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Abstract

The stimulated refraction effect (optical guiding) in a free-electron laser (FEL) is studied theoretically. The analysis of the 3-dimensional FEL wave equation is performed according to two different formal methods. First, the microwave field is expanded into vacuum waveguide modes, in which case the optical guiding appears as an active mode coupling effect. In the second method, the eigenmodes of the FEL radiation field are found, which yields both the gain of the system (complex eigenvalues) and the radial intensity distribution of the interacting waves, describing the guiding effect. Computer calculations show that optical guiding may have a strong influence on both the gain and mode content of millimeter-wave FELs.

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Free-electron lasers (FELs) are tunable sources of coherent electromagnetic radiation [1]. One of their most remarkable properties is the large phase shift [2-4] that the resonant beam-wave interaction induces on the amplified electromagnetic wave. Under proper circumstances, the refraction associated to this phase shift can be such that the electromagnetic wave is confined within the electron beam, in a manner somewhat akin to the guiding properties in an optical fiber. This stimulated refraction effect, referred to as "optical guiding" [5-13], has many important implications. For short wavelength FELs, it would mitigate the effects of diffraction, therefore allowing the length of FEL wigglers to exceed the Rayleigh range, and the obtention of higher single-pass gains. Such long wigglers are needed if FELs are to operate either in the vacuum-ultraviolet or at high efficiencies in the infrared wavelength regime. For millimeter-wave FELs, optical guiding can also be associated to higher gains due to a better coupling (overlap) between the electron beam and the FEL radiation mode (higher electromagnetic wave filling factor) ; in addition, the stimulated refraction effect can strongly affect the mode purity of waveguide FELs.

The purpose of this Letter is to present a simple theoretical analysis of optical guiding in a waveguide FEL. In contrast with the conventional vacuum waveguide mode expansion, we perform an eigenanalysis of the 3-D FEL wave equation, yielding the eigenmodes (guided modes) and eigenvalues of the problem. In particular the gain is calculated in a self-consistant manner, without using the so-called electromagnetic wave filling factor, by evaluating the imaginary part of the complex eigenwavenumber.

The FEL interaction can be described, within the framework of a linearized fluid model, by a set of 4 coupled PDEs describing the evolution of the 4-potential vector perturbation $\delta A_\mu \equiv (\delta\phi/c, \delta\vec{A})$

$$[\partial_t - \vec{v}_0 \cdot \vec{\nabla} + \vec{\nabla} \cdot \vec{v}_0 - \vec{\Omega}_0 \times + \gamma_0^2 (\vec{\Omega}_0 \times \vec{\beta}_0)\vec{\beta}_0 \cdot] \left(\Box \delta\vec{A} - \vec{\beta}_0 \Box \frac{\delta\phi}{c} \right) + \left[\left(\Box \delta\vec{A} - \vec{\beta}_0 \Box \frac{\delta\phi}{c} \right) \cdot \vec{\nabla} \right] \vec{v}_0$$
$$+ \frac{\omega_p^2}{c^2} \left\{ -\vec{\nabla}\delta\phi - \partial_t \delta\vec{A} + \vec{v}_0 \times \vec{\nabla} \times \delta\vec{A} + \vec{\beta}_0 (\vec{\nabla}\delta\phi + \partial_t \delta\vec{A}) \cdot \vec{\beta}_0 \right\} = \mathbf{0}, \tag{1}$$

$$\frac{1}{c^2} \partial_t \delta\phi + \vec{\nabla} \cdot \delta\vec{A} = 0, \tag{2}$$

where

$$\Box \equiv \partial_\mu \partial^\mu \equiv \vec{\nabla}^2 - \frac{1}{c^2} \partial_t^2$$

is the d'Alembertian operator (electromagnetic wave propagator). Here, the quantities indexed "0" refer to the unperturbed fluid equilibrium. Note that we can easily identify the different terms in equation (1) as a beam-mode type operator coupled to an electromagnetic wave propagator, and a beam coupling term proportional to the beam density (ω_p^2) and containing the ponderomotive force. Here, we have assumed that the equilibrium field is purely magnetic ($\vec{E}_0 = \mathbf{0}$), and we have defined the following parameters

$$\vec{\Omega}_0 = \frac{e\vec{B}_0}{\gamma_0 m_o} \quad , \quad \frac{\omega_p^2}{c^2} = \mu_o \frac{n_0 e^2}{\gamma_0 m_o} \quad , \quad \vec{\beta}_0 = \frac{\vec{v}_0}{c}.$$

At this point, different beam-wave interactions are characterized by different fluid equilibria and different boundary conditions for δA_μ and its derivatives.

