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FREE ELECTRON LASER RADIATION INDUCED BY A PERIODIC DIELECTRIC MEDIUM

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ABSTRACT

Stimulated emission by an unaccelerated electron beam propagating through a periodically modulated dielectric is studied. The laser gain in the low gain regime is calculated for the case of a cold, tenuous electron beam by applying the Einstein coefficient technique in the classical limit $\hbar \to 0$. In the high gain, strong pump regime equations for the evolution of the electron beam dynamics and of the radiation are developed using a self-consistent, one dimensional model for the interaction. Analytic calculations of the small signal gain, and numerical computations of the nonlinear saturation characteristics are presented.

I. INTRODUCTION

In a conventional free electron laser[1,2,3] (FEL), coherent electromagnetic radiation is produced by subjecting an electron beam in vacuum to a transverse periodic "wiggler" magnetic field which induces transverse periodic accelerations on the electrons. The undulating electron beam interacts with an incident electromagnetic wave, thereby generating an axially directed ponderomotive force which bunches the electrons. Subsequent interaction of the bunched beam with the incident wave leads to energy growth of the wave at the expense of the beam kinetic energy.

In this paper we examine an alternate concept in which an unaccelerated electron beam traveling at constant velocity traverses through, or passes in the immediate vicinity of a periodically modulated dielectric medium. The arrangement is illustrated schematically in Fig. 1. An electron beam traveling along the z axis with velocity $v_{\parallel}$ traverses a dielectric whose dielectric coefficient $K(\omega,z)$ undergoes periodic oscillations of the form

$$K(\omega,z) = K_0(\omega) + \Delta K(\omega)\cos(k_W z) \quad (1)$$

where $\Delta K(<<K_0)$ is the modulation amplitude and $k_W=2\pi/\ell_W$ is the "wiggler" wavenumber, with ℓ_W as the period. The dielectric coefficient may or may not be a function of the radiation frequency ω.

An electromagnetic wave

$$\vec{E} = \vec{E}_0\exp[j\omega t - jk_{\perp}x - j\int\kappa_{\parallel}(z)dz] \quad (2)$$

travels through the medium with the propagation vector $\vec{k}=\hat{x}k_{\perp}+\hat{z}\int\kappa_{\parallel}dz$ directed at an angle θ to the electron beam velocity. Such a wave may be a transverse electromagnetic (TEM) wave in an unbounded medium or a transverse magnetic (TM) waveguide mode. In either case with $\theta\neq 0$, the axial component of $\vec{E}$ now provides the "ponderomotive" force which bunches the electron beam and leads to wave growth.

This FEL mechanism is the stimulated manifestation of the incoherent, spontaneous "resonance transition radiation" studied by many workers.[4-8]

For that reason, and for the sake of brevity, we shall refer to it henceforth as the "Transition FEL". However, it is quite unlike the stimulated Čerenkov FEL[9-16] in which an electron beam passes through or near a homogeneous dielectric medium. Indeed, the well-known Čerenkov condition $(v_{\parallel}/c)\sqrt{K_0}\cos\theta=1(K_0>1)$ need not be satisfied in the Transition FEL, and stimulated emission can be achieved for dielectrics for which $K_0(\omega)$ is both greater and smaller than unity. The latter affords the possibility of generating coherent electromagnetic radiation by passing an electron beam through a modulated electron plasma whose (unperturbed) dielectric coefficient is given by

$$K_0(\omega) = 1 - \left(\omega_p/\omega\right)^2 \quad (3)$$

where $\omega_p=(Ne/m_0\varepsilon_0)^{\frac{1}{2}}$ is the electron plasma frequency of the medium. Such a modulated plasma could be created artificially, for example, by microwave or laser breakdown of a low pressure un-ionized gas in a quasioptical resonator; or by use of alternating slabs of material and operation at frequencies that exceed atomic frequencies for the materials in question. Indeed, use of such periodic stacks of material has led Piestrup and Finman[19] to examine propects for stimulated x-ray emission from such a system. Our analysis is a follow up on their work, but differs both in technique and in some of the results derived.

In section II of this paper we discuss the frequency characteristics of the Transition FEL. In section III we obtain the gain in the low gain regime for the case of a cold, tenuous electron beam. This gain is calculated by applying the Einstein coefficient technique in the classical limit $\hbar\to0$. In the high gain strong pump regime discussed in section IV, we develop equations for the electron beam dynamics and the radiation field using a self-consistent, one dimensional model for the interaction. Analytic calculations of the small signal gain, as well as numerical computations of the nonlinear saturation characteristics are presented. Section V summarizes the results.

II. RADIATION FREQUENCY

The emission from a Transition FEL results from the interaction between the ponderomotive wave on the beam,

$$\omega = \left(k_{\parallel} + nk_W\right)\beta_{\parallel}c \qquad (4)$$

with k_W as the dielectric periodicity, and an electromagnetic wave

$$\omega^2 K(\omega,z) = \left(k_{\parallel}^2 + k_{\perp}^2\right)c^2 = k_{\parallel}^2 c^2 + \omega^2 K_0(\omega)\sin^2\theta \qquad (5)$$

where $\beta_{\parallel}=v_{\parallel}/c$ is the normalized electron beam velocity, and n=0, ±1, ±2 ... is the harmonic number of the interaction. Maximum gain occurs near the frequency ω corresponding to crossing points of the above waves. Eliminating $k_{\parallel}$ between Eqs. (4) and (5), and setting $K(z)\simeq K_0$ subject to the assumption that $\Delta K/K \ll 1$, yields the radiation frequency

$$\omega = \frac{nk_W\beta_{\parallel}c}{1 - \beta_{\parallel}\sqrt{K_0(\omega)}\cos\theta} \qquad (6)$$

Equations (4) and (5) are similar to those encountered in the conventional FEL[1] using magnetic wigglers, for which n is typically unity, $K_0=1$ and $\theta=0$. The existence of the higher harmonics in the ponderomotive wave of Eq. (4) comes about, as we shall see later, from the periodicity of the wavenumber $\kappa_{\parallel}(z)$. The axial phase velocity $\omega/k_{\parallel}$ of the wave is greater than the electron beam velocity $v_{\parallel}$ just as it is in a conventional FEL driven by a magnetic wiggler. This is unlike the Čerenkov type[16] of FEL which must satisfy the resonance condition $\omega/k_{\parallel}=v_{\parallel}$. In fact, the Čerenkov condition for a homogeneous medium is recovered by setting k_W in Eq. (6) equal to zero, with the result that $\beta_{\parallel}\sqrt{K_0}\cos\theta=1$.

It is clear from Eq. (6) that in order to achieve high radiation frequencies, the electrons must have relativistic velocities with $\beta_{\parallel}\approx 1$. In this case also the spontaneously[4-8] emitted transition radiation is directed into a narrow cone subtending an angle $\theta\approx 1/\gamma$ with the z axis, with $\gamma=\left(1-\beta_{\parallel}^2\right)^{-\frac{1}{2}}$ =

$(1+eV/m_0c^2)$ as the relativistic energy factor, and V the beam voltage.

The FEL frequency tuning characteristics as a function of electron beam energy depend sensitively on the dispersion characteristics of the dielectric, and on whether K_0 is greater or less than unity. We consider two cases. First we take K_0 to be dispersionless and slightly greater than unity as is appropriate, say, to an un-ionized gas at frequencies well removed from atomic resonances. Writing $K_0-1\equiv\alpha<<1$, and choosing the angle of observation $\theta=\sin^{-1}(1/\gamma)$, one finds from Eq. (6) that

$$\omega = \frac{nk_w\beta_{\parallel}c\gamma^2}{1-(\beta_{\parallel}\gamma)^2(\alpha/2)} \qquad (7)$$

which is shown plotted in Fig. 2a for $\alpha=2\times10^{-3}$. The frequency scaling illustrates the $\sim\gamma^2$ upshift familiar to conventional FEL's, but differs from the Čerenkov FEL whose frequency generally decreases[17] with increasing γ.

