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THEORY OF EFFICIENCY ENHANCEMENT IN CARM AMPLIFIERS BY MAGNETIC FIELD TAPERING

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Theory of Efficiency Enhancement in CARM Amplifiers by Magnetic Field Tapering

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ABSTRACT

The efficiency of cyclotron autoresonance maser (CARM) amplifiers with piecewise linear tapering of the magnetic field is analyzed. In the low current limit, we find that increasing the magnetic field substantially enhances the efficiency if an effective detuning parameter is positive, while decreasing the magnetic field is advantageous when the detuning parameter is negative. For high current, high gain, CARM operation the efficacy of tapering is found to be reduced in a parameter regime where the saturation wave amplitude becomes of the order of an effective detuning parameter.

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The cyclotron autoresonance maser (CARM), potentially a tunable high power coherent radiation source from submillimeter to millimeter wavelengths, has attracted considerable attention. Extensive theoretical and computational effort has been made in the studies of the CARM interaction, including the theory of radiation amplification in the cyclotron resonance maser [1-3], the kinetic theory of CARM with both planar [4] and circular [5] electromagnetic waves, as well as waveguide mode configurations [6], the nonlinear efficiency studies [7], the stabilization of the CARM instability by intense electron beams [8] and momentum spread [9], absolute instabilities [10], and simulation studies of CARM amplifiers [11-15]. Experimental results on the CARM oscillator have been reported recently [16]. The CARM interaction takes place when the electrons undergoing cyclotron motion in a uniform axial magnetic field $B_0\hat{e}_z$ interact with an electromagnetic wave $(\omega, \vec{k})$ propagating along the z-direction. The cyclotron resonance condition is

$$\omega - k_{\|} v_z = \frac{s\omega_{c0}}{\gamma}, \tag{1}$$

where v_z and γ are respectively the axial velocity and relativistic mass factor of the electron beam, s is the harmonic number, $\omega_{c0} = eB_0/mc$ is the nonrelativistic cyclotron frequency, m and e are respectively the electron mass and charge, and c is the speed of light in vacuum. Making use of the phase velocity $v_\phi = \omega/k_{\|}$ and taking $s = 1$ (the fundamental cyclotron frequency), Eq. (1) becomes

$$\omega = \frac{\omega_{c0}}{\gamma(1 - \beta_{\|}/\beta_\phi)} \equiv \omega_D, \tag{2}$$

where $\beta_{\|} \equiv v_z/c$, $\beta_\phi \equiv v_\phi/c = \omega/ck_{\|}$, and ω_D is known as the Doppler shifted cyclotron frequency.

In this rapid communication we analyze, for an arbitrary choice of parameters, the efficiency of single mode CARM amplifiers with piecewise linear tapering of the magnetic

field. The efficiency, denoted by η, is defined as $\eta \equiv (< \gamma_0 > - < \gamma >)/(< \gamma_0 > -1)$, where $< \gamma_0 > mc^2$ and $< \gamma > mc^2$ are respectively the average initial and final energies of the electrons. In the low current limit, we find that increasing (or decreasing) the magnetic field substantially enhances the efficiency when an effective detuning parameter D_{eff} is positive (or negative). (The definition of D_{eff} will be given later.) For high current, high gain, CARM operation, this result remains applicable, except when the saturation wave amplitude is large compared with an effective detuning parameter.

Consider a beam of relativistic electrons of density n undergoing cyclotron motion in a magnetic field $\vec{B}_0(z,r)$ interact with a right-hand polarized electromagnetic wave $(\omega, k\hat{e}_z)$ described by the vector potential

$$\vec{A}(z,t) = A(z)[\hat{e}_x \cos\phi(z,t) - \hat{e}_y \sin\phi(z,t)], \tag{3}$$

where $\phi(z,t) = kz - \omega t + \delta(z)$ is the wave phase, and both the wave amplitude $A(z)$ and the phase shift $\delta(z)$ vary slowly within one wavelength. In polar coordinates, the magnetic field is assumed to have an azimuthally symmetric form [13]