We now consider a FEL with an axial guide field, pumped by a helically polarized wiggler. The waves propagate in a cylindrical waveguide of radius $r = a$. The externally applied fields are purely magnetic, and we neglect the self-fields generated by the beam. We have

$$\vec{B}_0(\theta, z) = \hat{z}B_\parallel + B_w(\hat{r}\cos\psi + \hat{\theta}\sin\psi), \tag{3}$$

where $B_\parallel$ is the strength of the axial guide magnetic field, B_w is the amplitude of the wiggler field and $\psi = k_w z - \theta$, where $\ell_w = 2\pi/k_w$ is the wiggler period. Note that here, for the sake of simplicity, we consider a radially uniform wiggler field and that we neglect the space-charge effects of the electron beam on the fluid equilibrium. The corresponding equilibrium fluid velocity field is

$$\vec{\beta}_0(\theta, z) = \hat{z}\beta_\parallel + \beta_\perp(\hat{r}\cos\psi + \hat{\theta}\sin\psi), \tag{4}$$

where the normalized fluid velocity components $\beta_\parallel$ and $\beta_\perp$ are constrained by energy conservation

$$\frac{1}{\gamma_0^2} = 1 - \beta_\parallel^2 - \beta_\perp^2, \tag{5}$$

and related by the following equation

$$\beta_\perp = \frac{\Omega_w \beta_\parallel}{\Omega_\parallel - \gamma_0 k_w \beta_\parallel c}. \tag{6}$$

Here, we have defined the relativistic cyclotron frequencies $\Omega_\parallel$ and Ω_w in the guide and wiggler fields, respectively. Because the divergence of the equilibrium fluid velocity field (4) is zero, the beam density profile can be described by a step function

$$n_0(r) = \begin{cases} n_0, & \text{for } r < r_b; \\ 0, & \text{for } r_b < r < a; \end{cases} \tag{7}$$

where r_b is the beam radius, which satisfies the equilibrium continuity equation everywhere, except at the edge of the beam. The surface effects can then be modelled by non-uniform surface charge and current distributions. Making use of equations (3) to (7) into the initial system (1-2) yields the 3-D FEL wave equation describing the evolution of the microwave laser field in the linear regime.

At this point, the 3-dimensional analysis of the FEL radiation field can be performed according to two different formal methods. One can either find the eigenmodes of the system, which yields both the gain (eigenvalue) of the system and the radial intensity distribution of the interacting waves, or expand the microwave field into vacuum waveguide modes and make use of the orthonormality of these modes to study their coupling. In the latter case, the guiding appears as an active mode conversion effect. We first briefly review the coupled-mode analysis. The general form of the 3-D wave equation is

$$\left[\Box + \frac{\omega_p^2(r)}{c^2}\mathcal{C}(r,\omega,k)\right]\delta\vec{A} = \mathbf{0}, \tag{8}$$

where the coupling operator $\mathcal{C}$ contains the space-charge waves; the beam dynamics effects, etc. The solution is expressed as a superposition of vacuum waveguide modes

$$\delta\vec{A} = \sum_m \delta A_m \vec{U}_m, \tag{9}$$

satisfying the following eigenvalue equation

$$\Box\vec{U}_m = \left(\frac{\omega^2}{c^2} - k^2 - x_m^2\right)\vec{U}_m. \tag{10}$$

