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Sources of intrinsic rotation in the low flow ordering

Felix I. Parra,* Michael Barnes and Peter J. Catto

*Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford, OX1 3NP, UK

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Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge MA 02139 USA

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Felix I Parra and Michael Barnes

Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford, OX1 3NP, UK

E-mail: `f.parradiaz1@physics.ox.ac.uk`

Peter J Catto

Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. A low flow, δf gyrokinetic formulation to obtain the intrinsic rotation profiles is presented. The momentum conservation equation in the low flow ordering contains new terms, neglected in previous first principles formulations, that may explain the intrinsic rotation observed in tokamaks in the absence of external sources of momentum. The intrinsic rotation profile depends on the density and temperature profiles, the up-down symmetry and the type of heating.

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1. Introduction

Experimental observations have shown that tokamak plasmas rotate spontaneously without momentum input [1]. This intrinsic rotation has been the object of recent work [1, 2] because of its relevance for ITER [3], where the projected momentum input from neutral beams is small, and the rotation is expected to be mostly intrinsic.

The origin of the intrinsic rotation is still unclear. There has been some theoretical work in turbulent transport of momentum using gyrokinetic simulations [4, 5, 6, 7, 8, 9, 10, 11, 12], and two main mechanisms have been proposed as candidates to explain intrinsic rotation. On the one hand, the momentum pinch due to the Coriolis drift [4] has been argued to transport momentum generated in the edge. On the other hand, it has also been argued that up-down asymmetry generates intrinsic rotation [7, 8]. However, neither of these explanations are able to account for all experimental observations. The up-down asymmetry is only large in the edge, generating rotation in that region that then needs to be transported inwards by the Coriolis pinch. Thus, intrinsic rotation in the core could only be explained by the pinch. The pinch of momentum is not sufficient because it does not allow the toroidal rotation to change sign in the core as is observed experimentally [13].

In this article we present a new model implementable in δf flux tube simulations [14, 15, 16, 17]. This model is based on the low flow ordering of [18], and self-consistently includes higher order contributions. As a result, new drive terms for the intrinsic rotation appear that depend on the gradients of the background profiles of density and temperature and on the heating mechanisms.

We present two new effects, related to the ion-electron collisions and the heating, that were not treated in the original work [18]. In addition, we recast the results from [18] in a form similar to the equations in the high flow ordering [19, 20]. These are the equations that have been implemented in most gyrokinetic codes that are employed to study momentum transport. For this reason, the new form of the equations is useful to identify the differences with previous models. Finally, we discuss how the new contributions drive intrinsic rotation and we show that the intrinsic rotation resulting from these new processes depends on density and temperature gradients and on the heating mechanisms.

In the remainder of this article we present the model, developed originally in [18], in a form more suitable for δf flux tube simulation. In Section 2 we give the complete model, and in Section 3 we discuss its implications for intrinsic rotation. Appendix A contains the details of the transformation from the equations in [18] to the formulation in this article. In Appendix B we discuss the treatment of the ion-electron collision operator.

2. Transport of toroidal angular momentum

The derivation of the transport of toroidal angular momentum in the low flow regime, including both turbulence and neoclassical effects, is described in detail in [18]. To simplify the derivation, the extra expansion parameter $B_p/B \ll 1$ was employed, with B the total magnetic field and B_p its poloidal component. In this section, we review the results of reference [18], recast them in a more convenient form and add a collisional term and a term that depends on the heating mechanisms that were not treated previously.

We assume that the turbulence is electrostatic and that the magnetic field is axisymmetric, i.e., $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$, where ψ is the poloidal magnetic flux, ζ is the toroidal angle, and we use a poloidal angle θ as our third spatial coordinate. With an axisymmetric magnetic field, in steady state and in the absence of momentum input, the equation that determines the rotation profile is $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}} = 0$, where $\overleftrightarrow{\mathbf{P}}_i = \int d^3v'\, f_i M\mathbf{v}'\mathbf{v}'$ is the ion stress tensor, M is the ion mass, R is the major radius, $\hat{\boldsymbol{\zeta}}$ is the unit vector in the toroidal direction, $\langle\ldots\rangle_\psi = (V')^{-1}\int d\theta\, d\zeta\,(...)/(\mathbf{B}\cdot\nabla\theta)$ is the flux surface average, $V' \equiv dV/d\psi = \int d\theta\, d\zeta\,(\mathbf{B}\cdot\nabla\theta)^{-1}$ is the derivative of the volume with respect to ψ, and $\langle\ldots\rangle_{\mathrm{T}}$ is the coarse grain or "transport" average over the time and length scales of the turbulence, much shorter than the transport time scale $\delta_i^{-2}a/v_{ti}$ and the minor radius a. Here $\delta_i = \rho_i/a \ll 1$ is the ion gyroradius ρ_i over the minor radius a, and v_{ti} is the ion thermal speed. Note that we use the prime in $\mathbf{v}'$ to indicate that the velocity is measured in the laboratory frame. Later we will find the equations in a convenient rotating frame where the velocity is $\mathbf{v} = \mathbf{v}' - R\Omega_\zeta\hat{\boldsymbol{\zeta}}$.

In reference [18] we derived a method to calculate $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}}$ to order $(B/B_p)\delta_i^3 p_i R|\nabla\psi|$, with p_i the ion pressure. We present the method again in different form to make it easier to compare with previous work in the high flow regime [19, 20]. In addition, instead of using the simplified ion Fokker-Planck equation of reference [18],

$$\frac{\partial f_i}{\partial t} + \mathbf{v}'\cdot\nabla f_i + \frac{Ze}{M}\left(-\nabla\phi + \frac{1}{c}\mathbf{v}'\times\mathbf{B}\right)\cdot\nabla_{v'} f_i = C_{ii}\{f_i\}, \tag{1}$$

where C_{ii} is the ion-ion collision operator, ϕ is the electrostatic potential, Ze is the ion charge, and e and c are the electron charge magnitude and the speed of light, in this article we use the more complete equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v}'\cdot\nabla f_i + \frac{Ze}{M}\left(-\nabla\phi + \frac{1}{c}\mathbf{v}'\times\mathbf{B}\right)\cdot\nabla_{v'} f_i = C_{ii}\{f_i\} + C_{ie}\{f_i, f_e\} + \mathcal{S}^{\mathrm{ht}}, \tag{2}$$

where $C_{ie}\{f_i, f_e\}$ is the ion-electron collision operator and $\mathcal{S}^{\mathrm{ht}} \sim \delta_i^2 f_i v_{ti}/a$ is a source that models the different heating mechanisms. Applying the procedure in reference [18] to equation (2) we find two additional terms in the expression for $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}}$ that were not considered in [18].

In subsection 2.1 we explain how we split the distribution function and the electrostatic potential into different pieces, and we present the equations to self-consistently obtain them. In subsection 2.2 we evaluate $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}}$ employing the pieces of the distribution function and the potential obtained in subsection 2.1.

Table 1. Pieces of the potential.

Potential	Size	Length scales	Time scales
$\phi_0(\psi, t)$	T_e/e	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi_1^{\rm nc}(\psi, \theta, t)$	$(B/B_p)\delta_i T_e/e$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi_2^{\rm nc}(\psi, \theta, t)$	$(B/B_p)^2\delta_i^2 T_e/e$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$\phi^{\rm tb}(\mathbf{r}, t)$	$\phi_1^{\rm tb} \sim \delta_i T_e/e$	$k_\perp \rho_i \sim 1$	$\partial/\partial t \sim v_{ti}/a$
	$\phi_2^{\rm tb} \sim (B/B_p)\delta_i^2 T_e/e$	$k_{\|\|} a \sim 1$	

Before presenting all the results, we emphasize that our results and order of magnitude estimates are valid for $B_p/B \ll 1$ and for collisionality in the range $\delta_i^2 \ll qR\nu_{ii}/v_{ti} \lesssim 1$ [18], where ν_{ii} is the ion-ion collision frequency and q is the safety factor.

2.1. Distribution function and electrostatic potential

The electrostatic potential is composed to the order of interest by the pieces in Table 1 [18]. The axisymmetric long wavelength pieces $\phi_0(\psi, t)$, $\phi_1^{\rm nc}(\psi, \theta, t)$ and $\phi_2^{\rm nc}(\psi, \theta, t)$ are the zeroth, first and second order equilibrium pieces of the potential. The lowest order component ϕ_0 is a flux surface function. The corrections $\phi_1^{\rm nc}$ and $\phi_2^{\rm nc}$ give the electric field parallel to the flux surface, established to force quasineutrality at long wavelengths (the superscript $^{\rm nc}$ refers to neoclassical because these are long wavelength contributions; however, we will show that turbulence can affect the final value of $\phi_1^{\rm nc}$ and $\phi_2^{\rm nc}$). We need not calculate $\phi_2^{\rm nc}$ because it will not appear in the final expression for $\langle\langle R\hat{\boldsymbol{\zeta}} \cdot \overleftrightarrow{\mathbf{P}}_i \cdot \nabla\psi\rangle_\psi\rangle_{\rm T}$. The piece $\phi^{\rm tb}(\mathbf{r}, t)$ is turbulent and includes both axisymmetric components (zonal flow) and non-axisymmetric fluctuations. It is small in δ_i but it has strong perpendicular gradients, i.e., $k_\perp \rho_i \sim 1$. Its parallel gradient is small, i.e., $k_{||} a \sim 1$. The function $\phi^{\rm tb}$ is calculated to order $(B/B_p)\delta_i^2 T_e/e$, i.e., $\phi^{\rm tb} = \phi_1^{\rm tb} + \phi_2^{\rm tb}$ with $\phi_1^{\rm tb} \sim \delta_i T_e/e$ and $\phi_2^{\rm tb} \sim (B/B_p)\delta_i^2 T_e/e$. It is convenient to keep both pieces together as $\phi^{\rm tb}$ as we do hereafter.