As our second example, we take $K_0(\omega)$ of Eq. (3), which refers to the highly dispersive electron plasmas. Inserting Eq. (3) in Eq. (6) and solving for the frequency ω gives

$$\omega = \frac{nk_w c\beta_{\parallel}}{(1-\beta_{\parallel}^2\cos^2\theta)}\left[1 \pm \beta_{\parallel}\cos\theta\sqrt{1-\frac{\omega_p^2(1-\beta_{\parallel}^2\cos^2\theta)}{(nk_w c\beta_{\parallel})^2}}\right] \qquad (8)$$

Here the FEL radiation frequency exhibits two branches, a high frequency branch corresponding to the positive sign, and a low frequency branch corresponding to the negative sign. Moreover, there exists a minimum electron beam energy γ below which no FEL interaction can take place. These features are illustrated in Fig. 3a which is plotted for the case $\theta=\sin^{-1}(1/\gamma)$ and $(\omega_p/nk_w c)^2=300$.

The reason for the two roots and the low energy cutoff are due to the dispersive nature of $K_0(\omega)$, as a result of which the electromagnetic wave of Eq. (5) now becomes

$$\omega^2 = (k_{\parallel}c/\cos\theta)^2 + \omega_p^2. \tag{9}$$

which shows that no propagation can occur at frequencies $\omega<\omega_p$. The ponderomotive wave given by the straight line relationship of Eq. (4) can now intersect the electromagnetic wave of Eq. (9) at two points. One then finds that at high energy γ, the two interacting waves exhibit phase synchronism at two well separated frequencies, one low and the other high. As γ is decreased the frequency separation decreases until a single, tangential intersection occurs corresponding to a minimum beam energy below which Eqs. (4) and (9) have no solution. These manifestations are similar to those also observed in conventional[20,21] FEL's.

The FEL gain is maximum near the tangential intersection discussed above (see section III). At this point then, and with $\theta=\sin^{-1}(1/\gamma)$, it follows from Eq. (8) that

$$\omega = \frac{nk_w c\beta_{\parallel}\gamma^2}{1+\beta_{\parallel}^2} = \frac{\omega_p\gamma}{(1+\beta_{\parallel}^2)^{\frac{1}{2}}} \tag{10}$$

III. FEL GAIN IN THE LOW GAIN REGIME

The power gain $G = \Delta P/P_i$ (change in output power divided by the total input power) can be conveniently calculated in the low gain regime ($G<1$) by means of the classical ($\hbar \to 0$) form of the Einstein coefficient method, which states that,[22]

$$G = \frac{8\pi^3 c^2 L}{K_0(\omega)\hbar\omega^3} \int \eta_\omega(\vec{p}')[f(\vec{p}') - f(\vec{p})]d^3p' \quad . \tag{11}$$

Here $G>0$ signifies power gain, and $G<0$, power loss; $\eta_\omega = (d^2W/d\omega d\Omega T)$ is the rate of spontaneous energy emission per unit frequency interval $d\omega$, per unit solid angle $d\Omega$, by one electron radiating for a time T; $f(\vec{p}')$ and $f(\vec{p})$ are the equilibrium momentum distribution functions for the beam electrons, with $\vec{p}'$ and $\vec{p}$ as the upper and lower momentum states associated with the transition; and d^3p' is a volume element in momentum space. L is the interaction length.

In the classical limit, we can expand $f(\vec{p}')$ around $f(\vec{p})$ in a Taylor series, retain the first two terms, and noting that $d^3p' \to d^3p$, obtain from Eq. (11)

$$G = \frac{8\pi^3 c^2 L}{K_0(\omega)\hbar\omega^3} \int \eta(\vec{p}) \frac{\partial f(\vec{p})}{\partial \vec{p}} \cdot \Delta\vec{p} \; d^3p \tag{12}$$

where $\Delta\vec{p} = (\vec{p}'-\vec{p})$ must be computed for the problem under investigation. Momentum conservation in a periodic system with periodicity $\vec{k}_w$ and no external fields other than the wiggler field requires that

$$\Delta\vec{p} = (\vec{p}' - \vec{p}) = \hbar(\vec{k} + n\vec{k}_w) \tag{13}$$

which leads to the sought-after classical form for the gain applicable to a variety of FEL systems:

$$G = \frac{8\pi^3 c^2 L}{K_0(\omega)\omega^3} \int \eta(\vec{p}) \frac{\partial f}{\partial \vec{p}} \cdot (\vec{k} + n\vec{k}_w) \; d^3p \quad . \tag{14}$$

We note in passing that on solving Eq. (13) together with the energy conservation equation

$$E' - E = \hbar\omega \tag{15}$$

where E is the total relativistic plus rest mass energy, yields,

$$\omega = (\vec{k} + n\vec{k}_w)\cdot\vec{\beta}c, \quad (\hbar \to 0) \tag{16}$$

which is just the vector form of the pondermotive wave of Eq. (4).

For present purposes we shall assume that the electrons have zero temperature in all directions, as a result of which the distribution function is a delta function distribution of the form,

$$f(p_\parallel, p_\perp) = \frac{N_b}{2\pi p_\perp}\,\delta(p_\perp - p_{0\perp})\,\delta(p_\parallel - p_{0\parallel}). \tag{17}$$

Here N_b is the number density of beam electrons, $p_{\parallel 0}=\gamma m_0\beta_\parallel c$ is the axial electron momentum and $p_\perp=\gamma m_0\beta_\perp c$ the transverse momentum. Inserting Eq. (17) in Eq. (14), noting that for our cylindrically symmetric geometry $d^3p=2\pi p_\perp dp_\perp dp_\parallel$, and integrating by parts gives

$$G = -\frac{8\pi^3c^2N_bL}{K_0(\omega)\omega^3}\left\{(k_\parallel + k_w)\left[\frac{\partial\eta(p_\parallel, p_\perp)}{\partial p_\parallel}\right]_{p_\parallel=p_{\parallel 0}} + k_\perp\left[\frac{\partial\eta(p_\parallel, p_\perp)}{\partial p_\perp}\right]_{p_\perp=p_{0\perp}}\right\} \tag{18}$$

which states that the FEL gain is proportional to the derivative with respect to the momentum of the spontaneous emission rate. This is a form of Madey's theorem.[23] Thus, we see that this theorem is a special application to free electron lasers of the Einstein coefficient method in its classical limit. The more general Eqs. (11) to (14), which allow for finite electron beam temperature, have also been applied to the cyclotron maser (gyrotron) instability,[22,24,25] the conventional FELs excited by magnetic wigglers[26,27] and to collisional instabilities.[22,28,29]

To complete the calculation of G we must derive the classical rate of spontaneous emission rate η. This is accomplished by means of the equation[30]

$$\eta = \frac{1}{T}\,\frac{e^2\omega^2}{16\pi^3\varepsilon_0 K_0(\omega)c^2}\left|\int_{-T/2}^{T/2}\hat{q}\times(\hat{q}\times\vec{\beta})e^{-j\omega t+j\int\kappa_{\parallel}dz}dt\right|^2 \tag{19}$$

where $\hat{q}=\vec{k}/|\vec{k}|$ and where the term $\exp[j\int\kappa_{\parallel}dz]$ is derived from Eqs. (1) and (5) followed by a series expansion in powers of $\Delta K/K$:

$$\exp[j\int\kappa_{\parallel}dz] = \sum_{n=-\infty}^{\infty} J_n(Q)\exp\left[jz\left\{\frac{\omega}{c}\sqrt{K_0(\omega)}\cos\theta + nk_w\right\}\right] \tag{20}$$

with

$$Q = \frac{\omega[\Delta K(\omega)]}{2k_w c\sqrt{K_0(\omega)}\cos\theta} \tag{21}$$

Inserting Eq. (20) in Eq. (19), setting $z=\beta_{\parallel}ct$ and integrating over time gives

$$\eta = \sum_{n=-\infty}^{\infty}\frac{e^2\omega^2\sqrt{K_0}\;T\beta_{\parallel}^2\sin^2\theta}{16\pi^3\varepsilon_0 c}\,J_n^2(Q)\,\frac{\sin^2 X_n}{X_n^2} \tag{22}$$

where

$$X_n = \frac{T}{2}\left[\omega - \omega\beta_{\parallel}\sqrt{K_0(\omega)}\cos\theta - nk_w\beta_{\parallel}c\right] \tag{23}$$

is the "detuning parameter"[31] and $T=L/\beta_{\parallel}c$ is the time the beam spends in the interaction region (L equals the system length).

