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Abstract

A set of constraints is derived for the operating characteristics of tokamak power reactors which are bulk heated by electron cyclotron resonance heating (ECRH). Four heating modes are considered: ordinary wave heating at the electron cyclotron frequency, Ω, and at the second harmonic frequency, 2Ω, and extraordinary wave heating at Ω and at 2Ω. For ordinary wave heating at Ω, which appears to be the most promising method, the wave frequency $\omega \approx \Omega$ must exceed the plasma frequency, ω_p, for wave penetration into the plasma. Our main conclusion is that the need for high density operation ($n_o > 4 \times 10^{20} m^{-3}$) in moderate size tokamak reactors, coupled with the wave accessibility condition $\Omega > \omega_p$, leads to the requirement of frequencies in the 200 GHz range for ECRH of reactor plasmas. A further condition on the heating frequency may be derived by consideration of the ignition condition using empirical scaling laws for the energy confinement time. This latter condition does not increase the heating frequency requirement unless impurities are present or the energy confinement degrades with increasing temperature. We also find that for ordinary wave heating at Ω, the average plasma β is limited to less than 0.039 for a central temperature below 15 keV, assuming parabolic density and temperature profiles. The use of extraordinary heating at Ω might lower the frequency requirement for ECRH of a reactor. However, it appears to be unattractive for reactor operation because, in order for the wave to penetrate to the center of the plasma, the heating ports must be located on the inside of the torus. High beta, lower field reactors can be heated from the outboard side of the torus with second harmonic radiation. However, these devices will have to be heated at frequencies which are as high or higher than those needed for devices which are heated with the ordinary wave at the fundamental frequency. Extrapolation of cyclotron resonance maser (gyrotron) technology to provide modular heating systems in the 200 GHz range is discussed.

1. Introduction

Considerable interest has developed in the use of electron cyclotron resonance heating (ECRH) for heating tokamak plasmas. The support for this technique is largely based on the results of ECRH experiments on TM-3[1] and on the development of efficient cyclotron resonance masers (gyrotrons) in the USSR.[2,3]

IF ECRH can be extrapolated to reactor grade plasmas it will present an attractive alternative to neutral beam injection and low frequency RF heating (e.g. lower hybrid, ICRH). The advantages of the technique include good penetration of the energy to the plasma center and capability for pressure and current profile control through localized energy deposition near the resonant surfaces. The fact that the absorption process is well understood in terms of straight-forward, linear theory lends a degree of confidence in the application of the technique. The validity of this theory is supported by the experimental observation that emission at the cyclotron frequency and second harminic is characteristic of an optically thick medium (black-body emission).[4] The suitability of simple waveguide antenna structures with small dimensions should facilitate the engineering of the reactor.

In this paper we determine the characteristics of ECRH tokamak reactors based on the requirements for wave propagation and absorption in the plasma. Heating at both the fundamental and the second harmonic is considered. Constraints on the toroidal beta and on the minimum magnetic field required to obtain suitable fusion power densities are derived. We also calculate the magnetic field and temperature necessary to achieve ignition based on the empirical scaling law for energy confinement time.[5] Frequency requirements are determined, and it is shown that frequencies of the order of 200 GHz are attractive for heating tokamak reactors of moderate size and thermal power levels. The extrapolation of high frequency gyrotron technology to develop appropriate heating sources is discussed.

2. Wave Propagation and Absorption

We will consider the requirements for propagation and absorption of electromagnetic radiation at the fundamental and second harmonic of the electron cyclotron frequency. The frequencies of interest are the cyclotron frequency

$$\Omega = \frac{eB}{m} \quad ;$$

the plasma frequency

$$\omega_p = \left(\frac{ne^2}{\varepsilon_o m}\right)^{1/2} \quad ;$$

the upper hybrid resonance frequency

$$\omega_{UH} = \left(\omega_p^2 + \Omega^2\right)^{1/2} \quad ;$$

and the right-hand and left-hand cut-off frequencies

$$\omega_\pm = \pm \left(\frac{\Omega}{2}\right) + \left[\left(\frac{\Omega}{2}\right)^2 + \omega_p^2\right]^{1/2} \quad .$$

We define the dimensionless parameter α as

$$\alpha \equiv \frac{\omega_{po}^2}{\Omega_o^2} = \frac{m}{\varepsilon_o} \frac{n_o}{B_o^2}$$

where ω_{po}, Ω_o , n_o and B_o are the values of these parameters evaluated on the plasma axis.

Four possible heating modes will be considered:

1) ordinary wave heating with frequency $\omega = \Omega_o$, where the wave is launched from the outside (low-field region);

2) extra-ordinary wave heating with $\omega = \Omega_o$, where the wave is launched from the inside of the torus (high field region);

3) extra-ordinary wave heating with $\omega = 2\Omega_o$, with the wave launched from the outside of the torus (low field region).

4) ordinary wave with $\omega = 2\,\Omega_o$.

The wave accessability for each case is illustrated in figure 1. Note that an extra-ordinary wave launched from the outside with a frequency corresponding to the central cyclotron frequency will always encounter the cut-off at ω_+ (right-hand cut-off) and be reflected before reaching the resonant layer. This mode is therefore unsuitable for central heating; the same restriction exists for heating at the upper hybrid resonance if the wave is to be launched toward the center from the low field side.

Absorption of the ordinary wave at the fundamental cyclotron resonance derives from the theory of propagation in a warm plasma with non-vanishing density and may be considered as a finite Larmor radius effect.[6,7] The wave damping increases with temperature and density and is at maximum for quasi-perpendicular propagation. The optical depth is approximately given by[6]

$$\Gamma_{10} \simeq \frac{\pi}{2}\,\frac{T_o}{mc^2}\,\alpha\,|k|R_o \qquad (1)$$

where the total absorption integrated along the wave trajectory is given by:

$$A_{10} = 1 - \exp(-\Gamma_{10})$$

and T_o is the central temperature, $|k|$ is the wave number at the resonance and R_o is the major radius. Numerical evaluation of (1) shows that any reasonable reactor grade plasma will be optically thick to the ordinary mode, insuring good absorption. The ordinary wave will propagate to the central resonance providing the plasma frequency does not exceed the wave frequency, i.e.

$$\alpha < 1 \tag{2}$$

Note that as $\alpha \to 1$ the wave number $k \to 0$ by definition of cut-off, and by (1) the optical depth $\Gamma \to 0$; hence operation exactly at the cut-off is not possible. For a given Ω, the value $\alpha \approx 2/3$ optimizes the optical depth when $\theta \approx \pi/2$; however, for reactor grade plasmas the numerical value of Γ_{10} is very large and optimization of this quantity is not required. Values of $\alpha > 2/3$ are therefore permissable; we note that near $\theta = \pi/2$ operation at $\alpha = 0.95$ reduces Γ_{10} by less than a factor or two below its optimum.

Propagation of the extra-ordinary wave to the resonant layer requires launching from the high field side as noted earlier. Then the wave propagates as long as the wave frequency everywhere exceeds that of the left-hand cut-off. For $\omega = \Omega_o$ this leads to the propagation condition

$$\alpha < 2 \tag{3}$$

The absorption of the extra-ordinary wave arises from resonant energy transfer to the gyrating electron from the right circularly polarized component of the wave. However, as the density is increased the extra-ordinary wave becomes predonimantly left-circularly polarized, leading to a decrease in absorption. The optical depth is approximately given by[6]

$$\Gamma_{1x} \simeq \frac{\pi}{2} \frac{T_o}{mc^2} \cos^2\theta \, \frac{1}{\alpha} \, [2 + \alpha(1 - \alpha)]^2 \, |k|R_o \tag{4}$$

where θ is the angle of propagation with respect to the magnetic field and (4) is valid for $(2T_o/mc^2)^{1/2} << \frac{\pi}{2} - \theta << 1$. Thus, depending on the angle θ, the resonant layer may become partially transparent to the extraordinary wave for values of α somewhat less than the cut-off value. In this case it is conceivable that the radiation would propagate through to the upper hybrid layer (see figure 1) where non-linear absorption may take place.

Finally we consider radiation at the second harmonic of the central cyclotron frequency. The propagation of the extraordinary mode is limited by the requirement $\omega > \omega_+$ everywhere. In terms of the parameter α this is equivalent to

$$\alpha < 2 \tag{5}$$

which is the same as equation (3). We here consider a wave launched from the low field side. Propagation from the high field side will have a smaller limiting α, since the maximum value of ω_+ will be located at $R < R_o$. The optical depth[8,9] has a simple form for values of α well below the cutoff and for $\theta = \pi/2$,

$$\Gamma_{2x} \simeq \frac{\pi}{2} \, \alpha \, \frac{T_o}{mc^2} \, |k|R_o \qquad \left(\theta = \frac{\pi}{2}\right) \tag{6}$$

The ordinary mode at the second harmonic will also be absorbed, with an optical depth smaller than the above by a factor of (T/mc^2) for perpendicular propagation[8]

$$\Gamma_{2o} \simeq \frac{\pi}{2} \, \alpha \left(\frac{T_o}{mc^2}\right)^2 |k|R_o \qquad \left(\theta = \frac{\pi}{2}\right) \tag{7}$$

For a reactor grade plasma $\Gamma_{20} >> 1$ so heating at this mode is also possible. Ordinary mode propagation requires only that $\omega = 2\Omega_o > \omega_{po}$ so we find

$$\alpha < 4 \quad . \tag{8}$$

Inequalities (2), (3), (5) and (8) represent cut-offs for electron cyclotron and second harmonic resonance heating for both modes of polarization. Heating will typically take place with α slightly below the cut-off value, in order to maintain strong absorption while optimizing the plasma parameters. An exception may be the X-mode at the fundamental, for which absorption decreases strongly as α approaches 2. In the following treatment we use the various cut-off values for simplicity; it should be understood that actual operation would in any case require α to be at least a few percent lower than this.

