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A DOUBLE PENDULUM PRECISION THRUST MEASUREMENT BALANCE

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Abstract

For the purpose of measuring the plasma momentum flux in a plasma system, a highly sensitive and precision balance has been developed. It can measure a force, an impulse or thrust as low as 0.1 mN (milli-Newton) free of mechanical noise, electrical and magnetic pick-ups. The Double Pendulum system consists of two parallel conducting plates. One or both of the plates can be suspended by needles. The needle suspended plate (or plates) can swing freely with negligible friction because of the sharp points of the needles. When one of the plates is impacted by an impulse it will swing relative to the fixed plate or other movable plate. The capacitance between the plates changes as a result of such a motion. The change of capacitance as a function of time is recorded as an oscillating voltage signal. The amplitude of such a voltage signal is proportional to the impacting force or impulse. The proportional factor can be calibrated. Therefore the forces can be read out from the recorded value of the voltage. The equation of motion for the pendulum system has been solved analytically. The circuit equation for the electronic measurement system has been formulated and solved numerically. The thrust at the exhaust of a Tandem Mirror plasma thruster has been measured. The analytical solution of the overall characteristics agrees greatly with the measurement.

I. Introduction

To make an accurate measurement of the momentum flux in a plasma system, such as the so called thrust in a plasma propulsion device or the plasma momentum flux into the divertor from the scrape layer of a tokamak fusion device, it is needed to develop a sensitive target balance. In electric propulsion research the common method to measure the thrust is a balance consisting of the suspended thruster and a displacement sensor [1] The thrust is determined from the displacement of the thruster when it recoils at the firing of the thruster. This method has several drawbacks: The thruster is heavy. It is tied to heavy electric power cables and gas feed line and therefore there is a very high motion resistance. There are frictional losses due to the suspension of the heavy thruster and due the transferring of displacement through pulleys to the sensor. In the future, the header will probably need to be cooled for high power operation. The motion resistance will be much higher with added cooling lines and heavier power cables. These factors will reduce the sensitivity and accuracy. This method is applicable to an open ended and relatively light weight and compact system only. For a closed, heavy and bulky system like tokamak, a target balance is needed.

There are two known target methods: One is to attach the target through a long shaft to the diaphragm of a commercial pressure transducer. The shaft is suspended by two strings [2]. The suspended target and the shaft system behaves like a pendulum and the pressure transducer serves as a displacement sensor. The other method is similar to the suspended target and shaft assembly whereas the sensor consists of a magnet and a pick-up coil. The force can be determined from the voltage induced on the pick-up coil from the motion of the magnet. The sensor unit has to be housed in a magnetically shielded box. The long arm of these two systems will make them difficult to be installed in the plasma stream in a closed system like a tokamak. Also it is obvious that the momentum flux that can be measured with these methods is unidirectional. Therefore they are not useful in the study of pressure balance at the plasma-gas boundary of a gas divertor in tokamaks. A new target system thus has to be developed that can meet this need as well.

None of the systems mentioned above has provided adequate means to deal with the problem of mechanical vibrations, and electric and magnetic interferences, particularly in the strong magnetic fields of tokamak systems.

The target balance system developed in this work consists of two identical plates suspended by needle points. The device is compact and light. It has the unique feature of being symmetric with respect to the equilibrium position as either or both plates are impinged by plasma from opposite directions. The system, therefore, can be inserted in the plasma stream in a closed system like a tokamak. The mechanical vibrations can be eliminated as a common mode to the two identical moving pendulums. The electric and magnetic pick-ups have been reduced to a negligible level by the use of differential amplifiers, shieldings, and dynamic bandpass filters. The detailed developmental process of the pendulum system, electronic circuit, the analysis and measurement are described in the following sections.

II. The Balance Development

The balance developmental work has been carried out on the Tandem Mirror Plasma propulsion device [3–5]. The experimental setup is shown in Figure 1. At present the device is operated in the low power level and at high specific impulse (about 13,000 sec). The thrust level is very low and initially was estimated to be about a few tens of mN. To measure such a low thrust the balance must have a sensitivity and accuracy of less than 1 mN. The balance has to be very compact so that it can be mounted in the limited space of the exhaust chamber.

The propulsion system is presently operated at pulsed mode. Plasma is created by microwave radiation and heated to high temperature by rf power, The magnetic impulse and mechanical vibrations, rf pick-ups, electric field, and electromagnetic noises from the power supply systems, are many orders of magnitude stronger than the anticipated thrust level. Undoubtedly they must be eliminated. At the beginning we were hoping to adapt the known methods. As described in the introduction none of them is adequate to our system and to the future application in tokamak divertors. The possibility of using torsion wire, or a strain gauge, has also been carefully examined and found not desirable either.

In order to keep the system as simple and compact as possible, we have given the simple pendulum a serious consideration. However, it loses simplicity and compactness when the position sensing system is incorporated. Any position sensing system almost has to be mounted inside the vacuum chamber, otherwise the vacuum penetration system and setup are elaborate and difficult. The sensing assembly has to be completely isolated mechanically, magnetically and electrically. Such a balance system can not be bidirectional as we are hoping to achieve.