The intensity of spontaneous emission is maximum whenever $X\to 0$. The case n=0 refers to Čerenkov radiation in a periodic dielectric medium. For a homogeneous, unmodulated dielectric, $\Delta K=0$, $k_w=0$, $Q=0$ and

$$\eta_c = \frac{e^2\omega^2\sqrt{K_0(\omega)}\;T\beta_{\parallel}^2\sin^2\theta}{16\pi^3\varepsilon_0 c}\,\frac{\sin^2 X_c}{X_c^2} \tag{24}$$

with $X_c=(\omega T/2)(1-\beta_{\parallel}\sqrt{K_0(\omega)}\cos\theta)$, which is the familiar expression for spontaneous Čerenkov radiation.

On substituting Eq. (22) in Eq. (18), performing the differentiation with respect to $\vec{p}$ and requiring that $L \gg 4\beta_{\parallel}c/\omega$ (interaction time >> period of the wave) yields the FEL gain for a length L of interaction region:

$$G = \sum_{n=-\infty}^{\infty} \frac{\omega_{pb}^2 L^3 \omega \sin^2\theta}{4\gamma^3\beta_{\parallel}^2\sqrt{K_0(\omega)}c^3} \; J_n^2(Q) \frac{d}{dX_n}\left(\frac{\sin^2 X_n}{X_n^2}\right) \tag{25}$$

where $\omega_{pb}=\left(N_b e^2/m_0\varepsilon_0\right)^{\frac{1}{2}}$ is the nonrelativistic beam plasma frequency. This result differs primarily from that of Ref. 19 in that the latter does not contain the Bessel function. Since $J_n(Q)$ for the problems of interest is typically less than $\sim$0.1 our gain is substantially less than that predicted by the previous authors.

Equation (25) contains gain information for several special cases. With n=0, it refers to the gain of a Čerenkov FEL in a periodic dielectric; and with $\Delta K=Q=k_w=0$, it refers to the Čerenkov gain in a homogeneous, unmodulated dielectric. Cases for which n=1,2,3 .... refer to the fundamental and higher harmonics of the Transition FEL of present interest. Maximum gain occurs when the detuning parameter $X_n=1.303$ for which value the quantity $\left(d/dX_n\right)\left(\sin^2X_n/X_n^2\right)$ has its maximum value equal to 0.5402. From Eq. (25) the maximum gain associated with the nth harmonic can then be written in terms of the electron beam current I and the electron beam cross sectional area $\sigma_b=\pi r_e^2$ as

$$\begin{aligned} G_n(\max) &= 0.8485 \left(\frac{Ik_w}{\sigma_b I_A}\right)\left(\frac{L}{\gamma\beta_{\parallel}}\right)^3 \left[\left(\frac{2\omega}{nk_w c}\right) \frac{\sin^2\theta}{\sqrt{K_0(\omega)}} \, nJ_n^2(Q)\right] \\ &= 0.8485 \left(\frac{Ik_w}{\sigma_b I_A}\right)\left(\frac{L}{\gamma\beta_{\parallel}}\right)^3 a_T^2 \end{aligned} \tag{26}$$

where $I_A=4\pi\varepsilon_0 m_0 c^3/e=17{,}000$ A is the "Alfven current". We see that the gain is proportional to the electron beam current and the interaction length L cubed.

The term in the large brackets denoted a_T^2 is a measure of the effective wiggler strength of the Transition FEL. The value of a_T as a function of the energy variable γ is shown plotted in Figs. 2b and 3b under the assumption that $\sin\theta=1/\gamma$. Figure 2b refers to the case of a modulated nondispersive medium, and Fig. 3b to a plasma medium. In Fig. 2b we assume that the modulated dielectric coefficient of Eq. (1) is given by

$$K = 1 + \alpha + \alpha(\cos k_w z) \tag{27}$$

with $\alpha=2\times10^{-3}$, and in Fig. 3b we assume that it is given by,

$$K = 1 - (\omega_p/\omega)^2 - (\omega_p/\omega)^2\cos(k_w z) \tag{28}$$

with ω_p normalized so that $(\omega_p/nk_w c)^2=300$. At the maximum gain frequency ω given by Eq. (10), $a_T \simeq n^{\frac{1}{2}} J_n(n/2)$, and is independent of the detailed physical parameters of the system; for $n=1$, $a_T \simeq 0.24$ (see Fig.3b).

It is instructive to compare the above results for the gain of the Transition FEL with the gain of a conventional FEL employing a magnetic wiggler, and operating in the so-called "low gain Compton regime":[1]

$$G_{FEL} = 0.8485 \left(\frac{Ik_w}{\sigma_b I_A}\right)\left(\frac{L}{\gamma\beta_{\parallel}}\right)^3 \left[a_{FEL}^2\right] \tag{29}$$

where $a_{FEL}=(\Omega_w/k_w c)$, and $\Omega_w=eB_w/m_0$ is the nonrelativistic cyclotron frequency associated with a wiggler magnetic field of amplitude B_w.

Equations (26) and (29) differ only in the form of the respective "wiggler amplitude" coefficients a_T and a_{FEL}. The value of a_T seen in Figs. 2b and 3b is typically in the range $\sim$0.1 to $\sim$1. This is to be compared with a_{FEL} of operating[1] FEL's, having $2\lesssim\ell_w\lesssim10$cm, and $2\lesssim B_w\lesssim 8$kG, for which a_w is typically in the range of 1 to 2. Therefore, for periodicities of a few centimeters, the gain of conventional FELs will be generally larger than for the Transverse FEL.

However, short periodicity wiggler systems,[32,33,34] which can attain high frequencies with moderate beam energies, are better suited to the Transition FEL. The reason is that whereas a_T is insensitive to the value of ℓ_w, a_{FEL} varies as ℓ_w. As a result, the gain-frequency product $G\omega$, which should generally be as large as possible, varies as $\sim 1/\ell_w^2$ for the Transition FEL but is independent of ℓ_w for the conventional FEL. One then finds that for periodicities $\ell_w \lesssim 1$cm the gain of the Transition FEL exceeds the gains of FEL's using magnetic or electromagnetic wigglers.[32,33,34]

IV. GAIN AND NONLINEAR SATURATION CHARACTERISTICS IN THE STRONG PUMP, HIGH GAIN REGIME

In this section we model the self-consistent generation of transition radiation by an electron beam in the high gain and/or nonlinear regimes. Our analysis follows that of Ref. 35 used in conventional free electron lasers. We assume that the beam is highly relativistic $\left[1/\gamma=(1-\beta_\parallel^2)^{\frac{1}{2}}<<1\right]$ and sufficiently tenuous so that space charge forces are small compared with the ponderomotive force of the wiggler.

The electron energy evolves according to the energy conservation equation

$$\frac{d\gamma}{dt} = -\left(\frac{e}{m_0 c^2}\right)\vec{v}\cdot\vec{E} \quad . \tag{30}$$

Here $\vec{v}$ is the electron velocity and $\vec{E}$ the self-consistent electric field given by

$$\vec{E} = \frac{1}{2}\left\{\sum_\ell E_\ell(z)J_\ell(Q)\exp\left[jk_\perp x + j(\ell k_w + k_\parallel)z - j\omega t\right] + c.c.\right\}\hat{e} \tag{31}$$

where Q is defined by Eq. (21), $k_\parallel=(\omega/c)\sqrt{K_0(\omega)}\cos\theta$, and $E_\ell(z)$ are slowly varying complex amplitudes which generally satisfy the inequality $dE_\ell/dz<<k_\parallel E_\ell$. In the absence of the electron beam all the amplitudes E_ℓ are equal to one another and are constant in time.

In order to reduce our equations to one dimension, we assume that the exponential factor $\exp(jk_\perp x)$ does not vary appreciably over the beam radius r_e. This implies that $k_\perp r_e=\left(k\sqrt{K_0(\omega)}\sin\theta\right)r_e\lesssim 1$. With $\theta\simeq 1/\gamma$, this means that $r_e\lesssim\gamma\lambda/2\pi\sqrt{K_0(\omega)}$ where λ is the radiation wavelength.

