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Accelerator

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The ion resonance stability properties of an intense electron beam in a modified betatron are investigated within the framework of the linearized Vlasov-Maxwell equations. The analysis includes the important influence of equilibrium electric and magnetic self fields as well as the applied toroidal field B_0 . It is shown that the maximum growth rate is an increasing function of the fractional charge neutralization f .

There is a growing literature on the equilibrium and stability properties¹⁻⁴ of high-current, relativistic electron beams in the modified betatron configuration, where an intense nonneutral ring of relativistic electrons is focussed and confined by combined betatron and toroidal magnetic fields¹⁻⁴. Depending on the operating regime, it is anticipated that the electron beam may be subject to various micro- and macro-instabilities. Although high vacuum conditions are intended for modified betatron operation, if we assume reasonably long acceleration times (*e.g.*, $\gtrsim 1\mu s$) then it is important to investigate the interaction of the beam electrons with the background ions that are created by ionization of the residual gas. In this regard, in the present analysis, we investigate the ion resonance instability⁵⁻⁸ for an intense electron beam interacting with background plasma ions in a modified betatron.

The equilibrium configuration consists of a relativistic nonneutral electron ring with major radius R_0 located at the midplane of an applied focussing betatron field $B_r(r, z)\hat{e}_r + B_z(r, z)\hat{e}_z$. In addition, the electron ring is located inside a toroidal conductor with minor radius a_c much smaller than the equilibrium radius R_0 . An applied toroidal magnetic field $B_\theta\hat{e}_\theta$ together with the betatron field act to confine the electron ring both axially and radially. In the present analysis, it is assumed that the external field index is $n = 1/2$, so that the radial and axial minor dimensions of the ring are equal, i.e., the minor cross section of the ring is circular. It is further assumed that $\nu/\gamma_b \ll 1$, where $\nu = (N_b/2\pi R_0)(e^2/m_b c^2)$ is Budker's parameter, N_b is the total number of electrons in the ring, and $e^2/m_b c^2$ is the classical electron radius. For convenience in the subsequent analysis, we introduce the toroidal polar coordinate system (ρ, Φ, θ) defined by³ $r - R_0 = \rho \cos\Phi$ and $z = \rho \sin\Phi$, where ρ is measured from the equilibrium ring location $(r, z) = (R_0, 0)$.

There are two exact single particle constants of the motion in the equilibrium field configuration. These are the total energy H and the canonical angular momentum P_θ . Within the context of the assumption of a thin ring with $a \ll R_0$, it has been shown³ that the canonical angular momentum $P_\Phi = \rho p_\Phi - (e/2c)B_\theta \rho^2$ in the plane perpendicular to the applied toroidal magnetic field B_θ is an approximate single-particle invariant for external field index $n = 1/2$ and sufficiently small spread in canonical angular momentum P_θ . Here, p_Φ is the mechanical momentum in the Φ -direction. For present purposes, we assume that the

equilibrium distribution function for component j ($j = b, i$) is of the form

$$f_j^0(x, p) = \left(\frac{\hat{n}_j}{2\pi\gamma_j m_j} \right) \delta(H - \omega_j P_\Phi - \hat{\gamma}_j m_j c^2) \delta(P_\theta - \gamma_j m_j \beta_j c R_0), \quad (1)$$

where \hat{n}_j is the particle density at $(r, z) = (R_0, 0)$, and $\omega_j, \beta_j, \gamma_j$, and $\hat{\gamma}_j$ are constants. From Eq. (1), the equilibrium density of component j is given by $n_j^0(\rho) = \hat{n}_j$ for $0 \leq \rho \leq a$, and $n_j^0(\rho) = 0$ for $\rho > a$. Here, the minor radius a of the electron beam is defined by $a^2 = 2c^2(\hat{\gamma}_j - \gamma_j)/[\gamma_j(\omega_j^+ - \omega_j)(\omega_j - \omega_j^-)]$, and the laminar rotation frequencies ω_j^\pm are defined by

$$\omega_j^\pm = -\frac{\epsilon_j}{2}\omega_{j\theta} \left\{ 1 \pm \left[1 - \frac{B_z^2}{B_\theta^2} - \frac{8\pi e_j}{\gamma_j m_j \omega_{j\theta}^2} \sum_k \hat{n}_k e_k (1 - \beta_j \beta_k) \right]^{1/2} \right\}, \quad (2)$$

where $\epsilon_j = \text{sgn } e_j$, the field components B_z and B_θ are evaluated at $(r, z) = (R_0, 0)$, and $\omega_{j\theta} = eB_\theta/\gamma_j m_j c$ is the cyclotron frequency in the toroidal field B_θ .

