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Citation: Levitov, L., A. Shtyk, and M. Feigelman. "Electron-Electron Interactions and Plasmon Dispersion in Graphene." Phys. Rev. B 88, no. 23 (December 2013). © 2013 American Physical Society

As Published: <http://dx.doi.org/10.1103/PhysRevB.88.235403>

Publisher: American Physical Society

Persistent URL: <http://hdl.handle.net/1721.1/88762>

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

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Electron-electron interactions and plasmon dispersion in graphene

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(Received 19 February 2013; published 2 December 2013)

Plasmons in two-dimensional electron systems with nonparabolic bands, such as graphene, feature strong dependence on electron-electron interactions. We use a many-body approach to relate plasmon dispersion at long wavelengths to Landau Fermi-liquid interactions and quasiparticle velocity. An identical renormalization is shown to arise for the magnetoplasmon resonance. For a model with $N \gg 1$ fermion species, this approach predicts a power-law dependence for plasmon frequency vs carrier concentration, valid in a wide range of doping densities, both high and low. Gate tunability of plasmons in graphene can be exploited to directly probe the effects of electron-electron interaction.

DOI: [10.1103/PhysRevB.88.235403](https://doi.org/10.1103/PhysRevB.88.235403)

PACS number(s): 72.80.Vp, 78.67.Wj

Plasmonics has emerged recently as an active direction in graphene research.^{1–5} Surface plasmons in two-dimensional (2D) electron systems are propagating charge density waves in which collective dynamics of clouds of charge is mediated by electric field in 3D.⁶ The dual matter-field nature of plasmons is a key ingredient for many interesting and important phenomena.^{7,8} Plasmons in graphene display a range of potentially useful properties, such as low Ohmic losses, a high degree of field confinement, and gate tunability.^{9,10} Gate tunability of plasmons in graphene was demonstrated recently.^{11,12}

The goal of this article is to investigate the density dependence of plasmons and relate it to the interaction effects in the electron system. The dependence of plasmon dispersion on carrier density arises due to several effects. It takes on the simplest form in the limit of weak electron-electron interactions,^{1–4}

$$\omega^2 = \frac{2e^2 E_F}{\kappa \hbar^2} q, \quad (1)$$

where $E_F \sim n^{1/2}$ is the Fermi energy of noninteracting massless Dirac particles, and n is carrier density. Here κ is an effective dielectric constant of the substrate and a long-wavelength limit is assumed, $q \ll p_F$. Furthermore, plasmon dispersion features strong dependence on interactions. Renormalization of the dispersion relation, Eq. (1), due to electron-electron interactions was predicted in Ref. 3, where perturbation expansion in a weak fine structure parameter $\alpha = e^2/\hbar v$ was employed. The results of Ref. 3 point to an interesting possibility to directly probe the effects of interactions by measuring plasmon dispersion relation. However, strong interactions in graphene, $\alpha \sim 2.5$, render the weak coupling approximation unreliable.

Acknowledging the difficulty of modeling the strong-coupling regime, it is beneficial to adopt a somewhat more general approach. Rather than attempting to make predictions based on a specific microscopic model, one can ask if a relation between the plasmon dispersion and some other fundamental characteristics of the system can be established. Below we point out that such a relation arises naturally from the Landau

theory of Fermi liquids.¹³ This theory affords a general, model-independent framework to describe systems of strongly interacting fermions at degeneracy. The effects of interactions are encoded in the Landau parameters, representing a “genetic code” of the Fermi liquid (FL). The parameter values can, in principle, be predicted from perturbation theory if interactions are weak. For systems with strong interactions, however, the most reliable way to obtain the Landau parameters is to use their relation with experimentally measurable quantities, such as compressibility, heat capacity, spin susceptibility, and dispersion of collective excitations.

Our many-body analysis upholds the conventional square-root dependence $\omega \sim q^{1/2}$. We show that all the effects of interactions are accumulated in the prefactor,

$$\omega^2 = Y \lambda q, \quad Y = (1 + F_1)v, \quad \lambda = \frac{N e^2 p_F}{2 \kappa \hbar^2}, \quad (2)$$

where p_F is Fermi momentum, $N = 4$ is the number of spin/valley flavors, and a long-wavelength limit is assumed, $q \ll p_F$. Here F_1 is the Landau interaction harmonic with $m = 1$, and v is the Fermi velocity renormalized by interactions. The quantity λ in Eq. (2) has units of frequency and depends only on the fundamental constants and carrier density via p_F . In some cases the dielectric constant κ may feature an essential q dependence. In particular, when image charges arise due to conducting boundaries or gates, a simple model yields¹⁴ $\kappa(q) = \frac{1}{2}[\kappa_1 + \kappa_2 \coth(qd)]$, giving an acoustic plasmon dispersion $\omega \sim q$.

Magnetic field alters the behavior, turning the gapless plasmon mode into a gapped mode. The magnetoplasmon dispersion relation obtained by adding Lorentz force to the FL dynamics takes on the form

$$\omega_B^2(q) = \omega_0^2(q) + Y^2 (eB/c p_F)^2, \quad q r_c \ll 1, \quad (3)$$

where $\omega_0(q)$ is plasmon dispersion at $B = 0$ given by Eq. (2), and r_c is the cyclotron radius. The dispersion relation becomes more complex at $q r_c \sim 1$ due to the presence of Bernstein modes.¹⁵ The size of the gap at $q = 0$ scales linearly with B , with a density dependent prefactor. Notably, the

magnetoplasmon dependence on the interactions is described by the same combination $Y = (1 + F_1)v$ as that appearing in Eq. (2).

The quantity Y describes the interaction dependence of plasmon dispersion. Measuring it as a function of carrier density can be used to determine the electron-electron interaction strength in the system. This behavior is in sharp departure with that for plasmons in two-dimensional systems with parabolic band dispersion, where Galilean invariance leads to an identity for Landau parameters, $(1 + F_1)v = v_0$,¹³ where $v_0 = mp_F$ is Fermi velocity of noninteracting particles at the same density [see discussion in Sec. IV]. As a result, the value $(1 + F_1)v$ is independent of interactions, leading to the “universal” long-wavelength plasmon dispersion in the parabolic case, $\omega_0^2(q) = \frac{2\pi e^2 n}{m\kappa(q)}q$. Similarly, at a finite magnetic field, Galilean invariance leads to a simple result for long-wavelength magnetoplasmons, $\omega_B^2(q) = \omega_0^2(q) + \omega_c^2$, where m is unrenormalized band mass, $\omega_c = eB/mc$ is the cyclotron frequency and $qr_c \ll 1$. These dependencies carry no information on the quantum effects or the interactions.

