 0$ and the function $\widehat{W}(\cdot; 0)$ in the preceding optimization is easily recovered.