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Requirements for Ohmic Ignition

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ABSTRACT

An analysis of ohmic ignition criteria is presented, giving the requirements on T, $n\tau$ and n/j in a form easily applicable to various confinement assumptions. For circular cross-section 'NeoAlcator' tokamaks with Spitzer resistivity a value of B^2a approximately equal to 250 T^2m is required. The outstanding uncertainties in schemes to lower this value are how much increase in current density is achievable by plasma shaping and what the exact NeoAlcator coefficient is.

Key Words: Ignition, Ohmic Heating, Confinement

Introduction

It is generally acknowledged that to obtain ignition of a magnetically confined plasma which is heated only by ohmic dissipation, due to DC currents in the plasma, is very difficult. Qualitatively this is because of the rapid decrease of the resistance with increasing temperature. However, recent unfavorable scalings of confinement observed with auxiliary heating on tokamaks have reawakened interest[1] in whether ohmic ignition could in fact be achieved. Despite previous studies[2,3] using specific confinement models, there appears to be no clear consensus on what are the conditions required for ignition of an ohmically heated plasma and what these plasma requirements demand of the machine engineering under various scaling assumptions. The purpose of this paper is to set forth these ignition requirements as clearly and succinctly as possible. By doing so, the engineering requirements and their sensitivity to physics assumptions can be directly and quantitatively deduced in terms of a few simple analytic formulae.

The uniform plasma power balance is analysed first leading to the rather simple theorem that, when the confinement time is independent of temperature and proportional to density, the optimum ignition point is at the temperature where Bremsstrahlung and alpha heating balance ($T \approx 4.4$ keV). Profile effects are then incorporated in order to obtain realistic machine requirements. Brief discussion of other issues and limitations is followed by a summary of conclusions.

Zero-Dimensional Analysis

Consider initially a uniform plasma in which the power balance equation can be written in terms of the power <u>densities</u> of ohmic (P_{OH}), alpha particle

(P_α) conduction loss (P_c) and bremsstrahlung (P_b) processes as

$$\frac{dW}{dt} = P_{OH} + P_\alpha - P_c - P_b$$

$$= \eta j^2 + n^2 \frac{Q_\alpha \langle \sigma v \rangle}{4} - \frac{3nT}{\tau} - Cn^2 T^{1/2}$$

$$= n^2 \left[\frac{H}{F^2} T^{-3/2} + R(T) - \frac{3T}{n\tau} - CT^{1/2}\right].$$

Here n is the electron density in an assumed 50-50 DT mixture with temperatures $T_e = T_D = T_T = T$; j is the current density, τ the (non-bremsstrahlung) energy confinement time, η the resistivity, Q_α the alpha energy (3.5 MeV), $\langle \sigma v \rangle$ the DT rate coefficient, R(T) the normalized fusion heating power $(Q_\alpha \langle \sigma v \rangle/4)$, F the ratio of density to current density (n/j), and H and C are the constant resistance and bremstrahlung coefficients. Cyclotron radiation is ignored because, as we shall see, the temperatures of interest to Ohmic ignition are rather low.

The normal steady-state equilibrium power balance condition is of course dW/dt = 0, which determines T if the other quantities are known. One needs to be somewhat cautious in defining the meaning of 'ignition' for an ohmically heated plasma because normally the ohmic power cannot be 'switched off' in the way that auxiliary power can. Physically the most rigorous definition comes about by recognizing that the stability of the power-balance equilibrium is determined by the sign of $\partial/\partial T$ (dW/dt). If this sign is negative, the equilibrium is stable, because a slight increase in temperature leads to a 'negative feedback' into the energy; i.e. a loss of energy (dW/dt<0). On the other hand if the sign is positive the equilibrium is unstable and a positive temperature perturbation will grow with time and lead to an (initially) exponential growth of temperature. Clearly, what is required for ohmic ignition is to reach the point where the usually stable low temperature equilibrium solution becomes marginally unstable; because then a small temperature perturbation will lead to a steadily increasing temperature moving the plasma into the regime where alpha heating is dominant. In other words, the ignition point of interest is the 'thermal runaway' point.

