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Laser and Cyclotron Maser Applications

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PERMANENT MAGNET HELICAL WIGGLER FOR FREE ELECTRON LASER
AND CYCLOTRON MASER APPLICATIONS

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ABSTRACT

A permanent magnet, "bifilar" helical wiggler for use in free electron lasers and cyclotron masers has been designed and tested experimentally. It is composed of a cylindrical array of staggered samarium-cobalt bar magnets, transversely magnetized and held in place in an axially grooved hollow metal cylinder. High quality helically polarized fields of several kilogauss can be readily achieved.

The magnetic wiggler, or undulator, is the principal ingredient in free electron lasers¹ (FELs) and high brightness synchrotron radiation sources.² It is also useful in spinning up³ relativistic electron beams for use in gyrotrons and cyclotron masers. At present, the favored permanent magnet system for FELs is composed of a linear array of Rare Earth Cobalt (REC) magnets arranged in the Halbach⁴ configuration. It produces a transverse, linearly polarized wiggler magnetic field. A helically polarized wiggler composed of glued segments of REC material has also been proposed,⁴ but, because of technical difficulties, has not been widely used. To be sure, helical wigglers offer advantages compared with linear wigglers. Because of their higher symmetry, the electron motion and thus the electromagnetic radiation has a low harmonic content. The FEL gain of the emitted circularly polarized radiation is larger than the corresponding gain associated with a linearly polarized wiggler of the same strength. And finally, the fact that a helical wiggler provides electron beam focussing in all transverse planes is a desirable feature since it eliminates the need for placing focussing quadrupole magnets or solenoids around the wiggler system.

In this Note we describe the design and construction of a novel "bifilar", helically polarized wiggler system composed of a cylindrical array of staggered permanent magnets. A prototype of the wiggler is illustrated in Fig. 1. Samarium-cobalt bar magnets with dimensions 0.4cm×0.4cm×4.8cm and magnetized at right angles to one of their broad faces, are inserted in an aluminum cylinder grooved⁵ on the outside with 12 straight channels running parallel to the cylinder axis. An external nonmagnetic metal cylinder fits over this structure, thereby keeping the magnets in place.

Figure 1(a) illustrates the direction of magnetization of our 12-period system as would be observed at some arbitrary cut made perpendicular to the cylinder axis z . It is seen that the dipole moments of six of the magnets

point radially out, and the dipole moments of the remaining six magnets point radially in. As one proceeds along the z axis, this pattern remains invariant except for an azimuthal rotation governed by the pitch of the helix. Magnets of length ℓ yield a wiggler periodicity $\ell_w = 2\ell$.

In order to achieve the desired pitch, the magnets are staggered in the z direction and their dipole moments alternated as is shown in Fig. 1(b). The stagger is provided by nonmagnetic spacers placed at the beginning of each channel and differing in length by ℓ_w/N where N is the number of channels (12 in our case). After filling the first 6 channels in this manner, a second identical set of spacers is used for the remaining six channels, except that here the directions of the dipole moments are reversed. Stuffing the channels with magnets is an easy task in view of the fact that neighboring magnets in a given channel attract one another.

Measurements of the magnetic field are shown in Fig. 2. These are carried out by means of a transverse Hall probe gaussmeter (Bell 610) mounted on a mechanical stage and motor driven along the wiggler axis. Figure 2(a) illustrates the fields over the central 2-period length of a 5-period long wiggler, and represents a tracing made from an x-y recorder chart. Each of the seven traces corresponds to a different azimuthal orientation of the Hall probe made in angular steps θ of 30° . The field profiles are seen to be very smooth, the field amplitude is uniform and the phase relation between successive scans in θ are as expected. Figure 2(b) shows the wiggler behavior at one of its ends.

As will be discussed below, our wiggler is a permanent magnet analogue of a bifilar system of current carrying conductors. As such, the axial magnetic field along the wiggler axis z should be zero. Using an axial Hall probe, we find that the amplitude of the axial field is indeed small, less than $\sim 50G$. This is to be compared with the transverse magnetic field amplitude of 1.16kG (see Fig. 2(a)). The observed B_z is attributed in a large part to the finite

transverse dimension of the Hall probe ($\sim 1\text{mm}$).