To write the distribution function it will be useful to consider the reference frame that rotates with toroidal angular velocity $\Omega_\zeta = -c\,\partial_\psi\phi_0 - (c/Zen_i)\partial_\psi p_i$, where $n_i(\psi, t)$ and $p_i(\psi, t)$ are the lowest order ion density and pressure. In this new reference frame it is easier to compare with previous formulations [19, 20]. To shorten the presentation, we perform the change of reference frame directly in the gyrokinetic variables. It is possible to do so easily because we are expanding in the parameter $B/B_p \gg 1$. We first present the gyrokinetic variables that we obtained for the laboratory frame and we argue later how they must be modified to give the gyrokinetic variables in the rotating frame. In [18] we used as gyrokinetic variables the gyrocenter position $\mathbf{R} = \mathbf{r} + \mathbf{R}_1 + \mathbf{R}_2 + \ldots$, the gyrokinetic kinetic energy $E = E_0 + E_1 + E_2 + \ldots$, the magnetic moment $\mu = \mu_0 + \mu_1 + \ldots$ and the gyrokinetic gyrophase $\varphi = \varphi_0 + \varphi_1 + \ldots$, where $E_0 = (v')^2/2$ is the particle kinetic energy in the laboratory frame, $\mu_0 = (v'_\perp)^2/2B$ is the lowest order magnetic moment, $\varphi_0 = \arctan(\mathbf{v}' \cdot \hat{\mathbf{e}}_2/\mathbf{v}' \cdot \hat{\mathbf{e}}_1)$ is the lowest order gyrophase, $\mathbf{R}_1 = \Omega_i^{-1}\mathbf{v}' \times \hat{\mathbf{b}} \sim \delta_i L$ is the first order correction to the gyrocenter position, $E_1 = Ze(\phi - \langle\phi\rangle)/M \sim \delta_i v_{ti}^2$ is the first order correction to the gyrokinetic kinetic energy, and the corrections $\mathbf{R}_2 \sim \delta_i^2 L$, $E_2 \sim \delta_i^2 v_{ti}^2$, $\mu_1 \sim \delta_i v_{ti}^2/B$ and $\varphi_1 \sim \delta_i$ are defined in [21]. Here $\Omega_i = ZeB/Mc$ is the ion gyrofrequency, $\hat{\mathbf{e}}_1(\mathbf{r})$ and $\hat{\mathbf{e}}_2(\mathbf{r})$ are two orthonormal vectors such that $\hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2 = \hat{\mathbf{b}}$, and $\langle \ldots \rangle = (2\pi)^{-1} \oint d\varphi\,(\ldots)|_{\mathbf{R},E,\mu,t}$ is the gyroaverage holding $\mathbf{R}$, E, μ and t fixed. When the ion distribution function is written as a function of these gyrokinetic variables, it does not depend on the gyrophase φ up to order $(B_p/B)\delta_i^2(qR\nu_{ii}/v_{ti})f_{Mi}$ [18, 21], where f_{Mi} is the lowest order distribution function that is a Maxwellian. For the magnetic moment and the gyrophase, only the first order corrections μ_1 and φ_1 are needed because the lowest order distribution function f_{Mi} does not depend on μ or φ. Moreover, since in [18] we expand on $B/B_p \gg 1$, the distribution function need only be known to order $(B/B_p)\delta_i^2 f_{Mi}$. Consequently, the piece of the distribution function that depends on the gyrophase, of order $(B_p/B)\delta_i^2(qR\nu_{ii}/v_{ti})f_{Mi}$, is negligible, and the gyrokinetic variables $\mathbf{R}$, E, μ and φ only need to be obtained to order $(B/B_p)\delta_i^2 L$, $(B/B_p)\delta_i^2 v_{ti}^2$, $(B/B_p)\delta_i v_{ti}^2/B$ and $(B/B_p)\delta_i$, respectively, implying that the corrections $\mathbf{R}_2$, E_2, μ_1 and φ_1 are not needed for the final result. To change to the new reference frame, where the velocity is $\mathbf{v} = \mathbf{v}' - R\Omega_\zeta\hat{\boldsymbol{\zeta}}$, the distribution function that is independent of φ has to be written as a function of the new gyrokinetic variables $\mathbf{R}$, ε and μ. Note that the gyrocenter position and the magnetic moment are the same in both reference frames to the order of interest. The reason is that the toroidal rotation has two components, one parallel to the magnetic field, $R\Omega_\zeta\hat{\boldsymbol{\zeta}} \cdot \hat{\mathbf{b}} = I\Omega_\zeta/B \sim (B/B_p)\delta_i v_{ti}$, and the other perpendicular, $R\Omega_\zeta|\hat{\boldsymbol{\zeta}} - \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \hat{\boldsymbol{\zeta}}| = |\nabla\psi|\Omega_\zeta/B \sim \delta_i v_{ti}$, and the parallel velocity is larger by $B/B_p \gg 1$. Since in [18] the gyrokinetic variables $\mathbf{R}$ and μ are to be obtained to order $(B/B_p)\delta_i^2 L$ and $(B/B_p)\delta_i v_{ti}^2/B$, and in $\mathbf{R}$ and μ only the perpendicular velocity $\mathbf{v}'_\perp = \mathbf{v}_\perp + R\Omega_\zeta(\hat{\boldsymbol{\zeta}} - \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \hat{\boldsymbol{\zeta}})$ enters, we can safely neglect the corrections due to the change of reference frame because they are of order $\delta_i^2 L$ and $\delta_i v_{ti}^2/B$, respectively. In contrast, the kinetic energy E as defined in [18] cannot be used in the rotating frame because it includes the parallel velocity $v'_{||} = v_{||} + I\Omega_\zeta/B$. We use a new kinetic energy variable ε that is related to the old kinetic energy variable by $\varepsilon = E - I\Omega_\zeta u'/B$, where $u' = \pm\sqrt{2(E - \mu B)}$ is the gyrokinetic parallel velocity in the laboratory frame. It is easy to check that $u = \pm\sqrt{2[\varepsilon - \mu B + (I/B)^2\Omega_\zeta^2/2]}$ is equal to $u = u' - I\Omega_\zeta/B$ and it is the gyrokinetic parallel velocity in the rotating frame. With this relation, we find that another way to interpret the new energy variable

$$\varepsilon = \frac{u^2}{2} + \mu B - \frac{R^2\Omega_\zeta^2}{2} \tag{3}$$

is realizing that it is the kinetic energy in the rotating frame plus the potential due to the centrifugal force. To write expression (3) we have used that $I/B \simeq R$ for $B_p/B \ll 1$. In Appendix A we rewrite the results in [18] using the new gyrokinetic kinetic energy ε.

The different pieces of the ion distribution function are given in Table 2 [18]. In this table, m is the electron mass. The functions f_{Mi}, H_{i1}^{nc}, H_{i2}^{nc}, H_{i2}^{tb}, H_{i2}^{ie} and H_{i2}^{ht} are axisymmetric long wavelength contributions. The Maxwellian $f_{Mi}(\psi(\mathbf{R}), \varepsilon)$ is uniform in a flux surface. The first and second order corrections H_{i1}^{nc} and H_{i2}^{nc} are neoclassical

Table 2. Pieces of the ion distribution function.

Distribution function	Size	Length scales	Time scales
$f_{Mi}(\psi(\mathbf{R}),\varepsilon,t)$	f_{Mi}	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i1}^{\rm nc}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)\delta_i f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm nc}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)^2\delta_i^2 f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm tb}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2 f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{ie}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)\delta_i\sqrt{m/M}f_{Mi}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{i2}^{\rm ht}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)(v_{ti}/qR\nu_{ii})\mathcal{S}^{\rm ht}a/v_{ti}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$f_i^{\rm tb}(\mathbf{R},\varepsilon,\mu,t)$	$f_{i1}^{\rm tb} \sim \delta_i f_{Mi}$	$k_\perp \rho_i \sim 1$	$\partial/\partial t \sim v_{ti}/a$
	$f_{i2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Mi}$	$k_{\|\|}a \sim 1$	

Table 3. Pieces of the electron distribution function.

Distribution function	Size	Length scales	Time scales
$f_{Me}(\psi(\mathbf{R}),\varepsilon,t)$	f_{Me}	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$H_{e1}^{\rm nc}(\psi(\mathbf{R}),\theta(\mathbf{R}),\varepsilon,\mu,t)$	$(B/B_p)\delta_i f_{Me}$	$ka \sim 1$	$\partial/\partial t \sim \delta_i^2 v_{ti}/a$
$f_e^{\rm tb}(\mathbf{R},\varepsilon,\mu,t)$	$f_{e1}^{\rm tb} \sim \delta_i f_{Me}$	$k_\perp \rho_i \sim 1$	$\partial/\partial t \sim v_{ti}/a$
	$f_{e2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Me}$	$k_{\|\|}a \sim 1$	

corrections, and they are not the functions $F_{i1}^{\rm nc}$ and $F_{i2}^{\rm nc}$ in [18] because we are now working in the rotating frame. The function $H_{i2}^{\rm tb}$ is an axisymmetric piece of the distribution function that originates from collisions acting on the ions transported by turbulent fluctuations into a given flux surface [18]. The functions H_{i2}^{ie} and $H_{i2}^{\rm ht}$ that were not included in [18] have their origin in the ion-electron collisions and in the heating mechanism. The function $f_i^{\rm tb}$ is the turbulent contribution. It will be determined self-consistently up to order $(B/B_p)\delta_i^2 f_{Mi}$, i.e., $f_i^{\rm tb} = f_{i1}^{\rm tb} + f_{i2}^{\rm tb}$ with $f_{i1}^{\rm tb} \sim \delta_i f_{Mi}$ and $f_{i2}^{\rm tb} \sim (B/B_p)\delta_i^2 f_{Mi}$. It is convenient to combine both pieces of the turbulent distribution function into one function $f_i^{\rm tb}$.

The electron distribution function is very similar to the ion distribution function. It will have its own gyrokinetic variables that can be easily deduced from the ion counterparts. To the order of interest in this calculation, the electron distribution function is determined by the pieces in Table 3. The long wavelength, axisymmetric pieces f_{Me} and $H_{e1}^{\rm nc}$ are the lowest order Maxwellian and the first order neoclassical correction. The second order long wavelength neoclassical correction is not needed for transport of momentum because of the small electron mass. The piece $f_e^{\rm tb}$ is the short wavelength, turbulent component that will be self-consistently calculated to order $(B/B_p)\delta_i^2 f_{Me}$.

We now proceed to describe how to find the different pieces of the distribution function and the potential. We use the equations in [18] but we change to the new gyrokinetic kinetic energy ε. The details of this transformation are contained in Appendix A.

