FUEL PREHEATING IN FREE-BURNING FIRES

By

Gerald K. Kwentus

B. S., Washington University
(1960)

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Doctor of Philosophy
at the

Massachusetts Institute of Technology

February 6, 1967

Signature of Author:

Signature redacted

Department of Chemical Engineering
February 6, 1967

Certified by:

Signature redacted

Thesis Supervisors

Accepted by:

Signature redacted

Chairman, Departmental Committee on
Graduate Theses

ABSTRACT

FUEL PREHEATING IN FREE-BURNING FIRES

By

Gerald K. Kwentus

Submitted to the Department of Chemical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy

The purpose of the work described in this thesis was to investigate the quantitative behavior of fuel preheating in free-burning fires, using a combination of experimental and mathematical modeling. Two types of steady-state, two-dimensional free-burning fires were considered; the first, a ground fire burning through compacted leaves and debris; the second, a brush fire.

Fuel preheating in ground fires was modeled mathematically with a partial differential equation, which upon integration provided a means of predicting fire propagation rates as a function of preheating fluxes, fuel bed properties, and ambient conditions. Preheating in brush fires was modeled with a set of simultaneous algebraic equations based upon a zoning technique similar to that employed in gas radiation problems. Solution of this set of equations yielded values of (1) fire propagation rate, (2) the configuration of the ignition interface in the fuel bed, and (3) fuel temperatures ahead of the ignition front as a function of parameters describing the fuel bed, the ambient, and flame plume and ember combustion.

The experimental modeling studies were carried out in a 4.5 ft.-diameter combustion wind tunnel, using shredded newsprint and wood excelsior to simulate, respectively, fuel beds in ground and brush fires. Data from the experimental studies were used to assess the adequacy of the mathematical models, and when possible, to evaluate parameters appearing in them.

Results from the ground fire studies suggested a means of resolving a disagreement between theory and experiment encountered in an earlier investigation of ground fire behavior. Those from the brush fire studies demonstrated the utility of the zoning technique for predicting the effects of action-at-a-distance phenomena in free-burning fires.

Thesis Supervisors:

Hoyt C. Hottel
Professor of Fuel Engineering

Glenn C. Williams
Professor of Chemical Engineering

Department of Chemical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

February 6, 1967

Professor William C. Greene
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Dear Professor Greene:

In accordance with the regulations of the Faculty, I herewith submit a thesis, entitled "Fuel Preheating in Free-Burning Fires", in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Chemical Engineering at the Massachusetts Institute of Technology.

Respectfully submitted,

Signature redacted

Gerald K. Kwentus

ACKNOWLEDGEMENTS

Use of modeling techniques to investigate the behavior of fuel preheating in free-burning fires was suggested to the author by Professors H. C. Hottel and G. C. Williams. Their advice and encouragement during the course of the modeling studies described in this thesis are gratefully acknowledged. A number of helpful suggestions from Professor A. F. Sarofim in the area of radiative heat transfer are similarly acknowledged.

The author wishes to thank Mr. H. A. Passler for his able assistance in constructing experimental apparatus. His talents as a machinist, welder, and sheet metal worker proved most useful.

Special thanks are due to Mrs. Elenore Kehoe, who typed this thesis; to Mr. and Mrs. A. E. Witt, who coordinated its reproduction; and to the author's fellow doctoral candidates--Messrs. W. Dalzell, J. Fahimian, R. Gurnitz, G. Miles, A. Morgan, J. Noble, D. Reed, K. Thompson, and A. Witt--who provided a congenial atmosphere in which to work.

The author is also grateful to the National Science Foundation for providing financial support for this work and to the M.I.T. Computation Center for providing facilities and time for the computer calculations in the mathematical modeling studies.

TABLE OF CONTENTS

	Page
SUMMARY	15
I INTRODUCTION	32
1.1 Background	32
1.2 Objective	33
1.3 Methods of Approach and Previous Work	34
II DEVELOPMENT OF A MATHEMATICAL MODEL FOR ENERGY TRANSFER IN THE UNBURNED FUEL IN A GROUND FIRE	38
2.1 Introduction	38
2.2 Linearization	39
2.3 Simulation of the Fuel Bed With a Homogeneous Slab	40
2.4 Energy Balance on Unburned Fuel	42
2.5 Fluxes Appearing in Boundary Conditions	46
2.6 Solution of Energy Balance Equation	48
2.7 Significance of Terms Which Appear in Integrated Energy Balance Equation	53
2.8 Use of Integrated Energy Balance to Describe the Behavior of Shredded Newsprint Fires	60
2.9 Comparison of Slab Model With Models in Combustion Symposium Paper	63
III APPARATUS AND PROCEDURE FOR EXPERIMENTAL MODELING OF FREE-BURNING FIRES	66
3.1 Introduction	66
3.2 Combustion Wind Tunnel	66
3.3 Radiometer	73

	Page
3.4 Fuel Bed	73
3.5 Experimental Procedure	75
IV MODELING OF ENERGY TRANSFER AHEAD OF THE FLAME FRONT IN GROUND FIRES--RESULTS AND DISCUSSION	77
4.1 Introduction	77
4.2 Radiative Flux From Flame Plume in Shredded Newsprint Fires	77
4.3 Radiative Flux From Heating Wire Arrays	83
4.4 Effect of Heating Wire Position Relative to Flame Front on Propagation Velocity	85
4.5 Effect of Randomly Oriented Fuel Particles	90
4.6 Effect of Heating Wire Radiation on Propagation Rate of Quiescent-Ambient Shredded Newsprint Fires	92
4.7 Effect of Heating Wire Radiation on Propagation Rate of Windblown Shredded Newsprint Fires	101
4.8 Analysis of Experimental Data From Shredded Newsprint Fires	108
V DEVELOPMENT OF A MATHEMATICAL MODEL FOR ENERGY TRANSFER IN THE UNBURNED FUEL IN A BRUSH FIRE	122
5.1 Introduction	122
5.2 Zoning of the Fuel Bed	123
5.3 Definition of Fuel-Bed Position Variables and Zone Numbers	126
5.4 Formulation of Energy Balance Equations	128
5.5 Enthalpy Flux Term	130
5.6 Radiation Interchange Between Fuel Bed Zones	131

	Page
5.7 Convective Transfer	133
5.8 Emission and Heat Sink Terms	134
5.9 Radiation From the Flame Plume	134
5.10 Solution of the Energy Balance Equations	139
5.11 Iteration Procedure	140
5.12 Newton-Raphson Iteration	141
5.13 Additional Comments on Solution of Energy Balance Equations	143
VI MODELING OF ENERGY TRANSFER AHEAD OF THE FLAME FRONT IN BRUSH FIRES--RESULTS AND DISCUSSION	147
6.1 Introduction	147
6.2 Effect of Radiation Absorption Coefficient k_b	148
6.3 Effect of Loading	153
6.4 Effect of Radiation Intensity From Flame Plume	157
6.5 Effect of Horizontal Displacement of Flame Plume	162
6.6 Effect of Plume Simulator Angle α_s	166
6.7 Comparison of Results With Emmon's Model	170
6.8 Significance of the Zoned Model	176
VII CONCLUSIONS	179
VIII RECOMMENDATIONS	181
8.1 Development of Burning Laws	181
8.2 Continued Experimental Modeling of Brush Fires	182
8.3 Investigation of Wood Pyrolysis and Piloted Ignition	182
APPENDIX	184
A Horizontal Conduction in the Slab Model	184

	Page
B Simulation of Flame Plume Radiation	191
C Air Distribution System	195
D Radiometer	201
D-1 Radiometer Design	201
D-2 Derivation of Radiometer Equation	203
D-3 Calibration	209
D-4 Radiometer Time Constant	214
E Derivation of an Approximate Expression for Surface Temperature as a Function of Enthalpy Temperature	217
F Effect of Anisotropy on Fuel Bed Radiation Interchange Factors	222
G Block Diagram and Listing of Fortran II Program for Brush Fire Model	228
H Summary of Experimental Data	250
I Location of Original Data	255
J Nomenclature	256
K Literature Citations	261
L Biographical Note	264

LIST OF FIGURES

Number	Title	Page
2.1	Cross-Sectional View of Fuel Bed of Shredded Newsprint	41
2.2	Cross Section of Slab Normal to Flame Front	43
2.3 (a,b)	Heat Retention Factors for a Single Line Source of Radiation Located L_w Units Directly Above Ignition Front	54
2.4 (a,b)	Heat Retention Factors for Flame Convection	56
2.5	Heat Retention Factors for a Semi-Infinite Slab	58
3.1 (a-c)	Test Section of Combustion Wind Tunnel	67
3.2	Fuel Bed and Heating Wires	72
3.3	Radiometer	74
4.1	Flame Plume Radiation From a Quiescent-Ambient Shredded Newsprint Fire	79
4.2	Correction Factor for Flame Plume Radiation Measurements	81
4.3	Flame Plume Geometries in Quiescent-Ambient Shredded Newsprint Fires for Two Values of $q'_A/4$	84
4.4	Radiative Flux Density Profile From an Array of Four Heating Wires in a Quiescent Ambient	86
4.5	Radiative Flux Density Profile From an Array of Six Heating Wires in a 1.6 ft/sec Wind	87
4.6	Effect of Heating Wire Position Relative to Flame Front on Propagation Velocity	88

Number	Title	Page
4.7	Heating Wire Arrays	93
4.8 (a-c)	Heat Retention Factors for Overhead Radiation	94
4.9	Effect of Heating Wire Radiation on Propagation	
	Rate of Quiescent-Ambient Shredded Newsprint Fires	98
4.10	Effect of Heating Wire Radiation on Propagation	
	Rate of Quiescent-Ambient Shredded Newsprint Fires	99
4.11	Effect of Heating Wire Radiation on Propagation	
	Rate of Quiescent-Ambient Shredded Newsprint Fires	
	(Data of Steward)	100
4.12	Effect of Heating Wire Radiation on Propagation	
	Rate of Windblown Shredded Newsprint Fires	102
4.13	Effect of Heating Wire Radiation on Propagation	
	Rate of Windblown Shredded Newsprint Fires	103
4.14	Effect of Heating Wire Radiation on Propagation	
	Rate of Windblown Shredded Newsprint Fires	104
4.15	Effect of Heating Wire Radiation on Propagation	
	Rate of Windblown Shredded Newsprint Fires	106
4.16	Flame Plume Geometries in Windblown Shredded	
	Newsprint Fires	107
4.17	Data of Figure 4.9 Plotted According to Equation 4.3	109
4.18	Data of Figure 4.10 Plotted According to Equation 4.3	110
4.19	Data of Figure 4.11 Plotted According to Equation 4.3	111
4.20	Data of Figure 4.12 Plotted According to Equation 4.3	112
4.21	Data of Figure 4.13 Plotted According to Equation 4.3	113

Number	Title	Page
4.22	Data of Figure 4.14 Plotted According to Equation 4.3	114
4.23	Ratio of Flame Plume Radiation in (1) Fires Burning Into a Headwind to (2) Fires Burning in a Quiescent Ambient	116
5.1	Cross Sectional View of Two-Dimensional Brush Fire Showing Fuel Bed Zones and Zone Numbers	125
5.2	Radiation Interchange Factors Between Zone <u>a</u> in Fuel Bed and Surrounding Zones	132
5.3	Perspective Sketch of Flame Simulator and Volume Element in Fuel Bed	136
5.4	View of Flame Simulator and Volume Element in the X-Y Plane	136
5.5	Cross Sectional View of Two-Dimensional Brush Fire Showing Half-Zone Grid and Zone Numbers	146
6.1	Effect of Radiation Absorption Coefficient k_b on Propagation Velocity	150
6.2	Fuel Bed Isotherms for Three Values of k_b	151
6.3	Effect of Fuel Loading on Propagation Velocity	155
6.4	Fuel Bed Isotherms for Three Fuel Loadings	156
6.5	Effect of Plume Radiation Intensity on Propagation Velocity	159
6.6	Fuel Bed Isotherms for Three Values of $(Q_w/L)/B_2$	160
6.7	Effect of Horizontal Displacement of Plume Radiation on Propagation Velocity	164

Number	Title	Page
6.8	Fuel Bed Isotherms for Three Values of ϵ/L_s	165
6.9	Effect of Plume Simulator Angle α_s on Propagation Velocity	168
6.10	Fuel Bed Isotherms for Three Values of α_s	169
6.11	Comparison of Burning Characteristic Predicted by Emmon's Model With Burning Characteristics Predicted by Zoned Model	174
6.12	Results of Fig. 6.11 Re-plotted According to Eq. 6.1	175
B-1	Two-Dimensional Flame-Plume Radiation Simulators	192
C-1	Components of Air Distribution System	196
C-2	Side View of Air Distribution System	197
C-3	Velocity Pattern in Cross Section of 16 in. O. D. Pipe at Upstream End of Combustion Tunnel	198
C-4	Velocity Pattern in Tunnel Cross Section Immediately Downstream of Air Distribution System	200
D-1	Perspective Sketch of Radiometer Sensing Head	204
D-2	Top View of Radiometer's Sensing Strips and Heat Sink	205
D-3	Radiometer Calibration	211
E-1	Cylindrical Fuel Particle	218
F-1	Torus and Its Projection Onto a Plane at an Angle ϕ Relative to the Horizontal	224
F-2	Two-Dimensional Radiation Interchange Factors in a Compressed Wood Excelsior Fuel Bed	227
G-1	Block Diagram of Fortran II Program for Brush Fire Model	229

LIST OF TABLES

Number	Title	Page
2.1	Comparison of Exact and Approximate Solutions for Semi-infinite Slab Model	64
4.1	Radiation Flux Density Measurements 0.6 in. Ahead of Flame Front and 9/32 in. Above Upper Surface of Fuel Bed for Six Values of $q'_{A/4}$	82
4.2	Heating-Wire Heat Retention Factors for the Data Points in Fig. 4.6	89
4.3	Effect of Random Orientation of Fuel Particles on Propagation Velocity	91
4.4	Parameters Obtained From Quiescent-Ambient Data	118
4.5	Parameters Obtained From Headwind Data	118
4.6	Ignition and Flame Convection Parameters in Quiescent-Ambient Shredded Newsprint Fires	120
4.7	Ignition and Flame Convection Parameters in Shredded Newsprint Fires Burning Into a 1.6 ft/sec Headwind	120
6.1	Values Assigned to Independent Parameters to Determine Effect of Radiation Absorption Coefficient k_b	149
6.2	Values Assigned to Independent Parameters to Determine Effect of Loading	154
6.3	Values Assigned to Independent Parameters to Determine Effect of Radiation Intensity From Flame Plume	158
6.4	Values Assigned to Independent Parameters to Determine Effect of Horizontal Displacement of Flame Plume	163

Number	Title	Page
6.5	Values Assigned to Independent Parameters to Determine Effect of Plume Simulator Angle α_s	167
6.6	Values Assigned to Independent Parameters to Determine Burning Characteristics	173
A-1	Effect of Horizontal Conduction on Slab Surface Temperature	189
D-1	Calibration Constants	213
H-1	Effect of Heating Wire Radiation on Propagation Rate of Quiescent-Ambient Shredded Newsprint Fires	250
H-2	Effect of Heating Wire Radiation on Propagation Rate of Windblown Shredded Newsprint Fires	251
H-3	Radiative Flux Density Measurements	252
H-4	Radiometer Calibration Data in Figure D-3	253

SUMMARY

The Woods Hole Summer Study of 1961 (22) outlined a number of goals toward which research on destructive, free-burning fires might profitably be directed. Central among these was the development of a predictive model for fire-spread. The function of a model of this kind would be to predict values of dependent variables in a free-burning fire -- variables such as propagation velocity, flame height, and flame depth -- from known values of independent parameters describing fuel loading, fuel properties, and meteorological conditions. Because of the model's predictive character, its use in combination with operations research techniques would provide a means of optimizing procedures for fighting and preventing unwanted fires.

The work described in this thesis, which is part of a continuing research program to develop a complete, predictive model for firespread, centered around the problem of energy transfer ahead of the flame front in free-burning fires. Its object was to model the quantitative behavior of fuel preheating in steady-state, two-dimensional fires. Two types of free-burning fires were considered; the first, a ground fire burning through compacted leaves and debris; the second, a brush fire.

The method of approach selected for use in this work combined experimental and mathematical modeling procedures.

The combination of modeling techniques was employed to offset partially the limitations which either would have had if used by itself. In the combined approach, use was made of simplifying assumptions to formulate approximate mathematical models, trading off rigor for tractability in the description of propagation. Data from small-scale experimental modeling studies were then utilized to assess the adequacy of the simplifying assumptions, and when possible, to evaluate parameters appearing in the models.

The experimental modeling studies were carried out in a 4.5 ft diameter combustion wind tunnel located in the Fuels Research Laboratory. The test section of the combustion tunnel, which had an axial length of 12 ft, was equipped with a traversing mechanism for moving the fuel bed during the course of fire experiments so that the flame front could be kept in a fixed position relative to the tunnel environment. The test section was also equipped with a series of electrical heating wires strung normal to its axis and parallel to the surface of the fuel bed. The function of the wires was to provide radiant preheat energy to the fuel bed in fire experiments. This was done to compensate, in part, for disproportionately low flame plume emissivities in laboratory-scale fires as compared to their large-scale counterparts. Radiation from the heating wires also proved to be a convenient independent parameter for use in checking the adequacy of mathematical models.

Radiant preheat energy to the upper surface of the fuel bed, from both the flame plume and heating wires, was measured with a water-cooled, air-shielded radiometer. The radiometer, which was designed to minimize its sensitivity to convective fluxes, was provided with its own traversing rig to move it along a line normal to the flame front. Other measurements made during the experimental work were fire front propagation velocity, ambient relative humidity, and wind velocity.

Mathematical modeling of fuel preheating in a ground fire typically gives rise to a differential equation. The form of the equation and its boundary conditions vary, depending upon the simplifying assumptions made in the model. Usually, these assumptions concern, among other things, the way in which flame-plume radiation is to be simulated, and the resistance and driving forces for convection.

Hottel, Williams, and Steward (14) presented four such models in a paper delivered at the Tenth Symposium (International) on Combustion. None of the models, however, seemed to agree with experimental data from steady-state, two-dimensional model fires burned in fuel beds of shredded newsprint. This disagreement served as the starting point for the ground fire studies reported in this thesis.

Each of the four models developed in the Symposium paper treated the unburned portion of the fuel bed as a single, homogeneous slab. In the first two models, the slab was infinitesimally thin, and in the latter two, infinitely thick. The other distinguishing feature of the models was the way in which each allowed for thermal losses from the fuel bed's upper surface. The first and third models assumed the thermal losses were a linear function of temperature, while the second and fourth allowed for non-linearities.

The ground fire model developed in this thesis -- hereafter referred to as the slab model -- retained the slab simulation of the fuel bed, but also allowed for intermediate cases wherein the slab was neither very thin nor infinitely thick. A linear formulation was chosen for thermal losses at the fuel bed's upper surface because the somewhat more rigorous non-linear formulation used in the second and fourth models in the Symposium paper had failed to resolve the disagreement mentioned earlier between theory and experiment. Other assumptions made in the slab model and their relation to those in the Symposium models are listed below.

1. All five models assumed that fuel ignites when its surface reaches a temperature T_{ig} which is independent of preheating conditions.

2. Convective preheat flux from hot gases in the flame plume to unburned fuel, designated q_c' , was assumed to be independent of fire front propagation rate in all five models.

3. The slab model assumed that the heat sink associated with pyrolysis reactions and/or mass transfer was zero order in mass concentration and a linear function of temperature. The Symposium models made no allowance for this type of heat sink.

4. The slab model simulated flame plume radiation with a series of line radiation sources; the Symposium models, with a vertical black plane.

5. In the slab model, heating wire radiation was simulated with an array of line radiation sources, distributed to correspond to the geometry of heating wire arrays used in model experiments. In the Symposium models, heating wire radiation was treated as though it arose from a single line source positioned directly above the ignition front.

Integration of the partial differential equation describing fuel preheating in the slab model generated a propagation equation suitable for use in predicting values of ground-fire propagation velocity as a function of preheating fluxes, fuel bed properties, and ambient conditions. In particular, the slab model's propagation equation -- like the propagation equations developed in the Symposium models -- provided a

means of predicting the effect of heating wire radiation q'_A on propagation velocity V .

Each of the Symposium models predicted that $dV/d(q'_A)$ should be a monotonically increasing function of q'_A . This dependence of V on q'_A , however, was not supported by the experimental data obtained by Steward (18) and reported in the Symposium paper. The slab model's propagation equation, while predicting effects which differed quantitatively from those of the Symposium models, also indicated that $dV/d(q'_A)$ should increase monotonically with q'_A . This suggested that the disagreement between theory and experiment in the Symposium paper arose because (1) the experimental data obtained by Steward were non-representative, and/or (2) the first two of the five assumptions listed earlier oversimplified the behavior of free-burning fires.

The next phase of the ground fire studies was experimental in nature. Its purpose was (1) to examine effects which might have led to errors in the data collected by Steward and the way in which they were analyzed and (2) to collect new data on the effect of q'_A on V in shredded newsprint fires.

Steward assumed, in analyzing his experimental data, that the contribution of plume radiation to fuel preheating could be ignored in shredded newsprint fires. This assumption was checked with the air-shielded radiometer by making plume radiation flux density measurements along the upper

surface of the fuel bed ahead of the ignition front and comparing them with flux densities from heating wire arrays. Steward's assumption proved to be a poor one, but not poor enough to account for the disagreement between theory and experiment.

Another possible source of error was the effect on propagation velocity of small departures of the heating wire array from its intended position relative to the ignition front. A careful examination of this effect in fire experiments, however, indicated that it was of little importance.

The fuel beds used by Steward in his model experiments were yet another possible source of the disagreement between theory and experiment. Steward prepared his fuel beds by randomly distributing newsprint shreds on a smooth, non-flammable substrate. The upper surface of the fuel bed, consequently, was not particularly smooth, and some of the fuel shreds lay in planes which were more nearly vertical than horizontal. The slab model, on the other hand, as well as the Symposium models, treated the upper surface of the fuel bed as a smooth horizontal plane. The importance of this effect on fire propagation velocity was checked by collecting new data on V as a function of q'_A from fires burned in fuel beds which had been tamped to make the fuel shreds lie as nearly as possible in horizontal planes. Data from the fires burned in tamped fuel beds and

those from randomly oriented fuel beds differed quantitatively, but not qualitatively. The new values of $dV/d(q'_A)$, like the old, were higher at low values of q'_A than at high.

The last phase of the ground fire studies was an investigation of the effect of q'_A on V in model fires burned in a strong headwind. All of the earlier experimental fires had been burned in a wind-free environment, with ambient flow depending upon buoyancy forces generated in the flame plume and above the electrically heated wires at the ignition front. The latter source of buoyancy varied with q'_A . It was thus possible that a large increase in q'_A could produce a substantial rise in ambient flow over the unburned portion of the fuel bed and into the foot of the flame plume, causing q'_C -- the convective preheat flux from the flame plume -- to decrease and h -- the convective loss coefficient -- to increase. Since both q'_C and h had been assumed to be independent of V and q'_A in the mathematical modeling studies, this effect may have contributed to the inability of the mathematical models to predict experimental results.

The strong headwinds provided an experimental environment wherein ambient velocity patterns were as nearly independent of q'_A and V as possible. (In a strong headwind, h should be constant; and q'_C , either constant or an increasing function of V .) Data from the headwind fires, however, did not differ substantially from the wind-free data. Once again, values of $dV/d(q'_A)$ were higher at low q'_A than at high. The data

did indicate, though, that $dV/d(q'_A)$ increased with q'_A over limited ranges of V .

Results from the headwind fire experiments implied, by a process of elimination, that T_{ig} (the fuel ignition temperature) and q'_C (convective preheat flux from the flame plume) were not independent of V as assumed in the slab and Symposium models. They implied, to the contrary, that both T_{ig} and q'_C increased with V and that the behavior of both parameters might be adequately approximated by assuming them to be step functions of V .

Allowance for a step-function dependence of both T_{ig} and q'_C on V resolved the disagreement between theory and experiment. This was done by applying the slab model piece-meal to experimental data in each of three velocity regions, assuming T_{ig} and q'_C to be invariant within a single region but to vary from region to region. (The division into three regions was suggested by inflections in the experimental data on V as a function of q'_A .) Values of q'_C and ignition energy Q''_i , the latter being a linear function of T_{ig} , were determined by this procedure for each of the three velocity regions. Both parameters were found to increase appreciably from the low velocity region to the high -- Q''_i , by a factor of approximately 5, and q'_C , by a factor of approximately 10.

The proposed explanation of this effect is that as propagation velocity increases, the separating distance between the water vapor wave evolved ahead of the flame front

and the flame front itself decreases until water vapor begins to dilute and then intermingles thoroughly with pyrolysis products. A higher rate of pyrolysis and consequently a higher fuel temperature would be required to compensate for the dilution with water vapor. Similarly, q'_c can be expected to increase with V because of higher pyrolysis rates and higher velocities of plume gases relative to the fuel bed.

Results from the ground fire studies served as a background for the investigation of fuel preheating in brush fires. In a brush fire, as in a ground fire, both plume radiation and plume convection contribute significantly to fuel preheating. Unlike a ground fire, however, the fuel bed in a brush fire also receives radiant preheat energy from hot, glowing embers behind the ignition front. The importance of this type of preheating in a brush fire is a consequence of the relatively high void volume of the fuel bed, which transmits thermal radiation as though it were a diathermous medium. Ember radiation thus exhibits action-at-a-distance behavior, and it shifts the mathematical description of fuel preheating from (1) the differential formulation of a ground fire to (2) an integro-differential formulation.

The brush fire model developed in this thesis approximated the integro-differential formulation with a finite set of simultaneous algebraic equations, where each member of the set was an energy balance around a fuel-bed zone of

finite size. The zoning procedure and the method used to set up the energy balances were analogous to those employed by Hottel and Cohen (13) in gas radiation problems.

A number of simplifying assumptions were made in the model for brush-fire fuel preheating -- hereafter referred to as the zoned model. The most important of these are listed below.

1. The fuel bed was assumed to be isotropic, with individual fuel particles having an emissivity of 1.0.
2. The temperature of each fuel particle was assumed to be uniform over its entire surface.
3. Fuel particles were assumed to ignite when their surface temperature reached a value T_{ig} .

The first two of the preceding assumptions allowed the radiation transport properties of the fuel bed to be simulated by those of a non-scattering grey gas with an absorption coefficient k_p numerically equal to one fourth of the surface area in a unit volume of fuel bed.

The zoning procedure divided the fuel bed into zones of rectangular cross section which were stationary with respect to the ignition front. Equation 5.1 is an energy balance written around one such zone in the unburned region of the fuel bed.

$$\begin{aligned}
 V \rho_b C_p B_1 (\bar{T}_i - \bar{T}_{i-1}) &= \sum_{\substack{j=1 \\ j \neq i}}^{N_2} [\overline{g_i g_j} / L] \sigma T_j^4 + \sum_{j=N_2+1}^{N_1} [\overline{g_i g_j} / L] \sigma T_B^4 \\
 &\text{increase in en-} \quad \text{radiation from} \quad \text{ember radiation from} \\
 &\text{thalpy of fuel} \quad \text{other zones} \quad \text{zones behind flame} \\
 &\text{as it moves} \quad \text{ahead of the} \quad \text{front} \\
 &\text{through zone } i \quad \text{flame front} \\
 &+ (\overline{Wg_i} / \delta L) (Q_w / L) + h_i (4k_b M_i B_1 B_2) (T_{A_i} - T_i) + h_{c_i} (4k_b M_i B_1 B_2) (T_F - T_{ig}) \\
 &\text{radiation from} \quad \text{convection from} \quad \text{convection from} \\
 &\text{flame plume} \quad \text{ambient in zone } i \quad \text{flame} \\
 &\text{simulator} \\
 &- \left\{ (4k_b M_i B_1 B_2) - [\overline{g_i g_i} / L] \right\} \sigma T_i^4 - (M_i B_1 B_2) Q_i \quad (5.1) \\
 &\text{radiative emission from} \quad \text{mass transfer-chemical} \\
 &\text{zone } i \quad \text{reaction heat sink}
 \end{aligned}$$

where B_1 = depth of zone in fuel bed

$\overline{g_i g_j} / L$ = radiation interchange length, fuel zone to fuel zone

h_{c_i} = flame convection heat transfer coefficient for zone i

h_i = ambient convection heat transfer coefficient for zone i

k_b = surface area of fuel/unit volume fuel bed, divided by 4

$M_i B_2$ = length of zone i in fuel bed as measured normal to infinite dimension.

N_1 = total number of zones in the fuel bed.

N_2 = number of zones ahead of flame front.

Q_i = mass transfer-chemical reaction heat sink.

Q_w/L = radiation emitted by a line heat source per unit length.

T_{A_i} = ambient temperature in zone i.

T_B = ember temperature of burning fuel.

T_F = flame temperature.

T_{ig} = fuel ignition temperature.

V = flame front propagation velocity.

$\overline{Wg}_i/\delta L$ = radiation interchange factor, flame plume simulator to zone i

$\rho_b C_p$ = heat capacity of fuel per unit total volume of fuel bed.

σ = Stephan-Boltzmann constant.

All temperatures appearing in Eq. 5.1 are absolute in value. Fuel temperatures are to be taken as surface temperatures with the exception of those with a bar over them. The latter are enthalpy mean temperatures.

Equation 5.1 is a representative member of the set of N_2 energy balances written around zones ahead of the ignition front. The dependent variables in this set of equations were: propagation velocity V , the configuration of the ignition interface in the fuel bed as defined in terms of three position parameters, and fuel bed temperatures T_i for all zones in front of the ignition interface except the zones in which ignition occurred. The system was determinate provided that parameters describing the flame plume and the ember burning region were treated as independent quantities.

