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Magnetic Topology Effects on Alcator C-Mod Flows

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Abstract

The effect of magnetic topology on ion and impurity flows in a tokamak is considered by investigating the consequences of (i) the reversal of toroidal and poloidal magnetic fields and currents, (ii) a switch from lower to upper X-point operation, (iii) poloidal magnetic field or plasma current reversal, and (iv) toroidal magnetic field reversal. The general symmetries associated with magnetic topology changes in tokamaks are employed to demonstrate that the flux surface flows inside and outside the separatrix observed in Alcator C-Mod [I. H. Hutchinson *et al.*, Phys. Plasmas **1**, 1511 (1994)] can be used to determine the flow features, including neoclassical and turbulent effects and in the presence of charge exchange.

I. INTRODUCTION

It is has probably been recognized ever since tokamaks were first constructed that they possess rather general symmetry properties with respect to magnetic field and plasma flow reversals. It is clear, for example, that turning over an up-down symmetric tokamak gives the same configuration as that obtained by reversing all currents and flows. Symmetry properties of up-down asymmetric tokamaks are more complicated. In particular, there has been a general recognition that the power balance between the outer and inner divertor plates in single null configurations is sensitive to magnetic topology [1 - 3]. However, to the best of our knowledge no orderly treatment existed until recently.

Cohen and Ryutov [4] appear to be the first to systematically observe that magnetic confinement devices with axisymmetric applied magnetic fields, conducting walls, and sources and sinks possess rather general symmetry properties with respect to reversals in the signs of the poloidal and/or toroidal magnetic fields. Associated with these symmetry properties are prescribed changes in the plasma flows since reversing poloidal and/or toroidal magnetic fields corresponds to reversing toroidal and/or poloidal currents and therefore flows. Cohen and Ryutov showed rigorously that simultaneously reversing all toroidal flows and currents and, therefore, poloidal magnetic fields, leaves a tokamak of arbitrary poloidal cross-section physically unchanged. The same pertains to simultaneously reversing toroidal magnetic field and poloidal flows and currents in an *up-down symmetric* tokamak.

We extend Cohen and Ryutov's formalism by investigating the response of the flow on a flux surface of *arbitrary cross-section* to magnetic field reversals. More importantly, we generalize their results to determine the toroidal and poloidal flow response associated with switching from lower to upper X-point operation. In addition, we demonstrate that understanding changes in the flows provides a means of measuring the radial electric field and the parts of the flow that are symmetric or asymmetric with respect to the equatorial plane.

The changes in magnetic topology that we consider herein alter the flows and radial electric field through reversal of (i) toroidal and poloidal magnetic fields (or equivalently poloidal and toroidal currents and flows), (ii) the X-point from lower single null to upper single null, (iii) the poloidal magnetic field (or equivalently all toroidal currents and flows), and (iv) the toroidal magnetic field (or equivalently poloidal currents and flows). Sensitivity to magnetic topology is observed in both the core [5, 6] and in the scrape-off layer [7, 8] flows of Alcator C-Mod.

In the next section we briefly review and extend the arguments of Cohen and Ryutov. The results given there hold in the presence of impurities (even with poloidal asymmetries due to toroidal rotation [9]), turbulence, and neutrals. Section III presents a neoclassical relation between flows of collisional high-Z trace impurities and background ions. Most core measurements are from the Doppler shifts of Argon or some other heavy trace impurities. Neoclassical theory provides the only means known for relating impurity and background ion flows. The expression given is obtained following the procedures of Refs. [10 - 12], with the details presented in the Appendix. In Sec. IV we demonstrate how the up-down symmetric and asymmetric portions of the flows can be measured by examining the flow changes with magnetic topology in both the core and scrape-off layer of Alcator C-Mod [5 - 8]. These flows can then be used to evaluate the radial electric field and determine the configuration with the optimum flow shear. We finish with a brief discussion of results and conclusions in Sec. V.

II. EFFECTS OF MAGNETIC TOPOLOGY ON ION FLOWS

We begin by introducing our tokamak conventions. We employ a coordinate system (ψ, ϑ, ζ) fixed in space, with ψ the flux function associated with the poloidal magnetic field, and ζ and ϑ are the toroidal and poloidal angle variables, respectively. We define the axisymmetric portion of the magnetic field by $\vec{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi \equiv \vec{B}_t + \vec{B}_p$ or equivalently by $\vec{B} = \nabla(\zeta - q\vartheta) \times \nabla\psi$, where $\vec{B}_t$ and $\vec{B}_p$ are the toroidal and poloidal magnetic fields, respectively, q $= (\vec{B}\cdot\nabla\zeta)/(\vec{B}\cdot\nabla\vartheta) = I/(R^2\vec{B}\cdot\nabla\vartheta)$ is the safety factor, and $I = I(\psi) = RB_t$ with R the cylindrical radial distance from the tokamak axis of symmetry and $B_t = |\vec{B}_t|$. We choose ζ in such a way that it increases in the clockwise direction when the tokamak is viewed from above. Normal operation corresponds to I > 0, toroidal magnetic field and toroidal plasma current in the same direction, and the poloidal angle ϑ increasing in the $\nabla\zeta \times \nabla\psi$ direction, with the magnetic drift towards a lower X-point.

Changes in the magnetic field correspond to changes in the current density and therefore the flows. The flow on a flux surface is most conveniently written in the form

$$\vec{V} = \omega R^2 \nabla\zeta + u\vec{B} = (\omega R^2 + uI)\nabla\zeta + u\nabla\zeta\times\nabla\psi. \tag{1}$$

In our notation a positive $V_{||} = \vec{B}\cdot\vec{V}/B = (\omega I/B) + uB$ indicates a parallel flow in the direction of $\vec{B}$. Our convention means that if $\vec{B}$ and $\vec{V}$ change sign together, $V_{||}$ does not change sign. Moreover, the toroidal flow is given by $V_\zeta = R\vec{V}\cdot\nabla\zeta = \omega R + (uI/R)$.