The situation is quite different in systems with nonparabolic dispersion, such as graphene. The density dependence in Y arises because the values v and F_1 are renormalized in an essentially different way. As an illustration, we analyze the limit of a large number of spin/valley flavors, $N \gg 1$, using a renormalization group (RG) approach. In this case, as we will see, Eq. (2) yields a power-law dependence on carrier density,

$$\omega^2 \sim A n^{(1-\beta)/2} q. \quad (4)$$

Here the exponent β is identical to that found from one-loop RG for velocity renormalization, $\beta = \frac{8}{N\pi^2}$,^{16–18} and the prefactor is $A \sim v_0 \frac{e^2}{\hbar\kappa} a^{-\beta}$, with $a \approx 0.142$ nm the carbon spacing. For a noninteracting system, $\beta = 0$. Crucially, the power law $n^{(1-\beta)/2}$ describes the dependence on carrier density not only near charge neutrality but also for all accessible n values.

Measurements of the density dependence of a plasmon resonance in graphene ribbons were reported in Ref. 11. The observed dependence approximately follows the relation $\omega^2 \propto q$, with the prefactor exhibiting an approximately linear dependence on $n^{1/2}$. However, the limited range of densities in which the dispersion was measured, as well as possible corrections due to the finite width of the ribbons, made it challenging to distinguish between $\beta = 0$ and $\beta \neq 0$. An attempt to experimentally determine the RG scaling exponents directly from transport measurements was made recently in Ref. 19. In this work, a systematic variation of the period of quantum oscillations with carrier density was interpreted in terms of Fermi velocity renormalization, giving a value $\beta = 0.5\text{--}0.55$. This value is considerably larger than the one-loop RG result, $\beta = \frac{8}{\pi^2 N} \approx 0.2$.^{16–18} This discrepancy is not yet understood.

We note parenthetically that the interaction effects are not expected to vanish in graphene bilayer despite the parabolic character of its band dispersion. Electronic states in graphene bilayer are Dirac-like rather than Schrödinger-like and hence do not admit Galilean transformation. For plasmons in this material we therefore expect a behavior similar to that in materials with nonparabolic band, described by Eqs. (2) and (3).

I. MICROSCOPIC FERMI-LIQUID ANALYSIS

The goal of this section is to relate plasmon dispersion with the standard quantities such as Landau FL interactions and renormalized velocity. The analysis proceeds by standard steps via resumming the ladder contributions to the dynamical polarization function, which account for the quasiparticle dynamics in Landau’s FL framework. In doing so, we keep ω and q small but finite, as appropriate for a plasmon dispersion analysis. This leads to a polarization response, $\Pi(q, \omega) \sim q^2/\omega^2$, describing plasmon excitations in the low-frequency and long-wavelength domain, $\omega \ll E_F$, $q \ll p_F$.

Charge carriers in single-layer graphene are described by the Hamiltonian for $N = 4$ species of massless Dirac particles. In second-quantized representation the Hamiltonian reads

$$\mathcal{H} = \sum_{\mathbf{p}, i} \psi_{\mathbf{p}, i}^\dagger v_0 \sigma \mathbf{p} \psi_{\mathbf{p}, i} + \mathcal{H}_{\text{el-el}}, \quad (5)$$

$$\mathcal{H}_{\text{el-el}} = \frac{1}{2} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{p}', i, j} V(\mathbf{q}) \psi_{\mathbf{p}+\mathbf{q}, i}^\dagger \psi_{\mathbf{p}-\mathbf{q}, j}^\dagger \psi_{\mathbf{p}', j} \psi_{\mathbf{p}, i}, \quad (6)$$

where $i, j = 1, \dots, N$, $v_0 \approx 10^6$ m/s is unrenormalized Fermi velocity and $V(\mathbf{q}) = 2\pi e^2/|\mathbf{q}|\kappa$ is the Coulomb interaction with the dielectric constant κ describing screening by the substrate. Here $\psi_{\mathbf{p}, i}$ is a two-component spinor describing the wave-function amplitude on the two sublattices of the graphene crystal lattice. The amplitudes associated with the two sublattices are usually referred to as pseudospin up and down components, with the (pseudo)spin- $\frac{1}{2}$ Pauli matrices in Eq. (5) acting on (pseudo)spinors $\psi_{\mathbf{p}, i}$.

Plasmons are collective excitations of 2D electrons coupled by the electric field in 3D. They can be described microscopically using the density correlation function

$$K(\mathbf{q}, \omega) = i \int dt \langle [\rho_{\mathbf{q}}(t), \rho_{\mathbf{q}}(t_0)] \rangle e^{i\omega(t-t_0)}, \quad (7)$$

where $\rho_{\mathbf{q}}(t) = \sum_{\mathbf{p}, i} \psi_{\mathbf{p}, i}^\dagger(t) \psi_{\mathbf{p}+\mathbf{q}, i}(t)$ are Fourier harmonics of the total electron density. The quantity K is expressed in a standard fashion¹³ through geometric series involving the polarization function $\Pi(\mathbf{q}, \omega)$ defined as the irreducible density-density correlator,

$$K(\mathbf{q}, \omega) = \frac{\Pi(\mathbf{q}, \omega)}{\tilde{\kappa}(\mathbf{q}, \omega)}, \quad \tilde{\kappa}(\mathbf{q}, \omega) = 1 - V(\mathbf{q})\Pi(\mathbf{q}, \omega). \quad (8)$$

Zeros of the dynamical screening function $\tilde{\kappa}(\mathbf{q}, \omega)$ give the poles of K , defining plasmon dispersion. To obtain the dispersion from the condition $\tilde{\kappa}(\mathbf{q}, \omega) = 0$ we need an input on $\Pi(\mathbf{q}, \omega)$ from a microscopic approach. In the long-wavelength limit, $q \ll p_F$, $\omega \ll E_F$, the behavior of the quantity $\Pi(\mathbf{q}, \omega)$ is dominated by excitations near the Fermi surface, which can be described in the FL framework.

The microscopic approach used to justify the FL picture involves several standard steps. We start, as usual, by isolating a quasiparticle pole contribution to the electron Green’s function $G(x - x') = -i \langle \psi(x) \psi^\dagger(x') \rangle$ near the Fermi surface,

$$G(\epsilon, \mathbf{p}) = G^{(\text{reg})}(\epsilon, \mathbf{p}) + G^{(\text{sing})}(\epsilon, \mathbf{p}). \quad (9)$$

The first term is a regular part of the Green’s function behaving as a smooth function near the Fermi level. The second term is

a singular contribution describing quasiparticles,

$$G^{(\text{sing})}(\epsilon, \mathbf{p}) = \frac{Z}{i\epsilon - \xi(p) + i\gamma \text{sgn}\epsilon}. \quad (10)$$

Here Z is a quasiparticle residue, γ is a quasiparticle decay rate, and $\xi(p) = v(p - p_F)$ is a quasiparticle energy dispersion, with v the renormalized velocity.