2.1.1. First order neoclassical distribution function and potential. The equation for $H_{i1}^{\rm nc}$ is

$$u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\left[H_{i1}^{\rm nc}+\frac{Ze\phi_1^{\rm nc}}{T_i}f_{Mi}+\left(\frac{M\varepsilon}{T_i}-\frac{5}{2}\right)\frac{Iuf_{Mi}}{\Omega_i T_i}\frac{\partial T_i}{\partial\psi}\right]-C_{ii}^{(\ell)}\{H_{i1}^{\rm nc}\}=0, \quad (4)$$

where $u=\pm\sqrt{2(\varepsilon-\mu B+R^2\Omega_\zeta^2/2)}\simeq\pm\sqrt{2(\varepsilon-\mu B)}$ is the gyrokinetic parallel velocity and $C_{ii}^{(\ell)}$ is the linearized ion-ion collision operator. The correction $H_{i1}^{\rm nc}$ gives the parallel component of the velocity [22, 23] $\mathbf{W}_i^{\rm nc}=\hat{\mathbf{b}}\int d^3v\,H_{i1}^{\rm nc}v_{||}=(kcI\mathbf{B}/Ze\langle B^2\rangle_\psi)\partial_\psi T_i$, where k is a constant that depends on the collisionality and the magnetic geometry. Interestingly, the density perturbation due to $H_{i1}^{\rm nc}$ is small for $qR\nu_{ii}/v_{ti}\ll 1$, i.e., $\int d^3v\,H_{i1}^{\rm nc}\sim(B/B_p)(qR\nu_{ii}/v_{ti})\delta_i n_i\ll(B/B_p)\delta_i n_i$ [18]. This will be important when determining $\phi_1^{\rm nc}$ below.

The equation for $H_{e1}^{\rm nc}$ is similar to (4) and it is given by [22, 23]

$$u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\left\{H_{e1}^{\rm nc}-\frac{e\phi_1^{\rm nc}}{T_e}f_{Me}-\left[\frac{1}{Zn_iT_e}\frac{\partial p_i}{\partial\psi}+\frac{1}{p_e}\frac{\partial p_e}{\partial\psi}+\left(\frac{M\varepsilon}{T_e}-\frac{5}{2}\right)\frac{1}{T_e}\frac{\partial T_e}{\partial\psi}\right]\frac{Iuf_{Me}}{\Omega_e}\right\}$$
$$-C_{ee}^{(\ell)}\{H_{e1}^{\rm nc}\}-C_{ei}^{(\ell)}\{H_{e1}^{\rm nc}\}=-\frac{ef_{Me}}{T_e}u\hat{\mathbf{b}}\cdot\mathbf{E}^A, \quad (5)$$

where $\Omega_e=eB/mc$ is the electron gyrofrequency, $\mathbf{E}^A$ is the electric field driven by the transformer, $C_{ee}^{(\ell)}$ is the linearized electron-electron collision operator and $C_{ei}^{(\ell)}$ is the linearized electron-ion collision operator. The lowest order solution for $H_{e1}^{\rm nc}$ is the Maxwell-Boltzmann response $(e\phi_1^{\rm nc}/T_e)f_{Me}\sim(B/B_p)\delta_i f_{Me}$. The rest of the terms are small because they are of order $(B/B_p)\delta_e f_{Me}\sim(B/B_p)\sqrt{m/M}\delta_i f_{Mi}\ll(B/B_p)\delta_i f_{Me}$, where $\delta_e=\rho_e/a$ is the ratio between the electron gyroradius ρ_e and the minor radius a.

Finally the poloidal variation of the potential is determined by quasineutrality,

$$Z\int d^3v\,H_{i1}^{\rm nc}+Z\int d^3v\,H_{i2}^{\rm tb}=\frac{e\phi_1^{\rm nc}}{T_e}n_e, \quad (6)$$

giving $e\phi_1^{\rm nc}/T_e\sim(B/B_p)(qR\nu_{ii}/v_{ti})\delta_i$. We have included the density $\int d^3v\,H_{i2}^{\rm tb}\sim(B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2 n_i$ because it becomes important for $qR\nu_{ii}/v_{ti}\lesssim(f_i^{\rm tb}/f_{Mi})\sqrt{a/\rho_i}\ll 1$ with $f_i^{\rm tb}/f_{Mi}\sim\rho_i/a$ [18].

2.1.2. Turbulent distribution function and potential. The turbulent piece of the ion distribution function is obtained using the gyrokinetic equation

$$\frac{Df_i^{\rm tb}}{Dt}+\left(u\hat{\mathbf{b}}+\mathbf{v}_M+\mathbf{v}_C+\mathbf{v}_E^{\rm tb}\right)\cdot\nabla_{\mathbf{R}}f_i^{\rm tb}-\left\langle C_{ii}^{(\ell)}\left\{h_i^{\rm tb}\right\}\right\rangle-\left\langle C_{ie}^{(\ell)}\left\{h_i^{\rm tb},h_e^{\rm tb}\right\}\right\rangle$$
$$=-\mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}}\psi\left[\frac{1}{n_i}\frac{\partial n_i}{\partial\psi}+\left(\frac{M\varepsilon}{T_i}-\frac{3}{2}\right)\frac{1}{T_i}\frac{\partial T_i}{\partial\psi}+\frac{MIu}{BT_i}\frac{\partial\Omega_\zeta}{\partial\psi}\right]f_{Mi}-\mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}}H_{i1}^{\rm nc}$$
$$-\frac{Zef_{Mi}}{T_i}\left(u\hat{\mathbf{b}}+\mathbf{v}_M+\mathbf{v}_C\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle+\frac{Ze}{M}\frac{\partial H_{i1}^{\rm nc}}{\partial\varepsilon}\left(u\hat{\mathbf{b}}+\mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle, \quad (7)$$

where $D/Dt=\partial_t+R\Omega_\zeta\hat{\boldsymbol{\zeta}}\cdot\nabla_{\mathbf{R}}$ is the time derivative in the rotating frame, $u=\pm\sqrt{2[\varepsilon-\mu B+R^2\Omega_\zeta^2/2]}\simeq\pm\sqrt{2(\varepsilon-\mu B)}$ is the parallel velocity in the rotating frame, $\mathbf{v}_M=(\mu/\Omega_i)\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B+(u^2/\Omega_i)\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}})$ are the ∇B and curvature drifts,

$\mathbf{v}_C = (2u\Omega_\zeta/\Omega_i)\hat{\mathbf{b}}\times[(\nabla R\times\hat{\boldsymbol{\zeta}})\times\hat{\mathbf{b}}]$ is the Coriolis drift, $\mathbf{v}_E^{\rm tb} = -(c/B)\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\times\hat{\mathbf{b}}$ is the turbulent $\mathbf{E}\times\mathbf{B}$ drift, $C_{ii}^{(\ell)}\{h_i^{\rm tb}\}$ is the linearized ion-ion collision operator, $C_{ie}^{(\ell)}\{h_i^{\rm tb}, h_e^{\rm tb}\}$ is the linearized ion-electron collision operator, and $\langle\ldots\rangle = (2\pi)^{-1}\oint d\varphi\,(\ldots)|_{\mathbf{R},E,\mu,t}$ is the gyroaverage holding $\mathbf{R}$, E, μ and t fixed. The ion-electron collision operator can be approximated by

$$C_{ie}^{(\ell)}\{h_i^{\rm tb}, h_e^{\rm tb}\} = \frac{n_e m\nu_{ei}(T_e - T_i)}{p_i M}\left(\frac{Mv^2}{T_i} - 3\right)\frac{e\phi^{\rm tb}}{T_e} f_{Mi} + \frac{n_e m\nu_{ei}}{n_i M}\nabla_v\cdot\left(\frac{T_e}{M}\nabla_v h_i^{\rm tb} + \mathbf{v}h_i^{\rm tb}\right) - \frac{1}{p_i}(\mathbf{F}_{ei}^{\rm tb} - n_e m\nu_{ei}\mathbf{W}_i^{\rm tb})\cdot\mathbf{v}f_{Mi}, \tag{8}$$

where $\nu_{ei} = (4\sqrt{2\pi}/3)Z^2e^4 n_i\ln\Lambda/m^{1/2}T_e^{3/2}$ is the electron-ion collision frequency, $\ln\Lambda$ is Coulomb's logarithm, $n_i\mathbf{W}_i^{\rm tb} = \int d^3v\, h_i^{\rm tb}\mathbf{v}$ is the turbulent ion flow, and

$$\mathbf{F}_{ei}^{\rm tb} = n_e m\nu_{ei}\mathbf{W}_i^{\rm tb} - \frac{2\pi Z^2 e^4 n_i\ln\Lambda}{m}\int d^3v_e\,\nabla_{v_e}\nabla_{v_e}v_e\cdot\nabla_v h_{e1}^{\rm tb} \tag{9}$$

is the friction force on the electrons due to collisions with ions. The functions that enter in the collision operators are

$$h_i^{\rm tb} = f_{ig}^{\rm tb} + \frac{Ze(\phi^{\rm tb} - \langle\phi^{\rm tb}\rangle)}{M}\left(-\frac{Mf_{Mi,0}}{T_i} + \frac{\partial H_{i1,0}^{\rm nc}}{\partial\varepsilon_0} + \frac{1}{B}\frac{\partial H_{i1,0}^{\rm nc}}{\partial\mu_0}\right) \tag{10}$$

and

$$h_e^{\rm tb} = f_{e0}^{\rm tb} - \frac{1}{\Omega_e}(\mathbf{v}\times\hat{\mathbf{b}})\cdot\nabla f_{e0}^{\rm tb} + \frac{mcf_{Me,0}}{BT_e}(\mathbf{v}\times\hat{\mathbf{b}})\cdot\nabla\phi^{\rm tb}. \tag{11}$$

Here the subscript $_g$ in $f_{ig}^{\rm tb} = f_i^{\rm tb}(\mathbf{R}_g, v^2/2, v_\perp^2/2B, t)$ indicates that we have replaced the variables $\mathbf{R}$, ε and μ by $\mathbf{R}_g = \mathbf{r} + \Omega_i^{-1}\mathbf{v}\times\hat{\mathbf{b}}$, $v^2/2$ and $v_\perp^2/2B$; similarly, the subscript $_0$ in $f_{Mi,0} = f_{Mi}(\psi(\mathbf{r}), v^2/2, t)$, $f_{Me,0} = f_{Me}(\psi(\mathbf{r}), v^2/2, t)$, $H_{i1,0}^{\rm nc} = H_{i1}^{\rm nc}(\psi(\mathbf{r}), \theta(\mathbf{r}), v^2/2, v_\perp^2/2B, t)$, $H_{e1,0}^{\rm nc} = H_{e1}^{\rm nc}(\psi(\mathbf{r}), \theta(\mathbf{r}), v^2/2, v_\perp^2/2B, t)$ and $f_{e0}^{\rm tb} = f_e^{\rm tb}(\mathbf{r}, v^2/2, v_\perp^2/2B, t)$ indicates that we have replaced the variables $\mathbf{R}$, ε and μ by $\mathbf{r}$, $v^2/2$ and $v_\perp^2/2B$.