To meet all our objectives the target and position sensor have to be one integrated unit. After weighing all the odds we decided on a system consisting of double plates as our best bet. We were encouraged by the quick success of a pendulum and fixed plate system as shown in Figure 2a. The octagonal shape is arbitrarily chosen for easy fabrication. Both the moving and fixed plates are identical in size and shape and are made of aluminium. The displacement of the pendulum is translated into variation of capacitance of the plates. The capacitance measuring method will be described later. The pendulum plate serves as the target as well as a sensor and the system is strikingly simple. The signals for the balance in the air and in the vacuum chamber are shown in Figure 3a and Figure 3b respectively. The oscillations in the air were produced by mechanical shock. The oscillation in the vacuum is produced by the pulsing of the magnetic field. As can be seen from these figures, the signal taken in the air damped quickly whereas there was almost no damping in the vacuum. This shows that the friction loss at the needle support is negligible. The magnitude of the magnetic impulse is very large and can be eliminated by dividing the plates into very thin strips.

It is within our expectation that the mechanical vibrations are the second order noise which appears when the magnetic impulse is removed. As shown in Figure 4 such a noise is random. The attempt to reduce this noise with mechanical suspension failed. It is worth to comment on the possibility of using of a gas charged optical table. The optical table can not be mounted inside the vacuum chamber and can not be used to isolate the entire system because of vacuum pumps and electrical and water connections. It is delightful to find that the mechanical noise can be eliminated by suspending both plates (Figure 2b) so that each plate swings as an independent pendulum. Since both plates respond identically to the noise, no net change in the capacitance is produced and therefore the noise signal can be eliminated as a common mode.

In the plasma stream, the conducting plate acts as a big Langmuir probe which will detect the electric field and current. These electric signals are the third order effects which appear when the vibrational noises are cancelled out. As shown in Figure 5, these electric signals are still very large. To reduce the electric field, insulating materials such as ceramic or silicon are used on the surface facing the plasma. Further, a set of plates with conducting strips are placed above the support. The electromagnetic noise from the inverse pendulums can then be used to cancel the noise picked up by the regular pendulums below the support. The inverse pendulum above the support swings opposite to the regular pendulum below so that the voltage signals from these two sets of capacitor plates are out of phase by 180, whereas the phases of pick-up voltages from electric and magnetic noise are not. By using a differential amplifier the pick-up signals are cancelled out and the real signals are summed. This system is shown in Figure 6.

Since the inverse pendulums are smaller than the regular ones, the noise can not be removed completely. To further avoid the electric field effect due to plasma, the plate in contact with the plasma is made entirely with an insulator, i.e., no conductor strips even on the back face. As depicted in Figure 7, the device is now a quadruple pendulums system where there is a set of double capacitor plates above the support and another set of double capacitor plates below the impact plates which are now made of insulators. The double capacitor plates consist of conductive metal strips used to monitor signals. These capacitor plates are enclosed in a conducting box to shield out the electric and magnetic pick-up. Any residue pick-up can be further eliminated with the use of differential amplifiers. This method effectively reduces the pick-up and enhances the sensitivity. The magnetic noise produced by the power supply was found to be the most stubborn to eliminate because it can transmit through all the cables. After exhaustive study of the characteristics of these noises we finally succeeded in eliminating them with the combination of shielding, wiring schemes and active bandpass filters.

III. The Signal Treatment

The simplified electronic circuit diagram is shown in Figure 8. To detect the small change in capacitance of the two pendulums a carrier radio frequency voltage at 30 kHz is applied to plates. There are two pairs of signals, one pair from the top capacitor plates and the other from the bottom capacitor plates. The signals from the each pair of plates are carried by a pair of twisted wires to the inputs of a differential amplifier. Since the inverse pendulum plates on the top swing in the opposite direction to the pendulum plates at the bottom, The signs of the signal from the output of the differential amplifiers are opposite. These outputs are again summed by the third differential amplifier. Therefore the overall signal is enhanced. The choice of the frequency of carrier signal is also very important. Generally the signal is larger at higher frequency, the choice of the frequency has to be made that all the noise frequencies fall outside the bandpass of the system and the gain of the amplifier has to be linear.

The carrier frequency is removed from the signal by rectifiers. The residual noise are practically cancelled by the differential amplifiers.

IV. Experimental

The two pendulums of the thrust balance are not exactly identical and, therefore, their natural frequencies are not equal. When a mechanical impulse is applied a beat as shown in Figure 9 is produced. The frequency of the beat is about one twentieth of the natural frequency or the period is 20 times longer. This beat frequency will become smaller when two natural frequencies are made to be closer. A great effort has been made to tune the two pendulums to make their natural frequencies match. However, perfect match can not be achieved and it can only make the beat period longer. The beat can not be eliminated completely. Currently the thruster is operated at pulsed mode and is fired in the first half cycle of the natural oscillation. The trace of the first cycle in Figure 9 is expanded as is shown in Figure 10. This expanded trace shows that the noise level is negligible. Figure 11 presents the relationship between the plasma density and temperature traces from a triple probe and the pendulum signal. This shot was made using the pendulum system shown in Figure 6 where the signal died out after one cycle. It should continue to oscillate without damping and therefore it may not be the valid signal. In order to determine the validity of the measurement it is necessary to carry out detailed analysis which will be given in the next section. An exhaustive study of the residual noise from the power supply reveals that they can sometime cause misleading false signal even at very low level. The complete elimination of such noise is necessary and was accomplished with the use of dynamic bandpass filters and shielding and wiring methods.