Since the toroidal conductor (radius $\rho = a_c$) has a large aspect ratio with $a_c \ll R_0$, the Laplacian operator ∇^2 can be approximated by

$$\nabla^2 = \rho^{-1}(\partial/\partial\rho)(\rho\partial/\partial\rho) - m^2/\rho^2 - k^2 \quad (3)$$

in the analysis of the eigenvalue equation. Here, $m = -i\partial/\partial\Phi$ is the poloidal mode number in the small cross section of the ring, and $k = \ell/R_0$ is the toroidal wave number associated with azimuthal harmonic number $\ell = -i\partial/\partial\theta$. Making use of Eqs. (2) and (3), the ion resonance stability analysis is carried out within the framework of the linearized Vlasov-Maxwell equations, including the important influence of both the betatron and toroidal magnetic fields, as well as the equilibrium self fields. After some straightforward algebra that parallels the analysis in Ref. 8, the resulting dispersion relation can be expressed as $\det\{D_{kj}(\omega)\} = 0$, where $\{D_{kj}(\omega)\}$ is the two-by-two matrix

$$D_{kj} = \delta_{kj} - [1 - (a/a_c)^{2m}](1 - \beta_k \beta_j)(\omega_{pj}^2 a^2 / 2m v_j^2) \Gamma_j(\omega),$$

where $k = b, i$ and $j = b, i$. Here δ_{kj} is the Kronecker delta, ω is the complex eigenfrequency, $\omega_{pj}^2 = 4\pi e_j^2 \hat{n}_j / \gamma_j m_j$ is the plasma-frequency squared, $v_j^2 = 2T_{j\perp} / \gamma_j m_j \equiv$

$(\omega_j^+ - \omega_j)(\omega_j - \omega_j^-)a^2$ is the characteristic thermal speed-squared, and $\Gamma_j(\omega)$ is defined by

$$\Gamma_j(\omega) = -1 + \left(\frac{\omega_j - \omega_j^+}{\omega_j^- - \omega_j^+} \right)^m \sum_{n=0}^m \frac{m!}{n!(m-n)!} \frac{(\omega - m\omega_j - k\beta_j c)}{[\omega - k\beta_j c - m\omega_j^- - n(\omega_j^+ - \omega_j^-)]} \left(\frac{\omega_j^- - \omega_j}{\omega_j - \omega_j^+} \right)^n. \quad (4)$$

The ion resonance stability properties for the dipole oscillation characterized by $m = 1$ has been investigated in considerable detail. Defining the normalized Doppler-shifted eigenfrequency and wave number by $\Omega = (\omega - k\beta_i c)/\omega_{b0}$ and $\xi = k(\beta_b - \beta_i)c/\omega_{b0}$, the dispersion relation can be expressed as

$$\left\{ (\Omega - \xi)^2 - (\Omega - \xi) - \frac{1}{2} \left[\frac{B_z^2}{B_\theta^2} + fK \right] \right\} \left\{ \Omega^2 + \left(\frac{\gamma_b m_b}{m_i} \right) \Omega - \frac{1}{2} \left[\frac{B_z^2}{B_\theta^2} + \left(\frac{\gamma_b m_b}{m_i} \right) K \right] \right\} = \frac{1}{4} f \frac{\gamma_b m_b}{m_i} K^2 \quad (5)$$

for $|\beta_i| \ll 1$ and $a \ll a_c$. In Eq. (5), $K = \omega_{pb}^2/\omega_{b0}^2$ and $f = \hat{n}_i/\hat{n}_b$ is the fractional charge neutralization.

The normalized growth rate $\Omega_i = Im\Omega$ and real oscillation frequency $\Omega_r = Re\Omega$ have been obtained numerically as a function of ξ from Eq. (5) for a broad range of system parameters, $f, K, \gamma_b m_b/m_i$, and B_z^2/B_θ^2 . To illustrate the influence of toroidal magnetic field B_θ on the ion resonance instability, in Fig. 1 we plot the $(\xi, B_z^2/B_\theta^2)$ stability boundaries obtained from Eq. (5) for $f = 0.3, K = 5$, and $\gamma_b m_b/m_i = 0.001$. The dashed curve in Fig. 1 corresponds to maximum growth rate. Note from Fig. 1 that the range of unstable ξ values decreases substantially with increasing B_z^2/B_θ^2 .