Mathematically, then, the ignition condition is that

$$\frac{\partial}{\partial T}\left(\frac{dW}{dt}\right) = 0 \quad , \quad \frac{dW}{dt} = 0$$

simultaneously. Since these derivatives assume n to be constant we could equally well replace dW/dt by $(1/n^2)$ dW/dt in these expressions.

We make the assumption, valid for most empirical tokamak ohmic confinement scalings, that τ is not explicitly dependent on T. Then we can immediately eliminate $n\tau$ from the ignition condition by writing

$$0 = \frac{\partial}{\partial T}\left(\frac{3}{n\tau}\right) = \frac{\partial}{\partial T}\left[\frac{H}{F^2} T^{-5/2} + \frac{R}{T} - CT^{-1/2} - \frac{1}{n^2 T}\frac{dW}{dt}\right]$$

$$= -\frac{5H}{2F^2} T^{-7/2} + \frac{d}{dT}\left(\frac{R}{T}\right) + \frac{C}{2} T^{-3/2} \quad .$$

Since R is a known function of T, we can regard this equation as indicating that for a given value of F (the n/j ratio), there is a unique temperature at which ohmic ignition occurs. It is more convenient, though, to solve the equation for F^2 with T as the independent variable. For any T, ohmic ignition can occur at that temperature only when

$$F^2 = \frac{5H}{2} / \left[T^{7/2} \frac{d}{dT}\left(\frac{R}{T}\right) + \frac{C}{2} T^2\right] \quad .$$

Having solved for F we can substitute back into the power balance to find what value of $n\tau$ is required for ignition at this temperature. We find immediately:

$$n\tau = 3/\left[\frac{2}{5} T \frac{d}{dT}\left(\frac{R}{T}\right) + \frac{R}{T} - \frac{4}{5} C T^{-1/2}\right] ,$$

at ignition. Whether or not any confined plasma in fact ignites depends upon whether its Lawson product $n\tau$ exceeds this value or not.

Of course there is presently no adequate a piori theory for how the confinement time should scale. However, a widespread observation is that the ohmic confinement time is proportional to density. So let us adopt a scaling of the form

$$\tau = K n$$

where all other dependences on such factors as field and size are incorporated into K. Then we write

$$n\tau = Kj^2 F^2$$

and recognize that, from the engineering viewpoint, there will generally be practical constraints on Kj^2 which will determine whether this equation can be satisfied, for a given scaling, in an achievable machine. Put the other way around, the ignition criteria just discussed prescribe the values of $n\tau$ and F^2 for ignition at any T, and hence demand a value of Kj^2 at least equal to $n\tau/F^2$. We can therefore regard Kj^2 as a required machine figure of merit. The least demand is placed upon engineering when ignition occurs at the minimum value of $n\tau/F^2$. From our previous equations we can readily evaluate this.

The minimum value of $n\tau/F^2$ occurs at that temperature which makes

$$n\tau \frac{d}{dT}\left(\frac{1}{n\tau}\right) + \frac{1}{F^2}\frac{d}{dT}(F^2) = 0 \qquad ,$$

where the total derivatives here mean derivatives along the marginal ignition condition. But, using the power balance equation,

$$\frac{d}{dT}\left(\frac{3}{n\tau}\right) = \frac{\partial}{\partial T}\left(\frac{3}{n\tau}\right)\Bigg|_{F\,=\,\mathrm{const}} + \frac{dF^2}{dT}\frac{\partial}{\partial F^2}\left(\frac{3}{n\tau}\right)\Bigg|_{T\,=\,\mathrm{const}} = -\frac{dF^2}{dT}\frac{HT^{-5/2}}{F^4}$$

Hence the minimum value of $n\tau/F^2$ at ignition occurs where

$$\frac{HT^{-3/2}}{F^2} - \frac{3T}{n\tau} = 0 \ ,$$

or, in other words, where the ohmic heating and conduction losses exactly balance. Note that since the ohmic and conduction powers balance, so also do the alpha and bremsstrahlung powers. We can therefore immediately write down the equation determining the optimum temperature for ignition as

$$R(T_m) - CT_m^{1/2} = 0,$$

and the required $n\tau$ as

$$(n\tau)_m = \frac{3F^2\,T_m^{5/2}}{H}$$

Notice that the solution for T_m, in the uniform plasma model is just the well known minimum possible temperature for idealized ignition (4.4 keV for $Z_{eff} = 1$). However, we do not require infinite confinement time for ignition there because we always have some ohmic heating to balance the non-bremsstrahlung losses.