Finally Figure 12 (solid curve) presents the signal obtained with the working pendulum system shown in Figure 7 using the electronic circuit shown in Figure 8. The signal continued to oscillate without damping but was not symmetric with respect to the baseline. This requires solid explanation in order to convince ourself that it is in fact a correct signal. The amplitude of the signal rises from negative value to a constant symmetric oscillation in after about three cycles. This asymmetric property is found to be due to the electric circuit (dashed curve) as demonstrated by the circuit analysis given in the next section.

V. **Analytical Solutions**

As discussed above in order to validate the measurement it needs to carry out a complete analysis of the pendulum system and electronic circuit. Let us write down the equations of motion for a simple pendulum as shown in Figure 13.

Assuming θ small, the equation of motion is

$$\frac{mld^2\theta}{dt^2} = F_T - F_f - F_g, \tag{1}$$

where

$$\begin{aligned} F_T &= Thrust, \\ F_f &= A\frac{d\theta}{dt}, \\ F_g &= mg\theta, \\ A &= a \text{ damping factor.} \end{aligned}$$

Let

$$\begin{aligned} \tau &= \text{pulse length}, \\ T_o &= \sqrt{l/g}. \end{aligned}$$

It should be noted that the real system consists of two compound pendulums and should be solved as such. However, we are interested in the qualitative characteristics and choose the simple pendulum for the sake of simplicity. Also, the frictional force at the needle points is negligible; the system may be used in the plasma stream or gas, thus the viscous force might be significant; Therefore we include the viscosity term only. This equation has been solved for other systems. To understand the characteristics of this system the procedure for solving the equation of motion and results are presented in the following.

By apply Laplace Transformation to equation (1)

$$\Theta(S) = \frac{L[F_T]}{(S^2ml + SA + mg)}. \qquad (2)$$

Examine the solutions in two regimes:

1. $\tau << T_o$

$$F_T(t) = F_T\tau\delta(t), \; a\ \delta - function,$$

$$L[F_T] = F_T\tau.$$

2. $\tau >> T_o$

$$F_T = F_T \cdot 1(t), \; a\ step\ function,$$

$$L[F_T] = \frac{F_T}{S}.$$

In each regime there are four different conditions:

1 $A = 0$, no damping,

2 $0 < A < \sqrt{4m^2gl}$, under damping,

3 $A = \sqrt{4m^2gl}$, critical damping,

4 $A > \sqrt{4m^2gl}$, over damping.

At present the plasma is pulsed, thus it is in $\tau << T_o$ regime, Apply Inverse Laplace Transformation to the equations for each condition, the solutions are

1 For $A = 0$,

$$\theta(t) = \frac{F_T\tau}{m\sqrt{gl}} \sin\sqrt{g/l}t. \qquad (3)$$

2 For $0 < A < \sqrt{4m^2gl}$,

$$\theta(t) = \frac{2F_T\tau}{\sqrt{4m^2gl - A^2}} \exp(-\frac{A}{2ml}t) \sin(\frac{\sqrt{4m^2gl - A^2}}{2ml}t). \qquad (4)$$

3 For $A = \sqrt{4m^2gl}$,

$$\theta(t) = \frac{F_T\tau}{ml} t \exp(-\sqrt{g/l}t). \qquad (5)$$

4 For $A > \sqrt{4m^2gl}$,

$$\theta(t) = \frac{F_T\tau}{\sqrt{A^2 - 4m^2gl}}\{\exp\left(\frac{-A}{2ml} + \frac{\sqrt{A^2 - 4m^2gl}}{2ml}\right)t,$$
$$- \exp\left(\frac{-A}{2ml} - \frac{\sqrt{A^2 - 4m^2gl}}{2ml}\right)t\}. \tag{6}$$

These solutions are plotted in Figure 14. Comparing the analytical solution and the experimental result presented in Figure 11 it appears that the experimental result can be explained by the damped solution. But it was shown previously that the friction at the needle support is negligible. The damping can not come from the pressure built-up in the chamber during the shot because it has been shown that there is no observable damping in the chamber at mtorr pressure range. Therefore this is not a true result. The result shown as solid curve in Figure 12 appears to be real and in agreement with the undamped solution except the asymmetric property in the first three cycle. We reason that this is an effect of the asymmetric relative motion of the capacitor plates and the electronic circuit. Because the relative motion of the pair of plates on the top is opposite to that of the pair of the bottom plates, the asymmetric effects nearly cancel out and are ignored for simplicity. The electronic circuit is very complicated and there are highly nonlinear elements. Fortunately, it can be represented by a drastically simplified diagram as shown in Figure 15 and the two simultaneous linear equations as follows:

$$\begin{cases} \frac{dy}{dt} = \frac{dy_1}{dt} + \frac{y_1}{T_1}; \\ \frac{dz}{dt} + \frac{z}{T_2} = \frac{y_1}{T_2}. \end{cases} \tag{7}$$

where $T_1 = R_1C_1$ and $T_2 = R_2C_2$, and y and z are the input and output signal respectively. The solution is shown as a dashed curve in Figure 12. Except a slight phase different the analytical solution closely matches the experimental data (solid curve). In any electric system phase shift is common and for this application we are not interested in its value. From this analysis we can confidently claim that the experimental result is the true measurement of the thrust free from mechanical, electrical and magnetic interferences.

