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Spatiotemporal Chaos**

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The Three Wave Interaction and Spatiotemporal Chaos

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Abstract

A tutorial account of spatiotemporal chaos (STC) in the nonlinear three wave interaction (3WI) is presented. The concept of STC is discussed and the 3WI is used as a paradigm for STC. Previous results of the 3WI, including time only solutions, low dimensional chaos, spacetime parametric interactions, solitons and the inverse scattering transform will be reviewed. These results will then provide the foundation to understanding STC in the 3WI.

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1 Introduction

In the recent past the area of nonlinear dynamics has witnessed two major discoveries – dynamical chaos and the inverse scattering transform (IST). Chaos is customarily defined to mean randomness in deterministic systems due to extreme sensitivity of initial conditions; considerable attention has been given to mostly low dimensional or few degree of freedoms dynamical systems [1, 2]. Although the ground work for chaos was laid out by Poincaré, it was the advent of computer simulations that led to the explosion of results and excitement in the last decade. The IST is a method to integrate certain special nonlinear partial differential and difference equations. These equations are usually associated with exhibiting solitons, nonlinear structures that preserve their form and collide elastically. Soliton solutions can be explicitly calculated with IST. The IST was first used by Gardner, Greene, Kruskal and Miura [3] to solve the Korteweg-deVries equation. It was later

discovered that their method was applicable to other PDE's [4, 5] and in the ensuing years the growth of the field has been prodigious.

With these two discoveries some natural questions come to mind: (1) What happens when more dimensions or degrees of freedom are added to a low dimensional chaotic system? (2) What happens when a spatially extended system, integrable by IST is perturbed slightly to break the integrability? Both of these questions will be addressed in this paper. The 3WI is a system that can be used to answer these questions. The conservative form of the 3WI in space and time is integrable by IST, and the nonconservative form in time only (spatially uniform) is chaotic. The dynamics of a non-conservative, nonintegrable spatially extended form of the 3WI was recently studied by us. The result is what has come to be called spatiotemporal chaos (STC). The term STC specifically refers to the chaotic evolution of patterns or coherent structures at a specific length scale in a spatially extended system [6, 7, 8, 9]. This is in contrast to fully developed turbulence where there is a cascade to smaller scales. The study of turbulence has been around for many decades and a variety of approaches have had only limited success in its description. It was once thought that chaos may have finally provided an answer. It is now believed that this is not the case [10]. Turbulent flow is full of coherent structures at all scales and is much more complicated than low dimensional chaotic dynamics can address. STC lies in a regime between the two extremes and is an interesting dynamical state in its own right.

The paper is organized as follows. The 3WI is introduced in section 2 and discussed in some detail. This is followed by the spatially uniform or time only dynamics in section 3; both the integrable and chaotic situations are reviewed. Section 4 is devoted to the spatiotemporal 3WI; the linearized parametric instability and the IST solutions of the nonlinear equations are reviewed. Finally in section 5, a discussion of STC in general and its manifestation in the 3WI is presented.

2 The Nonlinear Three Wave Interaction

The nonlinear 3WI appears in many contexts within the fields of plasma physics, nonlinear optics and hydrodynamics. Ref. [11] provides an excellent review of its applications. It can occur whenever: (1) a weakly nonlinear

medium supports a set of discrete waves

$$\omega_l = \omega_l(\mathbf{k}). \quad (1)$$

(2) The nonlinearity is manifested as a coupling of slowly varying linear field amplitudes, (3) The lowest order nonlinearity is quadratic in the field amplitudes, and (4) the three coupled waves satisfy the resonance conditions

$$\omega_i = \omega_j + \omega_k \quad (2)$$

$$\mathbf{k}_i = \mathbf{k}_j + \mathbf{k}_k. \quad (3)$$

These last two conditions are akin to conservation of energy and momentum. If these conditions are satisfied and the nonlinear coupling is conservative then a slowly varying amplitude or wave packet expansion will yield the conservative and integrable nonlinear 3WI [12, 13]

$$(\partial_t + \mathbf{v}_i \cdot \nabla) a_i = -K a_j a_k \quad (4)$$

$$(\partial_t + \mathbf{v}_j \cdot \nabla) a_j = K^* a_i a_k^* \quad (5)$$

$$(\partial_t + \mathbf{v}_k \cdot \nabla) a_k = K^* a_i a_j^*, \quad (6)$$

where the a 's are the slowly varying complex wave envelopes, K is a coupling coefficient and the \mathbf{v} 's are the group velocities. The above form of the 3WI is integrable by IST. Wave a_i will be referred to as the parent and the other two waves are the daughters. The solution will be discussed in section 4.

In many physical situations there will not be exact integrability. For instance the linear waves may have some growth or damping associated with them

$$\omega_l = \omega_l(\mathbf{k}_l) + i\gamma_l(\mathbf{k}_l) \quad (7)$$

There may also be the situation where the resonance is not exact so that

$$\omega_j + \omega_k - \omega_i = \delta \neq 0. \quad (8)$$

This results in a dephased interaction. From these considerations a nonconservative form of the 3WI can be derived [12, 14, 15]

$$\partial_t a_i + \mathbf{v}_i \cdot \nabla a_i = -K a_j a_k \exp(-i\delta t) + \gamma_i a_i + D \nabla^2 a_i \quad (9)$$

$$\partial_t a_j + \mathbf{v}_j \cdot \nabla a_j = K^* a_i a_k^* \exp(i\delta t) - \gamma_j a_j \quad (10)$$

$$\partial_t a_k + \mathbf{v}_k \cdot \nabla a_k = K^* a_i a_j^* \exp(i\delta t) - \gamma_k a_k, \quad (11)$$

Here the γ 's are growth or dissipation coefficients and D is a diffusion coefficient. This last term in Eq. (9) is usually not included in the 3WI; as will be detailed in section 4 its presence is essential for nonlinear saturation and the long time behavior of the equations. It arises if the growth of the parent is assumed to have a slow spatial variation. It then gives the simplest reflection invariant cutoff in wavenumber of the growth.