In order to obtain explicit solutions for the parameters we have discussed, we need to adopt an analytic form for R. Various forms are available, however it proves of considerable simplifying advantage later, in discussing profile effects, to adopt a power-law expression. The expression we shall use here is

$$R = A\,T^{3.5} \quad ; \quad A = 3.9 \times 10^{-23} \quad m^3 s^{-1}\ keV^{-2.5}$$

where we are choosing to measure all energies and temperatures in keV. This form approximates the DT reaction rate to within ~ 20% from T ≈ 2.5 to 10 keV and to within ~ 5% from ~ 3 to 7 keV, which covers the range of most interest. These errors are negligible compared to other uncertainties in the modelling. Also, in these units, the Spitzer resistivity with $\ln \Lambda = 16$ and the bremsstrahlung coefficient with $Z_{eff} = 1$ give

$$H = 1.64 \times 10^{8} \quad ms^{-1}A^{-2}\ keV^{5/2} \quad ; \quad C = 3.3 \times 10^{-21} \quad m^3 s^{-1}\ keV^{1/2} \quad .$$

With these forms we can write explicit expressions:

$$\frac{n^2}{j^2} = F^2 = \frac{5H}{2} \Big/ [2.5\,AT^5 + \frac{C}{2}\,T^2] = 10^{28}/[0.0024T^5 + 0.040T^2] \quad m^{-2}A^{-2}$$

$$n\tau = 3/[2AT^{2.5} - \frac{4}{5}\,CT^{-1/2}] = 10^{20}/[0.0026T^{2.5} - 0.088T^{-1/2}] \quad m^{-3}s$$

and also for the optimum ignition point:

$$T_m = (\frac{C}{A})^{1/3} = 4.4\ keV$$

$$(n\tau)_m = \frac{15\,(C/A)^{5/6}}{5A(C/A)^{5/3} + C\,(C/A)^{2/3}} = 1.6 \times 10^{21}\ m^{-3}s.$$

In Fig 1 are plotted the values of the parameters F^2 and $n\tau$ from these equations, together with the machine figure-of- merit $n\tau/F^2$ (= $\bar{K}j^2$),

$$\frac{n\tau}{F^2} = \frac{6\,[2.5\,AT^5 + CT^2/2]}{5H\,[2AT^{2.5} - 4CT^{-1/2}/5]} = 10^{-8}\,\frac{[0.0024\,T^5 + 0.040\,T^2]}{[0.0026\,T^{2.5} + 0.088\,T^{-1.2}]} \quad m^{-1}A^2 s.$$

The idealized ignition curve for $n\tau$ ($= 3/[AT^{2.5} - CT^{-1/2}]$) obtained by ignoring the ohmic heating term in the power balance is also shown. The ohmic ignition occurs at a noticeably lower curve than the idealized value.

The previous discussion is essentially completely general, applying to any confinement scheme. Even different assumptions about the n and T dependence of τ could be incorporated and the analysis suitably modified. However, there are various important additional factors which should be incorporated in order to obtain realistic results. Before considering some of these complicating factors let us end this discussion of the simple uniform plasma model by adopting a specific tokamak transport scaling: the NeoAlcator confinement[4],

$$\tau = Kn \quad ; \quad K = 2 \times 10^{-21} R^2 a \kappa^{1/2} \quad .$$

Then ignoring shaping ($\kappa = 1$) and using $j = 2B/\mu_0 Rq$ we get

$$\frac{n\tau}{F^2} = Kj^2 = 5 \times 10^{-9} (B^2 a)/q^2 \qquad (m^{-3}s)/(m^{-2}A^{-2})$$

with B in Tesla, a in meters and q the safety factor. Substituting the optimum value of $n\tau/F^2$, which is $3T_m^{5/2}/H$ we get the minimum required machine parameter as

$$\frac{B^2 a}{q^2} = 150 \qquad T^2 m.$$

This number is comparable to those quoted elsewhere[5] but it should be noted that simply to apply this criterion to the center of a tokamak (where $q \approx 1$) leads to a considerably too optimistic view of the ignition requirements, because of the importance of profile effects, which we now discuss.