$$\vec{B}_0(z,r) = B_{0z}(z)\hat{e}_z + B_{0r}(z,r)\hat{e}_r, \tag{4}$$

where $B_{0r}(z,r) = -(r/2)(dB_{0z}(z)/dz)$ is the radial component of the magnetic field. Using a system of units in which $e = m = c = B_{0z}(0) = 1$, i.e. introducing the dimensionless variables and parameters $B_{0z}(z) \to B_{0z}(z)/B_{0z}(0)$, $\omega_{c0}(z) \to \omega_{c0}(z)/\Omega$, $\omega_p \to \omega_p/\Omega$, $z \to cz/\Omega$, $v_z \to v_z/c$, $v_\phi \to v_\phi/c$, $p_z \to p_z/mc$, $p_t \to p_t/mc$, and $A \to eA/mc^2$, with $\Omega = eB_{0z}(0)/mc$, we can write the magnetically tapered CARM equations as

$$\frac{d\psi}{dz} = k - \frac{\omega\gamma}{p_z} + \frac{\omega_{c0}(z)}{p_z} + \frac{\omega A}{p_t}\left(\frac{1}{v_\phi} - \frac{\gamma}{p_z}\right)\sin\psi + \frac{d\delta(z)}{dz}, \tag{5a}$$

$$\frac{d\gamma}{dz} = \frac{\omega A p_t}{p_z}\cos\psi, \tag{5b}$$

$$\frac{dp_z}{dz} = \frac{kAp_t}{p_z}\cos\psi - \frac{p_t^2}{2p_z B_{0z}}\frac{dB_{0z}}{dz}, \tag{5c}$$

$$\frac{dA}{dz} = -\frac{\omega_p^2 v_{z0}}{2k}\langle\frac{p_t}{p_z}\cos\psi\rangle, \tag{5d}$$

$$\frac{d\delta}{dz} = \frac{\omega_p^2 v_{z0}}{2kA}\langle\frac{p_t}{p_z}\sin\psi\rangle, \tag{5e}$$

where $\omega_p^2 = 4\pi e^2 n/m$ is the electron plasma frequency squared, v_{z0} is the axial velocity of the electron beam while entering the $z \geq 0$ interaction region. Detailed derivations of the self-consistent equations describing the CARM amplifier with magnetic field tapering have been presented in Ref [13] and [14]. The first three equations describe the motion of the electrons in terms of the axial momentum p_z, energy γ, and phase $\psi \equiv kz - \omega t - \tan^{-1}(p_x/p_y) + \delta(z)$. The last two describe the wave evolution, where $\langle ... \rangle = N^{-1}\sum_{i=1}^{N} ...$ denotes the average over all the electrons. In this N-particle model there are total of $3N + 2$ equations since the transverse momentum p_t is solved from $\gamma = (1 + p_t^2 + p_z^2)^{\frac{1}{2}}$. From Eqs. (5b) and (5d), the total energy flux of the electron beam and electromagnetic wave field, $nv_{z0}\gamma + (1/4\pi)\omega k A^2$, is a constant of motion. Moreover, the magnetic moments of the electrons are adiabatically conserved in the absence of the wave field.

In the linear instability regime, the electrons undergo a transition from random to bunched phase distributions such that more electrons give up their kinetic energy to the electromagnetic wave. In the low current limit, since the wave amplitude and spatial growth rate are small, such phase bunching can be described as follows. Let us consider an untapered CARM amplifier operating at the frequency

$$\omega = \omega_D + \Delta\omega \equiv \frac{\omega_{c0}}{\gamma_0(1 - v_{z0}/v_\phi)} + \Delta\omega, \tag{6}$$

where $\Delta\omega$ and γ_0 are respectively the frequency detuning and initial beam energy. By differentiating Eq. (5a) and making use of Eqs. (5b), (5c), and (6), and the fact that $p_z(z) - p_{z0} = O(A)$ and $p_t(z) - p_{t0} = O(A)$, it is straight forward to show that the dynamics of the electron phase ψ is approximately govern by the pendulum equation