VI. **Experimental Results and discussion**

This balance has been used to measure the thrust from the Tandem Mirror propulsion device at MIT. The initial results are presented in Table I. The plasma temperature and density were monitored by a triple probe at a radial position two third of the plasma radius. A Langmuir probe biased at +150 V is used to measure the electron saturation current.

The specific impulse is 12,852 sec corresponding to the plasma temperature of 172 eV. The measured thrust is 76 mN in agreement with prediction. The propulsive efficiency is 68%. This can be considered as a milestone of this research.

Detailed discussion of the propulsion device and experiment is not the subject of this paper. However, this demonstrates the success and usefulness of the thrust balance as we set out to accomplish. This pendulum system is made in the laboratory for testing purpose and is the first of its kind. Therefore, there is room for improvement. The plates are made from printed circuit board which consists of G-10 type materials. G-10 outgases during the plasma shot which degrades the plasma conditions. The new plates will be made from silicon wafers. For short pulses the particles can all be absorbed by the G-10 material. For long pulses the surface reflection has to be kept to a minimum and its effect has to be factored into the measurement.

There are two major advantages of this balance: (1) Since the plates are not parts of propulsion system and are simple and inexpensive to build; they can be made sacrificial for high power operation. (2) The balance is symmetric in both directions, therefore, tt is particularly useful to study the pressure balance at the boundary layer between plasma and neutral gas in a gas divertor of tokamak reactors.

Table I

Plasma Parameters

T_i	$Ion temperature$	$172eV$
T_e	$Electron temperature$	$21.6eV$
n_e	$Plasma density$	$1.98 \times 10^{17} m^{-3}$
I_{sp}	$Extracted specific impulse$	$12,852s$
$Thrust$		$76mN$

Acknowledgement

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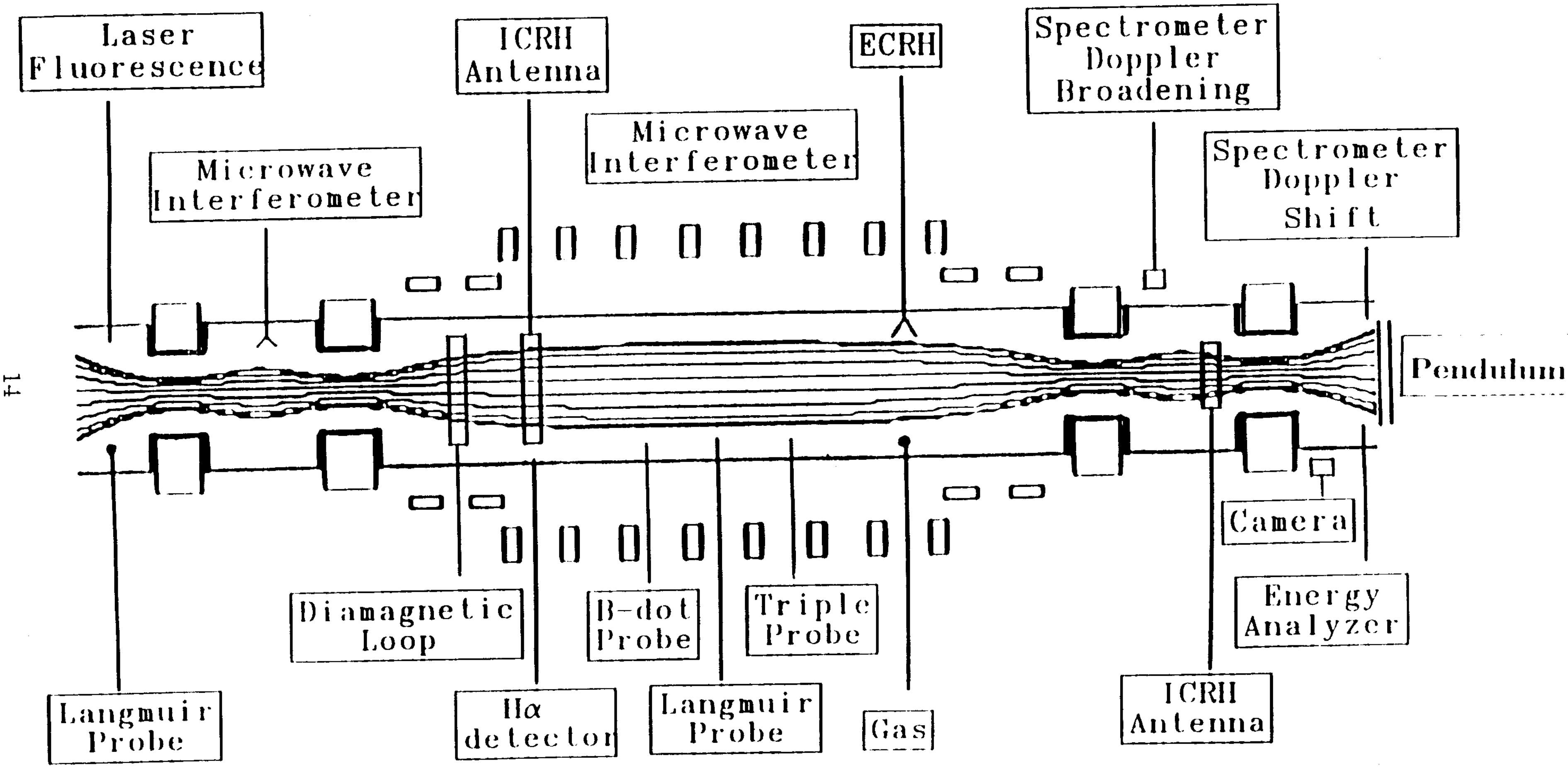


Figure 1. Experimental setup of the Tandem Mirror Plasma propulsion facility.