We note that the results of Fig. 2 are obtained with samarium-cobalt magnets⁵ whose remanence $B_r \approx 9000\text{G}$ can vary by as much as ± 1.5 percent and whose direction of magnetization (direction of the "easy axis") can vary by as much as $\pm 3^\circ$. No attempt has been made to sort or arrange the magnets to minimize errors, as is generally done for planar wigglers.⁶ The good performance of our wiggler is attributed in part to the large number of magnets per period, and in part to the overlapping of magnets along the axial direction, thus leading to smoothing and averaging over field inhomogeneities. As a check, we replace one full magnet in the wiggler center by a nonmagnetic spacer. The effects of this rather large perturbation are shown in Fig. 2(c). We see that the field profile remains quite smooth, although the local field amplitude drops by about 10 percent.

Because of the approximate straight line relationship⁴ between \vec{B} and $\mu_0 \vec{H}$ valid for REC magnets, and thus applicability of the principle of linear superposition of vacuum fields, one can obtain an approximate expression for the wiggler amplitude B_w on axis. The magnetization of each bar magnet can be represented by an effective circulating surface current density $J_s \propto B_r / \mu_0$ as is illustrated in Fig. 1(b). Superposing these currents in the continuum limit as $N \rightarrow \infty$ and the magnet widths and magnet spacings go to zero, one obtains helical current sheets flowing in opposite directions, and separated axially by a distance equal to $\lambda_w / 2$. The current sheets are infinitely thin in the axial direction, and have a thickness $(r_2 - r_1)$ equal to the bar magnet width in the r direction. Using the well-known result⁷ for a helical wiggler composed of infinitely thin current carrying conductors, and integrating over the thickness $r_2 - r_1$, yields the following expression for the field amplitude on axis of our REC magnet wiggler:

$$B_w = (2B_r / \pi) \left\{ U(k_w r_1) - U(k_w r_2) \right\} F \quad (1)$$

Here B_r is the remanence for our material (9kG) and $k_w = 2\pi/\lambda_w$ is the wiggler wave number; $U(x) = xK_1(x) + K_0(x)$ where K_0 and K_1 are modified Bessel functions of the second kind. Figure 3 shows a plot of $U(x)$ as a function of x .

The above equation is approximate. It fails to take account of the discrete properties of the magnets and thus can give no information concerning the amplitude of higher spatial harmonics. However, to allow for gaps between the magnets due to the finite width of the aluminum teeth separating magnet channels, we include in Eq. (1) a semiempirical filling factor $F = |N(r_2 - r_1) / \pi(r_2 + r_1)|$ equal to the cross sectional area of magnet material, divided by the total area subtended between radii r_1 and r_2 (see Fig. 1(a)). With $r_1 = 1.00\text{cm}$, $r_2 = 1.42\text{cm}$, and $N = 12$, $F = 0.64$. Inserting this value of F in Eq. (1) gives $B_w = 1.28\text{kG}$ for our wiggler periodicity $\lambda_w = 9.6\text{cm}$. This is to be compared with the experimental value $B_w = 1.16\text{kG}$. When we cut all of our magnets and thereby reduce the periodicity to 4.6cm , and again arrange the magnets in accordance with Fig. 1, we obtain $B_w = 0.93\text{kG}$. Equation (1) predicts a value equal to 1.05kG . Thus, we infer from the above comparisons and from several measurements in which we changed N and/or $(r_1 - r_2)$, that Eq. (1) yields a reasonably good estimate of the wiggler field amplitude, and can be used for purpose of scaling and system optimization.

In summary then, this Note reports on a novel REC magnet bifilar wiggler configuration which is capable of giving a high quality helically polarized magnetic field. As yet, no attempt has been made in these preliminary studies to optimize the system. For example, to increase the filling factor F one can envision bar magnets with a trapezoidal rather than square cross section. And to further increase B_w , one could decrease r_1 and increase r_2 (see Fig. 1) so as to increase the quantity of magnetic material and thereby optimize the function $U(k_w r_1) - U(k_w r_2)$ of Eq. (2). This could be accompanied by a reduction in N which would allow the entire magnet system to come nearer the z

axis where the electron beam is located. On the other hand, too large a reduction in N can result in an unacceptably large harmonic content. We believe that such helical REC magnet wigglers with wiggler strengths of several kilogauss are readily achievable. When one compares a permanent magnet helical wiggler with a corresponding current carrying system, the former has two obvious advantages. First, there is no need for a power supply. Secondly, wiggler amplitude and/or period tapering for purposes of adiabatic beam injection⁸ and FEL efficiency enhancement⁹ can be accomplished much more easily with the present arrangement.