The dependence of stability properties on the fractional charge neutralization f is illustrated in Fig. 2 where the maximum growth rate $[\Omega_i]_{MAX}$, which corresponds to the dashed curve in Fig. 1, is plotted versus B_z^2/B_θ^2 for several values of fractional charge neutralization f and parameters otherwise identical to Fig. 1. From Fig. 2, we note that the maximum growth rate $[\Omega_i]_{MAX}$ decreases with increasing values of B_z^2/B_θ^2 . Moreover, it is found that the maximum growth rate as well as the range of unstable ξ values are increasing functions of f for moderate values of fractional charge neutralization.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

- Fig. 1. Stability boundaries in parameter space $(\xi, B_z^2/B_\theta^2)$ for $f = 0.3, K = 5$ and $\gamma_b m_b/m_i = 0.001$. The dashed line represents the maximum growth rate.
- Fig. 2. Normalized maximum growth rate versus fractional charge neutralization f [Eq. (5)] for several values of f and parameters otherwise identical to Fig. 1.

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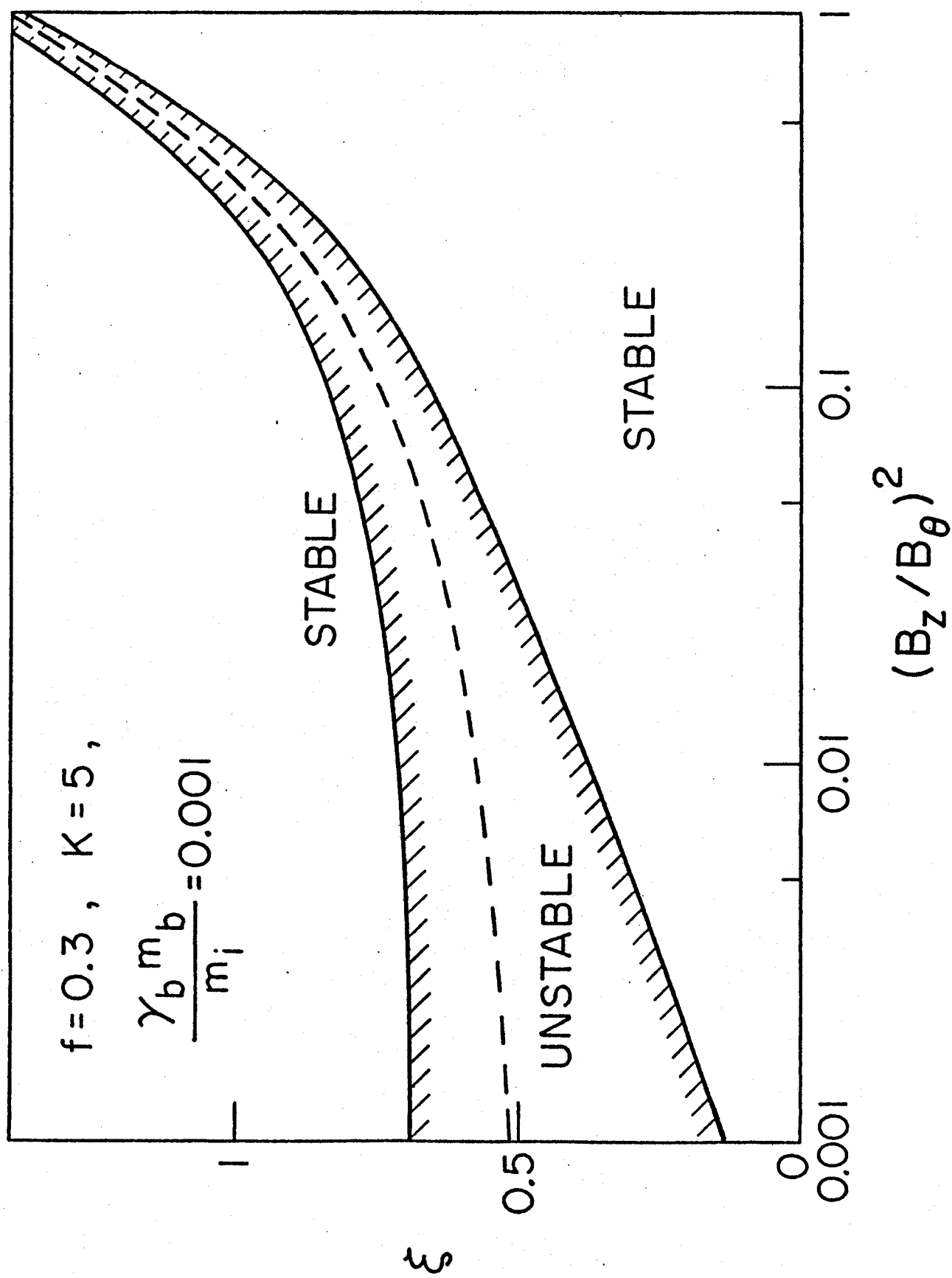


Figure 1.