The X-mode at the fundamental propagates only from the high-field side, which will lead to considerable access problems in a tokamak reactor. The other three modes considered are accessible from the outside (low-field side). All the modes considered can be shown to be optically thick for a reactor grade plasma. To be suitable for auxilliary heating, the radiation must also satisfy a start-up condition, namely that the plasma be optically thick at a relatively low temperature of 1 or 2 keV. This condition is easily satisfied by all the modes considered with the possible exception of the O-wave at $\omega = 2\Omega_o$, for which the optical depth scales with $(T/mc^2)^2$; for this mode the start-up capability is marginal, depending sensitively on the initial plasma parameters.

For each of the modes considered, the absorption will take place over a relatively narrow region of space near the resonant surface. The extent of the absorbing region and the deposition profile within that region depend on the absorption line shape and on the angle of which the propagation vector approaches the resonance, as well as the magnitude of the local absorption coefficient. The spatial width of the resonant region may be obtained from the line-width in frequency space. For a Doppler broadened absorption profile the width of the region is approximately equal to $n_{\parallel}(T/mc^2)^{1/2}R_o$, where $n_{\parallel} \equiv ck_{\parallel}/\omega$ is the parallel index of refraction. For propagation nearly perpendicular to the magnetic field, such that $n_{\parallel} \lesssim (T/mc^2)^{1/2}$, the absorption line shape is determined by the relativistic transverse Doppler effect; this effect may be considered as arising from the relativistic variation of the electron mass. The resulting line shape is asymmetric about the non-relativistic resonance, with the absorbing region having a width of approximately $(|k|c/\omega)(T/mc^2)R_o$ shifted toward lower field (low frequency). It should be noted that the direction of the propagation vector approaching the resonance depends not only on the launch angle but also on refraction of the wave inside the plasma, which in turn will depend on the density and density gradients. If the absorption coefficients are large, most of the power will be deposited in the wings of the resonance on the side from which the wave is launched, before reaching the nominal resonant surface. Since the absorption increases with temperature, this effect may result in a shift in the location of the absorption as the plasma is heated. In any case it is important to note that the energy deposited near a resonant layer is rapidly diffused over a flux surface. Therefore, the effective volume in which heating takes place is a toroidal shell defined by the flux surfaces passing through the energy deposition layer. For stable bulk heating and especially for control of the temperature profile it will be desireable to employ radiation sources operating over a range of frequencies, corresponding to different resonant layers in the plasma, so that the energy deposition profile can be varied in a controlled manner.

3. Implications for Reactor Parameters

As we have seen, the propagation limits for each possible heating mode can be expressed in terms of the parameter α. By casting the reactor quantities of interest in terms of this parameter we can derive the applicable operating regime for each case. In order to determine illustrative reactor parameters, we will consider a circular cross-section tokamak reactor with parabolic density and temperature profiles, i.e.

$$n = n_o(1 - r^2/a^2)$$

$$T = T_o(1 - r^2/a^2) = T_e = T_i \tag{9}$$

Where necessary we will further specialize to an aspect ratio $A = R_o/a = 5$ and a limiter safety factor $q(a) = aB_o/RB_p = 3.0$, where B_p is the poloidal field. The generalization to other values and profile shapes is straight-forward.

Let us first consider the scaling of average beta, $\langle\beta\rangle$, with the parameter α. Here $\langle\beta\rangle$ is defined as

$$\langle\beta\rangle = \frac{\langle n(T_e + T_i)\rangle}{B_o^2/2\mu_o} \tag{10}$$

where the brackets signify an average over the plasma volume. Combining (9) and (10) gives

$$\langle\beta\rangle = \frac{2}{3}\frac{n_oT_o}{B_o^2/2\mu_o}$$

$$= \frac{4}{3}\alpha\frac{T_o}{mc^2} \tag{11}$$

where we have made use of the definition $\alpha = mn_o/\varepsilon_o B_o^2$. The $<\beta>$ limits corresponding to the various cut-offs are shown in figure 2; note that for $\alpha = 1$, which is the limiting value for propagation of the ordinary mode at the fundamental, and a typical central reactor temperature of 15 keV, $<\beta>$ is 3.9%.

The average fusion power density $<P_f>$ is given by

$$\begin{aligned} <P_f> &= 7.04 \times 10^{-7} < n^2(\overline{\sigma v})_{DT} > \text{MW/m}^3 \\ &= n_o^2 f(T_o) \end{aligned} \tag{12}$$

where f is a function only of T_o and profile shape. By introducing the parameter α we can rewrite (12) in terms of the toroidal field and obtain the relation

$$B_o = \left(\frac{m}{\varepsilon_o \alpha}\right)^{1/2} \left(\frac{<P_f>}{f(T_o)}\right)^{1/4} \tag{13}$$

Thus we see that the operating field scales with $(<P_f>^{1/4}\alpha^{-1/2})$. Curves corresponding to typical values of $<P_f>$ as a function of B_o and T_o are shown in figure 3. The frequency appropriate to each field is shown on the right hand axis. Note that for ECRH heating to be utilized when the reactor is run at a moderate value of $<P_f> = 5$ MW/m^3 the technically attractive O-wave fundamental technique requires a central field greater than 6 Tesla for a central temperature of 15 keV. X-wave heating at Ω_o permits the lowest combination of field and frequency, but is unfavorable from an engineering standpoint beacuse of the difficulty of inside access. Second harmonic heating with the O-wave is consistent with a low field (3.5 T), high beta device, providing the target plasma is sufficiently hot to provide adequate absorption at start-up. The required field for a given $\langle P_f \rangle$ is reduced somewhat by operation at elevated temperature. However, there are disadvantages to operating at increased temperature which will be discussed below.

Ignition in a reactor plasma is achieved when the power deposited directly by charged fusion products exceeds losses due to thermal conduction and radiation. The ignition condition can be expressed in terms of the quantity $[n_o \tau_E]_{ign}$ as a function of temperature, where τ_E is the global energy confinement time. We introduce the empirical scaling law for τ_E[5]

$$[n_o \tau_E]_{emp} = 3.8 \times 10^{-21} (n_o a)^2 \ m^{-3} \ s \tag{14}$$

which may be applied provided $(n_o \tau_E)_{emp}$ does not exceed the neoclassical value. We can define the margin of ignition M_I as the ratio

$$M_I \equiv \frac{[n_o \tau_E]_{emp}}{[n_o \tau_E]_{ign}} \tag{15}$$

and the denominator can be written

$$[n_o \tau_E]_{ign} = \frac{n_o^2 T_o}{P_f^* - P_b} \tag{16}$$

where P_f^* is the fusion power density emitted in charged particles, equal to 20% of $\langle P_f \rangle$, and P_b is the radiation power due to bremmsstrahlung. Equation (16) is appropriate for plasmas with $Z_{eff} = 1.0$, i.e. where impurity radiation is insignificant. Since P_f^* is directly proportional to $\langle P_f \rangle$ and the scaling of P_b with temperature is well-known, we can evaluate the ratio M_I in terms of the propagation parameter α. The result is shown as a family of curves in the $B_o - T_o$ plane in figure 4, where we have taken the minor radius a = 1.2 meters, which is typical of recent tokamak reactor designs; we have also included the increased radiation loss due to the presence of alpha particles. From equations 14-16 we note that M_I scales as $B_o^4 a^2 \alpha^2$ for any given temperature. For α = 1(O-mode, $\omega = \Omega_o$) ignition at a central temperature of 15 keV requires an axial field of 5.3 T, corresponding to a frequency of 150 GHz.

The empirical scaling law (14) is derived from results in present experimental devices, which typically operate in the collisional or plateau regimes. Extrapolation of (14) to higher temperature and reduced collisionality is uncertain, and values of M_I greater than unity may be required to achieve ignition. Significant impurity radiation will also necessitate an increased M_I. If a margin of ignition of 3 is required, a field of 7 T and frequency of 200 GHz will be needed for $\alpha = 1$ and $\omega = \Omega_o$. Heating at the second harmonic reduces the magnetic field requirement, since α can be increased (figure 4b and 4c), but does not permit a reduction in frequency. As shown in figure 4b the frequency is reduced for the case of the X-mode at $\omega = \Omega_o$ and $\alpha \simeq 2$. However, the necessity of providing access on the inboard side of the tokamak for this mode may be very difficult to meet, and efficient absorption might not be realized for values of α much above 1, in which case the higher fields and frequencies of figure 4a will apply to this mode as well. We note that while increasing the temperature somewhat reduces the requirements to obtain a given M_I, the resulting lowered collisionality may degrade confinement such that a higher margin is needed to achieve ignition. Moreover, as will be discussed below, the attendant increase in plasma beta can adversely affect the MHD stability of the system.