$$\frac{d^2\psi}{dz^2} = D_{eff}(\frac{k^2\theta_{p0}}{p_{z0}})A(z)\cos\psi + O(A^2), \tag{7}$$

where $\theta_{p0} = p_{t0}/p_{z0}$ is the initial pitch angle of the electron beam. Here we have introduced an effective detuning parameter

$$D_{eff} = 1 - v_\phi^2 + [(\frac{v_\phi}{v_{z0}} - 1) + \frac{1}{\theta_{p0}^2}(\frac{v_\phi}{v_{z0}} - 1)^2]\frac{\Delta\omega}{\omega}. \tag{8}$$

Thus, the electrons bunch at the synchronous phase $\psi_r = \pi/2$ for $D_{eff} > 0$ since the stable fixed point is located at $(\psi, d\psi/dz) = (\pi/2, 0)$ and the unstable fixed point at $(\psi, d\psi/dz) = (3\pi/2, 0)$. For $D_{eff} < 0$, the synchronous phase is $\psi_r = 3\pi/2$. D_{eff} vanishes when $v_\phi = 1$ and $\Delta\omega = 0$, revealing the well-known cyclotron autoresonance phenomenon [7], where the electrons remain in synchronization with the electromagnetic wave in the course of evolution. Typically, the growth rate is small when $D_{eff} = 0$.

To get an intuitive picture of efficiency enhancement using magnetic field tapering, it suffices to analyze the motion of the electrons with phases close to the synchronous phase ψ_r. Since in the (ψ, p_z) plane the phase dependence of the axial momenta of these electrons can be approximated as $p_z(\psi) \approx p_z(\psi_r) + (dp_z(\psi_r)/d\psi)(\psi - \psi_r)$, the change in $d\psi/dz$ due to the magnetic field change δB_{0z} (or $\delta\omega_{c0}$) is then given by the Taylor expansion

$$\delta(\frac{d\psi}{dz}) = \frac{\delta\omega_{c0}}{p_z(\psi)} \approx \frac{\delta\omega_{c0}}{p_z(\psi_r)}[1 - \frac{1}{p_z(\psi_r)}\frac{dp_z(\psi_r)}{d\psi}(\psi - \psi_r)], \tag{9}$$

where $p_z(\psi_r)$ is the axial momentum of the electron with $\psi = \psi_r$. We shall argue in the following that increasing (or decreasing) the magnetic field enhances the efficiency when D_{eff} is positive (or negative). For $D_{eff} > 0$, both $d\gamma(\psi_r)/d\psi$ and $dp_z(\psi_r)/d\psi$ are negative before nonlinear saturation. This occurs because the synchronous electrons with $\psi > \psi_r = \pi/2$ lose energy and axial momenta while those with $\psi < \psi_r$ gain energy and axial momenta, as seen from Eqs. (5b) and (5c). Eq. (9) implies that as the magnetic field increases the synchronous electrons are forced to the right in the (ψ, γ) plane, so that the number of the electrons with phases situated in the interval $(\pi/2, 3\pi/2)$ increases or the electrons continue losing energy on the average. In short, increasing (or up tapering) the magnetic field yield efficiency enhancement when $D_{eff} > 0$. Similarly, decreasing (or down tapering) the magnetic field enhances the efficiency when $D_{eff} < 0$.