We conclude by noting that REC magnets are not easily demagnetized, and therefore our wiggler can be safely inserted⁵ in an axial guide magnetic field as high as ~ 10 kG. Such a combination of wiggler and guide fields is useful for FEL operation in the Ubitron and Raman regimes¹⁰ where the beam current is relatively high and the beam voltage low. It is also useful in giving electrons rotational motion as is required in gyrotrons and cyclotron masers.³

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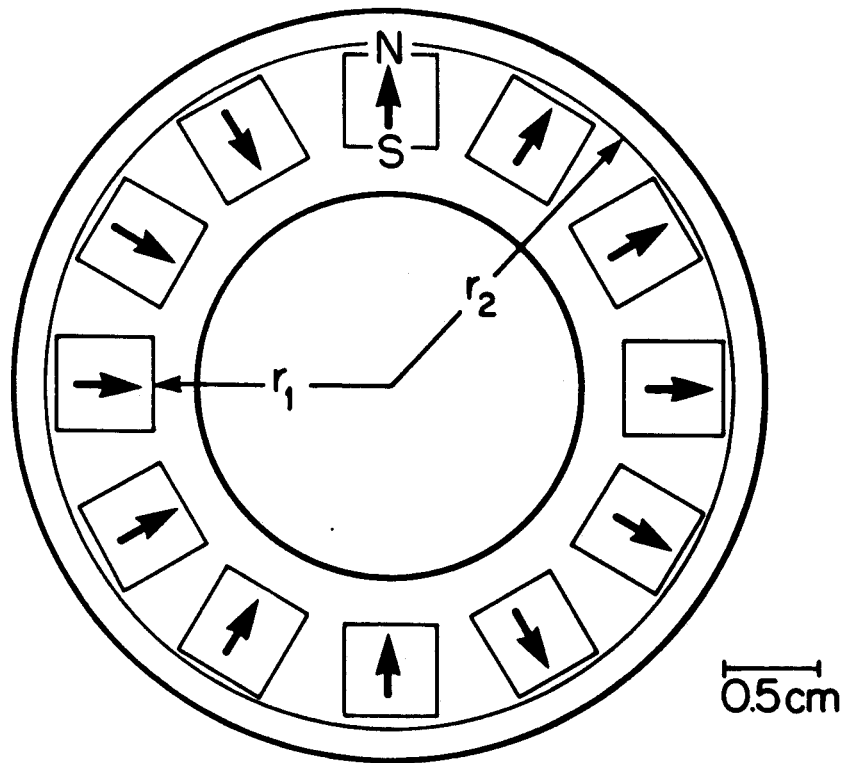
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FIGURE CAPTIONS

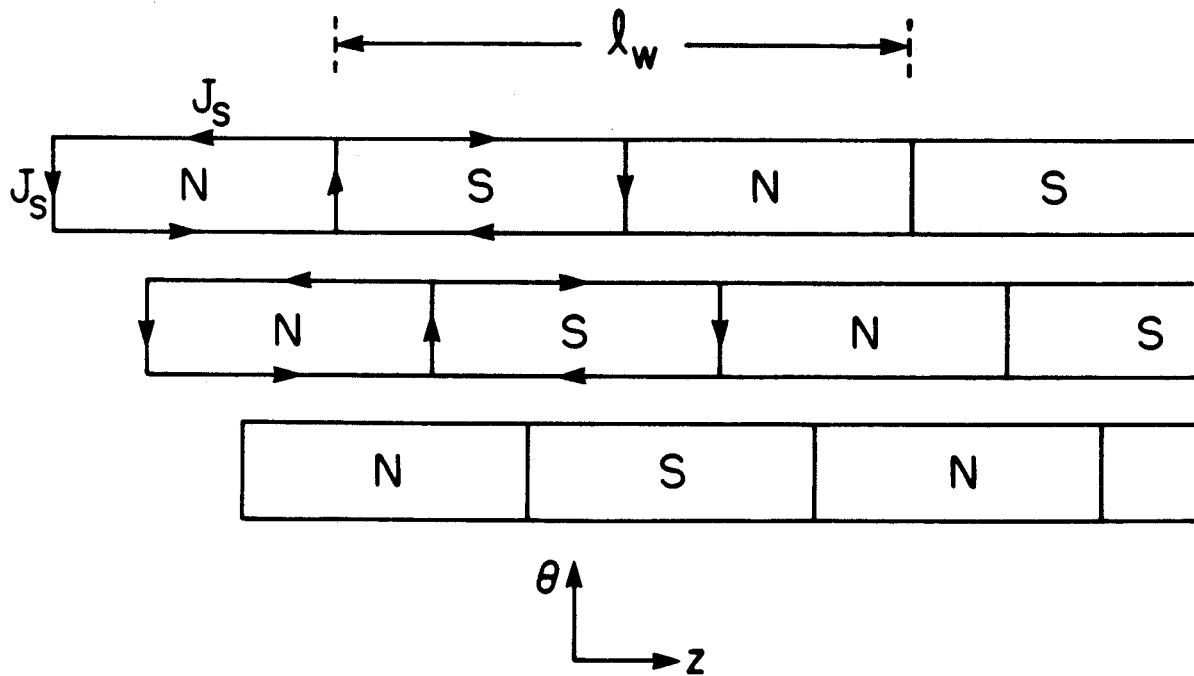
Fig. 1. Schematic of a permanent bar-magnet helical wiggler: (a) cross sectional view (to scale) showing the direction of magnetization of magnets; (b) side view (not to scale) after unrolling the cylinder.