The set of equations was solved by iteration on an IBM 7094 digital computer using a Fortran II program to set up the equations and to define the iteration procedure. For a single set of input parameters, i.e., parameters describing the fuel bed, ambient conditions, and preheating fluxes, iteration with a set of 35 equations converged to a solution in approximately one minute of computer time.

The zoned model, as described above, was used to predict the effect of several of the model's independent parameters on fire propagation. The independent parameters chosen for this purpose were (1) fuel-bed radiation absorption coefficient, (2) fuel bed loading, (3) plume radiation intensity, (4) plume simulator angle, and (5) plume displacement relative to the ignition front.

Qualitative data from model fires burned in the combustion wind tunnel, using wood excelsior to simulate the fuel bed in a brush fire, provided an experimental confirmation of the predicted effect of plume radiation intensity on the configuration of the fuel-bed ignition interface. Difficulties encountered in reaching steady state in the wood excelsior fires unfortunately prevented their use in checking other predicted effects. One additional check was made using experimental data collected by Curry and Fons (5). Their data indicated a trend of propagation velocity as a function of the fuel bed's radiation absorption coefficient which compared favorably with that predicted by the zoned model.

The last phase of the brush fire studies was a comparison of effects predicted by the zoned model and by a simplified model for fuel preheating developed by Emmons (8). Emmon's model was based on ember radiation preheating, treating this radiation as though it were collimated in the direction of fire propagation. All other preheat fluxes were treated as empirically determined corrections to ember radiation. The model consequently had the virtue of great simplicity. The comparison of predicted effects indicated, however, that the simplicity of Emmon's model was gained at an excessive sacrifice of its ability to describe adequately the behavior of free-burning fires.

Conclusions drawn from the modeling studies were as follows:

1. A zoning technique analagous to that employed in gas radiation problems can be used to describe the quantitative behavior of fuel preheating in brush fires.
2. The zoned model for brush fires provides a means of predicting values of (1) fire propagation velocity V , (2) the configuration of the ignition interface in the fuel bed, and (3) fuel bed temperatures ahead of the ignition interface as a function of parameters describing (1) the fuel bed's geometry, thermal properties, and ignition temperature, (2) ambient temperature and flow patterns, (3) the flame plume and ember combustion.

3. The criterion for piloted ignition which is based upon the assumption that fuel ignites when its surface reaches a critical temperature T_{ig} , where T_{ig} is independent of preheating rate, is adequate only over narrow ranges of preheating conditions.

4. Fuel ignition energies Q_i'' in shredded newsprint fires are four to six times higher at high preheating rates than at low.

5. The disagreement between theory and experiment reported by Hottel, Williams, and Steward (14) in their Combustion Symposium paper can be resolved by assigning higher values to both T_{ig} and the flame convection parameter q_c' at high values of V than at low.

6. Convection from the flame plume to unburned fuel in ground fires, i.e., q_c' , is a relatively strong increasing function of propagation velocity V , although it may be treated as constant over suitably restricted ranges of V . In shredded newsprint fires q_c' is of the order of 10 Btu/hr-ft at low V and of the order of 100 Btu/hr-ft at high V .

7. Radiation flux densities from small flame plumes can be measured to within 0.25 in. of the flame plume using a water-cooled radiometer provided with multiple sensing heads and an air shield.

Recommendations for future work included (1) the development of burning laws to describe the behavior of flame plume and ember combustion in free-burning fires, (2) a continuation of the brush-fire experimental modeling studies, and (3) an investigation of wood pyrolysis and piloted ignition.

I. INTRODUCTION

1.1 Background

"All things are an exchange for Fire, and Fire
for all things..."

Heraclitus of Ephesus

The philosopher Heraclitus, unlike earlier thinkers of Greek antiquity, recognized the principle of change and its universality. Fire embodied change, and he thus concluded that fire had to be the fundamental material of which all things were made. While this exalted view of fire's relation to matter has long since been discarded, Heraclitus' emphasis on the importance of fire as an agency of change, both good and ill, seems to have been justified.

The investigation of fire phenomena before the era of Heraclitus was haphazard and empirical and remained so until the Enlightenment. Since then much progress has been made toward a fundamental understanding of controlled fire phenomena. Uncontrolled, free-burning fires, on the other hand, have been investigated with few exceptions in an ad hoc manner, with more attention given to techniques of fire prevention than to an improved understanding of their behavior. The vast complexity of free-burning fires

served to discourage anything approaching a broad research program into their quantitative behavior. Fortunately, this situation has changed in recent years, largely because of the advent of new analytical techniques, including high-speed machine computation.

Part of the recent emphasis on the need for basic research into free-burning fire problems was provided by the Woods Hole Summer Study of 1961 (22). The Woods Hole Study proposed a number of goals toward which research on free-burning fires might profitably be directed. Central among these was the development of a predictive model for fire spread. The function of a predictive model of this kind would be to predict values of dependent variables in a free-burning fire, i.e., propagation velocity, flame height, and flame depth, from values of independent parameters such as fuel type, fuel loading, and meteorological conditions. When used in combination with operations research techniques, a predictive model would thus provide a means of optimizing fire fighting procedures and perhaps of preventing unwanted fires.

1.2 Objective

The work described in this thesis--admittedly very limited in scope--was directed toward an improved quantitative understanding of free-burning fires and

the eventual development of a predictive model for fire-spread. Its object was to model the quantitative behavior of energy transfer ahead of the flame front in steady-state, two-dimensional free-burning fires. Two types of fires were considered; the first, a ground fire burning through compacted leaves and debris; the second, a brush fire.

1.3 Methods of Approach and Previous Work

The development of a predictive model for the spread of a free-burning fire, or for the much simpler problem of energy transfer ahead of the flame front, could conceivably proceed along a number of different lines. One method of approach, for example, might make use of experimental modeling techniques. This would involve a series of model experiments and the subsequent correlation of experimental data as a function of relevant dimensionless groups such as the Grashof, Nusselt, and Reynolds numbers. The method of approach, however, has a serious drawback. Experimental modeling of a free-burning fire requires that a large number of modeling constraints be satisfied if all of the important aspects of a large-scale fire are to be adequately simulated in the laboratory. Unfortunately, some of these constraints, particularly those arising from buoyancy forces and plume radiation,

cannot be satisfied in laboratory-scale fire experiments. As a consequence, there is much uncertainty about how results obtained in the laboratory are to be scaled up so that they predict behavior in large fires.

The experimental modeling approach to free-burning fire problems has been used by a number of workers, most of them associated with the U. S. Forest Service, notably Fons and coworkers (9, 10) and Anderson and Rothermel (1). While much of the experimental work they have done has been carefully designed and well instrumented, they have not yet found a way around the scale-up problem.

Full-scale experimental fires or the accumulation of data taken during naturally occurring large-scale fires would avoid the scale-up problem. The first, unfortunately, is very costly. The second requires the accumulation of large amounts of carefully taken data to offset the absence of control over experimental parameters. Meaningful data from large-scale naturally occurring fires are non-existent at the present time, and a coordinated effort to remedy this situation has not yet been planned.

A second possible approach to the free-burning fire problem is to employ mathematical modeling along with what is known of the physics and chemistry of combustion processes. This too has its shortcomings. Any mathematical

description of a free-burning fire which attempts to give a reasonably rigorous account of transport processes and chemical reaction will consist of a set of complicated non-linear equations, one of which will be integro-differential in form. Uncertainty about the way in which some of the fluxes appearing in this set should be mathematically formulated, e.g., convection from plume gases to unburned fuel, in addition to their immense complexity makes even a first try at their solution of doubtful value at this time.

Yet a third approach to the development of a predictive model for free-burning fires is to use a combination of mathematical and experimental modeling. In this approach, the two types of modeling are combined in a way that partially offsets the weakness which each has when used by itself. Use is made of simplifying assumptions to formulate an approximate mathematical model, trading off rigor for tractability in the description of propagation. Since an approximate model of this kind may oversimplify the physical problem, its adequacy must be checked by some means. This can often be done with data from small-scale experimental modeling, which, for all its uncertainty with respect to scale-up, is still suitable for this purpose. Since the effect of scale is built into a mathematical model in the process of its formulation, the ability of a mathematical model to explain small-

scale fire data is strong indication (but not proof) of its adequacy to describe large-scale fires. The combined use of experimental and mathematical modeling also has the advantage that a complex fire problem can be conveniently broken up into a number of simpler problems. A forest fire, for example, often consists of three coupled regimes: a crown fire, a brush fire, and a ground fire. Each of the three regimes can be modeled in the absence of the other two, and the results of the model studies used to build up a predictive model for the overall forest fire by allowing mathematically for interaction between regimes.

The last of the methods of approach just discussed was chosen for use in the work described in this thesis. This approach was also used by Hottel, Williams, and Steward (14, 17) , whose work served as a basis for that described in the following chapters.

II. DEVELOPMENT OF A MATHEMATICAL MODEL FOR ENERGY TRANSFER IN THE UNBURNED FUEL IN A GROUND FIRE

2.1 Introduction

Compared to most other free-burning fire phenomena, a ground fire burning through compacted leaves and debris on the floor of a forest is neither awe-inspiring nor particularly destructive. Ground fires, none the less, are of considerable practical interest, since they often ignite and interact with other more destructive fire regimes. Ground fires are also a relatively uncomplicated type of free-burning fire, and their simplicity is a strong incentive to investigate their behavior before proceeding to more complex phenomena.

Simulation studies of ground fires have been reported by Hottel, Williams and Steward (14) in a recently published paper (hereafter referred to as the Combustion Symposium paper). Their simulation studies were both mathematical and experimental. The experimental simulation consisted of burning steady-state, two-dimensional fires in beds of shredded newsprint. In the mathematical simulation, attention was focused on energy transfer to the unburned fuel ahead of the ignition front. Four mathematical models were developed, corresponding to four different sets of simplifying assumptions. None of the four models agreed with results

obtained from the experimental studies.

The disagreement between theory and experiment reported in the Combustion Symposium paper was the starting point for the work described in this chapter and the two which follow. This chapter presents another mathematical model for energy transfer ahead of the flame front in ground fires, developed to check the adequacy of the simplifying assumptions used in the Combustion Symposium paper. The possibility that the disagreement between theory and experiment arose from non-representative experimental data rather than inadequacies in the mathematical modeling is considered in Chapter IV.

2.2 Linearization

In developing a mathematical model for energy transfer in a ground fire, a choice had to be made between the flexibility of a linear analysis and the rigour of one which allowed for non-linearities. A linear analysis was chosen for the following reasons. (1) The differential equation which was generated by the linear analysis could be integrated analytically for a quite general case; that generated by the non-linear analysis required numerical integration. (2) Superposition could be used to build up answers for new cases from existing solutions only for the linear analysis. (3) Allowance for a non-linear thermal loss in the second and fourth models in the Combustion

Symposium paper (the first and third allowed only for linear losses) did not resolve the disagreement between theory and experiment.

2.3 Simulation of the Fuel Bed With a Homogeneous Slab

The first two models in the Combustion Symposium paper treated the unburned portion of the fuel bed ahead of the ignition front as a thin, homogeneous slab; the last two, as a semi-infinite homogeneous slab. The model developed in the following sections (hereafter referred to as the slab model) retains the slab simulation but also allows for intermediate cases wherein the slab is neither very thin nor infinitely thick.

Before proceeding to the slab model's formulation, the question of the adequacy of using a homogeneous slab to simulate a composite fuel bed must be answered. Figure 2.1 is an idealized sketch of a fuel bed of shredded newsprint. The fuel bed shown in the figure, like the fuel bed in most ground fires, is a composite of alternate layers of fuel and air. Treating this type of fuel bed as a single homogeneous slab can be justified if the fuel elements have a thermal resistance (for conduction) in the vertical direction which is either (1) small compared to the thermal resistance of the air gaps in the fuel bed or (2) very large compared to the convective resistance at the upper surface of the fuel

Each newsprint shred is a planar slab
~1/3 in. by 1/3 in. by 0.003 in.

Figure 2.1 CROSS-SECTIONAL VIEW OF FUEL BED
OF SHREDDED NEWSPRINT

bed. What this means physically is that the uppermost layer of fuel in the bed must have very little thermal communication with anything below it. This is certainly so in shredded newsprint fires and would also seem to be the case in many large-scale ground fires.

2.4 Energy Balance on Unburned Fuel

A moving coordinate system and a stationary control volume were chosen as shown in Fig. 2.2 as a basis for formulating an energy balance for the unburned fuel. The coordinate system moves with the velocity V of the flame front; and its origin, which approaches the control volume as time θ increases, is at the lower surface of the slab directly below the ignition front. Since the process is steady state,

$$\frac{dy}{d\theta} = V$$

where y is the horizontal position of the control volume. Integrating the above expression,

$$y = y_m + V\theta$$

where y_m is a distance ahead of the flame front which is large enough so that the effect of the fire is not yet felt. (It can be shown that the precise value of y_m is unimportant; y_m need only be large compared to the flame height.)

In formulating the energy balance, conduction in the

Figure 2.2 CROSS SECTION OF SLAB NORMAL TO FLAME FRONT

y-direction is neglected since it will be small compared to other terms in the balance. (See APPENDIX A.) If, in addition, fuel properties are assumed to be independent of temperature, an energy balance around the control volume shown in Fig. 2.2 takes the form of Eq. 2.1.

$$\frac{\partial^2 (T-T_o)}{\partial x^2} - \frac{\rho C_p}{k} \frac{\partial (T-T_o)}{\partial \theta} - \frac{\gamma}{k} (T-T_o) = 0 \quad (2-1)$$

thermal
conduction
term

heat
storage
term

linearized
heat sink
term
(zero order
in mass
concentration)

where k = thermal conductivity of fuel

T = slab temperature at position (x,y)

T_o = slab temperature infinitely far ahead of flame
front

x = vertical position coordinate

γ = linearized heat sink coefficient

θ = time

ρC_p = volume heat capacity of fuel

Two sets of boundary and initial conditions, which respectively under- and overestimate heat leakage from the lower surface of the slab, will be used.

$$\text{Case I} \left\{ \begin{array}{l} (T-T_o)=0 \quad \text{for } \theta = 0, x \geq 0 \\ k \frac{\partial(T-T_o)}{\partial x} \Big|_{x=L} = -h(T-T_o)_{x=L} + q_L'' + q_s'' + q_c'' \\ k \frac{\partial(T-T_o)}{\partial x} \Big|_{x=0} = 0 \end{array} \right\} \text{for } \theta > 0$$

where h = loss coefficient at the upper surface of slab

q_s'' = flux density at upper surface of slab from plume
radiation simulator

q_c'' = flux density at upper surface of slab from flame
convection

$$q_L'' = h(T_A - T_o)$$

T_A = ambient temperature

$$\text{Case II} \left\{ \begin{array}{l} (T-T_o) = 0 \quad \text{for } \theta = 0, x \geq 0 \\ k \frac{\partial(T-T_o)}{\partial x} \Big|_{x=L} = -h(T-T_o)_{x=L} + q_L'' + q_s'' + q_c'' \\ k \frac{\partial(T-T_o)}{\partial x} \Big|_{x=0} = h(T-T_o)_{x=0} \end{array} \right\} \text{for } \theta > 0$$

2.5 Fluxes Appearing in Boundary Conditions

The fluxes q_i'' appearing in the boundary conditions are modeled in a manner similar to that used in the Combustion Symposium paper.

$$\begin{cases} q_L'' = h(T_A - T_O) = \text{constant} & y_m < y \leq 0 \\ q_L'' = 0 & y < y_m \end{cases}$$

$$q_S'' = \frac{\alpha q_S'}{2\pi M} \sum_{i=1}^M [L_{wi}^2 + (V\theta + y_m - \epsilon_i)^2]^{-1/2} L_{wi}$$

$$q_C'' = \frac{q_C'}{\ell} \exp \left[-\frac{\pi}{4} \left(\frac{y_m + V\theta}{\ell} \right)^2 \right]$$

where ℓ = scale of turbulence in flame convection

L_{wi} = height of radiation line source i above upper surface of slab

M = number of radiation line sources

q_S' = total radiation flux emitted by plume simulator, per unit length in infinite dimension

q_C' = total flame convection to unburned fuel, energy/unit time-unit length in infinite dimension

α = absorptivity of fuel

ϵ_i = horizontal separation between radiation line source i and the flame front

The sum $h(T - T_O)_{x=L} - q_L''$ describes total convective-radiative loss from the upper surface of the slab. The sum

linearizes radiative flux leaving the slab and combines the linearized coefficient with that for convective interchange with the ambient.

The flame convection term q_c'' is formed as the product of a constant coefficient and an exponential function which describes the assumed distribution of flame-eddy mixing lengths. The assumed driving force for flame convection is contained in the coefficient q_c' .

$$q_c' = \frac{k_c C_{g\rho_g} D (T_F - T_{ig})}{l}$$

where k_c = dimensionless proportionality constant for flame convection

$C_{g\rho_g}$ = volume heat capacity of gas in flame

D = eddy diffusivity

T_F = flame temperature

T_{ig} = fuel ignition temperature

l = scale of turbulence in flame convection

The term q_s'' simulates radiation from the flame plume-- as well as radiation from electrically heated wires used in small-scale ground fire experiments--with an array of line radiation sources. Other ways of simulating plume radiation are discussed briefly in APPENDIX B.

2.6 Solution of Energy Balance Equation

The subsidiary equation to Eq. 2.1 as obtained by Laplace transformation is

$$\frac{d^2 \bar{v}}{dx^2} - \left(\frac{P}{K} + \frac{\gamma}{k} \right) \bar{v} = 0 \quad (2.2)$$

where $K = \frac{k}{\rho C_p}$

P = Laplace time variable

$\bar{v}$ = Laplace transform of $(T - T_o)$

The transformed boundary conditions for Case I are

$$\left. \frac{d\bar{v}}{dx} \right|_{x=0} = 0 \quad (2.3)$$

and

$$\left. \frac{d\bar{v}}{dx} \right|_{x=L} = - \frac{h}{k} \bar{v}_{x=0} + \frac{1}{k} \bar{q}_i \quad (2.4)$$

where $\bar{q}_i$ = Laplace transform of $(q_L'' + q_C'' + q_S'')$

Equation 2.2 can be integrated for the above boundary conditions using standard techniques for the solution of ordinary linear differential equations.

$$\bar{v} = Ae^{rx} + Be^{-rx} \quad (2.5)$$

$$\left. \frac{d\bar{v}}{dx} \right|_{x=0} = rA - rB = 0 \quad (2.6)$$

$$\left. \frac{d\bar{v}}{dx} \right|_{x=L} = rAe^{rL} - rBe^{-rL} = -\frac{h}{k} (Ae^{rL} + Be^{-rL}) + \frac{1}{k} \bar{q}_i \quad (2.7)$$

where $r = \sqrt{\frac{P}{K} + \frac{Y}{k}}$

Using Eqs. 2.6 and 2.7 to eliminate A and B from Eq. 2.5,

$$\bar{v} = \frac{\frac{\bar{q}_i}{k} \frac{\cosh rx}{\cosh rL}}{r \tanh rL + \frac{h}{k}} \quad (2.8)$$

The term $\frac{\cosh rx}{\cosh rL}$ in Eq. 2.8 can be inverted

by use of the method of residues. It has no branch points, and it has poles at

$$r = i \beta_m, \quad m = 1, 2, 3, \dots$$

where $\pm \beta_m$, $m = 1, 2, 3, \dots$ are roots of $\beta \tan \beta L = \frac{h}{k}$

Applying the method of residues,

$$\mathcal{L}^{-1} \left[\frac{\cosh rx}{\cosh rL} \frac{1}{r \tanh rL + \frac{h}{k}} \right] = 2K \sum_{m=1}^{\infty} \frac{\beta_m^2 \frac{\cos \beta_m x}{\cos \beta_m L} \exp[-K(\beta_m^2 + \frac{Y}{k})\theta]}{\frac{h}{k} + L(\beta_m^2 + \frac{h^2}{k^2})}$$

Convolution integration can now be used to obtain the solution to Eq. 2.1

$$(T-T_o) = \frac{2K}{k} \int_0^\theta [q_L''(\lambda) + q_C''(\lambda) + q_S''(\lambda)]$$

$$\cdot \sum_{m=1}^{\infty} \frac{\beta_m^2 \frac{\cos \beta_m x}{\cos \beta_m L} \exp [-K(\beta_m^2 + \frac{\gamma}{k})(\theta-\lambda)]}{\frac{h}{k} + L(\beta_m^2 + \frac{h^2}{k^2})} d\lambda \quad (2.9)$$

Equation 2.9 can also be written as

$$V\rho C_P L(T-T_A) = \frac{\alpha q_S'}{4} \phi_S + q_C' \phi_C + V\rho C_P L(T_o-T_A) \phi_L \quad (2.10)$$

or, if $(T_o-T_A) = 0$, as

$$V\rho C_P L(T-T_o) = \frac{\alpha q_S'}{4} \phi_S + q_C' \phi_C \quad (2.11)$$

where $\phi_C \equiv 2 \frac{V\rho C_P L}{h\ell} \int_0^\theta \left\{ \sum_{n=1}^{\infty} \frac{[(\beta_n L)^2] \cdot \frac{\cos \beta_n x}{\cos \beta_n L} \exp[-(\beta_n L)^2 + \gamma L^2/k] \lambda_1}{1 + \frac{hL}{k} + \frac{k}{hL} (\beta_n L)^2} \right.$

$$\cdot \exp \left[-\frac{\pi}{4} \left(\frac{VL^2 \rho C_P}{k\ell} \right)^2 \left[\frac{k}{\rho C_P L^2} \left(\theta + \frac{\gamma_m}{V} \right) - \lambda_1 \right]^2 \right] d\lambda_1$$

$$\phi_S \equiv \frac{4}{\pi M} \frac{V\rho C_P L}{hL_w} \sum_{i=1}^M \int_0^\theta \left\{ \right\} \cdot \left[\frac{L_{wi}}{L_w} + \right.$$

$$\left(\frac{V\rho C_P L^2}{kL_w} \right)^2 \left(\frac{k}{\rho C_P L^2} \left(\theta + \frac{y_m}{V} - \frac{\varepsilon_i}{V} \right) - \lambda_1 \right)^2 \Big]^{-1} d\lambda_1$$

$$\phi_L = 1 - 2 \int_0^\varphi \left\{ \right\} d\lambda_1$$

$$\varphi = \frac{k}{V\rho C_P L^2} (y - y_m)$$

L_w = minimum value of L_{w_i} , $i=1, 2, 3, \dots, M$

When hL/k is small, the solution for Case II is almost identical to Eq. 2.10, differing only in that a $2h$ appears in the solution for Case II wherever an h appears in the solution given by Eq. 2.10. Thus, for small hL/k , a heat leak from the bottom of the slab can be allowed for by using Eq. 2.10 and taking h to be somewhat larger than would be the case if there were no heat leak from the bottom. When hL/k is not small, the heat leak from the slab's lower surface will be of less consequence, and Eq. 2.10 should again suffice.

If attention is restricted to slab surface temperatures at the ignition front, Eq. 2.10 becomes

$$V\rho C_P L (T_{ig} - T_A) = \frac{\alpha q'_s}{4} \phi_s + q'_c \phi_c + V\rho C_P L (T_O - T_A) \phi_L \quad (2.12)$$

In Eq. 2.12, $\phi_s = \phi_{s_{y=0}}^{x=L}$, $\phi_c = \phi_{c_{y=0}}^{x=L}$, and $\phi_L = \phi_{L_{y=0}}^{x=L}$.

ϕ_s , ϕ_c , and ϕ_L are heat retention factors and are analagous to the heat retention efficiencies defined in the Combustion Symposium paper. (When $hL/k = 0$ and $\gamma L^2/k = 0$, ϕ_s and ϕ_c are identically equal to the corresponding heat retention efficiencies.) ϕ_c is a function of

$$\frac{V\rho C_p L}{h \ell}, \frac{\gamma L^2}{k}, \text{ and } \frac{hL}{k}$$

while ϕ_s is a function of

$$\frac{V\rho C_p L}{hL_w}, \frac{\gamma L^2}{k}, \frac{hL}{k}, \text{ and } \frac{\epsilon_i}{L_{w_i}}, i=1, 2, 3, \dots M.$$

ϕ_L is a somewhat simpler function and can be expressed in closed form.

$$\phi_L = 1 - \frac{\frac{hL}{k}}{\sqrt{\frac{\gamma L^2}{k}} \tanh \sqrt{\frac{\gamma L^2}{k}} + \frac{hL}{k}}$$

Both ϕ_s and ϕ_c have been evaluated for a large number of cases using a digital computer to perform the required summations and integrations. Figures 2.3 (a,b), 2.4(a,b),

and 2.5 present part of the results of this computation. (The remainder of the results are presented in Chapter IV.) In using heat retention factors from Fig. 2.5 in Eq. 2.12, the slab thickness L in the equation is to be replaced by k/h .

2.7 Significance of Terms Which Appear in Integrated Energy Balance Equation

Equation 2.12 is a propagation law for ground fires. It equates (1) the increase in enthalpy of the upper surface of the fuel bed during preheating to (2) the fraction of the total energy input during preheating which remains within a differential depth of the surface at the time of ignition. The equation lists three sources of energy input: overhead radiation, flame plume convection, and a fuel-enthalpy input term arising from a possible difference in temperature between fuel and ambient temperatures far ahead of the ignition front.

Fuel far ahead of the ignition front will generally have the same temperature as the ambient, that is, T_o will be equal to T_A . If, however, a hot wind is blowing in the direction of fire propagation, the ambient temperature may be appreciably higher than the fuel temperature even at a distance y_m ahead of the flame front, where the

effects of both overhead radiation and flame convection are negligibly small. The last of the input terms in Eq. 2.12 allows for this possibility.

Each input term on the right-hand side of Eq. 2.12 is multiplied by a heat retention factor. The following description of a hypothetical fire serves to explain the physical significance of the factors. A fuel slab of thickness L is burning at a propagation rate V in an ambient in which the convective-radiative loss coefficient h is zero. The fuel's volumetric heat capacity is ρC_p ; its heat-sink coefficient γ is equal to zero; and it has no thermal gradients in the vertical direction. A macroscopic energy balance over the entire unburned portion of the fuel slab then has the form of Eq. 2.13.

$$V\rho C_p L(T_{ig} - T_O) = \frac{\alpha q'_s}{4} + q'_c \quad (2.13)$$

rate of increase of fuel enthalpy	total overhead radiation rate to unburned fuel	total flame con- vection rate to unburned fuel
--------------------------------------	--	--

or,

$$V\rho C_p L(T_{ig} - T_A) = \frac{\alpha q'_s}{4} + q'_c + V\rho C_p L(T_O - T_A) \quad (2.14)$$

The hypothetical fire described by Eq. 2.14 has no thermal losses. All of the preheat energy received by its fuel is utilized in elevating the fuel slab's temperature. On the other hand, Eq. 2.12 describes a fire with convective-radiative loss at the fuel slab's surface and a chemical reaction-mass transfer heat sink in its interior. A comparison of the two equations indicates that the heat retention factors in Eq. 2.12 are a measure of these losses relative to the behavior of the hypothetical fire.

As mentioned earlier, the fuel slab in the hypothetical fire does not have thermal gradients in the vertical direction. This implies that as much preheat energy is used to heat up the lower regions of the slab as the upper. The fuel slab of Eq. 2.12, however, may have large vertical thermal gradients, with more preheat energy stored near its upper surface than in its lower regions. The heat retention factors--since they are based upon the temperature of the slab's upper surface relative to the same quantity in the hypothetical fire--can thus have values greater than 1.0, as shown in Figs. 2.3, 2.4, and 2.5.

2.8 Use of Integrated Energy Balance to Describe the Behavior of Shredded Newsprint Fires

The integrated energy balance given by Eq. 2.12

affords a means of predicting the propagation rate of a ground fire provided that (1) fuel properties, (2) the convective-radiative loss coefficient, and (3) parameters describing the flame plume's temperature, geometry, emissivity, and eddy mixing length are known. (Flame plume parameters, in general, are dependent variables; and information about them can be obtained from "burning laws".) Equation 2.12, however, is not in a convenient form for describing ground fire propagation, since propagation velocity is not given as an explicit function of the other variables appearing in the equation. The following paragraphs present an explicit approximation to Eq. 2.12, for the special case of a ground fire burning through a fuel bed of newsprint shreds.

In the shredded newsprint fires to be described in Chapter IV,

$$\frac{hL}{k} \cong 0.02$$

$$\frac{V\rho C_P L}{hL_w} < 0.7$$

$$\text{and } \frac{\gamma L^2}{k} < 0.1$$

For the above range of conditions, the heat retention curves in Fig. 2.3b can all be superimposed upon the $\gamma L^2/k=0$ curve by replacing

$$\frac{V\rho C_p L}{hL_w} \quad \text{with} \quad \frac{V\rho C_p L}{h^*L_w}$$

where $h^*=h(1+ \frac{\gamma L}{h})$

The curve for $\gamma L^2/k=0$ in Fig. 2.3b is nearly linear with a slope of 1.0 in the region where $V\rho C_p L/hL_w < 0.7$. Thus,

$$\phi_s = A \frac{V\rho C_p L}{h^*L_w} \tag{2.15}$$

The factor A in Eq. 2.15 is a proportionality constant, and its value for a single line source of radiation located L_w units directly above the ignition front is 0.63.

The expected value of $V\rho C_p L/h\ell$ in shredded newsprint fires is greater than 1.0. In this region of dimensionless velocities, as shown in Fig. 2.4b, ϕ_c is near 1.0 and varies much more slowly with propagation velocity than does ϕ_s . In view of its weak dependence on propagation velocity as well as uncertainty in the value of ℓ ,

ϕ_c will be assigned a constant value of 1.0.