This general discussion can be specialized to the case of graphene as follows. The Green's function for electrons in graphene has a 2×2 matrix pseudospin structure. By projecting on the conduction and valence bands, it can be represented as

$$\hat{G}(\epsilon, \mathbf{p}) = G_{<}(\epsilon, \mathbf{p})\hat{P}_{<} + G_{>}(\epsilon, \mathbf{p})\hat{P}_{>}, \quad (11)$$

where $\hat{P}_{>(<)} = (1 \pm \sigma \mathbf{e}_p)/2$ are projectors for the two bands (here $\mathbf{e}_p$ is a unit vector in the direction of momentum $\mathbf{p}$). The quasiparticle excitations with low energies, which govern the low-frequency and long-wavelength response, reside near the Fermi level. Without loss of generality, we assume n -type doping, so that the Fermi level lies in the upper band, $E_F > 0$. In this case, excitations from the lower band do not appear explicitly in the FL theory and lead only to renormalization of various parameters such as the effective interactions and the quasiparticle velocity. The quasiparticle pole in Eq. (9) therefore arises only from the upper-band contribution $G_{>}$, whereas the lower-band contribution $G_{<}$ can be absorbed into the regular part $G^{(\text{reg})}$. Below the subscripts $>$ and $<$ will be omitted for brevity.

The next step, which is key for understanding the role of low-energy excitations, is the analysis of the polarization function $\Pi(q, \omega)$ at small ω and q . This is done by identifying the contributions due to pairs of Green's functions with proximal poles (the “dangerous” two-particle cross sections),¹³ which we write symbolically as $G^{(\text{sing})}G^{(\text{sing})} \sim Z^2 \frac{\mathbf{v}\mathbf{k}}{\omega - \mathbf{v}\mathbf{k}}$. One can represent $\Pi(q, \omega)$ as a sum of terms with different numbers of such contributions,

$$\begin{aligned} \Pi(q, \omega) &= \Pi_0(q, \omega) + \Pi_1(q, \omega) + \Pi_2(q, \omega) + \dots, \\ \Pi_1(q, \omega) &= T^\omega G G T^\omega, \\ \Pi_2(q, \omega) &= T^\omega G G \Gamma^\omega G G T^\omega \dots \end{aligned} \quad (12)$$

The corresponding graphs are shown in Fig. 1. Here we introduced so-called quasiparticle-irreducible quantities: the renormalized scalar vertex T^ω and the two-particle scattering

FIG. 1. Resummed Feynman graphs for the polarization operator $\Pi(\mathbf{q}, \omega)$. The non-quasiparticle contribution $\Pi_0(\mathbf{q}, \omega)$ and the FL ladder $\sum_{n \geq 1} \Pi_n(\mathbf{q}, \omega)$ are shown. Only the contributions $\Pi_1(\mathbf{q}, \omega)$ and $\Pi_2(\mathbf{q}, \omega)$ contribute to the low-energy plasmon dispersion; see text.

FIG. 2. Feynman graphs for the “quasiparticle-irreducible” quantities T^ω , Γ^ω . The bold lines represent a full Green's function, the thin lines represent a singular part $G^{(\text{sing})}$. The vertex Γ_0 represents the conventional irreducible vertex, whereas hatched blocks represent products of two Green's functions, with the contributions $G^{(\text{sing})}G^{(\text{sing})}$, which give a nonanalytical behavior at small $\omega, \mathbf{q}$, taken out (see text).

vertex Γ^ω [see Figs. 2(b) and 2(c)]. These quantities absorb all non-quasiparticle contributions in the upper band as well as the interband processes and the contribution of the states in the lower band.

We recall that the quasiparticle-irreducible quantities are distinct from the conventional irreducible quantities defined as sums of Feynman graphs that cannot be split in two by removing two electron lines.¹³ For example, the quasiparticle-irreducible vertex Γ^ω is obtained by summing all kinds of graphs except the ones with dangerous cross sections. The vertex Γ^ω (T^ω) can be obtained from the conventional irreducible vertex Γ_0 (T_0) by the resummation procedure pictured in Fig. 2, where the hatched blocks represent contributions due to pairs of Green's functions save for $G^{(\text{sing})}G^{(\text{sing})}$.

To analyze the dependence on ω and $\mathbf{q}$ in the long-wavelength limit, caution must be exercised by employing the quantities T^ω and Γ^ω taken at small but nonzero frequency and momentum values. We therefore adopt an approach similar to that used in Ref. 20: Our quasiparticle-irreducible quantities correspond to Luttinger's ω quantities, which are taken at finite ω and $\mathbf{q}$. They are distinct from the conventional ω quantities¹³ obtained in the limit $\omega, q \rightarrow 0$, ($\omega \gg v_F q$). This distinction, however, turns out to be inessential: Luttinger's ω quantities reproduce the conventional ω quantities in the limit $\omega, q \rightarrow 0$, which can be taken in arbitrary order since dangerous cross sections were left out of the definition of T^ω and Γ^ω .

Proceeding with the analysis, we note that the dependence on ω and q is very different for $\Pi_0(q, \omega)$ and $\Pi_{n \geq 1}(q, \omega)$. We first analyze the contribution $\Pi_0(q, \omega)$. This quantity does not contain dangerous cross sections which can generate a nonanalytic behavior at small ω and $\mathbf{q}$. Taking $\Pi_0(q, \omega)$ to be analytic, we can represent it as

$$\Pi_0(\mathbf{q}, \omega) = A(\omega) + B(\omega) \left(\frac{q}{p_F} \right)^2 + \dots, \quad (13)$$

where $A(\omega)$ and $B(\omega)$ are regular functions. Further, we recall that gauge invariance prohibits any physical response to spatially uniform time-dependent scalar field. Applying

this to the full polarization function, Eq. (12), we see that setting $q = 0$ yields $\Pi(\omega, 0) = 0$. Also, since the contributions of the dangerous cross-sections GG vanish at $q = 0$, all the quantities $\Pi_{n \geq 1}(q, \omega)$ do so. We therefore conclude that the function $A(\omega)$ vanishes, leaving us with $\Pi_0(\mathbf{q}, \omega) = (q/p_F)^2 B(\omega) + O(q^4)$. This gives an effective q -dependent permittivity:

$$\tilde{\kappa}(q, \omega) = 1 - V(q)\Pi_0(q, \omega). \quad (14)$$

The second term in Eq. (14) may be ignored in the long-wavelength limit. Indeed, since $V(q) = 2\pi e^2/\kappa q$ in 2D, whereas $\Pi_0 \propto q^2$, the quantity $\tilde{\kappa}$ equals unity in the limit $q/p_F \rightarrow 0$.