Summarizing Cohen and Ryutov [4] observations, and extending them to include X-point reversal and poloidal flow we find the following four cases for the impact of magnetic topology changes on ion flows.

(i) Total magnetic field reversed: $\vec{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi \rightarrow -I\nabla\zeta - \nabla\zeta\times\nabla\psi \equiv \vec{B}_i$.

If we first consider an up-down symmetric tokamak and turn it over (by rotating about a radial axis in the equatorial plane to make ζ and all currents reverse directions, along with the axis of symmetry), then with respect to the fixed coordinate system both the magnetic field and the flux surface flow must reverse, giving

$$\vec{V} = \omega R^2\nabla\zeta + u\vec{B} \rightarrow -\omega R^2\nabla\zeta - u\vec{B} = -\omega R^2\nabla\zeta + u\vec{B}_i$$

or $\omega \rightarrow -\omega$ and $u \rightarrow u$, where $\vec{B}_i = -I\nabla\zeta - \nabla\zeta\times\nabla\psi$. The sign of u is unchanged because $\vec{B} \rightarrow -\vec{B}$.

If a tokamak is not up-down symmetric we cannot simply turn it over. To reverse the total magnetic field the currents generating both the symmetric and asymmetric parts of $\vec{B}$ must be reversed so that the up-down symmetric and asymmetric parts of the flow also reverse while keeping the shape of the flux surface fixed to give

$$\vec{V} = \omega R^2\nabla\zeta + u\vec{B} \rightarrow -\omega R^2\nabla\zeta + u\vec{B}_i = \vec{V}_i. \quad (2)$$

An initially co-current toroidal flow ($\omega R^2 + uI > 0$) remains co-current upon field reversal since currents and flows reverse together. In addition, the poloidal flow and poloidal magnetic field reverse together.

Not surprisingly, the sum and difference of the two flows in terms of the original magnetic field are simply

$$\vec{V} + \vec{V}_i = 0 \quad (3)$$

and

$$\vec{V} - \vec{V}_i = 2\omega R^2\nabla\zeta + 2u\vec{B}. \quad (4)$$

A vanishing sum flow means that all currents and magnetic fields have been reversed in the presence of turbulence, heating, and/or neutrals. The difference expression is no more useful than Eq. (2).

(ii) X-point location reversed: $\vec{B} = \vec{B}_s + \vec{B}_a$.

Next, we consider the more interesting and subtle case where, for example, a lower single null configuration is changed to an upper single null without changing the direction of the toroidal or poloidal magnetic field and while maintaining the same mirror image cross sectional shape by letting $\psi(R,Z) \rightarrow \psi(R,-Z)$ where Z = 0 is the equatorial plane defined by the magnetic axis. In this case it is convenient to view the magnetic field as generated by symmetric (subscript s) and asymmetric (subscript a) toroidal currents so it contains symmetric and asymmetric portions, $\vec{B} = \vec{B}_s + \vec{B}_a$, as does the flow in a flux surface, $\vec{V} = \vec{V}_s + \vec{V}_a = (\omega_s+\omega_a)R^2\nabla\zeta + (u_s+u_a)\vec{B}$, where $\omega = \omega_s+\omega_a$ and $u = u_s+u_a$. In an up-down symmetric tokamak in the presence of up-down symmetric turbulence, neutrals, and heating $\omega_a = 0 = u_a$.

To switch from lower to upper null only the up-down asymmetric part of the toroidal current densities and flows are reversed to reverse the direction of the asymmetric part of the magnetic field. This case is equivalent to first letting $\vec{B} \to -\vec{B}$ to reverse all flows, and then turning the tokamak over to reverse $\vec{B}$ again along with the X-point, so that only the asymmetric portions of the flow are reversed. As a result, this case isolates the asymmetric parts of the flow by giving

$$\vec{V} = (\omega_s+\omega_a)R^2\nabla\zeta + (u_s+u_a)\vec{B} \to (\omega_s-\omega_a)R^2\nabla\zeta + (u_s-u_a)\vec{B}_{ii} = \vec{V}_{ii}, \tag{5}$$

where $\vec{B}_{ii} = \vec{B}_s - \vec{B}_a$, with $\vec{B}_s$ and $\vec{B}_a$ the up-down symmetric and asymmetric portions of the magnetic field. The fields $\vec{B}_s$ and $\vec{B}_a$ represent the double and single null portions of $\vec{B}$, respectively. If the two flux surfaces $\psi(R,Z)$ and $\psi(R,-Z)$ of interest are labeled by the same value of ψ, then only $\nabla\psi(R,Z)$ and $\nabla\psi(R,-Z)$ differ in $\vec{B}$ and $\vec{B}_{ii}$. Therefore, $\vec{B} = \vec{B}_s + \vec{B}_a = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi(R,Z)$ and $\vec{B}_{ii} = \vec{B}_s - \vec{B}_a = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi(R,-Z)$ give

$$\vec{B}_s = I\nabla\zeta + (1/2)\nabla\zeta\times\nabla[\psi(R,Z) + \psi(R,-Z)] \equiv I\nabla\zeta + \nabla\zeta\times\nabla\psi_s$$

and

$$\vec{B}_a = (1/2)\nabla\zeta\times\nabla[\psi(R,Z) - \psi(R,-Z)] \equiv \nabla\zeta\times\nabla\psi_a ,$$

and we see that $\vec{B}_s\cdot\nabla\zeta = I/R^2$ and $\vec{B}_a\cdot\nabla\zeta = 0$.

From Eq. (5) we see that an initially co-current toroidal flow ($\omega R^2 + uI > 0$) only remains co-current upon the X-point location reversal if $(\omega_s - \omega_a)R^2 + (u_s - u_a)I > 0$, and the poloidal flow and poloidal magnetic field are (mis)aligned before and after the reversal if $u_s + u_a > 0$ (< 0) and $u_s - u_a > 0$ (< 0), respectively. For the outboard or low field side poloidal flow to change direction from into a lower X-point ($u_s + u_a < 0$) to into an upper X-point ($u_s - u_a > 0$) outside the separatrix, requires that u_a dominates over u_s and $u_a < 0$.

