MHD Equilibrium and Stability Properties

of a Bipolar Current Loop

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ABSTRACT

A study is made of equilibrium and stability properties of a semi-toroidal current loop imbedded in a high temperature plasma. The loop has a toroidal current density and poloidal current density. By explicitly including the global curvature of the loop, the net Lorentz and pressure forces acting along the major radius are calculated. Requirement of equilibrium force-balance gives rise to conditions that must be satisfied by physical parameters such as current, pressure, magnetic field and geometry. On the basis of these conditions, we deduce a class of equilibrium semi-toroidal current loops satisfying $c^{-1}\underline{J}\times\underline{B} - \underline{\nabla}p = 0$. Furthermore, this class of equilibria is shown to be stable to a number of destructive MHD modes. Application of the theoretical results to solar bipolar current loops is discussed.

I. INTRODUCTION

Among the diverse variety of possible current and magnetic structures in the solar atmosphere, one configuration which has received wide attention is that of bipolar current loops. Not only are such loops relatively amenable to quantitative treatment but also they may play an important role in solar flares and other surface phenomena. Indeed, the recent Skylab observations have shown that a significant number of flare events are associated with bipolar loops in preflare active regions.

An important property of the bipolar loop structures is that they often appear to be quasi-stationary on the time scale of hours. This suggests that such magnetic and current loops can be considered to be in equilibrium for the purpose of understanding their large-scale structures. Accordingly, considerable effort has been devoted to the investigation of the nature of the supposed equilibrium current and magnetic structures. In addition, some attempts have also been made to incorporate equilibrium bipolar loops into flare models. As a result, a large body of literature on equilibrium loop models already exists, and can be found in a number of recent and well-known review papers and books on solar flare physics and related topics. These references include Brown and Smith (1980), Priest (1981), Sturrock (1980) and Švestka (1976, 1981).

Another important attribute of bipolar current loops is the intrinsic curvature of the configuration. A bipolar current loop typically has a major radius R of 10^4km to 10^5km and a minor radius a of 10^3km to 10^4km with the aspect ratio R/a of the order of 10. A number of simple model bipolar loops have been investigated with the emphasis placed on calculating the minor radial profiles of current and pressure satisfying the equation $c^{-1}\underline{J}\times\underline{B} - \underline{\nabla}p = 0$ (Chiuderi, et al., 1977; Hood and Priest, 1979). For this purpose, straight-cylinder approximations have been used, and the question of the major radial force-balance has not been addressed. Although these approximations are generally adequate for calculating local fields, there exist effects which arise from the global curvature and which influence the forces on the current even in the large aspect ratio limit. In the present paper, we extend the previous works to include the finite major radius of curvature and investigate the effects which are important for the determination of the equilibrium structure. Using a model current loop, we explicitly calculate the major radial forces which are primarily due to the curved nature of the structure and identify a class of semi-toroidal equilibrium current loops imbedded in a background plasma. General equilibrium conditions relating the minor radial profile to the geometry will be discussed.

The apparent long-life of bipolar loops imposes another constraint on the possible configurations: the loops must not only be in equilibrium but also be stable to gross MHD instabilities that would otherwise destroy the magnetic configurations on the fast MHD time scales (typically, tens of seconds). Considerable work has also been done in this area (see, for example, Van Hoven, 1981, for a detailed review). In the present paper, we test the model current loop for stability against some destructive MHD modes.

The curvature of a current loop is shown to be important for equilibrium and stability properties in a way that has generally not been recognized previously. The new theoretical results will be discussed in the context of bipolar current loops in the solar atmosphere.

The organization of the paper is as follows: In Section II, we describe a model current loop imbedded in a background plasma and derive an equilibrium force-balance condition. Section III discusses the basic MHD stability properties of the model current loop. In Section IV, the the results and possible application to solar bipolar current loops will be considered.