It is instructive to compare this behavior of $\tilde{\kappa}(q, \omega)$ with that arising for $q \gg p_F$. In this case the effects of finite doping are negligible and we can estimate the polarization function using the result obtained for massless Dirac particles at zero doping,

$$\Pi(q, \omega) = -\frac{N}{16} \frac{q^2}{\sqrt{q^2 v^2 - \omega^2}}, \quad (15)$$

where $N = 4$ is the number of spin/valley flavors. In the limit $qv \gg \omega$, we obtain a well-known renormalized permittivity

$$\tilde{\kappa}(q, \omega) = 1 + \frac{\pi N \alpha}{8}. \quad (16)$$

This q -independent expression describes the effect of intra-band polarization in undoped graphene. We stress, however, that while $\tilde{\kappa} > 1$, the above expression is obtained for q and ω values which are not relevant for plasmon excitations. This is so because plasmons do not exist for such $\Pi(\mathbf{q}, \omega)$, as the plasmon dispersion terminates for $q \gtrsim p_F$. In contrast, the permittivity in Eq. (14), evaluated in the long-wavelength limit relevant for plasmons, $q \ll p_F$, $\omega \ll E_F$, equals unity.

Next, we proceed with the analysis of the remaining terms, $\Pi_{n \geq 1}(q, \omega)$, which give a leading contribution to the low-energy plasmon dispersion. This is the part of polarization which depends on the quasiparticle contributions. The corresponding Feynman graphs are given by a ladder with rungs consisting of two quasiparticle lines separated by vertex parts, as shown in Fig. 1. This gives geometric series that can be easily summed up,

$$\Pi(\mathbf{q}, \omega) = N \oint \frac{d\theta}{2\pi} \mathcal{T}^\omega \left(\frac{vZ^2}{\omega - \mathbf{q}\mathbf{v}(1 + \hat{F})} \mathbf{q}\mathbf{v} \mathcal{T}^\omega \right), \quad (17)$$

where $\hat{F}$ is an integral operator,

$$\hat{F}f(\theta) = vZ^2 \oint \frac{d\theta'}{2\pi} \Gamma^\omega(\omega, \mathbf{q}, \theta, \theta') f(\theta'). \quad (18)$$

Here $v = p_F/(2\pi\hbar^2 v)$ is the density of states per flavor and θ (θ') is an angle between $\mathbf{p}$ ($\mathbf{p}'$) and $\mathbf{q}$. For zero external momentum $\mathbf{q} \rightarrow 0$ the kernel of the operator $\hat{F}$ depends only on the angle between $\mathbf{p}$ and $\mathbf{p}'$. In what follows, we will need the quantity $\Gamma^\omega(0, 0, \theta, \theta') \equiv \Gamma^\omega(\theta - \theta')$.

The scalar vertex $\mathcal{T}^\omega$ takes on a simple form on the Fermi surface. For small external frequency and momentum values ω , $v_F q \ll \varepsilon_F$ the vertex can be decomposed as

$$\mathcal{T}^\omega(\omega, \mathbf{q}, \theta) = \mathcal{T}_0 + \mathcal{T}_1 \omega + \mathcal{T}_2 q \cos \theta + \dots, \quad (19)$$

where $\mathcal{T}_0 = Z^{-1}$ by virtue of Ward's identity.¹³ The linear terms are potentially relevant, if judged by power counting.

However, these contributions drop out, because for external frequency ω and momentum $\mathbf{q}$, the expressions in question contain both $\mathcal{T}^\omega(\omega, \mathbf{q}, \theta)$ and $\mathcal{T}^\omega(-\omega, -\mathbf{q}, \theta)$. This leads to a cancellation of the terms linear in ω and $\mathbf{q}$.

Continuing with the analysis, we note that only the first two terms of the series in Eq. (12) are relevant for long-wavelength plasmons with $v_F q \ll \omega \ll \varepsilon_F$. Anticipating the square-root dependence for plasmon frequency vs wave number, we expand in qv/ω to obtain

$$\begin{aligned} \Pi_1 &= \oint \frac{d\theta}{2\pi} \mathcal{T}^\omega(\omega, \mathbf{q}, \theta) \frac{vZ^2 v q \cos \theta}{\omega - v q \cos \theta} \mathcal{T}^\omega(-\omega, -\mathbf{q}, \theta) \\ &= vZ^2 \mathcal{T}_0^2 \oint \frac{d\theta}{2\pi} \frac{(v q \cos \theta)^2}{\omega^2} \\ &= \frac{v}{2} \frac{v^2 q^2}{\omega^2}, \end{aligned} \quad (20)$$

$$\begin{aligned} \Pi_2 &= \oint \frac{d\theta d\theta'}{(2\pi)^2} \mathcal{T}^\omega(\omega, \mathbf{q}, \theta) \frac{vZ^2 v q \cos \theta}{\omega} \Gamma^\omega(\omega, \mathbf{q}, \theta, \theta') \\ &\quad \times \frac{vZ^2 v q \cos \theta'}{\omega} \mathcal{T}^\omega(-\omega, -\mathbf{q}, \theta') \\ &= v^2 Z^4 \mathcal{T}_0^2 \frac{v^2 q^2}{\omega^2} \oint \frac{d\theta d\theta'}{(2\pi)^2} \Gamma^\omega(\theta - \theta') \cos \theta \cos \theta' \\ &= \frac{v}{2} \frac{v^2 q^2}{\omega^2} vZ^2 \oint \frac{d\theta}{2\pi} \Gamma^\omega(\theta) \cos \theta. \end{aligned} \quad (21)$$

The terms $\Pi_{n \geq 3}$, expanded in qv/ω , yield contributions which are higher order in q . The same is true for contributions arising from expanding $\mathcal{T}^\omega$, Γ^ω in powers of $\mathbf{q}$, and ω [with the exception for potentially relevant linear terms $\mathcal{T}_1$, $\mathcal{T}_2$ in Eq. (19), which merely cancel out]. These terms are therefore not essential in the long-wavelength limit.