Fig. 2. Magnetic field amplitude as a function of axial distance for different Hall probe rotations, $\theta=0, 30^\circ, 60^\circ, \dots, 180^\circ$; (a) at wiggler center; (b) at wiggler end; (c) after removal of 1 bar magnet.

Fig. 3. The function $U(x)$ of Eq. (1).



(a)



(b)

Fig. 1
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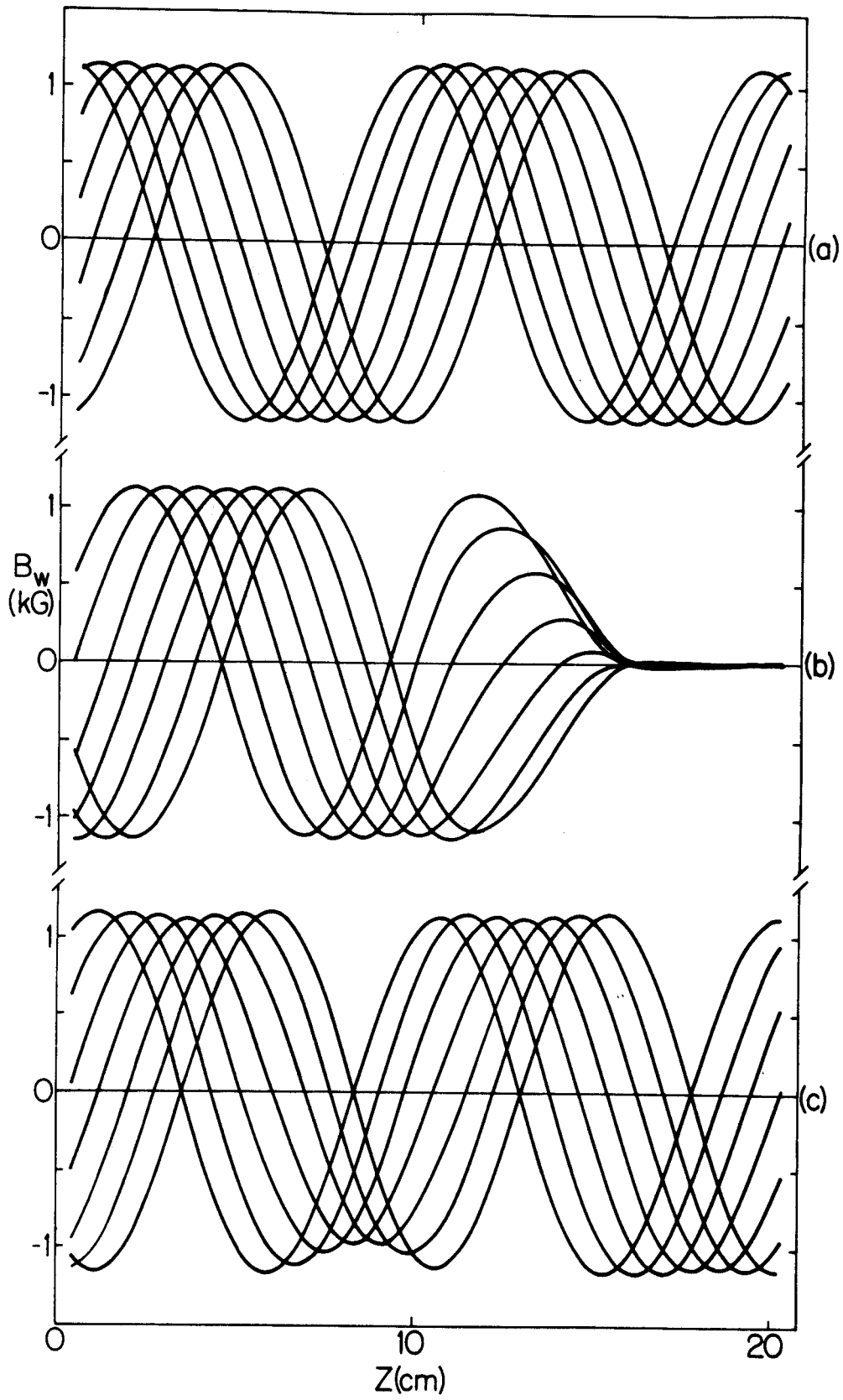