Profile Effects and Shaping

In order to determine quantitatively the effects of non-uniform profiles of plasma parameters on the ignition condition we adopt a model in which the plasma is taken to be cylindrical and all profiles are taken to be proportional to a parabola to some power:

$$T = T_o\,(1 - r^2/a^2)^{\alpha} \quad ; \quad n = n_o\,(1 - r^2/a)^{\beta} \quad .$$

The convenience of this model is that the volume average of any quantity $f = f_o\,(1 - r^2/a^2)^k$ is readily shown to be

$$\langle f \rangle = f_o \frac{1}{k+1} \quad .$$

This enables us to generalize the previous treatment very rapidly. We simply recognize that the correct total power-balance equation is the volume average of equation 1, which then becomes

$$\frac{d}{dt} \langle W \rangle = \langle\, [HT^{-3/2}\, j^2 + AT^{3.5}\, n^2 - \frac{3nT}{\tau} - CT^{1/2}\, n^2]\, \rangle$$

$$= n_o^2\, [\, \frac{H}{(3\alpha/2+1)} \frac{T_o^{-3/2}}{F_o^2} + \frac{A}{(3.5\alpha+2\beta+1)} T_o^{3.5} - \frac{3T_o}{n_o\tau(\alpha+\beta+1)} - \frac{C}{(\alpha/2+2\beta+1)} T_o^{1/2}\,] \quad .$$

Here we have assumed $j \propto T^{3/2}$ and put $F_o = n_o/j_o$. Given this equation, we can see that the previous analysis goes through just as before except that the coefficients must be replaced by modified values

$$H' = H/(\frac{3\alpha}{2} + 1)$$

$$A' = A/(3.5\alpha + 2\beta + 1)$$

$$C' = C/(\frac{\alpha}{2} + 2\beta + 1) \quad .$$

So to obtain the central values appropriate to the profile-corrected ignition condition we simply replace the parameters in all previous equations by their primed alternates defined above. For example:

$$n_o\tau\ (\alpha+\beta+1) = 3/[\frac{2AT^{2.5}}{(3.5\alpha+2\beta+1)} - \frac{4CT^{-1/2}}{(\alpha/2+2\beta+1)}]\ ,$$

$$T_m = (\frac{C}{A})^{1/3} (\frac{3.5\alpha+2\beta+1}{\alpha/2+2\beta+1})^{1/3} ,$$

$$(\frac{n\tau}{F^2})_m = 3\ (\frac{C}{A})^{5/6}\ \frac{1}{H}\ (\frac{3.5\alpha+2\beta+1}{\alpha/2+2\beta+1})^{5/6} (\frac{3\alpha/2+1}{\alpha+\beta+1}) ,$$

where we leave the subscript zero as understood from now on.

In Fig. 2 we show the correction factors for T_m and $(n\tau/F^2)_m$ as a function of α for three typical values of β (0,1/2 and 1). It should be noted that in a Tokamak $(3\alpha/2+1)$ is equal to q_a/q_o, the ratio of edge to central safety factor in this model. Therefore α is strongly contrained by known physics. The value of β is less certain but peaking the density profile by increasing β helps to lower the required value of $n\tau/F^2$.

As an optimistic example consider a tokamak with $q_a = 2$, $q_o = 1$, $(\alpha = 2/3)$, peaked density, $\beta = 1$. We include the correction $n_o = 1.5\ \bar{n}$ in the Neo Alcator formula, which applies to the chord averaged density $\bar{n}$. Then we find the $n\tau/F^2$ requirement calls for

$$B^2a = 150\ (\frac{3.5\alpha+2\beta+1}{\alpha/2+2\beta+1})^{5/6} (\frac{3\alpha/2+1}{\alpha/2+2\beta+1})\ 1.5 = 250\ T^2m.$$

This value is approximately the lowest plausible value for a circular tokamak with no neoclassical resistivity enhancement.