Inserting Eq. (31) in Eq. (30), and changing from t to z as the independent variable yields

$$d\gamma/dz = -\frac{1}{2}\sin\theta\left[\tilde{E}_n \exp(j\psi) + c\cdot c\right]$$

$$+\frac{1}{2}\sin\theta\left[\sum_{\ell\neq 0}\tilde{E}_\ell \exp\left(j\psi + j\ell k_w z\right) + c\cdot c\right] \tag{32}$$

where $\tilde{E}_\ell$ is a normalized electric field amplitude defined as

$$\tilde{E}_\ell = \left(e/m_0c^2\right)J_\ell(Q)\ E_\ell \quad . \tag{33}$$

The phase ψ evolves according to

$$\frac{d\psi}{dz} = nk_w + k_\| - \left(\omega/c\beta_\|\right) = -\ 2X_n/L \tag{34}$$

where X_n is the detuning parameter given by Eq. (23). Since the electron beam is assumed to be resonant with only the n^{th} harmonic, we average Eq. (32) over the periodicity ℓ_w. This eliminates the rapidly varying second term on the right-hand side of Eq. (32) with the result that,

$$d\gamma/dz = -\frac{1}{2}\sin\theta\left[\tilde{E}_n \exp(j\psi) + c\cdot c\right] \quad . \tag{35}$$

The evolution of the radiation field is found by inserting Eq. (31) in the wave equation

$$\nabla\times(\nabla\times\vec{E}) - (\omega/c)^2K(\omega,z)\vec{E} = j\omega\mu_0\vec{J} \tag{36}$$

where $K(\omega,z)$ is given by Eq. (1). As a result,

$$\sum_\ell 2j\left(\ell k_w + k_\|\right)\left(\frac{d\tilde{E}_\ell}{dz}\right)\exp\left[j\left(\ell k_w + k_\|\right)z-\omega t\right] = -\ j\omega\mu_0 e\ \sin\theta J_\|/m_0c^2\ . \tag{37}$$

The current density $J_\|=-Nev_\|$ is determined self-consistently from the electron dynamics. We multiply by the factor $\exp\left[-j\left(nk_w+k_\|\right)+j\omega t\ \right]$ and average, assuming that the electron beam is periodic in $\psi=\left(nk_w+k_\|\right)z-\omega t$. Thus,

$$\frac{d\tilde{E}_n}{dz} = \frac{\omega_{pb}^2\omega\ \sin\theta}{2c^2\left(nk_w+k_\|\right)}\left\langle \exp\left[-j\psi z\right]\right\rangle \tag{38}$$

where the average $\langle\ \rangle$ is over the electrons in one periodicity wavelength $2\pi/(nk_w+k_\parallel)$.

Equations (34), (35), and (38) comprise a closed system. They are identical to the equations describing a conventional FEL that uses a magnetic wiggler, except that now the quantity $\sin\theta$ replaces the familiar wiggle amplitude parameter $\Omega_w/k_w c\gamma\,(=a_{FEL}/\gamma)$ (see section III). We can, therefore, immediately linearize these equations following the method of Bonifacio, Pellegrini, and Narducci.[36] Assuming a zeroth order energy distribution function with energy spread $\Delta\gamma$ of the form

$$f(\gamma)\begin{cases} = (1/2\Delta\gamma) & |\gamma-\gamma_0| < \Delta\gamma \\ = 0 & \text{, otherwise} \end{cases} \tag{39}$$

one obtains the following cubic dispersion relation for the (complex) spatial growth rate Γ of the electromagnetic wave amplitude:

$$\Gamma^3 + 2k_0\Gamma^2 + \left[k_0^2 - n^2k_w^2(\Delta\gamma/\gamma)^2 + A_T\right]\Gamma + nA_Tk_w = 0 \tag{40}$$

Here A_T is the coupling strength of the Transition FEL given by

$$A_T = \frac{\omega_{pb}^2 \sin^2\theta}{2c^2\gamma_0}\ , \tag{41}$$

and k_0 is the detuning parameter

$$k_0 = nk_w + k_\parallel - \left(\omega/c\beta_{\parallel 0}\right) = -\,2X_n/L\ . \tag{42}$$

We note that the growth rate of a conventional FEL employing a magnetic wiggler is governed by the same dispersion equation as that given by Eq. (40) with n=1 and $A_{FEL}=\omega_{pb}^2 a_{FEL}^2/2c^2\gamma_0^3$.

For the case of a cold beam ($\Delta\gamma=0$) and near the point of maximum growth ($k_0\approx 0$), Eq. (40) can be approximated by

$$\Gamma^3 \simeq -\,nA_Tk_w \tag{43}$$

a result that, for the special case n=1, closely resembles that of Ref. 19. Of the three roots represented by the above dispersion equation, only one exhibits an exponentially growing solution with a growth rate given by the real part of Γ, namely,

$$\Gamma_r = (\sqrt{3}/2)(nA_T k_w)^{1/3} \tag{44}$$

Thus, the gain $G_n = P_0/P_i$, defined as the ratio of RF output power $P_0(z=L)$ in harmonic n, to the total RF input power $P_i(z=0)$ in all harmonics n is

$$G_n = J_n^2(Q)\left[\frac{1}{3} + \frac{1}{3}e^{-\Gamma_r L} + \frac{1}{3}e^{\Gamma_r L}\right]^2 \tag{45}$$

Here $J_n^2(Q)$ represents the fraction of the input power in harmonic n available for interaction with the electron beam; and the three terms in the brackets represent the distribution of amplitudes amongst the three (active and passive) waves. Since only one wave exhibits exponential growth, the quantity $J_n^2(Q)/9$ denotes the "launching loss". The variable Q is defined by Eq. (21).

The fact that the growth rate Γ_r increases with harmonic numbers n appears to make the Transition FEL attractive for operation at higher harmonic frequencies. However, this must be weighed against the increase in launching loss that occurs as n increases. For example, when the periodic dielectric is a plasma, and the radiation frequency is given by Eq. (10) where the growth is maximum, the launching loss equals $J_n^2(0.5n)/9$ and $\Gamma_r = \sqrt{3}(nk_w)/2\gamma^{1/3}$. The launching loss of successive harmonics n=1, 2, 3 is $\sim$22, 28, and 34dB, respectively.

The saturation efficiency η caused by phase trapping of the electrons in the potential wells of the ponderomotive potential can be estimated from the relationship[1]

$$\eta \approx \Delta E/m_o c^2(\gamma_0 - 1) \approx 2(\beta_{0\parallel}\gamma)^3(c\Gamma_i)/(\gamma_0 - 1)\omega \tag{46}$$

where Γ_i is the imaginary part of the growth rate Γ given by Eq. (43) and ω is the radiation frequency. Substituting Γ_i from Eq. (43) yields, (for $\gamma_0 \gg 1$),

$$\eta = \left(c\gamma_0^5 n k_w \omega_{pb}^2 \sin^2\theta / 2\omega^3\right)^{1/3} \quad . \tag{47}$$

For the special case of the modulated plasma dielectric, at the frequency for maximum gain specified by Eq. (10), one finds that

$$\eta = \frac{1}{\gamma_0}\left(\frac{2\omega_{pb}}{nk_w c}\right)^{2/3} \qquad (\gamma >> 1) \ . \tag{48}$$

The efficiency of a conventional FEL at the corresponding maximum gain point is given by $\eta = (2^{-1/3})\gamma_0^{-1}\,(\omega_{pb}/k_w c)^{2/3}\, a_{FEL}^{2/3}$. Typically, $a_{FEL} \sim 1$ and therefore the efficiencies of the Transition FEL and the conventional FEL are of comparable magnitudes.