II. EQUILIBRIUM PROPERTIES

A. A Model Bipolar Current Loop

Consider an equilibrium current loop carrying a "toroidal" current density J_t and "poloidal" current density J_p as shown schematically in Figure 1. The loop is imbedded in a high-temperature plasma of pressure p_a and we consider the case where there is no reverse current. The components J_t and J_p produce magnetic field components B_p and B_t, respectively. The current distribution below the photosphere does not directly affect the equilibrium consideration but is included to indicate the conservation of current.

As the figure shows, a bipolar loop is intrinsically curved regardless of its detailed internal structure. As a first approximation, we model the geometry with a semi-torus having a major radius R and a uniform circular cross-section of radius a. Here, the aspect ratio R/a is roughly 10. Clearly, bipolar loops are generally not true semi-tori with uniform cross-sections. However, observations seem to indicate that the minor cross-sections often vary slowly along the loops in the corona (Foukal, 1976; Krieger, 1977). In any case, the corrections introduced by this simplication will turn out to be unimportant for the consideration of equilibrium forces and will be discussed as we develop our analysis in the next section.

The class of equilibrium semi-toroidal (henceforth called toroidal) loops considered in this paper consists of those configurations in which the Lorentz force $c^{-1}(\underline{J}x\underline{B})$ and the pressure force $(-\underline{\nabla}p)$ are balanced. If we denote by $\underline{f}$ the force density acting on the fluid elements, then

$$\underline{f} = \frac{1}{c}\, \underline{J}x\underline{B} - \underline{\nabla}p, \tag{1}$$

$$\underline{J} = \frac{c}{4\pi}\, \underline{\nabla}x\underline{B},$$

with $\underline{f} = 0$ everywhere in equilibrium. In this equation, the gravitational force which is not due to curvature effects is neglected in comparison with the Lorentz force.

The basic structure of the current loop described in the preceding paragraphs is similar to those of the model solar bipolar loops considered in a number of previous works, including Chiuderi, et al (1977), Hood and Priest (1979), Van Hoven (1981) and references therein. The significant new ingredient in the present paper is the consideration of major radial forces in the intrinically toroidal structure. Although the model geometry is a simple one, the underlying basic physics is applicable to a wide variety of curved current distributions imbedded in a plasma, and the qualitative results will be shown to be insensitive to the detailed structure.

B. Equilibrium Forces

If a toroidal current loop is in equilibrium given by Equation (1) with f = 0, then the total force acting on the loop along the major radius must vanish. However, the existence of curvature gives rise to non-zero contributions from the Lorentz and pressure forces. In order to calculate the net major radial force including the curvature effects (henceforth called toroidal effects), we integrate $\underline{f}$ over the semi-torus. The result can be conveniently expressed as follows (Shafranov, 1966):

$$F = \frac{I_t^2}{c^2 R}\left[\ell n\left(\frac{8R}{a}\right) + \beta_p - \frac{3}{2} + \frac{\ell_i}{2}\right] , \tag{2}$$

where F is the total major radial force per unit length of the loop and I_t is the total toroidal current. In equilibrium F = 0. The quantity β_p is defined as

$$\beta_p \equiv \frac{\overline{p} - p_a}{B_p^2/8\pi} , \tag{3}$$

where $\overline{p}$ is the average internal pressure of the loop, p_a is the ambient pressure and $B_p = B_p(a)$. (This quantity β_p should not be confused with the plasma β.) The quantity ℓ_i represents the internal inductance per unit length of the loop, characterizing the detailed profile of the current distribution, and is given by

$$\ell_i \equiv \frac{2\int_0^a dr\ r\ B_p^2(r)}{a^2\ B_p^2(a)} ,$$

where the integration is across the minor radius. The ℓ_i is of order unity and takes on the values $\ell_i = 0$ for a surface current model and $\ell_i = 1/2$ for a solid current model. A detailed derivation of Equation (2) including some simple applications to astrophysical situations can be found in Shafranov (1966) and, therefore, will not be repeated here. However, we will give in the next section a more physically transparent heuristic derivation, illuminating the significance of the equation.