By combining the above relationships with possible technological and economic boundaries we can now characterize a tokamak power reactor which is heated with ECRH. We will consider a thermal power rating of 1500 MW_{th}, including exothermal tritium breeding interactions in the blanket such that $Q_{fus} = 22.4$ MeV. The thermal power output for a circular plasma is then written as

$$W_{th} = 2\pi^2 Aa^3 \langle P_f \rangle \left(\frac{22.4}{17.6}\right) = 1500 \quad MW_{th} \tag{17}$$

Since we have taken A = 5, equation (17) provides a relationship between the minor radius a and $\langle P_f \rangle$, which in turn can be written as a function of B_o, T_o, and the propagation parameter α. We now introduce an engineering limit due to the neutron wall loading and assume that the neutron power flux through the first wall should not exceed 4 MW/m^2. Taking the first wall radius to be $r_W = (1.15\ a)$ and making use of (17) gives the inequality

$$a \geq 1.02 \text{ m} \tag{18}$$

An upper limit on the size of a reactor may be imposed by cost considerations. For illustrative purposes we will require that the minor radius satisfy

$$a \leq 1.5 \text{ m} \tag{19}$$

which is consistent with current tokamak power reactor designs.[10] It should be observed that since we have fixed the reactor thermal power, the size limitations of (18) and (19) may be alternately expressed as bounds on the permissible fusion power density. In this form we may rewrite the wall loading restriction as

$$\langle P_f \rangle \leq 11.25 \text{ MW/m}^3 \tag{18a}$$

while the maximum size implies that

$$\langle P_f \rangle \geq 3.54 \text{ MW/m}^3 \tag{19a}$$

The technology of the radiation source, assumed to be a gyrotron-type cyclotron resonance maser, imposes a limit on the frequency utilized. The power generated by a single tube can be expected to decrease strongly with increasing frequency. For this discussion, we assume $\nu_{RF} \leq 300$ GHz for adequate power capability (see Section 4, below). This corresponds to a maximum central field of

$$B_o \leq \begin{cases} 10.7 \text{ Tesla } (\omega = \Omega_o) \\ 5.36 \text{ Tesla } (\omega = 2\Omega_o) \end{cases} \tag{20}$$

It should be pointed out that we are here considering only that class of reactors for which ECRH is used to heat the plasma to the operating point, where ignition or near-ignition driven operation is assumed to take place. One could conceive of a class of "alpha-boosted" reactors in which ignition occurs for parameters corresponding to values of $\langle P_f \rangle$ lower than (19a). In this case excess power in the emitted alpha particles would be used to increase the temperature and density to "boost" the system to an acceptable operating point. Since ECRH would be used only for the first stage of the two-step heating process, the final operating point would not need to satisfy the propagation conditions (2), (3),

(5) or (8). Since only ECR heating to ignition is required for an alpha boosted reactor, the curves of figure 4 alone define the relevant parameters. Another possible means of relaxing the ECRH requirement would be the use of adiabatic compression in major radius following initial heating at low magnetic field.[11] Although the parameter α would not be changed by compression, heating at low magnetic field would reduce the ECRH frequency requirement by C, where C is the ratio of the major radius of the initial plasma to the major radius of the final plasma. However, the use of large compression ratios in power reactors is unattractive.

For a 1500 MW_{th} reactor heated to the operating point by ECRH, the boundaries of the operating regimes given by (18)-(20) are depicted in figure 5. Three non-intersecting regions corresponding to the propagation limits $\alpha = 1$, 2, and 4 are shown. Additional constraints on the operating space arise from considerations of ignition and MHD stability. Within the bounds depicted in figure 5, the ignition condition is met or exceeded if the empirical scaling law for confinement applies. However, as noted earlier, the validity of this law at high temperature is uncertain. Dashed lines in figure 5 indicate the restrictions introduced by assuming that $M_I = 3$ is required to achieve ignition. Horizontal lines at $B_o = 10.7$ and 5.36 T define the probable high frequency limit of 300 GHz, for fundamental and second harmonic interaction respectively, as indicated in (20). For ordinary wave heating ($\alpha = 1,4$) these lines intersect the operating space at the high-field, low temperature extreme; the region appropriate for $\alpha = 2$, corresponding to extraordinary wave heating at either the fundamental or harmonic, however, lies partially above the high frequency limit for the harmonic. Heating with the X-wave at $2\Omega_o$ and $\alpha \gtrsim 2$ will be restricted unless CRM technology is extended well into the submillimeter band.

Vertical bars of constant $\langle\beta\rangle$ are indicated for each region (see eqn (11), figure 2). We note that the $\alpha = 1$ island lies mostly to the left of the line representing $\langle\beta\rangle = 4\%$, which represents a possible limit imposed by the onset of the ballooning mode.[12] For the X-wave island, only the section below $T_o = 7.7$ keV satisfies this condition, while essentially the entire region for $\alpha = 4$ has $<\beta> \geq 8\%$. Thus, the latter two cases will be viable only if stable high $<\beta>$ equilibria can be produced, as for example, in a flux conserving tokamak configuration.[13,14]

4. Source Requirements for ECR Heating

The power, P, required to heat the tokamak plasma to ignition temperature can be written approximately as:

$$P = \frac{3 <n\ (\Delta T_e + \Delta T_i)> V}{2\tau_E} \qquad (21)$$

where ΔT_e and ΔT_i are the temperature rise of electrons and ions, respectively, and V is the total plasma volume, ignoring alpha particle heating and radiation loss. Using the empirical scaling relation for $n_o \tau_E$ vs. $n_o^2 a^2$, Eq. (21) leads to the simple result:

$$P \approx 0.8\ R\ \Delta T_o \ \ \text{MW} \qquad (22)$$

where R is the major radius in meters and ΔT_o is in keV. For a tokamak power reactor with a major radius of 6.0 m and a central temperature of 15 keV the required heating power is approximately 75 MW.

Eq. (21) is predicated on 100% coupling of the ECR radiation to the plasma, and any reduction in coupling will require an increase in P. Poor coupling and confinement will result, for example, if heating causes a tail of hot electrons.[6] Such tail heating was not observed in experiments at lower plasma densities[1], but may occur at higher heating intensities.

The most promising source for providing ECRH radiation is the gyrotron,[2,3] but a major extension of gyrotron performance will be required for plasma heating to ignition. Although the technology of the required gyrotrons is still not certain, we may make the following rough projections of the devices required. The power emitted by a single gyrotron unit may have to be equal to or greater than about 100 kW for reasons of cost, but may not be much higher because of the difficulty of achieving high power at high frequency. Although we refer to the required 100 kW device as a gyrotron, which is a single cavity oscillator, the required device may also be a more complex tube based upon the same principles. A group of about 50 gyrotrons, each emitting 100 kW, could be placed in an array with their waveguides converging on a single port of the reactor, to form a 5 MW unit. For a rough estimate, we assume that the gyrotron waveguide windows can withstand 2 kW/cm^2 of CW radiation, so that each port for a 5 MW unit would be 0.25 m^2 in area. The port diameter of 0.56m is then small compared to the minor diameter of the plasma $2a \approx 2.4$ m. The total power of 75 MW could be achieved by distributing 15 units about the reactor. The RF ports would subtend in total 1.3% of the exterior area of the torus.

A power level of 100 kW at 200 GHz will require a significant extension of gyrotron technology. The highest reported output powers for high frequency gyrotrons are, for CW operation, 22 kW at 150 GHZ and for pulsed operation (of order of 1 msec pulses) 1.1 MW at 100 GHz and 210 kW at 125 GHz.[15] High power gyrotrons are also currently under development in the U.S.[16,17] Some of the major considerations in achieving operation at high frequency are efficiency, output power, output mode, polarization, window and waveguide losses, tube lifetime and cost. Using the theory of Nusinovich and Erm[18], one can show that an overall efficiency of greater than 30% can be achieved[19] at a frequency of 200 GHz if the gyrotron is operated at the fundamental frequency (ω_c). The predicted scaling of gyrotron efficiency with wavelength is shown in figure 6. The required field of about 7.5 T for a 200 GHz device could be provided by superconducting magnets. The required output power of 100 kW is very high and may not be attainable with a single gyrotron cavity oscillator. Possibly, a variation of the basic gyrotron configuration will be required to achieve this power level. Very few results have been reported on high frequency gyrotrons to date; consequently, we cannot yet completely characterize the devices required for plasma heating. However, there is every reason to be optimistic that a suitable configuration based on the gyrotron concept can be developed, and the resulting devices should have the high reliability associated with microwave tubes.

5. Conclusions

Based on this simple treatment we can make several statements about operating regimes, as well as about the probable microwave source requirements.

1) For ordinary wave heating ($\alpha \leq 1$) at $\omega = \Omega_o$, $\langle\beta\rangle$ is limited to less than .039 for T_o < 15 keV. We may derive two constraints on the characteristics of a reactor, one based on the fusion power density and the other based on the margin of ignition. To heat typical tokamak reactors operating at central temperatures of around 15 keV and fusion power densities of 5 MW/m^3 requires a magnetic field of 6.25 T and an ECRH frequency of 175 GHz. Based upon the empirical scaling law for the energy confinement time a magnetic field of 5.4 T and an ECRH frequency of 150 GHz are needed to insure ignition for typical tokamak parameters of $a = 1.2$ m and $T_o = 15$ keV. Higher magnetic fields and ECRH frequencies will be necessary if impurities are present of if the energy confinement degrades with increasing temperature.

2) Heating with the extraordinary mode at $\omega = \Omega_o$ close to $\alpha = 2$ can be used to lower the magnetic field and ECRH frequency requirements if high $\langle\beta\rangle$ (> 4%) MHD stability is allowed. The frequency requirement would be around 125 GHz for typical reactor parameters. However, the necessity of launching this wave from the inboard side of the vacuum chamber may severely complicate the engineering of the reactor.