Here, x_m corresponds to the cutoff frequency of the m-th vacuum mode, determined from the boundary conditions at the waveguide walls. For TE_{ln} modes in a cylindrical pipe of radius a, $x_{ln} = \chi'_{ln}/a$. The wave equation is then reduced to a set of coupled linear equations by integrating over the waveguide cross-section and using the orthonormality of the U_m's

$$\int_S dS \quad \vec{U}_n^* \cdot \sum_m \left[\left(\frac{\omega^2}{c^2} - k^2 - x_m^2\right) + \frac{\omega_p^2(r)}{c^2}\mathcal{C}(r,\omega,k)\right]\delta A_m \vec{U}_m = 0, \tag{11}$$

to obtain

$$\left[\left(\frac{\omega^2}{c^2} - k^2 - x_n^2\right) + \frac{\omega_{p0}^2}{c^2}C_{nn}(\omega,k)\right]\delta A_n + \frac{\omega_{p0}^2}{c^2}\sum_{m\neq n} C_{mn}(\omega,k)\delta A_m = 0. \tag{12}$$

Here, the guiding effect appears as active mode coupling. In the conventional theory, one neglects the coupling terms C_{mn} and introduces the electromagnetic wave filling factor, defined as

$$p_2 = \int_S dS \quad \vec{U}_n^* \cdot \left[\frac{\omega_p(r)}{\omega_{p0}}\right]^2 \vec{U}_n, \tag{13}$$

to obtain the usual FEL dispersion relation for cylindrical TE_{ln} modes

$$\left[\frac{\omega^2}{c^2} - k^2 - \left(\frac{\chi'_{ln}}{a}\right)^2\right]\left[\frac{\omega}{c} - \beta_\parallel(k+k_w) + \frac{\omega_{p0}}{\gamma_\parallel c}\sqrt{\phi}\right]\left[\frac{\omega}{c} - \beta_\parallel(k+k_w) - \frac{\omega_{p0}}{\gamma_\parallel c}\sqrt{\phi}\right]$$
$$= p_2\left[\beta_\perp^2 \frac{\omega_{p0}^2}{c^2}\frac{\phi}{\beta_\parallel}kk_w\right]. \tag{14}$$

Here, ϕ is a correction to the space-charge dispersion equation due to the combined presence of the axial and wiggler magnetic fields [14]. From this relation, it is straightforward to evaluate the Raman growth rate

$$\delta k^2 \simeq -p_2\left[\frac{1}{2}\frac{\beta_\perp}{\beta_\parallel}\gamma_\parallel\left(\frac{\omega_{p0}k_w}{\gamma_\parallel c}\sqrt{\phi}\right)^{1/2}\right]^2 = p_2(i\Gamma_0)^2. \tag{15}$$

We now study the eigenmodes of the system, for TE ($\delta A_z = \delta\phi = 0$) geometry. The interaction region is divided into two areas. Region 1 ($r_b < r < a$) corresponds to the vacuum surrounding the electron beam. The plasma frequency $\omega_p(r) = 0$, and the general solution to the 3-D wave equation for the azimuthal component of the 4-potential vector is of the form

$$\delta A_\theta(r,\theta,z,t) = [AJ'_{l_1}(x_1 r) + BY'_{l_1}(x_1 r)]\exp[i(\omega_1 t - k_1 z + l_1\theta)], \tag{16}$$

where k_1 and x_1 are constrained by the vacuum dispersion relation

$$D_1(k_1, x_1) = \frac{\omega_1^2}{c^2} - k_1^2 - x_1^2 = 0. \tag{17}$$

Note that in this region of space, the Bessel function of the second kind can be included in the general solution because $r > r_b \neq 0$. Region 2 ($0 < r < r_b$) corresponds to the electron beam. Here the plasma frequency is constant $\omega_p(r) = \omega_{p0}$, and the general solution for δA_θ is of the form

$$\delta A_\theta(r,\theta,z,t) = CJ'_{l_2}(x_2 r)\exp[i(\omega_2 t - k_2 z + l_2\theta)], \tag{18}$$

where k_2 and x_2 are constrained by the beam dispersion relation $[D_2(k_2, x_2) = 0]$,

$$\left[\frac{\omega_2^2}{c^2} - k_2^2 - x_2^2\right]\left[\frac{\omega_2}{c} - \beta_{\|}(k_2 + k_w) + \frac{\omega_{p0}}{\gamma_{\|} c}\sqrt{\phi}\right]\left[\frac{\omega_2}{c} - \beta_{\|}(k_2 + k_w) - \frac{\omega_{p0}}{\gamma_{\|} c}\sqrt{\phi}\right] = \beta_{\perp}^2 \frac{\omega_{p0}^2}{c^2}\frac{\phi}{\beta_{\|}} k_2 k_w. \quad (19)$$