Before we proceed, it is of importance to note that Equation (2) is derived for an equilibrium current loop in a background plasma of pressure p_a [Section 5, Shafranov (1966)]. Thus, in spite of some superficial similarities, Eq. (2) differs significantly from the force equation used to describe a tokamak-type system in which the current carrying plasma is surrounded by vacuum, which in turn is surrounded by a metallic containment vessel. Moreover, in a tokamak, equilibrium is established by an applied "vertical" field and stabilized by a strong applied toroidal magnetic field. Shafranov's work was originally motivated by applications to laboratory plasma physics but the treatment itself is a general one, of which Equation (2) and the "tokamak equation" (Bateman, 1978) are but two special cases.

At this point, we assess the consequences of the assumptions regarding the geometry. First, Eq. (2) shows that the effects of the constant cross-section approximation enter the force consideration through the term $\ln(8R/a)$. Thus the analysis is indeed insensitive to the approximation, which can also be justified on the basis of observation (Sec. II.A). Secondly, the fact that there is only one-half of a torus introduces a correction of the order of a/R (cf. the Biot-Savart law). For bipolar current loops under consideration, a/R is typically of the order of 0.1. Thus, the geometrical corrections do not alter the results materially.

C. A Heuristic Derivation

Equation (2) is generally valid to a high degree of accuracy for large aspect ratio (R/a) and low β (less than unity) toroidal plasmas. In order to simplify the derivation while retaining the essential physics, we note that the internal inductance ℓ_i, which describes the current profiles, contributes a fraction of unity in the square brackets. Thus, the detailed current distribution in the loop has only a relatively small effect on the force analysis. In this section, we adopt a particularly simple surface current model with $\ell_i = 0$ in order to elucidate the physical significance of the equation. A more realistic profile can be thought of as a superposition of thin current layers, a tedious but straightforward method of constructing diffuse profiles.

Let the current J_t and J_p be distributed over a thin layer at the minor radial boundary $r = a$. It is useful to define I_t and I_p by

$$I_t \equiv 2\pi \int_o^a dr\ r\ J_t \ ,$$

and

$$I_p \equiv 2\pi\ R \int_o^a drJ_p \ ,$$

where r is the minor radial coordinate. For a relatively large aspect ratio ($R/a \sim 10$) torus, the magnetic field inside the loop is

$$B_t = \frac{2I_p}{cR} \ , \qquad (4)$$

and the field outside the loop is

$$B_p = \frac{2I_t}{cr} \ . \tag{5}$$

These expressions describe the local equilibrium magnetic field in the plasma in the absence of reverse currents. The correction terms in Equations (4) and (5) are of the order of (a/R) and have been neglected. The local pressure balance condition can be given by

$$\left(\frac{d}{dr} \quad p + \frac{B_t^2 + B_p^2}{8\pi}\right) = - \frac{1}{4\pi} \frac{B_p^2}{r} \ . \tag{6}$$

Integrating this equation across the boundary at $r = a$, we obtain

$$\overline{p} - p_a = \frac{B_p^2}{8\pi} - \frac{B_t^2}{8\pi} \ , \tag{7}$$

where $\overline{p}$ is the pressure inside the loop, p_a is the ambient plasma pressure outside the loop and $B_p = B_p(a)$. (In the corona, p_a is essentially constant around the minor circumference because the gravitational scale height $H \simeq 1.5\text{x}10^5$km is much greater than the typical minor radial scale of 10^3km to 10^4km. This is equivalent to neglecting the gravitational force (Section II.A) for the solar case.)

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In equilibrium, there are no time-dependent fields. Then, the total magnetic energy of the semi-torus is

$$\varepsilon = \varepsilon_t + \varepsilon_p$$

where

$$\varepsilon_t = \frac{B_t^2}{8\pi} (\pi^2 a^2 R) \quad , \tag{8}$$

and

$$\varepsilon_p \equiv \frac{1}{4} L \; I_t^2 \; . \tag{9}$$

Here, L is the self-inductance of a circular current in a plasma given by (for example, Shafranov, 1966),

$$L = \frac{4\pi R}{c^2} \left[\ell n \left(\frac{8R}{a} \right) - 2 \right] \quad , \tag{10}$$

where the internal inductance ℓ_i has been neglected.