3) Based upon propagation requirements second harmonic absorption using the extraordinary mode permits heating at α close to 2 at relatively high $\langle\beta\rangle$ (.05 - .10) and low field. However, ECRH frequencies greater than those required for ordinary mode fundamental heating will be needed to achieve the same fusion power densities and ignition margin. For example, for T(0) = 15 keV and P_f = 5 MW/m^3 a field of only 4.4 T is sufficient but a frequency of 250 GHz is necessary; beta for this case would be .078.

4) An extremely high $\langle\beta\rangle$ (10 - 15%), low field reactor is consistent with O-wave heating at $\omega = 2\Omega_o$, and propagation parameters $\omega_{po}^2/\Omega_o^2 \approx 4$. However, although this technique reduces magnetic field requirements relative to ordinary mode fundamental heating, the gyrotron frequency requirement is not decreased. Furthermore, this technique depends on creating an initial target plasma of sufficiently high temperature to absorb the microwave radiation; central temperatures of the order of 2-3 keV would have to be achieved during the ohmic heating phase. Thus O-wave heating at $\omega = 2\Omega_o$ may have very limited applicability.

5) A two-stage reactor, heated by ECRH to ignition at low $\langle P_f \rangle$ and low $\langle\beta\rangle$ and subsequently driven to an operating point at higher plasma density by alpha particle heating alone can be envisioned. In this case the magnetic field and the ECRH frequency requirements would no longer be determined by the desired fusion power density. However, the inability to heat at the operating point can significantly restrict the options for controlling thermal equilibrium and stability. Furthermore, since $\langle\beta\rangle$ must be substantially increased by the alpha heating, its value at ignition should be relatively low to insure MHD stability at the operating point. Because $\langle\beta\rangle$ is proportional to (αT_o), and T_o required for

ignition must exceed 6 keV, α may well be restricted during the ECRH heating phase to values of order 1, regardless of the propagation limit for the specific mode used. From figure 4a we see that even for a two-stage reactor central fields of the order of 5 Tesla or greater will be required, even if ignition occurs at $M_I = 1$.

6) Each of the ECRH reactor models discussed requires the use of high RF frequencies, with the possible exception of X-mode heating at the fundamental. The most favorable approach, i.e. the one satisfying the most conservative assumptions, as well as the most straight-forward, is for a reactor heated by the ordinary mode at $\omega = \Omega_o$. High fields will probably be needed since $\langle\beta\rangle$ will be limited to modest values by the propagation requirement. The required frequency will thus be of the order of 200 GHz. Lower field, high beta reactors heated at the second harmonic would require as high or higher frequencies, (approaching 300 GHz). Hence, it is likely that the development of high frequency ($\geq$ 200 GHz) gyrotron sources will be necessary for the use of ECRH to heat tokamak power reactors.

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Figure Captions:

1. Wave accessibility for the extraordinary (a) and ordinary (b) modes at $\omega = \Omega_o$, $2\Omega_o$ (labeled ω_c, $2\omega_c$ in the figure) and at $\omega = \omega_{UH}$, the upper hybrid frequency (extraordinary mode only). A parabolic density profile is assumed, with $\omega_{po}^2/\Omega_o^2 = 0.5$ and $a/R_o = 0.3$. The cutoff frequencies are discussed in the text.

2. Average toroidal beta is shown as a function of central temperature for the three limiting values of the propagation parameter $\alpha = \omega_{po}^2/\Omega_o^2$, for plasmas with parabolic density and temperature profiles.

3. Representative curves of fusion power density are shown as functions of B_o and T_o for parabolic density and temperature profiles, for the limiting values of the propagation parameter $\alpha = \omega_{po}^2/\Omega_o^2$:

   a) $\alpha = 1$, corresponding to the cut-off for the ordinary wave at $\omega = \Omega_o$

   b) $\alpha = 2$, corresponding to the cut-off for the extraordinary wave at $\omega = \Omega_o$ or $2\ \Omega_o$.

   c) $\alpha = 4$, corresponding to cut-off for the ordinary wave at $\omega = 2\Omega_o$

   Frequencies of the relevant resonance for each toroidal field are shown on the right-hand axis.

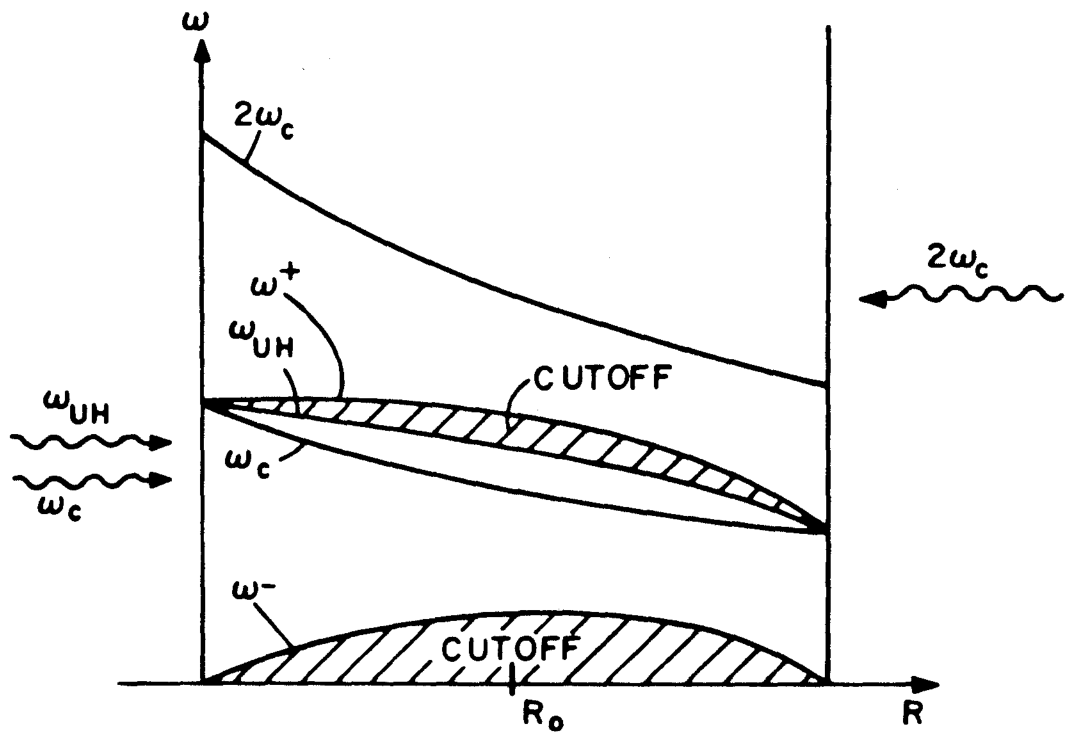
4. Fields and frequencies required to give ignition margins of 1 and 3 are shown as a function of central temperature for limiting values of α, computed for parabolic density and temperature profiles and a minor radius of 1.2 meters. The resonant heating frequencies are shown on the right-hand axis and average

toroidal beta on the upper scale. a) $\alpha = 1$, b) $\alpha = 2$, c) $\alpha = 4$.

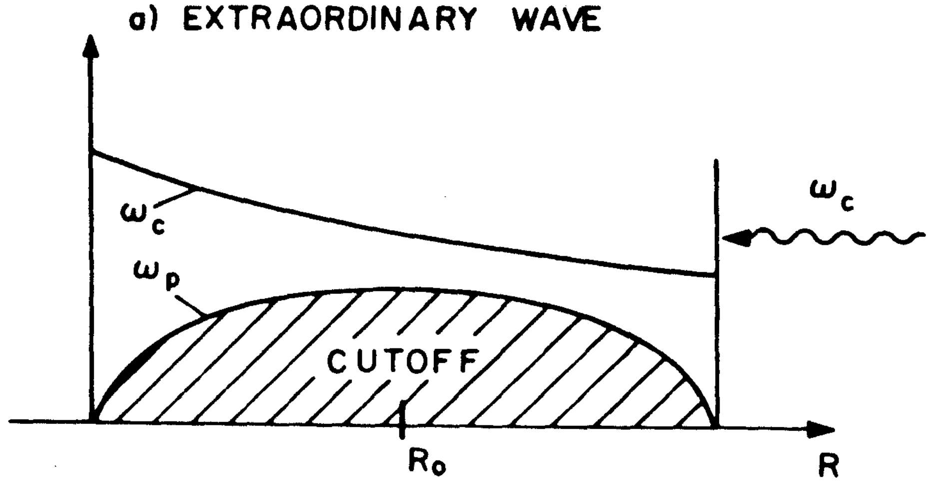
5. Typical option space for a tokamak power reactor producing 1500 MW_{th}, heated to the operating point by ECRH, with propagation parameters near cutoff for each available mode. Imposed constraints are: aspect ratio $A = R_o/a = 5$; neutron wall loading $P_{wall} \leq 4\ MW/m^2$ (defines the upper curved, hatched line); minor radius $a \leq 1.5$ m (lower curved boundary); heating frequency $\omega/2\pi \leq 300$ GHz (horizontal dashed line). Dashed vertical lines indicate the temperatures for which average beta equals 0.04, and in the case of $\alpha = 4$, $\langle\beta\rangle = 0.08$. The curved dashed lines labeled $M_I = 3$ are the lower bounds of the restricted option space available if confinement is a factor of three poorer than predicted by the empirical scaling law and ignited operation is required.