Equation 2.12 now can be rewritten as

$$V\rho C_P L(T_{ig}-T_O) = \frac{\alpha q'_S}{4} A \frac{V\rho C_P L}{h^* L_w} + q'_C \quad (2.16)$$

or, equivalently,

$$V = \frac{-q'_C}{\frac{\alpha q'_S}{4} A \frac{\rho C_P L}{h^* L_w} - \rho C_P L(T_{ig}-T_O)}$$

Equation 2.16 will be used in Chapter IV to analyze data from shredded newsprint fires.

2.9 Comparison of Slab Model with Models in Combustion Symposium Paper

The slab model of Eq. 2.12 is a generalization of the first and third models in the Combustion Symposium paper. When $hL/k=0$, $T_A=T_O$, $\gamma = 0$, $M=1$, and $\epsilon_1=0$, the slab model is identical to the first model in the Symposium paper. The third model in the Symposium paper is another limiting case of the slab model; when $L = \infty$, $T_A = T_O$, $\gamma=0$, $M=1$, and $\epsilon_1=0$, the two models are identical.

The third model was originally formulated as an integral equation. Its approximate solution is given by Eq. 2.17.

$$V\rho C_P \frac{k}{h} (T_{ig} - T_O) = \frac{\alpha q'_S}{4} \left[\frac{0.795 V_S^*}{V_S^{*0.5} + 1.18} \right] + q'_C [1.08 V_C^{*0.5}] \quad (2.17)$$

where $V_S^* = \frac{V\rho C_P k}{h^2 L_w}$ and $V_C^* = \frac{V\rho C_P k}{h^2 l}$

Equation 2.18, which is a limiting case of Eq. 2.12, is the exact solution of the equivalent differential equation formulation.

$$V\rho C_P \frac{k}{h} (T_{ig} - T_O) = \frac{\alpha q'_S}{4} \phi_S + q'_C \phi_C \quad (2.18)$$

The exact and approximate solutions are compared in Table 2.1.

TABLE 2.1
Comparison of Exact and Approximate Solutions for
Semi-infinite Slab Model

V_S^*	ϕ_S	$\frac{0.795V_S^*}{V_S^{*0.5} + 1.18}$	V_C^*	ϕ_C	$1.08V_C^{*0.5}$
0.01	0.00580	0.00621	0.01	0.00908	0.108
0.10	0.0522	0.0530	0.10	0.0816	0.341
1.00	0.364	0.364	1.00	0.556	1.08
10.0	1.84	1.83	10.0	2.66	3.41
100.0	7.22	7.10	100.0	10.2	10.8

As shown in the table, the approximate solution is very good at high values of dimensionless propagation velocity. At low values, however, it departs markedly from the exact solution.

Trends predicted by the slab model differ quantitatively but not qualitatively from those predicted by the four models in the Combustion Symposium paper. In all the models, the fraction of preheat energy which remains at the fuel slab's upper surface increases monotonically with increasing propagation velocity. The experimental data reported in the Symposium paper are consistent with this trend if (1) flame convection decreases with an increase in propagation velocity, or (2) fuel ignition temperature increases with propagation velocity. Both possibilities are considered in Chapter IV.

III. APPARATUS AND PROCEDURE FOR EXPERIMENTAL MODELING OF FREE-BURNING FIRES

3.1 Introduction

The development of experimental apparatus and procedure described in this chapter had two purposes. The first was to produce data of the type Steward (18) obtained but under more carefully controlled conditions and over a wider range of independent parameters. The second, to provide a facility for future experimental studies of small-scale wind blown fires. Results obtained from the experimental simulation of ground and brush fires are presented and discussed in Chapters IV and VI, respectively.

3.2 Combustion Wind Tunnel

The combustion wind tunnel, shown in Figs. 3.1a, 3.1b, and 3.1c, was assembled from the test section of Quench Tank No. 1 in the Fuels Research Laboratory. The test section, cylindrical in shape with a diameter of 4.5 ft and an axial length of 12 ft, was converted to a combustion tunnel by the addition of several components. The added components were (1) an air distribution system, (2) a transport mechanism for the fuel bed,

Tunnel Door Closed

Tunnel Door Open

Figure 3.1a

TEST SECTION OF COMBUSTION WIND TUNNEL

drilled plate (air distribution system)

honeycomb (air distribution system)

shredded newsprint fuel bed

radiometer

radiometer
traversing rig

Figure 3.1b

TEST SECTION OF COMBUSTION WIND TUNNEL

Test Section as Observed Looking Upstream
From a Vantage Point Directly Above the
Tunnel Axis
(Tunnel Door Open)

Test Section as Observed Looking Downstream
From a Vantage Point Directly Above the
Tunnel Axis
(Tunnel Door Open)

(3) a heating wire rig, and (4) a radiometer and traversing rig.

Air flow in the combustion tunnel was provided by the quench tank's air compressor system, which was capable of supplying air at flow rates from 0 to 26 lb/sec. Air discharging from the compressor system first passed through a venturi where its flow rate was measured and then discharged through a 16 in.-diameter pipe into the air distribution system at the upstream end of the test section. The function of the distribution system was to expand the flow emerging from the 16 in. pipe uniformly across the portion of the 4.5 ft-diameter cross section lying above the fuel bed's transport mechanism. This was done in an axial distance of about 3 ft by using a combination of drilled plates, flow-dividing funnels, and a honeycomb. (See APPENDIX C)

A transport mechanism for the fuel bed was located in the lower portion of the test section. Its purpose was to move the fuel bed so that the position of the flame front of a fire burning in the bed could be kept stationary relative to the environment of the test section. The mechanism consisted of a shallow cement-board box, 6 ft long and 2.5 ft wide, mounted on two 12 ft rails. A wire

rope, pulleys, and shaft connected the box to a crank outside the tunnel so that it could be moved either upstream or downstream during the course of an experiment. Fiber-glass aircraft insulation placed in the box served as a substrate for the fuel bed. A closeup view of the fiber-glass substrate and a shredded newsprint fuel bed is shown in Fig. 3.2.

A series of electrically heated 20 gauge Kanthal wires was stretched across the fuel bed at the flame front as shown in Fig. 3.2. The function of the heated wires was to provide overhead radiation to the unburned fuel in addition to that from the flame plume. The reason for their use was to compensate for the disproportionately low plume radiation in small-scale fires as compared to their large-scale counterparts. Several different heating wire arrays were used; that shown in Fig. 3.2 was a horizontal array of six wires 1.25 in. above the fuel bed with a center to center distance between wires of 0.2 in. In each case, the heating wire array was connected through a variable transformer to a circuit capable of providing 15 amps at 220 volts. Sagging of the heated wires because of thermal expansion was prevented by individually spring loading each wire in the array.

Heating Wires and Shredded Newsprint
Fuel Bed as Viewed Through Window
in Tunnel Door
(Heating Wire Circuit Open)

Shredded Newsprint Fire Burning
Into a 1.6 ft/sec Headwind
(Heating Wire Circuit Closed)

Figure 3.2

FUEL BED AND HEATING WIRES

3.3 Radiometer

Radiative flux from the heating wires and flame plume incident upon the upper surface of the unburned fuel was measured by a water-cooled radiometer shown in Fig. 3.3. The radiometer was attached to a traversing rig, as shown in Fig. 3.1b, so that it could be moved along a horizontal line perpendicular to the flame front. The problem of sensitivity to convective flux usually encountered in this type of instrument was overcome by using a combination of an air shield and multiple sensing strips. APPENDIX D presents a brief description of the instrument's design and calibration.

3.4 Fuel Bed

The fuel used in most of the experimental work was shredded newsprint, which was burned as a means of simulating ground fires. A second fuel, wood excelsior, was used in a small number of qualitative experiments to simulate brush fires. The newsprint fuel consisted of planar shreds, approximately $1/3$ in. by $1/3$ in. by 0.003 in., cut from the Boston Globe. When distributed on the fuel bed substrate, this material formed a rather compact fuel bed capable of transmitting little if any thermal radiation. Wood excelsior, on the other hand, made a very porous fuel

Top View

Side View

Front View

The distance between numbered divisions on the scale is 1.0 in.

Figure 3.3

RADIOMETER

bed. The excelsior, which had a porosity in excess of 95%, was composed of strands of poplar wood, each of which was several inches long and had a rectangular cross section of 0.017 in. by 0.035 in.

A fuel loading of 0.0625 lb/ft^2 was used in all the shredded newsprint fires, corresponding to a bed depth of 6 layers of newsprint. The length of the newsprint fuel bed as measured parallel to the direction of propagation was 4 ft, and its width was 12 in. for both headwind and quiescent-ambient fires and 20 in. for fires burning with a tailwind. In the wood excelsior fires, the fuel loading was 0.17 lb/ft^2 ; and the fuel bed's depth, width, and length were respectively 2 in., 20 in., and 5 ft. Both types of fuel were allowed to equilibrate with ambient temperature and moisture level for several hours before burning.

3.5 Experimental Procedure

The following steps were used in the experimental procedure.

- (1) Air flow in the combustion tunnel was adjusted to the desired rate and its relative humidity measured.

(2) A weighed amount of fuel was then distributed uniformly on the fuel bed substrate, and the fuel bed moved as far as possible to either the upstream or downstream end of the tunnel, depending upon whether the fire was to burn into a headwind or with a tailwind.

(3) The edge of the fuel bed nearest the heating wire array was wetted with igniter fluid, which was a mixture of pentane and methanol.

(4) After closing the tunnel door, the heating wire circuit was closed and its power dissipation adjusted by means of a variable transformer. (The tunnel door was left open for fires burned in a quiescent ambient.)

(5) The wetted end of the fuel bed was next moved under the heating wire array where the igniter fluid was lit with a flame from a propane burner.

(6) Following ignition, the fire was allowed to burn several inches along the fuel bed to reach steady state, the fuel bed being moved so that the position of the flame front remained constant relative to the heating wire array. After reaching steady state, measurements of both radiation flux density and propagation velocity were made, the latter by recording blips on a moving strip chart recorder for every inch of fuel bed that moved under the heating wire array.

IV. MODELING OF ENERGY TRANSFER AHEAD OF THE FLAME FRONT
IN GROUND FIRES--RESULTS AND DISCUSSION

4.1 Introduction

In the Combustion Symposium paper (14), both flame convection and ignition temperature were assumed to be independent of propagation velocity. Either -- or both -- of these assumptions could have led to the disagreement between theory and experiment reported in the paper. Other possible sources of the disagreement are related to the Symposium paper's experimental data and the procedure used by Steward (19) to obtain these data.

This chapter considers each of the above possible sources of disagreement between theory and experiment. The early sections of the chapter examine effects which may have led to errors in Steward's data or the way in which they were analyzed. The later sections present new data of the type Steward obtained and analyze both sets of data using the slab model of Chapter II.

4.2 Radiative Flux From Flame Plume in Shredded
Newsprint Fires

In analyzing his data from shredded newsprint

fires, Steward assumed that flame plume radiation was of negligible importance in fuel preheating. This assumption has since been checked using the radiometer described in Chapter III. Two types of measurements were made, the first, to determine total plume radiation flux to the unburned fuel in the absence of artificial radiation, that is, with $q_A'/4 = 0$; the second, to determine the effect of $q_A'/4$ on plume radiation. (The total artificial radiation rate--provided by electrically heated wires at the flame front--to the unburned portion of the fuel bed is given by $q_A'/4$; and the total radiation rate from the flame plume, by q_F' . Thus, $q_S'/4 = q_A'/4 + q_F'$.)

The plume radiation flux density profile for a fire burning without artificial radiation is shown in Fig. 4.1. If the flux density profile is extrapolated as shown in the figure and if a correction is made for radiation from the part of the plume lying below the radiometer, the area under the curve is equal to q_F' , the total plume radiation flux received by the unburned fuel. The uncorrected area in Fig. 4.1 is equivalent to 37 Btu/hr-ft. When corrected for the part of the plume below the radiometer, the area is increased by a factor of 1.8 and corresponds to a total plume radiation rate to the unburned fuel of

Figure 4.1 FLAME PLUME RADIATION FROM A QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRE

67 Btu/hr-ft.

The way in which the correction factor of 1.8 was calculated is indicated in Fig. 4.2. The calculation assumes that the flame plume can be simulated by a **diffuse emitter** plane and then uses the crossed strings method described by Hottel (12) to evaluate the ratio of (1) flux received by the unburned fuel to (2) flux received at the horizontal plane along which measurements were made. From Fig. 4.2,

$$\begin{aligned} \text{Correction Factor} &= \frac{\frac{s_1 s_2}{L}}{\frac{s_3 s_4}{L}} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{\overline{AE} + \overline{BC} - \overline{AD}} \\ &= \frac{y_m + \frac{L_F}{\sin \varphi} - \sqrt{L_F^2 + \left[y_m + \frac{L_F}{\tan \varphi}\right]^2}}{y_m + \sqrt{(L_F - \delta)^2 + \left[\frac{L_F}{\tan \varphi}\right]^2} - \sqrt{(L_F - \delta)^2 + \left[y_m + \frac{L_F}{\tan \varphi}\right]^2}} \\ &\cong \frac{1 - \cos \varphi}{\sqrt{\left[1 - \frac{\delta}{L_F}\right]^2 \sin^2 \varphi + \cos^2 \varphi} - \cos \varphi} \end{aligned}$$

If $\frac{\delta}{L_F} = \frac{9/32}{5/4} = 0.225$ and $\varphi = 35^\circ$, Correction Factor = 1.8.

Radiation from a black plane is not a particularly good simulation of that from a flame plume. However, since the black-plane simulator was used to form both the numerator and

$\frac{\overline{s_1 s_j}}{L}$ = radiation interchange area, per unit length in infinite dimension, between surface ① and surface ②
 $\overline{MN}$ = length of string stretched from point M to point N

$$\frac{\overline{s_1 s_2}}{L} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{2}$$

$$\frac{\overline{s_3 s_4}}{L} = \frac{\overline{AE} + \overline{DF} - \overline{EF} - \overline{AD}}{2} = \frac{\overline{AE} + \overline{BC} - \overline{AD}}{2}$$

$$\text{Correction Factor} = \frac{\frac{\overline{s_1 s_2}}{L}}{\frac{\overline{s_3 s_4}}{L}} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{\overline{AE} + \overline{BC} - \overline{AD}}$$

Figure 4.2 CORRECTION FACTOR FOR FLAME PLUME RADIATION MEASUREMENTS

denominator of the correction factor ratio, the error introduced by the simulation should tend to cancel out.

The effect of artificial radiation on flame plume flux density is presented in Table 4.1.

TABLE 4.1
RADIATION FLUX DENSITY MEASUREMENTS 0.6 IN.
AHEAD OF FLAME FRONT AND 9/32 IN. ABOVE
UPPER SURFACE OF FUEL BED FOR SIX VALUES OF $q'_A/4$

$q'_A/4$ Btu/hr-ft	Flux Density From Heating Wires Btu/hr-ft ²	Flux Density From Heating Wires + Flame Plume Btu/hr-ft ²	Flux Density From Flame Plume Btu/hr-ft ²
0	0	210	210
82	490	720	230
126	760	1040	280
145	870	1140	270
196	1170	1420	250
223	1330	1590	260

The variation of plume flux density with increasing $q'_A/4$ is relatively small and does not follow a consistent pattern. In all probability, the variation reflects uncertainty in the measurement procedure, and it is reasonable to conclude

that plume flux density is independent of $q'_A/4$. The effect of $q'_A/4$ on the size and shape of the flame plume as shown in Fig. 4.3 also support this conclusion. Figure 4.3 indicates that flame depth increased appreciably with $q'_A/4$ but that plume height and plume shape near the ignition front were independent of $q'_A/4$. Since the rear of the flame plume has a poor view of the unburned fuel bed, an increase in the depth of the plume from 2 to 6 in. (without altering its height or configuration near the ignition interface) should not appreciably increase plume radiation to the unburned fuel.

Steward's assumption that plume radiation can be neglected is thus adequate only when $q'_A/4 \gg 67$ Btu/hr-ft. Since Steward took all of his data at values of $q'_A/4 < 480$ Btu/hr-ft, his assumption proves to be a poor one. More will be said about radiation and its contribution to fuel preheating in the following sections.

4.3 Radiative Flux From Heating Wire Arrays

Radiation flux density profiles from two different heating wire arrays were measured with the radiometer to determine the adequacy of treating the arrays as non-interacting line sources of radiation. Measurements were made along a horizontal plane below each of the arrays and

are presented in Figs. 4.4 and 4.5. In both figures, the solid line represents measured values of dimensionless flux density, and the dashed line, calculated values of the same quantity obtained by treating each wire in the array as though it were an independent line source of radiation. As shown in the figures, the calculated values closely approximate the measured, even when the heating wires are close together, as in Fig. 4.4.

4.4 Effect of Heating Wire Position Relative to Flame Front on Propagation Velocity

The effect of heating wire position relative to the flame front on propagation velocity is shown in Fig. 4.6. The effect was investigated for two reasons. The first was to determine if a small departure of the heating wire array from its intended position could lead to a large error in the measured propagation rate; the second, to provide experimental data with which to check results predicted by the slab model.

As indicated in Fig. 4.6, small departures of the wire array from its intended position have an appreciable effect on propagation rate only when the intended position is less than -0.2 , as measured in units of ϵ_1/L_w . Most of the experimental data reported in this chapter, including the

Figure 4.4 RADIATIVE FLUX DENSITY PROFILE FROM AN ARRAY OF FOUR HEATING WIRES IN A QUIESCENT AMBIENT

Figure 4.5 RADIATIVE FLUX DENSITY PROFILE FROM AN ARRAY OF SIX HEATING WIRES IN A 1.6 FT/SEC WIND

$$q_A'/4 = 470 \text{ Btu/hr-ft}$$

Wind Velocity = 0 ft/sec

◇ - 27% Relative Humidity

○ - $37 \pm 2\%$ Relative Humidity

Figure 4.6 EFFECT OF HEATING WIRE POSITION RELATIVE TO FLAME FRONT ON PROPAGATION VELOCITY

data of Steward, was taken at values greater than -0.2; and consequently, inadvertent lags (or leads) in heating wire position should have introduced little, if any, error into the measured propagation velocities.

The terms in the slab model which should vary most with changes in heating wire position are propagation velocity V and the heat retention factor ϕ_s , with V increasing monotonically as a function of ϕ_s . This functional dependence was checked by calculating a value of ϕ_s for each data point in Fig. 4.6. The results of the calculation are presented in Table 4.2, where values of V and ϕ_s are listed in order of increasing V .

TABLE 4.2

HEATING-WIRE HEAT RETENTION FACTORS FOR THE

DATA POINTS IN FIG. 4.6

<u>V ft/hr</u>	<u>ϕ_s^1</u>	<u>ϵ_1/L_w</u>
38.2	0.117	- 0.80
45.0	0.161	- 0.48
47.6	0.178	- 0.32
52.5	0.199	- 0.16
52.6	0.199	0.0
61.8	0.213	0.32
64.6	0.235	0.16
66.2	0.245	0.0

$$^1 \frac{\rho C_p L}{h^* L_w} = \frac{(14)(0.003)}{(5)(1.25)} = 0.00672 \text{ hr/ft}$$

As shown in the preceding table, V increases monotonically with ϕ_s , as predicted.

4.5 Effect of Randomly Oriented Fuel Particles

The last three sections have examined effects which, if ignored, could lead to errors in either (1) experimental procedure or (2) analysis of experimental data. This section follows along the same lines.

The slab model derived in Chapter II, as well as the four models presented in the Combustion Symposium paper, treat the upper surface of the fuel bed as a smooth horizontal plane. Fuel beds of randomly oriented shreds of newsprint, however, are not particularly smooth; and some of the shreds lie in planes which are more nearly vertical than horizontal. The shredded newsprint fuel beds used by Steward (18) contained randomly oriented fuel particles. Consequently, fuel preheating in his fire experiments may have departed substantially from that predicted by slab models. As a means of estimating the effect of random orientation on propagation velocity and hence on fuel preheating, a series of four fire experiments was performed. The fuel in the first two experiments was randomly oriented. In the other two experiments, the fuel was carefully tamped after it was

distributed on the fuel bed substrate to make the fuel particles lie as nearly as possible in horizontal planes. Propagation velocities measured in the four experiments are listed in Table 4.3.

TABLE 4.3

EFFECT OF RANDOM ORIENTATION OF FUEL PARTICLES
ON PROPAGATION VELOCITY¹

<u>$q_A'/4$</u> <u>Btu/hr-ft</u>	<u>V (random)</u> <u>ft/hr</u>	<u>V (tamped)</u> <u>ft/hr</u>
220	38.6	33.5
470	58.6	52.6

¹Relative humidity = $37 \pm 2\%$. Other experimental conditions were identical to those in Figure 4.6. $\epsilon_1/L_w=0$

The data in Table 4.3 indicate that randomly oriented fuel burned more rapidly than fuel which had been tamped. The probable cause was twofold. (1) The randomly oriented fuel, on the average, had larger interchange factors for both flame-plume and heating-wire radiation. (2) The randomly oriented fuel had a higher flame convection coefficient as a consequence of the greater irregularity of the fuel bed surface.

All of the data in this chapter, except that in Table 4.3 and the data of Steward, were taken from fires burned in tamped fuel beds. This was done to make experimental conditions correspond as closely as possible to the slab model of Chapter II.

4.6 Effect of Heating Wire Radiation on Propagation Rate of Quiescent-Ambient Shredded Newsprint Fires

Most of the data reported by Steward from quiescent-ambient shredded newsprint fires were measurements of propagation rate as a function of heating-wire radiation intensity. This section presents data of the same type.

The heating wire arrays used to investigate the effect of radiation intensity on propagation rate are shown in Fig. 4.7. The first six arrays were used to obtain the tamped-fuel-bed data reported in this section and the following; Array No. VII and Array No. VIII were used by Steward. Heat retention factors for each of the eight arrays were computed from the slab model of Chapter II to provide a means of comparing and analyzing data. The retention factors are shown in Fig. 4.8 (a-c).

Measurements of propagation velocity from fires burned in tamped fuel beds are presented in Figs. 4.9 and 4.10;

Figure 4.7

HEATING WIRE ARRAYS

and Steward's data, in Fig. 4.11. The tamped-fuel-bed data differ quantitatively, but not qualitatively, from Steward's data.

From Eq. 2.16,

$$V = \frac{q'_C}{\rho C_P L (T_{ig} - T_O) - \left(\frac{q'_A}{4} + q'_F \right) \alpha A \frac{\rho C_P L}{h^* L_w}} \quad (4.1)$$

Assuming that q'_C , q'_F , T_{ig} , and h^* are all constant,

$$\frac{dV}{d\left(\frac{q'_A}{4}\right)} = \frac{\alpha A \frac{\rho C_P L}{h^* L_w} q'_C}{\left[\rho C_P L (T_{ig} - T_O) - \left(\frac{q'_A}{4} + q'_F \right) \alpha A \frac{\rho C_P L}{h^* L_w} \right]^2} \quad (4.2)$$

Equation 4.2 predicts that the slope of a smooth curve through the data in Figs. 4.9, 4.10, and 4.11 should increase monotonically with increasing $q'_A/4$. The data in the figures, however, do not agree with this prediction. The disagreement suggests that some combination of the quantities q'_C , h^* , and T_{ig} --or, equivalently, q'_C , h , γ and T_{ig} --varied with propagation velocity in the fire experiments. (As shown in Section 4.2, q'_F is virtually independent of propagation velocity in shredded

Relative Humidity: 29-31%

Heating Wire Array: No. I

Figure 4.9 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRES

Relative Humidity: 69-71%

Heating Wire Array: No. III

Figure 4.10 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRES

- △ - 27% Relative Humidity, Heating Wire Array No. VII
- - 37% Relative Humidity, Heating Wire Array No. VIII
- ◇ - 37-39% Relative Humidity, Heating Wire Array No. VII
- - 47-49% Relative Humidity, Heating Wire Array No. VII
- ◊ - 48% Relative Humidity, Heating Wire Array No. VIII

Figure 4.11 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRES (DATA OF STEWARD)

newsprint fires and thus should not have contributed to the disagreement.) The following section presents data taken to determine if the disagreement arose because of variation of q'_C or h .

4.7 Effect of Heating Wire Radiation on Propagation Rate of Windblown Shredded Newsprint Fires

The convection parameters q'_C and h both depend upon ambient velocity patterns near the surface of the fuel bed. In a wind-free fire, the flow of ambient is dependent on buoyancy driving forces generated in the flame plume and above the electrically-heated wires at the ignition front. The latter source of buoyancy varies with $q'_A/4$. It is thus possible that a large increase in $q'_A/4$ could produce a substantial rise in ambient flow rate over the unburned fuel and into the foot of the flame plume, causing h to increase and q'_C to decrease.

A series of shredded newsprint fires was burned into a strong headwind to provide experimental conditions wherein ambient velocity patterns were as nearly independent of $q'_A/4$ and V as possible. (In the presence of a strong headwind, h should be constant; and q'_C , either constant or an increasing function of V .) Data from the fires are shown in Figs. 4.12, 4.13, and 4.14. The trends

Relative Humidity: $27 \pm 2\%$

Heating Wire Array: No. I

Wind Velocity: 1.6 ft/sec headwind

Figure 4.12 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF WINDBLOWN SHREDDED NEWSPRINT FIRES

Relative Humidity: $49 \pm 2\%$

Heating Wire Array: No. II

Wind Velocity: 1.6 ft/sec headwind

Figure 4.13 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF WINDBLOWN SHREDDED NEWSPRINT FIRES

Figure 4.14 EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF WINDBLOWN SHREDDED NEWSPRINT FIRES

indicated by the data do not differ substantially from those in Figs. 4.9, 4.10, and 4.11. The windblown data, as well as the wind-free, disagree with Eq. 4.2, although the former do indicate that $dV/d(q'_A/4)$ may increase monotonically over limited ranges of V .

Figure 4.15 presents data from a series of fires burned with a tailwind. These data are too few in number to infer anything about $dV/d(q'_A/4)$ and are presented only for purposes of comparison with headwind fire data. Flame plume geometries for both types of windblown fires are shown in Fig. 4.16. The presence of a tailwind, as indicated in the figure, drastically alters plume geometry, causing the plume to bend over the unburned fuel, enhancing both plume-radiation and plume-convection preheating.

The headwind fire data imply, by a process of elimination, that T_{ig} and/or γ increased as V increased. Both of these quantities and their behavior as a function of V are closely related to the problem of what constitutes an adequate criterion for piloted ignition. The remainder of this chapter considers the problem of fuel ignition and analyzes the data presented thus far using the slab model of Chapter II.

Relative Humidity: $49 \pm 2\%$

Heating Wire Array: No. IV

Wind Velocity: 0.45 ft/sec tailwind

Figure 4.15

EFFECT OF HEATING WIRE RADIATION ON PROPAGATION
RATE OF WINDBLOWN SHREDDED NEWSPRINT FIRES

Fuel Ahead of Ignition Front
Fuel Behind Ignition Front

The flame plume, fuel bed, and heating wire array are shown below in cross section for two fires, the first burning into a headwind, and the second, with a tailwind. The cross sections lie in the x-y plane, and all their dimensions are in inches.

$q'_A/4 = 400 \text{ Btu/hr-ft}$
 wind velocity = 1.6 ft/sec

$q'_A/4 = 120 \text{ Btu/hr-ft}$
 wind velocity = 0.45 ft/sec

Figure 4.16 FLAME PLUME GEOMETRIES IN WIND-BLOWN SHREDDED NEWSPRINT FIRES

4.8 Analysis of Experimental Data from Shredded Newsprint Fires

Use of a surface temperature criterion for ignition along with a heat sink coefficient for fuel preheating which is independent of propagation velocity represents an attempt to provide a simple answer to the quite complex question of what determines whether or not a piece of fuel will ignite after being exposed to a given thermal flux. This type of criterion is doubtlessly adequate over a suitably restricted range of preheating conditions. When, however, the range of preheating conditions is large, it may no longer suffice, as suggested by the data of the preceding section.

Equation 4.1 can be inverted and written as

$$\frac{1}{V} = \left(\frac{q'_A}{4} + q'_F \right) \frac{\alpha_A}{q'_C} \frac{\rho C_P L}{h^* L_w} + \frac{\rho C_P L (T_{ig} - T_o)}{q'_C} \quad (4.3)$$

If T_{ig} and γ are constant over a small range of V , data taken in this range and plotted according to Eq. 4.3 should fall along a straight line whose slope is inversely proportional to $h^* q'_C$ and whose intercept is directly proportional to $(T_{ig} - T_o)/q'_C$. Figures 4.17 through 4.22 contain the data of Figs. 4.9 through 4.14, respectively, re-plotted according to Eq. 4.3. As shown in the figures, straight

Figure 4.17

DATA OF FIGURE 4.9 PLOTTED ACCORDING TO EQUATION 4.3

Figure 4.18 DATA OF FIGURE 4.10 PLOTTED ACCORDING TO EQUATION 4.3

Figure 4.19 DATA OF FIGURE 4.11 PLOTTED ACCORDING TO EQUATION 4.3

Figure 4.20 DATA OF FIGURE 4.12 PLOTTED ACCORDING TO EQUATION 4.3

Figure 4.21 DATA OF FIGURE 4.13 PLOTTED ACCORDING TO EQUATION 4.3

Figure 4.22 DATA OF FIGURE 4.14 PLOTTED ACCORDING TO EQUATION 4.3

lines can be drawn through the data over limited ranges of V . (The subscript n in Fig. 4.19 refers to normalization of values of $q_A'/4$ relative to heating wire Array No. VII; and in Fig. 4.22, normalization relative to heating wire Array No. IV.)