These equations ignore wave particle 'quasilinear' interactions whose lowest order effect is also second order in the field amplitudes. The particles are assumed to be nonresonant with the waves. In a generalized amplitude expansion the 3WI is the lowest order nonlinear effect and so the 3WI will dominate other nonlinear effects if the resonance conditions (2) and (3) can be satisfied. For instance the celebrated nonlinear Schrödinger equation would come in at a higher order. Only one spatial dimension will be considered in the paper.

3 Time Only Evolution

The 3WI in time only has the form

$$\dot{a}_i = \gamma_i a_i - K a_j a_k e^{-i\delta t} \quad (12)$$

$$\dot{a}_j = -\gamma_j a_j + K^* a_i a_k^* e^{i\delta t} \quad (13)$$

$$\dot{a}_k = -\gamma_k a_k + K^* a_i a_j^* e^{i\delta t} \quad (14)$$

The conservative, resonant interactions ($\gamma_l = 0, \delta = 0$) are easily solved in terms of Jacobi elliptic functions [17, 18, 16]. The solutions are oscillatory with a period

$$T \simeq \frac{1}{2K|a_i(0)|} \ln \frac{|a_i(0)|}{|a_j(0)|}. \quad (15)$$

With the addition of the nonconservative terms closed form solutions do not exist. However, we can consider an initial situation where the daughter wave amplitudes are small. Then the equations can be linearized. This is known as a *parametric interaction*. Linearizing (12) yields

$$a_i(t) = a_i(0)e^{\gamma_i t} \quad (16)$$

The other two equations become

$$\dot{a}_j + \gamma_j a_j = K^* a_i(t) a_k^* \quad (17)$$

$$\dot{a}_k^* + \gamma_k a_k^* = K a_i(t)^* a_j \quad (18)$$

Now assume that $a_i(t)$ is very slowly varying and substitute in the following

$$a_j = A_j e^{pt}, \quad a_k^* = A_k^* e^{pt}, \quad \gamma = |K a_i(t)| \quad (19)$$

This then yields the dispersion relation

$$(p + \gamma_j)(p + \gamma_k) - \gamma^2 = 0. \quad (20)$$

The threshold for instability is given by $\gamma^2 > \gamma_j \gamma_k$. So for a slowly growing high-frequency mode a_i there will always be a parametric interaction instability. The question is what happens nonlinearly.

This question was studied in references [19, 20, 21]. It was shown that there is no nonlinear saturation unless the interaction is off resonance (i.e. $\delta \neq 0$). In those works the damping rates of the two daughters were chosen equal. The essential parameters governing the dynamics were the dephasing δ and the ratio of the dissipation to the growth γ_j/γ_i . Depending on the values of these parameters they observed regions of no saturation, stable equilibrium, period doubling route to chaos, and intermittency.

4 Spacetime Evolution

The spacetime conservative 3WI is integrable by Inverse Scattering Transforms (IST) [11, 22, 23, 24]. Ref. [11] provides a complete review of the solution. IST is a transform technique to solve certain classes of nonlinear partial differential equations, and difference equations. Other equations integrable by IST include the Korteweg-de Vries, sine-Gordon, and nonlinear Schrödinger equations. One notable feature of IST theory is the ability to explicitly calculate soliton solutions.

In the 3WI the order of the group velocities gives different behavior. The case where parent wave has the middle group velocity is known as the Soliton Decay Instability. The IST solutions for the conservative case on the infinite domain show that solitons exist but they do not necessarily belong uniquely to a particular envelope [11, 22, 24]. Solitons in the parent wave tend to deplete to solitons in the daughters which propagate away. The simplest soliton solution for decay shows that a soliton of the form $|a_i| = 2\eta \text{sech} 2\eta x$,

will decay into solitons in the daughters of the form $|a_j| = \sqrt{2}\eta \text{sech}\eta(x + v_j t)$, where η is the IST spectral parameter for the Zakharov-Manakov [23] scattering problem. The spectral parameter is also the eigenvalue for a bound state in the Zakharov-Shabat [5] scattering problem with the parent pulse as the potential function. In the WKB limit η is related to the area of the parent pulse through the Bohr quantization condition [11, 14, 5]

$$\int_a^b |a_i^2 - \eta^2|^{1/2} dx = \pi/2, \quad (21)$$

where $[a, b]$ are turning points for a local pulse. A collision between a daughter pulse and a parent soliton is necessary to induce the decay of the parent [11, 24]. For arbitrary shaped parent pulses that exceed the area threshold, the soliton content will be transferred to the daughters leaving the radiation behind. Collisions between daughter solitons are elastic. The depletion of the parent into solitons is the nonlinear saturation of an absolute instability. The case where the parent wave has the highest (or lowest) group velocity is known as Stimulated Back Scatter. In this case the daughters can possess solitons but they are not transferred between the envelopes.