6. The theoretical efficiency of a gyrotron is evaluated for ω_c and $2\omega_c$ operation in the wavelength range 0.5 - 5.0 mm using the theory of Nusinovich and Erm (ref.18). Assumed parameters are a TE_{031} cavity mode and a 30 kV, 1A electron beam with $\beta_\perp/\beta_\parallel = 1.5$; other parameters, such as cavity length, are optimized. Experimental data of Zaytsev et al., ref. 3, at ω_c (highest efficiency point) and $2\omega_c$ (lower two points) are shown for comparison.

WAVE ACCESSIBILITY

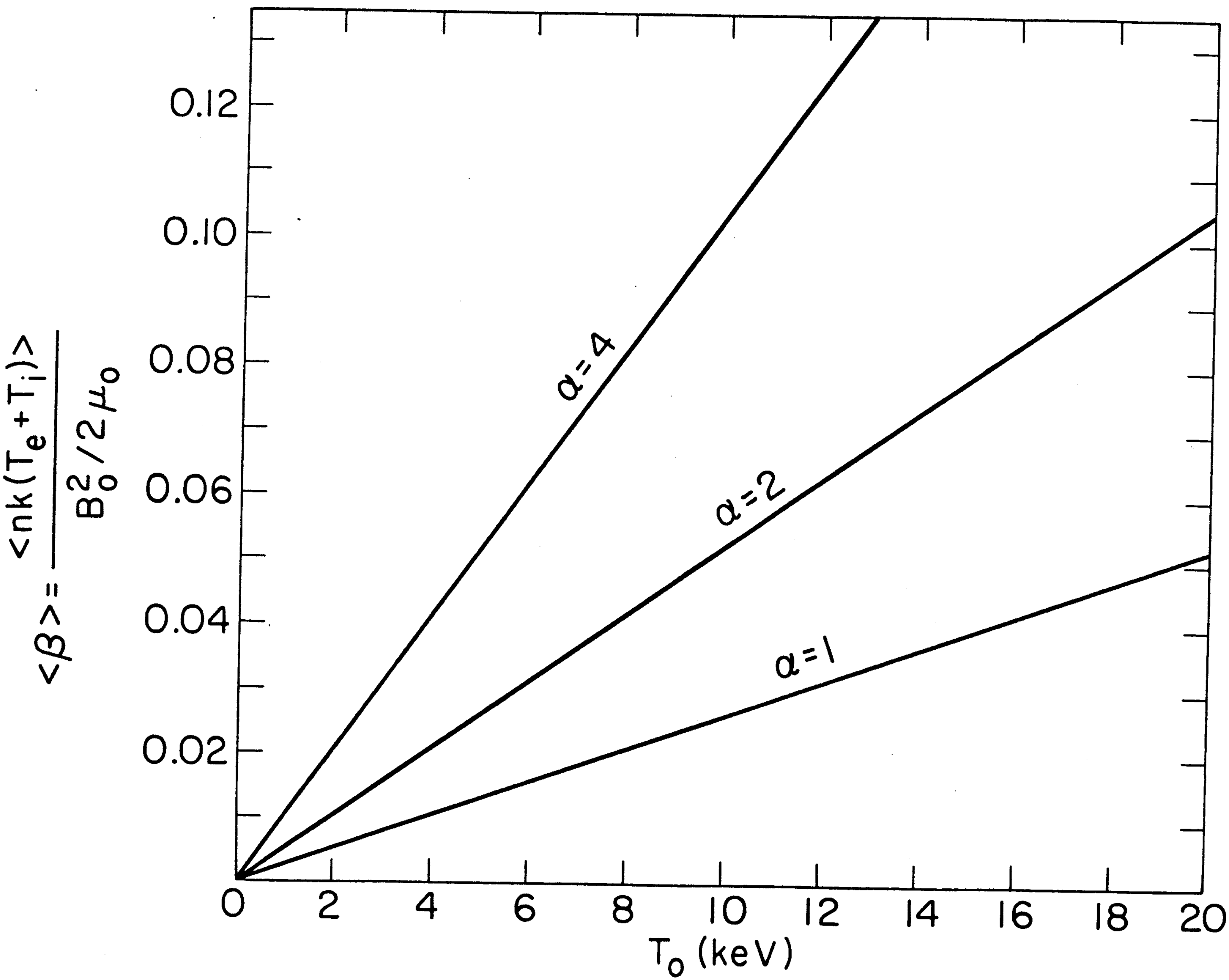


a) EXTRAORDINARY WAVE



b) ORDINARY WAVE

Fig. 1



$\langle\beta\rangle = \dfrac{\langle nk(T_e+T_i)\rangle}{B_o^2/2\mu_o}$
0.12
0.10
0.08
0.06
0.04
0.02
0
2
4
6
8
10
12
14
16
18
20
T_o (keV)
α=4
α=2
α=1