Fig. 1a shows the untapered and corresponding optimally tapered efficiencies as functions of the relative frequency detuning $\Delta\omega/\omega$, obtained from self-consistent simulations with Eq. (5), cold electron beams, and piecewise linear tapering. For the results in this figure the simulations had $v_\phi = 1$, corresponding to the CARM operating in vacuum, $\gamma_0 = 2.37$, and $\theta_{p0} = 0.53$. The dimensionless electron plasma frequency $\omega_p = 0.05$, which is representative of the low current limit. In order to obtain the tapered efficiencies, upward tapering was used for the high frequency branch ($\omega > \omega_D$), while downward tapering for the low frequency branch ($\omega < \omega_D$). To achieve optimal efficiency we start tapering somewhat before nonlinear saturation occurs. Usually the absolute value of the slope for optimal tapering ranges from 0.04 to $0.08\Gamma B_{0z}(0)$. A similar plot is shown in Fig. 1b for $v_\phi = 1.03$ and $\omega_p = 0.005$, where only the negative D_{eff} branch is plotted since the growth rate is small for the positive D_{eff} branch and low current. This figure corresponds to the CARM operating with a waveguide mode. (Since the transverse variation of the rf field and the forces due to the longitudinal rf field can be neglected under the conditions that $1 - v_\phi^{-2} << 1 - v_{z0}/v_\phi$, the electron Larmor radius $r_L << 1/k_\perp$, and the electron beam radius $r_b << 1/k_\perp$ [13], the one-dimensional model given by Eq. (5) provides a good description for the CARM

operating in waveguide.)

For high current CARM operation, the validity of the pendulum equation (7) breaks down because the term of the order of A^2 can no longer be ignored in the expansion. Indeed, when deriving Eq. (7), one has to differentiate for instance the sine term in Eq. (5a), which contributes a term of the order of $A^2\sin(2\psi)$ to the right hand side of Eq. (7). For example, if the term proportional to $A^2\sin(2\psi)$ dominates, then the electrons will equally bunch at two phases differing by π. To the lowest order, tapering does not yield net gain in energy extraction because the electrons bunched at ψ_r give up energy, while those at $\psi_r + \pi$ gain energy, or vice versa. Typically, multiple phase bunching occurs and the efficacy of tapering is reduced when the saturation wave amplitude A_{sat} is of the order of $D_{eff}k^2\theta_{p0}/p_{z0}$. A qualitative criterion for tapering to result in efficiency enhancement is

$$A_{sat} < |D_{eff}|(\frac{k^2\theta_{p0}}{p_{z0}}). \tag{10}$$

In Fig. 2 we plot the untapered efficiency (dashed curve), corresponding tapered efficiency (solid curve), and dimensionless ratio $A_{sat}p_{z0}/|D_{eff}|k^2\theta_{p0}$ (dotted curve) as functions of D_{eff}, where downward tapering is used and the parameters are the same as in Fig. 1b, except now $\omega_p = 0.3$.

We conclude that in the low current limit the efficiency of CARM amplifiers can be substantially enhanced by increasing (or decreasing) the magnetic field if an effective detuning parameter is positive (or negative). Moreover, for high current, high gain, CARM operation this result remains valid, as long as condition (10) holds approximately. When condition (10) is violated, the effect of tapering on the coupling between the electrons and electromagetic wave field becomes delicate and requires further investigations.

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FIGURE CAPTIONS

Figure 1

The untapered (dashed line) and corresponding optimal tapered (solid line) efficiencies are plotted as functions of detuning parameters in the low current limit. a) The choice of $v_\phi = 1$ corresponds to the CARM operating in vacuum. Other parameters used in the simulations are $\omega_p = 0.05$, $\gamma_0 = 2.37$, and $\theta_{p0} = 0.53$. Upward tapering was used for the high frequency branch with $\Delta\omega > 0$, and downward tapering for the low frequency branch with $\Delta\omega < 0$. b) A similar plot for $\omega_p = 0.005$ and $v_\phi = 1.03$, corresponding to the CARM operating in a waveguide mode for the negative D_{eff} branch.

Figure 2

The untapered efficiency (dashed line), corresponding optimal tapered efficiency (solid line) and ratio $A_{sat}p_{z0}/|D_{eff}|k^2\theta_{p0}$ (dotted line) as functions of D_{eff} for $v_\phi = 1.03$, $\omega_p = 0.3$, $\gamma_0 = 2.37$, and $\theta_{p0} = 0.53$.