Fig. 2
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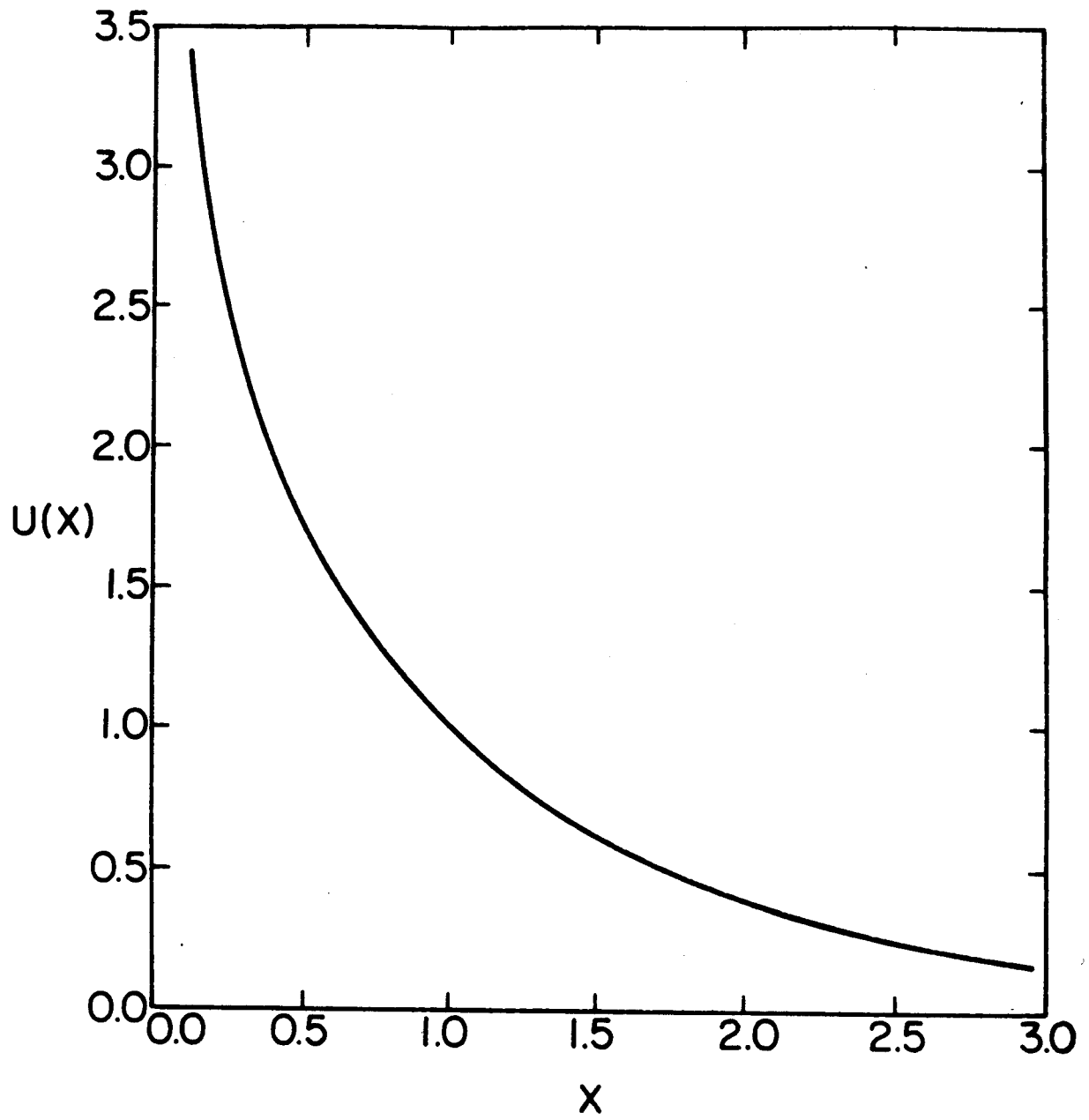


Fig. 3
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