The boundary conditions are the following. At the waveguide wall ($r = a$), the tangential electric field component ($\delta E_\theta = -i\omega_1 \delta A_\theta$) must be zero, which yields

$$\delta A_\theta(r = a) = 0. \quad (20)$$

At the beam edge ($r = r_b$), we require that the 4-vector potential be continuous

$$\Delta\delta A_\theta(r = r_b) = 0, \quad (21)$$

which is equivalent to having δE_θ and δB_r continuous. Finally, we use the dielectric boundary condition

$$\Delta\partial_r \delta A_\theta(r = r_b) = 0. \quad (22)$$

This last condition implies that we have neglected the surface current distributions at the beam edge. The continuity of δA_θ at the beam edge (21) yields the following relations

$$k_1 = k_2 = k, \quad (23)$$

$$l_1 = l_2 = l, \quad (24)$$

$$\omega_1 = \omega_2 = \omega, \quad (25)$$

which correspond to the fact that continuity must hold at any z, θ and t, respectively. In addition, we have

$$AJ_l'(x_1 r_b) + BY_l'(x_1 r_b) = CJ_l'(x_2 r_b). \quad (26)$$

The continuity of $\partial_r \delta A_\theta$ at $r = r_b$ (22) yields

$$x_1[AJ_l''(x_1 r_b) + BY_l''(x_1 r_b)] = x_2 C J_l''(x_2 r_b). \quad (27)$$

Finally, the boundary condition at the waveguide wall (20) results in

$$AJ_l'(x_1 a) + BY_l'(x_1 a) = 0. \quad (28)$$

Combining equations (26), (27) and (28), and upon elimination of the constants A, B and C, we obtain a third relation $[D_0(x_1,x_2)=0]$,

$$\left[J_l''(x_1r_b)-J_l'(x_1a)\frac{Y_l''(x_1r_b)}{Y_l'(x_1a)}\right]-\frac{x_2J_l''(x_2r_b)}{x_1J_l'(x_2r_b)}\left[J_l'(x_1r_b)-J_l'(x_1a)\frac{Y_l'(x_1r_b)}{Y_l'(x_1a)}\right]=0. \quad (29)$$

Equation (29) and the two dispersion relations in vacuum (17) and inside the beam (19) form a system of 3 nonlinear equations in the complex eigenvalues k, x_1 and x_2

$$D_0(x_1,x_2)=0 \quad , \quad D_1(k,x_1)=0 \quad , \quad D_2(k,x_2)=0.$$

We can solve these equations as functions of the frequency ω to obtain the complex eigen-wavenumber $k(\omega)$, where $\mathrm{Im}(k)$ is the 3-D growth rate of the FEL instability, and $x_1(\omega)$ and $x_2(\omega)$ which determine the radial profile of the electromagnetic waves (eigenmodes) in the interaction region, and therefore describe the optical guiding effect in a waveguide FEL.