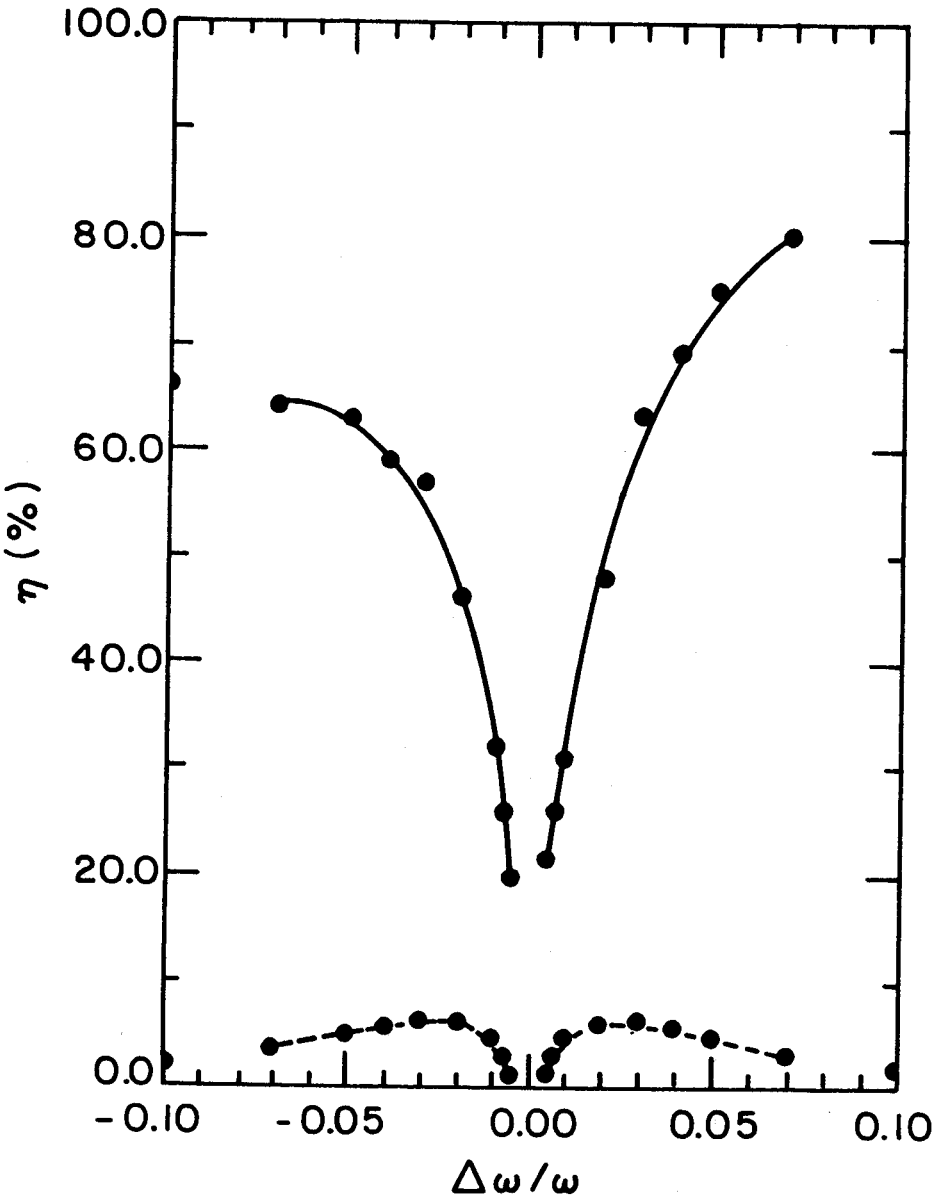

Figure 1(a)