The equation for electrons is equivalent to the one for the ions, giving

$$\frac{Df_e^{\rm tb}}{Dt} + \left(u\hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_E^{\rm tb}\right)\cdot\nabla_{\mathbf{R}}f_e^{\rm tb} - \langle C_{ee}^{(\ell)}\{h_e^{\rm tb}\}\rangle - \left\langle C_{ei}^{(\ell)}\{h_e^{\rm tb}\}\right\rangle = -\mathbf{v}_E^{\rm tb}\cdot\nabla_{\mathbf{R}}\psi\left[\frac{1}{n_e}\frac{\partial n_e}{\partial\psi} + \left(\frac{M\varepsilon}{T_e} - \frac{3}{2}\right)\frac{1}{T_e}\frac{\partial T_e}{\partial\psi}\right]f_{Me} + \frac{ef_{Me}}{T_e}\left(u\hat{\mathbf{b}} + \mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle, \tag{12}$$

where $C_{ee}^{(\ell)}$ is the linearized electron-electron collision operator and $C_{ei}^{(\ell)}$ is the linearized electron-ion collision operator. If we were to neglect the effect of the trapped electrons, the solution to this equation would simply be the adiabatic response $f_e^{\rm tb} \simeq (e\langle\phi^{\rm tb}\rangle/T_e)f_{Me}$.

Finally, the electrostatic potential $\phi^{\rm tb}$ is obtained from the quasineutrality equation

$$Z\int d^3v\,\frac{Ze(\phi^{\rm tb} - \langle\phi^{\rm tb}\rangle)}{M}\left[-\frac{Mf_{Mi,0}}{T_i} + \left(\frac{\partial H_{i1,0}^{\rm nc}}{\partial\varepsilon_0} + \frac{1}{B}\frac{\partial H_{i1,0}^{\rm nc}}{\partial\mu_0}\right)\right] + Z\int d^3v\, f_{ig}^{\rm tb} = \int d^3v\, f_{eg}^{\rm tb}. \tag{13}$$

2.1.3. Second order, long wavelength distribution function. The long wavelength pieces $H_{i2}^{\rm nc}$, $H_{i2}^{\rm tb}$, H_{i2}^{ie} and $H_{i2}^{\rm ht}$ are given by

$$u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}H_{i2}^{\alpha}-C_{ii}^{(\ell)}\{H_{i2}^{\alpha}\}=\mathcal{S}^{\alpha}-\Bigg\langle\int d^3v\,\mathcal{S}^{\alpha} + \left(\frac{2M\varepsilon}{3T_i}-1\right)\int d^3v\,\mathcal{S}^{\alpha}\left(\frac{M\varepsilon}{T_i}-\frac{3}{2}\right)\Bigg\rangle_{\psi}\frac{f_{Mi}}{n_i}, \tag{14}$$

where $\alpha=$ nc, tb, ie, ht, and

$$\begin{aligned}\mathcal{S}^{\rm nc}=&-\frac{MIuf_{Mi}}{BT_i}\frac{\partial\Omega_\zeta}{\partial\psi}\mathbf{v}_M\cdot\nabla_{\mathbf{R}}\psi-\left(\mathbf{v}_C-\frac{c}{B}\nabla_{\mathbf{R}}\phi_1^{\rm nc}\times\hat{\mathbf{b}}\right)\cdot\nabla_{\mathbf{R}}\psi\left(\frac{M\varepsilon}{T_i}-\frac{5}{2}\right)\frac{f_{Mi}}{T_i}\frac{\partial T_i}{\partial\psi}\\&-\mathbf{v}_M\cdot\nabla_{\mathbf{R}}H_{i1}^{\rm nc}+\frac{I}{n_iM\Omega_i}\frac{\partial p_i}{\partial\psi}\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}H_{i1}^{\rm nc}-\frac{Zef_{Mi}}{T_i}\left(u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\phi_2^{\rm nc}+\mathbf{v}_M\cdot\nabla_{\mathbf{R}}\phi_1^{\rm nc}\right)\\&+\frac{Ze}{M}\left(u\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\phi_1^{\rm nc}-\frac{1}{Zen_i}\frac{\partial p_i}{\partial\psi}\mathbf{v}_M\cdot\nabla_{\mathbf{R}}\psi\right)\frac{\partial H_{i1}^{\rm nc}}{\partial\varepsilon}+\left\langle C_{ii}^{(n\ell)}\{H_{i1}^{\rm nc},H_{i1}^{\rm nc}\}\right\rangle;\end{aligned} \tag{15}$$

$$\mathcal{S}^{\rm tb}=-\frac{|u|}{B}\nabla_{\mathbf{R}}\cdot\left(\frac{B}{|u|}\left\langle f_i^{\rm tb}\mathbf{v}_E^{\rm tb}\right\rangle_{\rm T}\right)+\frac{Ze}{M}\frac{|u|}{B}\frac{\partial}{\partial\varepsilon}\left(\frac{B}{|u|}\left\langle f_i^{\rm tb}\left(u\hat{\mathbf{b}}+\mathbf{v}_M\right)\cdot\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\right\rangle_{\rm T}\right); \tag{16}$$

$$\mathcal{S}^{ie}=\frac{n_em\nu_{ei}}{n_iM}\left\langle\nabla_v\cdot\left(\frac{T_e}{M}\nabla_vH_{i1}^{\rm nc}+\mathbf{v}H_{i1}^{\rm nc}\right)\right\rangle-\frac{u}{p_i}(\mathbf{F}_{ei}^{\rm nc}-n_em\nu_{ei}\mathbf{W}_i^{\rm nc})\cdot\hat{\mathbf{b}}f_{Mi}, \tag{17}$$

where $n_i\mathbf{W}_i^{\rm nc}=\hat{\mathbf{b}}\int d^3v\,H_{i1}^{\rm nc}v_{||}$ is the axisymmetric long wavelength ion flow and

$$\mathbf{F}_{ei}^{\rm nc}=n_em\nu_{ei}\mathbf{W}_i^{\rm nc}-\frac{2\pi Z^2e^4n_i\ln\Lambda}{m}\int d^3v_e\,\nabla_{v_e}\nabla_{v_e}v_e\cdot\nabla_vH_{e1}^{\rm nc} \tag{18}$$

is the axisymmetric long wavelength friction force on the electrons due to collisions with ions; and $\mathcal{S}^{\rm ht}$ is the source in the kinetic equation that mimics the heating mechanism. For radiofrequency heating, $\mathcal{S}^{\rm ht}$ can be obtained from the quasilinear models that are widely used. It is also possible to use model sources. For example, in [24, 25] simplified sources were employed to study for the first time the effect of radiofrequency heating on transport of momentum.

2.2. Calculation of the momentum transport

We obtain an equation for $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\mathbf{P}}_i\cdot\nabla\psi\rangle_\psi\rangle_{\rm T}$ similar to equation (39) of [18] by employing the same procedure that was used in that reference, but starting from the more complete Fokker-Planck equation (2). The final result is as in equation (39) of [18] plus the new terms

$$-\frac{M^2c}{2Ze}\left\langle\left\langle\int d^3v'\,C_{ie}\{f_i\}R^2(\mathbf{v}'\cdot\hat{\boldsymbol{\zeta}})^2+\int d^3v'\,\mathcal{S}^{\rm ht}(\mathbf{r},\mathbf{v}')R^2(\mathbf{v}'\cdot\hat{\boldsymbol{\zeta}})^2\right\rangle_\psi\right\rangle_{\rm T}. \tag{19}$$

Adding these new terms to expression (39) from [18] and using that for $B/B_p\gg1$, $R\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\simeq Iv_{||}/B$, we find

$$\langle\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\mathbf{P}}_i\cdot\nabla\psi\rangle_\psi\rangle_{\rm T}=\Pi_{-1}^{\rm tb}+\Pi_0^{\rm tb}+\Pi_{-1}^{\rm nc}+\Pi_0^{\rm nc}+\Pi_{-1}^{ie}+\Pi_0^{ie}+\Pi^{\rm ht}+\frac{Mc\langle R^2\rangle_\psi}{2Ze}\frac{\partial p_i}{\partial t}, \tag{20}$$

Table 4. Contributions to transport of momentum.