To illustrate the derivations presented above, we have done some computer calculations, comparing the FEL gain and radiation field profile with and without guiding. The example we study corresponds to the design parameters of a FEL experiment planned at MIT in collaboration with TTE. The beam voltage is $V=500$ kV, the beam current density is kept constant at $j=0.337$ kA/cm^2. The nominal wiggler field amplitude is $B_w=500$ G, with a period $\ell_w=3.0$ cm. The guide field strength is $B_\parallel=2.5$ kG, and the waveguide radius is $a=10$ mm. The microwave are launched in the fundamental TE_{11} cylindrical mode. For these parameters, the operation frequency is $\omega_+/2\pi\simeq 42$ GHz. On the drawings, the TE_{11} mode and the eigenmode are normalized so that they have the same power flux through the waveguide. In Figure 1, we show the radial profile of the normalized azimuthal electric field component for a 5-mm-radius electron beam, and 3 different amplitudes of the pump : $B_w=0$, 100 and 500 G. As the optical activity (gain and phase shift of the FEL interaction) of the beam increases, the guiding effect appears clearly and the eigenmode deviates substantially from the vacuum TE_{11} mode. In Fig. 2, the wiggler field is held constant at $B_w=500$ G, and we plot the normalized microwave intensity profile as the beam radius is varied from 4 mm to 9 mm. The guiding effect is strong at small beam radii (strong transverse "gradient" of the anomalous refractive index) and, as expected, the eigenmode relaxes towards the vacuum TE_{11} mode as the beam radius increases. Note that if the beam fills the waveguide, no guiding is expected. Finally, in Fig. 3 we plot the ratio of the imaginary part of the eigen-wavenumber $\mathrm{Im}(k)$ to the TE_{11} gain $\Gamma_0\sqrt{p_2}$, calculated from Eq. (15). For large beam radii, the guiding is small and both gains are comparable and close to the 1-D gain, Γ_0. However, as the beam radius decreases, the electromagnetic wave filling factor for the TE_{11} mode p_2 is strongly reduced, while $\mathrm{Im}(k)$ remains close to Γ_0. At still smaller radii, the radiation appears to decouple from the beam, which may be due to strong diffraction effects. It should be noted, however, that at the present time our model does not include the effects of the surface charge and current distributions, which clearly become predominant at small beam radii.

In conclusion, we have described a theoretical model of the FEL interaction in the linear regime that includes the stimulated refraction effect which is the physical phenomenon at the origin of optical guiding. Indeed, we find that the eigenmodes of the problem can substantially deviate from the vacuum waveguide modes used in other models. In addition, the imaginary part of the complex eigen-wavenumber directly yields the gain, making the calculation of the so-called electromagnetic wave filling factor an unnecessary step. We find that this gain can be higher than that calculated from the vacuum mode theory. Finally, it should be pointed out that our model can be improved by using more complete boundary conditions at the beam/vacuum interface, in particular by including the non-uniform charge and current distributions at the beam edge resulting from the wiggling of the beam.

References

[1] C.W. Roberson and P. Sprangle, Phys. Fluids **B1**, 3 (1989).

[2] J. Fajans and G. Bekefi, Phys. Fluids **29**, 3461 (1986).

[3] J. Fajans, J.S. Wurtele, G. Bekefi, D.S. Knowles and K. Xu, Phys. Rev. Lett. **57**, 579 (1986)

[4] T.J. Orzechowski, E.T. Scharlemann and B.D. Hopkins, Phys. Rev. **A35**, 2184 (1987).

[5] P. Sprangle and C.M. Tang, Appl. Phys. Lett. **39**, 677 (1981).

[6] W.M. Kroll, P.L. Morton and M.W. Rosenbluth, IEEE J. Quantum Electron. **17**, 1436 (1981).

[7] J.M. Slater and D.D. Lowenthal, J. Appl. Phys. **52**, 44 (1981).

[8] D. Prosnitz, A. Szoke and V.K. Neil, Phys. Rev. **A24**, 1436 (1981).

[9] E.T. Scharlemann, A.M. Sessler and J.S. Wurtele, Phys. Rev. Lett. **54**, 1925 (1985).

[10] G.T. Moore, Opt. Commun. **52**, 46 (1984).

[11] G.T. Moore, Nucl. Instrum. Methods Phys. Res. Sect. **A239**, 19 (1985).

[12] F. Hartemann, K. Xu, G. Bekefi, J.S. Wurtele and J. Fajans, Phys. Rev. Lett. **59**, 1177(1987).

[13] A. Fruchtman, Phys. Rev. **A37**, 2989 (1988).