In order to observe the FEL evolution towards its nonlinear regime, we performed a computer simulation. Ten kilowatts of radiant power at a wavelength of 10μm were injected into the first harmonic, n=1, at position z=0. The electron beam had a current density of 1kA/cm^2, an energy of 11.1MeV (γ_0= 22.7) and an initial energy spread $\Delta\gamma$=0.003. The unperturbed dielectric coefficient K_0=1.002 ($\alpha = 2\times10^{-3}$ in Eq. (27)). The modulated dielectric had a periodicity ℓ_w=2cm and was 100 periods long. Resonance was achieved for an angle θ between $\vec{k}$ and $\vec{z}$ (see Fig. 1) equal to 3.24×10^{-2} radians.

The computer simulations are illustrated in Figs. 4a and 4b. In Fig. 4a we plot the radiation output power as a function of the axial distance z. One observes an initial exponential growth followed by saturation, and synchrotron oscillations of the electrons trapped in the ponderomotive wells. Figure 4b shows the electron phase space (γ,ψ), after saturation, at position z=110cm; strong electron bunching is clearly visible. The synchrotron wavelength ≃ 100cm. The nonlinear gain (P_{out}/P_{in}) equals 23db, excluding that portion of the launching loss which is associated with the Bessel function $J_1^2(Q)$. The electron efficiency $(P_{radiation}/P_{beam})$ equals 0.43%.

Figure 5 shows what happens when one shortens the periodicity ℓ_w. We again inject 10kW of power at a wavelength of 10μm, with $\Delta\gamma$=0.003. The beam current density is now 7.75ka/cm^2 and the beam energy is 5.7MeV (γ_0=12.1). A 200 period long "microwiggler" with K_0=1.002 and ℓ_w=0.2cm is employed. Resonance occurs at an injection angle θ=7.1×10^{-2}rad. Saturation occurs at z≃35 cm. The nonlinear gain is 33dB, and the electronic efficiency is 0.46%.

V. DISCUSSION

The purpose of the present study is to elucidate some of the physical phenomena involved in the generation of stimulated emission caused when an unaccelerated electron beam propagates through a periodically modulated dielectric medium. The basic radiation mechanism is stimulated transition radiation. We employ the formalism developed over the years for use with conventional free electron lasers energized by magnetic wigglers. We then obtain for the "Transition FEL" the frequency tuning characteristics, the linear and nonlinear gains, and the system efficiency.

Some of the results we obtain are in interesting contrast to the conventional FELs. For example, in the Transition FEL the coupling strength A of Eq. (40) is determined by the angle θ between the electron beam direction and the wave propagation direction; it takes the place of the wiggle amplitude parameter $a_{FEL}/\gamma=\Omega_w/\gamma k_w c$ of the conventional FEL. Thus, in the Transition FEL, the growth rate Γ does not depend on the amplitude ΔK of the wiggle modulation, unlike the conventional FEL where Γ is determined by the wiggle amplitude B_w through the term a_{FEL}. On the other hand, the launching loss in the Transition FEL is a strong function of the modulation amplitude ΔK, but is independent of the amplitude B_w in the conventional FEL. Finally, in the Transition FEL, the growth rate Γ increases with the harmonic number n and with k_w, in contrast to the conventional FEL where Γ generally decreases with n (if it depends on n at all), and also decreases with increasing k_w.

In our paper we assume implicitly that the electron traverses the periodic dielectric medium. Of course, stimulated transition radiation can also be induced in an arrangement in which an electron beam in vacuum passes in the immediate vicinity of the modulated dielectric. This eliminates the problem of electron beam scattering within the dielectric. The interaction then takes place between the beam and fringing electromagnetic fields just outside the dielectric surface. An example of this is a sheet beam skirting a periodically modulated dielectric slab of dielectric coefficient K_0 and thickness a, which satisfies the (unmodulated) dispersion equation[17]

$$K_0 \left(k_{\parallel}^2 - \frac{\omega^2}{c^2}\right)^{\frac{1}{2}} = \left(\frac{\omega^2 K_0}{c^2} - k_{\parallel}^2\right)^{\frac{1}{2}} \tan \left\{\left[\frac{\omega^2 K_0}{c^2} - k_{\parallel}^2\right]^{\frac{1}{2}} a\right\} \tag{47}$$

where $k_{\parallel}$ is the propagation constant parallel to the electron beam direction. Elimination of $k_{\parallel}$ between Eqs. (4) and (47) yields the frequency versus γ tuning curves illustrated in Fig. 6. Those portions of the curves where ω increases with energy γ are associated with stimulated transition radiation (cf Fig. 2). The portions in which ω decreases with increasing γ are typical of stimulated Cerenkov radiation.[17] To achieve good coupling between the fringing fields and the electrons, and thus good gain, the electrons must be within a distance $\sim\gamma\lambda$ from the dielectric surface (λ is the radiation wavelength).

Probably the most interesting aspect of the coherent emission mechanism discussed in this paper is the possibility of using a dense, periodically modulated plasma as the wiggler. By creating such a fully ionized medium, the problem of electron scattering in the dielectric is greatly diminished. For example, it has been demonstrated[37] that by optically mixing two CO_2 laser lines one can generate a plasma wave with a plasma density of $1.15\times10^{17}\text{cm}^{-3}$ ($\omega_p = 1.9\times10^{13}\text{sec}^{-1}$). Stimulated transition radiation could then be achieved by injecting a relativistic electron beam into the medium at the appropriate angle θ. To be sure, all our calculations have presupposed a stationary periodic dielectric. However, by analogy with electromagnetic wigglers,[1,33,38,39] we believe that stimulated transition radiation should be possible from a nonstationary dielectric such as a propagating or standing plasma wave.

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FIGURE CAPTIONS

Fig. 1. Schematic drawing of a Transition Free Electron Laser.

Fig. 2. Radiation characteristics of a Transition FEL as a function of the electron beam energy parameter γ for $K = 1 + 0.002\ [1 + \cos(k_w z)]$. Frequency tuning (top), and the wiggler strength parameter a_T (bottom), as calculated from Eqs. (7) and (26), respectively for $\sin\theta = 1/\gamma$. Bottom figure applies only to the fundamental, $n = 1$.

Fig. 3. Radiation characteristics of a Transition FEL as a function of the electron beam energy parameter γ for $K = 1 - (\omega_p/\omega)^2 [1 + \cos(k_w z)]$ with $(\omega_p/nk_w c)^2 = 300$ and $\sin\theta = 1/\gamma$. Top: frequency tuning obtained from Eq. (8). Bottom: wiggler strength parameter a_T obtained from Eq. (26). Bottom figure applies only to the fundamental $n = 1$.

Fig. 4. Computer simulation of the Transition FEL for a 11.1MeV, 1kA/cm^2 electron beam traversing, at an angle $\theta = 3.24\times10^{-2}$rad., a modulated dielectric $K = 1 + 0.002(1 + \cos[100\pi z(m)])$. Initial energy spread $\Delta\gamma = 0.003$.

Fig. 5. Computer simulation of the Transition FEL for a 5.7MeV, 7.75kA/cm^2 electron beam traversing, at an angle $\theta = 7.1\times10^{-2}$rad., a modulated dielectric $K = 1 + 0.002(1 + \cos[1000\pi z(m)])$. Initial energy spread $\Delta\gamma = 0.003$.

Fig. 6. Frequency tuning characteristics of a Transition FEL for the case of a sheet electron beam skirting a modulated dielectric slab, as calculated from Eqs. (4) and (47). Curve (a) is for $K_0 = 2$ and $k_w a = 2\pi\times10^{-4}$; curve (b) is for $K_0 = 2.5$ and $k_w a = 2\pi\times10^{-4}$; curve (c) is for $K_0 = 2$ and $k_w a = 2\pi\times10^{-3}$ (a is the slab thickness).