The inclusion of growth and dissipation breaks the integrability of the 3WI just as it did in the time only case. However, the linearized parametric instability where the daughters are initially small can be studied as in the time only case [12]. Transform to the frame of the parent wave ($v_i = 0$) and consider $a_i(t) = \text{const} \gg a_j, a_k$ so eq's (10, 11) can be linearized [25]

$$(\partial_t + v_j \partial_x - \gamma_j) a_j = \gamma a_k^* \quad (22)$$

$$(\partial_t + v_k \partial_x - \gamma_k) a_k = \gamma a_j^*, \quad (23)$$

where $\gamma = |K^* a_i|$ and only one spatial dimension is considered. The dispersion relation is simply

$$D = (\omega - kv_j + i\gamma_j)(\omega - kv_k + i\gamma_k) + \gamma^2 = 0. \quad (24)$$

There is a threshold of instability

$$\gamma^2 > \gamma_j \gamma_k \equiv \gamma_c^2. \quad (25)$$

For $v_j v_k > 0$ this is a convective instability. For $v_j v_k < 0$ (parent has middle group velocity) there is an additional threshold for an absolute instability

[25]

$$\frac{\gamma}{|v_j v_k|^{1/2}} \equiv \alpha > \frac{1}{2}(|\alpha_j| + |\alpha_k|) \quad (26)$$

where $\alpha = \gamma_l/v_l$.

If the parent is spatially varying then one must solve a boundary value problem. For $v_j v_k < 0$ the condition for an absolute instability is the existence of a growing normal mode. The WKB condition is given by [25]

$$\left(n - \frac{1}{2}\right) \pi \leq \int_a^b [\alpha^2 - (|\alpha_j| + |\alpha_k|)^2/4]^{1/2} dx \leq \left(n + \frac{1}{2}\right) \pi \quad (27)$$

where a and b are turning points. Notice that for no damping this condition is identical to the condition for soliton possession.

5 Spatiotemporal Chaos

5.1 Definition

The term spatiotemporal chaos has acquired a more specific meaning than simply chaos in space and time. Although there is no official definition, STC has come to refer to the chaotic behavior of coherent structures or patterns. This is in contrast to the more familiar low dimensional chaos and fully developed turbulence. The distinction can be made on the basis of length scales. Following Hohenberg and Shraiman [6], for any chaotic dynamical system there exist certain length scales. There is: (a) the excitation length l_E , the length scale at which energy is put into the system; (b) the dissipation length l_D , the scale at which energy is dissipated; (c) the system size L ; and (d) the coherence length ξ . Systems where energy is created and destroyed at the same length scale, $L \sim l_E \sim l_D$, $\xi > L$, correspond to low dimensional chaos. The system is completely spatially correlated. On the other extreme, in fully developed turbulence energy is usually injected at some large scale and dissipated at a small length scale, $L > l_E \gg l_D$, and the so called inertial range lies between the length scales. Also in fully developed turbulence, coherent structures exist at all scale lengths and the correlation length is not well defined. However systems where energy is injected and dissipated at the same length scale and the correlation length is much smaller than the system size, $L > l_E \sim l_D$, $\xi \ll L$, corresponds to the regime of STC. In

STC there is no inertial range yet spatial degrees of freedom are very important. The clean separation of scales also allows a statistical description in that correlation functions are well defined.

5.2 STC in the 3WI

This finally leads us to STC in the 3WI. We studied the dynamics of the 3WI in one spatial dimension x and time t . For weakly growing and damped waves without dephasing the nonconservative form of the 3WI Eqs (9-11) is

$$\partial_t a_i - D \partial_{xx} a_i - \gamma_i a_i = -a_j a_k \quad (28)$$

$$\partial_t a_j - \partial_x a_j + \gamma_j a_j = a_i a_k^* \quad (29)$$

$$\partial_t a_k + \partial_x a_k + \gamma_k a_k = a_i a_j^*, \quad (30)$$

where the a 's are complex wave envelopes, the γ 's are growth or damping coefficients, and D is a diffusion coefficient. We have transformed to the frame of the parent wave and normalized the magnitude of the daughter group velocities to one. We will consider the case where the daughter waves have equal damping (i.e. $\gamma_j = \gamma_k$). The length and time can then be rescaled so that the damping coefficient is unity [26].

The group velocities satisfy the condition $v_k > v_i > v_j$ (i.e. the highest frequency parent wave has the middle group velocity, see [27]). In the absence of growth, damping and diffusion ($\gamma_i = D = 0$) the IST solutions for this group velocity ordering is described by soliton exchange between wavepackets [11, 22, 23, 24].

We numerically simulated the system on the domain $x \in [0, L)$ with periodic boundary conditions. We began with random real initial conditions and evolved until the transients died away before the system was analysed. It can be shown that for real valued initial conditions the envelopes remain real for all time [11, 15]. We were interested in the large system, long time limit. We considered the case with parameters $D = 0.001$, $\gamma_i = 0.1$, $\gamma_j = \gamma_k = 1$, and $L = 20$. These parameters were chosen because they exhibit STC and fall into a regime where perturbation theory is possible. However, the system is extremely rich and different parameters do lead to vastly different behaviour. Aspects of these different regimes will be touched upon later and details are

given in [15]. We measured the correlation function, $S_i(x, t) = \langle a_i(x - x', t - t') a_i(x', t') \rangle$, where the angled brackets denote time averages.