In order to calculate the electromagnetic force, we apply an infinitesimal virtual displacement to the major radius R, holding the current I_t and I_p unchanged. After some straightforward algebra and using Equation (4) and the pressure balance condition, we obtain

$$F_1 = \frac{I_t^2}{2c^2R}(\beta_p - 1) \tag{11}$$

and

$$F_2 = \frac{I_t^2}{c^2R}\left[\ln\left(\frac{8R}{a}\right) - 1\right] . \tag{12}$$

where $F_1 = (\pi R)^{-1}\partial\varepsilon_t/\partial R$ and $F_2 = (\pi R)^{-1}\partial\varepsilon_p/\partial R$, representing forces per unit length of the loop. Equations (11) and (12) are the integrated contributions from J_pB_t and J_tB_p, respectively. The total Lorentz force F_{em} along the major radius is obtained by adding these equations, giving

$$F_{em} = \frac{I_t^2}{c^2R}\left[\ln\left(\frac{8R}{a}\right) + \frac{1}{2}\beta_p - \frac{3}{2}\right] , \tag{13}$$

where F_{em} is expressed in units of force per unit length. In order to calculate the major radial pressure force, we integrate the pressure over the minor radial boundary including the toroidicity (e.g. Golant, et al, 1980). Using Equation (7), the pressure balance condition, we find the net pressure force per unit length

$$F_p = \frac{I_t^2}{2c^2R}\beta_p . \tag{14}$$

Adding Equations (13) and (14), we obtain Equation (2). Thus, Equation (2) is the sum of the contributions from $J_t B_p$, $J_p B_t$, the pressure inside the loop $\bar{p}$ and the pressure outside the loop p_a.

Note that F_{em} and F_p are primarily due to the toroidicity of the loop and not to the detailed internal structure. This point is also born out by the fact that F_{em} and F_p occur even though B_t and B_p are obtained using the large aspect ratio approximation. In a true straight cylinder, the forces per unit length, F_{em} and F_p, properly vanish as they should. The important point is that the balance of the above forces must be treated with care in a toroidal system in a tenuous plasma such as the corona. Note also that the equilibrium values of $\ln(8R/a)$ and β_p both diverge for infinite R. Inspection of the force equations shows that $\ln(8R/a)$ and β_p represent total forces. As R increases to infinity, so do the total forces as well as other physical quantities such as mass. It is more physically meaningful to interpret the results in terms of the relevant quantities per unit length which must be well behaved in any physically acceptable systems.

The basic physics described here has also been used in connection with a number of other effects in the solar atmosphere. For example, Anzer (1978) and Van Tend (1979) described the motion of a loop-like coronal transient using the Lorentz force acting on a current ring (Equation (7) of Anzer, also expressed as the force per unit length). The model current profile chosen is a uniform toroidal current loop in which poloidal current, the ambient coronal gas and the pressure gradients are neglected (but gravity is included). It is straightforward to see that Anzer's equation (7) is identical to our Equation (12) by setting F_1, the poloidal current contribution, equal to zero and by setting $\ell_i = 1/2$ for a solid current model. The magnetic force described by Equation (12) is well known and is often called the "hoop stress".

The above heuristic derivation is exact for the simple current profile. The emphasis has been placed on a detailed exposition of the underlying physics. However, consistent with the basic model described in Sec. II.A, Eq. (2) can be proved quite generally (Shafranov, 1966) for a wide range of distributed current profiles imbedded in a back-ground plasma with the caveat that $\overline{p}$ and B_t are now the internal pressure and toroidal magnetic field averaged over the minor cross-section. The fact that it is possible to reproduce Eq. (2) using a simple current model is yet another confirmation that the equation describes a detail-independent aspect of the physics of toroidal equilibrium forces.