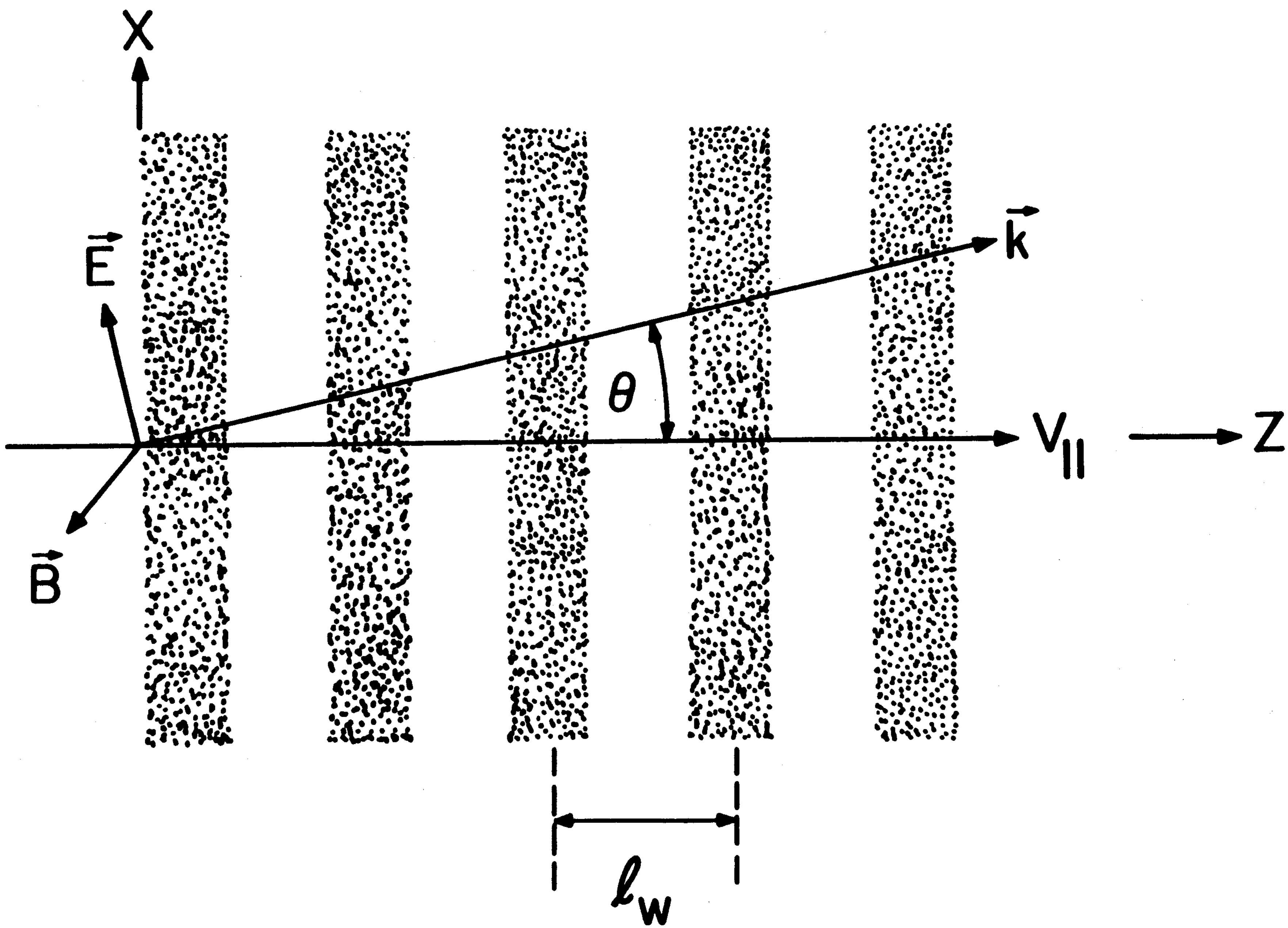

Fig. 1
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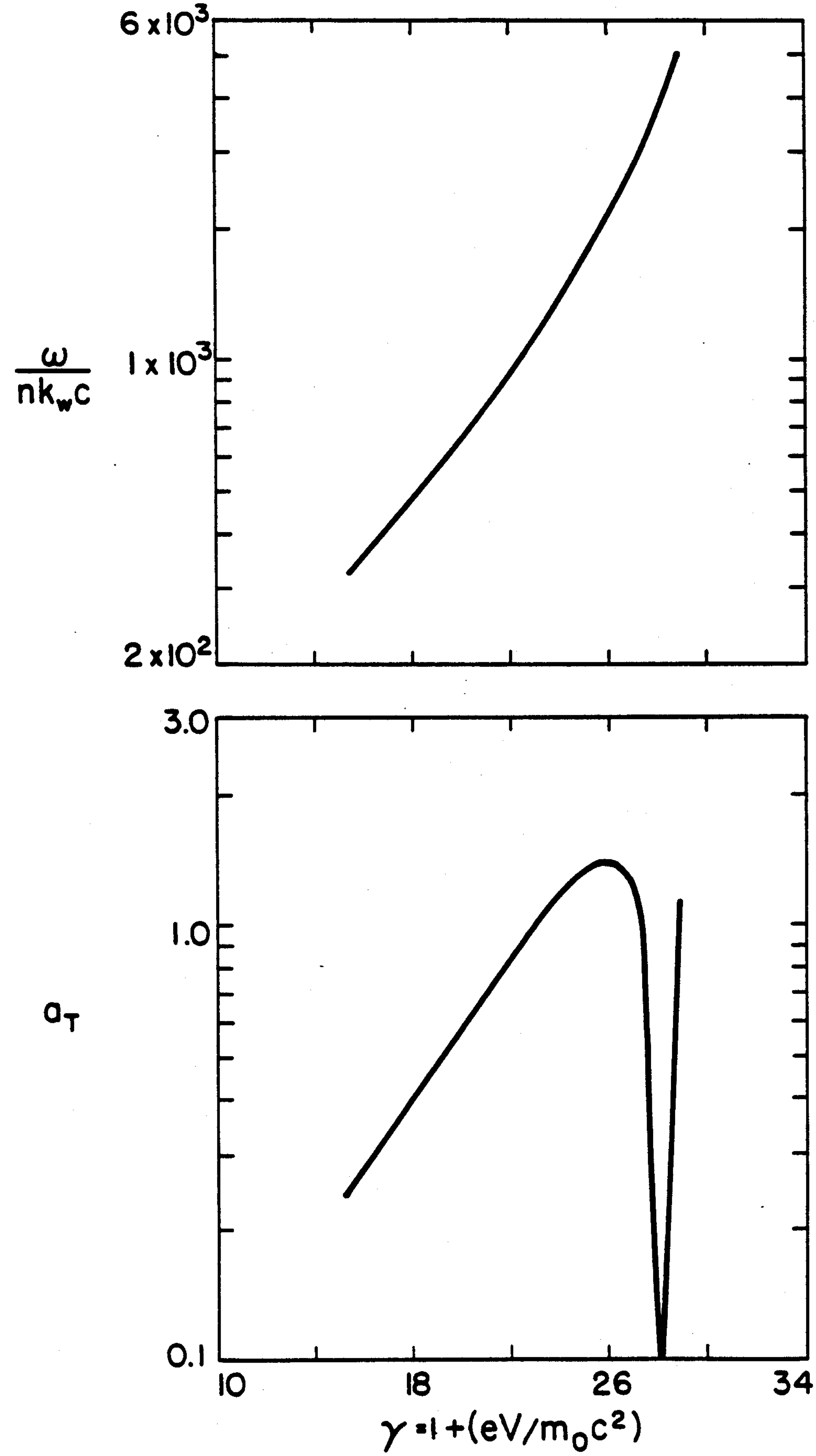