Fig. 2

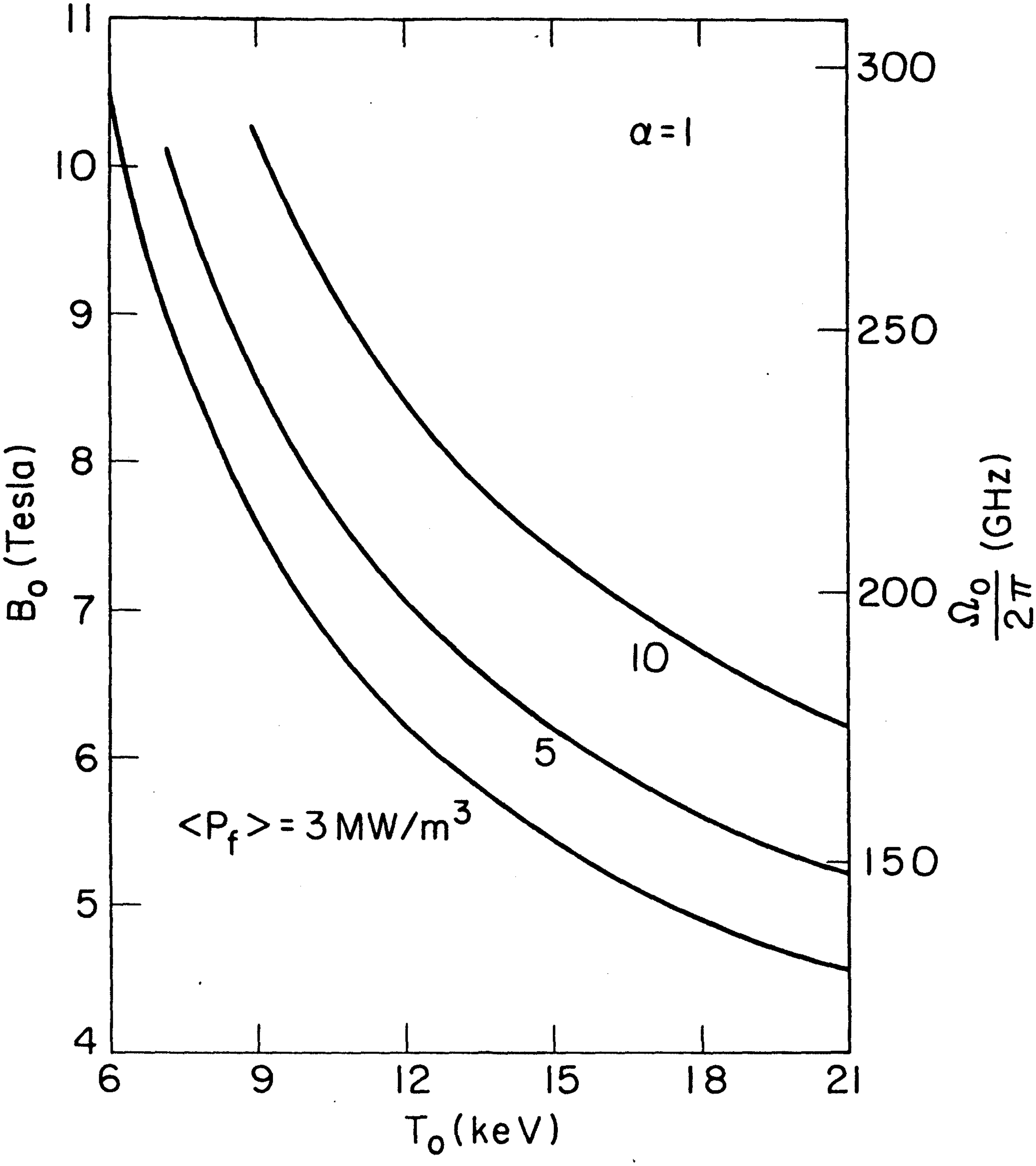
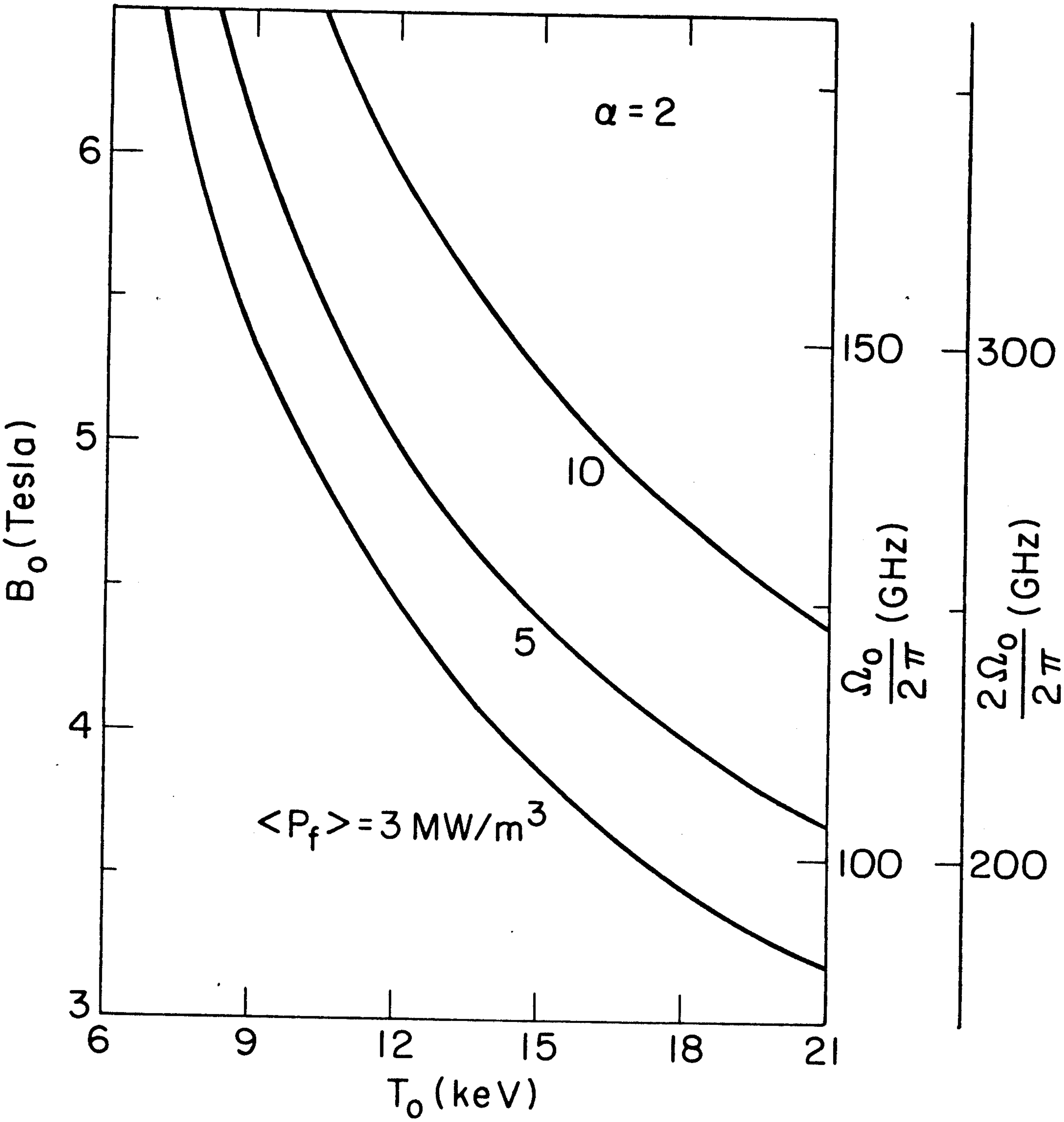
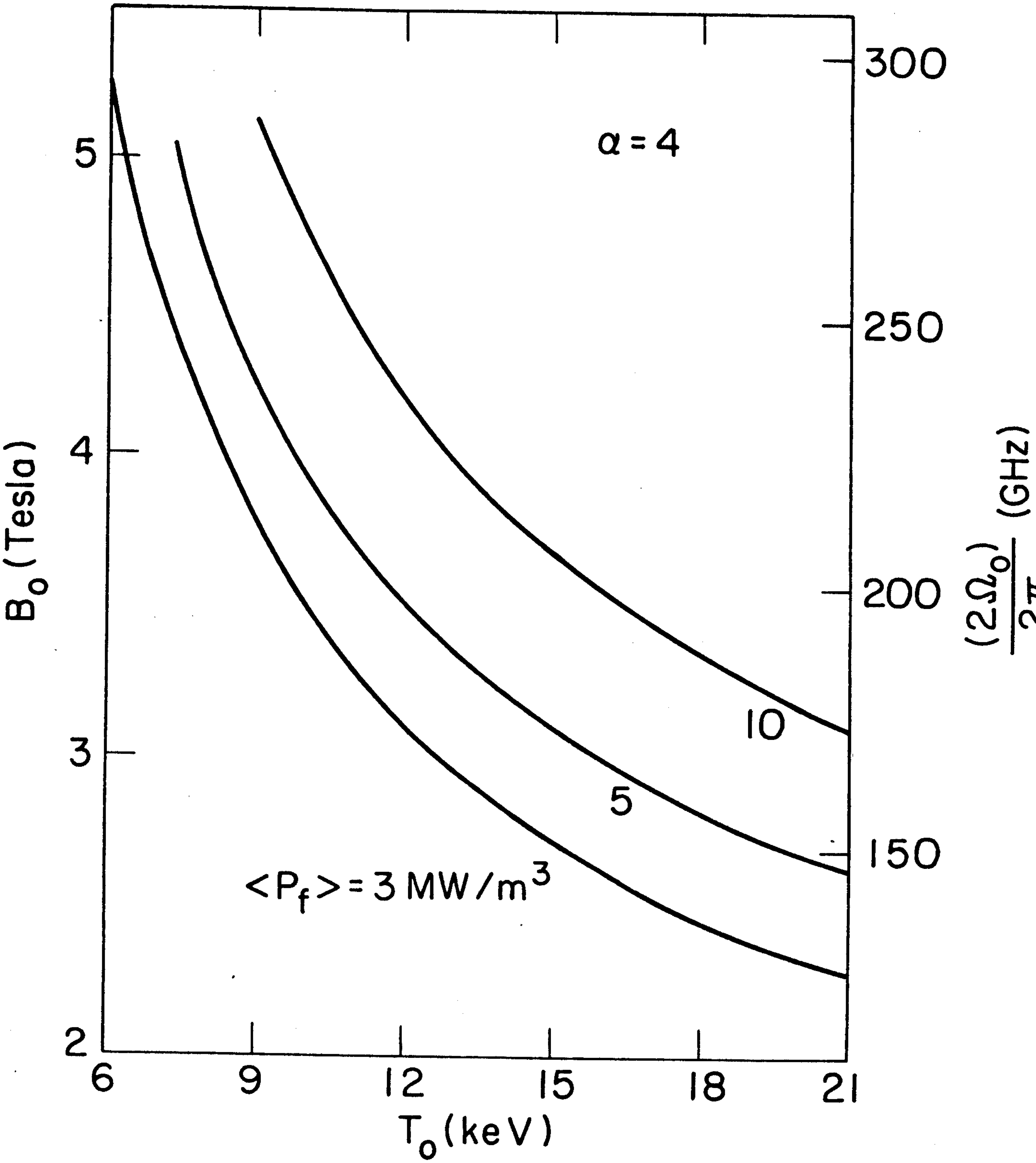

α=1
10
5
<P_f> = 3 MW/m^3
B_o (Tesla)
Ω_o/2π (GHz)
T_o (keV)
4
5
6
7
8
9
10
11
6
9
12
15
18
21
150
200
250
300

Fig.3a



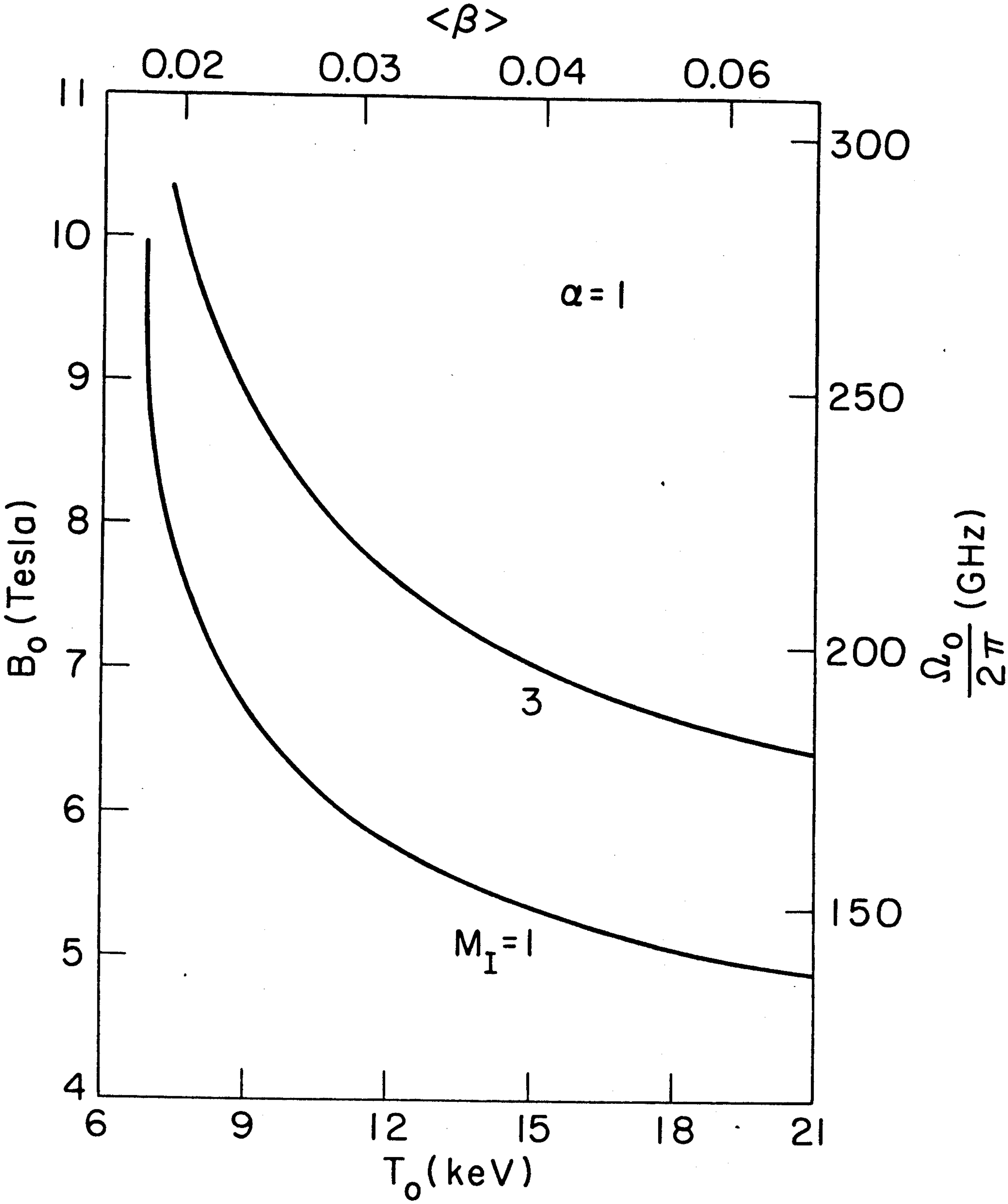
α = 2
10
5
<Pf> = 3 MW/m3
6
5
4
3
B0 (Tesla)
6
9
12
15
18
21
T0 (keV)
150
100
Ω0/2π (GHz)
300
200
2Ω0/2π (GHz)