Combining all the above results for Π_0 and $\Pi_{n \geq 1}$, we find the long-wavelength asymptotic behavior for the net polarization function:

$$\Pi(\mathbf{q}, \omega) = \frac{1}{2} v(1 + F_1) \frac{v_F^2 q^2}{\omega^2}, \quad (22)$$

$$F_1 = vZ^2 \oint \frac{d\theta}{2\pi} \Gamma^\omega(\theta) \cos \theta. \quad (23)$$

The quantity F_1 also gives the eigenvalues of the integral operator $\hat{F}$ corresponding to eigenfunctions $\cos \theta$ and $\sin \theta$. We can therefore write $F_1 \cos \theta = \hat{F}[\cos \theta]$, which gives a Fourier harmonic of the operator kernel identical to Eq. (23).

Plasmon dispersion can now be obtained from the relation $1 - V(q)\Pi(q, \omega) = 0$, giving Eq. (2). The effects of interaction are encoded in the quantity Y , which equals Fermi velocity v_0 in the absence of interactions and is renormalized to a different value in an interacting system.

We note a difference between the quantities F_m used in the FL literature¹³ and those used here, which is manifest in their sign. The difference arises due to the long-range character of the $1/r$ interaction. In our case the density-density interaction $F(\theta - \theta')$ accounts for the effects due to exchange correlation but not for the Hartree effects. The Hartree contribution is expressed through the $1/r$ interaction taken at the plasmon momentum $\mathbf{q}$, corresponding to the Feynman graphs which can be disconnected by cutting a single interaction line. These contributions are incorporated

in the dynamically screened interaction, Eq. (8), and hence not included in the definition of Γ^ω above. In contrast, for Fermi liquids with short-range interactions, the Landau interactions describing density-density response are dominated by the Hartree effects. As a result, they have positive sign for weak repulsive interactions. In contrast, our F_m are negative, since they are dominated by exchange effects. In particular, we expect $F_1 < 0$. The negative sign, expected from this general reasoning, is also borne out by a microscopic analysis at weak coupling; see below.

We also note an interesting analogy between the approach developed in this section and the analysis of superconducting FLs by Larkin and Migdal²¹ and Leggett.²² References 21 and 22 were concerned with FL renormalization of the quantities such as superfluid density in a metal with BCS pairing. Their analysis focused on the current correlation function which determines the response of current to vector potential, and followed similar steps as in the above discussion of $\Pi(\mathbf{q}, \omega)$. The renormalization effects were expressed through a combination of FL parameters, featuring a cancellation for a system with a parabolic band.

II. DENSITY DEPENDENCE FROM ONE-LOOP RG

In this section we derive plasmon dispersion for a simple model describing strongly interacting Dirac particles. This is done by employing the renormalization group analysis developed in Refs. 16–18. We treat the two-body scattering vertex by accounting for dynamical screening of the Coulomb interaction in the random-phase approximation (RPA),

$$U_{\mathbf{q}, \omega} = \frac{V(\mathbf{q})}{\tilde{\kappa}(\mathbf{q}, \omega)}, \quad \tilde{\kappa}(\mathbf{q}, \omega) = 1 - V(\mathbf{q})\Pi(\mathbf{q}, \omega). \quad (24)$$

Here the quantity $\tilde{\kappa}(\mathbf{q}, \omega)$ which describes dynamical screening is identical to that introduced in the above discussion of the dynamical density correlator, Eq. (8). Here $\Pi(\mathbf{q}, \omega)$ is the polarization function²

$$\Pi(\mathbf{q}, \omega) = N \sum_{\mathbf{k}, s, s'} |F_{\mathbf{k}, \mathbf{k}+\mathbf{q}}|^2 \frac{f(\epsilon_{\mathbf{k}, s}) - f(\epsilon_{\mathbf{k}+\mathbf{q}, s'})}{i\omega + \epsilon_{\mathbf{k}, s} - \epsilon_{\mathbf{k}+\mathbf{q}, s'} + i0}, \quad (25)$$

with the band indices $\{s, s'\} = \pm$ and the coherence factors $|F_{\mathbf{k}, \mathbf{k}'}|^2 = |\langle \mathbf{k}', s' | \mathbf{k}, s \rangle|^2$ describing overlaps of different pseudospin states. The polarization function is a sum of interband and intraband contributions, $\Pi = \Pi_1 + \Pi_2$, described by $s' \neq s$ and $s' = s$, respectively. The quantity $\tilde{\kappa}(\mathbf{q}, \omega)$ describes the effect of intrinsic screening in graphene arising due to both the interband and intraband polarization.

For undoped graphene, only interband transitions contribute, giving $\Pi_1(\mathbf{q}, \omega) = -\frac{N}{16} \frac{q^2}{\sqrt{v^2 q^2 + \omega^2}}$. This expression is sufficient for our RG analysis [for a comprehensive treatment of the quantity $\Pi(\mathbf{q}, \omega)$ we refer to Ref. 2].^{16–18}

The full RG analysis of log-divergent corrections to Green's functions and vertices was performed in Refs. 16–18. Below we use the results for one-loop RG calculation for large N . The RG flow for the quasiparticle velocity takes the form

$$\frac{dv}{d\ell} = \beta v, \quad \beta = \frac{8}{N\pi^2}, \quad (26)$$

where $\ell = \ln(p_0/p)$ is the RG time parameter (here the UV cutoff is set by interatomic spacing in graphene lattice, $p_0 \sim a^{-1}$). This gives a power-law dependence

$$v(p) = (p/p_0)^{-\beta} v_0. \quad (27)$$

For $N = 4$ we find $\beta \approx 0.2$. This value is obtained from a one-loop RG which employs $1/N$ as a small parameter. The results for $N \sim 1$ are qualitatively similar; however, the mathematical expressions are more cumbersome. Acknowledging an approximate character of the scaling dimensions obtained from one-loop RG, we leave the exponent β unspecified in the analytic expressions.