As a general remark, we point out that the magnetic "tension", which has occasionally been invoked in connection with equilibrium considerations is a part of the Lorentz force (Jackson, 1962) and is already included consistently in Eq. (2).

D. Equilibrium Conditions

In this section, we infer a number of general conditions on the relevant physical parameters without reference to any particular minor radial profiles. In equilibrium where the major radius remains quasi-stationary, the force F per unit length must vanish. Then,

$$\beta_p = -\ln\left(\frac{8R}{a}\right) + \frac{3}{2}\ , \tag{15}$$

where the unimportant internal inductance term ℓ_i has been neglected. For aspect ratios of the order of 10, β_p is negative and takes on values of roughly -3. A negative β_p implies

$$\bar{p} < p_a . \tag{16}$$

In a diffuse current distribution, this means that the average internal pressure is less than the ambient pressure. However, since p = 2nkT, the density inside the loop need not be less than the ambient density. Equation (15) has not been discussed in connection with equilibrium bipolar current loops and describe a specific relationship showing the geometrical constraint on the relevant physical parameters. However, we point out that equilibrium toroidal current loops with positive pressure gradients have been investigated by a number of researchers

(Chiuderi, et al, 1977; Hood and Priest, 1979), motivated by observational indications for such loops (Foukal, 1976). Our results are not based on any particular observation but are derived as consequences of major radial force-balance, giving a theoretical basis for the importance of the type of configurations considered in this paper and in the above references.

III. MHD STABILITY PROPERTIES

If a bipolar current loop persists for an extended period of time, the loop must not only be in equilibrium but also be stable to gross MHD instabilities that would destroy its overall magnetic configuration on fast MHD time scales. Considerable effort has been devoted to this subject (see, for example, Van Hoven, 1981, and references therein). In this section, we test the class of equilibrium loops described in the preceding section for stability against the following destructive instabilities: the "sausage" mode ($m = 0$), "kink" mode ($m = 1$) and the local Suydam modes. Since the treatment of these instabilities and the underlying physics are standard in textbooks, we will only show the relevant results. Interested readers are referred to, for example, Krall and Trivelpiece (1970), Schmidt (1979) and Kadomtsev (1966).

Equations (3), (7) and (15) show that, for $R/a \approx 10$, we have $B_t \approx 2\ B_p$. This immediately allows one to conclude that $m = 0$ (sausage) mode is stable since the stability condition is given by $B_t > B_p/\sqrt{2}$. For the $m = 1$ (kink) mode, we consider the long wavelength modes since these are the most destructive ones. Because the footpoints are essentially immobile on the relevant time scales due to the high mass density in the photosphere, the longest wavelength for the semi-torus is πR (another important deviation from the tokamak case) so that $ka << 1$. Then, the approximate eigenfrequency can be given by (Kadomtsev, 1966)

$$\omega^2 = \frac{B_p^2 k^2}{4\pi\rho}\left[1 - \left(\frac{B_p}{B_t}\right)^2 \ln\left(\frac{1}{ka}\right)\right] ,$$

where k is the toroidal wavenumber and ρ is the mass density of the loop. Using the equilibrium magnetic field, we find that the quantity in the square brackets is positive so that $\omega^2 > 0$. Thus, the loop is stable to the $m = 1$ mode. This behavior is qualitatively similar to the stabilizing influence of $dp/dr > 0$ (Giachetti, et al, 1977; Van Hoven, et al, 1977).

In order to discuss the local pressure driven instability which tends to destroy the magnetic surfaces, we first define the safety factor q(r) by

$$q(r) = \frac{r}{R}\frac{B_t}{B_p} .$$

For any current profile, B_p vanishes linearly in r near $r = 0$ and q decreases from some finite value near $r = 0$ to zero outside the loop. For the loops under consideration, the typical value is $q \approx 0.2$, remaining less than unity inside the loop. The condition for stability against local perturbation is given by the Suydam condition (Suydam, 1958) appropriately modified for the toroidal geometry (Mercier, 1960);

$$\frac{1}{4}\left(\frac{1}{q}\frac{dq}{dr}\right)^2 + \frac{1 - q^2}{B_t^2/8\pi}\left(\frac{1}{r}\frac{dp}{dr}\right) > 0 \ .$$

For the loops under consideration, we have $dp/dr > 0$ and $q < 1$. We see that the stability condition is satisfied throughout the loop interior. This should be contrasted with a tokamak plasma in which $dp/dr > 0$ so that the $q < 1$ region near the center is generally unstable. As a general remark, we emphasize the fundamental differences exhibited by laboratory toroidal plasmas and bipolar current loops that may exist in an environment such as the solar atmosphere.