Equation (5) indicates for case (ii) that the ω_a and u_a portions of the toroidal flow and the u_a part of the poloidal flow reverse. Of course, the symmetric parts of the flow are unchanged. The sum and difference flows near the equatorial plane and on the same flux surface for the X-point switch to become

$$\vec{V}+\vec{V}_{ii}=2\omega_s R^2\nabla\zeta+2u_s\vec{B}_s+2u_a\vec{B}_a \tag{6}$$

and

$$\vec{V}-\vec{V}_{ii}=2\omega_a R^2\nabla\zeta+2u_s\vec{B}_a+2u_a\vec{B}_s, \tag{7}$$

and can be used to isolate the contributions to the flows.

(iii) Poloidal magnetic field reversed: $\vec{B}=I\nabla\zeta+\nabla\zeta\times\nabla\psi\rightarrow I\nabla\zeta-\nabla\zeta\times\nabla\psi=\vec{B}_{iii}$.

If we reverse all toroidal currents (including those in vertical and shaping coils) and flows while keeping the shape of the flux surfaces fixed, then the poloidal magnetic field must reverse: $\vec{B}_p\rightarrow-\vec{B}_p$. Cohen and Ryutov [4] demonstrate that this corresponds to reflection across a poloidal plane by which $\zeta\rightarrow-\zeta$, $v_\zeta\rightarrow-v_\zeta$, $B_R\rightarrow-B_R$, $B_Z\rightarrow-B_Z$, and $E_\zeta\rightarrow-E_\zeta$, leaving the Maxwell equations and equations of motion (including the Fokker-Planck and Boltzmann equations) for each charged and neutral species unchanged.

Reversing the toroidal current reverses the toroidal flow so that $\omega\rightarrow-\omega$ and $u\rightarrow-u$. As a result, reversing the poloidal magnetic field yields

$$\vec{V}=\omega R^2\nabla\zeta+u\vec{B}\rightarrow-\omega R^2\nabla\zeta-u\vec{B}_{iii}=\vec{V}_{iii} \tag{8}$$

where $\vec{B}_{iii}=I\nabla\zeta-\nabla\zeta\times\nabla\psi$ is the new magnetic field having $\vec{B}_p$ reversed. Notice that the poloidal flow direction is unchanged, becoming opposed to the poloidal magnetic field.

The sum and difference flows for poloidal magnetic field reversal become

$$\vec{V}+\vec{V}_{iii}=2u\vec{B}_p \tag{9}$$

and

$$\vec{V}-\vec{V}_{iii}=2(\omega R^2+uI)\nabla\zeta, \tag{10}$$

giving poloidal or toroidal results, respectively, that isolate the poloidal and toroidal components of the flow.

(iv) Toroidal magnetic field reversed: $\vec{B}=I\nabla\zeta+\nabla\zeta\times\nabla\psi\rightarrow-I\nabla\zeta+\nabla\zeta\times\nabla\psi\equiv\vec{B}_{iv}$.

Reversing the toroidal magnetic field has to be equivalent to reversing the total magnetic field as in (i) and then reversing the poloidal magnetic field as in (iii). Therefore, in response to toroidal magnetic field reversal the poloidal flow reverses giving

$$\vec{V}=\omega R^2\nabla\zeta+u\vec{B}\rightarrow\omega R^2\nabla\zeta-u\vec{B}_{iv}=\vec{V}_{iv}, \tag{11}$$

where $\vec{B}_{iv} = -I\nabla\zeta + \nabla\zeta\times\nabla\psi$ is the magnetic field with its toroidal component reversed. We get the same result by noting that to reverse the toroidal magnetic field we must reverse the poloidal current density and flow velocities. From Eq. (11) we see that the poloidal flow reverses to oppose the poloidal magnetic field, while an initially co-current toroidal flow ($\omega R^2 + uI > 0$) remains co-current.

In this toroidal magnetic field reversal case the sum and difference flows are

$$\vec{V} + \vec{V}_{iv} = 2(\omega R^2 + uI)\nabla\zeta \tag{12}$$

and

$$\vec{V} - \vec{V}_{iv} = 2u\vec{B}_p, \tag{13}$$

thereby isolating the toroidal and poloidal contributions to the flow. Notice that the sum and difference expressions in this case are the same as the difference and sum expressions for case (iii).

Before closing this section we remark that in the absence of island formation any magnetic fluctuations are expected to be very small compared to the axisymmetric tokamak magnetic field $\vec{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi$. As a result, to a very good approximation the perpendicular ion flow is $\vec{E}\times\vec{B}$ plus ion diamagnetic in the axisymmetric magnetic field, giving only toroidal and parallel flow components on a flux surface. Consequently, the toroidal rotation frequency ω is expected to be of the form

$$\omega = -c[\partial\Phi/\partial\psi + (en)^{-1}\partial p_i/\partial\psi] , \tag{14}$$

with Φ the electrostatic potential, p_i the ion pressure, n the plasma density, e the magnitude of the charge of an electron (we assume singly charged background ions), and c the speed of light. As a result, knowing $\omega = \omega_s + \omega_a$, the plasma density, and the ion pressure profile allows the radial electric field to be determined.