Π	Size $[(B/B_p)\delta_i^3 p_i R\vert\nabla\psi\vert]$	Dependences
$\Pi_{-1}^{\rm tb}$	$(B_p/B)\Delta_{ud}\delta_i^{-1}$ for $\Delta_{ud} \gtrsim (B/B_p)\delta_i$	$\partial_\psi\Omega_\zeta, \Omega_\zeta, \Delta_{ud}, \partial_\psi T_i, \partial_\psi n_e, \partial_\psi T_e, \partial_\psi^2 T_i$
	1 for $\Delta_{ud} \lesssim (B/B_p)\delta_i$	
$\Pi_0^{\rm tb}$	1	$\partial_\psi T_i, \partial_\psi n_e, \partial_\psi T_e, \partial_\psi^2 T_i, \partial_\psi^2 n_e, \partial_\psi^2 T_e$
$\Pi_{-1}^{\rm nc}$	$\Delta_{ud}(qR\nu_{ii}/v_{ti})\delta_i^{-1}$ for $\Delta_{ud} \gtrsim (B/B_p)\delta_i$	$\partial_\psi\Omega_\zeta, \Delta_{ud}, \partial_\psi T_i, \partial_\psi n_e, \partial_\psi^2 T_i$
	$(B/B_p)(qR\nu_{ii}/v_{ti})$ for $\Delta_{ud} \lesssim (B/B_p)\delta_i$	
$\Pi_0^{\rm nc}$	$(B/B_p)(qR\nu_{ii}/v_{ti})$	$\partial_\psi T_i, \partial_\psi n_e, \partial_\psi^2 T_i$
Π_{-1}^{ie}	$(B_p/B)(qR\nu_{ii}/v_{ti})\delta_i^{-2}\sqrt{m/M}$	$T_i - T_e$
Π_0^{ie}	$(qR\nu_{ii}/v_{ti})\delta_i^{-1}\sqrt{m/M}$	$\partial_\psi T_i, \partial_\psi n_e, \partial_\psi T_e, \mathbf{E}^A$
$\Pi^{\rm ht}$	$\delta_i^{-2}(\mathcal{S}^{\rm ht} a/v_{ti} f_{Mi})$	Heating

with

$$\Pi_{-1}^{\rm tb} = -\left\langle\left\langle \frac{c}{B}(\nabla\phi^{\rm tb}\times\hat{\mathbf{b}})\cdot\nabla\psi \int d^3v\, f_{ig}^{\rm tb}\left(\frac{IMv_{||}}{B} + MR\Omega_\zeta\right)\right\rangle_\psi\right\rangle_{\rm T}, \tag{21}$$

$$\begin{aligned}\Pi_0^{\rm tb} = &-\frac{M^2c}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\left\langle \frac{c}{B}(\nabla\phi^{\rm tb}\times\hat{\mathbf{b}})\cdot\nabla\psi\int d^3v\, f_{ig}^{\rm tb}\frac{I^2v_{||}^2}{B^2}\right\rangle_{\rm T}\right\rangle_\psi \\ &+\left\langle\left\langle\frac{cI}{B}\hat{\mathbf{b}}\cdot\nabla\phi^{\rm tb}\int d^3v\, f_{ig}^{\rm tb}\frac{IMv_{||}}{B}\right\rangle_\psi\right\rangle_{\rm T} - \frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i2,0}^{\rm tb}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi, \end{aligned}\tag{22}$$

$$\Pi_{-1}^{\rm nc} = -\frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i1,0}^{\rm nc} + H_{i2,0}^{\rm nc}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi, \tag{23}$$

$$\begin{aligned}\Pi_0^{\rm nc} = &-\frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(n\ell)}\{H_{i1,0}^{\rm nc}, H_{i1,0}^{\rm nc}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi \\ &-\frac{M^3c^2}{6Z^2e^2}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i1,0}^{\rm nc}\}\frac{I^3v_{||}^3}{B^3}\right\rangle_\psi, \end{aligned}\tag{24}$$

$$\Pi_{-1}^{ie} = \frac{n_e mc\nu_{ei}}{Ze}\left\langle R^2\left(1 + \frac{e\phi_1^{\rm nc}}{T_e}\right)\right\rangle_\psi (T_i - T_e), \tag{25}$$

$$\Pi_0^{ie} = -\frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i2,0}^{ie}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi + \frac{p_e mc\nu_{ei}}{Zen_i}\left\langle\frac{I^2}{B^2}\int d^3v\, H_{i1}^{\rm nc}\left(\frac{Mv_{||}^2}{T_e} - 1\right)\right\rangle_\psi \tag{26}$$

and

$$\Pi^{\rm ht} = -\frac{M^2c}{2Ze}\left\langle\int d^3v\, C_{ii}^{(\ell)}\{H_{i2,0}^{\rm ht}\}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi - \frac{M^2c}{2Ze}\left\langle\int d^3v\,\mathcal{S}^{\rm ht}\frac{I^2v_{||}^2}{B^2}\right\rangle_\psi. \tag{27}$$

Recall that the subscript $_g$ indicates that $\mathbf{R}$, ε and μ have been replaced by $\mathbf{R}_g$, $v^2/2$ and $v_\perp^2/2B$, and the subscript $_0$ that they have been replaced by $\mathbf{r}$, $v^2/2$ and $v_\perp^2/2B$. In Table 4 we summarize the size of all these contributions compared to the reference size $(B/B_p)\delta_i^3 p_i R|\nabla\psi|$, and we write what they depend on. To obtain these dependences, we use equations (4), (5), (6), (7), (12), (13) and (14). Most of the size estimates are taken from [18], except for Π_{-1}^{ie}, Π_0^{ie} and Π^{ht} that are trivially found from the results here. We use Δ_{ud} to denote a measure of the flux surface up-down asymmetry. It ranges from zero for perfect up-down symmetry to one for extreme asymmetry. Notice that for extreme up-down asymmetry, Π_{-1}^{tb} and Π_{-1}^{nc} clearly dominate. The contribution Π_{-1}^{ie} is formally very large for $qR\nu_{ii}/v_{ti} \sim 1$, but since the ion energy conservation equation requires that $(T_i - T_e)/T_i \sim (B/B_p)(v_{ti}/qR\nu_{ii})\delta_i^2\sqrt{M/m}$, it will always be comparable to $(B/B_p)\delta_i^3 p_i R|\nabla\psi|$.

3. Discussion

We finish by showing how this new formalism gives a plausible model for intrinsic rotation. Until now, models have only considered the contribution Π_{-1}^{tb}, with f_i^{tb} and ϕ^{tb} obtained by employing equations (7) and (13) without the terms that contain H_{i1}^{nc}. This is acceptable for $R\Omega_\zeta \sim v_{ti}$ or high up-down asymmetry $\Delta_{ud} \sim 1$. In this limit, $\Pi_{-1}^{\mathrm{tb}}(\partial_\psi\Omega_\zeta, \Omega_\zeta) \simeq -\nu^{\mathrm{tb}}\partial_\psi\Omega_\zeta - \Gamma^{\mathrm{tb}}\Omega_\zeta + \Pi_{ud}^{\mathrm{tb}}$. To obtain this last expression we have linearized around $\partial_\psi\Omega_\zeta = 0$ and $\Omega_\zeta = 0$ for $R\Omega_\zeta/v_{ti} \ll 1$. Here ν^{tb} is the turbulent diffusivity, Γ^{tb} is the turbulent pinch of momentum and $\Pi_{ud}^{\mathrm{tb}} \sim \Delta_{ud}\delta_i^2 p_i R|\nabla\psi|$ is the value of Π_{-1}^{tb} at $\Omega_\zeta = 0$ and $\partial_\psi\Omega_\zeta = 0$, and is zero for perfect up-down asymmetry when equations (7) and (13) are solved without the terms that contain H_{i1}^{nc} [26]. Notice then that imposing $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}} \simeq \Pi^{\mathrm{tb}} = -\nu^{\mathrm{tb}}\partial_\psi\Omega_\zeta - \Gamma^{\mathrm{tb}}\Omega_\zeta + \Pi_{ud}^{\mathrm{tb}} = 0$ gives intrinsic rotation only for up-down asymmetry or if momentum is pinched into the core from the edge.

The complete model described in this article includes contributions that have not been considered before. On the one hand, the gyrokinetic equations (7) and (13) have new terms with H_{i1}^{nc}, giving $\Pi_{-1}^{\mathrm{tb}} \simeq -\nu^{\mathrm{tb}}\partial_\psi\Omega_\zeta - \Gamma^{\mathrm{tb}}\Omega_\zeta + \Pi_{ud}^{\mathrm{tb}} + \Pi_{-1,0}^{\mathrm{tb}}$, where $\Pi_{-1,0}^{\mathrm{tb}} \sim (B/B_p)\delta_i^3 p_i R|\nabla\psi|$ is a new contribution due to the new terms in the gyrokinetic equation. On the other hand, there are the new terms Π_{-1}^{nc}, Π_0^{nc}, Π_{-1}^{ie}, Π_0^{ie} and Π^{ht}. As we did for Π_{-1}^{tb}, we can linearize $\Pi_{-1}^{\mathrm{nc}}(\partial_\psi\Omega_\zeta)$ around $\partial_\psi\Omega_\zeta = 0$ to find $\Pi_{-1}^{\mathrm{nc}} \simeq -\nu^{\mathrm{nc}}\partial_\psi\Omega_\zeta + \Pi_{ud}^{\mathrm{nc}} + \Pi_{-1,0}^{\mathrm{nc}}$, where $\Pi_{ud}^{\mathrm{nc}} \sim \Delta_{ud}(B/B_p)(qR\nu_{ii}/v_{ti})\delta_i^2 p_i R|\nabla\psi|$ and $\Pi_{-1,0}^{\mathrm{nc}} \sim (B/B_p)^2(qR\nu_{ii}/v_{ti})\delta_i^3 p_i R|\nabla\psi|$. Combining all these results and imposing that $\langle\langle R\hat{\boldsymbol{\zeta}}\cdot \overleftrightarrow{\mathbf{P}}_i \cdot\nabla\psi\rangle_\psi\rangle_{\mathrm{T}} = 0$, we obtain

$$\Omega_\zeta = -\int_\psi^{\psi_a} d\psi' \left.\frac{\Pi^{\mathrm{int}}}{\nu^{\mathrm{tb}} + \nu^{\mathrm{nc}}}\right|_{\psi=\psi'} \exp\left(\int_\psi^{\psi'} d\psi'' \left.\frac{\Gamma^{\mathrm{tb}}}{\nu^{\mathrm{tb}} + \nu^{\mathrm{nc}}}\right|_{\psi=\psi''}\right)$$
$$+\Omega_\zeta|_{\psi=\psi_a} \exp\left(\int_\psi^{\psi_a} d\psi' \left.\frac{\Gamma^{\mathrm{tb}}}{\nu^{\mathrm{tb}} + \nu^{\mathrm{nc}}}\right|_{\psi=\psi'}\right), \qquad (28)$$

where ψ_a is the poloidal flux at the edge, $\Omega_\zeta|_{\psi=\psi_a}$ is the rotation velocity in the edge and $\Pi^{\rm int} = \Pi^{\rm tb}_{ud} + \Pi^{\rm tb}_{-1,0} + \Pi^{\rm tb}_0 + \Pi^{\rm nc}_{ud} + \Pi^{\rm nc}_{-1,0} + \Pi^{\rm nc}_0 + \Pi^{ie}_{-1} + \Pi^{ie}_0 + \Pi^{\rm ht}$. Notice that this equation gives a rotation profile that depends on $\Pi^{\rm int}$ that in turn depends on the gradient of temperature and density, the geometry and the heating mechanism. The typical size of the rotation is $\Omega_\zeta \sim (B/B_p)\delta_i v_{ti}/R$ for $\Delta_{ud} \lesssim (B/B_p)\delta_i$ and $\Omega_\zeta \sim \Delta_{ud} v_{ti}/R$ for $\Delta_{ud} \gtrsim (B/B_p)\delta_i$.