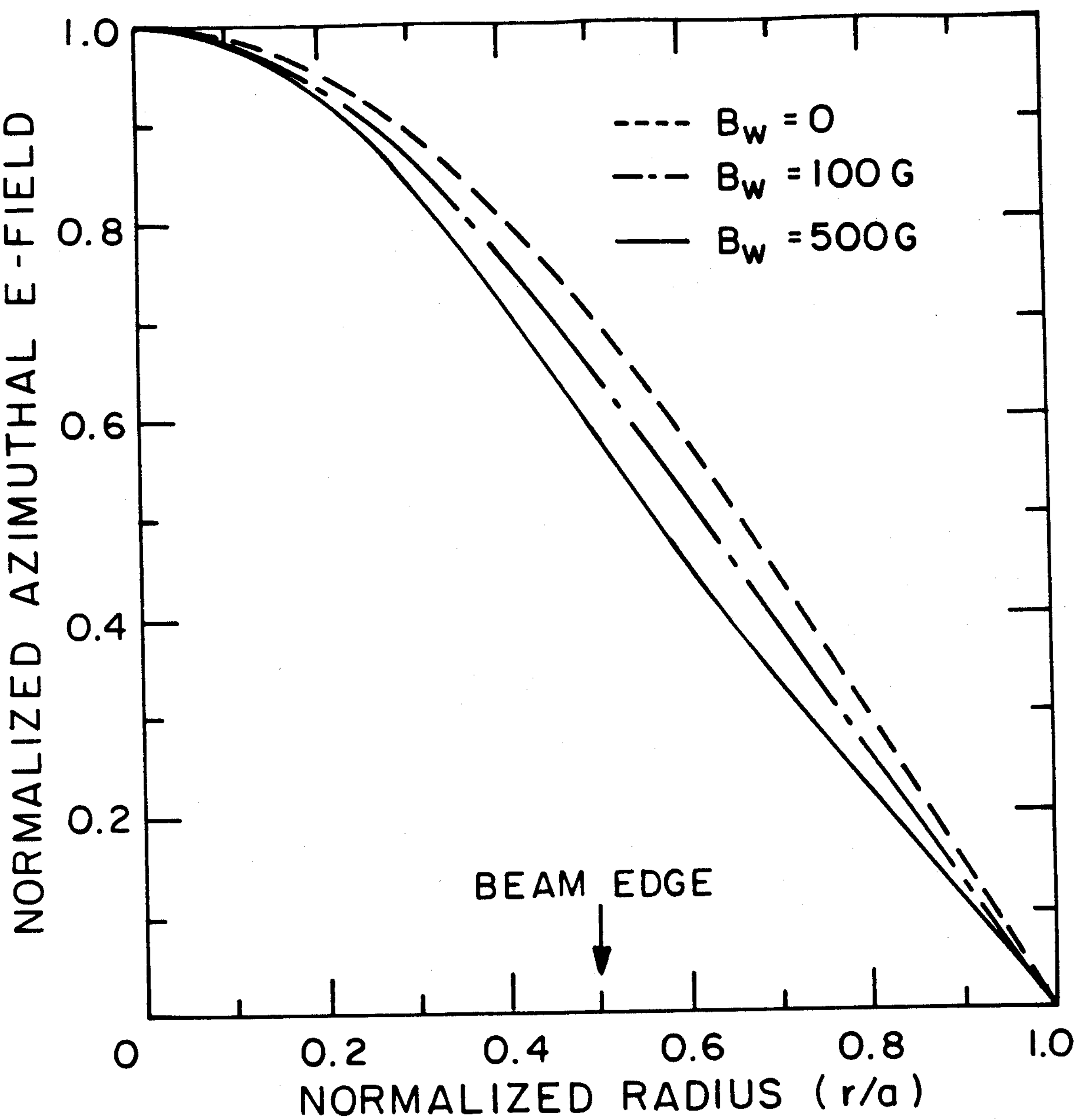
[14] H.P Freund and P. Sprangle, Phys. Rev. **A28**, 1835 (1985).

Figure Captions

Fig. 1 Normalized azimuthal E-field component of the eigenmode as a function of the normalized radius for $B_w = 0$, 100 and 500 G. The beam radius is $r_b = 5$ mm.