In the case of interest (doped graphene) the interband contribution Π_1 follows the above dependence for large momenta and frequencies, $|\mathbf{q}| \gg p_F$, $\omega \gg E_F$, which dominate the RG flow. The intraband contribution Π_2 is much smaller than Π_1 at such $\mathbf{q}$ and ω , with the two contributions becoming comparable for $|\mathbf{q}| \sim p_F$, $\omega \sim E_F$. In the static limit, $\omega \ll E_F$, the polarization is dominated by the Π_2 contribution. In the range $q < 2p_F$, which is where we need it below, it is identical to that for 2D systems with parabolic band,

$$\Pi(|\mathbf{q}| < 2p_F) = -N\nu, \quad (28)$$

$\nu = p_F/2\pi v$ (we refer to Ref. 2 for the analysis of other regimes). This gives a standard expression for the static RPA-screened interaction,

$$U_{\mathbf{q}, 0} = \frac{2\pi e^2}{\kappa|\mathbf{q}| + 2\pi N\nu e^2}. \quad (29)$$

We can obtain the two-particle scattering vertex Γ^ω by taking the interaction on the Fermi surface, $\mathbf{q} \sim p_F$, $\omega \ll E_F$. This gives

$$\Gamma^\omega(\theta, \theta') = -g_{\mathbf{p}, \mathbf{p}'} [\mathcal{T}(\Delta\mathbf{p})]^2 U_{\Delta\mathbf{p}, 0}, \quad \Delta p = 2p_F \sin \frac{\Delta\theta}{2}, \quad (30)$$

where $g_{\mathbf{p}, \mathbf{p}'} = |\langle \mathbf{p}'\alpha' | \mathbf{p}\alpha \rangle|^2 = \cos^2(\Delta\theta/2)$ is the coherence factor describing the overlap of (pseudo)spinors describing quasiparticles at different points of the Fermi surface (here $\Delta\theta = \theta - \theta'$). The minus sign in Eq. (30) arises because this expression represents a contribution from an exchange part of the two-particle vertex.¹³

The FL interaction can now be obtained from its relation with the vertex Γ^ω , Eq. (18). Combining Eqs. (18) and (30), we find

$$F(\theta - \theta') = -\nu g_{\mathbf{p}, \mathbf{p}'} Z^2 [\mathcal{T}(\Delta\mathbf{p})]^2 U_{\Delta\mathbf{p}, 0}. \quad (31)$$

In the large N limit, the static RPA-screened interaction can be approximated as $U_{\mathbf{q}, \omega} \approx -\frac{1}{\Pi(\mathbf{q}, \omega)} = \frac{1}{N\nu}$, where we take into account that $|\Delta p| < 2p_F$.

Both Z and $\mathcal{T}$ flow under RG; however, their product remains equal to unity because of the Ward identity. As a result, FL interactions do not undergo a power-law renormalization. Starting from $Z(p)\mathcal{T}(p) = 1$, where both $Z(p)$ and $\mathcal{T}(p)$ are given by power laws drawn from RG, we set $p = p_F$. This gives

$$F(\theta - \theta') = -\frac{1}{N} \left(\frac{\mathcal{T}(\Delta\mathbf{p})}{\mathcal{T}(p_F)} \right)^2 \cos^2 \left(\frac{\theta - \theta'}{2} \right). \quad (32)$$

We therefore conclude that, up to a remnant dependence on p_F which may arise in the angle dependence due to the ratio $T(\Delta\mathbf{p})/T(p_F)$, the function F does not flow under RG.

The function $F(\theta - \theta')$ is essentially independent of doping, whereas the velocity has a power-law dependence on doping, $v \propto p_F^{-\beta}$, given by Eq. (27) for $p = p_F$. Combining these results, we find a power law dependence for plasmon dispersion,

$$\omega^2 = A|\mathbf{q}|, \quad A = \frac{Ne^2 p_F v(p_F)}{2\kappa\hbar^2} (1 + F_1) \propto p_F^{1-\beta}. \quad (33)$$

This result is valid for plasmons with long wavelengths, $q \ll p_F$. The predicted power-law dependence holds in a wide range of carrier densities, both large and small, except very near the neutrality point where spatial inhomogeneity and thermal broadening play a role.

To conclude, plasmon renormalization results from competition of two effects: Plasmons tend to stiffen due to RG enhancement of velocity and to soften due to the negative sign of F_1 . However, since F_1 does not flow under RG, whereas velocity v does, the net effect of interactions is to stiffen plasmon dispersion. The predicted dependence $A \propto n^{(1-\beta)/2}$ can be used for extracting the exponent β from measurement results.

III. MAGNETOPLASMON IN A FERMIL LIQUID

Below we analyze plasmon dispersion using FL transport equations. We first deal with plasmons in the absence of magnetic field, then proceed to add a B field. Some of the relevant quantities, such as the Landau FL interaction $F(\theta - \theta')$, have already been introduced and analyzed, here we discuss them again to make a connection to the microscopic derivation in Sec. I.

In a semiclassical picture, the main effect dominating the FL behavior is forward scattering, wherein the whole system of interacting particles acts as a refractive medium in which a quasiparticle energy is a function of occupancies of other particles. This is described by so-called Landau functional,¹³

$$\delta\epsilon(\mathbf{p}) = \int \frac{d^2 p'}{(2\pi)^2} f(\mathbf{p}, \mathbf{p}') \delta n(\mathbf{p}', \mathbf{r}), \quad (34)$$

where $\delta n(\mathbf{r}, t)$ accounts for deviation of quasiparticle distribution from equilibrium.

Since deviation from equilibrium occurs in a narrow band of states near Fermi surface, it is convenient to write the Landau functional by setting $|\mathbf{p}| = |\mathbf{p}'| = p_F$ and parameterizing the Fermi surface by a unit vector $\mathbf{n} = \hat{\mathbf{p}}$. Introducing the dimensionless Landau interaction $F(\mathbf{p}, \mathbf{p}') = v f(\mathbf{p}, \mathbf{p}')$, where $v = p_F/(2\pi\hbar^2 v)$ is the density of states per flavor, we write

$$\epsilon(\mathbf{p}, \delta n) = \epsilon_0(\mathbf{p}) + \int \frac{d\theta'}{2\pi} F(\mathbf{p}, \mathbf{p}') \delta \tilde{n}(\mathbf{p}', \mathbf{r}). \quad (35)$$

Here $\epsilon_0(\mathbf{p}) = v(p - p_F)$ is linearized quasiparticle energy, the angle θ' describes orientation of $\mathbf{p}'$, and $\delta \tilde{n}(\mathbf{p})$ is obtained by integrating $\delta n(\mathbf{p})$ along the Fermi surface normal. The expression (35) can be treated as a Hamiltonian of one quasiparticle moving in a self-consistent field of other quasiparticles. Equations of motion can then be obtained from

Hamiltonian formalism via $\partial_t n = \{H, n\}$. This gives

$$(\partial_t + \mathbf{v}\nabla)\delta n(\mathbf{p}, \mathbf{r}, t) = -\mathbf{v}\nabla \hat{F} \delta n(\mathbf{p}, \mathbf{r}, t), \quad (36)$$

where $\hat{F}$ is the integral operator defined in Eq. (35).