The values of q_F' used in Figs. 4.17-4.22 were 67 and 35 Btu/hr-ft for quiescent-ambient and headwind fires, respectively. The former value was obtained from Section 4.2, and the latter from Section 4.2 and Fig. 4.23. From Fig. 4.23,

$$\frac{\frac{s_1 s_2}{L}}{\frac{s_3 s_4}{L}} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{\overline{DE} + \overline{EF} - \overline{DF}} \approx \frac{L_{F_1}}{L_{F_2}} \frac{\frac{1}{\sin \varphi_1} - \frac{1}{\tan \varphi_1}}{\frac{1}{\sin \varphi_2} - \frac{1}{\tan \varphi_2}}$$

If $L_{F_1}/L_{F_2} = 1$, $\varphi_1 = 18.5^\circ$, and $\varphi_2 = 35^\circ$,

$$\frac{\frac{s_1 s_2}{L}}{\frac{s_3 s_4}{L}} = 0.52 \approx \frac{35}{67}$$

The data in Figs. 4.17-4.22 seem to fall into three regions--corresponding to low, intermediate, and high propagation velocities. One possible explanation of this effect is that as propagation velocity increases, the separating distance between the water vapor wave evolved ahead of the

$\frac{s_i s_j}{L}$ = radiation interchange area, per unit length in infinite dimension, between surface ① and surface ③
 $\overline{MN}$ = length of string stretched from point M to point N

$$\frac{s_1 s_2}{L} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{2}$$

$$\frac{s_3 s_4}{L} = \frac{\overline{DE} + \overline{EF} - \overline{DF}}{2}$$

$$\frac{\frac{s_1 s_2}{L}}{\frac{s_3 s_4}{L}} = \frac{\overline{AB} + \overline{BC} - \overline{AC}}{\overline{DE} + \overline{EF} - \overline{DF}}$$

Figure 4.23 RATIO OF FLAME PLUME RADIATION IN (1) FIRES BURNING INTO A HEADWIND TO (2) FIRES BURNING IN A QUIESCENT AMBIENT

flame front and the flame front itself decreases until water vapor appreciably dilutes and then thoroughly intermingles with pyrolysis products. A higher rate of pyrolysis and thus a higher fuel temperature would be required to compensate for the dilution with water vapor. Similarly, flame-convection eddy diffusivity and hence q'_C can be expected to increase with increasing propagation velocity as a consequence of higher pyrolysis rates and higher velocities of plume gases relative to the fuel bed. (An alternative explanation is that fuel ignition is kinetically limited, i.e., pyrolysis reactions may not be able to keep in step with increased rates of fuel preheating.)

The three velocity regions in Figs. 4.17-4.22 are reflected in the values listed in Tables 4.4 and 4.5, which were obtained from the slopes and intercepts of the dashed lines in the figures. As shown in the tables, the group $h^*(T_{ig}-T_o)$, which contains both the heat sink parameter γ and the fuel ignition temperature T_{ig} , is four to six times larger in the high velocity region than in the low. On the other hand, the ratio $\rho C_p L(T_{ig}-T_o)/q'_C$ is appreciably smaller in the high velocity region than in the low, implying that the increase in q'_C is greater than that in $\rho C_p L(T_{ig}-T_o)$.

TABLE 4.4

PARAMETERS OBTAINED FROM QUIESCENT-AMBIENT DATA

Approximate Relative Humidity %	$\frac{\rho C_P L (T_{ig} - T_o)}{q'_c}$ hr/ft			$h^* (T_{ig} - T_o)$ Btu/hr-ft ²		
	Inter-			Inter-		
	mediate V			mediate V		
	Low V	High V		Low V	High V	
30	0.076	0.048	0.033	1370	2870	5170
30		0.019			3800	
40		0.021			4150	
50		0.022			4310	
70	0.165	0.065		1700	4700	

TABLE 4.5

PARAMETERS OBTAINED FROM HEADWIND DATA

Approximate Relative Humidity %	$\frac{\rho C_P L (T_{ig} - T_o)}{q'_c}$ hr/ft			$h^* (T_{ig} - T_o)$ Btu/hr-ft ²		
	Inter-			Inter-		
	mediate V			mediate V		
	Low V	High V		Low V	High V	
30	0.094	0.054	0.036	1160	3370	6570
40	0.154	0.080		1700	4500	
50			0.0371			9810

The quantity $\rho C_p L(T_{ig} - T_o)$ can be thought of as an ignition energy and will be referred to hereafter as Q_i'' . Values of Q_i'' and q_c' obtained from Tables 4.4 and 4.5, assuming h^* to have a constant value of 5 Btu/hr-ft²-°R, are presented in Tables 4.6 and 4.7. Since $h^* = h + \gamma L$, the assumption that h^* is constant is equivalent to assuming that γ , but not T_{ig} , is independent of V over its entire range.

From a survey of ignition energies reported in the literature, Steward (20) concluded that the total energy input required to produce piloted ignition of newsprint is approximately 10 Btu/ft². Since a substantial fraction of the total energy input will be (1) lost by convection and back radiation and (2) consumed in a chemical reaction-mass transfer heat sink, the sensible energy increase of the newsprint required for piloted ignition--and hence the size of Q_i'' --will be much less than 10 Btu/ft². The values of Q_i'' in Tables 4.6 and 4.7 for the high velocity region (this region corresponds most closely to conditions in the literature) call for fractional losses of the order of 0.5 if they are to be consistent with the number cited by Steward. This is reasonable. The increase of Q_i'' and q_c' at high propagation

TABLE 4.6
IGNITION AND FLAME CONVECTION PARAMETERS IN
QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRES

Approximate Relative Humidity %	Q_i'' Btu/ft ²			q_c' Btu/hr-ft		
	Inter- mediate V			Inter- mediate V		
	Low V		High V	Low V		High V
30	0.96	2.0	3.6	13	42	110
30		2.7			140	
40		2.9			140	
50		3.0			140	
70	1.2	3.3		7.2	51	

TABLE 4.7
IGNITION AND FLAME CONVECTION PARAMETERS IN
SHREDDED NEWSPRINT FIRES BURNING INTO A 1.6 FT/SEC HEADWIND

Approximate Relative Humidity %	Q_i'' Btu/ft ²			q_c' Btu/hr-ft		
	Inter- mediate V			Inter- mediate V		
	Low V		High V	Low V		High V
30	0.81	2.4	4.6	8.7	43	130
40	1.2	3.1		7.7	40	
50			6.9			180

velocities also agrees with the postulated merging of the water vapor front and the ignition front. (The high values of q'_C for the data obtained by Steward are a consequence of the randomness of the fuel beds he used. See Section 4.5.)

The results discussed above and in the earlier sections of this chapter provide a means of resolving the disagreement between theory and experiment reported in the Combustion Symposium paper. They also serve as a background for the investigation of fuel preheating in brush fires, which is the topic of the next two chapters.

V. DEVELOPMENT OF A MATHEMATICAL MODEL FOR ENERGY TRANSFER
IN THE UNBURNED FUEL IN A BRUSH FIRE

5.1 Introduction

In a brush fire, as in a ground fire, both plume radiation and plume convection contribute significantly to fuel preheating. Unlike a ground fire, however, a brush fire also preheats its fuel with radiation from hot, glowing embers behind the ignition front. The importance of this type of preheating in a brush fire is a consequence of the relatively high void volume of the fuel bed, which transmits thermal radiation--from both hot fuel particles as well as the flame plume--as though it were a diathermous medium. Ember-radiation preheating thus exhibits action-at-a-distance behavior, and it shifts the mathematical description of energy transfer in the unburned fuel from

- (1) the differential formulation of a ground fire to
- (2) an integro-differential formulation.

The model developed in this chapter approximates the integro-differential formulation with a finite set of simultaneous algebraic equations, where each member of the set is an energy balance around a fuel-bed zone of finite size. Sections 5.2 through 5.13 describe the zoning technique and the procedure for setting up and solving

the set of energy balance equations. Results predicted by this model, hereafter referred to as the zoned model, as well as a discussion of their significance are reserved for Chapter VI.

Other work which bears relation to the zoned model has been done by Emmons (8), Hottel and Cohen (13), and Einstein (7). Emmons has developed a simplified mathematical model for energy transfer ahead of the flame front in brush-type fires; this model is discussed at some length in Chapter VI. Hottel and Cohen considered the problem of radiative transfer within a gas-filled enclosure and devised a procedure for zoning the system to approximate energy transfer in it. In a similar manner, Einstein used a zoning technique to describe energy transfer in a grey gas flowing between two parallel plates. The results of both Hottel and Cohen and Einstein were used as a basis for the present work.

5.2 Zoning of the Fuel Bed

The adequacy of a zoning technique for describing energy transfer in a brush-fire fuel bed depends upon the size and shape of the zones which are used. In general, the zones should be small enough so that temperature does not vary appreciably within an individual zone, and their

shape (as well as size) should be such that all parts of any one zone have approximately the same view of the surroundings. However, the use of a large number of small-size zones as suggested by the above criteria can cause severe computational problems since the effort required to solve the set of zoned equations increases with the square of the number of zones.

The zoning scheme chosen for the brush fire model is a compromise between use of small zones and computational ease. As shown in Fig. 5.1, the fuel bed is divided into two-dimensional zones which are rectangular in cross section and which are stationary with respect to the flame front. The elemental zone is B_1 units high and B_2 units long, where

$$B_1 = \frac{L_b}{4}$$

$$B_2 \leq \frac{L_f}{4}$$

L_b = depth of fuel bed

L_f = flame height

The zoning grid divides the fuel bed into Eulerian control volumes which move relative to the fuel bed at the velocity of the flame front.

Figure 5.1 CROSS SECTIONAL VIEW OF TWO-DIMENSIONAL BRUSH FIRE SHOWING FUEL BED ZONES AND ZONE NUMBERS

Near the flame front, where temperature gradients are largest, the zone size is kept relatively small. Further ahead of the flame front, however, zone sizes are increased to avoid having an unwieldy total number of zones. Zoning is terminated at approximately $17B_2$ ahead of the ignition front since the effect of the fire will be felt only weakly beyond this point.

5.3 Definition of Fuel-Bed Position Variables and Zone Numbers

The zoning of the fuel bed into four vertical regions implies that ignition occurs in a total of four zones, one for each level. The position of the uppermost of these zones is an independent variable since the zoning grid and the position of the flame plume are defined relative to it. The positions of the other three ignition zones, however, are dependent variables whose values are to be determined by iteration. To carry out this iteration, it is necessary to define position variables and to assign zone numbers in an orderly fashion.

The four variables which locate the horizontal position of the ignition zones are designated, respectively, N_3 , N_4 , N_5 , N_6 , beginning at the uppermost level of the fuel bed and ending at the lowest. Each of the variables is numerically equal to the number of zones in a horizontal row

between the ignition interface and the unzoned region far ahead of the flame front. (In Fig. 5.1, $N_3 = 7$, $N_4 = 8$, $N_5 = 9$, $N_6 = 11$.) Four additional position variables designated N_7 , N_8 , N_9 , N_{10} locate the ember burn-out interface in the fuel bed behind the flame front, with each of the variables being equal to the number of zones in a horizontal row between the burn-out interface and the unzoned region ahead of the flame front. (In Fig. 5.1, $N_7 = 12$, $N_8 = 13$, $N_9 = 14$, $N_{10} = 15$.) Unlike the ignition interface position variables, N_7 , N_8 , N_9 and N_{10} are to be treated as independent parameters. (If the ember burning law is known, they can be treated as dependent; more will be said about this in Chapter VI.)

The ignition and burn-out position variables are used to assign a unique zone number to each of the zones ahead of the ignition front as well as to each zone in the ember burning region. The way in which zone numbers are assigned is shown in Fig. 5.1. As indicated in the figure, the four ignition zones are numbered N_3 , $(N_3 + N_4)$, $(N_3 + N_4 + N_5)$, $(N_3 + N_4 + N_5 + N_6)$. These zone numbers, as well as those for other zones in the fuel bed, will appear as subscripts in the energy balance equations.

5.4 Formulation of Energy Balance Equations

Equation 5.1 is an energy balance around zone i ahead of the flame front.

$$V\rho_b C_p B_1 (\bar{T}_i - \bar{T}_{i-1}) = \sum_{\substack{j=1 \\ j \neq i}}^{N_2} [\bar{g}_i \bar{g}_j / L] \sigma T_j^4 + \sum_{j=N_2+1}^{N_1} [\bar{g}_i \bar{g}_j / L] \sigma T_B^4$$

increase in enthalpy of fuel as it moves through zone i radiation from other zones ahead of the flame front ember radiation from zones behind flame front

$$+ (\bar{W} \bar{g}_i / \delta L) (Q_w / L) + h_i (4k_b M_i B_1 B_2) (T_{A_i} - T_i) + h_{c_i} (4k_b M_i B_1 B_2) (T_F - T_{ig})$$

radiation from flame plume simulator convection from ambient in zone i convection from flame

$$- \{ (4k_b M_i B_1 B_2) - [\bar{g}_i \bar{g}_i / L] \} \sigma T_i^4 - (M_i B_1 B_2) Q_i \quad (5.1)$$

radiative emission from zone i mass transfer-chemical reaction heat sink

where B_1 = depth of zone in fuel bed

$\bar{g}_i \bar{g}_j / L$ = radiation interchange length, fuel zone to fuel zone

h_{c_i} = flame convection heat transfer coefficient for zone i

h_i = ambient convection heat transfer coefficient for zone i

k_b = surface area of fuel/unit volume fuel bed,
divided by 4

$M_i B_2$ = length of zone i in fuel bed as measured normal
to infinite dimension.

N_1 = number of zones ahead of flame front + number of
zones in ember burning region.

N_2 = number of zones ahead of flame front.

Q_i = mass transfer-chemical reaction heat sink.

Q_w/L = radiation emitted by a line heat source per
unit length.

T_{A_i} = ambient temperature in zone i

T_B = ember temperature of burning fuel

T_F = flame temperature

T_{ig} = fuel ignition temperature

V = flame front propagation velocity

$\overline{Wg}_i/\delta L$ = radiation interchange factor, flame plume
simulator to zone i

$\rho_b C_p$ = heat capacity of fuel per unit total volume
of fuel bed

σ = Stephan-Boltzmann constant

All temperatures appearing in Eq. 5.1 are absolute in value.

Fuel temperatures are to be taken as surface temperatures
with the exception of temperatures with a bar over them.

The latter are enthalpy mean temperatures.

The four assumptions made in formulating Eq. 5.1 are as follows. (1) The fuel is isotropically arrayed in space. (2) The emissivity of fuel particles is 1.0. (3) The temperature of an individual fuel particle is uniform over its entire surface and can be simply related to its bulk mean temperature. (4) A fuel particle ignites when its surface temperature reaches a value T_{ig} . A more complete description of Eq. 5.1 is given in the following sections.

5.5 Enthalpy Flux Term

The enthalpy flux term on the left-hand side of Eq. 5.1 contains enthalpy average temperatures which in general will not be equal to the surface temperature of the fuel. For the case of fuel particles whose thermal resistance is small compared to convective resistance at the fuel surface, enthalpy temperatures can be replaced by surface temperatures without introducing appreciable error. For intermediate values of thermal resistance, semi-theoretical or empirical correlations relating surface temperature to bulk enthalpy temperatures can be used. (See APPENDIX E.) However, when fuel thermal resistance is large, Eq. 5.1 no longer applies. The assumption that the

temperature of an individual fuel particle is uniform over its entire surface breaks down. If this assumption is discarded, an energy balance must be written around a differential volume in the interior of a fuel particle, giving rise to a partial differential equation with a conduction term normal to the particle surface. One of the boundary conditions will be integro-differential in form and will contain terms similar to the first six terms on the right-hand side of Eq. 5.1

5.6 Radiation Interchange Between Fuel Bed Zones

The terms appearing in brackets in Eq. 5.1 are radiation interchange lengths between zone i and zone j . (The two-dimensionality of the problem gives rise to interchange areas per unit length in the infinite dimension, or equivalently, interchange lengths.) Since the fuel bed is isotropic with respect to radiation transfer and fuel particles have an emissivity of 1.0, its radiation transport properties are equivalent to those of a non-scattering grey gas. Taking the absorption coefficient k_b of the fuel bed to be one fourth of the surface area per unit volume of fuel bed, gas-gas interchange factors such as those computed by Cohen (4) can be used to calculate radiation transfer in the fuel bed. Figure 5.2 presents

interchange lengths, computed by summing Cohen's results for interchange between gas cubes, for zones which are separated by distances less than $4B_1$. The interchange lengths in Fig. 5.2 have been computed for zones which are square in cross section; interchange lengths for zones rectangular in cross section can be readily evaluated from the figure by means of a simple summation process described by Cohen (3).

The effect of an anisotropic fuel bed on radiation interchange is considered briefly in APPENDIX F. Five-fold numerical integration is required to evaluate the anisotropic interchange factors. However, once they have been calculated, they can be incorporated immediately in the model without altering it in any way.

5.7 Convective Transfer

The first of the two convective heat transfer terms in Eq. 5.1 describes transfer between ambient and fuel in zone i and is flexible enough to allow for variation of both heat transfer coefficients and ambient temperature from zone to zone. If the ambient temperature field and the spatial variation of heat transfer coefficients are not known apriori, they can be determined (at least in principle) by use of energy, force, and mass balances on the ambient.

An additional convection term allows for interchange between hot flame gases and unignited fuel. The driving force for this term is $(T_F - T_{ig})$, the difference between flame and ignition temperatures.

5.8 Emission and Heat Sink Terms

The emission term in Eq. 5.1 allows for self-absorption of radiation originating in zones of finite size. As the optical thickness of the zone increases, the self-absorption length $[\overline{g_i g_i}/L]$ becomes increasingly large.

The form of the heat sink term has been left unspecified. It can be either endo- or exothermic and can also be a function of temperature.

5.9 Radiation from the Flame Plume

The plume radiation simulator chosen for use in the zoned model is a line source of radiation surrounded by a cold grey gas. In selecting this simulator, sophistication was traded off for flexibility and simplicity. Its use, however, does not restrict the generality of the zoned model, since it can be readily replaced by any other simulator once the appropriate interchange factors have been evaluated. The term $(\overline{Wg_i}/\delta L)$ in Eq. 5.1 is the dimensionless interchange factor between the plume simulator and

fuel bed zone i. The way in which this factor is computed for a line source of radiation surrounded by an elliptical cylinder of cold grey gas is presented in the following paragraphs.

Figures 5.3 and 5.4 show the flame plume radiation simulator and a volume element $L \, dA$ in the fuel bed. The interchange factor between a length dZ of plume simulator at position (X_2, Y_2, Z_2) and the fuel bed element at (X_1, Y_1, Z_1) is

$$d^3(\bar{W}_g/\delta L) = \frac{\cos \phi_1}{\pi R^2} dA \cdot \exp(-k_g S_g - k_b S) \cdot k_b dZ \quad (5.2)$$

fraction of energy leav- ing element of line source di- rected towards $L \, dA$	fractional de- crease in inten- sity because of attenuation in intervening grey gas and fuel	fraction absorbed in fuel bed element
--	---	--

where $R^2 = (X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2 = x^2 + y^2 + z^2$

ϕ_1 = angle between R and a unit vector normal to the line source

$R_C^2 = x^2 + y^2$

k_g = absorption coefficient of grey gas

k_b = absorption coefficient of fuel bed

S_g = length of absorption path through gas cylinder

S = length of absorption path in fuel bed

Figure 5.3 PERSPECTIVE SKETCH OF FLAME SIMULATOR AND VOLUME ELEMENT IN FUEL BED

Figure 5.4 VIEW OF FLAME SIMULATOR AND VOLUME ELEMENT IN THE X-Y PLANE

Referring to Figs. 5.3 and 5.4, it can be shown from the similarity of triangles that

$$S = S_c R/y$$

and

$$S_g = S_{gc} R/R_c$$

Thus,

$$d^3(\bar{W}g/\delta L) = \frac{k_b R_c}{\pi (R_c^2 + z^2)^{3/2}} \left\{ \exp \left[- (k_b S_c / y + k_g S_{gc} / R_c) \cdot \sqrt{R_c^2 + z^2} \right] \right\} dA dz \quad (5.3)$$

Making the substitutions

$$R_c \sinh \theta = z$$

$$R_c^2 \cosh^2 \theta = z^2 + R_c^2$$

and

$$S' = (k_b S_c / y + k_g S_{gc} / R_c) R_c$$

Eq. 5.3 becomes

$$d^3(\bar{W}g/\delta L) = \frac{k_b dA}{\pi R_c} \cdot \frac{\exp(-S' \cosh \theta)}{\cosh^2 \theta} d\theta$$

or, integrating with respect to θ ,

$$d^2(\bar{W}g/\delta L) = \frac{2k_b dA}{\pi R_c} \int_0^\infty \frac{\exp(-S' \cosh \theta)}{\cosh^2 \theta} d\theta \quad (5.4)$$

Steward (21) has numerically evaluated

$$\int_0^{\infty} \frac{\exp(-S' \cosh \theta)}{\cosh^2 \theta} d\theta$$

and presents the results as mean beam length ratios β_r

$$\text{where } \beta_r = -\frac{1}{S'} \ln \left[\int_0^{\infty} \frac{\exp(-S' \cosh \theta)}{\cosh^2 \theta} d\theta \right]$$

For the two integrations remaining in Eq. 5.4, the integrand on the right-hand side can be treated as a constant since $R_c > 2B_1$. Thus,

$$(\overline{Wg}/\delta L) = \frac{2k_b}{\pi R_c} B_1 B_2 e^{-\beta_r S'}$$

To facilitate machine computations using $(\overline{Wg}/\delta L)$, an equation has been fitted through Steward's mean beam length results.

$$\beta_r = \left(\frac{\pi}{2} - 1 \right) \left[\frac{1}{0.936(S')^{0.685} + 1} \right] + 1$$

where $S' = (k_b S_c / y + k_g S_{gc} / R_c) R_c$

and

$$S_{gc} = \frac{a}{\sqrt{(a/c)^2 + [1 - (a/c)^2] [y^2 \cos^2 \alpha_s + x^2 \sin^2 \alpha_s - 2xy \sin \alpha_s \cos \alpha_s] / R_c^2}}$$

The expression for S_{gc} follows from the geometry of an ellipse with major axis a and minor axis c .

5.10 Solution of the Energy Balance Equations

Equation 5.1 is a representative member of the set of N_2 energy balances written around zones ahead of the flame front. The dependent variables in this set of equations are: propagation velocity V ; the three ignition front position variables N_4 , N_5 , N_6 ; and fuel bed temperatures T_i for all zones in front of the ignition interface except the four zones in which ignition occurs. The system is determinate provided that parameters describing flame-plume and ember radiation can be treated as independent quantities.

Solution of the set of equations is most readily accomplished by iteration. Machine computation is required to carry out the iteration, and a Fortran II program was written for the IBM 7094 both to set up the equations and to solve them. Sections 5.11 and 5.12 describe the iteration procedure, and APPENDIX G is a block diagram and listing of the Fortran II program. For a single set of input parameters, that is, flame simulator parameters, ignition temperature, flame temperature, etc., a 35-zone case (35 zones ahead of the ignition interface) converges to a solution in approximately one minute of computer time.

5.11 Iteration Procedure

The following steps are used in the iteration procedure for solving the set of energy balance equations.

(1) Trial values for the position of the ignition front and temperature field ahead of the flame front are assumed. To facilitate bookkeeping in the iterative process, these trial values are assigned zero pre-subscripts: ${}_0^{N_4}$, ${}_0^{N_5}$, ${}_0^{N_6}$, ${}_0^{T_1}$, ${}_0^{T_2}$, $\dots$, ${}_0^{T_{N_2}}$. The pre-subscripts will be promoted by 1 unit for each subsequent iteration so that, for example, ${}_P^{T_N}$ will be the temperature in zone N after the Pth iteration.

(2) A trial value of propagation velocity, designated ${}_0V$, is calculated using the equation for zone N_3 and the trial values assumed in Step (1).

(3) Newton-Raphson iteration is used to compute the temperature field ahead of the flame front, with the temperatures assumed in Step (1) as input for the first iteration.

(The Newton-Raphson iteration is an iteration loop which is "inside" the loop described by the steps listed in this section, and it is described separately in Section 5.12.) The temperatures computed in this step are designated ${}_1^{T_1}$, ${}_1^{T_2}$, ${}_1^{T_3}$, $\dots$, ${}_1^{T_{N_2}}$.

(4) If $p^{T_{N_3}}$ differs appreciably from T_{ig} , e.g., by more than $30^\circ R$, a new trial value of propagation velocity is computed from the equation for zone N_3 using temperatures $p^{T_1}, p^{T_2}, \dots, p^{T_{N_2}}$. This value is then multiplied by $p^{T_{N_3}}/T_{ig}$ and designated p^V .

(5) If $p^{T_{N_3+N_4}}$ is appreciably smaller than T_{ig} , N_4 is increased in size by one zone, i.e., $p^{N_4} = p^{N_4} + 1$. On the other hand, if $p^{T_{N_3+N_4}}$ is appreciably larger than T_{ig} , $p^{N_4} = p^{N_4} - 1$. If $p^{T_{N_3+N_4}} \cong T_{ig}$, $p^{N_4} = p^{N_4}$.

(6) Values of p^{N_5} and p^{N_6} are computed using the procedure of Step (5) and temperatures $p^{T_{N_3+N_4+N_5}}$, and $p^{T_{N_2}}$, respectively.

(7) The temperature field, $p^{+1}T_1, p^{+1}T_2, p^{+1}T_3, \dots, p^{+1}T_{N_2}$, is computed by Newton-Raphson iteration.

(8) Steps (4) through (7) are repeated until comparison of $p^{+1}V, p^{+1}N_4, \dots, p^{+1}T_1, \dots, p^{+1}T_{N_2}$ with the corresponding values from the P th iteration indicates that the solution has converged.

5.12 Newton-Raphson Iteration

Defining the coefficients of the T^4 terms in Eq. 5.1 to be $C_{i,j}/\sigma$ where σ is the Stephan-Boltzmann

constant, and the coefficients of the T terms to be $D_{i,j}$, Eq. 5.1 can be written in shorthand form as

$$\sum_{j=1}^{N_2} C_{i,j} E_j + \sum_{j=i-1}^i D_{i,j} T_j = R_i \quad (5.5)$$

In the above equation, $E_j = \sigma T_j^4$, and R_i is the algebraic sum of all terms in Eq. 5.1 containing known temperatures and/or fluxes.

The set of N_2 equations, of which Eq. 5.5 is the i th, can be written in matrix notation as

$$[C_{i,j}] \cdot \vec{E} + [D_{i,j}] \cdot \vec{T} = \vec{R} \quad (5.6)$$

The initial trial value of $\vec{E}$, called ${}^{\circ}\vec{E}$, where ${}^{\circ}E_1 = \sigma_p T_1^4$, ${}^{\circ}E_2 = \sigma_p T_2^4$, ..., ${}^{\circ}E_{N_2} = \sigma_p T_{N_2}^4$, is used to calculate a residue vector ${}^{\circ}\vec{H}$. (The pre-^{super}script is used as an aid in bookkeeping and is promoted by 1 unit for each Newton-Raphson iteration. The pre-subscript defined in the preceding section does not change during Newton-Raphson iteration.)

$${}^m\vec{H} = - [C_{i,j}] \cdot {}^m\vec{E} - [D_{i,j}] \cdot {}^m\vec{T} + \vec{R} \quad (5.7)$$

where

$${}^m T_i = ({}^m E_i / \sigma)^{0.25}$$

A correction vector ${}^m\vec{\Delta E}$ is next calculated from the linear matrix equation

$$[{}^m A_{i,j}] \cdot {}^m\vec{\Delta E} = {}^m\vec{H} \quad (5.8)$$

where $m_{A_{i,j}} = C_{i,j} + \frac{1}{4} D_{i,j} \frac{m_{T_j}}{m_{E_j}}$

A new trial value of the emissive power vector is computed using the correction vector.

$$m_{\vec{E}}^{m+1} = m_{\vec{E}} + m_{\Delta \vec{E}} \quad (5-9)$$

The next iteration is carried out by using $m_{\vec{E}}^{m+1}$ as the unput value of $\vec{E}$ in Eq. 5.7. The iteration is continued until $(m_{\vec{E}}^{m+1} - m_{\vec{E}})$ is arbitrarily small.

The pre-subscripts on the temperature field, propagation velocity, and position variables do not change in value during the Newton-Raphson iteration. However, after the iteration has converged, the temperature field pre-subscripts are promoted by 1 unit to distinguish the newly computed temperature field from that which was used as the initial input in Eq. 5.7, i.e., $p^{T_1}, p^{T_2}, p^{T_3}, \dots, p^{T_{N_2}}$. Assuming that convergence occurs after the q th Newton-Raphson iteration, the new temperature field is $p_{+1}^{T_1}, p_{+1}^{T_2}, p_{+1}^{T_3}, \dots, p_{+1}^{T_{N_2}}$, where $p_{+1}^{T_N} = (q_{E_N}/\sigma)^{0.25}$.

5.13 Additional Comments on Solution of Energy Balance Equations

The iteration procedure described in the preceding sections causes the assumed horizontal position of each of

the lower three ignition zones to advance or retreat by increments of a full zone. Convergence of the iteration to an exact solution, however, may require that the assumed position of the ignition zones be altered by a distance less than that of a full zone. Under these conditions, use of full-zone increments will make the iteration scheme oscillate about the exact solution rather than converge to it in the mathematical sense.

Two provisions have been incorporated in the Fortran II computer program to handle the oscillation problem. The first is as follows. Each time the computer passes through the iteration cycle of Section 5.11, it prints out the values it has calculated in Steps (4) through (7). If the computer is unable to converge to the exact solution, it automatically stops after one full oscillation about the exact solution, leaving a record of the attempted solutions obtained during the oscillation cycle. The attempted solutions are next ranked in order of increasing mean square deviation of the calculated temperatures in the assumed ignition zones, relative to T_{ig} . The best of the attempted solutions as determined by minimum mean square deviation can then be used to obtain an approximation to the exact solution by interpolation.