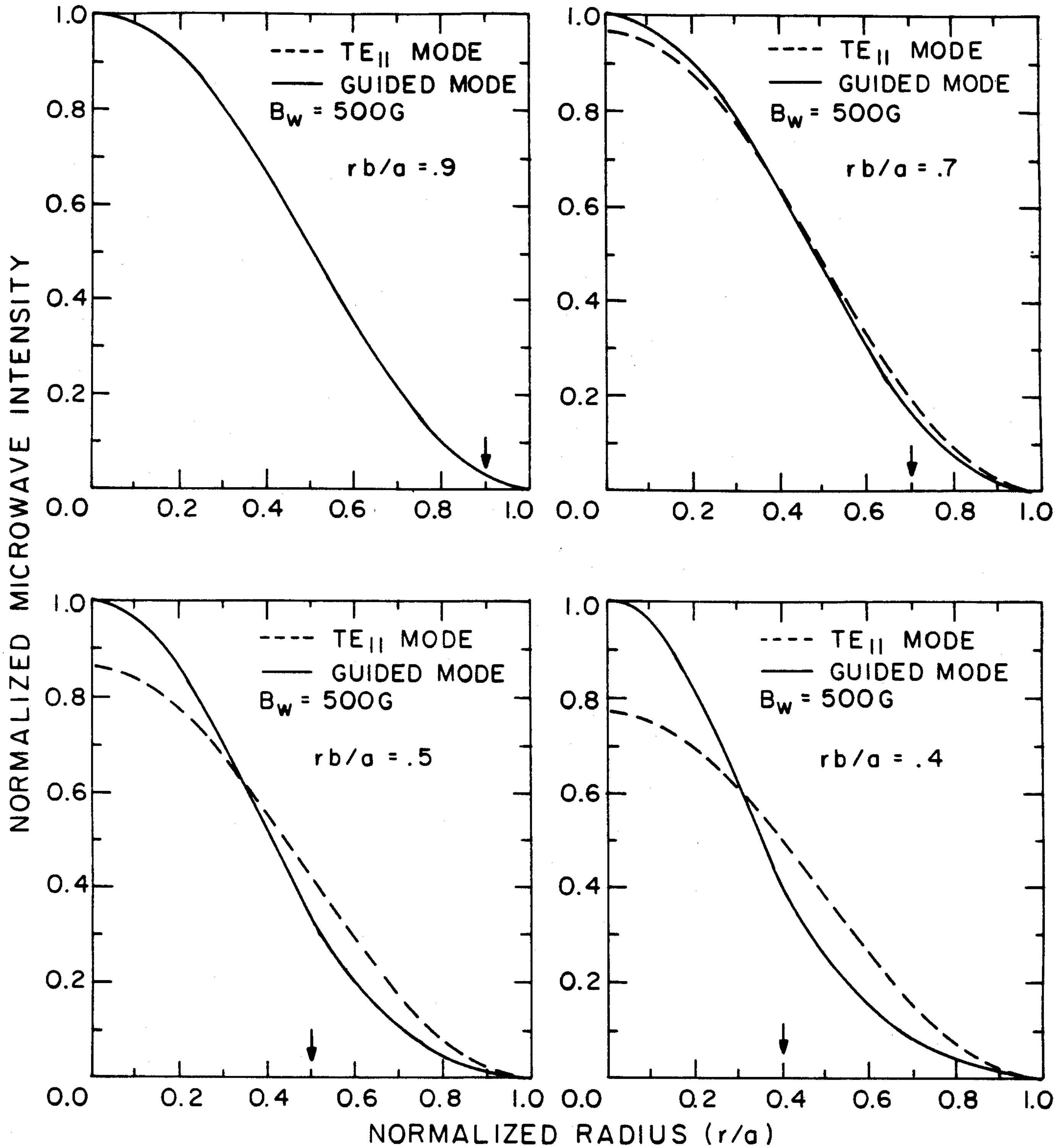
Fig. 2 Normalized microwave intensity profile for the eigenmode (solid line) and the TE_{11} mode (dashed line) for different beam radii. The wiggler field $B_w = 500$ G. The beam edges are marked by the arrows.