In a system with rotational symmetry, such as graphene and 2D electron gases, the functional F depends only on the angle between $\mathbf{p}$ and $\mathbf{p}'$:

$$\hat{F} \delta n(\theta) \rightarrow \oint \frac{d\theta'}{2\pi} F(\theta - \theta') \delta n(\theta'). \quad (37)$$

This expression defines a Hermitian operator in the space of functions on the Fermi surface with the inner product

$$\langle f_1(\theta) | f_2(\theta) \rangle = \oint \frac{d\theta}{2\pi} f_1^*(\theta) | f_2(\theta). \quad (38)$$

The eigenvalues of $\hat{F}$ are simply given by the Fourier coefficients

$$F_m = \overline{F(\theta)e^{-im\theta}} = \oint \frac{d\theta}{2\pi} F(\theta)e^{-im\theta}. \quad (39)$$

The quantities F_m parameterize FL interactions of a 2D system.

To describe plasmons, we add to Eq. (36) a long-range electric field arising due to oscillating charge density,

$$[\partial_t + \mathbf{v}\nabla(\hat{1} + \hat{F})]\delta n(\mathbf{p}, \mathbf{r}, t) + e\mathbf{E}\nabla_{\mathbf{p}} n_0(\mathbf{p}) = 0, \quad (40)$$

where $n_0(\mathbf{p})$ is the equilibrium Fermi distribution. Here $\mathbf{E} = -\nabla\Phi$, where $\Phi(\mathbf{r})$ is the potential

$$\Phi(\mathbf{r}) = \sum_i \int d^2 r' \oint \frac{d\theta'}{2\pi} \frac{e}{\kappa |\mathbf{r} - \mathbf{r}'|} \delta n(\theta', \mathbf{r}', t).$$

Here the sum is taken over N spin/valley flavors, and the dielectric constant κ accounts for screening by substrate. Performing Fourier transform, $\delta n(\theta, \mathbf{r}, t) = \iint \frac{d\omega d^2 k}{(2\pi)^3} \delta n_{\omega, \mathbf{q}}(\theta) e^{-i\omega t + i\mathbf{q}\mathbf{r}}$, we arrive at an eigenvalue equation of the form identical to that found in Sec. I by analyzing poles of the dynamical screening function,

$$1 - V(\mathbf{q})\Pi(\mathbf{q}, \omega) = 0. \quad (41)$$

The quantity $\Pi(\mathbf{q}, \omega)$ is identical to that found above by summation of FL-type ladder graphs,

$$\Pi(\mathbf{q}, \omega) = Nv \text{Tr}_{\theta} \left[\frac{1}{\omega - \mathbf{q}\mathbf{v}(1 + \hat{F})} \mathbf{q}\mathbf{v} \right], \quad (42)$$

where trace is taken with respect to the inner product defined by Eq. (38).

Plasmon dispersion in the long wavelength limit can be found by expanding in the ratio $v|\mathbf{q}|/\omega$. We obtain

$$\begin{aligned} \Pi(\mathbf{q}, \omega) &\approx \frac{v}{\omega^2} \langle \mathbf{q}\mathbf{v} | (1 + \hat{F}) | \mathbf{q}\mathbf{v} \rangle \\ &= \frac{vq^2 v^2}{\omega^2} \langle \cos \theta | (1 + \hat{F}) | \cos \theta \rangle. \end{aligned} \quad (43)$$

Expressing the angle-averaged quantity through the Fourier coefficient F_1 and using the relation $v = p_F/(2\pi v)$ we rewrite this result as

$$\Pi(\mathbf{q}, \omega) = \frac{Np_F q^2}{4\pi\omega^2} v(1 + F_1). \quad (44)$$

Plugging this into Eq. (41) and restoring Planck's constant, we obtain the same expression for plasmon dispersion as above; see Eq. (2).

This analysis can be easily generalized to a system in the presence of an external magnetic field. This is done by accounting for the Lorentz force in the $\nabla_{\mathbf{p}}n$ term,

$$[\partial_t + \mathbf{v} \nabla (\hat{1} + \hat{F})] \delta n(\mathbf{p}, \mathbf{r}, t) + \left(e\mathbf{E} + \frac{e}{c} \tilde{\mathbf{v}} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{p}} n = 0, \quad (45)$$

where the velocity $\tilde{\mathbf{v}} = \nabla_{\mathbf{p}} \epsilon(\mathbf{p}, \delta n)$ includes the contributions accounting for the distribution function change $\delta n(\mathbf{p})$. This equation can be linearized as above, $n(\mathbf{p}, \mathbf{r}, t) = n_0(\mathbf{p}) + \delta n(\mathbf{p}, \mathbf{r}, t)$. In doing so, particular caution must be taken with the Lorentz force term since it is affected by the FL interactions. Accounting for the term in the velocity $\tilde{\mathbf{v}}$ that depends on $\delta n(\mathbf{p})$, we write

$$\tilde{\mathbf{v}}(n_0 + \delta n) = \nabla_{\mathbf{p}} \epsilon(n_0 + \delta n) = \nabla_{\mathbf{p}} \epsilon + \nabla_{\mathbf{p}} \hat{F} \delta n = \mathbf{v} + \hat{F} \nabla_{\mathbf{p}} \delta n. \quad (46)$$

Here we used Eq. (35), performing integration by parts in the last term.

Terms linear in δn can arise both from $\nabla_{\mathbf{p}} n_0$ and $\tilde{\mathbf{v}}$. Taking a solution in a plane wave form $\delta n(\mathbf{r}, \mathbf{p}, t) \propto e^{-i\omega t + i\mathbf{q} \cdot \mathbf{r}} \nabla_{\mathbf{p}} n_0(\mathbf{p}) f(\theta)$, where θ is the angle between $\mathbf{q}$ and $\mathbf{v}$, we have

$$[i\omega - ivq \cos \theta (1 + \hat{F})] f(\theta) - \frac{e}{c} (\mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{p}} f(\theta) - [\hat{F} \nabla_{\mathbf{p}} f(\theta) \times \mathbf{B}] \cdot \mathbf{v} = ivV(q)qv \cos \theta \oint \frac{d\theta'}{2\pi} f(\theta'). \quad (47)$$

This equation can be simplified as follows:

$$[i\omega - ivq \cos \theta (1 + \hat{F}) - \omega_B (1 + \hat{F}) \partial_{\theta}] f(\theta) = ivV(q)qv \cos \theta \oint \frac{d\theta'}{2\pi} f(\theta'), \quad \omega_B = \frac{evB}{p_F c}. \quad (48)$$