Fig. 3b



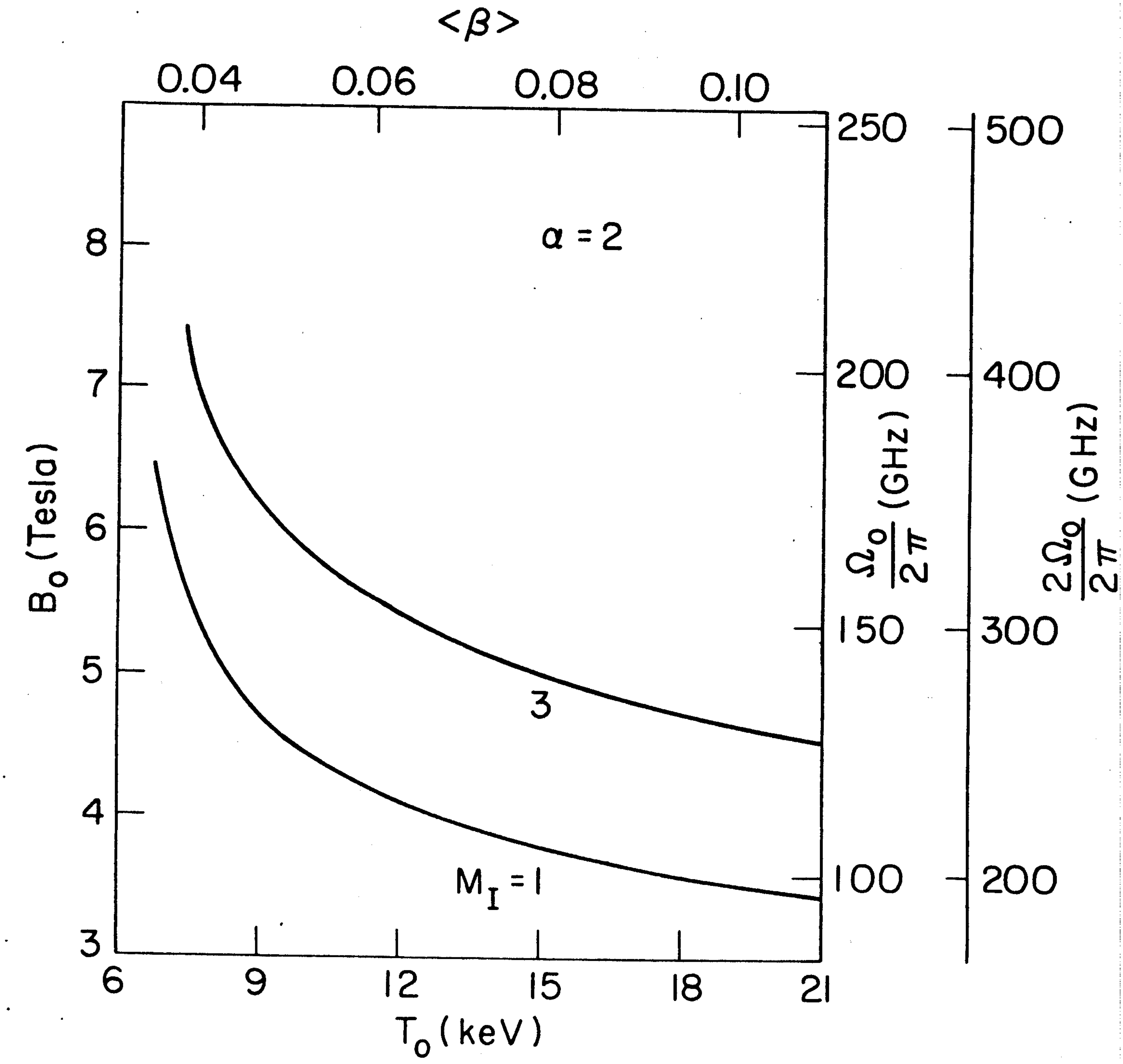
α = 4
<P_f> = 3 MW/m^3
10
5
B_o (Tesla)
5
4
3
2
6
9
12
15
18
21
T_o (keV)
300
250
200
150
(2Ω_o/2π) (GHz)

Fig. 3c



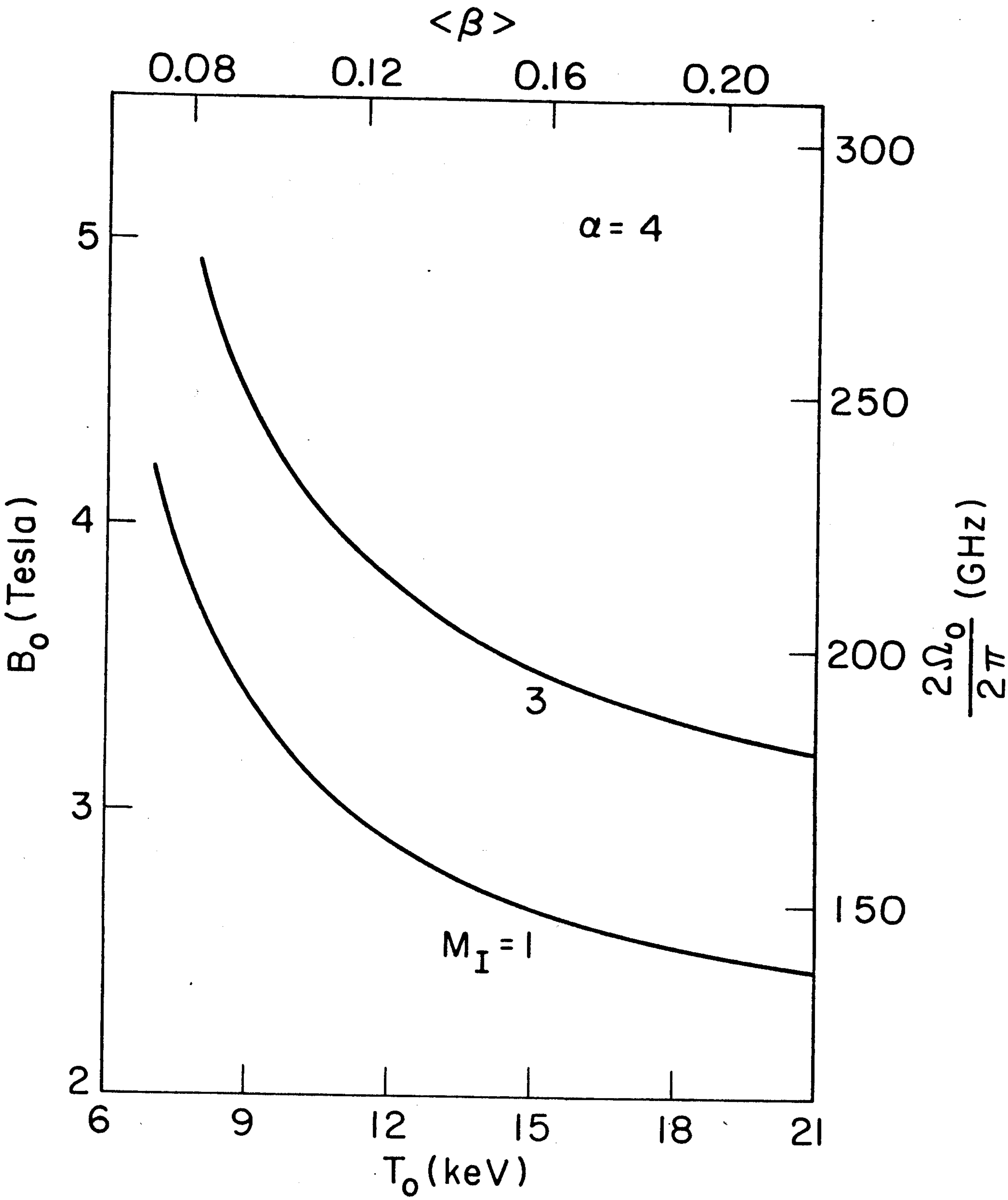
$<\beta>$
0.02
0.03
0.04
0.06
11
10
9
8
7
6
5
4
300
250
200
150
B_o (Tesla)
$\frac{\Omega_o}{2\pi}$ (GHz)
$\alpha = 1$
3
$M_I = 1$
6
9
12
15
18
21
T_o (keV)