If we allow shaping, profile effects are more difficult to deal with. Vertical elongation appears to have a favorable effect for two main reasons. One is the increase in NeoAlcator τ as $\kappa^{1/2}$ ($\kappa \equiv b/a$, the vertical elongation); although the data base for this additional scaling is small. The other is the permitted increase in current density at fixed q and B. In so far as shaping increases the current density <u>on axis</u>, the B^2a requirement is decreased proportional to j_o^{-2}, presuming other profile effects are unchanged. On the other hand, a stronger effect of shaping is to increase the mean current density for fixed $q\psi$ at the plasma edge. Quite how that affects the profile factors is uncertain and may depend on details of the plasma configuration. In the absence of a well-established model for the profile-effects of shaping, an estimate may be gained by supposing the shaping simply to allow a lowering of the edge circular q-value ($q_c \equiv 2\pi a^2 B/\mu_o R I_p$). If the central current-density rises by only a relatively smaller amount, then what shaping allows is a broadening of the temperature and current profiles, i.e. a modification of α. For example we might suppose that at $q_c = 1$ we should take $\alpha = 0$, in which case we get $B^2a = 110\ T^2m$ in our previous example. This crude estimate should be regarded as little more than a rough indication that shaping may have the potential to ease the ignition engineering demands by a significant factor. Further experimental work on shaping of ohmic Tokamaks is essential to give a firmer basis for shaping estimates.

<u>Other Corrections and Limitations</u>

Modifications of the coefficients due to additional physical effects are easily incorporated. For example, trapped-particle neoclassical corrections to the resistivity may increase the value of H by a factor which, averaged over the profile, may be as large as 2[3]. This increases the F^2 value and decreases the $n\tau/F^2$ value proportionately but does not affect the value of $n\tau$ or of T_m, since they depend only on A and C.

Loss of some fraction of the alpha power due to unconfined drift orbits can be incorporated as a decrease in A, which increases $n\tau/F^2$ as $A^{-5/6}$.

A more complex situation arises when noticeable amounts of impurity are present. Assuming the contribution to radiative losses to be dominantly bremsstrahlung Z_{eff} modification - an assumption which will prove false for heavy impurities - we can calculate how a small amount of impurity of charge Z modifies the ignition conditions. There are three effects: increase of ohmic power by a factor which is approximately $(0.65\ Z_{eff} + 0.35)$, decrease of alpha power, due to dilution, by a factor $[(Z - Z_{eff})/(Z - 1)]^2$, and increase of radiation by Z_{eff}. All that we require is to modify the coefficients H, A, C by these factors. In particular the $n\tau/F^2$ correction factor is

$$f_z = \left[\frac{Z_{eff}\,(Z-1)^2}{(Z - Z_{eff})^2}\right]^{5/6} \frac{1}{0.65\ Z_{eff} + 0.35} .$$

If we differentiate this expression with respect to Z_{eff} we get

$$\frac{Z_{eff}}{f_z}\frac{df_z}{dZ_{eff}} = 5/6 + \frac{5}{3}\frac{Z_{eff}}{Z - Z_{eff}} - 0.65\frac{Z_{eff}}{0.65\ Z_{eff} + 0.35}$$

which at $Z_{eff} = 1$ is always positive: $0.18 + 1.67/(Z - 1)$. Therefore impurities always make it more difficult to achieve ignition even on this optimistic assumption about radiation.

An issue which so far we have not addressed concerns whether the F values we require are achievable. Normally tokamaks experience a 'density-limit' which cannot be exceeded. This is often expressed in terms of a 'Murakami number' nR/B. The basis of this is that for fixed q the mean current density is proportional to B/R. Therefore, at least for circular tokamaks, the F parameter is proportional to the Murakami number. Now the optimum F value is

$$F = \left[\frac{5H}{5A(C/A)^{5/3} + C(C/A)^{2/3}}\right]^{1/2} = 4.6 \times 10^{13} \ \mathrm{m}^{-1}\ \mathrm{A}^{-1},$$

ignoring profile effects. This translates (for q = 1) to a Murakami number of $2F/\mu_o = 7.3 \times 10^{19}\ \mathrm{m}^{-2}\ \mathrm{T}^{-1}$. It is interesting to note that this value corresponds to the high range of density operation for most Tokamaks but not significantly exceeding typical achievable values. It should also be noted that a smaller F-value by a factor of up to 2 does not greatly alter the engineering factor $n\tau/F^2$. Therefore the required ignition density generally is less than present expectations of what should be achievable. The density limit is not a serious problem. Of course, if significant confinement degradation occurs near the density limit, that is a problem.