This new model for intrinsic rotation has been constructed such that the pinch and the up-down symmetry drive, discovered in the high flow ordering, are naturally included. By transforming to the frame rotating with Ω_ζ we have made this property explicit.

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Appendix A. Equation for the distribution function in the rotating frame

In this Appendix we derive equations (4), (7) and (14) for the different pieces of the ion distribution function, equations (5) and (12) for the different pieces of the electron distribution function, and equations (6) and (13) for the different pieces of the potential. These equations are valid in the frame rotating with angular velocity Ω_ζ, and we deduce them from the results in [18], obtained in the laboratory frame.

In reference [18] we showed that in the limit $B_p/B \ll 1$, and neglecting the ion-electron collisions and the effect of the heating, the ion distribution function is given by $f_i(\mathbf{R}, E, \mu, t) = f_{Mi}(\psi(\mathbf{R}), E, t) + F^{\rm nc}_{i1}(\psi(\mathbf{R}), \theta(\mathbf{R}), E, \mu, t) + F^{\rm nc}_{i2}(\psi(\mathbf{R}), \theta(\mathbf{R}), E, \mu, t) + f^{\rm tb}_i(\mathbf{R}, E, \mu, t)$, where the size of these different pieces is $F^{\rm nc}_{i1} \sim (B/B_p)\delta_i f_{Mi}$, $F^{\rm nc}_{i2} \sim (B/B_p)^2 \delta_i^2 f_{Mi}$ and $f^{\rm tb}_i = f^{\rm tb}_{i1} + f^{\rm tb}_{i2}$, with $f^{\rm tb}_{i1} \sim \delta_i f_{Mi}$ and $f^{\rm tb}_{i2} \sim (B/B_p)\delta_i^2 f_{Mi}$. The equations for the different pieces were obtained from the gyrokinetic equation

$$\frac{\partial f_i}{\partial t} + \dot{\mathbf{R}} \cdot \nabla_{\mathbf{R}} f_i + \dot{E}\frac{\partial f_i}{\partial E} = \langle C_{ii}\{f_i\}\rangle, \tag{A.1}$$

where the time derivative $\dot{\mathbf{R}}$ is

$$\dot{\mathbf{R}} = u'\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}'_M - \frac{c}{B}\nabla_{\mathbf{R}}\langle\phi\rangle \times \hat{\mathbf{b}} \tag{A.2}$$

and the time derivative $\dot{E}$ is

$$\dot{E} = -\frac{Ze}{M}[u'\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}'_M] \cdot \nabla_{\mathbf{R}}\langle\phi\rangle. \tag{A.3}$$

Here, $u' = \pm\sqrt{2(E-\mu B)}$ is the gyrokinetic parallel velocity in the laboratory frame, and

$$\mathbf{v}'_M = \frac{\mu}{\Omega_i}\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B + \frac{(u')^2}{\Omega_i}\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}}) \tag{A.4}$$

are the ∇B and curvature drifts in the laboratory frame. Equations (19) and (20) of [18] for F_{i1}^{nc} and equation (24) of [18] for F_{i2}^{nc} are obtained from the long wavelength axisymmetric contributions to (A.1) of order $\delta_i f_{Mi} v_{ti}/a$ and $(B/B_p)\delta_i^2 f_{Mi} v_{ti}/a$, respectively. Equation (25) of [18] for F_{i2}^{tb} is also a long wavelength axisymmetric component of (A.1). In particular, it is the contribution of order $\delta_i^2 f_{Mi} v_{ti}/a$ that does not become of order $(B/B_p)\delta_i^2 f_{Mi}\nu_{ii}$ when the equation is orbit averaged. Equation (55) of [18] for f_i^{tb} is the sum of the short wavelength components of (A.1) of order $\delta_i f_{Mi} v_{ti}/a$ and $(B/B_p)\delta_i^2 f_{Mi} v_{ti}/a$.

In this article, we extend equation (A.1) to account for ion-electron collisions and the effect of the different heating mechanisms. For this reason, we use

$$\frac{\partial f_i}{\partial t} + \dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}} f_i + \dot{E}\frac{\partial f_i}{\partial E} = \langle C_{ii}\{f_i\}\rangle + \langle C_{ie}\{f_i\}\rangle + \mathcal{S}^{\mathrm{ht}}, \tag{A.5}$$

where $C_{ie}\{f_i\} \sim \sqrt{m/M}\,\nu_{ii} f_i$ is the ion-electron collision operator, treated in detail in Appendix B. Moreover, we want to write the equation in the rotating frame, that is, we need to use the new gyrokinetic variable $\varepsilon = E - I\Omega_\zeta u'/B$. Thus, the new gyrokinetic equation is

$$\frac{\partial f_i}{\partial t} + \dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}} f_i + \dot{\varepsilon}\frac{\partial f_i}{\partial \varepsilon} = \langle C_{ii}\{f_i\}\rangle + \langle C_{ie}\{f_i\}\rangle + \mathcal{S}^{\mathrm{ht}}. \tag{A.6}$$

The time derivative of the new gyrokinetic variable ε is

$$\dot{\varepsilon} = \dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}\varepsilon + \dot{E}\frac{\partial\varepsilon}{\partial E}. \tag{A.7}$$

In $\dot{\mathbf{R}}$, using $u' = u + I\Omega_\zeta/B$, with $u = \pm\sqrt{2(\varepsilon - \mu B + R^2\Omega_\zeta^2/2)}$, leads to

$$\dot{\mathbf{R}} = u\hat{\mathbf{b}} + \frac{I\Omega_\zeta}{B}\hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_C - \frac{c}{B}\nabla_{\mathbf{R}}\langle\phi\rangle\times\hat{\mathbf{b}} + O\left(\frac{B^2}{B_p^2}\delta_i^3 v_{ti}\right), \tag{A.8}$$

with

$$\mathbf{v}_M = \frac{\mu}{\Omega_i}\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B + \frac{u^2}{\Omega_i}\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}}) \tag{A.9}$$

the ∇B and curvature drifts in the rotating frame, and $\mathbf{v}_C = (2uI\Omega_\zeta/B\Omega_i)\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}})$ the Coriolis drift. To obtain this expression for $\dot{\mathbf{R}}$ we have used $(u')^2 = u^2 + 2I\Omega_\zeta u/B + O[(B/B_p)^2\delta_i^2 v_{ti}^2]$ to write $\mathbf{v}'_M = \mathbf{v}_M + \mathbf{v}_C + O[(B^2/B_p^2)\delta_i^3 v_{ti}]$. The usual result for the Coriolis drift $\mathbf{v}_C = (2u\Omega_\zeta/\Omega_i)\hat{\mathbf{b}}\times[(\nabla_{\mathbf{R}}R\times\hat{\boldsymbol{\zeta}})\times\hat{\mathbf{b}}]$ can be recovered by realizing that for $B_p/B \ll 1$, $\hat{\mathbf{b}}\simeq\hat{\boldsymbol{\zeta}}$, $\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}} \simeq -\nabla_{\mathbf{R}}R/R$ and $I/B\simeq R$, giving

$$\mathbf{v}_C = \frac{2Iu\Omega_\zeta}{B\Omega_i}\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}}) \simeq \frac{2u\Omega_\zeta}{\Omega_i}\hat{\mathbf{b}}\times[(\nabla_{\mathbf{R}}R\times\hat{\boldsymbol{\zeta}})\times\hat{\mathbf{b}}]. \tag{A.10}$$

In addition, using $I\hat{\mathbf{b}}/B = R\hat{\boldsymbol{\zeta}} + \hat{\mathbf{b}}\times\nabla\psi/B$, $\phi = \phi_0 + \phi_1^{\rm nc} + \phi_2^{\rm nc} + \phi^{\rm tb}$, $\langle\phi_0\rangle = \phi_0(\psi(\mathbf{R}), t) + O(\delta_i^2 T_e/e)$, $\langle\phi_1^{\rm nc}\rangle = \phi_1^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), t) + O[(B/B_p)\delta_i^3 T_e/e]$ and $\langle\phi_2^{\rm nc}\rangle = O[(B^2/B_p^2)\delta_i^2 v_{ti}]$, we can simplify equation (A.8) to

$$\dot{\mathbf{R}} = u\hat{\mathbf{b}} + R\Omega_\zeta\hat{\boldsymbol{\zeta}} + \mathbf{v}_M - \frac{1}{n_i M\Omega_i}\frac{\partial p_i}{\partial\psi}\hat{\mathbf{b}}\times\nabla\psi + \mathbf{v}_C - \frac{c}{B}\nabla_{\mathbf{R}}\phi_1^{\rm nc}\times\hat{\mathbf{b}}$$
$$-\frac{c}{B}\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\times\hat{\mathbf{b}} + O\left(\frac{B^2}{B_p^2}\delta_i^3 v_{ti}\right). \quad \text{(A.11)}$$

The time derivative $\dot{\varepsilon}$ in (A.7) can be written as

$$\dot{\varepsilon} = \dot{E} - \frac{Iu'}{B}\frac{\partial\Omega_\zeta}{\partial\psi}\dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}\psi - \Omega_\zeta\dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) - \frac{I\Omega_\zeta}{Bu'}\dot{E}. \quad \text{(A.12)}$$

To simplify this equation we use $\phi = \phi_0 + \phi_1^{\rm nc} + \phi_2^{\rm nc} + \phi^{\rm tb}$, $\langle\phi_0\rangle = \phi_0(\psi(\mathbf{R}), t) + O(\delta_i^2 T_e/e)$, $\langle\phi_1^{\rm nc}\rangle = \phi_1^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), t) + O[(B/B_p)\delta_i^3 T_e/e]$, $\langle\phi_2^{\rm nc}\rangle = \phi_2^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), t) + O[(B^2/B_p^2)\delta_i^4 T_e/e]$, $u' = u + O[(B/B_p)\delta_i v_{ti}]$ and

$$\dot{\mathbf{R}}\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) = u'\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) + \mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) - \frac{c}{B}(\nabla_{\mathbf{R}}\langle\phi\rangle\times\hat{\mathbf{b}})\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right)$$
$$= \frac{Ze}{Mc}\mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\psi + \frac{ZeI}{MBu'}\mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\langle\phi\rangle + O\left(\frac{B_p}{B}\delta_i^2 v_{ti}^2\right). \quad \text{(A.13)}$$