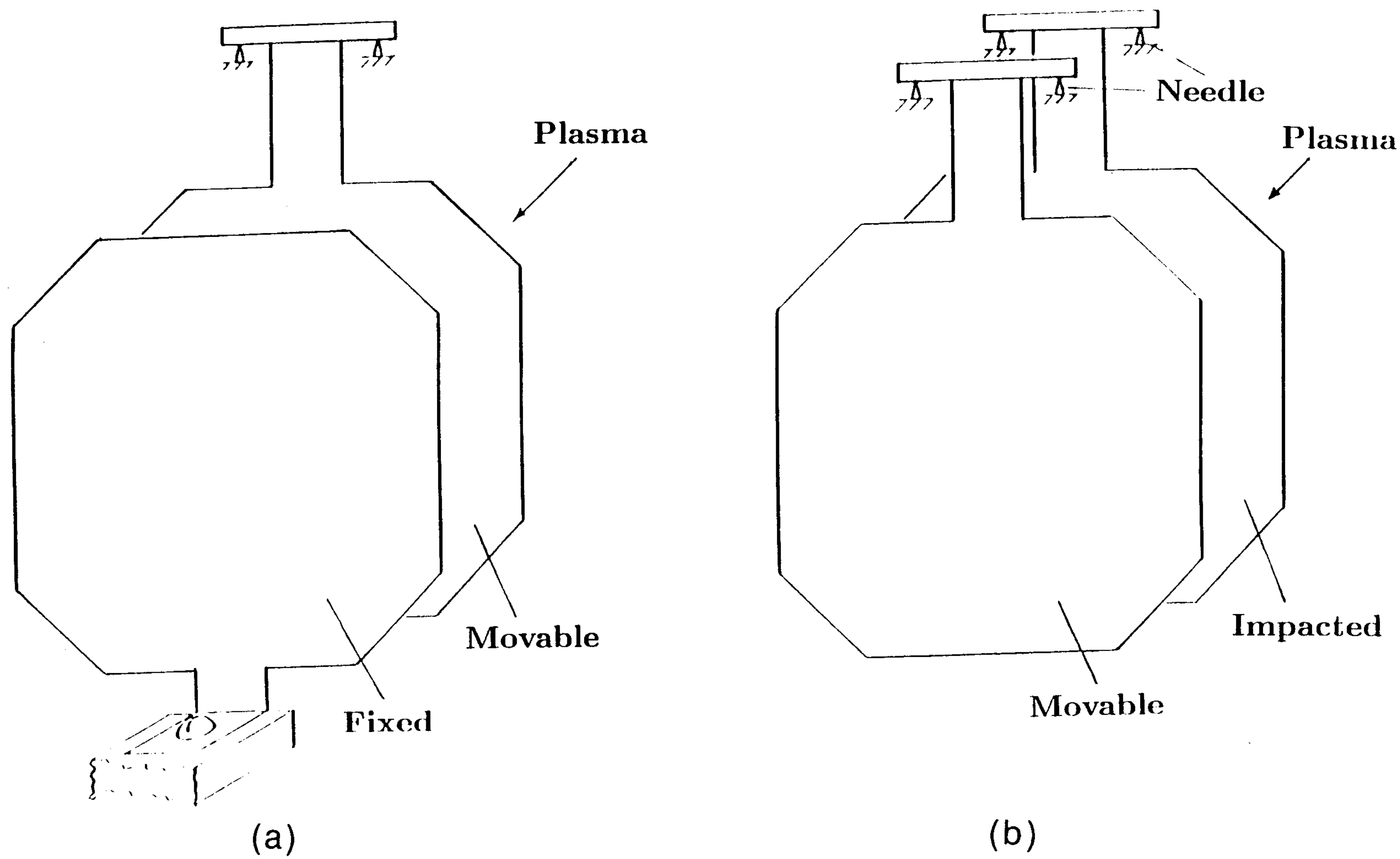


Figure 2. A thrust balance consisting of a simple pendulum and a fixed plate(a) and a double pendulum thrust balance consisting of two identical movable plates (b).