Fig. 4a



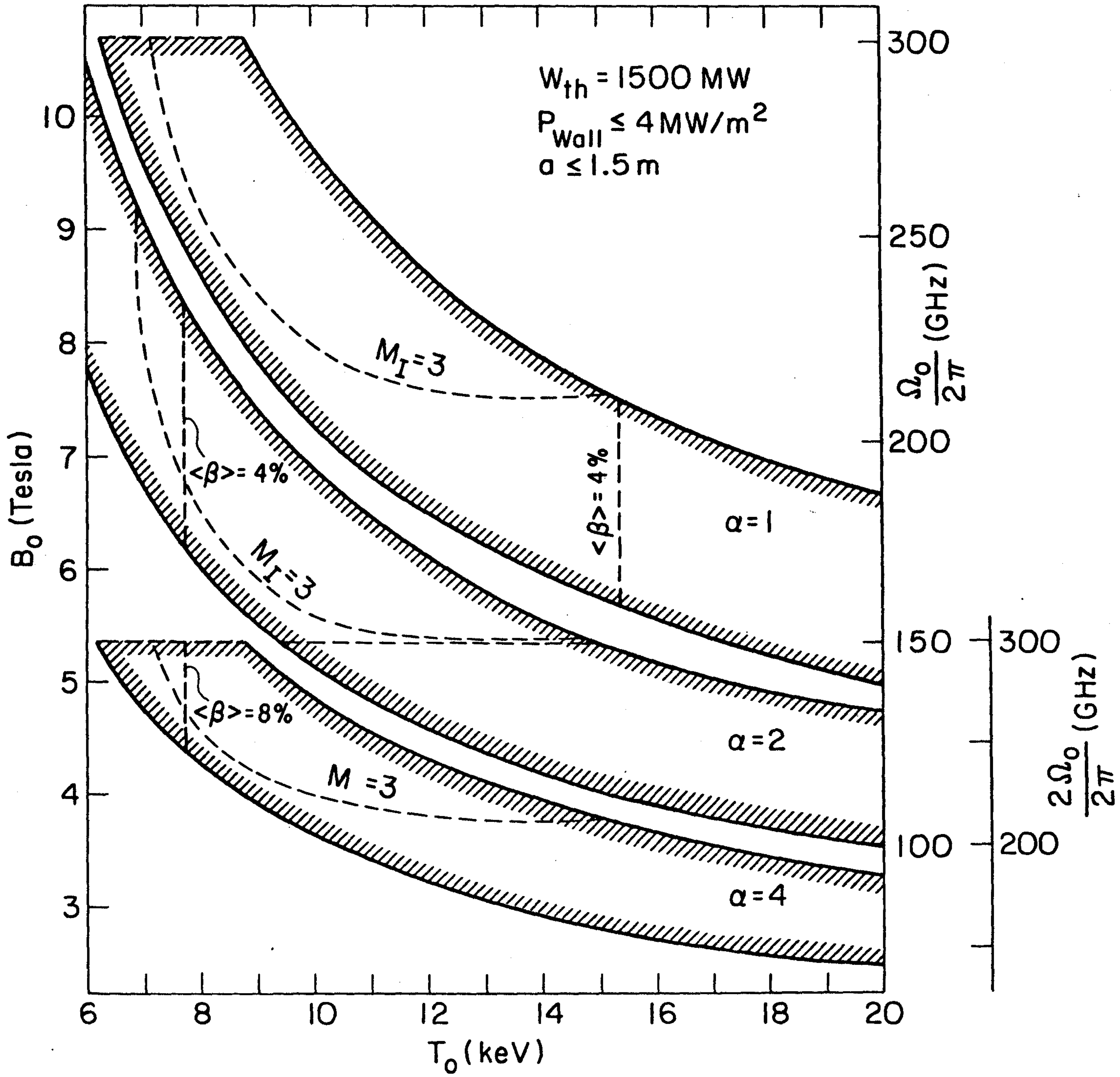
<β>
0.04
0.06
0.08
0.10
α = 2
3
M_I = 1
B_o (Tesla)
8
7
6
5
4
3
6
9
12
15
18
21
T_o (keV)
250
200
150
100
Ω_o/2π (GHz)
500
400
300
200
2Ω_o/2π (GHz)

Fig. 4b



<β>
0.08
0.12
0.16
0.20
300
250
200
150
α = 4
5
4
3
2
B_o (Tesla)
2Ω_o/2π (GHz)
3
M_I = 1
6
9
12
15
18
21
T_o (keV)

Fig. 4c



W_th = 1500 MW
P_Wall ≤ 4 MW/m^2
a ≤ 1.5 m
M_I = 3
<β> = 4%
<β> = 4%
M_I = 3
<β> = 8%
M = 3
α = 1
α = 2
α = 4
B_o (Tesla)
T_o (keV)
Ω_o/2π (GHz)
2Ω_o/2π (GHz)

Fig. 5

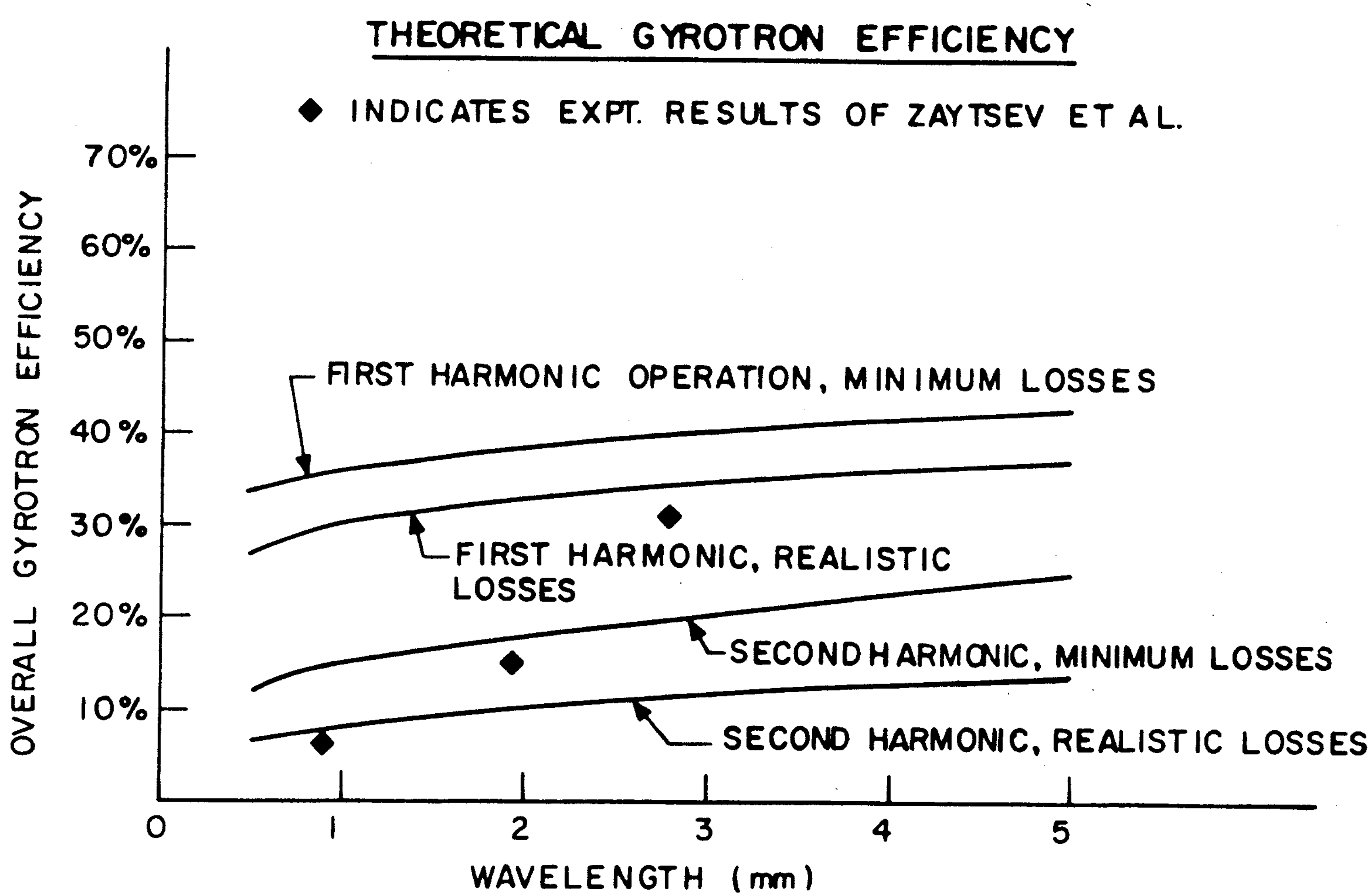

THEORETICAL GYROTRON EFFICIENCY
◆ INDICATES EXPT. RESULTS OF ZAYTSEV ET AL.
OVERALL GYROTRON EFFICIENCY
70%
60%
50%
40%
30%
20%
10%
FIRST HARMONIC OPERATION, MINIMUM LOSSES
FIRST HARMONIC, REALISTIC LOSSES
SECOND HARMONIC, MINIMUM LOSSES
SECOND HARMONIC, REALISTIC LOSSES
0
1
2
3
4
5
WAVELENGTH (mm)

Fig. 6