A sample of the spatiotemporal evolution profiles in the STC regime of the parent and daughter envelopes is given in Fig. 1. The length shown is one half the system size and $t = 0$ is an arbitrary time well after the transients have decayed. The profile of the parent wave is irregular but spatial and temporal scales can be observed. There are coherent structures of a definite length scale that can be seen to grow, deplete and collide with one another. The profile of the daughter wave shows a sea of structures convecting to the left. We only show one daughter, the other will be similar but with structures convecting to the right. The correlation functions for both the parent and the daughter waves are given in Fig. 2. The parent correlation function shows a gradual decay in time. Spatially, there is a definite length scale seen in Fig. 1. The daughter function is calculated along the characteristic. It has a fast decay followed by a slow decay in both time and space. The approach to zero in correlations in both space and time indicates STC. Figure 3 shows the spectrum of static fluctuations $S_i(t = 0, q)$. For the parent wave there is a cutoff near $q \simeq 10$ and a range of modes show up as a prominent hump. The cutoff reflects the length scale seen in the spacetime profile. For q below the hump the spectrum is flat. The daughter spectrum has a cutoff around $q \simeq 6$ again indicating a length scale. Figure 4 shows the local power spectrum $S_i(\omega, x = 0)$. The spectrum for the parent clearly shows two time scales. The spectrum bends over near $\omega \simeq 0.02$ which gives a long time scale and a shoulder at $\omega \simeq 0.3$ gives a short time scale. Longer runs with these parameters hint that there may be a very slow power law rise of undetermined exponent for frequencies below the low ω bend similar to that observed in the Kuramoto-Sivashinsky equation [6]. The short time scale appears as the growth and depletion cycle observed in the spatiotemporal profile. The daughter power spectrum has two peaks at high ω . One is where the shoulder of the parent spectrum is and the other is at twice this frequency. The spectrum begins to bend over and flatten out at $\omega \simeq 0.007$. This bend is more pronounced in longer runs. It is not known whether the spectrum becomes flat or has a power law rise like the parent for frequencies below the bend.

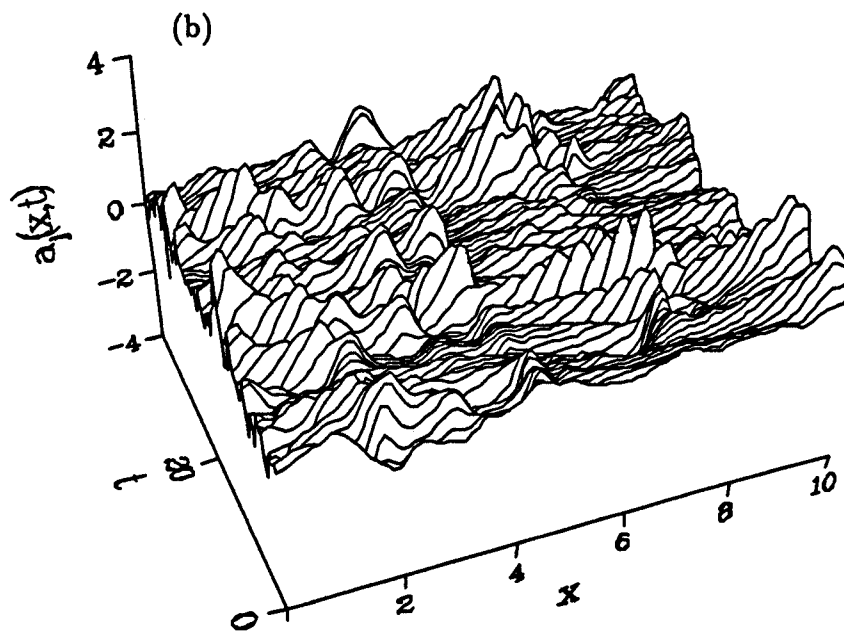
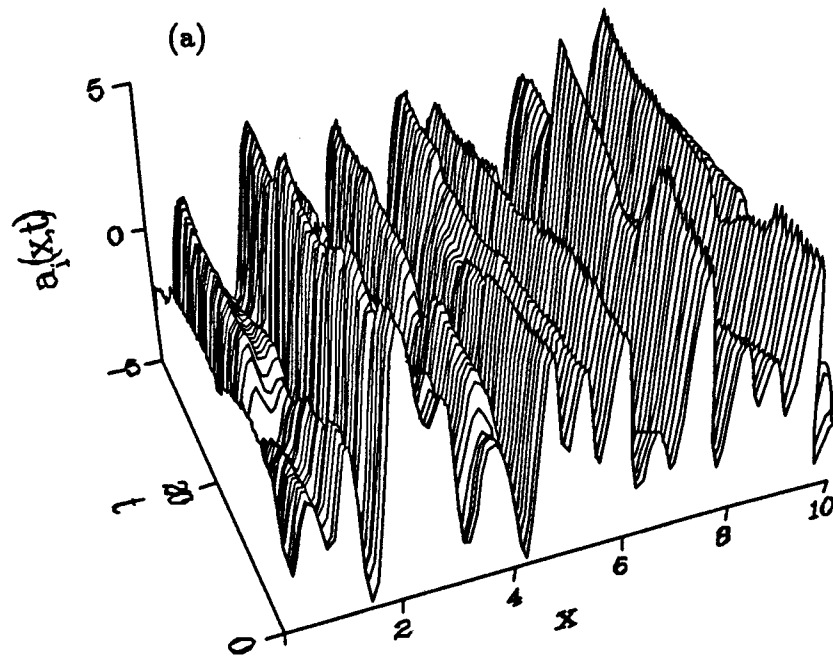


Figure 1: Spatiotemporal profiles of (a) the parent wave $a_i(x,t)$ and (b) the daughter wave $a_j(x,t)$.