III. NEOCLASSICAL RELATION BETWEEN IMPURITY AND ION FLOWS

Alcator C-Mod has toroidal flow measurements both inside [5, 6] and outside [7, 8] the separatrix for cases (i) and (ii). The core flow measurements in C-Mod are from the Doppler shifts of Argon (Ar^{16+} and Ar^{17+}) X-ray transitions. Therefore, it is the flow of the trace Argon impurity that is determined. The flows of the trace impurity and the main plasma ions are quite different. However, for Pfirsch-Schlüter high-Z trace impurities they can be related to each other by the following simple neoclassical formula [10 - 12] (the applicability conditions and some details of the derivation are given in the Appendix):

$$\vec{V}_z = \vec{V} + (k-1)u\vec{B} + \frac{c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_z}\frac{\partial p_z}{\partial \psi}\right)\left(R^2\nabla\zeta - \frac{I\vec{B}}{\langle B^2\rangle}\right), \tag{15}$$

where Ze, n_z, and p_z are the charge, density, and pressure of the impurity, respectively. The constant k and the flow coefficient u depend on the collisionality regime of the main plasma ions. For Pfirsch-Schlüter ions k=1, while for banana regime ions $k = 1 + 0.59(f_t/f_c)$, with f_t and f_c denoting the trapped and circulating particle fractions, respectively. The neoclassical expressions for u inside the separatrix are well known [10, 13]. In the Pfirsch-Schlüter and banana regimes they are given approximately by

$$u|_{ps} \approx -1.8(cI/e\langle B^2\rangle)\partial T_i/\partial\psi \quad \text{and} \quad u|_b \approx \frac{1.2(cI/e\langle B^2\rangle)\partial T_i/\partial\psi}{1+0.46(f_t/f_c)}, \tag{16}$$

with T_i the ion temperature, $B=|\vec{B}|$, and $\langle ...\rangle$ denoting the flux surface average. These neoclassical forms for u only change sign when I does, that is, for cases (iii) and (iv). The impurity flow must change with magnetic topology in the same way as the ion flow. The impurity pressure gradient term is approximately Z >> 1 times smaller than the ion pressure gradient term, so can sometimes be neglected (depending on the localization of the impurity). If the background ions are also in the Pfirsch-Schlüter regime, then the toroidal ion and impurity flows are approximately equal when the $1 - I^2/R^2\langle B^2\rangle$ factor multiplying the pressure terms is small. Neoclassical expressions for an arbitrary tokamak cross section are available for both the symmetric and asymmetric contributions to ω in the Pfirsch-Schlüter regime inside the separatrix [14]. In the banana regime, neoclassical up-down symmetric large aspect ratio expressions for ω for subsonic plasma flows [15] or general neoclassical expressions for ω valid when the flow is on the order of the sound speed [16] are available inside the separatrix.

The impurity flow given by Eq. (15) changes with magnetic topology (as described in Sec. II) for measurements near the equatorial plane and on the *same* flux surface in the following ways:

$$\vec{V}_{zi} = \vec{V}_i + (k-1)u\vec{B}_i - \frac{c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta + \frac{I\vec{B}_i}{\langle B^2 \rangle}\right), \qquad (17a)$$

$$\vec{V}_{zii} = \vec{V}_{ii} + (k-1)u\vec{B}_{ii} + \frac{c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta - \frac{I\vec{B}_{ii}}{\langle B^2 \rangle}\right), \qquad (17b)$$

$$\vec{V}_{ziii} = \vec{V}_{iii} - (k-1)u\vec{B}_{iii} - \frac{c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta - \frac{I\vec{B}_{iii}}{\langle B^2 \rangle}\right), \qquad (17c)$$

and

$$\vec{V}_{ziv} = \vec{V}_{iv} - (k-1)u\vec{B}_{iv} + \frac{c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta + \frac{I\vec{B}_{iv}}{\langle B^2 \rangle}\right). \qquad (17d)$$

Only for topology change (i) is it possible to obtain an impurity flow sum (or difference) expression in which the impurity characteristics do not enter explicitly:

$$\vec{V}_Z + \vec{V}_{zi} = \vec{V} + \vec{V}_i. \qquad (18a)$$

In all other cases the ion and impurity pressure profiles and densities enter:

$$\vec{V}_Z - \vec{V}_{zi} = \vec{V} - \vec{V}_i + 2(k-1)u\vec{B} + \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta - \frac{I\vec{B}}{\langle B^2 \rangle}\right), \qquad (18b)$$

$$\vec{V}_Z + \vec{V}_{zii} = \vec{V} + \vec{V}_{ii} + 2(k-1)u\vec{B}_s + \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2\nabla\zeta - \frac{I\vec{B}_s}{\langle B^2 \rangle}\right), \qquad (18c)$$

$$\vec{V}_Z - \vec{V}_{zii} = \vec{V} - \vec{V}_{ii} + 2(k-1)u\vec{B}_a - \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\frac{I\vec{B}_a}{\langle B^2 \rangle}, \qquad (18d)$$

$$\vec{V}_Z + \vec{V}_{ziii} = \vec{V} + \vec{V}_{iii} + 2(k-1)u\vec{B}_p - \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\frac{I\vec{B}_p}{\langle B^2 \rangle}, \qquad (18e)$$

$$\vec{V}_Z - \vec{V}_{ziii} = \vec{V} - \vec{V}_{iii} + 2(k-1)uI\nabla\zeta + \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2 - \frac{I^2}{\langle B^2 \rangle}\right)\nabla\zeta, \qquad (18f)$$

$$\vec{V}_Z + \vec{V}_{ziv} = \vec{V} + \vec{V}_{iv} + 2(k-1)uI\nabla\zeta + \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\left(R^2 - \frac{I^2}{\langle B^2 \rangle}\right)\nabla\zeta, \qquad (18g)$$

and

$$\vec{V}_Z - \vec{V}_{ziv} = \vec{V} - \vec{V}_{iv} + 2(k-1)u\vec{B}_p - \frac{2c}{e}\left(\frac{1}{n}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_Z}\frac{\partial p_Z}{\partial \psi}\right)\frac{I\vec{B}_p}{\langle B^2 \rangle}. \qquad (18h)$$

In some cases either $Z >> 1$ or $1 - I^2/R^2\langle B^2\rangle << 1$ may be used to try to justify the neglect of the localized impurity contribution or all pressure gradient modifications, respectively.