Note that the results regarding the stability behavior discussed above do not include the toroidicity directly. Rather, the toroidal equilibrium condition (Equation (15)) and Equations (3) and (7) determine the overall toroidal configuration, which then determines the stability properties.

It is useful to note that singular current distributions such as our surface current model are generally less stable than diffuse current profiles. We expect loops with distributed current to possess even more favorable stability characteristics.

IV. DISCUSSION

In the preceding sections, we have described a class of semi-toroidal equilibrium current loops imbedded in a background plasma. This class of loops satisfies the equation $c^{-1}\underline{J}\times\underline{B} - \underline{\nabla}p = 0$ with the major radial forces explicitly balanced. Here, the current density is given consistently by $\underline{J} = (c/4\pi)\ \underline{\nabla}\times\underline{B}$. The principal purpose of the paper is to illustrate the nature and effects of the significant toroidal forces acting on the intrinsically curved equilibrium structure. For this purpose, a simple model current loop (Section II.A) has been used to derive a number of equilibrium conditions (Equation (15) and Inequality (16)). In addition, this class of loops is seen to possess favorable MHD stability properties (Section III).

For the solar applications, we note that the Sun manufactures a diverse variety of structures that may appear to be bipolar loops. Our analysis is intended to model a subclass of possible configurations consistent with the description of Sec. II.A. As noted earlier, a number of previous works have investigated similar model loops with respect to minor radial profiles but without including the major radial force balance. The origin of the new results found in the present paper can be traced to the inclusion of the toroidicity of the loop. It is well-known (Bateman, 1978; Van Hoven, 1981) that a straight cylindrical plasma is an ill-constrained configuration. The results given in this paper are characteristic of toroidal equilibrium magnetic and current structures imbedded in a background plasma, and constitute geometrical constraints on the physical quantities for the class of loops described in Section II.A. If bipolar loops of this type are present in the solar corona, then the above equilibrium conditions or qualitatively similar conditions are expected to hold. Conversely, these conditions can be used to distinguish this class of equilibria from other types of equilibria that may exist in the solar corona. For example, Cheng (1980) gives a detailed observational description of a number of loops in an active region which appear to have average internal pressures that are slightly higher than the ambient pressure of approximately 2 dyn/cm^2. These loops may require additional observational and theoretical considerations. However, the basic physics of toroidal forces still should be accounted for in the equilibrium force considerations.

Note that the physical dimensions R and a in the description of the model loop refer to the magnetic and current structure, which may differ from the "visible" loop structure in some cases. In addition, our analysis uses local physical quantities such as currents and magnetic fields in the corona. The observational inferences of these quantities have not been established definitively. However, magnetic fields of the order of 10G are frequently quoted as estimates in connection with loop structures (e.g. Levine and Withbroe, 1977; Chiuderi, et al, 1977; Hood and Priest, 1979). Smaller and larger values may also occur.