Concerning the Beta limits, if we operate at the optimum F value this implies that

$$\beta_t \equiv \frac{2\mu_o nT}{B^2} = \frac{2\mu_o\ F_m T_m j}{B^2} = \frac{4F_m T_m}{qRB} \quad .$$

using the circular q-value. Substituting the zero-dimensional values and q = 1 we get

$$\beta_t = \frac{0.13}{BR} .$$

Thus at the optimum ignition point in a high-field machine the central β_t value will be quite modest, e.g. 1% for B_t = 13T, R = 1m. Since high field will be essential to achieving ignition anyway, this shows that the beta-limit should not be a problem.

We can confirm that an ohmic ignition Tokamak has sufficient current ($\gtrsim$ 5MA) to confine the alphas. Using again the circular q value which we take to be 2 we get

$$I = q \frac{2\pi a^2 B}{\mu_o R} = 10 \frac{a^2 B}{R} \quad \text{MA.}$$

Since B^2a is determined by the ignition requirement (which for present purposes we take to be $B^2a \simeq 100$) we can eliminate B to get

$$I \simeq 100\, a^{1/2} \frac{a}{R} \quad \text{MA}$$

Thus the current is more than enough, in even quite small machines to confine the alphas.

Also we should verify that neoclassical ion transport, which is presumably the irreducible mimimum, is low enough to permit the ignition we require. Using the approximate formula[6] for the Banana regime:

$$n\tau_{NC} = 10^7\, I^2\, T^{1/2}\, (a/R)^{1/2}$$

we find, for T = 4.4 keV and (a/R) = 1/4 that in order to achieve the required $n\tau(\simeq 1.6 \times 10^{21} m^{-3} s$ on the zero-d model) we need at least 12

MA of current. This will be present provided a $\gtrsim 0.23$ m, from the previous equation. So again, even a relatively modest sized machine should provide sufficient neoclassical confinement. Of course, if there were appreciable enhancement of the ion transport above Neoclassical, ignition might be prevented.

Summary of Conclusions

(1) Ohmic ignition, physically defined, takes place at a temperature which is a unique function of the ratio (F) of density to current density. Independent of any assumptions about confinement other than that τ is not explicitly a function of temperature.

(2) The $n\tau$ required for ignition is similarly a unique function of F, and hence of T, lying slightly below the idealized fusion ignition curve (see Fig. 1).

(3) The quantity $n\tau/F^2$ constitutes an engineering figure-of-merit when confinement time scales proportional to n. It is then proportional to the square of the current-density and its required value is likewise a unique function of ignition temperature.

(4) The minimum value of $n\tau/F^2$ occurs, for a uniform plasma, at T = 4.4 keV where the bremsstrahlung and alpha powers balance. The $n\tau$ required to reach this is $1.6 \times 10^{21} m^{-3} s$.

(5) NeoAlcator scaling allows the $n\tau/F^2$ to be reexpressed as a $B^2 a$ requirement; requiring $B^2 a > 150$ in a uniform plasma but more realistically $\gtrsim 250$ in a circular tokamak (not accounting for trapped particle resistivity enhancement) when profile effects are accounted for.

(6) Impurity effects are always detrimental to the ability to achieve igntion.

(7) Density limits, beta limits, alpha drift orbit confinement and neoclassical ion transport do not appear to be serious limiting factors in practical sized machines. If machine parameters are such as to achieve ignition, these limits will not be significantly stretched.

(8) The most crucial areas of uncertainty in determining the engineering requirements for igniton are (a) the neoAlcator coefficient and (b) the degree to which the current density, particularly on axis, can be increased by shaping. Our present degree of uncertainty in these issues, particularly shaping, leave the B^2a uncertainty large enough to encompass values (from $\lesssim 100$ to ≈ 300) which range from plausibly achievable to presently inconceivable.