The second provision for the oscillation problem refines the zoning grid by dividing the horizontal dimension of zones near the ignition front by a factor of 2.0. The half-zone grid, shown in Fig. 5.5, can be used in lieu of the interpolation procedure referred to above. The refining of the zoning grid occurs only after the solution with a full-zone grid has failed to converge, and requires that both full-zone and half-zone radiation interchange factors be read into the computer before iteration begins. If the iteration also fails to converge with the half-zone grid, the best of the attempted solutions obtained with the half-zone grid can be employed to find an approximation to the exact solution by means of interpolation.

VI. MODELING OF ENERGY TRANSFER AHEAD OF THE FLAME FRONT
IN BRUSH FIRES -- RESULTS AND DISCUSSION

6.1 Introduction

The zoned model for energy transfer in the unburned fuel in a brush fire was used to determine the effect of several of the model's independent parameters on (1) propagation velocity, (2) configuration of the ignition interface in the fuel bed, and (3) fuel-bed temperatures ahead of the ignition front. The independent parameters chosen for this purpose were (1) fuel-bed radiation absorption coefficient, (2) fuel bed loading, (3) plume radiation intensity, (4) plume simulator angle, and (5) plume displacement relative to the ignition front. The effects predicted by the model when each of the preceding parameters is varied are reported in Sections 6.2 through 6.6. The predicted effects are presented as (1) graphs of propagation velocity V as a function of the independent parameter in question and (2) plots of isotherms in a cross-section of the fuel bed taken normal to its infinite dimension. When possible, experimental data are compared to results predicted by the model.

Section 6.7 compares results obtained from the zoned model with those predicted by a simplified model developed

by Emmons (8). The concluding section discusses the significance of the model and the way it fits into the overall research program in free-burning fires.

6.2 Effect of Radiation Absorption Coefficient k_b

The radiation absorption coefficient k_b is directly proportional to the total surface area of fuel per volume of fuel bed. The effects produced by its variation, holding all other independent parameters constant at the values listed in Table 6.1, are shown in Figures 6.1 and 6.2.

Five values of k_b were used, corresponding to the five open circles in Fig. 6.1. (Each value was multiplied by the fuel-bed height L_b to make it dimensionless.) The figure shows that propagation velocity, when expressed in terms of

$$\frac{V\rho_F}{S_F B_2} ,$$

where B_2 = length of zone in fuel bed

S_F = surface-area-to-volume ratio of a fuel particle

V = flame front propagation rate

ρ_F = density of a fuel particle,

decreases almost linearly with increasing k_b . The reason for this trend is that an increase in k_b at constant S_F gives rise to a larger fuel preheating load. Although

TABLE 6.1

VALUES ASSIGNED TO INDEPENDENT PARAMETERS

TO DETERMINE

EFFECT OF RADIATION ABSORPTION COEFFICIENT k_b

FUEL BED PARAMETERS

$$R/k = 0 \quad B_2/B_1 = 1.0 \quad T_{ig} = 1000^\circ R \quad T_o/T_{ig} = 0.570$$

$$Q_i = 0 \text{ for all } i$$

EMBER COMBUSTION PARAMETERS

$$N_7 = 12 \quad N_8 = 13 \quad N_9 = 14 \quad N_{10} = 15$$

$$T_B/T_{ig} = 2.4$$

FLAME PLUME PARAMETERS

$$h_{c1} = 0 \quad h_{c2} = 0 \quad h_{c3} = 0 \quad h_{c4} = 0$$

$$T_F/T_{ig} = 2.4 \quad L_s/L_b = 1.0 \quad \epsilon/L_s = 0 \quad \alpha_s = \pi/4$$

$$a/c = 3.0 \quad k_g a = 0.5 \quad (Q_w/L)/L_s = 5.21 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

AMBIENT PARAMETERS

$$T_{A_i}/T_{ig} = 0.570 \text{ for all } i \quad h_i = 1 \text{ Btu/ft}^2\text{-hr-}^\circ R \text{ for all } i$$

Figure 6.1 EFFECT OF RADIATION ABSORPTION COEFFICIENT k_b ON PROPAGATION VELOCITY

Figure 6.2 FUEL BED ISOTHERMS FOR THREE VALUES OF k_b

the amount of ember radiation also goes up, its increase is not big enough to satisfy the larger preheating requirements. As a consequence, the propagation velocity drops until the available net flux integrated over the time interval before ignition meets the preheating requirements.

Figure 6.2 presents fuel bed isotherms for three values of k_b . The 1000°R isotherm is the ignition interface; the region to its left is at the uniform ember burning temperature of 2400°R while the region far to the right of the 600°R isotherm is at the uniform temperature of 570°R . (The dashed line indicates the position of the ember burn-out interface.) As k_b increases, the isotherms move closer together because radiation is attenuated more rapidly in the fuel.

The experimental work of Curry and Fons (5), as reported by Anderson and Rothermel (2), agrees with the zoned model's predicted trend of propagation velocity with increased k_b . Their experimental data suggest that V is directly proportional to $\left(\frac{S_F}{4k_b} - 1\right)^{0.5}$

Since $S_F/(4k_b)$ is usually much larger than 1, this amounts to a direct proportionality between V and $\left(\frac{S_F}{4k_b}\right)^{0.5}$.

Figure 6.1, within the limits of accuracy of the model and the experimental data, agrees quite favorably with this proportionality.

6.3 Effect of Loading

Dimensionless fuel loading in the zoned model is given by $\frac{\rho_b L_b}{4\rho_F/S_F}$. Its effect on propagation velocity and fuel bed isotherms is shown in Figs. 6.3 and 6.4, respectively. All other parameters were held constant at the values listed in Table 6.2.

The open hexagons in Fig. 6.3 represent predicted velocities for five values of $\frac{\rho_b L_b}{4\rho_F/S_F}$. As indicated in the figure, propagation velocity drops with increased loading, and the reason for this trend, once again, is an increase in fuel preheating requirements. In this instance, however, flux from the ember region to the unburned fuel does not rise as rapidly with increased $\frac{\rho_b L_b}{4\rho_F/S_F}$ as it does with k_b . As a consequence, propagation velocity drops at a faster rate with increased fuel loading than with increased k_b .

Figure 6.4 shows fuel bed isotherms corresponding to the first, third, and fifth points in Fig. 6.3. As shown in the figure, the 1000°R isotherm (representing the configuration of the ignition interface) is canted forward

TABLE 6.2
VALUES ASSIGNED TO INDEPENDENT PARAMETERS
TO DETERMINE
EFFECT OF LOADING

FUEL BED PARAMETERS

$$k_b B_2 = 0.3 \quad R/k = 0 \quad T_{ig} = 1000^\circ R \quad T_o/T_{ig} = 0.570$$

$$Q_i = 0 \text{ for all } i$$

EMBER COMBUSTION PARAMETERS

$$N_7 = 15 \quad N_8 = 15 \quad N_9 = 15 \quad N_{10} = 15$$

$$T_B/T_{ig} = 2.4$$

FLAME PLUME PARAMETERS

$$h_{c1} = 0 \quad h_{c2} = 0 \quad h_{c3} = 0 \quad h_{c4} = 0$$

$$T_F/T_{ig} = 2.4 \quad L_s/B_2 = 4.0 \quad \epsilon/L_s = 0 \quad \alpha_s = \pi/4$$

$$a/c = 3.0 \quad k_g a = 0.5 \quad (Q_w/L)/L_s = 5.21 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

AMBIENT PARAMETERS

$$T_{A_i}/T_{ig} = 0.570 \text{ for all } i \quad h_i = 1 \text{ Btu/ft}^2\text{-hr-}^\circ R \text{ for all } i$$

Figure 6.3 EFFECT OF FUEL LOADING ON PROPAGATION VELOCITY

more strongly at low loadings than at high. This in turn reflects the greater importance of plume radiation (which was held constant for all cases) at low loadings.

6.4 Effect of Radiation Intensity from Flame Plume

Radiation intensity from the flame plume is directly proportional to $(Q_w/L)/B_1$. The effect of varying this quantity, while holding all other independent parameters constant at the values listed in Table 6.3, on the zoned model's dependent variables is shown in Figs. 6.5 and 6.6.

The propagation velocity curves in Fig. 6.5 are concave upwards near the origin as expected, since in this region losses do not keep in step with increased overhead radiation levels. Further out, the curves approach linearity with an extrapolated intercept near the origin. In this region, the terms of primary importance in the energy balance equations are the enthalpy flux terms and the flame plume simulator terms.

Figure 6.6 presents fuel bed isotherms for three values of $(Q_w/L)/B_1$, all at $k_b L_b = 2.0$. The isotherms in the figure are nearly vertical for low levels of $(Q_w/L)/B_1$. However, as $(Q_w/L)/B_1$ increases, the isotherms tilt forward and their upper ends bend over towards the horizontal.

A qualitative experimental confirmation of this effect

TABLE 6.3
VALUES ASSIGNED TO INDEPENDENT PARAMETERS
TO DETERMINE
EFFECT OF RADIATION INTENSITY FROM FLAME PLUME

FUEL BED PARAMETERS

$$\begin{aligned} R/k &= 0 & B_2/B_1 &= 1.0 & T_{ig} &= 1000^\circ\text{R} & T_o/T_{ig} &= 0.570 \\ Q_i &= 0 \text{ for all } i \end{aligned}$$

EMBER COMBUSTION PARAMETERS

$$\begin{aligned} N_7 &= 15 & N_8 &= 15 & N_9 &= 15 & N_{10} &= 15 \\ T_B/T_{ig} &= 2.4 \end{aligned}$$

FLAME PLUME PARAMETERS

$$\begin{aligned} h_{c_1} &= 0 & h_{c_2} &= 0 & h_{c_3} &= 0 & h_{c_4} &= 0 \\ T_F/T_{ig} &= 2.4 & L_s/L_b &= 1.0 & \epsilon/L_s &= 0 & \alpha_s &= \pi/4 \\ a/c &= 3.0 & k_g a &= 0.5 \end{aligned}$$

AMBIENT PARAMETERS

$$\begin{aligned} T_{A_i}/T_{ig} &= 0.570 \text{ for all } i & h_i &= 1 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F for all } i \end{aligned}$$

Figure 6.5 EFFECT OF PLUME RADIATION INTENSITY
ON PROPAGATION VELOCITY

was observed in wood excelsior fires. In the first of two experiments, the wood excelsior was treated prior to burning with a flame depressant. When ignited the excelsior was consumed in a glowing combustion with no visible flame present. (The absence of a flame would correspond to a zero value of $(Q_w/L)/B_1$.) The position of the ignition interface, and hence of the T_{ig} isotherm, as observed from a position slightly overhead and off to the side of the fuel bed, was nearly vertical. In the second experiment, the wood excelsior was left untreated; and when ignited burned with a large overhead flame plume. The combustion interface in this experiment was canted forward very strongly, particularly near the upper surface of the fuel bed.

This result is not at all surprising. When flame radiation is increased, most of the added radiation is absorbed by the upper region of the fuel bed. The upper region then tends to burn more quickly than before, and in effect has to drag the lower regions along with it. At steady state, the lag between upper and lower regions of the fuel bed is such that the increased flame radiation levels available to the upper regions is matched by an increased level of ember radiation received by the lower levels. This increased ember radiation is a consequence

of an "improved view" of the ember combustion region afforded by the canted ignition interface.

6.5 Effect of Horizontal Displacement of Flame Plume

The horizontal displacement of the flame plume relative to the ignition interface in the zoned model is given by ϵ , where ϵ is measured from (1) the point on the ignition interface which is $L_g/8$ units below the bed's upper surface to (2) the plume's radiation center. The effect of varying ϵ , holding all other independent parameters constant at the values of Table 6.4, is shown in Figs. 6.7 and 6.8.

Figure 6.7 indicates that displacement of the plume radiation center in the direction of propagation causes the propagation velocity to pass through a maximum at a dimensionless displacement of approximately +1.3. Further displacement causes propagation velocity to decrease because plume radiation received by unburned fuel is back-radiated and lost by convective cooling before the fuel reaches the ignition interface. This effect is shown at the bottom of Fig. 6.8 where the 700, 800 and 900°R isotherms become nearly horizontal between the ignition front and the highly displaced plume radiation center. A displacement of the plume radiation center of this type

TABLE 6.4

VALUES ASSIGNED TO INDEPENDENT PARAMETERS

TO DETERMINE

EFFECT OF HORIZONTAL DISPLACEMENT OF FLAME PLUME

FUEL BED PARAMETERS

$$k_b L_b = 2.0 \quad B_2/B_1 = 1.0 \quad T_{ig} = 1000^\circ R \quad T_o/T_{ig} = 0.570$$

$$R/k = 0 \quad Q_i = 0 \quad \text{for all } i$$

EMBER COMBUSTION PARAMETERS

$$N_7 = 15 \quad N_8 = 15 \quad N_9 = 15 \quad N_{10} = 15$$

$$T_B/T_{ig} = 2.4$$

FLAME PLUME PARAMETERS

$$h_{c_1} = 0 \quad h_{c_2} = 0 \quad h_{c_3} = 0 \quad h_{c_4} = 0$$

$$T_F/T_{ig} = 2.4 \quad L_s/L_b = 1.0 \quad k_g a = 0.5 \quad \alpha_s = \pi/4$$

$$a/c = 3.0 \quad (Q_w/L)/L_s = 5.21 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

AMBIENT PARAMETERS

$$T_{A_i}/T_{ig} = 0.570 \quad \text{for all } i \quad h_i = 1 \text{ Btu/ft}^2\text{-hr-}^\circ R \quad \text{for all } i$$

Figure 6.7 EFFECT OF HORIZONTAL DISPLACEMENT OF PLUME RADIATION ON PROPAGATION VELOCITY

corresponds roughly to the effect of wind on flame plume geometry in a fire, with negative displacements corresponding to fires burning into a headwind, and positive displacements, to a fire burning with a tailwind.

The flatness of the maximum in Fig. 6.7 suggests that experiments using artificial radiation be designed so that the radiation center is far enough ahead of the flame front to be near this maximum. If this is done, the effect of any uncertainty in the position of the artificial radiation source relative to the ignition interface will be minimized.

6.6 Effect of Plume Simulator Angle α_s

The effect of varying plume simulator angle α_s on propagation velocity and temperature field ahead of the flame front is shown in Figs. 6.9 and 6.10. Figure 6.9 indicates that the maximum propagation rate occurs at an α_s of $\pi/4$ and the minimum, at approximately $7\pi/8$, corresponding to a shift in the maximum level of flux density received by the fuel bed from (1) a point somewhat ahead of the flame front to (2) a point behind the flame front. The effect of this change in α_s is thus analagous to the effect of displacing the plume radiation center as shown in Fig. 6.7.

TABLE 6.5

VALUES ASSIGNED TO INDEPENDENT PARAMETERS

TO DETERMINE

EFFECT OF PLUME SIMULATOR ANGLE α_s

FUEL BED PARAMETERS

$$k_b L_b = 2.0 \quad B_2/B_1 = 1.0 \quad T_{ig} = 1000^\circ R \quad T_o/T_{ig} = 0.570$$

$$R/k = 0 \quad Q_i = 0 \quad \text{for all } i$$

EMBER COMBUSTION PARAMETERS

$$N_7 = 15 \quad N_8 = 15 \quad N_9 = 15 \quad N_{10} = 15$$

$$T_B/T_{ig} = 2.4$$

FLAME PLUME PARAMETERS

$$h_{c1} = 0 \quad h_{c2} = 0 \quad h_{c3} = 0 \quad h_{c4} = 0$$

$$T_F/T_{ig} = 2.4 \quad L_s/L_b = 1.0 \quad \epsilon/L_s = 0 \quad k_g a = 0.5$$

$$a/c = 3.0 \quad (Q_w/L)/L_s = 5.21 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

AMBIENT PARAMETERS

$$T_{A_i}/T_{ig} = 0.570 \quad \text{for all } i \quad h_i = 1 \text{ Btu/ft}^2\text{-hr-}^\circ R \quad \text{for all } i$$

Figure 6.9 EFFECT OF PLUME SIMULATOR ANGLE α_s ON PROPAGATION VELOCITY

Figure 6.10 FUEL BED ISOTHERMS FOR THREE VALUES OF α_s

While changes in α_s hence affect propagation rate in a manner similar to changes in ϵ/L_s , variation of α_s produces much smaller changes in the 700, 800, and 900°R isotherms than does a change in ϵ/L_s as can be seen by comparing Figs. 6.8 and 6.10.

6.7 Comparison of Results with Emmon's Model

Emmons (8) has developed a simplified model for energy transfer to the unburned fuel in a steady state brush or forest fire. The model is based upon radiative transfer from hot embers behind the flame front, treating this radiation as though it were collimated in the direction of fire propagation. The model equates (1) total radiation from the ember region directed toward the unburned fuel to (2) the energy required to raise the fuel to ignition. The propagation equation arising from the model is given by Eq. 6.1.

$$\frac{V_b C_p (T_{ig} - T_o)}{\sigma T_B^4} = 1 - e^{-k_b L_c} \quad (6.1)$$

C_p = heat capacity of fuel, Btu/lb-°R

k_b = radiation absorption coefficient of fuel bed, ft⁻¹

L_c = depth of ember combustion region, as measured parallel to direction of propagation, ft.

T_B = ember combustion temperature, °R

T_{ig} = ignition temperature, °R

T_O = temperature of fuel infinitely far ahead of
ignition front

V = propagation velocity, ft/hr

ρ_b = density of fuel bed, lb/ft³ of total volume

σ = Stephan-Boltzmann constant

In using Eq. 6.1, Emmons suggests that k_b be treated as an empirical constant whose value is chosen so that the equation allows in an approximate way for all phenomena present. That is to say, k_b should be chosen so that it includes the effect of convection, plume radiation, fire brands, and presumably the three dimensionality of radiative transfer as well as back-radiation from unignited fuel.

The zoned model makes explicit allowance for each of these phenomena-- with the exception of fire brands. As such it provides a means for checking the reasonableness of Emmon's suggestion that Eq. 6.1 with a suitably chosen value of k_b be used to describe propagation of free-burning fires.

A relatively simple way of comparing Eq. 6.1 with results obtained from the zoned model is to vary $k_b L_c$ in both the zoned model and in Eq. 6.1 and then to compare

the effect of this variation on V . This was done, and the results are shown in Fig. 6.11. (Values assigned to independent parameters are listed in Table 6.6.) The dashed line in the figure corresponds to Eq. 6.1, while the circles, triangles and squares correspond to results predicted by the zoned model.

As shown in Fig. 6.11, the zoned model predicts results which depart appreciably from those predicted by Eq. 6.1. The reason for this departure is twofold.

(1) The zoned model makes allowance for radiative transfer in 4π steradians, while Eq. 6.1 treats radiation as though it were collimated in the direction of propagation.

(2) The zoned model allows for back-radiation from unignited fuel; Eq. 6.1 does not.

Following Emmon's suggestion, an attempt was next made to find values of k_b to make the curves in Fig. 6.11 fall as nearly as possible on top of the dashed line. This was done by plotting

$$\log \left[1 - \frac{V\rho C_p(T_{ig} - T_o)}{\sigma T_B^4} \right] \text{ as a function of } -k_b L_c$$

as shown in Fig. 6.12. The strong non-linearity of the curves through the circles, squares and triangles in this figure

TABLE 6.6

VALUES ASSIGNED TO INDEPENDENT PARAMETERS

TO DETERMINE

BURNING CHARACTERISTICS

FUEL BED PARAMETERS

$$B_2/B_1 = 1.0 \quad k_b L_b = \begin{cases} 0.8 \\ 1.6 \\ 2.0 \end{cases} \quad T_{ig} = 1000^\circ R \quad T_o/T_{ig} = 0.570$$

$$R/k = 0 \quad Q_i = 0 \text{ for all } i$$

EMBER COMBUSTION PARAMETERS

$$T_B/T_{ig} = 2.4 \quad N_7 = N_8 = N_9 = N_{10} = (L_c/B_2 + 7)$$

FLAME PLUME PARAMETERS

$$\begin{aligned} h_{c_1} &= 0 & h_{c_2} &= 0 & h_{c_3} &= 0 & h_{c_4} &= 0 \\ T_F/T_{ig} &= 0 & L_s/B_2 &= 4.0 & \epsilon/L_s &= 0 & \alpha_s &= \pi/4 \\ a/c &= 3 & k_g a &= 0.5 & (Q_w/L)/L_s &= 0 & & \end{aligned}$$

AMBIENT PARAMETERS

$$T_{A_i}/T_{ig} = 0 \text{ for all } i \quad h_i = 0 \text{ for all } i$$

$$\text{BURNING CHARACTERISTIC } P = \frac{V \rho_b C_p (T_{ig} - T_o)}{\sigma T_B^4 k_b L_c}$$

$$\text{FIRE FRONT VELOCITY} = k_b L_c$$

Figure 6.11 COMPARISON OF BURNING CHARACTERISTIC PREDICTED BY EMMON'S MODEL WITH BURNING CHARACTERISTICS PREDICTED BY ZONED MODEL

Figure 6.12 RESULTS OF FIG. 6.11 RE-PLOTTED ACCORDING TO EQ. 6.1

suggests that the corresponding curves in Fig. 6.11 cannot be made to fall on top of the dashed curve over any appreciable range of $k_b L_c$, unless k_b is allowed to vary with $k_b L_c$ in some artificial manner. If k_b has to be a function of $k_b L_c$ in order to make Eq. 6.1 describe propagation in the simple case where plume radiation and all convection are absent, the adequacy of Eq. 6.1 for describing more complex fires is very doubtful.

In fairness to Emmons, it must be mentioned that he was aware that his model might have this deficiency. His method of approach is sound; unfortunately, the assumptions which he made in order to derive a propagation equation in closed form seem to over-simplify the problem.

6.8 Significance of the Zoned Model

The zoned model as here presented is not intended to be a definitive account of energy transfer ahead of the flame front in brush-type fires. For all its complexity, the model still contains a number of restrictive simplifying assumptions. The foremost of these are: (1) The fuel is isotropically arrayed in space, and (2) The surface temperature of each fuel particle is uniform over its entire surface and can be simply related to the bulk mean temperature of the fuel particle in question. As

mentioned in Section 5.6, the first of these assumptions can be replaced by a less restrictive one without altering the model or the Fortran computer program. The second, however, presents greater difficulties. Replacing it with a less restrictive assumption probably entails a major revision of the model.

On the positive side of the ledger, the model probably does a better job of describing energy transfer ahead of the flame front than anything proposed thus far. This assertion, however, is based only on the relative rigor of the model and remains to be affirmed or denied by quantitative experimental data. Another aspect of the model which seems to have some merit is its demonstration of the applicability of zoning techniques to free-burning fire problems. The importance of radiative transfer in free-burning fires makes it necessary to give an explicit account of plume and ember radiation and hence to make allowance for action-at-a-distance phenomena. The zoning technique, which approximates the rigorous integro-differential action-at-a-distance formulation, does just this.

The question of how the zoned model--or an improved version of it-- can be incorporated into an overall

predictive model for free-burning fires can be answered as follows. The zoned model provides a means of predicting propagation velocity V , the configuration of the fuel ignition interface, and the temperature field ahead of the flame front provided that (1) flame plume geometry and temperatures and (2) the depth of the ember burning region (measured parallel to the propagation direction) are known. These parameters are in general dependent variables, and information about them can be obtained from "burning laws". If flame-plume and ember burning laws are known, they can be coupled to the zoned model (or its equivalent) to form a predictive model for free-burning fires, that is, a model which will enable one to predict values of propagation velocity, flame height, and flame depth from knowledge of the values of independent parameters describing fuel loading, fuel geometry, and meteorological conditions.

VII. CONCLUSIONS

1. A zoning technique analagous to that employed in gas radiation problems can be used to describe the quantitative behavior of fuel preheating in brush fires.
2. The zoned model for brush fires provides a means of predicting values of (1) fire propagation velocity V , (2) the configuration of the ignition interface in the fuel bed, and (3) fuel bed temperatures ahead of the ignition interface as a function of parameters describing (1) the fuel bed's geometry, thermal properties, and ignition temperature, (2) ambient temperature and flow patterns, (3) the flame plume and ember combustion.
3. The criterion for piloted ignition which is based upon the assumption that fuel ignites when its surface reaches a critical temperature T_{ig} , where T_{ig} is independent of preheating rate, is adequate only over narrow ranges of preheating conditions.
4. Fuel ignition energies Q_i'' in shredded newsprint fires are four to six times higher at high preheating rates than at low.
5. The disagreement between theory and experiment reported by Hottel, Williams, and Steward (14) in their Combustion Symposium paper can be resolved by assigning higher values to both T_{ig} and the flame convection parameter q_c' at high

values of V than at low.

6. Convection from the flame plume to unburned fuel in ground fires -- i.e., q'_c -- is a relatively strong increasing function of propagation velocity V , although it may be treated as constant over suitably restricted ranges of V . In shredded newsprint fires q'_c is of the order of 10 Btu/hr-ft at low V and of the order of 100 Btu/hr-ft at high V .

7. Radiation flux densities from small flame plumes can be measured to within 0.25 in. of the flame plume using a water-cooled radiometer provided with multiple sensing heads and an air shield.

VIII. RECOMMENDATIONS

8.1 Development of Burning Laws

The modeling work reported in the preceding chapters centered around propagation laws based upon energy transfer ahead of the flame front. In this work, it proved useful to treat all flame plume and most ember burning parameters as independent variables. However, in free burning fires -- other than those burned under laboratory conditions -- all of these parameters are dependent. Part of the additional information which allows one to treat them as dependent is provided by a burning law (or laws).

Two important parameters which fall into this category are (1) the width of the burning region within the fuel bed (as measured parallel to the direction of propagation) and (2) the width of the flame plume at some characteristic height (also measured parallel to the propagation direction). Both of these parameters strongly influence fire-spread, and information about their size and the way in which they are coupled to firespread will be given by burning laws.

Development of a burning law of this sort may well be more empirical than work done on propagation laws. The energy transfer problems encountered in the study of fuel preheating are also present here, and the situation is further complicated by mass transfer and chemical reaction.

This is not an easy problem. However, it is time that it be approached quantitatively.

8.2 Continued Experimental Modeling of Brush Fires

Model experiments simulating windblown brush fires by burning wood excelsior in the combustion wind tunnel have thus far produced only qualitative results. Additional experiments to yield quantitative data are needed. Use of another model fuel with a lower ratio of surface area to mass than that of wood excelsior should make this possible.

The parameters which are recommended for study are:

(1) wind velocity, using both tail- and headwinds, (2) fuel-bed void fraction, (3) surface area of fuel per unit volume of fuel bed, (4) fuel loading, and (5) fuel moisture content. The effect of each of these variables on propagation velocity and ignition interface configuration can be measured experimentally and also predicted mathematically by use of the zoned model for fuel preheating. Comparison of predicted and measured results will further test the adequacy of the mathematical model and provide a means of evaluating parameters which appear in it.

8.3 Investigation of Wood Pyrolysis and Piloted Ignition

Results from the modeling of ground fires indicate that a constant surface temperature criterion for piloted ignition is adequate only over limited ranges of preheating

conditions. An investigation to find a criterion less sensitive to preheating conditions is needed. A criterion of this type is likely to depend upon the concentration of volatile combustible materials generated during fuel preheating. This in turn suggests that a study be made of wood pyrolysis under conditions similar to those encountered in free-burning fires.

APPENDIX

A. Horizontal Conduction in the Slab Model

If allowance is made for horizontal conduction in the slab model, the model's energy balance as given by Eq. 2.1 assumes the form of Eq. A-1.

$$\frac{\partial^2 (T-T_O)}{\partial x^2} + \frac{\partial^2 (T-T_O)}{\partial y^2} - \frac{\gamma}{k} (T-T_O) = \frac{\rho C_P}{k} \frac{\partial (T-T_O)}{\partial \theta} \quad (A-1)$$

The above equation cannot be integrated analytically for the boundary conditions of Eq. 2.1. It can, however, be integrated for the following set of conditions.

$$(T-T_O) = 0, \quad \theta = 0 \quad (A-2)$$

$$(T-T_O) = T_A - T_O, \quad \theta \rightarrow \infty, \quad y = 0 \quad (A-3)$$

$$k \left. \frac{\partial (T-T_O)}{\partial x} \right|_{x=L} = -h(T-T_A), \quad \theta > 0 \quad (A-4)$$

$$k \left. \frac{\partial (T-T_O)}{\partial x} \right|_{x=0} = 0, \quad \theta \rightarrow 0 \quad (A-5)$$

The initial and boundary conditions listed in Eqs. A-2 through A-5 are equivalent to those of Case I in Section 2.4 when $q_s'' = q_c'' = 0$. Integration of Eq. A-1 for the above conditions followed by comparison with the integrated form of Eq. 2.1 when $q_c'' = q_s'' = 0$ thus affords a means of measuring

the effect of conduction in the y-direction.

From Section 2.4, $\frac{dy}{d\theta} = V$ and $y = y_m + V\theta$. Let $z = y - y_m$. Then $\frac{dz}{d\theta} = V$ and $z = V\theta$. Equations A-1 through A-5 can now be rewritten as follows. (The heat sink coefficient γ is set equal to zero for the sake of simplicity.)

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} - \frac{V\rho C_P}{k} \frac{\partial v}{\partial z} = 0 \quad (\text{A-6})$$

$$v=0, \quad z=0 \quad (\text{A-7})$$

$$v=v_A, \quad z \rightarrow -\infty \quad (\text{A-8})$$

$$k \left. \frac{\partial v}{\partial x} \right|_{x=L} = -h(v-v_A) \quad (\text{A-9})$$

$$k \left. \frac{\partial v}{\partial x} \right|_{x=0} = 0 \quad (\text{A-10})$$

where $v = T - T_o$

$$v_A = T_A - T_o$$

Equation A-6 can be integrated using separation of variables.