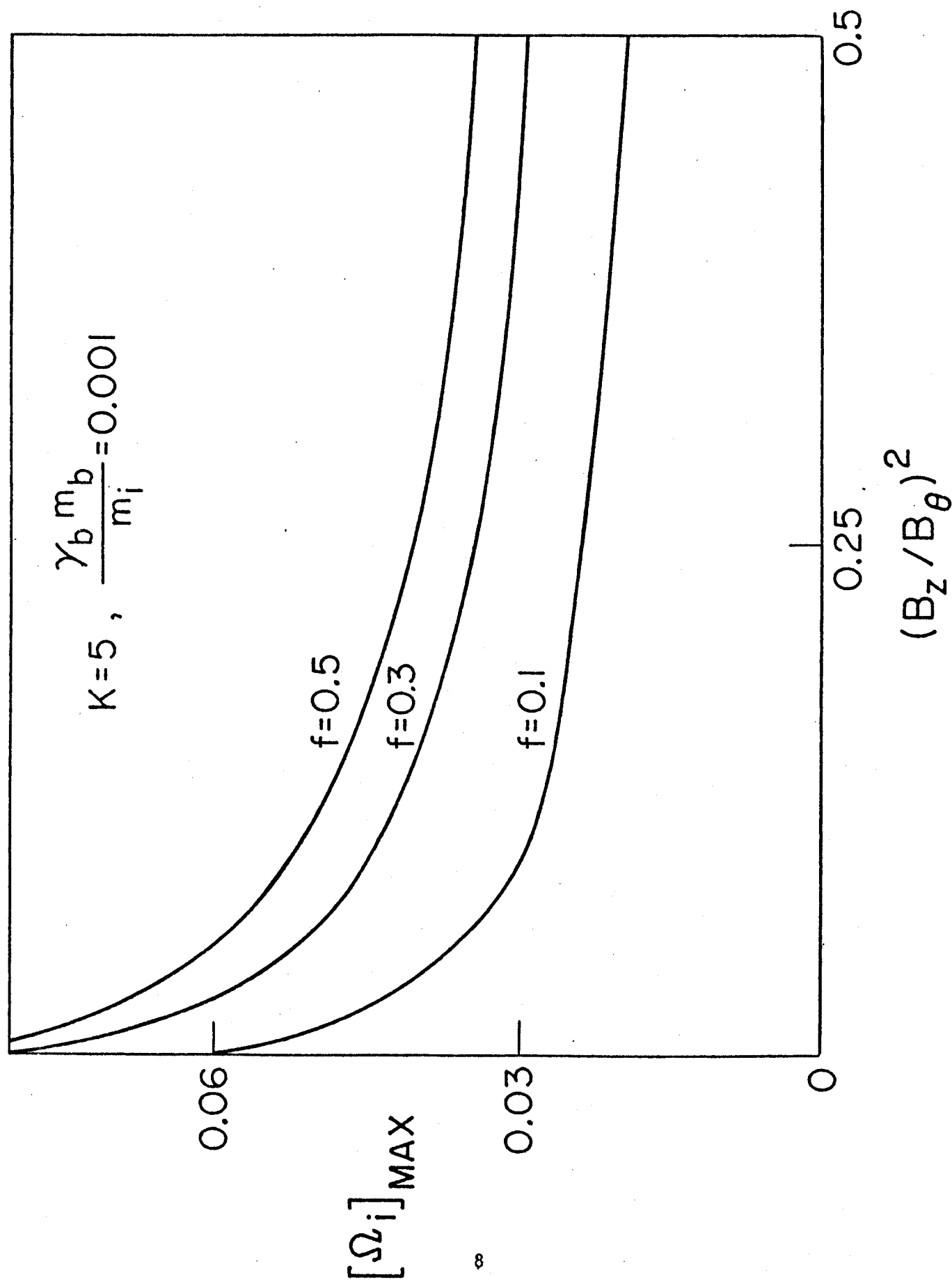


Figure 2.