Fig. 2
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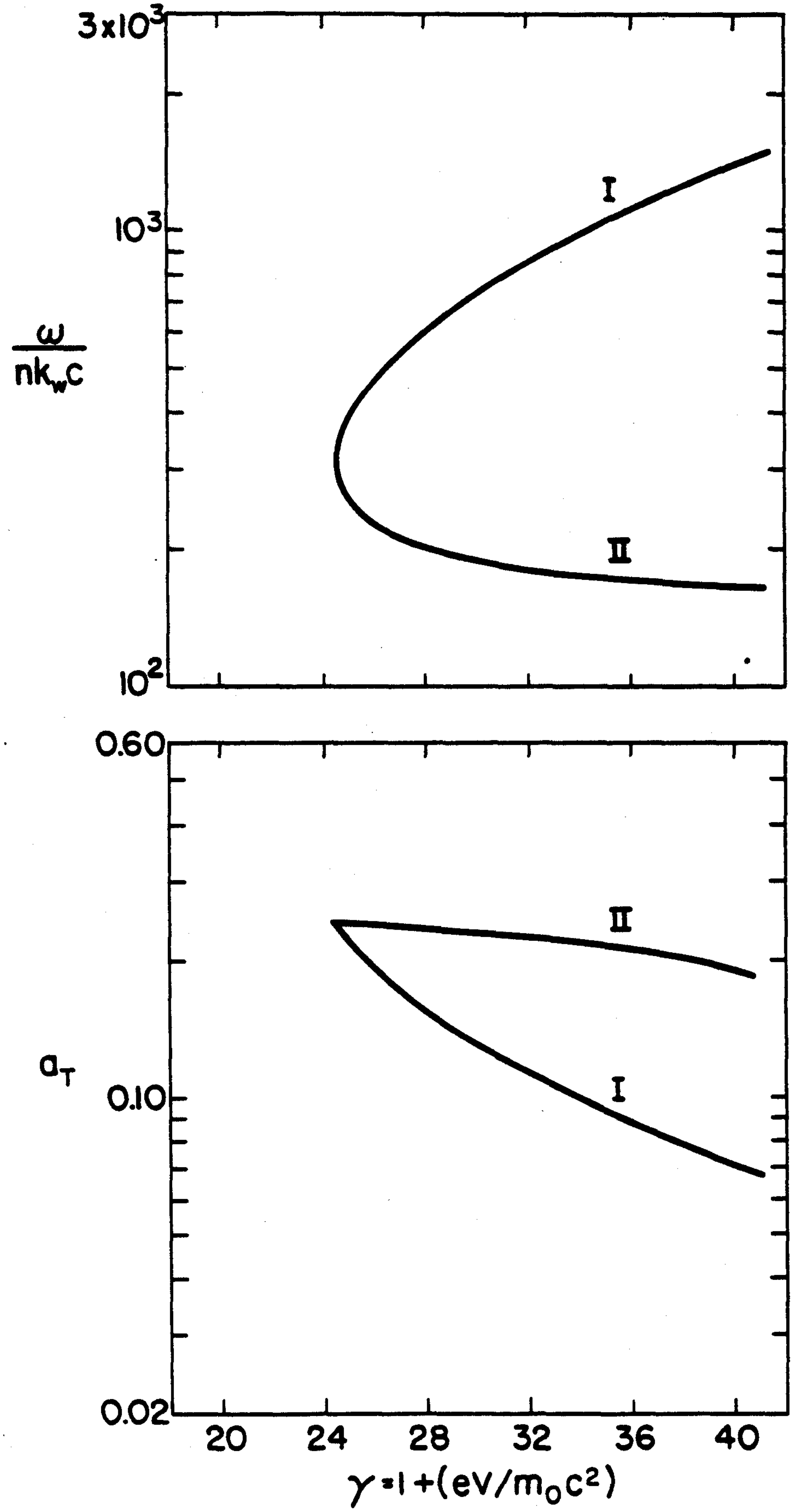

Fig. 3
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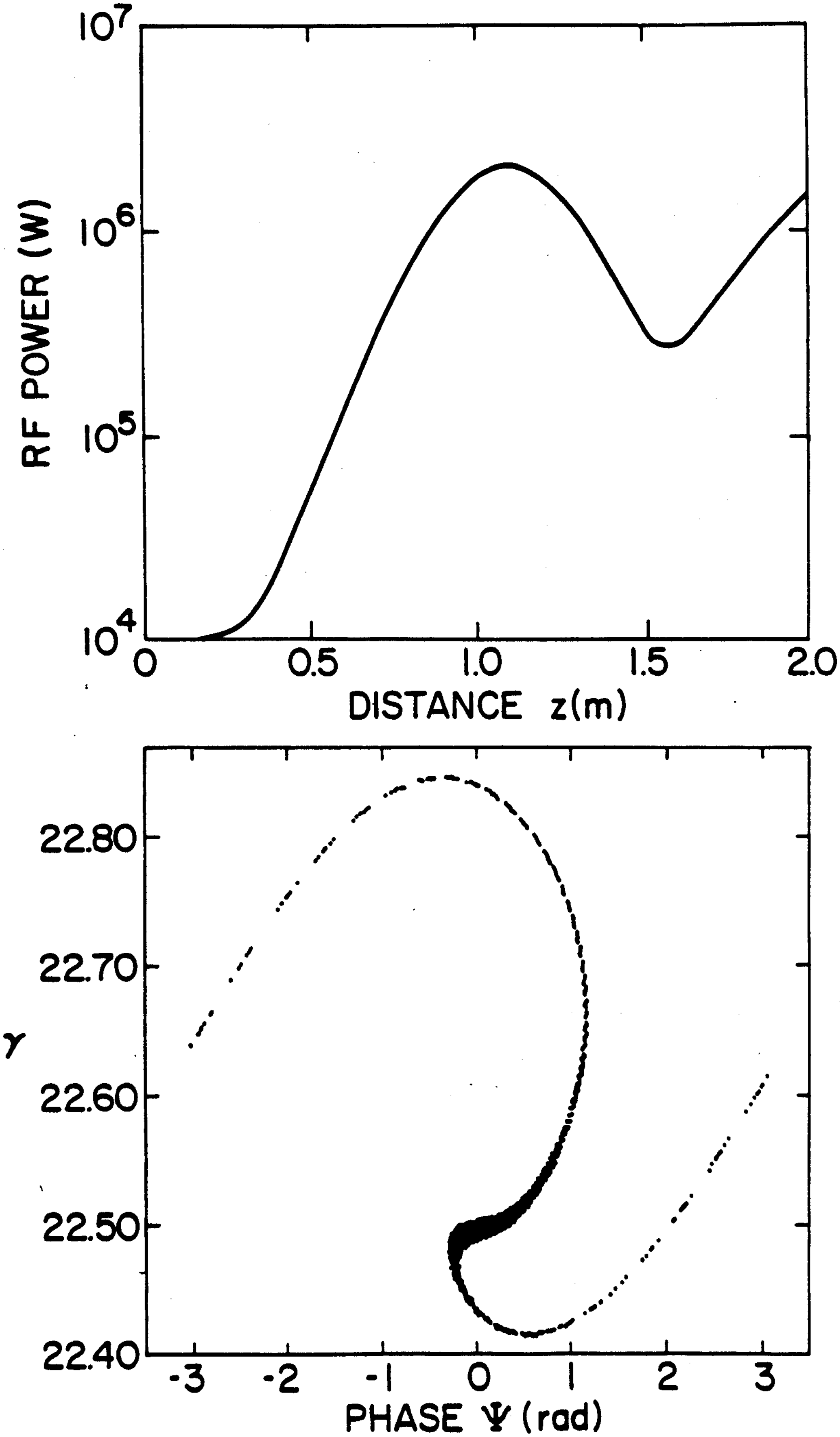

