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Pfirsch-Schlüter Electric Field in a Tokamak

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Abstract

A concise and complete differential equation determining the Pfirsch-Schlüter radial electric field in an up-down symmetric tokamak is presented in the limit of weak poloidal magnetic field.

The differential equation for the Pfirsch-Schlüter electric field in a tokamak has been evaluated for large aspect ratio circular flux surfaces [1], for general cross sections [2,3], and for tokamaks having the poloidal magnetic field B_p small compared to the toroidal magnetic field B_t with a weak ion diamagnetic drift [4]. Recent measurements in Alcator C-Mod [5] and DIII-D [6] for subsonic ion flows indicate that the ion diamagnetic flow must be retained since it tends to cancel with the $\vec{E}\times\vec{B}$ drift velocity, where $\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi$ is the axisymmetric tokamak magnetic field with ψ the flux function, ζ is the toroidal angle variable, $I = RB_t$, R is the major radius, and $\vec{E} = -(\partial\Phi/\partial\psi)\nabla\psi$ is the lowest order electric field with the electrostatic potential Φ a flux function to lowest order. Consequently, it is convenient to use the results of [2-4] to generalize the $B_p << B_t$ expression of [4] so that it retains the ion diamagnetic drift. To do so is very straightforward since the result of [4] is consistent with the $B_p << B_t$ limit of [2,3] within the ion diamagnetic drift term that now has to be retained. Defining the ion density, temperature and pressure as n, T and p, and taking Z = 1 for the ion charge number, the lowest order ion flow velocity $\vec{V}$ on a flux surface is the usual Pfirsch-Schlüter result of Hazeltine [7]:

$$\vec{V} = \omega R^2\nabla\zeta + u(\psi)\vec{B} \tag{1}$$

with

$$\omega = -c[(\partial\Phi/\partial\psi) + (en)^{-1}(\partial p/\partial\psi)], \tag{2}$$

and

$$u = -\frac{cI}{e}\left[\frac{1.78}{\langle B^2\rangle} + \frac{0.057\langle(\nabla_{\|}\ell n B)^2\rangle}{\langle(\nabla_{\|}B)^2\rangle}\right]\frac{\partial T}{\partial\psi}\ , \qquad (3)$$

where $B = |\vec{B}|$, $\nabla_{\|} = B^{-1}\vec{B}\cdot\nabla$, and $\langle\ldots\rangle$ denotes a flux surface average. The departure of $\partial\Phi/\partial\psi$ from a generalized radial Maxwell-Boltzmann relation (ω = constant) for an up-down symmetric tokamak is then found from the radial flux of toroidal angular momentum by employing

$$-\frac{5}{6p\nu}\langle R^2\nabla\zeta\cdot\vec{\vec{\pi}}\cdot\nabla\psi\rangle = \left\langle\frac{R^4B_p^2}{\Omega^2}\right\rangle\frac{\partial\omega}{\partial\psi} + 0.19\frac{cI^2T_e(\partial T/\partial\psi)^2}{eT(T+T_e)\langle\Omega^2\rangle}\left\langle R^2\left(1-\frac{B^2}{\langle B^2\rangle}\right)\right\rangle$$

$$-0.53\frac{cT^{1/2}}{en^2}\frac{\partial}{\partial\psi}\left\{\frac{I^2n^2\langle B^2\rangle}{T^{1/2}\langle\Omega^2\rangle}\frac{\partial T}{\partial\psi}\left[\langle R^2\rangle\left\langle\left(\frac{1}{B}-\frac{B}{\langle B^2\rangle}\right)^2\right\rangle - \left\langle\frac{R^2}{B^2}\left(1-\frac{\langle R^2\rangle}{R^2}\right)\right\rangle\right]\right\}\ , \qquad (4)$$

where T_e is the electron temperature, $\nu = (4\pi^{1/2}e^4 n\ell n\Lambda)/3M^{1/2}T^{3/2}$ is the ion-ion collision frequency, and $\vec{\vec{\pi}}$ is the ion stress tensor. Setting

$$\langle R^2\nabla\zeta\cdot\vec{\vec{\pi}}\cdot\nabla\psi\rangle = 0 \qquad (5)$$

gives the differential equation determining the steady state Pfirsch-Schlüter radial electric field once a boundary condition is specified. Except for the retention of the $\partial p/\partial\psi$ term in the ω appearing in (4), Eqs. (2) and (4) are the same as Eqs. (68) and (75) of Ref. [4] for an up-down symmetric tokamak. Moreover, Eqs. (2) and (4) correspond precisely to the $B_p << B_t$ limit of Eq. (60) of Ref. [2], and are consistent with the $T_e = T$ large aspect ratio result given in Ref. [1] for which the terms in the second line of Eq. (4) may be neglected as smaller than those in the first line by an inverse aspect ratio squared. The up-down asymmetric term of Refs. [2,4] may be added into (4) as necessary.

The correct isothermal limit is now recovered [8] by (5), while for the more typical non-isothermal cases the $\partial p/\partial\psi$ term in ω is comparable to the small numerical coefficient second term on the right when $T\partial[n^{-1}\partial p/\partial\psi] \sim 0.2q^2(\partial T/\partial\psi)^2$, with q the safety factor. In fact, the ion pressure gradient term is required to understand the Alcator C-Mod measurements in collisional to semi-collisional pedstals [5] because it nearly cancels the $\partial\Phi/\partial\psi$ term in ω. Moreover, based on the entropy production argument of [9], in a banana regime pedestal of poloidal ion gyroradius width, $T\partial p/\partial\psi >> p\partial T/\partial\psi$ is required, as has been observed in DIII-D [6].

The temporal evolution of the electrostatic potential is found from the flux surface average of conservation of toroidal angular momentum:

$$\frac{\partial}{\partial t}\left[Mn(\omega\langle R^2\rangle + Iu)\right] + \frac{1}{V'}\frac{\partial}{\partial\psi}\left[V'\langle R^2\nabla\zeta\cdot\vec{\pi}\cdot\nabla\psi\rangle\right] = 0 \ , \qquad (6)$$

where $V' = \langle ... \rangle$.

In summary, we have presented a concise and complete expression for the differential equation determining the Pfirsch-Schlüter radial electric field in an up-down symmetric tokamak in the limit of weak poloidal magnetic field.

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