With these results, we obtain

$$\dot{\varepsilon} = -\frac{Ze}{M}[u\hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}_M + \mathbf{v}_C]\cdot\left(-\frac{1}{Zen_i}\frac{\partial p_i}{\partial\psi}\nabla_{\mathbf{R}}\psi + \nabla_{\mathbf{R}}\phi_1^{\rm nc} + \nabla_{\mathbf{R}}\phi_2^{\rm nc} + \nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\right)$$
$$-\frac{Iu}{B}\frac{\partial\Omega_\zeta}{\partial\psi}\left(\mathbf{v}_M - \frac{c}{B}\nabla_{\mathbf{R}}\langle\phi^{\rm tb}\rangle\times\hat{\mathbf{b}}\right)\cdot\nabla_{\mathbf{R}}\psi + O\left(\frac{\delta_i^2 v_{ti}^2}{a}\right). \quad \text{(A.14)}$$

To obtain the result in (A.13), we have employed $\mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\psi = u'\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}(Iu'/\Omega_i)$;

$$-\frac{c}{B}(\nabla_{\mathbf{R}}\langle\phi\rangle\times\hat{\mathbf{b}})\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) = \frac{ZeI}{MBu'}\left[\frac{\mu}{\Omega_i}\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B - \frac{(u')^2}{\Omega_i}\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}\ln\left(\frac{I}{B}\right)\right]\cdot\nabla_{\mathbf{R}}\langle\phi\rangle$$
$$= \frac{ZeI}{MBu'}\left[\frac{\mu}{\Omega_i}\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}B + \frac{(u')^2}{\Omega_i}\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}})\right]\cdot\nabla_{\mathbf{R}}\langle\phi\rangle + O\left(\frac{B_p}{B}\delta_i v_{ti}^2\right)$$
$$= \frac{ZeI}{MBu'}\mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\langle\phi\rangle + O\left(\frac{B_p}{B}\delta_i v_{ti}^2\right), \quad \text{(A.15)}$$

where we have used $I/B = R + O[(B_p^2/B^2)R]$ and $\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\hat{\mathbf{b}} = -\nabla_{\mathbf{R}}R/R + O[(B_p/B)R^{-1}]$; and

$$\mathbf{v}'_M\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) = \frac{u'}{\Omega_i}\left[\nabla_{\mathbf{R}}\times(u'\hat{\mathbf{b}}) - u'\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\times\hat{\mathbf{b}}\right]\cdot\nabla_{\mathbf{R}}\left(\frac{Iu'}{B}\right) = O\left(\frac{B_p^2}{B^2}\delta_i v_{ti}^2\right), \quad \text{(A.16)}$$

where we have used $\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}\times\hat{\mathbf{b}} \sim (B_p/B)a^{-1}$, $\hat{\mathbf{b}}\cdot\nabla_{\mathbf{R}}(Iu'/B) \sim Rv_{ti}/qR \sim (B_p/B)(R/a)v_{ti}$ and $\nabla_{\mathbf{R}}\times(u'\hat{\mathbf{b}})\cdot\nabla_{\mathbf{R}}(Iu'/B) = \nabla_{\mathbf{R}}\cdot[u'\hat{\mathbf{b}}\times\nabla_{\mathbf{R}}(Iu'/B)] = \nabla_{\mathbf{R}}\cdot[(Iu'/B)\nabla_{\mathbf{R}}\zeta\times\nabla_{\mathbf{R}}(Iu'/B)] + \nabla_{\mathbf{R}}\cdot[(u'/B)(\nabla\zeta\times\nabla\psi)\times\nabla_{\mathbf{R}}(Iu'/B)] = \nabla_{\mathbf{R}}\cdot\{\nabla_{\mathbf{R}}\zeta\times\nabla_{\mathbf{R}}[I^2(u')^2/2B^2]\} - \partial_\zeta[(u'/R^2B)\nabla\psi\cdot\nabla_{\mathbf{R}}(Iu'/B)] = 0$.

With equations (A.6), (A.11) and (A.14), we can now easily obtain equations (4), (7) and (14) for $H_{i1}^{\rm nc}$, $f_i^{\rm tb}$, $H_{i2}^{\rm nc}$, $H_{i2}^{\rm tb}$, H_{i2}^{ie} and $H_{i2}^{\rm ht}$. To obtain (4), we take the long wavelength axisymmetric contribution to (A.6) to order $\delta_i f_{Mi} v_{ti}/a$, giving

$$u\hat{\mathbf{b}} \cdot \nabla_{\mathbf{R}} H_{i1}^{\rm nc} + \mathbf{v}_M \cdot \nabla_{\mathbf{R}} f_{Mi} - \frac{Ze}{M}\frac{\partial f_{Mi}}{\partial \varepsilon}\left(u\hat{\mathbf{b}} \cdot \nabla_{\mathbf{R}} \phi_1^{\rm nc} - \frac{1}{Zen_i}\frac{\partial p_i}{\partial \psi}\mathbf{v}_M \cdot \nabla_{\mathbf{R}}\psi \right) = C_{ii}^{(\ell)}\{H_{i1}^{\rm nc}\}. \quad \text{(A.17)}$$

This equation differs from equations (19) and (20) of [18], and gives a function $H_{i1}^{\rm nc}$ different from the function $F_{i1}^{\rm nc}$ defined in [18]. The reason is that $f_{Mi}(\psi(\mathbf{R}), \varepsilon) + H_{i1}^{\rm nc} + H_{i2}^{\rm nc}$ must be equal to the function $f_{Mi}(\psi(\mathbf{R}), E) + F_{i1}^{\rm nc} + F_{i2}^{\rm nc}$ defined in [18] to the order of interest, but how the terms of first and second order in δ_i are assigned to one or the other piece differs depending on the frame. For this reason, we have changed the name of the functions. The final result in (4) is obtained from (A.17) by using $\mathbf{v}_M \cdot \nabla_{\mathbf{R}}\psi = u\hat{\mathbf{b}} \cdot \nabla_{\mathbf{R}}(Iu/\Omega_i)$ for $u = \pm\sqrt{2(\varepsilon - \mu B + R^2\Omega_\zeta^2/2)} \simeq \pm\sqrt{2(\varepsilon - \mu B)}$.

Equation (7) is the sum of the short wavelength contributions to (A.6) of order $\delta_i f_{Mi} v_{ti}/a$ and $(B/B_p)\delta_i^2 v_{ti}/a$. The equation is almost straightforward if we apply the same methodology as in [18]. Only two terms require some care. On the one hand, the ion-electron collision operator that was not treated in [18] is now considered in detail in Appendix B. On the other hand, in $\dot{\mathbf{R}}$ there is a drift $-(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)$ that is not included in (7). To study the effect of the perpendicular drift $-(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)$, it is better to consider the local approximation. In the local approximation, the length scale of the turbulence is so small that the background quantities can be represented by their local value and their local derivative, i.e., the drift $-(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)$ is given by its value $-(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)|_{\psi=\psi_0}$ at the point $\psi = \psi_0$ around which we want to calculate the turbulent fluctuations and the linear dependence $-(\psi - \psi_0)\partial_\psi[(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)]|_{\psi=\psi_0}$. The characteristic scale of the turbulence is the ion gyroradius, giving $\psi - \psi_0 \sim \rho_i R B_p$ and $-(\psi - \psi_0)\partial_\psi[(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)]|_{\psi=\psi_0} \sim \delta_i^2 v_{ti}$. Since we only need to keep terms up to order $(B/B_p)\delta_i^2 f_{Mi} v_{ti}/a$, $-(\psi - \psi_0)\partial_\psi[(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)]|_{\psi=\psi_0} \cdot \nabla_{\mathbf{R}} f_i^{\rm tb} \sim \delta_i^2 f_{Mi} v_{ti}/a$ is negligible. Only the constant drift $-(n_i M\Omega_i)^{-1}\partial_\psi p_i(\hat{\mathbf{b}} \times \nabla\psi)|_{\psi=\psi_0}$ remains. A constant drift does not change the character of the short wavelength structures and can be safely ignored.

Equation (14) is found from the long wavelength axisymmetric components of (A.6) to order $\delta_i^2 f_{Mi} v_{ti}/a$. Note that to this order we have the time derivative $\partial_t f_{Mi}$ [18]. Using $\partial_t f_{Mi} = [n_i^{-1}\partial_t n_i + (M\varepsilon/T_i - 3/2)T_i^{-1}\partial_t T_i] f_{Mi}$ and realizing that $\partial_t n_i = \sum\langle\int d^3v\, \mathcal{S}^\alpha\rangle_\psi$ and $(3/2)\partial_t(n_i T_i) = \sum\langle\int d^3v\, \mathcal{S}^\alpha M\varepsilon\rangle_\psi$, where the summations are over α = nc, tb, ie, ht, we find the final form in (14). The equations for $H_{i2}^{\rm nc}$ and $H_{i2}^{\rm tb}$ are obtained in the same way as equations (24) and (25) in [18], i.e., the equation for $H_{i2}^{\rm nc}$ is the axisymmetric long wavelength component of (A.6) of order $(B/B_p)\delta_i^2 f_{Mi} v_{ti}/a$, and the equation for $H_{i2}^{\rm tb}$ is the axisymmetric long wavelength component of order $\delta_i^2 f_{Mi} v_{ti}/a$ that when it is orbit averaged does not reduce to a piece of order $(B/B_p)\delta_i^2 \nu_{ii} f_{Mi}$. The equations for H_{i2}^{ie} and $H_{i2}^{\rm ht}$ where not considered in [18].

The equation for H_{i2}^{ie} is the axisymmetric long wavelength contribution that includes the ion-electron collision operator that is treated in detail in Appendix B. The equation for $H_{i2}^{\rm ht}$ is the axisymmetric long wavelength contribution that has the source $\mathcal{S}^{\rm ht}$.