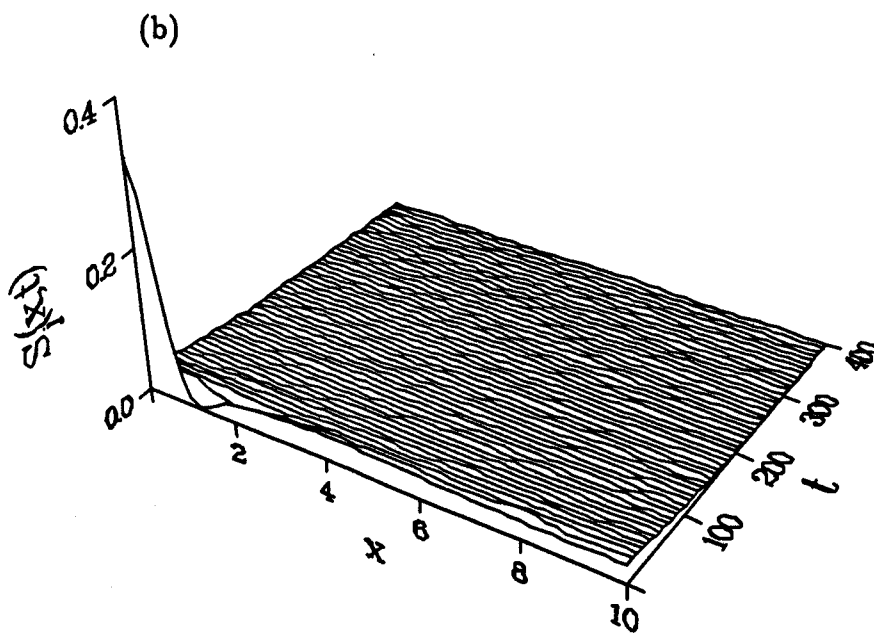
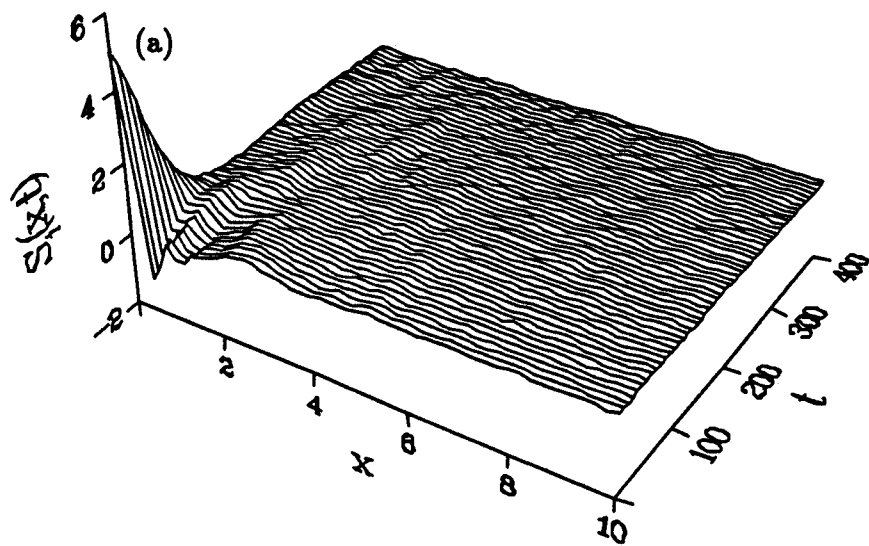


Figure 2: Correlation Function $S_i(x,t)$ of (a) the parent wave and (b) the daughter wave.