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Figure Captions

Fig. 1 The values of $n/j(=F)$ and $n\tau$ required for ohmic ignition at a temperature T in a uniform plasma. The idealized ignition $n\tau$ requirement (ignoring ohmic heating) is shown for comparison. The ratio $n\tau/F^2$ is the machine figure-of-merit requirement. It is minimized at the optimum temperature T_m.

Fig. 2 The profile correction factors for the central optimum temperature and $n\tau/F^2$ requirements as a function of the temperature and density profile indices (α and β).

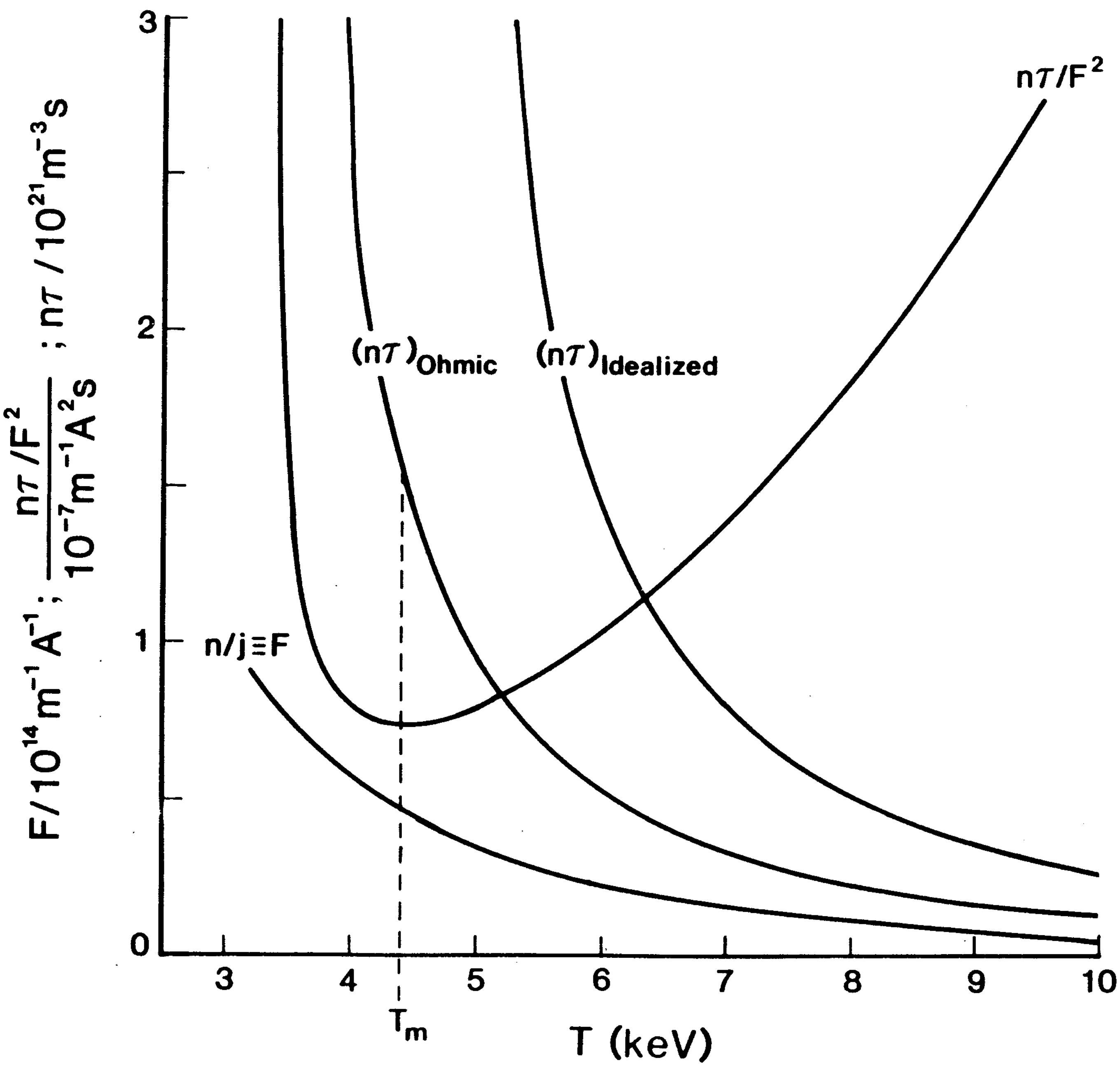

Figure 1

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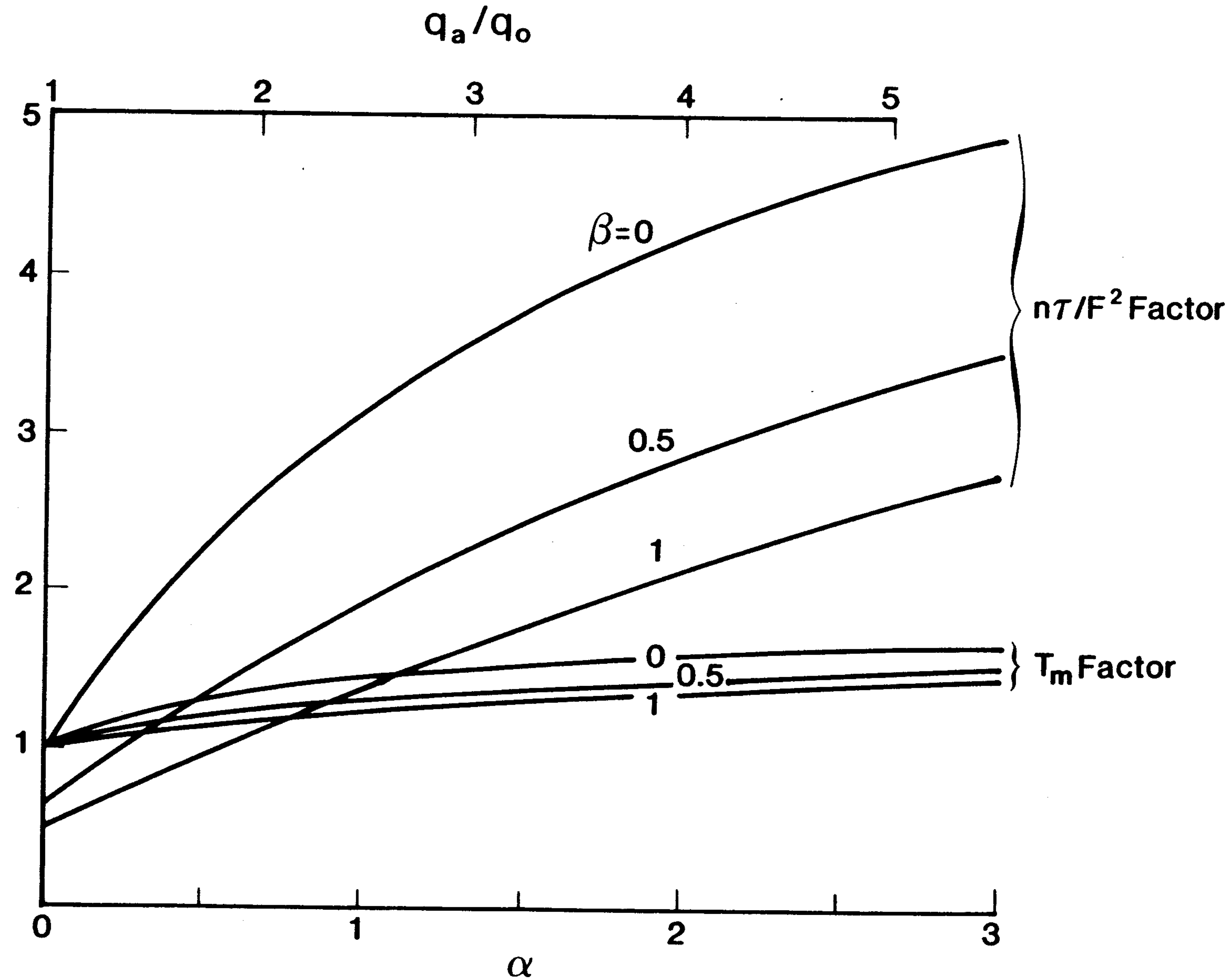

Figure 2

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