IV. FLOW OBSERVATIONS IN ALCATOR C-MOD

When all the magnetic fields are reversed in Alcator C-Mod (the toroidal and poloidal magnetic fields have not been reversed separately to date), as in case (i), the core toroidal rotation reverses to lowest order to remain co-current in Ohmic and radio frequency heated high confinement mode (H mode) discharges [5]. This behavior appears to be in broad agreement with the prediction of Eqs. (3) and (18a). The absence of complete reversal is presumably due to differences in the time dependence of stored energy and heating power levels (see Fig. 1 of Ref. [5]).

If the flow is assumed neoclassical with the background ions in the banana regime so that $u|_b < 0$ may be employed, then to determine ω and thereby the radial electric field, we require the ion pressure and temperature profiles, the impurity pressure profile, and the plasma and impurity densities, as can be seen by recalling Eqs. (16) and (18b). To obtain some information we consider the toroidal components of Eqs. (4) and (18b). Assuming $1 - I^2/R^2\langle B^2\rangle << 1$ for these on-axis measurements, we obtain

$$V_{\zeta z} - V_{\zeta zi} \approx V_{\zeta} - V_{\zeta i} + 2(k-1)uI/R = 2[\omega R + (kuI/R)] \ . \qquad (19)$$

Notice the impurity and ion difference flows differ in the banana regime since $k = 1 + 0.59(f_t/f_c)$. If the neoclassical expression for u is employed, then Eq (19) provides a measure of ω and, upon using Eq. (14), the radial electric field.

In Ref. [6] the topology in Alcator C-Mod was changed from lower single null (LSN) to upper single null (USN) during the same Ohmic low confinement mode (L mode) discharges to find that the core toroidal flow became more strongly counter-current (see Fig. 3 of Ref. [6]). In the absence of up-down asymmetry in ω and u, Eqs. (7) and (18d) give the obvious result that the flows are unchanged. The fact that they do change indicates the presence of an asymmetry in ω and/or u, which is consistent with the discharges being single null. Equations (7) and (18d) along with $\vec{B}_s \cdot \nabla\zeta = I/R^2$ and $\vec{B}_a \cdot \nabla\zeta = 0$ require $V_{\zeta z} - V_{\zeta zii} = V_{\zeta} - V_{\zeta ii} = 2[\omega_a R + (u_a I/R)] > 0$ to be consistent with Fig.

3. This expression and the results from Fig. 3 allow the asymmetric contribution to the radial electric field to be estimated if the ion pressure profile and the density are known and u is assumed to be neoclassical so that $u_a = 0$ and $u_s = u|_b$ as given by Eq. (16).

To estimate the symmetric contribution to the radial electric field we must use Eqs. (6) and (18c) to determine ω_s. Assuming $1 - I^2/R^2\langle B^2\rangle << 1$ we obtain

$$V_{\zeta z} + V_{\zeta zii} \approx V_\zeta + V_{\zeta ii} + 2(k-1)u_s I/R = 2[\omega_s R + (ku_s I/R)] \; . \tag{21}$$

Knowing the ion pressure and temperature profiles and the density, Eq. (21) allows the symmetric contribution to the radial electric field to be determined if $u_s = u|_b$ is again assumed.

The results shown in Fig. 3 of Ref. [5] provide convincing evidence that asymmetric flows are present in single null configurations. Some idea of the neoclassical asymmetry can be found by considering the Pfirsch-Schlüter regime [14] where the normalized asymmetry factor A plotted in Fig. 1 and given by

$$A = -\frac{R_0^3\langle R^2\vec{B}\cdot\nabla\ell nB\rangle}{B_0\left\langle R^2 B_p^2 B^{-4}\left(R^2B^2 + 3I^2\right)\right\rangle} \tag{22}$$

for LSN operation in C-Mod acts as a part of the drive for ω_a. In Eq. (22) R_0 and B_0 are the on axis values of R and B. Notice that the corresponding asymmetric contribution to the flow does not necessarily vanish in the core of C-Mod, even though the numerator of A becomes small, because the poloidal field dependence of the denominator also causes it to become small.

The ion flow measurements in the scrape-off layer (SOL) are simpler to deal with because impurities are not involved so only the background ions need be considered when analyzing measurements of the parallel ion velocity and/or its projection onto the toroidal direction. On the other hand SOL flows are more complicated because neoclassical results do not hold [although Eq. (14) is still expected to be valid] outside the separatrix and neutrals can be playing a role as well as turbulence and heating.

The SOL measurements for all fields reversed, case (i), indicate that the low field side flows do not quite reverse in C-Mod as seen by Fig. 10 of Ref. [7]. Presumably the discrepancy is related to the asymmetries introduced by the scanning probes being located above the equatorial plane or small differences in the profiles. The difference measurements from Eq. (4) give $V_{||} - V_{||i} = (2\omega I/B) + 2uB$, but the relative sizes of ω and u cannot be determined unless measurements are made at another location on the same flux surface.

The SOL measurements for switching from lower to upper single null, case (ii), are available on both the low and high field side [7, 8] so one can determine all four quantities ω_s, ω_a, u_s, and u_a. To illustrate this, we first form the parallel flows from Eq. (5) to find