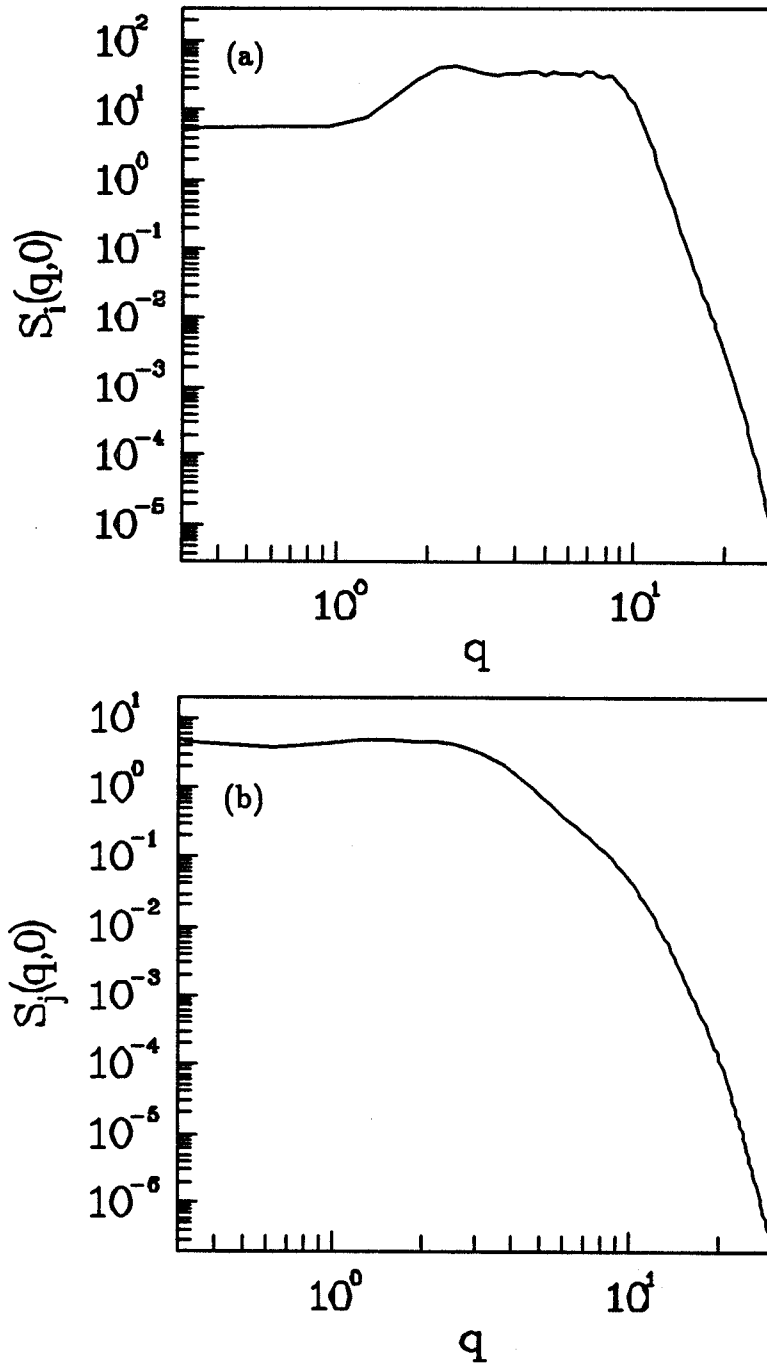


Figure 3: Spectrum of static fluctuations $S_i(q, t = 0)$ of (a) the parent wave and (b) the daughter wave.

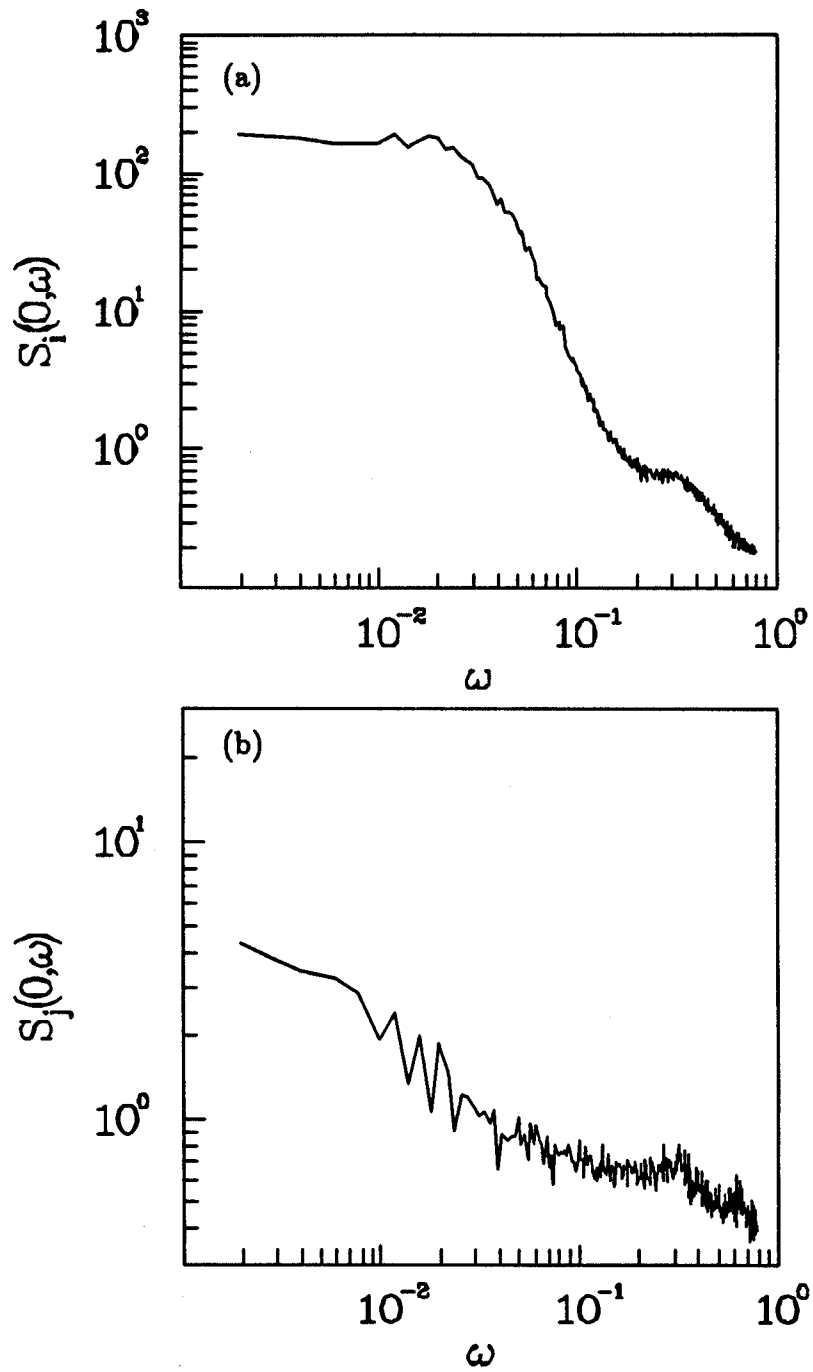


Figure 4: Local power spectrum $S_i(x = 0, f)$ of (a) the parent wave and (b) the daughter wave.