Finally, keeping in mind the above potential uncertainties, we describe an order-of-magnitude illustration of the toroidal effects, consistent with the equilibrium conditions of Section II. Consider an equilibrium loop of $R \approx 5 \times 10^9$cm and $a \approx 5 \times 10^8$cm, carrying a current of $\approx 10^{10}$A. This particular example corresponds to (coronal) magnetic fields of the order of 10G. Then, the major radial hoop stress is $F_2 \approx 6 \times 10^8$ dyn/cm (Equation (12)) with the Lorentz force of $F_{em} \approx 3 \times 10^8$ dyn/cm (Equation (13)), where the equilibrium value of $\beta_p \approx -2.8$ has been used (Equation (15)). In the papers of Anzer (1978) and Van Tend (1979), it is also the hoop stress that is used (see Section II.C). If the ambient pressure is ≈ 2 dyn/cm^2 (e.g. a solar active region), corresponding to, for example, $n \approx 5 \times 10^9$cm^{-3} and $T \approx 2 \times 10^6$K, then the total pressure force is $F_p \approx -3 \times 10^8$ dyn/cm (Equation (14)). In equilibrium as in this example, the net force per unit length is obviously zero.

In this paper, we have used a simple model geometry to illustrate the basic physics of toroidal equilibria in a background plasma. Modifications for more complex geometry may be useful for a better understanding of the observed structures. The dynamic behavior of an initial equilibrium loop is also being studied (Xue and Chen, 1980; Chen and Xue, 1981). More detailed work will be reported in a future report.

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REFERENCES

Anzer, U.: 1978, Solar Phys. 57, 111.

Bateman, G.: 1978, MHD Instabilities, The MIT Press, Cambridge, Massachusetts.

Brown, J.C. and Smith, D.F.: 1980, Rep. Prog. Phys. 43, 125.

Chen, J. and Xue, M.L.: 1981, Bu-l. Am. Astron. Soc. 13, 889.

Cheng, C.C.: 1980, Astrophys. J. 238, 743.

Chiuderi, C., Giachetti, R. and Van Hoven, G.: 1977, Solar Phys. 54, 107.

Foukal, P.V.: 1976, Astrophys. J. 210, 575.

Giachetti, R., Van Hoven, G., and Chiuderi, C.: 1977, Solar Phys. 55, 371.

Golant, V.E., Zhilinsky, A.P., and Sakharov, I.E.: 1980, Fundamentals of Plasma Physics, John Wiley & Sons, NY.

Hood, A.W. and Priest, E.R.: 1979, Astron. Astrophys. 77, 233.

Jackson, J.D.: 1962, Classical Electrodynamics, John Wiley & Sons, New York.

Kadomtsev, B.B.: 1966, in Reviews of Plasma Physics, Vol. 2, Consultants Bureau, New York.

Krall, N.A. and Trivelpiece, A.W.: 1973, Principles of Plasma Physics, McGraw-Hill, New York.

Krieger, A.S.: 1977, in Proceedings of the OSO-8 Workshop, University of Colorado, Boulder.

Levine, R.H. and Withbroe, G.L.: 1977, Solar Phys. 51, 83.

Mercier, C.: 1960, Nuclear Fusion 1, 47.

Priest, E.R.:(ed.), 1981, Solar Flare Magnetohydrodynamics, Gordon and Breach, London.

Schmidt, G.: 1979, Physics of High Temperature Plasmas, 2nd ed., Academic Press, New York.

Shafranov, V.D.: 1966, in Reviews of Plasma Physics, Vol. 2, Consultants Bureau, New York.

Sturrock, P.A. (ed.): 1980, Solar Flares, Colorado Assoc. Univ. Press, Boulder, Colorado.

Švestka, A.: 1976, Solar Flares, D. Reidel Publishing Co., Dordrecht, Holland.

Švestka, A.: 1981, in Priest, E.R. (ed.) Solar Flare Magnetohydrodynamics, Gordon and Breach, London.

Van Hoven, G., Chiuderi, C., and Giachetti, R.: 1977, Astrophys. J. 213, 869.