$$V_{||} = (\omega_s + \omega_a)(I/B) + (u_s + u_a)B$$

and

$$V_{||ii} = (\omega_s - \omega_a)(I/B_{ii}) + (u_s - u_a)B_{ii} .$$

Figure 2 of Ref. [7] is for LSN operation. At about 2.5 mm beyond the separatrix the low and high field side flows are roughly equal, which is compatible with the assumption that ω and u are flux functions (thereby keeping the flow incompressible) since $(\omega I/B^{hi}) + uB^{hi} \approx (\omega I/B^{lo}) + uB^{lo} \approx 15\,\text{km/sec}$ can be satisfied as long as $\omega > 0$ and $u > 0$ (giving parallel and poloidal flows at the equatorial plane on the low field side away from the lower X-point, but into the X-point on the high field side). For the estimates here we assume $B_p << B$. Considering the USN operation results of Fig. 3 of Ref. [7] next, we observe that on the low field side the flow vanishes at about 2.5 mm beyond the separatrix giving $(\omega_s - \omega_a)(I/B_{ii}^{lo}) + (u_s - u_a)B_{ii}^{lo} \approx 0$. Using these results, noting from Fig. 3 that the high field side flow is strongly counter-current (giving a parallel or poloidal flow into the upper X-point on the high field side) such that $(\omega_s - \omega_a)(I/B_{ii}^{hi}) + (u_s - u_a)B_{ii}^{hi} \approx -40$ km/sec, taking $B_{ii}^{hi} \approx B^{hi} \approx 2B^{lo} \approx 2B_{ii}^{lo}$, and solving these equations gives in km/sec: $\omega_s I/B^{lo} \approx 18$, $\omega_a I/B^{lo} \approx -8$, $u_s B^{lo} \approx -11$, and $u_a B^{lo} \approx 16$. These estimates indicate that asymmetry is playing an important role. Notice that the high field side poloidal flows are towards the X-points, but weaker for a lower X-point than for an upper one. In addition, for a lower X-point the ion flow is onto the inner target on the high field side and will reverse to be onto the outer target if the magnetic field is reversed as might be expected from Ref. [3] (presumably flux surface heat flows will have a similar behavior).

Recalling Eq. (14), we see that the flow makes symmetric and asymmetric contributions to the radial electric field. By evaluating the flow components throughout the SOL the toroidal and poloidal flow shears can be determined and possibly related to the L to H mode power threshold.

V. DISCUSSION

In the preceding sections we have investigated the effect of magnetic topology on the flows in the core and scrape-off layer of a tokamak for (i) reversal in the direction of toroidal and poloidal magnetic fields, (ii) switching between lower single and upper single null operation, (iii) reversal of the poloidal magnetic field, and (iv) reversal of the toroidal magnetic field. We have demonstrated how the changes in the flows with magnetic topology can be used to measure the up-down symmetric and asymmetric portions of the flows and radial electric field, and thereby gain insight into the flows and flow shear associated with each configuration.

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APPENDIX: RELATION BETWEEN ION AND IMPURITY NEOCLASSICAL FLOWS

This appendix derives Eq. (15) and discusses its applicability conditions. The derivation is based upon classical Refs. [10 - 12] and is given here mainly to make the paper self-contained.

Dotting by $\vec{B}$, both the $\vec{v}$ and $v^2\vec{v}$ moments of the species (a) kinetic equation, flux surface averaging, and assuming that the characteristic time scales and flow velocities are on the main plasma ion diamagnetic drift frequency and diamagnetic velocity scales, respectively, we obtain

$$\langle (Z_a e n_a E_{||}^{(A)} + F_{a1})B\rangle = \langle \vec{B}\cdot(\vec{\nabla}\cdot\vec{\vec{\pi}}_a)\rangle, \quad \langle F_{a2}B\rangle = \langle \vec{B}\cdot(\vec{\nabla}\cdot\vec{\vec{\theta}}_a)\rangle, \tag{A1}$$

where $\vec{\vec{\pi}}_a = M_a\int(\vec{v}\,\vec{v} - \vec{\vec{I}}\,v^2/3)f_a d^3v$, $\vec{\vec{\theta}}_a = M_a\int(\vec{v}\,\vec{v} - \vec{\vec{I}}\,v^2/3)(M_a v^2/2T_a - 5/2)f_a d^3v$, $F_{a1} = \vec{B}\cdot M_a\int\vec{v}C_a d^3v$, $F_{a2} = \vec{B}\cdot M_a\int\vec{v}(M_a v^2/2T_a - 5/2)C_a d^3v$, and $Z_a e$, n_a, M_a, f_a, and C_a are the species (a) charge, density, mass, distribution function, and collision operator, respectively. In deriving Eq. (A1) we take into account the fact that n_a, T_a, and Φ are flux functions to leading order (which can be demonstrated *a posteriori*), so that in particular $\langle Z_a e n_a \vec{E}\cdot\vec{B}\rangle \approx \langle Z_a e n_a E_{||}^{(A)}B\rangle$, where $E_{||}^{(A)}B \equiv -c^{-1}\vec{B}\cdot\partial\vec{A}/\partial t$, with $\vec{A}$ the electromagnetic vector potential.

Using the $\vec{v}$ and $v^2\vec{v}$ moments of the species (a) kinetic equation to evaluate the first order [in $\delta_a \equiv \rho_a/L_\perp \ll 1$ with ρ_a the (a) species gyroradius] perpendicular flow and heat flux, and employing the results in the species continuity and energy conservation equations, respectively, we obtain for the first order parallel flow and heat flux

$$V_{||a} = \omega_a(\psi)I/B + u_a(\psi)B, \quad q_{||a} = (5/2)p_a[s_a(\psi)I/B + g_a(\psi)B], \tag{A2}$$

so that

$$\vec{V}_a = \omega_a(\psi)R^2\nabla\zeta + u_a(\psi)\vec{B}, \quad \vec{q}_a = (5/2)p_a[s_a(\psi)R^2\nabla\zeta + g_a(\psi)\vec{B}], \tag{A3}$$

where

$$\omega_a = -c[\partial\Phi/\partial\psi + (Z_a e n_a)^{-1}\partial p_a/\partial\psi], \quad s_a = -(c/Z_a e)\partial T_a/\partial\psi, \tag{A4}$$

and the flux functions $u_a(\psi)$ and $g_a(\psi)$ are to be determined from Eqs. (A1).