Fig. 4
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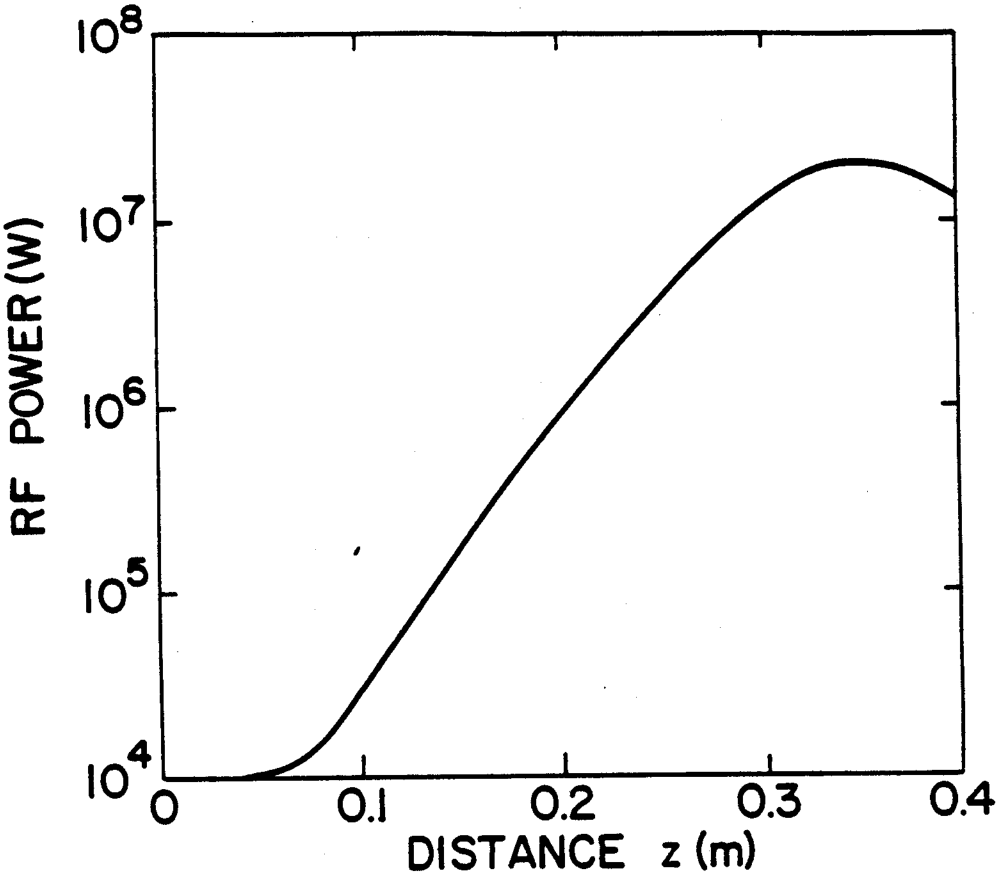

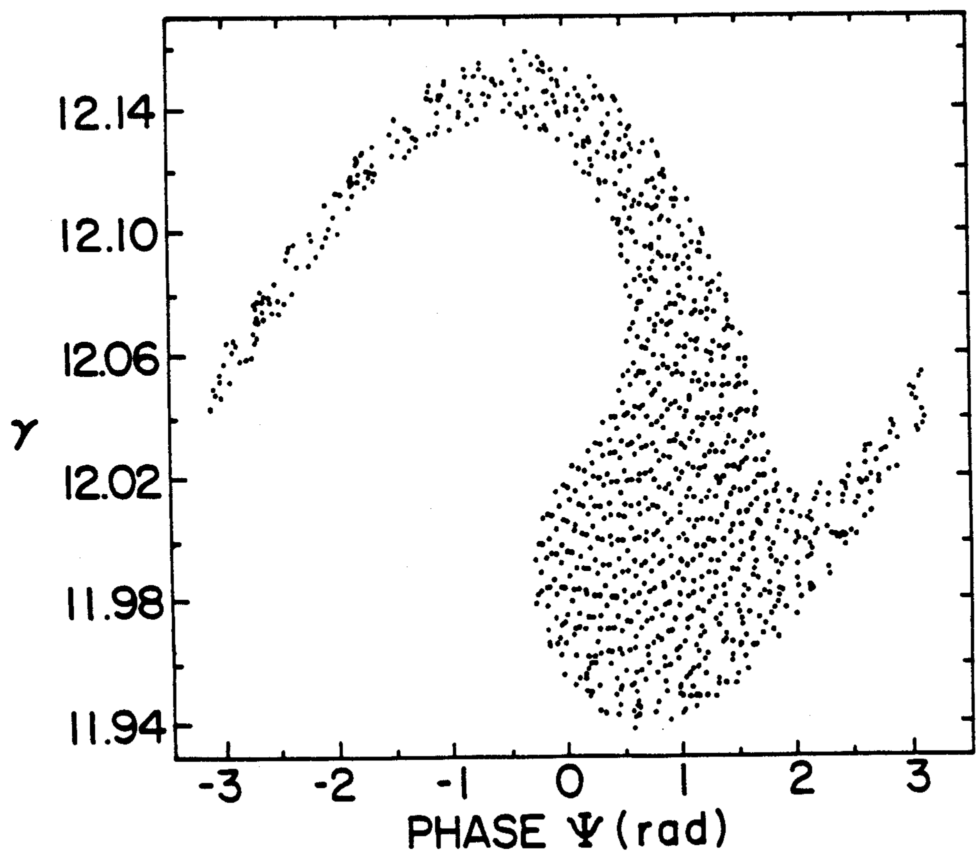

Fig. 5
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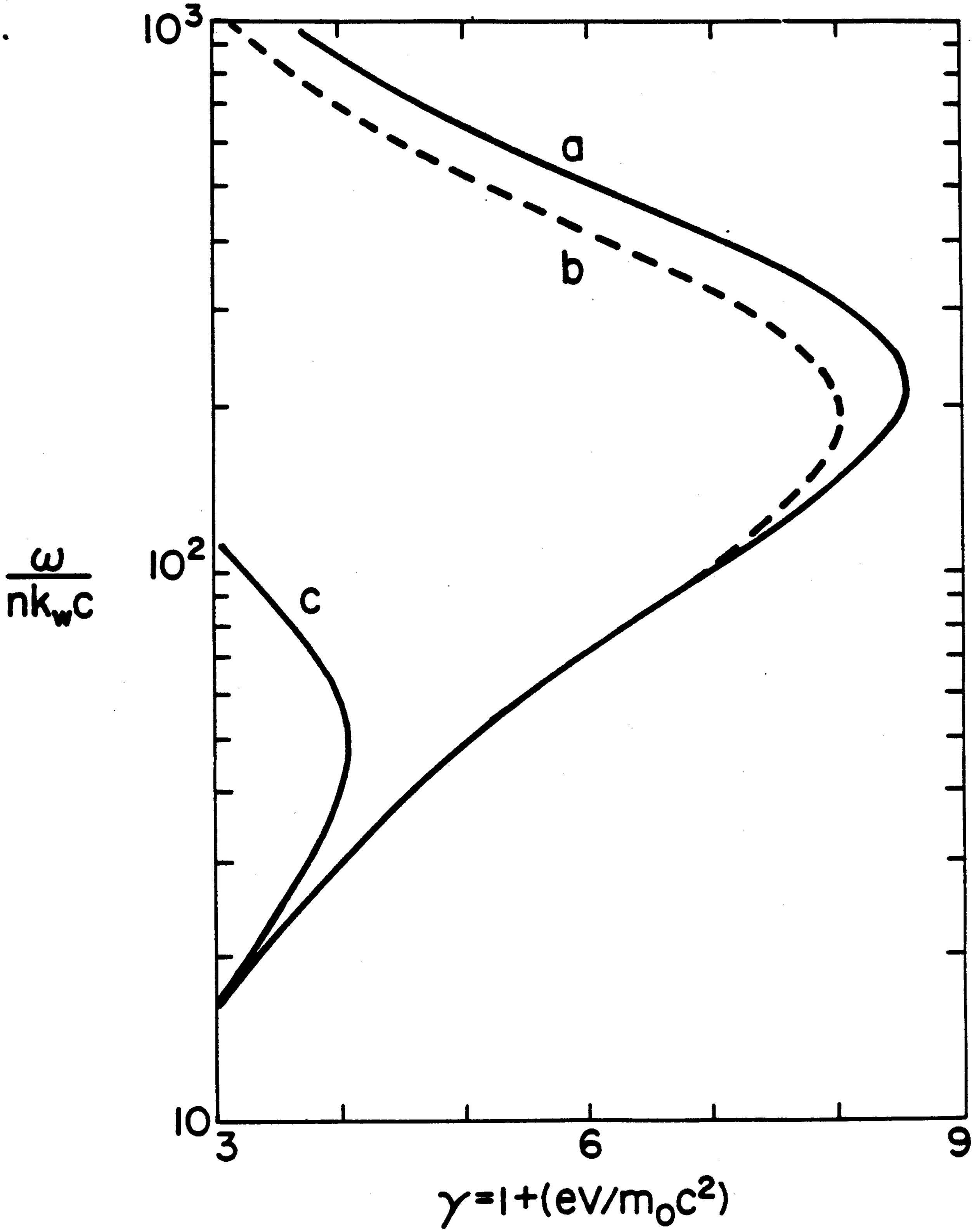

Fig. 6
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