This gives an eigenvalue problem with ω a spectral parameter and $f(\theta)$ an eigenfunction. Inverting the operator on the left-hand side gives a self-consistency equation,

$$\oint \frac{d\theta}{2\pi} \frac{ivV(q)qv}{\omega + i\omega_B(1 + \hat{F})\partial_{\theta} - qv \cos \theta (1 + \hat{F})} \cos \theta = 1,$$

where $\frac{1}{\omega + \dots}$ is a shorthand for operator inverse. Magnetoplasmon dispersion can be obtained via perturbation theory in the parameter $qv/\omega \ll 1$, giving

$$\omega^2(\mathbf{q}) = (1 + F_1)^2 \omega_B^2 + (1 + F_1)^{\frac{1}{2}} V(\mathbf{q}) v \mathbf{q}^2 v^2. \quad (49)$$

This analysis ignores Bernstein modes which appear for $qr_c \sim 1$, where r_c is the cyclotron radius.¹⁵ The validity of Eq. (49) is henceforth limited to long wavelengths, $qr_c \ll 1$. Using the notation $Y = (1 + F_1)v$ we arrive at Eq. (3). Magnetoplasmon dependence on interactions is therefore described by the parameter Y identical to that found for plasmons at $B = 0$.

As discussed above, the density dependence of the quantities v and $1 + F_1$ can be linked to their flow under RG. The power-law RG flow of velocity leads to stiffening of plasmon dispersion, which overwhelms the effect of softening due to the negative sign of F_1 .

IV. COMPARISON TO SYSTEMS WITH PARABOLIC BAND DISPERSION

To put the above results in perspective, we recall some important aspects of long-wavelength plasmons in 2D electron systems with a parabolic band. Such plasmons afford a simple description in terms of classical equations of motion for collective “center-of-mass” variables describing oscillating charge density.⁶ The result is expressed in a general form through unrenormalized band mass and electron interaction as

$$m\omega^2 = nV(q)q^2, \quad (50)$$

where n is carrier density and $V(q) = \frac{2\pi e^2}{|q|\kappa}$ for two-dimensional systems. An identical result is found for the quantum problem, since Heisenberg evolution generates classical equations of motion for the operators corresponding to the center-of-mass variables describing collective charge dynamics.

The absence of renormalization of plasmon dispersion, Eq. (50), can be linked to Galilean invariance. In quantum systems, Galilean invariance is a symmetry of the Hamiltonian generated by the transformation $x' = x + vt$, $t' = t$. This symmetry, which holds for any system with parabolic band dispersion and instantaneous interactions, ensures a complete cancellation of the effects of interaction, rendering plasmon dispersion unrenormalized. As discussed above, the cancellation of FL corrections follows from the FL identity¹³ which relates renormalized velocity with the quantity F_1 ,

$$Y = (1 + F_1)v = v_0, \quad (51)$$

where $v_0 = p_F/m$ is Fermi velocity for noninteracting particles. Crucially, the validity of this identity depends on the band structure being parabolic on the scales $\epsilon \gtrsim E_F$ and $\epsilon \sim E_F$ which determine the FL interactions.

The relation between unrenormalized plasmon dispersion and Galilean symmetry also holds in the presence of a magnetic field, wherein gapless plasmons turn into gapped magnetoplasmons. The magnetoplasmon dispersion is $\omega_B^2(q) = \omega_0^2(q) + \omega_c^2$, where $\omega_0(q)$ is given by Eq. (50) and $\omega_c = eB/mc$ is unrenormalized cyclotron frequency. In this case, the absence of renormalization is guaranteed by Kohn's theorem.²³ The Kohn's theorem is established by treating collective charge dynamics in magnetic field using the center-of-mass variables in complete analogy with the derivation of Eq. (50). Because of the Galilean invariance, Heisenberg equations of motion for the center-of-mass variables obey classical dynamics with unrenormalized cyclotron frequency.

Unrenormalized plasmon dispersion also arises in other space dimensions, with $V(q) = \frac{4\pi e^2}{q^2 \kappa}$ for 3D systems and $V(q) = \frac{2e^2}{\kappa} \ln \frac{1}{|q|a}$ for 1D systems. In the latter case, plasmon dispersion matches that of charge modes in 1D Luttinger liquids. We stress that, in a general Luttinger liquid framework, the effective interaction for 1D plasmons is distinct from the bare interaction. Nevertheless, due to Galilean invariance, plasmon dispersion in a 1D system with parabolic bands is expressed through unrenormalized bare interaction. As noted above, what matters here is the character of the overall band structure rather than the linear dispersion in a system linearized near the Fermi points.

In contrast to systems with parabolic dispersion, plasmons in graphene are sensitive to interactions. This is so because Galilean invariance is a nonsymmetry for particles with linear dispersion, and hence the absence of renormalization is not guaranteed by any general principles. As a result, plasmons in graphene feature a nontrivial dependence on interactions. As we have seen above, plasmon dispersion is expressed through the Fermi velocity value v which is renormalized by the interaction effects, and also through the FL interaction via a factor $1 + F_1$. We parenthetically note that electronic spectrum of graphene bilayer, while featuring parabolic band dispersion, does not obey Galilean invariance. We therefore expect plasmon dispersion in a bilayer to exhibit a full-fledged FL renormalization, similar to graphene monolayer.

To summarize, renormalization of electron properties due to interactions results in a nonclassical dependence of plasmon frequency on carrier density. Using a nonperturbative approach based on the FL theory, we show that plasmon dispersion can be expressed through Landau FL interactions. Measurements of plasmon resonance can therefore be used to

extract the interaction parameters in a model-free way, which is particularly useful for studying strongly interacting systems such as graphene. Our results indicate a significant deviation from the $n^{1/4}$ power law dependence predicted for weakly interacting electrons in Refs. 1–4. The density dependence predicted by our approach derives from the RG flow of the quantity $Y = (1 + F_1)v$, where the RG “time” parameter value tracks the Fermi momentum. As an illustration, we consider RG for a large number of fermion flavors, which yields a power law of the form $n^{(1-\beta)/4}$, $\beta > 0$. The density dependence of the plasmon resonance can therefore provide a direct, model-free probe of the RG theory of interaction effects in graphene.

ACKNOWLEDGMENTS

We thank A. V. Chaplik, M. I. Dyakonov, F. H. L. Koppens, I. V. Kukushkin, and M. Yu. Reizer for useful discussions. This work was supported in part under the MIT Skoltech Initiative, a collaboration between the Skolkovo Institute of Science and Technology (Skoltech), the Skolkovo Foundation, and the Massachusetts Institute of Technology.

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