The main features of the behaviour can be understood if we consider the growth and dissipation as perturbations about the conservative 3WI, as discussed in Sec. 4. With the addition of weak growth and dissipation, parent pulses deplete provided they satisfy the WKB threshold condition (27). In the normalization of Eqs. (28-30) this condition is [14, 25]

$$\int_a^b |a_i^2 - \gamma_j^2|^{1/2} dx > \pi/2. \quad (31)$$

The decay products in the daughters are quasi-solitons; they damp as they propagate away and do not collide elastically. The soliton content of the parent is not completely transferred to the daughters. The parent wave with some initial local eigenvalue η will deplete and be left with some remaining area. This area is due to the conversion of soliton content into radiation by the perturbations. This left over area can be represented by an effective 'eigenvalue' η' . This remaining part of the parent will then grow until it exceeds the threshold for decay. This time denoted by t_c is given by

$$t_c \simeq \frac{1}{\gamma_i} \ln \frac{\eta}{\eta'}. \quad (32)$$

The cycling time observed in the spacetime profiles is this time plus the time required to deplete. The depletion time from IST theory is on the order $1/2\eta$ and for $\gamma_i \ll 2\eta$ this can be neglected and t_c gives the cycling time. By treating the damping and growth as a slow time scale perturbation of the IST soliton decay solution described above and ignoring the effects of diffusion on this short time scale, a multiple-time scale perturbation analysis about the IST soliton solution was used to estimate η' . In this calculation the ordering $\gamma_i \ll \gamma_j \ll 2\eta$ was chosen. The small parameter is $\gamma_j/2\eta$ but by simply rescaling in time and space either γ_j or η can be scaled to $O(1)$. To leading order this yields [15]

$$\eta' \simeq \gamma_j. \quad (33)$$

The derivation assumes that the decay time for a soliton is very much faster than the growth and damping time. Simulations for parent soliton initial conditions verify Eq. (33) [15]. In order to complete the calculation for the cycling time t_c it is necessary to estimate the threshold local η required for decay. By comparing the Bohr quantization condition (21) with the WKB

condition for decay with damping (31) we know that $\eta > \gamma_j$. Using the IST scattering space perturbation theory developed by Kaup [11, 28, 29] and recently reviewed in Ref. [30], we constructed the time dependence of the IST scattering data due to the perturbation. The same ordering as the multiple scale calculation was chosen. From this we were able to estimate η to leading order to be [15]

$$\eta \simeq 2\gamma_j + 4\xi_p\gamma_i, \quad (34)$$

where ξ_p is the parent correlation length and will be defined later. Equation (34) is sensitive to the amplitudes of the colliding daughter amplitude that induces the decay. The calculation assumes the decay is induced by collisions with quasi-solitons with the same phase from each daughter generated two correlation lengths away. The relative phases of the colliding daughters is very important. Consider real amplitudes for the moment, Eq. (28) shows that two daughter quasi-solitons with opposite signs (phase) actually reinforce the parent rather than make it deplete. Thus expression (34) should be considered more of a lower bound. In the simulation, radiation and diffusive effects will be relevant and may also further delay the decay of the parent. From η we are able to estimate the daughter correlation length. This is given by the quasi-soliton width $\xi_d \simeq 2/\eta$.

The long time behaviour is governed by the diffusion. The trivial fixed point of Eqs. (28)-(30) is given by

$$\partial_{xx}a_i + q_0^2a_i = 0, \quad a_j = a_k = 0, \quad (35)$$

where $q_0 = \sqrt{\gamma_i/D}$. Modes with $q > q_0$ will damp and those with $q < q_0$ will grow. Thus the fixed point is always unstable to long wave length fluctuations. However, when a local area between two turning points of the parent wave contains a bound state with eigenvalue η it will deplete. In the depletion process broad parent pulses will be decimated. The growth in the $q < q_0$ modes are thus saturated nonlinearly. This results in long wavelength distortions beyond lengths $2\pi/q_0$. The principal mode q_0 was observed as the cutoff in the spectrum of static fluctuations (fig. 3a). The mode q_0 defines the correlation length for the parent, $\xi_p \simeq 2\pi/q_0$. If $D = 0$ there will not be any nonlinear saturation of the instability because q_0 would become infinite and so would the amplitude required to fulfill the area threshold (31).

The long time scale for the parent τ_p is given by the diffusion time across a length ξ_p giving $\tau_p \simeq (2\pi)^2/\gamma_i$. This is the time scale in which the local parent structures will shift position, collide with other structures or diffuse away. The long correlation time observed in the daughters is associated with the interaction of the daughter quasi-solitons with the parent structures. Whenever quasi-solitons collide with the parent structures they may induce a decay and create a new quasi-soliton where the collision occurred. This would lead to a long correlation time for the daughters. As the parent structures drift so would the creation location of new quasi-solitons. However because the quasi-solitons have a different width than the parent structures, the long time scale for the daughters would be given by the diffusion time across a quasi-soliton width yielding $\tau_d \simeq 4/(\eta^2 D)$. The newly created quasi-soliton damps while it continues to propagate along the characteristic. However when it collides with another parent structure it could induce a decay and repeat the process. The parent structures act as amplifiers regenerating damped quasi-solitons that collide with them.

Using the above analysis for the parameters of the simulation we obtain the following estimates: $\tau_p \simeq 400$, $q_0 = 10$, $\xi_p \simeq 0.6$, $\eta' \simeq 1$, $\eta \simeq 2.2$, $t_c \simeq 8$, $\xi_d \simeq 0.9$, $\tau_d \simeq 800$. These estimates corroborate fairly well with the simulation. The estimate for t_c is a bit low compared to the shoulder in the parent power spectra at $\omega \sim 0.3$ corresponding to $t \simeq 20$. However the spacetime profiles in Fig. 1 do show some of the parent structures cycling near the predicted time scale, so the calculation does predict a lower bound.