Assume a solution of the form $v - v_A = G(x)F(z)$.

$$\text{Then } \frac{\partial v}{\partial x} = G_x(x)F(z)$$

$$\frac{\partial^2 v}{\partial x^2} = G_{xx}(x)F(z)$$

$$\frac{\partial v}{\partial z} = G(x)F_z(z)$$

$$\frac{\partial^2 v}{\partial z^2} = G(x) F_{zz}(z)$$

From Eq. A-6,

$$G_{xx}(x)F(z) + G(x)F_{zz}(z) - \frac{V\rho C_P}{k} G(x)F_z(z) = 0 \quad (A-11)$$

or, equivalently,

$$-\frac{G_{xx}(x)}{G(x)} = \frac{F_{zz}(z)}{F(z)} - \frac{V\rho C_P}{k} \frac{F_z(z)}{F(z)} = \pm \beta^2 \quad (A-12)$$

Equation A-12 can be separated into two ordinary differential equations.

$$G_{xx}(x) + \beta^2 G(x) = 0 \quad (A-13)$$

$$F_{zz}(z) - \frac{V\rho C_P}{k} F_z(z) + \beta^2 F(z) = 0 \quad (A-14)$$

The solutions of Eqs. A-12 and A-13 are, respectively,

$$G(x) = C_1 \sin \beta x + C_2 \cos \beta x \quad (A-15)$$

and

$$F(z) = C_3 \exp\left[\left(\frac{1}{2}V_1 + \frac{1}{2}\sqrt{V_1^2 + 4\beta^2}\right)z\right] \\ + C_4 \exp\left[\left(\frac{1}{2}V_1 - \frac{1}{2}\sqrt{V_1^2 + 4\beta^2}\right)z\right] \quad (A-16)$$

where C_1 , C_2 , C_3 , and C_4 are constants of integration

$$\text{and } V_1 = \frac{V\rho C_P}{k}$$

Therefore,

$$v - v_A = \{C_1 \sin \beta x + C_2 \cos \beta x\} \{C_3 \exp\left[\left(\frac{1}{2}V_1 + \frac{1}{2}\sqrt{V_1^2 + 4\beta^2}\right)z\right] \\ + C_4 \exp\left[\left(\frac{1}{2}V_1 - \frac{1}{2}\sqrt{V_1^2 + 4\beta^2}\right)z\right]\} \quad (A-17)$$

From the conditions of Eqs. A-8 and A-10, $C_3 = C_1 = 0$.

Let $C_5 = C_2 \cdot C_4$. Then

$$v-v_A = \{C_5 \cos \beta x\} \exp\left[\left(\frac{1}{2}V_1 - \frac{1}{2} \sqrt{V_1^2 + 4\beta^2}\right)z\right] \quad (A-18)$$

Substituting Eq. A-18 into Eq. A-9,

$$-\beta \sin \beta L = -\frac{h}{k} \cos \beta L$$

Hence, β_n , $n=1, 2, 3, \dots$ is a root of the transcendental equation

$$\beta_n \tan \beta_n L = \frac{h}{k}, \quad n=1, 2, 3, \dots$$

The temperature distribution becomes

$$T-T_A = \sum_{n=1}^{\infty} \{C_n \cos \beta_n x\} \exp\left[\left(\frac{1}{2}V_1 - \frac{1}{2} \sqrt{V_1^2 + 4\beta_n^2}\right)z\right] \quad (A-19)$$

where a value of C_n corresponds to each value of β_n .

Values of C_n can be determined as follows. Assume T_o can be expanded in an infinite series:

$$T_o - T_A = C_1 \cos \beta_1 x + C_2 \cos \beta_2 x + C_3 \cos \beta_3 x + \dots$$

If both sides of the above expression are multiplied by $\cos \beta_n x$ and integrated from 0 to L, the following is obtained:

$$(T_o - T_A) \int_0^L \cos \beta_n x \, dx = \int_0^L C_n \cos^2 \beta_n x \, dx$$

which integrates to

$$C_n = \frac{2 (T_o - T_A) \frac{hL}{k}}{\left[\frac{hL}{k} + \left(\frac{hL}{k} \right)^2 + \beta_n^2 L^2 \right] \cos \beta_n L} \quad (A-20)$$

The solution of Eq. A-6 is thus

$$\frac{T - T_A}{T_o - T_A} = 2 \frac{hL}{k} \sum_{n=1}^{\infty} \frac{\cos \beta_n x}{\cos \beta_n L} \exp \left\{ \frac{1}{2} \left[\frac{V_p C_P L}{k} - \sqrt{\left(\frac{V_p C_P L}{k} \right)^2 + 4 \beta_n^2 L^2} \right] \frac{z}{L} \right\} \frac{\frac{hL}{k} + \left(\frac{hL}{k} \right)^2 + \beta_n^2 L^2}{\cos \beta_n L} \quad (A-21)$$

A comparison of dimensionless temperatures predicted by Eq. A-21 and Eq. 2.9 when $q_s''(y) = q_c''(y) = 0$ is presented in Table A-1.

Table A-1 indicates that conduction in the horizontal direction has little effect on slab surface temperatures as long as

$$\frac{V_p C_P L}{k} \cdot \frac{k}{hL} > 5$$

The above inequality is satisfied in shredded newsprint fires. It is also satisfied in other types of ground fires, provided that their propagation velocities are higher than 10 ft/hr.

One additional comment needs to be made. The linearity of Eq. A-6 and its boundary conditions suggests that Eq. A-21 be used to build up a general solution for the boundary conditions of Eq. 2.1 by means of superposition. This can not

TABLE A-1
EFFECT OF HORIZONTAL CONDUCTION ON SLAB SURFACE TEMP.

$\frac{hL}{k}$	$\frac{V\rho C_P L}{k}$	$\frac{z}{L}$	$\frac{x}{L}$	$\left[\frac{T-T_A}{T_O-T_A} \right]_{\text{Eq. A-21}}$	$\left[\frac{T-T_A}{T_O-T_A} \right]_{\text{Eq. 2.9}}$
0.02	0.01	0.0000001	1.0	0.99966	0.99944
0.02	0.01	0.0001	1.0	0.99944	0.99257
0.02	0.01	1.0	1.0	0.86685	0.13600
0.02	0.01	100.0	1.0	0.00000	0.00000
0.02	0.01	100,000	1.0	0.00000	0.00000
0.02	0.1	0.0000001	1.0	0.99966	0.99959
0.02	0.1	0.0001	1.0	0.99948	0.99741
0.02	0.1	1.0	1.0	0.89907	0.81399
0.02	0.1	100.0	1.0	0.00005	0.00000
0.02	0.1	100,000	1.0	0.00000	0.00000
0.02	1.0	0.0000001	1.0	0.99966	0.99964
0.02	1.0	0.0001	1.0	0.99956	0.99895
0.02	1.0	1.0	1.0	0.97412	0.97348
0.02	1.0	10.0	1.0	0.81709	0.81399
0.02	1.0	100.0	1.0	0.14127	0.13600
0.02	1.0	100,000	1.0	0.00000	0.00000
0.02	10.0	0.0000001	1.0	0.99966	0.99966
0.02	10.0	0.0001	1.0	0.99959	0.99944
0.02	10.0	1.0	1.0	0.99273	0.99257
0.02	10.0	100.0	1.0	0.81402	0.81399
0.02	10.0	100,000	1.0	0.00000	0.00000
1.0	0.01	0.0000001	1.0	0.99976	0.98878
1.0	0.01	1.0	1.0	0.31515	0.00000
1.0	0.01	10.0	1.0	0.00014	0.00000
1.0	0.01	100.0	1.0	0.00000	0.00000
1.0	1.0	0.0000001	1.0	0.99976	0.99884
1.0	1.0	1.0	1.0	0.45256	0.34820
1.0	1.0	10.0	1.0	0.00517	0.00045
1.0	1.0	100.0	1.0	0.00000	0.00000
1.0	10.0	0.0000001	1.0	0.99976	0.99961
1.0	10.0	1.0	1.0	0.73064	0.72360
1.0	10.0	10.0	1.0	0.35009	0.34820
1.0	10.0	100.0	1.0	0.00047	0.00045
1.0	100.0	0.0000001	1.0	0.99976	0.99985
1.0	100.0	1.0	1.0	0.89675	0.89647
1.0	100.0	10.0	1.0	0.72368	0.72360
1.0	100.0	100.0	1.0	0.34823	0.34820

be done, however, since the derivative

$$\left. \frac{\partial (T - T_A)}{\partial z} \right]_{z=0}$$

is not identically equal to zero.

B. Simulation of Flame Plume Radiation

A number of different ways of simulating thermal radiation transfer from a flame plume to an underlying surface have been reported in the literature (14), (17). These simulators fall broadly into three categories as shown in Fig. B-1: hot grey gas, black plane, and line radiation source.

The relative merit of the simulators can be gauged by several criteria, four of which are: (1) congruence of the flux density profile at the fuel bed's upper surface as predicted by the simulator and the profile from a flame plume, (2) congruence of the simulator's polar diagram at the fuel bed's surface and the corresponding polar diagram of a flame plume, (3) ability of the simulator to allow for windblown plume geometries and/or plume temperature gradients, (4) calculational simplicity. When the fuel bed's porosity is not high enough to allow radiation to penetrate its upper surface, the second of the above criteria can be ignored.

The hot grey gas simulator satisfies the first two criteria -- perhaps more adequately than any other simulator-- but suffers from computational complexity. If the underlying fuel bed is porous, a five-fold integration is required to evaluate each flame-to-fuel-bed-zone interchange group. While the simulator can allow for windblown plumes and plume temperature gradients, the calculations for these cases are

quiescent

windblown

HOT GREY GAS

quiescent

windblown

BLACK PLANE

multiple
line
source

LINE RADIATION SOURCE

line source
+ cold grey
gas

Figure B-1

TWO-DIMENSIONAL FLAME-PLUME RADIATION SIMULATORS

even more time consuming than those for a quiescent plume.

Black plane simulators do a much poorer job of simulating flux density profiles and polar diagrams of a flame plume than do hot grey gas simulators. In particular, the flux density pattern at the foot of the plane -- the region of greatest importance -- can depart markedly from the pattern produced by a plume, particularly if the foot of the plume is cold relative to its upper regions. In addition, black plane simulators have the disadvantage of requiring a quadruple numerical integration for each interchange group when the fuel bed's porosity is high.

Multiple line radiation source simulators provide a means of modeling a flame plume wherein the first criterion is satisfied and the second, approximately so; and the computation requires only single integrations. (In the limit, if enough line sources are used, the second criterion can be satisfied exactly; however, the number of line sources will become cumbersome large, and triple numerical integrations will have to be performed.) Both windblown plume shapes and plume temperature gradients can be approximated by properly positioning the line sources.

A simulator comprised of a line radiation source surrounded by an elliptical cylinder of cold grey gas permits approximate simulation of a number of different plume geometries with but a single line source. As was the

case for multiple line sources, evaluation of interchange factors requires only single integrations.

Line source simulators were chosen for use in the mathematical modeling described in this thesis. Their flexibility and simplicity were felt to be strong enough advantages to more than offset any shortcoming in simulating polar diagrams from a plume.

C. Air Distribution System

The air distribution system used in the combustion wind tunnel is shown in Figs. C-1 and C-2. The system's components and their contribution to the development of a uniform air flow within the tunnel are described in the paragraphs below.

The small drilled plate in Fig. C-1 was positioned inside the 16 in. O.D. pipe at the upstream end of the tunnel to smooth the air flow inside the pipe before it discharged into the tunnel. The velocity pattern in the pipe prior to installation of the drilled plate is shown in Fig. C-3. The irregularity of this pattern was probably caused by a right-angle bend in the pipe a short distance upstream of its discharge point. (The direction of the bend in the pipe is indicated by the arrow in the upper left hand part of the figure.) Insertion of the drilled plate smoothed the pattern to within $\pm 10\%$ of the average value.

The upstream end of the nest of sheet metal funnels in Fig. C-1 was butted up against the small drilled plate as shown in Fig. C-2. The function of the funnel system was to divide the air flow in the 16 in. pipe into a central core and four annular rings. The funnels were shaped so that the flow in each of the five area was expanded into a corresponding fraction of the 4.5 ft. diameter cross section of the tunnel.

HONEYCOMB

8.5 in. long soda
straws stacked in
a sheet metal hoop
with axes parallel
to flow direction

DRILLED PLATE

$\frac{3}{4}$ in. thick ply-
wood with 0.25 in.
diameter holes on
0.674 in. triangular
centers

FLOW-DIVIDING FUNNELS

DRILLED PLATE

$\frac{3}{8}$ in. thick ply-
wood with 0.185 in.
diameter holes on
0.445 in. triangular
centers

Figure C-1

COMPONENTS OF AIR DISTRIBUTION SYSTEM

The intersections of the dashed lines correspond to points at which velocity measurements were made. The number at each intersection is the local velocity in ft/sec as measured with a pitot tube.

Figure C-3 VELOCITY PATTERN IN CROSS SECTION OF 16 IN. O. D. PIPE AT UPSTREAM END OF COMBUSTION TUNNEL

The large drilled plate in Fig. C-1 was butted up against the downstream end of the funnel system. Its purpose was to smooth the air flow leaving the funnel system. Because of the adverse pressure gradient in the funnels, however, the flow pattern downstream of the large drilled plate was somewhat erratic. The adverse pressure gradient is thought to have produced large-scale eddy formation, resulting in a rotational component in the flow. Since rotational flows tend to be self-sustaining, the drilled plate with an L/D ratio of only 3.0--where L/D is the ratio of hole length to hole diameter--would not have removed the rotation induced in the funnels.

The honeycomb in Figs. C-1 and C-2, with an L/D ratio of 60, was added to the air distribution system to remove rotational components in the flow. The honeycomb was fabricated from a reinforced sheet metal hoop filled with soda straws. A fine mesh wire screen was soldered to the downstream side of the hoop to hold the straws in place.

The flow pattern in the tunnel downstream of the air distribution system is shown in Fig. C-4. As indicated in the figure, air flow in the portion of the tunnel's cross section lying above the fuel bed was uniform to within approximately $\pm 10\%$ of the average value.

The intersections of the dashed lines correspond to points at which velocity measurements were made. The number at each intersection is the local velocity in ft/sec as read by a vane anemometer.

Figure C-4 VELOCITY PATTERN IN TUNNEL CROSS SECTION
IMMEDIATELY DOWNSTREAM OF
AIR DISTRIBUTION SYSTEM

D. Radiometer

D-1 Radiometer Design

The radiometer described in Chapter III measured total black body flux density from a thermal radiation source by means of a temperature gradient developed along a thin, blackened metal strip. One end of the strip was attached to a low-absorptivity heat sink in the radiometer; the other, to a fine wire thermocouple. When placed in a radiation field, the end of the strip at the thermocouple became hotter than the end at the heat sink, with the temperature difference between ends of the strip depending (in part) upon the intensity of the radiation field.

Temperature gradients along a metal strip (or fin) were also used by Gardon (11) and de Rochechouart (16) to measure flux densities. Their radiometers, however, were both bulky and sensitive to convection at low levels of flux density. In radiometers of this type, the temperature difference between ends of the strip is not a unique function of the amount of radiation received by the strip; it also depends upon the temperature of the ambient contacting the strip, T_A , and upon the convective heat transfer coefficient, H , between the strip and ambient. Both of these quantities, in general, are unknown and can be expected to vary from measurement to measurement. This difficulty can be avoided by carefully blowing a

stream of shielding air across the strip at a controlled velocity and temperature. If the velocity of the shielding air stream is high enough, both H and T_A will have values determined by the shielding air stream and will not be influenced by the ambient external to the radiometer.

In the radiometer of Chapter III, a shielding air stream was blown over the sensing strip at a relatively low velocity to minimize interaction between the shielding air and the flames and heating wire arrays whose radiation was being measured. The shielding provided by the low velocity stream was imperfect. Consequently, the ambient external to the radiometer influenced H and T_A , causing their values to depart slightly from those for the case of perfect air shielding.

To account for possible variations in T_A or H , two additional strips differing in size and emissivity from the first strip were attached to the heat sink. The combination of the three strips and a shielding air stream permitted accurate determination of flux densities from measurements of temperature differences between the heat sink and the far end of each of the three strips. These temperature differences were measured thermoelectrically, with the strips themselves serving as elements in the circuitry. (A radiometer designed by McGuire and Wraight (15) also used more than one sensing element.

Its elements, one of high emissivity and one of low, were not air shielded and allowed only an approximate correction for convection.)

Sketches of the radiometer and its sensing elements are shown in Figs. D-1 and D-2. The equation which describes its steady-state behavior is derived in the following section. The "working form" of this equation is

$$q_s'' = A_1 E_1 + \left[\frac{C_1 C_3 (A_2 E_2 - A_1 E_1) (E_3 - E_1) - C_1 C_2 (A_3 E_3 - A_1 E_1) (E_2 - E_1)}{C_1 C_3 (E_3 - E_1) - C_2 C_3 (E_3 - E_2) - C_1 C_2 (E_2 - E_1)} \right] \quad (D-1)$$

In the above equation, q_s'' is the flux density being measured; E_1 , E_2 , and E_3 are emf signals generated by differential thermocouples in the radiometer; A_1 , A_2 , A_3 , C_1 , C_2 , and C_3 are instrument parameters whose values can be determined by calibration. The term in brackets is a correction for convection. If the radiometer's air shielding is adequate, it will be much smaller than $A_1 E_1$.

D-2. Derivation of Radiometer Equation

A one-dimensional steady-state energy balance is written for each of the three strips in Fig. D-2.

Figure D-1 PERSPECTIVE SKETCH OF RADIOMETER SENSING HEAD

Strip No. 1: blackened Chromel P, 0.035 in. thick, 0.35 in. long
 Strip No. 2: bare Chromel P, 0.003 in. thick, 0.375 in. long
 Strip No. 3: blackened Chromel P, 0.003 in. thick, 0.375 in. long

Figure D-2 TOP VIEW OF RADIOMETER'S SENSING STRIPS AND HEAT SINK

$$k_r t_i \frac{d^2(T_i - T_A)}{dx^2} - H_i(T_i - T_A) = - e_i q_s''$$

(D-2)

(i = 1,2,3)

The boundary conditions are: $T_i = T_c$ at $x = 0$ and

$$\frac{dT_i}{dx} = 0 \text{ at } x = L_i$$

where

- H_i = convective heat transfer coefficient
between ambient and sensing strip i
- k_r = thermal conductivity of sensing strips
- L_i = distance between free end of strip i
and heat sink
- t_i = thickness of strip i
- T_i = temperature of strip i at position x
- T_A = temperature of ambient contacting
sensing strips
- T_c = temperature of heat sink
- q_s'' = radiation flux density at sensing strips
- x = position variable (distance along strip
from heat sink)
- e_i = emissivity of strip i

Integration of Eq. D-2 gives

$$\frac{e_1 q_s''}{H_1} = \frac{\cosh \sqrt{\frac{H_1 L_1^2}{k_r t_1}}}{\left(\cosh \sqrt{\frac{H_1 L_1^2}{k_r t_1}} \right) - 1} (T_{L1} - T_c) + (T_c - T_A) \quad (D-3)$$

where $T_1 = T_{L1}$ at $x = L_1$.

The coefficient of $(T_{L1} - T_c)$ in Eq. D-3 can be approximated by

$$1 + \frac{2k_r t_1}{H_1 L_1^2}$$

with an error of less than 2%. Noting that $e_1 = e_3 = 1$, the above approximation can be incorporated in Eq. D-3 to yield a separate equation for each of the three strips,

$$q_s'' = \frac{2k_r t_1}{L_1^2} (T_{L1} - T_c) + H_1 (T_{L1} - T_A) \quad (D-4)$$

$$q_s'' = \frac{2k_r t_2}{e_2 L_2^2} (T_{L2} - T_c) + \frac{H_2}{e_2} (T_{L2} - T_A) \quad (D-5)$$

$$q_s'' = \frac{2k_r t_3}{L_3^2} (T_{L3} - T_c) + H_3(T_{L3} - T_A) \quad (D-6)$$

Assuming both H_2/H_1 and H_3/H_1 are independent of changes in H_1 , Eqs. D-4, D-5, and D-6 can be rewritten as follows.

$$q_s'' = A_1 E_1 + C_1 H_1 E_1 + H_1(T_c - T_A) \quad (D-7)$$

$$q_s'' = A_2 E_2 + C_2 H_1 E_2 + \frac{C_2 H_1}{C_1} (T_c - T_A) \quad (D-8)$$

$$q_s'' = A_3 E_3 + C_3 H_1 E_3 + \frac{C_3 H_1}{C_1} (T_c - T_A) \quad (D-9)$$

where

$$A_1 = \frac{2k_r t_1 C_1}{L_1^2}, \quad A_2 = \frac{2k_r t_2 C_1}{e_2 L_2^2}, \quad A_3 = \frac{2k_r t_3 C_1}{L_3^2}$$

C_1 = temperature difference corresponding to one unit of thermoelectric emf

$$C_2 = C_1 H_2/H_1 e_2, \quad C_3 = C_1 H_3/H_1$$

$$E_1 = (T_{L1} - T_c)/C_1, \quad E_2 = (T_{L2} - T_c)/C_1$$

$$E_3 = (T_{L3} - T_c)/C_1$$

The "working form" of the radiometer equation, as given by Eq. D-1, is obtained by using Eqs. D-8 and D-9 to eliminate both H_1 and $(T_c - T_A)$ from Eq. D-7.

D-3 Calibration

The radiometer was calibrated with an electrically heated furnace in a manner similar to that described by de Rochechouart (16). A small, sharp-edged hole at one end of the furnace cavity approximated a black body radiation source. The fraction of radiation leaving this aperture which was received by the radiometer's sensing elements was calculated using interchange factors reported by Eckert and Drake (6). The calibration conditions were as follows.

(1) The furnace temperature, as measured with both an optical pyrometer and a platinum, platinum-rhodium thermocouple, was 1710 °R.

(2) The convection parameter T_A was held constant at 80°F, as measured with a thermocouple in the shielding air stream.

(3) The convection heat transfer coefficient H_1 was held constant at a value corresponding to a shielding air flow rate of 4 standard cubic feet per hour.

Both q_s'' and $(T_c - T_A)$ were used as independent variables during the calibration. The former was varied by changing the position of the radiometer's sensing

elements relative to the furnace aperture; the latter, by changing the cooling water rate to the radiometer. By a trial-and-error adjustment of the cooling water rate, it was thus possible to make $(T_c - T_A)$ negative enough to cause $(T_{L1} - T_A)$ to approach zero. From Eqs. D-4 and D-7,

$$A_1 = \left(\frac{q_s''}{E_1} \right) T_{L1} = T_A$$

The calibration data in Fig. D-3 indicate that $A_1 = 1890$ BTU/hr-ft²-millivolt. Considerable care was exercised in obtaining these data since A_1 is the most important constant appearing in Eq. D-1.

From Eqs. D-5 and D-8,

$$A_2 = \left(\frac{q_s''}{E_2} \right) T_{L2} = T_A$$

Similarly, from Eqs. D-6 and D-8,

$$A_3 = \left(\frac{q_s''}{E_3} \right) T_{L3} = T_A$$

The product $C_1 H_1$ was evaluated by blocking the radiometer's view of the furnace and adjusting the cooling water rate until $(T_{L1} - T_A)$ reached a steady, relatively large negative value. If the thermoelectric emf

corresponding to $(T_c - T_A)$ is designated E_4 ,

$$C_1 H_1 = \left[\frac{A_1 E_1}{(E_1 + E_4)} \right] q_s'' = 0$$

In an analagous manner,

$$C_2 H_1 = \left[\frac{A_2 E_2}{(E_2 + E_4)} \right] q_s'' = 0$$

and

$$C_3 H_1 = \left[\frac{A_3 E_3}{(E_3 + E_4)} \right] q_s'' = 0$$

Values of the radiometer's instrument parameters, determined by the calibration procedures just described, are listed in Table D-1.

TABLE D-1

CALIBRATION CONSTANTS

Constant	Numerical Value Btu/hr-ft ² -millivolt
A ₁	1890
A ₂	961
A ₃	738
C ₁ H ₁	95.6
C ₂ H ₁	59.5
C ₃ H ₁	55.1

If the preceding values of A₁, A₂, A₃, C₁H₁, C₂H₁, and C₃H₁ are substituted in Eq. D-1, H₁ will cancel out of the term in brackets, and the equation can be written as

$$q_s'' = 1890 E_1 + \left[\frac{(961E_2 - 1890E_1)(E_3 - E_1) - (796E_3 - 2040E_1)(E_2 - E_1)}{(E_3 - E_1) - 0.621(E_3 - E_2) - 1.079(E_2 - E_1)} \right]$$

D-4 Radiometer Time Constant

The transient behavior of Sensing Strip Number 1 is described by Eq. D-10.

$$k_r t_1 \frac{\partial^2 T_1}{\partial x^2} - H_1(T_1 - T_A) - (\rho C_P)_r t_1 \frac{\partial T_1}{\partial \theta} = - q_s'' \quad (D-10)$$

where $(\rho C_P)_r$ = volumetric heat capacity of sensing strip
 θ = time variable

If attention is restricted to the free end of the strip where $T_1 = T_{L1}$ and $x = L_1$, Eq. D-10 can be approximated with the following ordinary differential equation.

$$\frac{2 k_r t_1}{L_1^2} (T_{L1} - T_c) + H_1(T_{L1} - T_c) + (\rho C_P)_r t_1 \frac{d(T_{L1} - T_c)}{d\theta} = q_s'' - H_1(T_c - T_A) \quad (D-11)$$

Equation D-11 integrates to Eq. D-12

$$(T_{L1} - T_c) = \left\{ (T_{L1} - T_c)_o - \left[\frac{q_s'' - H_1(T_c - T_A)}{\frac{2k_r t_1}{L_1^2} + H_1} \right] \right\} \cdot \exp \left[- \frac{2k_r t_1 + L_1^2 H_1}{(\rho C_P)_r t_1 L_1^2} \theta \right] + \frac{q_s'' - H_1(T_c - T_A)}{\frac{2k_r t_1}{L_1^2} + H_1}$$

(D-12)

where $(T_{L1} - T_c)_o$ = value of dependent variable at time zero .

From Eq. D-12, or equivalently, from Eq. D-4,

$$(T_{L1} - T_c)_F = \frac{q_s'' - H_1(T_c - T_A)}{\frac{2k_r t_1}{L_1^2} + H_1} \quad (D-13)$$

where $(T_{L1} - T_c)_F$ = value of dependent variable as $\theta \rightarrow \infty$.

Combining Eqs. D-12 and D-13,

$$\frac{(T_{L1} - T_c) - (T_{L1} - T_c)_F}{(T_{L1} - T_c)_O - (T_{L1} - T_c)_F} = \exp \left[- \frac{2k_r t_1 + L_1^2 H_1}{L_1^2 t_1 (\rho C_P)_r} \theta \right]$$

(D-14)

The left-hand side of Eq. D-14 is equal to $(1.0 - e^{-1})$ when

$$\theta = 0.460 \frac{L_1^2 t_1 (\rho C_P)_r}{2k_r t_1 + L_1^2 H_1}$$

If $L_1 = 0.35$ in.

$t_1 = 0.035$ in.

$(\rho C_P)_r = 57.4$ Btu/ft³-°R

$k_r = 10$ Btu/hr-ft-°R

$H_1 = 3$ Btu/ft²-hr-°R

the radiometer's time constant is equal to

$$(0.460) \frac{(0.35)^2 (0.035) (57.4) (3600)}{(2)(10)(0.035)(144) + (0.35)^2 (3)(12)}$$

= 3.9 seconds

The predicted value of 3.9 seconds compares favorably with the experimentally measured value of approximately 4 seconds.

E. Derivation of an Approximate Expression for Surface Temperature as a Function of Enthalpy Temperature

An idealized cylindrical fuel particle is shown in Fig. E-1. An energy balance over a volume shell of thickness dr in Fig. E-1 has the form

$$\frac{\partial}{\partial r} (2\pi r k \frac{\partial t}{\partial r}) = 2\pi r \frac{\rho_b C_P}{1 - \Lambda} \frac{\partial t}{\partial \theta} \quad (E-1)$$

thermal con-
duction normal
to cylindrical
surface

energy
storage

where k = thermal conductivity of fuel particle

t = temperature in fuel particle at distance
 r from center line

$\rho_b C_P$ = volume heat capacity of fuel/unit
total volume of fuel bed

Λ = void fraction in fuel bed

Equation E-1 assumes that energy transfer within the particle is one-dimensional. If, in addition, it is assumed that $\frac{\partial t}{\partial \theta}$ is independent of r , Eq. E-1 can be integrated with respect to r to give Eq. E-2:

Figure E-1 CYLINDRICAL FUEL PARTICLE

$$2\pi rk \frac{\partial t}{\partial r} = \frac{\pi r^2 \rho_b C_P}{1 - \Lambda} \frac{\partial t}{\partial \theta} \quad (E-2)$$

At the fuel surface, $r = R$ and $t = T$. Thus,

$$2\pi Rk \left[\frac{\partial t}{\partial r} \right]_{r=R} = \frac{\pi R^2 \rho_b C_P}{1 - \Lambda} \frac{\partial t}{\partial \theta} \quad (E-3)$$

Let q''_{sp} = net flux at surface of particle. It then follows that

$$q''_{sp} = k \left[\frac{\partial t}{\partial r} \right]_{r=R}$$

Equation E-3 can be rewritten using the above definition as

$$\frac{\rho_b C_P}{1 - \Lambda} \frac{\partial t}{\partial \theta} = \frac{2q''_{sp}}{R}$$

Substituting the above expression in Eq. E-2,

$$k \frac{\partial t}{\partial r} = \frac{rq''_{sp}}{R} \quad (E-4)$$

Equation E-4 is integrated with respect to r from $r = 0$, where $t = t_c$, to $r = r$.

$$t - t_c = \frac{r^2 q''_{sp}}{2kR} \quad (E-5)$$

Equation E-5 can now be used to find the enthalpy average temperature $\bar{T}$ of the fuel as follows.

$$\begin{aligned}\pi R^2 \bar{T} &= 2\pi \int_0^R r t \, dr = 2\pi \int_0^R \left(\frac{r^3 q_{sp}''}{2kR} + r t_c \right) dr \\ &= 2\pi \left(\frac{R^3 q_{sp}''}{8k} + \frac{R^2 t_c}{2} \right)\end{aligned}$$

Thus,

$$\bar{T} - t_c = \frac{q_{sp}'' R}{4k} \quad (E-6)$$

Referring back to Eq. E-5 and recalling that $t = T$ when $r = R$,

$$T - t_c = \frac{q_{sp}'' R}{2k} \quad (E-7)$$

Eliminating t_c from Eqs. E-6 and E-7,

$$T - \bar{T} = \frac{q_{sp}'' R}{4k} \quad (E-8)$$

Equation E-8 predicts a difference in surface and enthalpy temperatures which is directly proportional to the product of fuel thermal resistance and net thermal flux at the surface.