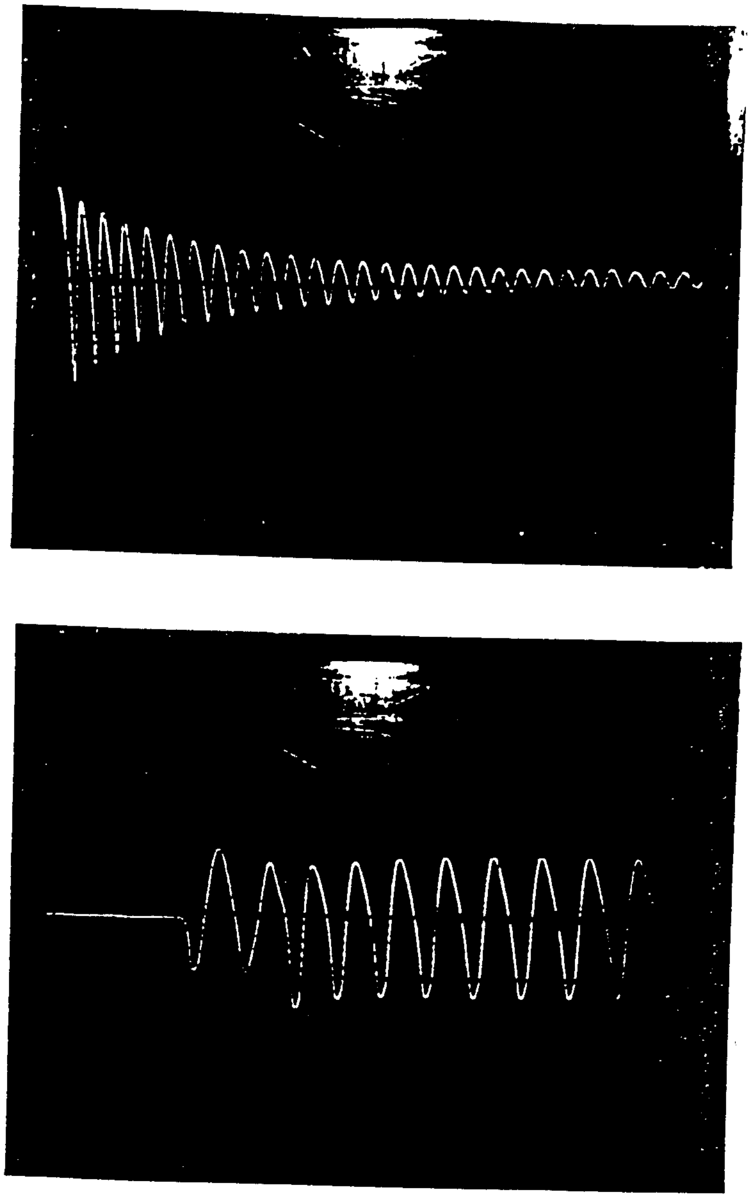


Figure 3. Signals from the thrust balance: (a) A damped oscillation from a mechanical shock in the air and (b) a nondamping oscillation produced by magnetic impulse in the vacuum.

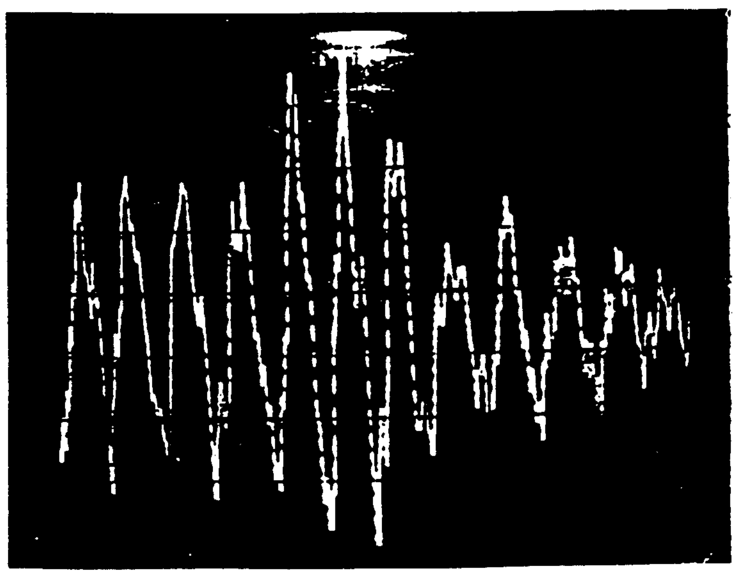


Figure 4. Noise signal from mechanical vibrations in vacuum.

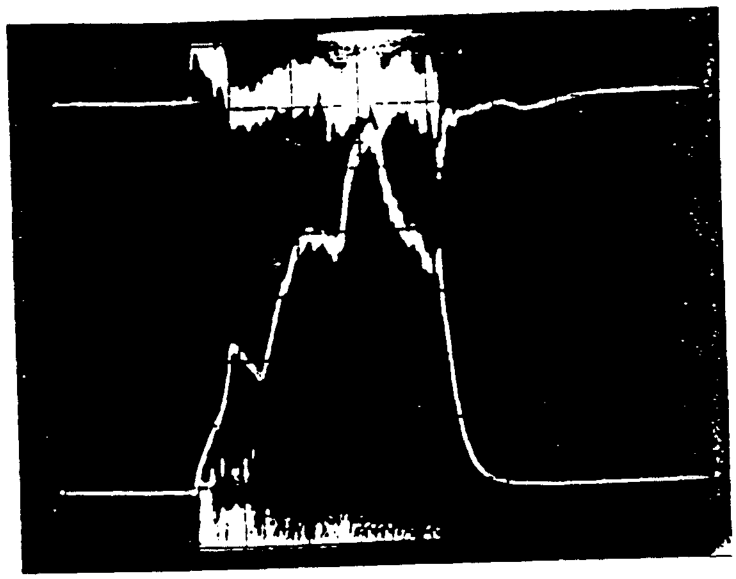


Figure 5. Noise signal from electric pick-ups.