A word should be said about the system size. It is clear with the very long correlation times for the daughters that they cycle the box many times before correlations decay away. Thus for long times, the temporal correlation function along the characteristic or at a single spatial location would be the same. This was borne out in the simulation. It is unknown what the precise boundary effects are since it would be impossible to numerically test a system large compared to this long time scale. However with other runs of varying length, it was found that the above time scales seem to be unaffected by the box size as long as the box is much larger than ξ_p . The power law rise for the parent power spectrum below $2\pi/\tau_p$, seems to decrease in exponent as the system increases.

We chose parameters where perturbation theory about the IST solutions could be applied to try to understand the dynamics. However the behavior does dramatically change for different parameter regimes [15]. For strong growth rates, the long time scales observed tend to disappear and only the growth and depletion cycling time is evident. The parent grows strongly and depletes violently preventing the structures to become established. The larger the growth rate the larger the amplitudes of the quasi-solitons [15]. Another regime is when the diffusion is large so the parent structures are much broader than the damping length of the daughters. In this situation the daughters grow and damp within the confines of a parent pulse. Spatial exchange of information between these pulses is very slow. These and other regimes are reported in ref. [15]. It is quite clear that the 3WI in spacetime is an extremely rich system. For weak growth and dissipation, it exhibits STC and perturbation theory is able to estimate the length and time scales.

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References

- [1] P. Bergé, Y. Pomeau, C. Vidal, *Order within Chaos*, (Hermann, Paris, 1984).
- [2] R.L. Devaney, *An Introduction to Chaotic Dynamical Systems*, (Addison-Wesley, Reading, Massachusetts, 1989).
- [3] C.S. Gardner, J.M. Greene, M.D. Kruskal and R.M. Miura, *Phys. Rev. Lett.* **19**, 1095 (1967).
- [4] M.J. Ablowitz, D.J. Kaup, A.C. Newell and H. Segur, *Stud. Appl. Math.* **53**, 249 (1974).
- [5] V.E. Zakharov and A.B. Shabat, *Zh. Eksp. Teor. Fiz.* **61**, 118 (1971) [*Sov. Phys. – JETP* **34**,62 (1971)]
- [6] P.C. Hohenberg and B.I. Shraiman, *Physica D* **37**, 109 (1989) and references therein.

- [7] P. Couillet, C. Elphick, and D. Repaux, *Phys. Rev. Lett.* **58**, 431 (1987).
- [8] F.T. Arrechi, G. Giacomelli, P.L. Ramazza, and S. Residori, *Phys. Rev. Lett.* **65**, 2531 (1990).
- [9] S. Ciliberto and M. Caponeri, *Phys. Rev. Lett.* **64**, 2775 (1990).
- [10] R. Kraichnan, *Physica D* **37**, 160 (1989).
- [11] D.J. Kaup, A. Reiman, and A. Bers, *Rev. Mod. Phys.* **51**, 915 (1979) and references therein.
- [12] A. Bers in *Plasma Physics-Les Houches 1972*, edited by C. De Witt and J. Peyraud (Gordon and Breach, New York, 1975).
- [13] D.J. Benney and A.C. Newell, *J. Math. Phys.* **46**, 133 (1967).
- [14] C.C Chow, A. Bers, A.K. Ram, to appear in *Research Trends in Chaotic Dynamis and Transport in Fluids and Plasmas* conf. proc., La-Jolla, CA, AIP 1991.
- [15] Carson C. Chow, Ph.D. Thesis, M.I.T., 1991.
- [16] N. Bloembergen, *Nonlinear Optics*, (Bengamin, New York, 1965).
- [17] R.Z. Sagdeev and A.A. Galeev, *Nonlinear Plasma Theory*, (Benjamin, New York, 1969).
- [18] R.C. Davidson, *Methods in Nonlinear Plasma Theory*, (Academic, New York, 1972).
- [19] S.Y. Vyshkind and M.I. Rabinovich, *Zh. Eksp. Teor. Fiz* **71**, 557 (1976) [*Sov. Phys. - JETP* **44**, 292 (1976)].
- [20] J. Wersinger, J. Finn, and E. Ott, *Phys. Fluids* **23**, 1142 (1980).
- [21] C. Meunier, M. Bussac and G. Laval, *Physica* **4D**, 236 (1982).
- [22] D.J. Kaup, *Stud. Appl. Math.* **55**, 9 (1976).
- [23] V.E. Zakharov and S.V. Manakov, *Zh. Eksp. Teor. Fiz. Pis'ma Red.* **18**, 413 (1973) [*Sov. Phys.-JETP Lett.* **18**,243 (1973)].

- [24] A. Bers, D.J Kaup, and A.H. Reiman, *Phys. Rev. Lett.* **37**, 182 (1976).
- [25] A. Bers in *Handbook of Plasma Physics*, edited by M.N. Rosenbluth and R.Z. Sagdeev (North Holland, Amsterdam, 1983), Vol. 1, Chap. 3.2.
- [26] An equivalent set of 3WI equations can be written in two spatial dimensions (e.g. x and y) for nonlinear interactions in the steady state [11]; the equations are of the same form as (28)-(30) where t is y and, in each equation, all other terms are divided by the y -component of the group velocity of the wave. Thus the solutions we describe (x, t) apply also to (x, y) with appropriate boundary conditions.
- [27] In the two-dimensional steady-state, see [26] above, this condition involves only the ratios of group velocity components.
- [28] D.J. Kaup, *SIAM J. Appl. Math.* **31**, 121 (1976).
- [29] D.J. Kaup and A.C. Newell, *Proc R. Soc. Lond. A.* **361**, 413 (1978).
- [30] Y.S. Kivshar and B. A. Malomed, *Rev. Mod. Phys.* **61**, 763 (1989).