A term such as $(\bar{T}_1 - \bar{T}_{1-1})$ in an energy balance around fuel bed zone 1 can be approximated using Eq. E-8 in the following manner:

$$(\bar{T}_1 - \bar{T}_{1-1}) = (T_1 - T_{1-1}) - \frac{R}{4k} (q''_{sp_1} - q''_{sp_{1-1}})$$

where

$$\begin{aligned} q''_{sp_1} = & \frac{1}{4k_b M_1 B_1 B_2} \left(\sum_{\substack{j=1 \\ j \neq 1}}^{N_2} \left[\overline{g_1 g_j / L} \right] \cdot \sigma T_j^4 \right. \\ & + \sum_{j=N_2+1}^{N_1} \left[\overline{g_1 g_j / L} \right] \cdot \sigma T_B^4 + (\overline{W g_1} / \delta L) (Q_w / L) \\ & + h_1 (4k_b M_1 B_1 B_2) (T_{A_1} - T_1) + h_{c_1} (4k_b M_1 B_1 B_2) (T_F - T_{1g}) \\ & \left. - \left\{ (4k_b M_1 B_1 B_2) - \left[\overline{g_1 g_1 / L} \right] \right\} \cdot \sigma T_1^4 \right) \end{aligned}$$

F. Effect of Anisotropy on Fuel Bed Radiation Interchange Factors

The value assigned to the absorption coefficient k_b in an anisotropic fuel bed depends upon the location of the receiving zone relative to the source zone, but not upon their separating distance. In a spherical coordinate system, this is equivalent to the statement that k_b is a function of the coordinate angles θ and ϕ .

The interchange factor arising from an anisotropic fuel bed and its location-dependent absorption factor is given by Eq. F-1.

$$d^5 \left[\frac{\bar{g}_i \bar{g}_j}{L} \right] = 4k_b(\theta, \phi) \cdot dV_i \cdot \frac{dA_j k_b(\theta, \phi)}{4\pi R^2} \cdot \exp \left[-k_b(\theta, \phi) \cdot R \right]$$

emission from
volume dV_i
per unit
of its emissive
power

fraction
absorbed
by unit
length of
zone whose
cross section
is dA_j

fraction trans-
mitted through
distance R

(F-1)

Equation F-1 applies to a two-dimensional system and assumes that the concentration of fuel is uniform throughout the bed. The origin of the coordinate system used in the equation's formulation is located at the center of zone j .

If the fuel bed is not anisotropic in all directions, the form of Eq. F-1 can be simplified somewhat. Consider a fuel bed of wood excelsior which is initially isotropic. If the fuel bed is compressed in the direction normal to the plane in which the angle θ is measured, the bed will become anisotropic with respect to the azimuth angle ϕ but will remain isotropic with respect to θ . This permits $k_b(\theta, \phi)$ to be replaced by $k_b(\phi)$ wherever it appears in Eq. F-1.

An approximation to the functional form of $k_b(\phi)$ for the case of a wood excelsior fuel bed compressed along its vertical axis can be obtained as follows. The compressed fuel bed is idealized as a collection of tori lying in horizontal planes with their centers of gravity randomly arrayed within the fuel bed. The projected area of each torus onto a plane normal to the azimuth angle ϕ will be an elliptical ring as shown in Fig. F-1. The aggregate area of elliptical rings per unit volume of fuel bed is equal to $k_b(\phi)$.

Referring again to Fig. F-1, the projected area of a torus onto a horizontal plane, i.e., a plane for which $\phi = 0$, is

$$2\pi P t$$

The projected area for both zero and non-zero values of ϕ is

Figure F-1 TORUS AND ITS PROJECTION ONTO A PLANE AT AN ANGLE ϕ
RELATIVE TO THE HORIZONTAL

$$4Pt \int_0^P \left[\frac{P^2 - (\sin^2 \phi) x^2}{P^2 - x^2} \right]^{1/2} \frac{dx}{P}$$

which is equivalent to

$$4Pt \left\{ \int_0^{\pi/2} \left[1 - \sin^2 \phi \sin^2 \lambda \right]^{1/2} d\lambda \right\}$$

The term in braces in the above expression is a complete elliptical integral of the second kind and is tabulated in standard mathematical tables.

If the total projected area onto a horizontal plane of all tori in a unit volume of fuel bed is designated A_0 , then

$$k_b(\phi) = \frac{2A_0}{\pi} \int_0^{\pi/2} \left[1 - \sin^2 \phi \sin^2 \lambda \right]^{1/2} d\lambda$$

or, in terms of surface area of fuel per unit volume of fuel bed, designated S_0 ,

$$k_b(\phi) = \frac{2S_o}{\pi^2} \int_0^{\pi/2} \left[1 - \sin^2 \phi \sin^2 \lambda \right]^{1/2} d\lambda \quad (F-2)$$

Alternatively, the compressed wood excelsior fuel bed can be idealized as a collection of cylinders lying in horizontal planes but randomly oriented in all other respects. If the total surface area per unit volume of fuel bed is once again designated S_o , $k_b(\phi)$ is still described by Eq. F-2.

Substitution of Eq. F-2 into Eq. F-1 followed by integration along a line parallel to the direction in which the system is invariant gives rise to the interchange factors shown in Fig. F-2. The curves in the figure indicate that interchange factors in a compressed wood excelsior fuel bed are a relatively strong function of the azimuth angle ϕ between zone centers. These factors can be used without further integration in Eq. 5.1 for zones which are far apart compared to the square root of the zones' cross sectional area. For zones which are close together, however, four additional numerical integrations must be performed before they can be used in Eq. 5.1.

G. Block Diagram and Listing of Fortran II Program for
Brush Fire Model

The block diagram of Fig. G-1 indicates the major computational and logical operations in the Fortran II program written for the brush fire model of Chapter V. Details of the program are not shown in the figure; they are, however, included in the Fortran listing in the latter part of this section. Variables and parameters appearing in the Fortran input and output statements are defined as follows in terms of the nomenclature of Chapter V.

A(K,1) $A_{K,1}$

AOC a/c

ASPR B_2/B_1

C(K,L) $C_{K,L}$

COR empirical interpolation factor for use in computing $V_p C_p$ when $(T_{N_3} - T_{1g}) \neq 0$. The value assigned to COR to obtain the results of Chapter VI was 1.03.

COSA $\cos \alpha_s$

D(K,L) $D_{K,L}$

DET input parameter for XSIMEQ subroutine. DET should be set equal to 1.0 for this program.

Figure G-1

BLOCK DIAGRAM OF FORTRAN II PROGRAM FOR BRUSH FIRE MODEL

DKL	$V_p C_p$
DKLC	value of DKL computed using COR interpolation factor
$E(L)$	q_{E_L}
$E1(L)$	$q + 1_{E_L}$
$GF(IM, JM)$	$\frac{\overline{g_i g_j}}{L} \cdot \frac{1}{B_1} \text{ , where } B_2/B_1 = 1.0$ <p>The $GF(IM, JM)$ array is read into the computer for a grid which is 25 zones long and 4 zones deep, with all zones being equal in size.</p>
$GH(IM, JM)$	$\frac{\overline{g_i g_j}}{L} \cdot \frac{1}{B_1} \text{ , where } B_2/B_1 = 0.5.$ <p>This array is the half zone counterpart of $GF(IM, JM)$ and is read into the computer for a grid which is 25 zones long and 4 zones deep.</p>
$GPUN(IM, JM)$	<p>fuel bed radiation interchange factors for a full zone grid. The $GPUN(IM, JM)$ array is more general in form than $GF(IM, JM)$. The array need not have the dimensions of $GF(IM, JM)$, nor need it be read in for a 25 by 4 grid. B_2/B_1 can have any positive value.</p>

GPUNH(IM,JM) fuel bed radiation interchange factors
for a half zone grid. GPUNH(IM,JM) is the
half zone counterpart of GPUN(IM,JM).

H h

HC1 h_{c_1}

· ·

· ·

· ·

HC4 h_{c_4}

IHP length of grid, in units of B_2 , for which
GPUN(IM,JM) data are to be read into
computer

IM dimensionless vertical position in fuel
bed, in units of B_1

INPUT input flag. The value assigned to INPUT
should be + 1 if $B_2/B_1 = 1.0$; or, 0 if
 $B_2/B_1 \neq 1.0$.

IVP depth of grid, in units of B_1 , for which
GPUN(IM,JM) data are to be read into
computer.

JALL input flag to read in data for new cases wherein a complete set of all input parameters in the program is to be read into the computer for each new case. The value assigned to JALL should be equal to the total number of such cases, including the original case. JALL is independent of JEMB, JPKB, JTEM, and J99.

JEMB input flag to read in data for new cases wherein values assigned to the set of parameters N_7 , N_8 , N_9 , N_{10} , NH_3 --or any of its subsets--are to be changed. The value assigned to JEMB should be equal to the total number of such cases, including the original case.

JM dimensionless horizontal position in fuel bed, in units of B_2

JPKB input flag to read in data for new cases wherein only $GF(IM, JM)$ and $GH(IM, JM)$ are to be changed. The value assigned to JPKB should be equal to the total number of such cases, including the original case.

JTEM input flag to read in data for new cases wherein values assigned to the set of parameters T_A , T_B , T_{ig} , T_o , h--or any of its subsets--are to be changed. The value assigned to JTEM should be equal to the total number of such cases, including the original case.

J99 input flag to read in data for new cases wherein values assigned to the set of parameters L_s/B_1 , x_s/B_1 , $\cos \alpha_s$, $k_g a$, $k_b B_2$, a/c B_2/B_1 , $(Q_w/L)/B_1$ --or any of its subsets--are to be changed. The value assigned to J99 should be equal to the number of such cases, including the original case.

K2 K_2

 .

 .

 .

K10 K_{10}

LF $N_3 + N_4$

LF2 $N_3 + N_4 + N_5$

MC debugging parameter. The value assigned to MC should be + 1 if early print out is desired; otherwise, it should be 0.

N2 N_2

· ·
· ·
· ·

N10 N_{10}

NH3 half zone grid parameter. $NH3 = \begin{cases} 0 & \text{if } N_3 < 6 \\ 1 & \text{if } N_3 = 6 \\ 0 & \text{if } N_3 = 7 \\ (7-N_3) & \text{if } N_3 > 7 \end{cases}$

P(K) H_K

PIHH $GPUN(IM, JM)/GF(IM, JM)$

PIHO $GPUN(IM, JM)/GF(IM, JM)$

PKA k_g^a

PKB $k_b B_2$

PNL L_s/B_1

PNOR maximum value of $|P(K)|$, for all K

PNW x_s/B_1

QW $(Q_w/L)/B_1$

R(K) R_K

SCALE normalization factor to prevent XSIMEQ subroutine from overflowing. A value of 1×10^{-21} was used to obtain the results of Chapter VI.

SINA $\sin \alpha_s$

SGOL(K) $\overline{Wg_K}/(\delta L)$

T(L) fuel surface temperature in zone L

TA T_A

TB T_B

TDIFF $T_{N_3} - T_{N_3} + N_4$

TEST1 value used in Newton-Raphson convergence test.
TEST1 should be set equal to the maximum allowable value of

$$\left| \frac{q_{E_L}^{+1} - q_{E_L}}{q_{E_L}} \right|$$

TEST2 value used in testing for convergence of temperature in zone N_3 . TEST2 should be set equal to the maximum allowable value of

$$\left| T_{N_3} - T_{ig} \right|$$

TEST3 value used in testing for convergence of temperature in lower three ignition zones.
Its value should be equal to or greater than that of TEST2.

TIG

T_{ig}

TO

T_o

V*RHO*CP

DKLC


```

*M4098-3438, FMS, RESULT, 5, 5, 5000, 0
*   XEQ
*   LIST
C ENERGY TRANSFER IN A POROUS FUEL BED   E4
  DIMENSION T(60), T1(60), R(60), P(60), NNK(60), E(60), E1(60),
  18OL(60), SGOL(60), G(4,25), C(60,60), D(60,60), TIGN(60),
  2A(60,60), GG(60,60), AH(60), NT(30), GF(4,25), GH(4,25),
  3RHAL(60), NNF(60), TH(4,10), KNN(60), GPUN(25,25), GPUNH(25,25)
  COMMON A, C, D, GG, GPUN, GPUNH
100 FORMAT (9H OVERFLOW)
101 FORMAT (36H SINGULAR MATRIX OR WRONG DIMENSIONS)
102 FORMAT (10I4)
103 FORMAT (5X,1HE,8X,2HE1,7X,1HE,8X,2HE1,7X,1HE,8X,2HE1,
  17X,1HE,8X,2HE1,7X,1HE,8X,2HE1,7X,1HE,8X,2HE1)
104 FORMAT (5X,1HT,9X,1HR,6X,4HSGOL,5X,6HA(K,1),4X,1HT,9X,1HR,6X,
  14HSGOL,5X,6HA(K,1),4X,1HT,9X,1HR,6X,4HSGOL,5X,6HA(K,1))
105 FORMAT (5X,1HC,8X,1HD,8X,1HC,8X,1HD,8X,1HC,8X,1HD,8X,1HC,8X,
  11HD,8X,1HC,8X,1HD,8X,1HC,8X,1HD)
106 FORMAT (8F8.4, E12.4)
107 FORMAT (5F10.2)
108 FORMAT (2I4, 5F9.4, E12.4)
109 FORMAT (10F7.5)
110 FORMAT (3X,4HTB =,F7.1,5X,5HTIG =,F7.1,6X,4HTA -,F7.1,6X,4HTO =,
  1F7.1,7X,3HH =,E9.3,5X,4HQW =,E10.4)
111 FORMAT (3X,4HN7 =,I3,10X,4HN8 =,I3,10X,4HN9 =,I3,9X,5HN1 =,I3,7X,
  17HTEST1 =,E9.3,2X,7HTEST2 =,E9.3,2X,7HTEST3 =,E9.3)
112 FORMAT (2X,5HPNL =,F5.2,7X,5HPNW =,F5.2,6X,6HCOSA =,F5.3,7X,
  15HPKA =,F5.3,7X,5HPKB =,F5.3,8X,5HAOC =,F5.2,6X,7HB2/B1 =,F6.3)
113 FORMAT (4F10.4)
114 FORMAT (2X,5HHC1 =,E9.3,3X,5HHC2 =,E9.3,3X,5HHC3 =,E9.3,3X,
  15HHC4 =,E9.3)
115 FORMAT (5I4)
120 FORMAT (E11.3, 11E9.3)
121 FORMAT (3X,5HDKL =,E10.3,2X,6HPNOR =,E10.3,2X,4HN3 =,I3,2X,
  14HN4 =,I3,2X,4HN5 =,I3,2X,4HN6 =,I3)
123 FORMAT (15F8.1)
124 FORMAT (33H TEMPERATURE FIELD - T(L), L = 1,,I3)
125 FORMAT (3X,5HDKL =,E10.3,2X,6HPNOR =,E10.3,2X,4HK3 =,I3,2X,
  14HK4 =,I3,2X,4HK5 =,I3,2X,4HK6 =,I3)
127 FORMAT (1H4, 16HINPUT PARAMETERS)
128 FORMAT (1H0, 6HOUTPUT)
129 FORMAT (5X, I2, 7X, I2, 33X, F7.4)
130 FORMAT (8E10.4)
133 FORMAT (2X, 10HV*RHO*CP =, E10.4, 5X, 18H(T(N3) T(N3+N4)) =,
  1E11.4)
134 FORMAT (53H THE ABOVE SOLUTION IS BETTER THAN THOSE PRECEDING IT)
135 FORMAT (2X, 31HVELOCITY INTERPOLATION FACTOR =, F6.3)
136 FORMAT (1H1, 57HINTERCHANGE FACTORS--((GF(IM,JM), JM = 1, 25), IM
  1= 1, 4))
137 FORMAT (2X, 10E11.4)
139 FORMAT (58H INTERCHANGE FACTORS--((GH(IM,JM), JM = 1, 25), IM = 1,
  1 4))
150 READ 102, N2, N3, N4, N5, N6, N7, N8, N9, N10, NH3
  READ 106, PNL, PNW, COSA, SINA, PKA, PKB, AOC, ASPR, QW
  READ 107, TB, TIG, TA, TO, H

```

```

      READ 113, HC1, HC2, HC3, HC4
      READ 108, MC, INPUT, DET, TEST1, TEST2, TEST3, COR, SCALE
      READ 115, J99, JEMB, JPKB, JTEM, JALL
      READ 107, (T(L), L = 1, N2)
      IF (INPUT) 151, 160, 151
151 DO 152 IM = 1, 25
      DO 152 JM = 1, 25
      GPUN(IM,JM) = 0.
152 GPUNH(IM,JM) = 0.
      IF (ASPR - 1.01) 153, 153, 156
153 READ 129, IVP, IHP, PIHO
      READ 130, ((GPUN(IM,JM), JM = 1, IVP), IM = 1, IHP)
      READ 129, IVP, IHP, PIHH
      READ 130, ((GPUNH(IM,JM), JM = 1, IVP), IM = 1, IHP)
      DO 154 IM = 1, 4
      DO 154 JM = 1, 25
      GH(IM,JM) = GPUNH(IM,JM)/PIHH
154 GF(IM,JM) = GPUN(IM,JM)/PIHO
      GO TO 170
156 READ 129, IVP, IHP, PIHO
      READ 130, ((GPUN(IM,JM), IM = 1, IVP), JM = 1, IHP)
      READ 129, IVP, IHP, PIHH
      READ 130, ((GPUNH(IM,JM), IM = 1, IVP), JM = 1, IHP)
      DO 157 IM = 1, 4
      DO 157 JM = 1, 25
      GF(IM,JM) = GPUN(IM,JM)/PIHO
157 GH(IM,JM) = GPUNH(IM,JM)/PIHH
      GO TO 170
160 READ 109, ((GF(IM,JM), IM = 1, 4), JM = 1, 25)
      READ 109, ((GH(IM,JM), IM = 1, 4), JM = 1, 25)
170 PRINT 136
      PRINT 137, ((GF(IM,JM), JM = 1, 25), IM = 1, 4)
      PRINT 139
      PRINT 137, ((GH(IM,JM), JM = 1, 25), IM = 1, 4)
C ZONE NUMBERS FOR HALF-ZONE STATIONARY GRID AS A FUNCTION OF ZONE
C NUMBERS FOR FULL-ZONE STATIONARY GRID (700 THROUGH 701)
      700 DO 701 I = 1, 4
      NNF(15*I-10) = 15*I - 14
      NNF(15*I-9) = 15*I - 13
      701 NNF(15*I-8) = 15*I - 12
C PHAL = 1.0 CORRESPONDS TO FULL-ZONE GRID--PHAL = 0.5, TO HALF-ZONE
      740 PHAL = 1.0
      TO1 = TO
      TO2 = TO
      TO3 = TO
      TO4 = TO
C RHAL(L) IS RADIATION INPUT TERM TO ZONE L FROM UNBURNED ZONES
C WHOSE TEMPERATURES ARE GREATER THAN TO
C (WHEN PHAL = 1.0, RHAL(L) = 0.0)
      DO 741 L = 1, 60
      741 RHAL(L) = 0.0
      DO 742 IM = 1, 4
      DO 742 JM = 1, 25

```

```
742 G(IM,JM) = GF(IM,JM)
745 DO 750 L = 1, N2
750 E(L) = .1713*(T(L)/100.)**4
      NT(1) = 400*N6 + 20*N5 + N4
      DEVO = 1000000.
760 PRINT 127
      PRINT 110, TB, TIG, TA, TO, H, QW
      PRINT 111, N7, N8, N9, N10, TEST1, TEST2, TEST3
      PRINT 112, PNL, PNW, COSA, PKA, PKB, AOC, ASPR
      PRINT 114, HC1, HC2, HC3, HC4
      PRINT 135, COR
      IT = 1
C INTERCHANGE AREA CALCULATIONS (800 THROUGH 900)
      DO 900 JJJ = 1, 15
      IF (PHAL - 0.75) 809, 999, 800
800 IF (JJJ - 5) 802, 801, 801
801 J11 = 1
      J12 = JJJ + 10
      PJJ = JJJ
      X = 7.5 - PJJ - PNW
      GO TO 840
802 IF (JJJ - 4) 804, 803, 801
803 J11 = 2
      J12 = 13
      X = 4. - PNW
      GO TO 840
804 IF (JJJ - 3) 806, 805, 803
805 J11 = 3
      J12 = 10
      X = 6.5 - PNW
      GO TO 840
806 IF (JJJ - 2) 808, 807, 805
807 J11 = 4
      J12 = 6
      X = 10. - PNW
      GO TO 840
808 J11 = 5
      J12 = 1
      X = 14.5 - PNW
      GO TO 840
809 IF (JJJ - 10) 811, 811, 810
810 J11 = 2
      J12 = 2*(JJJ-10) + 11
      PJJ = JJJ
      X = 7.5 - PJJ - PNW
      GO TO 840
811 IF (JJJ - 3) 813, 812, 812
812 J11 = 1
      J12 = JJJ + 2
      PJJ = JJJ
      X = 0.5*(4.5 - PJJ) - PNW
      GO TO 840
813 J11 = 2
```

```

      J12 = 2*JJJ - 1
      PJJ = JJJ
      X = 3.5 - PJJ - PNW
840 DO 900 NNN = 1, 15
      IF (PHAL - 0.75) 859, 999, 845
845 IF (NNN - 5) 852, 851, 851
851 N11 = 1
      N12 = NNN + 10
      GO TO 865
852 IF (NNN - 4) 854, 853, 851
853 N11 = 2
      N12 = 13
      GO TO 865
854 IF (NNN - 3) 856, 855, 853
855 N11 = 3
      N12 = 10
      GO TO 865
856 IF (NNN - 2) 858, 857, 855
857 N11 = 4
      N12 = 6
      GO TO 865
858 N11 = 5
      N12 = 1
      GO TO 865
859 IF (NNN - 10) 861, 861, 860
860 N11 = 2
      N12 = 2*(NNN - 10) + 11
      GO TO 865
861 IF (NNN - 3) 863, 862, 862
862 N11 = 1
      N12 = NNN + 2
      GO TO 865
863 N11 = 2
      N12 = 2*NNN - 1
C INTERCHANGE AREA CALCULATIONS--FLAME SIMULATOR TO ZONE IN FUEL BED
C (865 TO 871)
865 DO 900 I = 1, 4
      JJ = (I-1)*15 + JJJ
      PJ11 = J11
      IF (NNN - 1) 870, 870, 871
870 PI = I
      AH(JJ) = H*PJ11
      Y = PNL + (PI - .5)/ASPR
      RC = SQRTF(Y**2 + X**2)
      COSQ = COSA**2*(Y/RC)**2+SINA**2*(X/RC)**2-2.*COSA*SINA*X*Y/RC**2
      SPR = PKA*SQRTF(1./((AOC**2 + (1. - AOC**2)*COSQ))
      TSPR = PKB/PHAL*(PI - .5)/ASPR*RC/Y + SPR
      BETA = .571*(1./(.936*TSPR**0.685 + 1.)) + 1.
      SGOL(JJ) = .6366*PJ11*PKB*EXP(-BETA*TSPR)/RC
C INTERCHANGE AREA CALCULATION--FUEL BED ZONE TO FUEL BED ZONE
C (871 THROUGH 900)
871 DO 900 M = 1, 4
      NN = (M - 1)*15 + NNN

```

```

      IM = XABSF(I - M) + 1
      GG(JJ,NN) = 0.
      DO 873 JD = 1, J11
      DO 873 ND = 1, N11
      JM = XABSF(N12 + ND - J12 - JD) + 1
873  GG(JJ,NN) = GG(JJ,NN) + G(IM,JM)
      IF (JJ - NN) 900, 874, 900
874  GG(JJ,JJ) = GG(JJ,JJ) - 4.*PKB*PJ11
900  CONTINUE
C  ZONE NUMBERS FOR STATIONARY GRID AS A FUNCTION OF VARIABLE GRID
C  ZONE NUMBERS (449 THROUGH 457)
449  DO 450 K = 1, N3
450  NNK(K) = K
      KI = N3 + 1
      KF = N3 + N4
      DO 451 K = KI, KF
451  NNK(K) = 15 + K - N3
      KI = N3 + N4 + 1
      KF = N3 + N4 + N5
      DO 452 K = KI, KF
452  NNK(K) = 30 + K - N3 - N4
      KI = N3 + N4 + N5 + 1
      KF = N2
      DO 453 K = KI, KF
453  NNK(K) = 45 + K - N3 - N4 - N5
      KI = N2 + 1
      KF = N2 + N7 - N3
      DO 454 K = KI, KF
454  NNK(K) = N3 + K - N2
      KI = N2 + N7 - N3 + 1
      KF = N2 + N7 + N8 - N3 - N4
      DO 455 K = KI, KF
455  NNK(K) = 15 + N4 + K - N2 - N7 + N3
      KI = N2 + N7 + N8 - N3 - N4 + 1
      KF = N2 + N7 + N8 + N9 - N3 - N4 - N5
      DO 456 K = KI, KF
456  NNK(K) = 30 + K - N6 - N7 - N8
      KI = N6 + N7 + N8 + N9 + 1
      KF = N7 + N8 + N9 + N10
      DO 457 K = KI, KF
457  NNK(K) = 45 + K - N7 - N8 - N9
C  ZONE NUMBERS FOR VARIABLE GRID AS A FUNCTION OF ZONE NUMBERS FOR
C  STATIONARY GRID (458 THROUGH 459)
458  DO 459 L = 1, 60
      KOK = NNK(K)
459  KNN(KOK) = K
C  CALCULATION OF C AND D MATRICES AND R VECTOR (460 THROUGH 481)
460  DO 462 K = 1, KF
      DO 462 L = 1, KF
      KOK = NNK(K)
      LOL = NNK(L)
      D(K,L) = 0.
462  C(K,L) = GG(KOK,LOL)

```

```

463 KI = N2 + 1
    KF = N7 + N8 + N9 + N10
464 DO 466 K = 1, N2
    R(K) = 0.
    DO 465 L = KI, KF
465 R(K) = R(K) + C(K,L)
    KOK = NNK(K)
466 R(K) = -R(K)*.1713*(TB/100.):**4 - SGOL(KOK)*QW 4.*PKB*AH(KOK)*TA
    1- RHAL(KOK)
467 DNEW = 0.
    DO 468 L = 1, N2
468 DNEW = DNEW + C(N3,L)*E(L)
    DKL = (DNEW-R(N3)-4.*PKB*H*TIG+4.*PKB*HC1*(TB-TIG)+C(N3,N3)
    1*(.1713*(TIG/100.):**4-E(N3)))/(T(N3)-T(N3-1))*T(N3)/TIG
    DO 480 L = 1, N2
    LOL = NNK(L)
    IF (L - 1) 472, 472, 469
469 IF (L - N3 - 1) 477, 473, 470
470 IF (L - N3 - N4 - 1) 477, 474, 471
471 IF (L - N3 - N4 - N5 - 1) 477, 475, 477
472 R(L) = R(L) - DKL*TO1
    GO TO 476
473 R(L) = R(L) - DKL*TO2
    GO TO 476
474 R(L) = R(L) - DKL*TO3
    GO TO 476
475 R(L) = R(L) - DKL*TO4
476 D(L,L) = -DKL - 4.*PKB*AH(LOL)
    GO TO 480
477 D(L,L) = -DKL - 4.*PKB*AH(LOL)
    D(L,L-1) = DKL
480 CONTINUE
    R(N3) = R(N3) - 4.*PKB*HC1*(TB - TIG)
    KOM = N3 + N4
    R(KOM) = R(KOM) - 4.*PKB*HC2*(TB - TIG)
    KOM = N3 + N4 + N5
    R(KOM) = R(KOM) - 4.*PKB*HC3*(TB - TIG)
481 R(N2) = R(N2) - 4.*PKB*HC4*(TB - TIG)
C SOLUTION OF SIMULTANEOUS EQUATIONS--NEWTON-RAPHSON ITERATION
C (1 THROUGH 35)
    IC = 0
    1 IC = IC + 1
    DO 2 K = 1, N2
    DO 2 L = 1, N2
    2 A(K,L) = (C(K,L) + 0.25*T(L)/E(L)*D(K,L))*SCALE
    DO 4 K = 1, N2
    P(K) = 0.
    DO 3 L = 1, N2
    3 P(K) = P(K) + C(K,L)*E(L) + D(K,L)*T(L)
    4 P(K) = R(K) - P(K)
    PNOR = 0.
    DO 5 K = 1, N2
    PKAY = ABSF(P(K))