The equations (5) and (12) for the electron distribution function in the rotating frame are derived in the same way as the equations for the ion distribution function. The only differences are that the Coriolis drift $\mathbf{v}_C$ and the term in (A.14) that is proportional to $\partial_\psi \Omega_\zeta$ are small by $\sqrt{m/M}$ and hence negligible, and that we include the electric field $\mathbf{E}^A$ driven by the transformer, leading to a modified time derivative for the energy

$$\dot{\varepsilon} = \frac{e}{m} u \hat{\mathbf{b}} \cdot \mathbf{E}^A + \frac{e}{m} [u \hat{\mathbf{b}}(\mathbf{R}) + \mathbf{v}_M] \cdot \left(- \frac{1}{Zen_i} \frac{\partial p_i}{\partial \psi} \nabla_{\mathbf{R}} \psi + \nabla_{\mathbf{R}} \phi_1^{\rm nc} + \nabla_{\mathbf{R}} \langle \phi^{\rm tb} \rangle \right). \qquad (A.18)$$

Finally, the equations for the different pieces of the electrostatic potential (6) and (13) are easily deduced from the results in [18] by realizing that moving to a rotating reference frame does not modify the quasineutrality equation.

Appendix B. Ion-electron collisions

In this Appendix we discuss how we treat the ion-electron collision operator, given by

$$C_{ie}\{f_i, f_e\} = \gamma_{ie} \nabla_v \cdot \left[\int d^3 v_e \, \nabla_g \nabla_g g \cdot \left(f_e \nabla_v f_i - \frac{M}{m} f_i \nabla_{v_e} f_e \right) \right], \qquad (B.1)$$

where $\gamma_{ie} = 2\pi Z^2 e^4 \ln \Lambda / M^2$, $\mathbf{g} = \mathbf{v} - \mathbf{v}_e$ and $\nabla_g \nabla_g = (g^2 \overleftrightarrow{\mathbf{I}} - \mathbf{g}\mathbf{g})/g^3$. Both the ion velocity, $\mathbf{v}$, and the electron velocity, $\mathbf{v}_e$, are measured in the rotating frame.

We must write the ion-electron collision operator up to order $\sqrt{m/M}\nu_{ii}\delta_i f_{Mi}$. For the wavelengths of interest, between the minor radius a and the ion gyroradius, $f_i(\mathbf{R}, \varepsilon, \mu, t) = f_{Mi}(\psi(\mathbf{R}), \varepsilon, t) + H_{i1}^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t) + f_i^{\rm tb}(\mathbf{R}, \varepsilon, \mu, t) + \ldots$ and $f_e(\mathbf{R}, \varepsilon, \mu, t) = f_{Me}(\psi(\mathbf{R}), \varepsilon, t) + H_{e1}^{\rm nc}(\psi(\mathbf{R}), \theta(\mathbf{R}), \varepsilon, \mu, t) + f_e^{\rm tb}(\mathbf{R}, \varepsilon, \mu, t) + \ldots$, where $H_{e1}^{\rm nc} = (e\phi_1^{\rm nc}/T_e) f_{Me} + O[(B/B_p)\delta_e f_{Me}]$ and $f_e^{\rm tb} = (e\phi^{\rm tb}/T_e) f_{Me} + O(\delta_e f_{Me})$. The electron distribution function is then a Maxwell-Boltzmann response $(e\phi/T_e) f_{Me} \sim \delta_i f_{Me}$ plus a correction of order $\delta_e f_{Me} \sim \sqrt{m/M}\delta_i f_{Me}$ that is smaller by $\sqrt{m/M}$. Expanding the ion distribution function around $\mathbf{R}_g = \mathbf{r} + \Omega_i^{-1} \mathbf{v} \times \hat{\mathbf{b}}$, $v^2/2$ and $v_\perp^2/2B$, we obtain $f_i = f_{Mi,0} + H_{i1,0}^{\rm nc} + h_i^{\rm tb}$, where $h_i^{\rm tb} = f_{ig}^{\rm tb} - [Ze(\phi^{\rm tb} - \langle \phi^{\rm tb} \rangle)/T_i] f_{Mi,0}$. For the electrons, since we are considering wavelengths larger than the electron gyroradius, it is possible to expand around $\mathbf{r}$, $v^2/2$ and $v_\perp^2/2B$, giving $f_e = f_{Me,0} + H_{e1,0}^{\rm nc} + h_e^{\rm tb}$, where $h_e^{\rm tb} = f_{e0}^{\rm tb} - \Omega_e^{-1}(\mathbf{v} \times \hat{\mathbf{b}}) \cdot \nabla f_{e0}^{\rm tb} + [mc(\mathbf{v} \times \hat{\mathbf{b}}) \cdot \nabla \phi^{\rm tb}/BT_e] f_{Me,0}$. Here, the subscript $_0$ in $f_{Mi,0}$, $f_{Me,0}$, $H_{i1,0}^{\rm nc}$, $H_{e1,0}^{\rm nc}$ and $f_{e0}^{\rm tb}$ indicates that the variables $\mathbf{R}$, ε and μ have been replaced by $\mathbf{r}$, $v^2/2$ and $v_\perp^2/2B$. The subscript $_g$ in $f_{ig}^{\rm tb}$ indicates that $\mathbf{R}$, ε and μ are replaced by $\mathbf{R}_g = \mathbf{r} + \Omega_i^{-1} \mathbf{v} \times \hat{\mathbf{b}}$, $v^2/2$ and $v_\perp^2/2$. Using these expressions for f_i and f_e, and the relation

$$\nabla_g \nabla_g g = \nabla_{v_e} \nabla_{v_e} v_e - \mathbf{v} \cdot \nabla_{v_e} \nabla_{v_e} \nabla_{v_e} v_e + O\left(\frac{m}{M} \frac{1}{v_{te}} \right), \qquad (B.2)$$

we find that

$$C_{ie}\{f_i, f_e\} = -\frac{M\gamma_{ie}}{T_i}\nabla_v \cdot \left[\left(1+\frac{e\phi_1^{\rm nc}}{T_e}+\frac{e\phi^{\rm tb}}{T_e}\right) f_{Mi}\mathbf{v}\cdot\int d^3v_e\, f_{Me}\nabla_{v_e}\nabla_{v_e}v_e\right]$$
$$+\gamma_{ie}\nabla_v\cdot\left[(\nabla_v H_{i1}^{\rm nc}+\nabla_v h_i^{\rm tb})\cdot\int d^3v_e\, f_{Me}\nabla_{v_e}\nabla_{v_e}v_e\right]$$
$$-\frac{M\gamma_{ie}}{m}\nabla_v\cdot\left[f_{Mi}\int d^3v_e\,\nabla_{v_e}\nabla_{v_e}v_e\cdot(\nabla_{v_e}H_{e1}^{\rm nc}+\nabla_{v_e}h_e^{\rm tb})\right]$$
$$-\frac{M\gamma_{ie}}{T_e}\nabla_v\cdot\left[\left(1+\frac{e\phi_1^{\rm nc}}{T_e}+\frac{e\phi^{\rm tb}}{T_e}\right) f_{Mi}\mathbf{v}\cdot\int d^3v_e\, f_{Me}\nabla_{v_e}\nabla_{v_e}\nabla_{v_e}v_e\cdot\mathbf{v}_e\right]$$
$$-\frac{M\gamma_{ie}}{T_e}\nabla_v\cdot\left[(H_{i1}^{\rm nc}+h_i^{\rm tb})\mathbf{v}\cdot\int d^3v_e\, f_{Me}\nabla_{v_e}\nabla_{v_e}\nabla_{v_e}v_e\cdot\mathbf{v}_e\right]$$
$$+O\left(\frac{m}{M}\nu_{ii}\delta_i f_{Mi}\right). \qquad \text{(B.3)}$$

Here we have used that $\nabla_{v_e} f_{Me} = -(m\mathbf{v}_e/T_e)f_{Me}$ and $\nabla_{v_e}\nabla_{v_e}v_e\cdot\mathbf{v}_e = 0$. Using $\nabla_{v_e}\nabla_{v_e}\nabla_{v_e}v_e\cdot\mathbf{v}_e = -\nabla_{v_e}\nabla_{v_e}v_e$ and $\int d^3v_e\, f_{Me}\nabla_{v_e}\nabla_{v_e}v_e = (2/3)\overleftrightarrow{\mathbf{I}}\int d^3v_e\, f_{Me}/v_e = (2\sqrt{2}/3\sqrt{\pi})n_e\sqrt{m/T_e}\,\overleftrightarrow{\mathbf{I}}$, we find

$$C_{ie}\{f_i, f_e\} = \frac{n_e m\nu_{ei}}{n_i M}\left(1+\frac{e\phi_1^{\rm nc}}{T_e}+\frac{e\phi^{\rm tb}}{T_e}\right)\left(\frac{T_e}{T_i}-1\right)\left(\frac{Mv^2}{T_i}-3\right)f_{Mi}$$
$$+\frac{n_e m\nu_{ei}}{n_i M}\nabla_v\cdot\left[\frac{T_e}{M}(\nabla_v H_{i1}^{\rm nc}+\nabla_v h_i^{\rm tb})+\mathbf{v}(H_{i1}^{\rm nc}+h_i^{\rm tb})\right]$$
$$-\frac{1}{p_i}[\mathbf{F}_{ei}^{\rm nc}+\mathbf{F}_{ei}^{\rm tb}-n_e m\nu_{ei}(\mathbf{W}_i^{\rm nc}+\mathbf{W}_i^{\rm tb})]\cdot\mathbf{v}f_{Mi}$$
$$+O\left(\frac{m}{M}\nu_{ii}\delta_i f_{Mi}\right). \qquad \text{(B.4)}$$

Here $\nu_{ei} = (4\sqrt{2\pi}/3)Z^2e^4 n_i\ln\Lambda/m^{1/2}T_e^{3/2}$ is the electron-ion collision frequency, $\mathbf{F}_{ei}^{\rm nc}$ is the long wavelength axisymmetric friction force on electrons due to collisions with ions, given in (18), $\mathbf{F}_{ei}^{\rm tb}$ is the short wavelength turbulent friction force, given in (9), $\mathbf{W}_i^{\rm nc} = n_i^{-1}\hat{\mathbf{b}}\int d^3v\, H_{i1}^{\rm nc}v_{||}$ is the long wavelength axisymmetric ion average velocity in the rotating frame, and $\mathbf{W}_i^{\rm tb} = n_i^{-1}\int d^3v\, h_i^{\rm tb}\mathbf{v}$ is the short wavelength turbulent ion average velocity.

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