To determine these two flux functions we notice that in neoclassical theory the distribution function f_a is a Maxwellian f_{Ma} to lowest order. After expanding f_a in powers of δ_a, $f_a = f_{Ma} + f_a^{(1)} + f_a^{(2)} + \cdots$, we may next expand $f_a^{(1)}$ in generalized Laguerre polynomials $L_j^{(3/2)}(x_a^2)$, j=0,1,2,$\cdots$, $x_a^2 \equiv v^2/v_a^2$, $v_a \equiv (2T_a/M_a)^{1/2}$:

$$f_a^{(1)} = 2f_{Ma}\vec{v}/v_{tha} \cdot \left[\sum_{j=0}^{+\infty} \vec{C}_{aj} L_j^{(3/2)}(x_a^2)\right], \qquad \text{(A5)}$$

$$\vec{C}_{aj} \equiv 3\int L_j^{(3/2)}(x_a^2) f_a^{(1)}\,\vec{v}\,d^3v \Big/ 2\int [L_j^{(3/2)}(x_a^2)\,x_a]^2\,d^3v\,,$$

with $\vec{C}_{a0} = \vec{V}_a$, $\vec{C}_{a1} = -2\vec{q}_a/5p_a$. It turns out that the infinite series (A5) can be safely truncated after j=1. Substituting this truncated expansion (A5) into definitions of F_{a1} and F_{a2} we obtain

$$F_{a1} = \sum_b [\ell_{11}^{ab} V_{\|b} - \ell_{12}^{ab}(2q_{\|b}/5p_a)], \quad F_{a2} = \sum_b [-\ell_{21}^{ab} V_{\|b} + \ell_{22}^{ab}(2q_{\|b}/5p_a)], \qquad \text{(A6)}$$

where the coefficients ℓ_{ij}^{ab}, i=1,2, j=1,2, are independent of collisionality regime and are derived in Ref. [10] for a plasma consisting of electrons and multiple ion species. For a plasma consisting of electrons, singly-charged (hydrogen) ions, and a single high-Z impurity species the matrices

$$\vec{\vec{L}}_{ab} \equiv \begin{pmatrix} \ell_{11}^{ab} & -\ell_{12}^{ab} \\ -\ell_{21}^{ab} & \ell_{22}^{ab} \end{pmatrix}$$

are given by

$$\vec{\vec{L}}_{ee} = -(1+\alpha)\frac{M_e n_e}{\tau_{ei}}\begin{pmatrix} 1 & -3/2 \\ -3/2 & 13/4 + \sqrt{2}/(1+\alpha) \end{pmatrix}, \ \vec{\vec{L}}_{ei} = \frac{M_e n_e}{\tau_{ei}}\begin{pmatrix} 1 & 0 \\ -3/2 & 0 \end{pmatrix}, \ \vec{\vec{L}}_{ez} = \alpha\vec{\vec{L}}_{ei}\,,$$

$$\vec{\vec{L}}_{ii} = -\frac{M_i n_i}{\tau_{iz}}\begin{pmatrix} 1 & -3/2 \\ -3/2 & 13/4 + \sqrt{2}/\alpha \end{pmatrix}, \ \vec{\vec{L}}_{ie} = \frac{M_e n_e}{\tau_{ee}}\begin{pmatrix} 1 & -3/2 \\ 0 & 0 \end{pmatrix}, \ \vec{\vec{L}}_{iz} = \frac{M_i n_i}{\tau_{iz}}\begin{pmatrix} 1 & 0 \\ -3/2 & 0 \end{pmatrix},$$

$$\vec{\vec{L}}_{zz} = -\frac{M_i n_i}{\tau_{iz}}\begin{pmatrix} 1 & 0 \\ 0 & (T_i/T_z)[15/2 + \alpha\sqrt{2}\,v_z/v_i] \end{pmatrix}, \ \vec{\vec{L}}_{zi} = \frac{M_i n_i}{\tau_{iz}}\begin{pmatrix} 1 & -3/2 \\ 0 & 0 \end{pmatrix}, \ \vec{\vec{L}}_{ze} = \alpha\vec{\vec{L}}_{ie}\,,$$

where $\tau_{ab} \equiv (3/16\sqrt{\pi})(M_a^2 v_a^3/n_b Z_a^2 Z_b^2 e^4 \ln\Lambda)$ with the Coulomb logarithm $\ln\Lambda$ assumed to be the same for all species, $\alpha \equiv Z_{eff} - 1 = Z^2 n_z/n_e$ with Z>>1 but

$Zn_z / n_e << 1$, and the zero entries in the matrices denoting quantities of order $(M_e / M_i)^{1/2} \ll 1$, $(M_e / M_z)^{1/2} \ll 1$, or $(M_i / M_z)^{1/2} \ll 1$.

Finally, it can be shown [10] that

$$\langle \vec{B} \cdot (\vec{\nabla} \cdot \vec{\pi}_a) \rangle = 3\langle (\vec{B} \cdot \vec{\nabla} \ln B)^2 \rangle [\mu_{a1} u_a + \mu_{a2} g_a], \tag{A7}$$

$$\langle \vec{B} \cdot (\vec{\nabla} \cdot \vec{\theta}_a) \rangle = 3\langle (\vec{B} \cdot \vec{\nabla} \ln B)^2 \rangle [\mu_{a2} u_a + \mu_{a3} g_a],$$

where the coefficient μ_{a1}, μ_{a2}, and μ_{a3} are functions of the collisionality regime of species (a) only [10]. Introducing the matrix of viscosity coefficients

$$\vec{\vec{M}}_a \equiv \begin{pmatrix} \mu_{a1} & \mu_{a2} \\ \mu_{a2} & \mu_{a3} \end{pmatrix}$$

for a plasma consisting of electrons, singly-charged ions, and a single high-Z impurity, we find that for the Pfirsch-Schlüter regime

$$\vec{\vec{M}}_i^{PS} = \frac{p_i \tau_{ii}}{1 + 2.39\alpha + 1.08\alpha^2} \begin{pmatrix} 1.36 + 1.91\alpha & 2.19 + 4.27\alpha \\ 2.19 + 4.27\alpha & 6.92 + 12.13\alpha \end{pmatrix},$$

$$\vec{\vec{M}}_e^{PS} = \frac{p_e \tau_{ee}}{1 + 1.02\alpha + 0.24\alpha^2} \begin{pmatrix} 0.73 + 0.43\alpha & 1.45 + 0.96\alpha \\ 1.45 + 0.96\alpha & 4.26 + 2.72\alpha \end{pmatrix},$$