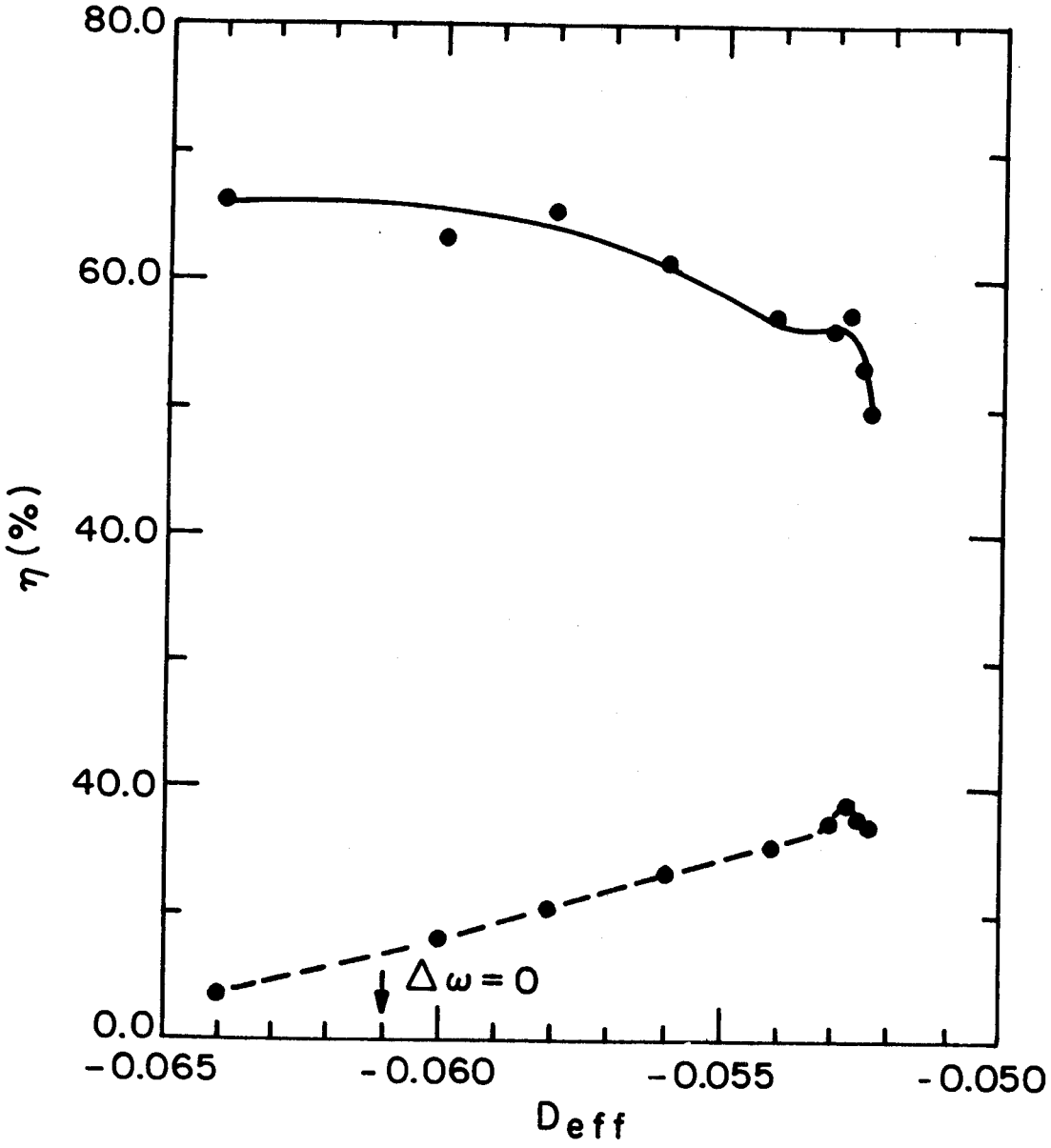

Figure 1(b)