```

```
5 PNOR = MAX1F(PNOR,PKAY)
6 DO 6 K = 1, N2
  P(K) = P(K)*SCALE/PNOR
  LL = XSIMEQF(60, N2, 1, A, P, DET, BOL)
  GO TO (9, 7, 8), LL
7 PRINT 100
  GO TO 300
8 PRINT 101
  GO TO 300
9 DO 10 L = 1, N2
  E1(L) = E(L) + A(L,1)*PNOR
C EARLY PRINT OUT OPTION FOR DEBUGGING (11 TO 12)
  IF (IC - MC) 11, 11, 12
11 PRINT 103
  PRINT 120, (E(L),E1(L), L = 1, N2)
  PRINT 104
  PRINT 120, (T(K), R(K), SGOL(K), A(K,1), K = 1, N2)
  PRINT 105
  PRINT 120, ((C(K,L), D(K,L), L = 1, N2), K = 1, N2)
  PRINT 121, DKL, PNOR, N3, N4, N5, N6
C IGNITION ZONE REPOSITIONING (12 TO 30)
12 IC1 = 0
  IC2 = 0
  IC3 = 0
  LI = N3 + 1
  LF = N3 + N4
  DO 14 L = LI, LF
    IF (E1(L) - 50.) 13, 14, 14
13 IC1 = IC1 + 1
14 CONTINUE
  LI2 = N3 + N4 + 1
  LF2 = N3 + N4 + N5
  DO 16 L = LI2, LF2
    IF (E1(L) - 50.) 15, 16, 16
15 IC2 = IC2 + 1
16 CONTINUE
  LI3 = N3 + N4 + N5 + 1
  LF3 = N2
  DO 18 L = LI3, LF3
    IF (E1(L) - 50.) 17, 18, 18
17 IC3 = IC3 + 1
18 CONTINUE
  IF (IC1) 19, 19, 21
19 IF (IC2) 20, 20, 21
20 IF (IC3) 30, 30, 21
21 DO 22 L = 1, 60
22 T1(L) = TO
23 DO 24 L = LI3, LF3
  LLL = L + IC1 + IC2 + IC3
24 T1(LLI) = T(L)
  DO 25 L = LI2, LF2
  LLL = L + IC1 + IC2
25 T1(LLI) = T(L)
```

```
DO 26 L = LI, LF
  LLL = L + IC1
26 T1(LLL) = T(L)
DO 27 L = 1, N3
27 T1(L) = T(L)
  N4 = N4 + IC1
  N5 = N5 + IC2
  N6 = N6 + IC3
  N2 = N3 + N4 + N5 + N6
DO 28 L = 1, N2
  T(L) = T1(L)
28 E(L) = .1713*(T(L)/100.)**4
  GO TO 449
C TEST FOR NEWTON-RAPHSON CONVERGENCE (30 TO 35)
30 PMAX = 0.
  DO 31 L = 1, N2
  DEL = ABSF(E1(L) - E(L))/E(L)
31 PMAX = MAX1F(PMAX, DEL)
  DO 33 L = 1, N3
  IF (E1(L)) 32, 32, 33
32 E1(L) = 50.
33 CONTINUE
  DO 34 L = 1, N2
  T(L) = (E1(L)/(.1713*.00000001))**.25
34 E(L) = E1(L)
35 IF (PMAX - TEST1) 36, 36, 1
C TEST TO DETERMINE IF ASSUMED IGNITION ZONES ARE NEAR TIG (36 TO 70)
36 IF (ABSF(T(N3)-TIG) - TEST2) 37, 37, 463
37 PRINT 128
  IF (PHAL - 0.75) 38, 999, 39
38 PRINT 125, DKL, PNOR, N3, N4, N5, N6
  GO TO 40
39 PRINT 121, DKL, PNOR, N3, N4, N5, N6
40 IF (T(N3) - TIG) 41, 44, 42
41 RATIO = TIG/T(N3) - 1.
  RATIO = (1. + RATIO)/(1. + COR*RATIO)
  GO TO 43
42 RATIO = T(N3)/TIG - 1.
  RATIO = (1. + COR*RATIO)/(1. + RATIO)
43 DKLC = RATIO*DKL*T(N3)/TIG
  TDIFF = T(N3) - T(LF)
  PRINT 133, DKLC, TDIFF
  PRINT 124, N2
44 PRINT 123, (T(L), L = 1, N3)
  PRINT 123, (T(L), L = LI, LF)
  PRINT 123, (T(L), L = LI2, LF2)
  PRINT 123, (T(L), L = LI3, N2)
  IF (ABSF(T(LF)-TIG) - TEST3) 45, 45, 50
45 IC1 = 0
46 IF (ABSF(T(LF2)-TIG) - TEST3) 47, 47, 54
47 IC2 = 0
48 IF (ABSF(T(N2)-TIG) - TEST3) 60, 60, 57
50 IF (T(LF) - TIG) 51, 54, 52
```



```
51 IC1 = 1
   GO TO 46
52 IC1 = -1
   GO TO 46
54 IF (T(LF2) - TIG) 55, 57, 56
55 IC2 = 1
   GO TO 48
56 IC2 = -1
   GO TO 48
57 IF (T(N2) - TIG) 58, 999, 59
58 IC3 = 1
   GO TO 61
59 IC3 = -1
   GO TO 61
60 IC3 = 0
61 IF (IC1) 70, 62, 70
62 IF (IC2) 70, 63, 70
63 IF (IC3) 70, 300, 70
70 LI = N3 + 1
   LF = N3 + N4
   LI2 = N3 + N4 + 1
   LF2 = N3 + N4 + N5
   LI3 = N3 + N4 + N5 + 1
71 LF3 = N2
201 DEV = (T(N3)-TIG)**2 + (T(LF)-TIG)**2 + (T(LF2) TIG)**2
      1+ (T(N2)-TIG)**2
   IF (DEV - DEVO) 202, 202, 206
202 PRINT 134
   IF (PHAL - 0.75) 206, 999, 203
203 DEVO = DEV
   DO 204 I = 1, N2
204 TIGN(I) = T(I)
   N3OPT = N3
   N4OPT = N4
   N5OPT = N5
   N6OPT = N6
   LFOPT = N3 + N4
   LF2OPT = N3 + N4 + N5
   N2OPT = N2
   IC1PT = IC1
   IC2PT = IC2
   IC3PT = IC3
206 IF (N6 + IC3 - N10) 208, 208, 300
C TEST TO DETERMINE IF SOLUTION IS OSCILLATING ABOUT ANSWER
C (208 TO 216)
208 NTES = 400*(N6+IC3) + 20*(N5+IC2) + N4 + IC1
   IOSC = 0
   DO 212 ITT = 1, IT
   IF (NT(ITT) - NTES) 212, 210, 212
210 IOSC = IOSC + 1
212 CONTINUE
   IF (IOSC) 216, 214, 216
214 IT = IT + 1
```

```
      NT(IT) = NTES
      GO TO 21
C  ZONE NUMBER RE-ASSIGNMENT--FULL-ZONE TO HALF-ZONE GRID
C  (216 THROUGH 264)
216 IF (PHAL - 0.75) 600, 999, 218
218 N3 = N3OPT - NH3 - 3
      IF (N4OPT - 10) 222, 220, 220
220 N4 = N4OPT + IC1PT
      GO TO 228
222 IF (N4OPT - 6) 224, 226, 226
224 N4 = N4OPT - 4 + IC1PT
      GO TO 228
226 N4 = 2*(N4OPT - 6) + 2 + IC1PT
228 IF (N5OPT - 10) 232, 232, 230
230 N5 = N5OPT + IC2PT
      GO TO 238
232 IF (N5OPT - 6) 234, 234, 236
234 N5 = N5OPT - 4 + IC2PT
      GO TO 238
236 N5 = 2*(N5OPT - 6) + 2 + IC2PT
238 IF (N6OPT - 10) 242, 242, 240
240 N6 = N6OPT + IC3PT
      GO TO 248
242 IF (N6OPT - 6) 244, 244, 246
244 N6 = N6OPT - 4 + IC3PT
      GO TO 248
246 N6 = 2*(N6OPT - 6) + 2 + IC3PT
248 LF = N3 + N4
      LF2 = LF + N5
      N2 = LF2 + N6
      IF (N7 - 10) 256, 250, 250
250 IF (N8 - 10) 258, 252, 252
252 IF (N9 - 10) 260, 254, 254
254 IF (N10 - 10) 262, 264, 264
256 N7 = 2*N7 - 10
      GO TO 250
258 N8 = 2*N8 - 10
      GO TO 252
260 N9 = 2*N9 - 10
      GO TO 254
262 N10 = 2*N10 - 10
264 CONTINUE
C  SHIFT FROM FULL-ZONE TO HALF-ZONE GRID (265 THROUGH 298)
265 PHAL = 0.5
      PKB = PKB*0.5
      T01 = TIGN(4)
      DO 266 N = 1, 4
266 TH(1,N) = TIGN(4)
      DO 268 N = 5, 10
268 TH(1,N) = TIGN(3)
      T(1) = TIGN(5)
      T(2) = TIGN(6)
      MMI = 3
```

```
MMF = N3
IAVE = 2
DO 270 I = MMI, MMF, 2
IAVE = IAVE + 1
JK = IAVE + 3
JL = JK + 1
T(I) = 0.5*(TIGN(JK) + TIGN(JL))
270 T(I+1) = TIGN(JL)
T(N3) = TIGN(N3OPT)
TO2 = TIGN(N3OPT+4)
LIL = N3OPT + 4
DO 272 N = 1, 4
272 TH(2,N) = TIGN(LIL)
LIL = N3OPT + 3
DO 274 N = 5, 10
274 TH(2,N) = TIGN(LIL)
T(N3+1) = TIGN(N3OPT+5)
T(N3+2) = TIGN(N3OPT+6)
MMI = N3 + 3
MMF = LF
IAVE = MMI - 1
DO 275 I = MMI, MMF, 2
IAVE = IAVE + 1
JK = N3OPT - N3 + 3 + IAVE
JL = JK + 1
T(I) = 0.5*(TIGN(JK) + TIGN(JL))
275 T(I+1) = TIGN(JL)
IF (N4 - 10) 278, 278, 276
276 DO 277 I = 11, N4
NML = I + N3
NMLK = I + N3OPT
277 T(NML) = TIGN(NMLK)
278 T(LF-1) = 0.5*(TIGN(LFOPT) + TIGN(LFOPT-1))
T(LF) = TIGN(LFOPT)
TO3 = TIGN(LFOPT+4)
LIL = LFOPT + 4
DO 279 N = 1, 4
279 TH(3,N) = TIGN(LIL)
LIL = LFOPT + 3
DO 280 N = 5, 10
280 TH(3,N) = TIGN(LIL)
T(LF+1) = TIGN(LFOPT+5)
T(LF+2) = TIGN(LFOPT+6)
MMI = LF + 3
MMF = LF2
IAVE = MMI - 1
DO 282 I = MMI, MMF, 2
IAVE = IAVE + 1
JK = LFOPT - LF + 3 + IAVE
JL = JK + 1
T(I) = 0.5*(TIGN(JK) + TIGN(JL))
282 T(I+1) = TIGN(JL)
IF (N5 - 10) 285, 285, 283
```

```

283 DO 284 I = 11, N5
    NML = I + LF
    NMLK = I + LFOPT
284 T(NML) = TIGN(NMLK)
285 T(LF2-1) = 0.5*(TIGN(LF2OPT) + TIGN(LF2OPT-1))
    T(LF2) = TIGN(LF2OPT)
    T04 = TIGN(LF2OPT+4)
    LIL = LF2OPT + 4
    DO 286 N = 1, 4
286 TH(4,N) = TIGN(LIL)
    LIL = LF2OPT + 3
    DO 287 N = 5, 10
287 TH(4,N) = TIGN(LIL)
    T(LF2+1) = TIGN(LF2OPT+5)
    T(LF2+2) = TIGN(LF2OPT+6)
    MMI = LF2 + 3
    MMF = N2
    IAVE = MMI - 1
    DO 288 I = MMI, MMF, 2
    IAVE = IAVE + 1
    JK = LF2OPT - LF2 + 3 + IAVE
    JL = JK + 1
    T(I) = 0.5*(TIGN(JK) + TIGN(JL))
288 T(I+1) = TIGN(JL)
    IF (N6 - 10) 291, 291, 289
289 DO 290 I = 11, N6
    NML = I + LF2
    NMLK = I + LF2OPT
290 T(NML) = TIGN(NMLK)
291 T(N2-1) = 0.5*(TIGN(N2OPT) + TIGN(N2OPT-1))
    T(N2) = TIGN(N2OPT)
C CALCULATION OF RHAL(L) FOR HALF-ZONE GRID (292 THROUGH 297)
292 DO 294 I = 1, 4
    DO 294 J = 5, 6
    JJ = (I - 1)*15 + J
    JJJJ = NNF(JJ)
    DO 293 M = 1, 4
    DO 293 N = 1, 4
    NN = (M - 1)*15 + N
    LOL = KNN(NN)
293 RHAL(JJJJ) = RHAL(JJJJ) + GG(JJ,NN)*0.1713*(TIGN(LOL)/10.0)**4
294 CONTINUE
    DO 297 I = 1, 4
    DO 297 J = 1, 2
    JJ = (I - 1)*15 + 5 - J
    DO 296 M = 1, 4
    IM = XABSF(I - M) + 1
    DO 296 N = 1, 10
    JM = XABSF(J - N) + 7
296 RHAL(JJ) = RHAL(JJ) + GH(IM,JM)*0.1713*(TH(M,N)/100.0)**4
297 CONTINUE
    DO 298 IM = 1, 4
    DO 298 JM = 1, 25

```

```

298 G(IM,JM) = GH(IM,JM)
    GO TO 745
C SHIFT FROM HALF-ZONE GRID TO FULL-ZONE GRID (600 THROUGH 618)
600 DO 602 I = 1, N2OPT
602 T(I) = TIGN(I)
    N3 = N3OPT
    N4 = N4OPT
    N5 = N5OPT
    N6 = N6OPT
    N2 = N2OPT
    IF (N7 - 10) 610, 604, 604
604 IF (N8 - 10) 612, 606, 606
606 IF (N9 - 10) 614, 608, 608
608 IF (N10 - 10) 616, 618, 618
610 N7 = 5 + N7/2
    GO TO 604
612 N8 = 5 + N8/2
    GO TO 606
614 N9 = 5 + N9/2
    GO TO 608
616 N10 = 5 + N10/2
618 CONTINUE
    PKB = PKB/PHAL
    GO TO 301
300 IF (PHAL - 0.75) 600, 999, 301
C VARIATION OF PLUME RADIATION PARAMETERS
301 J99 = J99 - 1
    IF (J99) 304, 304, 302
302 READ 106, PNL, PNW, COSA, SINA, PKA, PKB, AOC, ASPR, QW
    GO TO 740
C VARIATION OF DEPTH OF EMBER ZONE
304 JEMB = JEMB - 1
    IF (JEMB) 308, 308, 306
306 READ 115, N7, N8, N9, N10, NH3
    GO TO 740
C VARIATION OF OPTICAL DEPTH OF FUEL BED ZONES
308 JPKB = JPKB - 1
    IF (JPKB) 312, 312, 310
310 READ 109, ((GF(IM,JM), IM = 1, 4), JM = 1, 10)
    READ 109, ((GH(IM,JM), IM = 1, 4), JM = 1, 10)
    GO TO 740
C VARIATION OF TEMPERATURE PARAMETERS
312 JTEM = JTEM - 1
    IF (JTEM) 316, 316, 314
314 READ 107, TB, TIG, TA, TO, H
    GO TO 740
C NEW CASE
316 JALL = JALL - 1
    IF (JALL) 999, 999, 150
999 CALL EXIT
END

```

002

0689

TOTAL 0689*

H Summary of Experimental Data

Table H-1

EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE OF QUIESCENT-AMBIENT SHREDDED NEWSPRINT FIRES

Data in Figure 4.9 *		Data in Figure 4.10 **	
$q' / 4$ A Btu/hr-ft	V ft/hr	$q' / 4$ A Btu/hr-ft	V ft/hr
0	16.6	0	8.5
14	18.1	38	10.9
24	19.0	73	17.4
36	21.2	96	21.0
55	22.5	152	24.2
65	24.8	202	30.0
82	26.5	284	36.0
104	29.1		
126	31.7		
154	34.6		
175	36.6		
220	42.2		
279	48.1		
339	50.5		
405	56.0		
473	66.2		

* 29-31% relative humidity, Heating Wire Array No. I

** 69-71% relative humidity, Heating Wire Array No. III

Table H-2

EFFECT OF HEATING WIRE RADIATION ON PROPAGATION RATE
OF WINDBLOWN SHREDDED NEWSPRINT FIRES

Data in Figure 4.12 *		Data in Figure 4.13 **		Data in Figure 4.14 ***		Data in Figure 4.15 ****	
$q_A^{1/4}$	V	$q_A^{1/4}$	V	$q_A^{1/4}$	V	$q_A^{1/4}$	V
Btu/hr-ft	ft/hr	Btu/hr-ft	ft/hr	Btu/hr-ft	ft/hr	Btu/hr-ft	ft/hr
0	12.3	438	46.8	0	7.7	0	26.1
0	13.0	520	55.7	8	7.9	42	28.8
12	13.3	695	65.6	33	9.9	77	37.3
25	13.2	758	74.7	54	11.2	120	40.8
39	16.2	811	83.5	75	14.8	174	51.3
61	20.3			148	19.2		
87	22.1			241	28.1		
115	23.7						
158	26.3			0	8.2		
200	29.7			42	10.6		
204	29.3			74	13.2		
256	33.3			122	15.2		
258	33.9			305	25.1		
332	39.0						
390	42.8			130	13.6		
449	47.8			266	22.5		
527	50.1			377	27.3		
618	58.7						
696	63.5						

* $27 \pm 2\%$ relative humidity, 1.6 ft/sec headwind,
Heating Wire Array No. I

** $49 \pm 2\%$ relative humidity, 1.6 ft/sec headwind,
Heating Wire Array No. II

*** $40 - 43\%$ relative humidity, 1.6 ft/sec headwind,
Heating Wire Arrays No. IV, VI, V

**** $49 \pm 2\%$ relative humidity, 0.45 ft/sec tailwind,
Heating Wire Array No. IV

Table H-3

RADIATIVE FLUX DENSITY MEASUREMENTS

Data in Figure 4.1		Data in Figure 4.4		Data in Figure 4.5	
y	q''_s	$\frac{y}{L_w}$	$\frac{2\pi L_w q''_A}{q''_A}$	$\frac{y}{L_w}$	$\frac{2\pi L_w q''_A}{q''_A}$
in.	Btu/hr-ft ²				
-2.00	75	-3.23	0.088	-3.60	0.112
-1.75	95	-2.90	0.112	-3.36	0.129
-1.50	112	-2.56	0.173	-3.09	0.152
-1.25	143	-2.23	0.208	-2.84	0.178
-0.85	228	-1.90	0.262	-2.58	0.210
-0.45	285	-1.57	0.337	-2.45	0.230
-0.25	340	-1.23	0.535	-2.32	0.255
		-0.90	0.694	-2.19	0.279
		-0.57	0.848	-2.06	0.315
		-0.40	0.878	-1.94	0.354
				-1.81	0.396
				-1.68	0.456
				-1.55	0.510
				-1.42	0.574
				-1.29	0.629
				-1.16	0.672
				-1.10	0.689
				-1.03	0.692

Table H-4

RADIOMETER CALIBRATION DATA IN FIGURE D-3*

q''_s Btu/hr-ft ²	E_1 millivolts	$\frac{q''_s}{E_1}$ $\frac{\text{Btu/hr-ft}^2}{\text{millivolt}}$
39.5	0.020	1975
57.0	0.030	1900
69.5	0.037	1880
88.5	0.047	1880
114	0.062	1840
132	0.070	1885
154	0.080	1923
182	0.0975	1868
221	0.116	1905
246	0.131	1879
271	0.143	1893
304	0.168	1810
335	0.180	1860
384	0.205	1872
435	0.230	1890
499	0.252	1978
577	0.300	1925
672	0.345	1945

* $T_{L1} = T_A$

Other quantitative experimental data are presented in tabular form in Tables 4.1, 4.2, 4.3, and D-1.

I Location of Original Data

All of the original data, including that obtained as computer output in the mathematical modeling studies, are in the possession of the author.

J Nomenclature

A	- defined in Equation 2.15
$A_{K,L}$	- defined in Equation 5.8
a	- major axis of cross section in zoned model's plume simulator
B_1	- zone height
B_2	- zone length
C_g	- specific heat of flame plume gases
$C_{K,L}$	- defined in Equation 5.5
C_P	- specific heat of fuel
c	- minor axis of cross section in zoned model's plume simulator
D	- eddy diffusivity in plume convection
$D_{K,L}$	- defined in Equation 5.5
E_i	- emf signal generated by sensing strip no. i in radiometer
E_L	- emissive power of zone L in fuel bed
$\overline{g_i g_j} / L$	- two-dimensional radiation interchange factor between zone i and zone j
H_i	- convective loss coefficient at upper surface of sensing strip no. i in radiometer
H_K	- defined in Equation 5.7
h	- radiation-convection loss coefficient at upper surface of fuel bed in slab model
h^*	- $h + \varnothing L$
h_{c_i}	- flame convection coefficient in zone i
h_i	- ambient convection coefficient in zone i
K	- $k / (\rho C_P)$
K_j	- half-zone position variable, defined in Figure 5.5

- k - thermal conductivity of fuel
- k_b - fuel-bed radiation absorption coefficient
- k_c - dimensionless coefficient in flame plume convection
- k_g - radiation absorption coefficient of gas in zoned model's
flame plume simulator
- k_r - thermal conductivity of sensing strips in radiometer
- L - slab thickness
- L_b - depth of fuel bed, numerically equal to $4 B_1$
- L_c - length of ember combustion region
- L_F - flame height in slab model
- L_f - flame height in zoned model
- L_i - length of sensing strip no. i in radiometer
- L_s - height of flame plume simulator above upper surface of
fuel bed
- L_w - height of lowest wire in heating wire array above upper
surface of fuel bed
- L_{w_i} - height of wire i in heating wire array above upper surface
of fuel bed
- M - number of wires in heating wire array
- $M_1 B_2$ - length of zone in fuel bed, defined in Figure 5.1
- $\overline{MN}$ - defined in Figure 4.2
- N_j - full-zone position variable, defined in Section 5.3
- P - Laplace time variable
- Q_i - mass transfer-chemical reaction heat sink in zone i ,
defined in Equation 5.1

- Q_i'' - ignition energy per unit area of fuel bed, defined in Section 4.8
- Q_w/L - rate of radiative emission, per unit length, of zoned model's plume simulator
- q_A' - wire radiation flux
- q_c' - flame-convection preheat flux
- q_c'' - flame-convection preheat flux density
- q_F' - flame-radiation preheat flux in slab model
- $\bar{q}_i$ - Laplace transform of $(q_L'' + q_s'' + q_c'')$
- q_L'' - convective-radiative flux density from upper surface of fuel bed in slab model
- q_s' - $q_A' + 4q_F'$
- q_s'' - radiative preheat flux density at upper surface of fuel slab
- q_{sp}'' - net flux density at surface of fuel particle
- R - radius of cylindrical fuel particle
- R_c - defined in Equation 5.2
- R_K - defined in Equation 5.5
- r - defined in Equation 2.7
- S' - defined in Section 5.9
- S_F - surface-area-to-volume ratio of a fuel particle
- S_o - surface area of fuel per unit volume of fuel bed
- $\overline{s_i s_j}/L$ - defined in Figure 4.2
- T - fuel temperature in slab model
- T_A - ambient temperature
- T_{A_i} - ambient temperature in zone i

T_B	- ember combustion temperature
T_c	- heat sink temperature in radiometer
T_F	- flame temperature
T_i	- surface temperature of fuel in zone i
$\overline{T}_i$	- enthalpy mean temperature of fuel in zone i
T_{ig}	- ignition temperature
T_{Li}	- temperature at free end of sensing strip no. i in radiometer
T_o	- fuel temperature far ahead of ignition front
t_i	- thickness of radiometer sensing strip no. i
V	- fire propagation velocity
V_c^*	- defined in Equation 2.17
V_s^*	- defined in Equation 2.17
V_l	- $V \rho C_p / k$
v	- $(T - T_o)$
$\overline{v}$	- Laplace transform of $(T - T_o)$
v_A	- $(T_A - T_o)$
$\overline{Wg}_i / \delta L$	- radiation interchange factor from plume simulator to zone i in fuel bed
x	- vertical position variable in slab model
x_s	- defined in Figure 5.4
y	- horizontal position variable in slab model
y_m	- horizontal distance ahead of the flame front at which T is arbitrarily close to T_o
z	- $(y - y_m)$
ℓ	- scale of turbulence in flame convection

α	- fuel bed absorptivity in slab model
α_s	- plume simulator angle, defined in Figure 5.4
β_m	- m th root of transcendental equation $\beta \tan \beta L = h/k$
β_r	- mean beam length ratio, defined in Section 5.9
γ	- chemical reaction-mass transfer heat sink coefficient
ϵ	- horizontal displacement of flame plume relative to ignition front
ϵ_i	- horizontal position of heating wire i relative to ignition front
θ	- time
ρ	- fuel density
ρ_b	- density of fuel bed in zoned model
ρ_F	- density of single fuel particle
ρ_g	- density of gases in flame plume
$(\rho C_p)_r$	- volumetric heat capacity of sensing strips in radiometer
σ	- Stephan-Boltzmann constant
$\bar{\Phi}_c$	- heat retention factor for flame convection
$\bar{\Phi}_L$	- heat retention factor for ambient convection
$\bar{\Phi}_s$	- heat retention factor for overhead radiation
ϕ_c	- defined in Equation 2.11
ϕ_L	- defined in Equation 2.11
ϕ_s	- defined in Equation 2.11

K. Literature Citations

1. Anderson, H. E. and R. C. Rothermel, "Influence of Moisture and Wind Upon the Characteristics of Free-Burning Fires," Tenth Symposium (International) on Combustion, pp 1009-1019, The Combustion Institute, Pittsburgh (1965).
2. Ibid, p 1012.
3. Cohen, E. S., "Effect of Gas Temperature Gradients on Radiant Heat Transmission," pp 55-58, Sc. D. thesis, M.I.T., Cambridge, Mass. (1955).
4. Ibid, pp 22-85.
5. Curry, J. R. and W. L. Fons, "Forest Fire Behavior Studies," Mechanical Engineering, 62, 219 (1939).
6. Eckert, E. R. G., and R. M. Drake, "Heat and Mass Transfer," 2nd ed., p. 298 McGraw-Hill, New York (1959).
7. Einstein, T. H., "Radiant Heat Transfer to Absorbing Gases Enclosed Between Parallel Flat Plates with Flow and Conduction," Technical Report R-154, National Aeronautics and Space Administration (1963).
8. Emmons, H. W., "Fundamental Problems of the Free Burning Fire," Tenth Symposium (International) on Combustion, pp 951-964, The Combustion Institute, Pittsburgh (1965).
9. Fons, W. L., H. D. Bruce, W. Y. Pong, and S. S. Richards, "Project Fire Model," Summary Progress Report, Forest Service - U. S. Department of Agriculture (1960).

10. Fons, W. L., H. B. Clements, E. R. Elliott, and P. M. George, "Project Fire Model," Summary Progress Report II, Forest Service - U. S. Department of Agriculture (1962).
11. Gardon, R., "An Instrument for the Direct Measurement of Intense Thermal Radiation," The Review of Scientific Instruments, Vol. 24, No. 5, pp 366-370 (1953).
12. Hottel, H. C., "Radiant Heat Transmission," in "Heat Transmission," W. H. McAdams, 3rd ed., p 66, McGraw-Hill, New York (1954).
13. Hottel, H. C., and E. S. Cohen, "Radiant Heat Exchange in a Gas-Filled Enclosure," Am. Soc. Mech. Engrs. Paper No. 54-HT -- 23 (1957).
14. Hottel, H. C., G. C. Williams, and F. R. Steward, "The Modeling of Firespread Through a Fuel Bed," Tenth Symposium (International) on Combustion, pp 997-1007, The Combustion Institute, Pittsburgh (1965).
15. McGuire, J. H. and H. Wraight, "Radiometer for Field Use," Journal of Scientific Instruments, 37, 128-130 (1960).
16. deRochechouart, C. L., "Radiation from a Line Fire," S. M. Thesis, M.I.T., Cambridge, Mass. (1961).
17. Steward, F. R., "Flame Spread Through Solid Fuel," Sc.D. thesis, M.I.T., Cambridge, Mass. (1962).
18. Ibid., pp 15-29.
19. Ibid., p 16.

20. Ibid., p 21.
21. Ibid., pp 30-33.
22. "A Study of Fire Problems," Publication 949, National Academy of Sciences-National Research Council (1961).

I Biographical Note

The author was born in St. Louis, Missouri, on January 10, 1937. He received his secondary education in Kirkwood, Missouri, and graduated from the local high school in June 1955. In the Fall of 1956, he entered the School of Engineering at Washington University and graduated four years later with a B. S. degree in chemical engineering. In September 1960, he entered M.I.T. as a graduate student in the Department of Chemical Engineering.

The author was an N.S.F. Fellow from September 1960 to September 1963 and was subsequently a Research Assistant and then a Teaching Assistant. He is currently employed as a research engineer in Monsanto's Organic Chemicals Division in St. Louis.