Fig. 3 Ratio of the imaginary part of the eigenvalue k to the TE_{11} gain as a function of the normalized beam radius, for different values of B_w.



NORMALIZED AZIMUTHAL E-FIELD
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
NORMALIZED RADIUS (r/a)
$B_W = 0$
$B_W = 100$ G
$B_W = 500$ G
BEAM EDGE

Fig. 1



TE11 MODE
GUIDED MODE
B_W = 500G
rb/a = .9
rb/a = .7
rb/a = .5
rb/a = .4
NORMALIZED MICROWAVE INTENSITY
NORMALIZED RADIUS (r/a)

Fig. 2

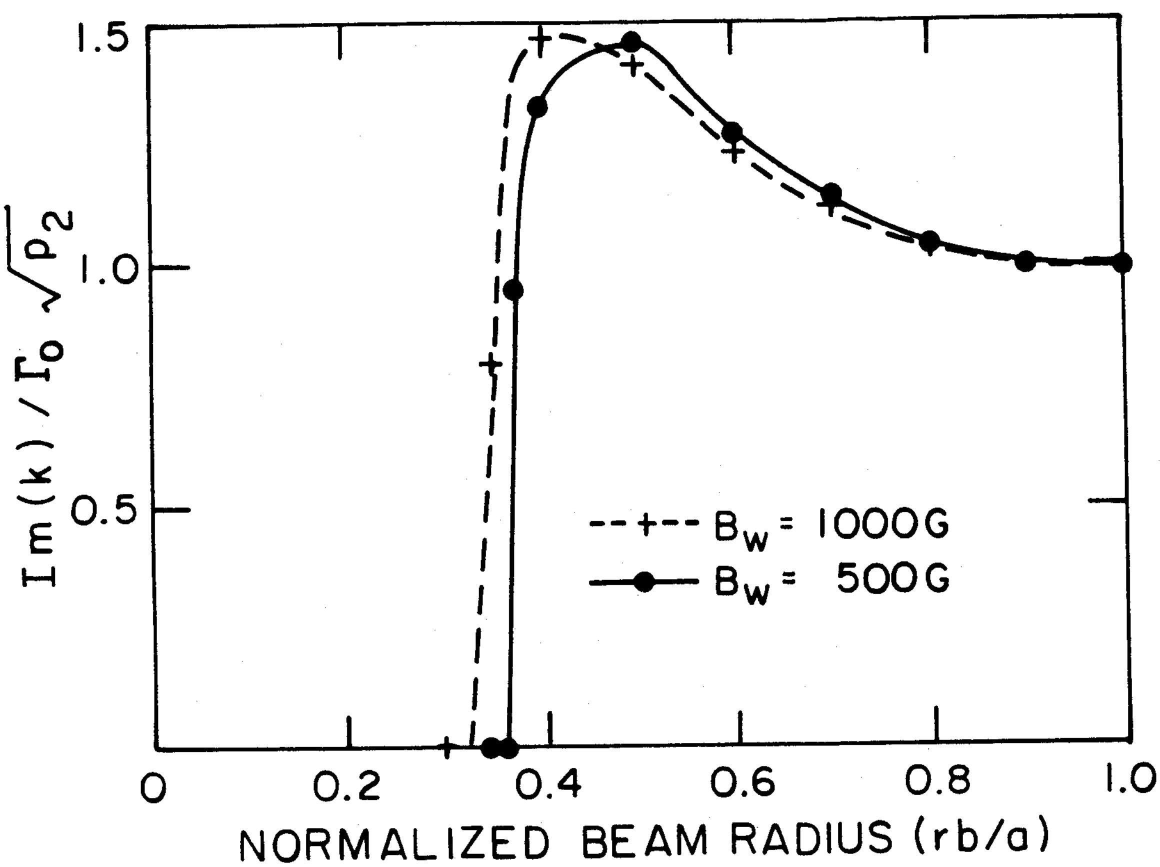

1.5
1.0
0.5
Im(k) / Γo √p2
0
0.2
0.4
0.6
0.8
1.0
NORMALIZED BEAM RADIUS (rb/a)
B_W = 1000G
B_W = 500G

Fig. 3