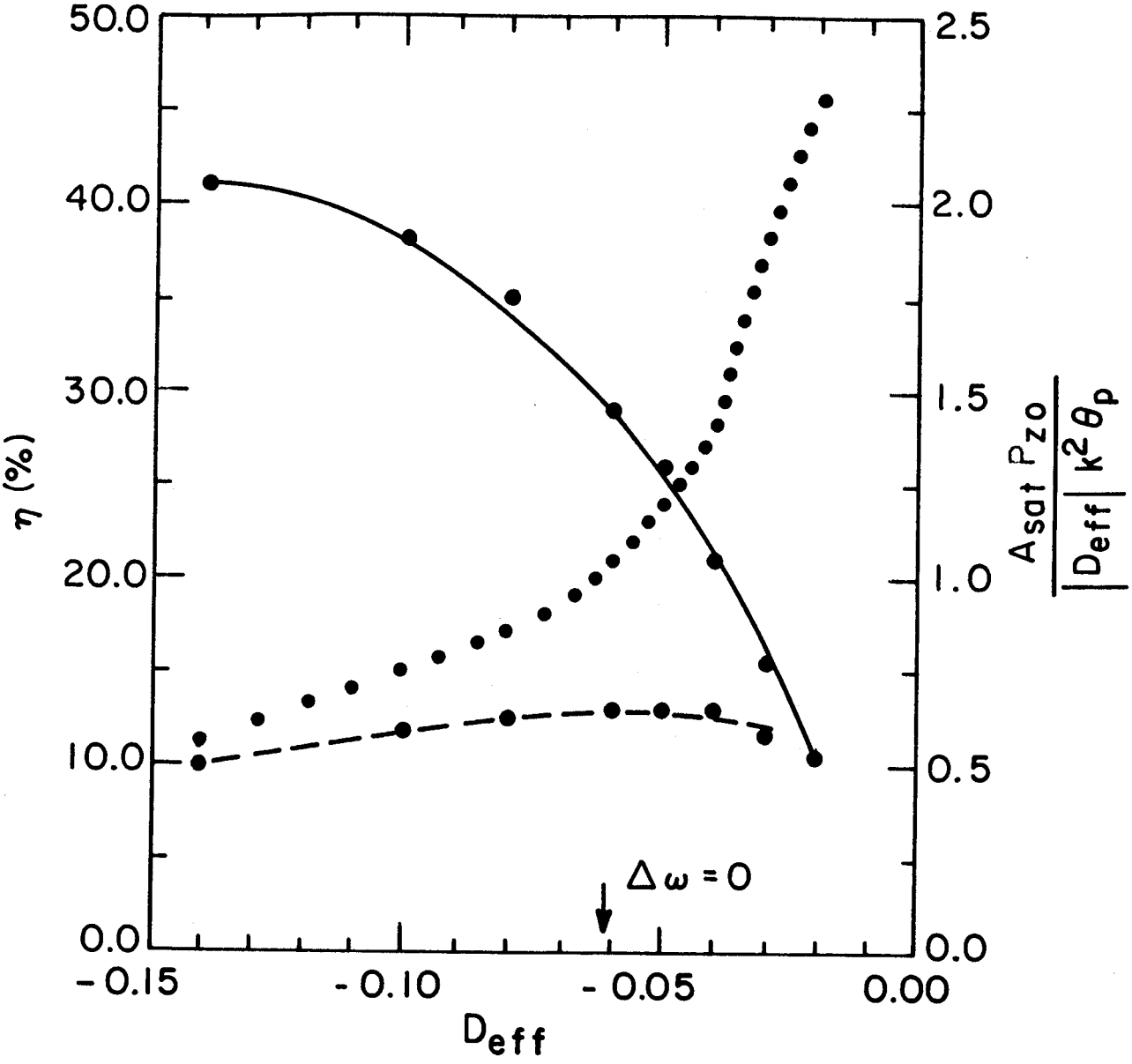
50.0
40.0
30.0
20.0
10.0
0.0
η (%)
2.5
2.0
1.5
1.0
0.5
0.0
$\frac{A_{sat} P_{zo}}{|D_{eff}| k^2 \theta_p}$
Δω = 0
-0.15
-0.10
-0.05
0.00
D_{eff}

Figure 2