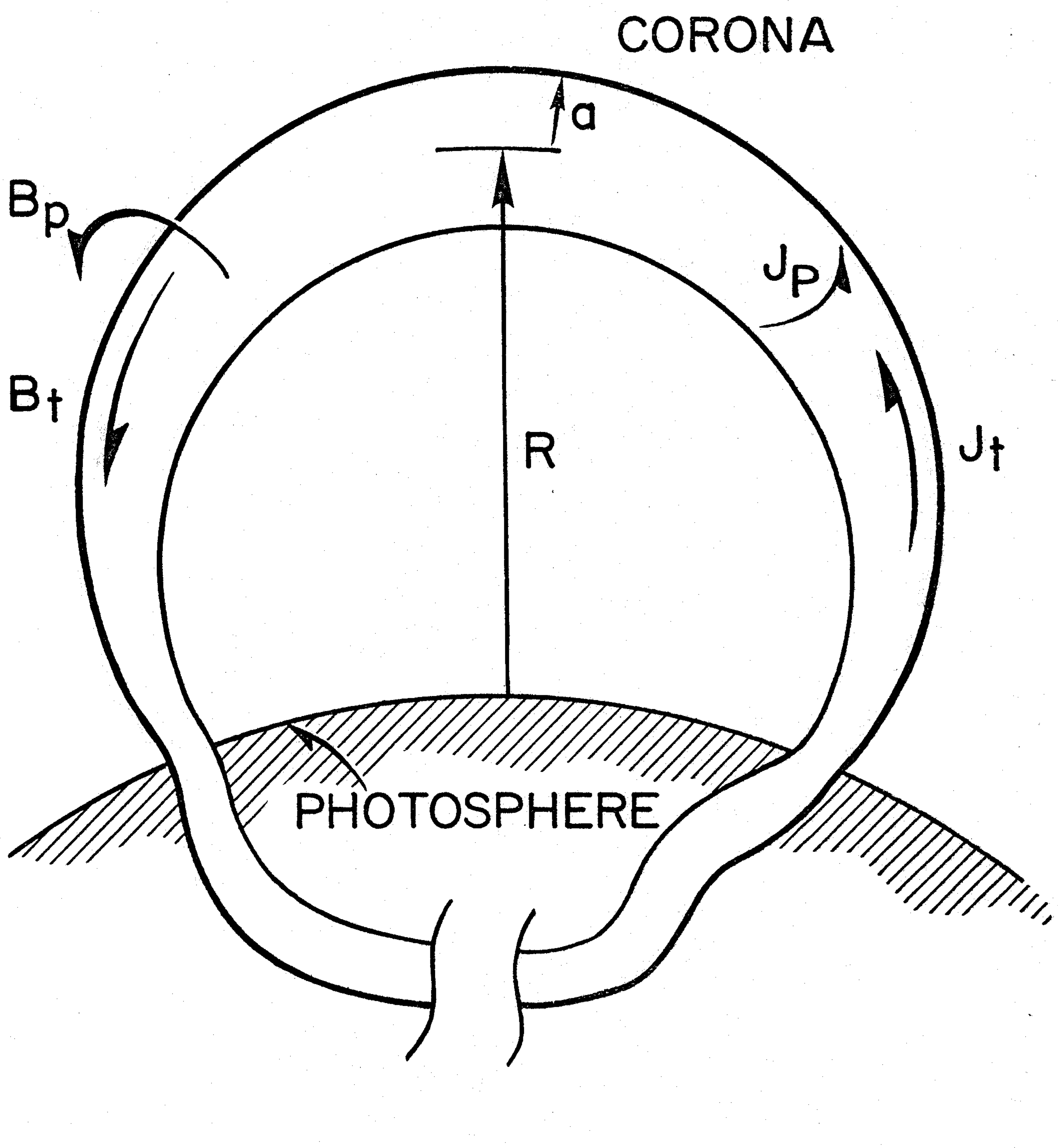
Van Hoven, G.: 1981, in Priest, E.R. (ed.) Solar Flare Magnetohydrodynamics, Gordon and Breach, London.

Van Tend, W.: 1979, Solar Phys. 61, 89.

Xue, M.L. and Chen, J.: 1980, PFC/JA-80-27, Massachusetts Institute of Technology, Cambridge, MA.

FIGURE CAPTION

Fig. 1 A schematic representation of a simple bipolar current loop imbedded in the corona. No particular structure is specified below the photosphere. See Section II.A for detail.


CORONA
a
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