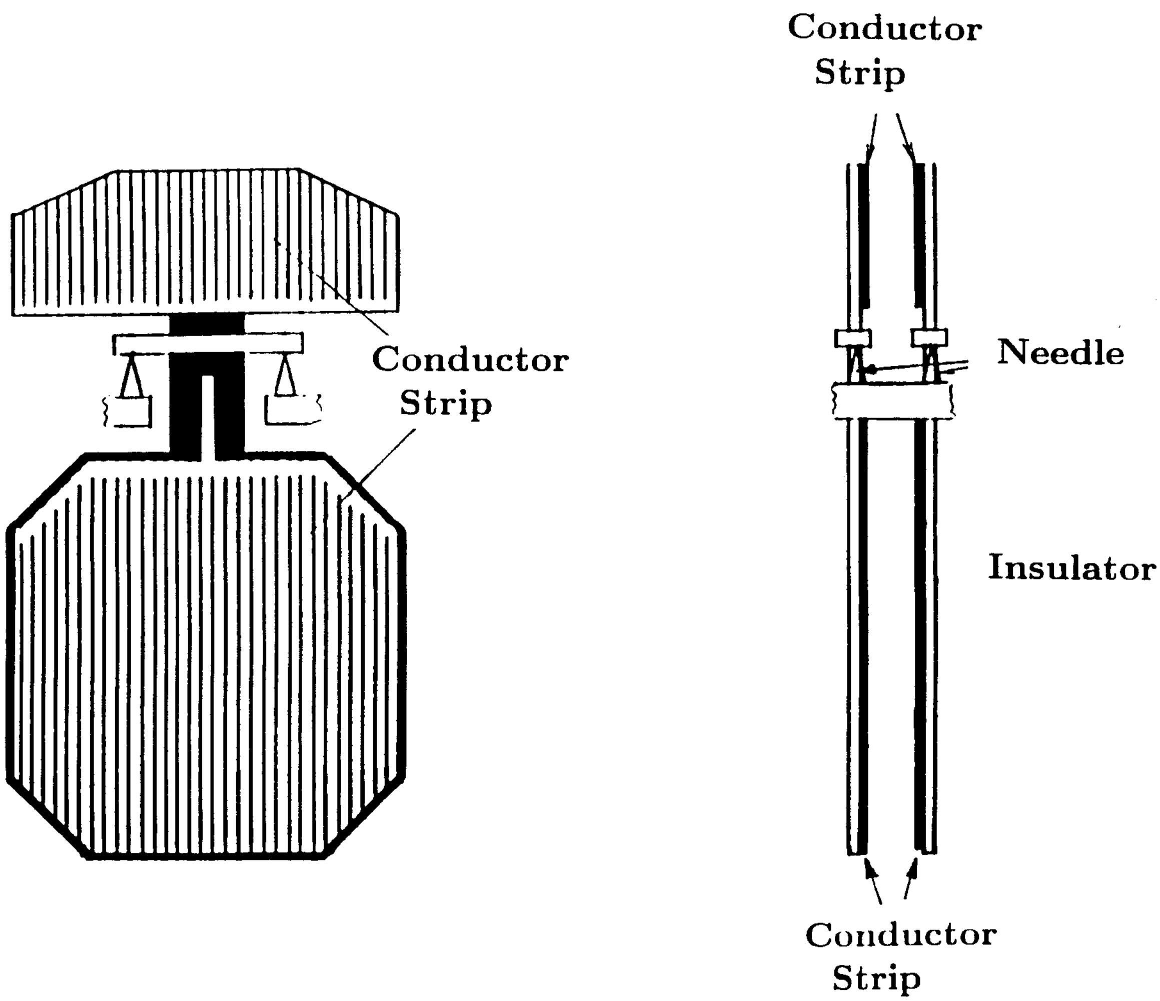


Figure 6. A thrust balance with inverse pendulums consisting of insulator and fine conductor strips.