$$\vec{\vec{M}}_z^{PS} = p_z \tau_{zz} \begin{pmatrix} 1.36 & 2.19 \\ 2.19 & 6.92 \end{pmatrix} + p_z \tau_{zz} O(\alpha^{-1} \sqrt{M_i / M_z} \sqrt{T_z / T_i}),$$

and for the banana regime

$$\vec{\vec{M}}_i^{b} = \frac{f_t}{f_c} \frac{n_i M_i}{\tau_{ii}} \frac{\langle B^2 \rangle}{3\langle (\vec{B} \cdot \vec{\nabla} \ln B)^2 \rangle} \begin{pmatrix} 0.53 + \alpha & -0.63 - 1.50\alpha \\ -0.63 - 1.50\alpha & 1.39 + 4.25\alpha \end{pmatrix},$$

$$\vec{\vec{M}}_e^{b} = \frac{f_t}{f_c} \frac{n_e M_e}{\tau_{ee}} \frac{\langle B^2 \rangle}{3\langle (\vec{B} \cdot \vec{\nabla} \ln B)^2 \rangle} \begin{pmatrix} 0.53 + (1+\alpha) & -0.63 - 1.50(1+\alpha) \\ -0.63 - 1.50(1+\alpha) & 1.39 + 4.25(1+\alpha) \end{pmatrix},$$

$$\vec{\vec{M}}_z^{b} = \frac{f_t}{f_c} \frac{n_z M_z}{\tau_{zz}} \frac{\langle B^2 \rangle}{3\langle (\vec{B} \cdot \vec{\nabla} \ln B)^2 \rangle} \begin{pmatrix} 0.53 & -0.63 \\ -0.63 & 1.39 \end{pmatrix},$$

where $f_c \equiv (3/4)\langle B^2 \rangle \int_0^{B_{max}^{-1}} \lambda \langle (1 - \lambda B)^{1/2} \rangle^{-1} d\lambda$ is the fraction of circulating particles with λ the pitch angle variable and B_{max} the maximum value of B on the flux surface, and $f_t \equiv 1 - f_c$.

Substituting expressions (A6) and (A7) into Eqs. (A1) and employing Eqs. (A2) to simplify the result, we arrive at the following system of equations for $u_a(\psi)$ and $g_a(\psi)$:

$$3\langle B^2\rangle^{-1}\langle(\vec{B}\cdot\vec{\nabla}\ln B)^2\rangle\vec{M}_a\cdot\vec{U}_a=\sum\nolimits_b\vec{L}_{ab}\cdot(\vec{U}_b+\vec{S}_b)+\vec{E}_a\,, \qquad \text{(A8)}$$

where $\vec{U}_a=(u_a,g_a)$, $\vec{S}_a=I\langle B^2\rangle^{-1}(\omega_a,s_a)$, and $\vec{E}_a=(Z_a e n_a\langle B^2\rangle^{-1}\langle E_{\|}^{(A)}B\rangle,0)$. The last term on the right-hand side of Eq. (A8) is assumed small and so it will be neglected in what follows.

Next, we evaluate the quantities u_i and u_z by solving Eqs. (A8) in the banana and Pfirsch-Schlüter regimes. While doing so we assume that the impurity is merely a trace impurity (i.e. $\alpha<<1$) and is always in the Pfirsch-Schlüter collisionality regime. To simplify the calculation we will follow Ref. [11] and consider the situation when the electron effects (both the viscosity and friction) can be neglected, i.e. when $\alpha\gg(M_e/M_i)^{1/2}(T_i/T_e)^{3/2}$. We also assume that impurity viscosity is negligible compared with impurity-impurity and impurity-ion friction forces, requiring that both $\kappa^b\equiv(3/2)[v_i^2\tau_{ii}^2\langle(\vec{B}\cdot\vec{\nabla}\ln B)^2\rangle/\langle B^2\rangle](M_z/M_i)^{1/2}(T_z/T_i)^{5/2}(\alpha Z^4)^{-1}\ll 1$ and $\kappa^{PS}\equiv(M_z/M_i)^{1/2}(T_z/T_i)^{5/2}Z^{-4}\ll 1$. To neglect the impurity viscosity in the Pfirsch-Schlüter regime we have to also assume that $\alpha\gg(M_i/M_z)^{1/2}(T_z/T_i)^{1/2}$.

Solving Eqs. (A8) in the banana and Pfirsch-Schlüter regimes of collisionality of the main hydrogen ions and electrons we obtain

$$u_i^b=\frac{cI}{e\langle B^2\rangle}\frac{1.18}{1+0.46(f_t/f_c)}\frac{\partial T_i}{\partial\psi}, \qquad \text{(A9)}$$

$$u_z^b=\frac{cI}{e\langle B^2\rangle}\left[\frac{1}{Zn_z}\frac{\partial p_z}{\partial\psi}-\frac{1}{n_i}\frac{\partial p_i}{\partial\psi}+\frac{1.18+0.70(f_t/f_c)}{1+0.46(f_t/f_c)}\frac{\partial T_i}{\partial\psi}\right],$$

and

$$u_i^{PS}=-1.62\frac{cI}{e\langle B^2\rangle}\frac{\partial T_i}{\partial\psi},\ u_z^{PS}=\frac{cI}{e\langle B^2\rangle}\left[\frac{1}{Zn_z}\frac{\partial p_z}{\partial\psi}-\frac{1}{n_i}\frac{\partial p_i}{\partial\psi}-1.62\frac{\partial T_i}{\partial\psi}\right], \qquad \text{(A10)}$$

respectively. Expressions for u_i^b and u_z^b for α~1 are given in Ref [12]. A slightly more accurate calculation gives -1.69 rather than -1.62 in Eq. (A10) [10].

Employing Eqs. (A9) and (A10) to rewrite u_z in terms of u_i and using the results obtained in Eq. (A3) we arrive at Eq. (15).

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FIGURE CAPTION

The asymmetry factor A(ψ) for a lower single null equilibrium in Alcator C-Mod.