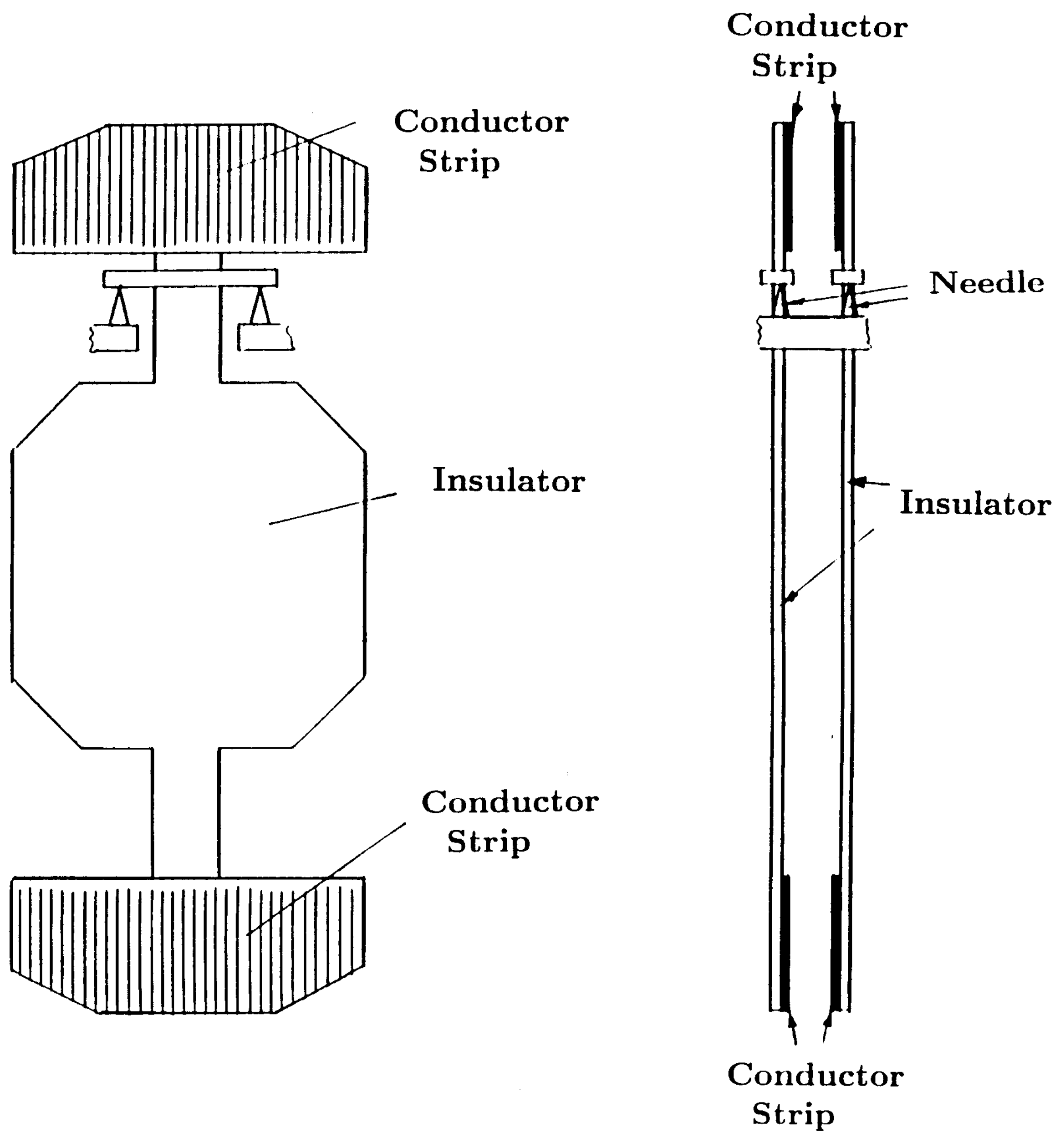
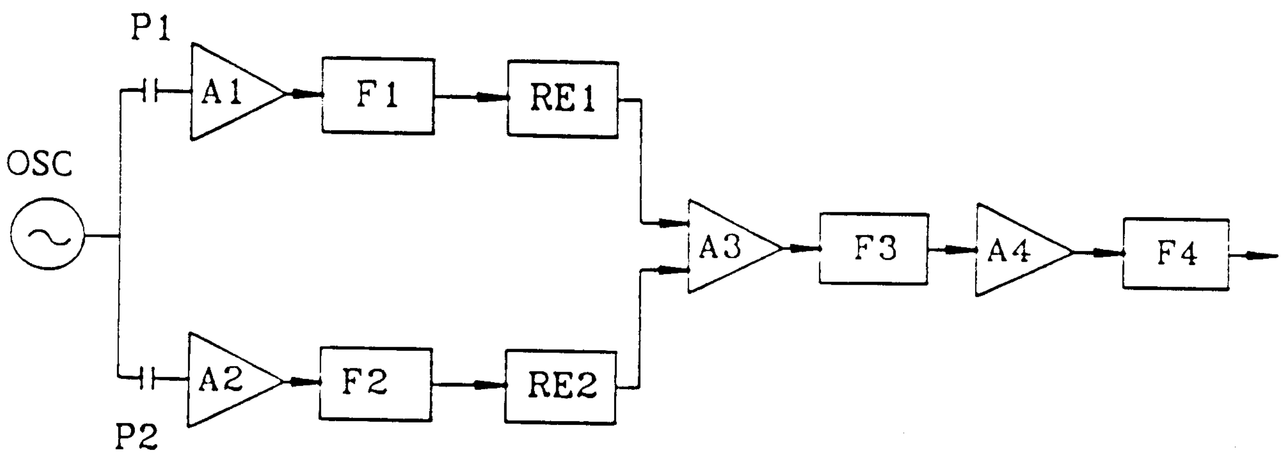


Figure 7. The working version of the thrust balance consisting of nonconducting target plates and sensor capacitor plates with fine conducting strips enclosed in shielded boxes on the top and bottom.



Legend

OSC	Oscillator, 40kHz, 100V	A3	Differential Amplifier
P1,P2	Double Pendulum	A4	Output Amplifier
F1,F2	Active Filter	F3	Anti-DC-Drift Filter
RE1,RE2	Rectifier and Filter	F4	Low Pass Filter

Figure 8. Block diagram of the electronic circuit.

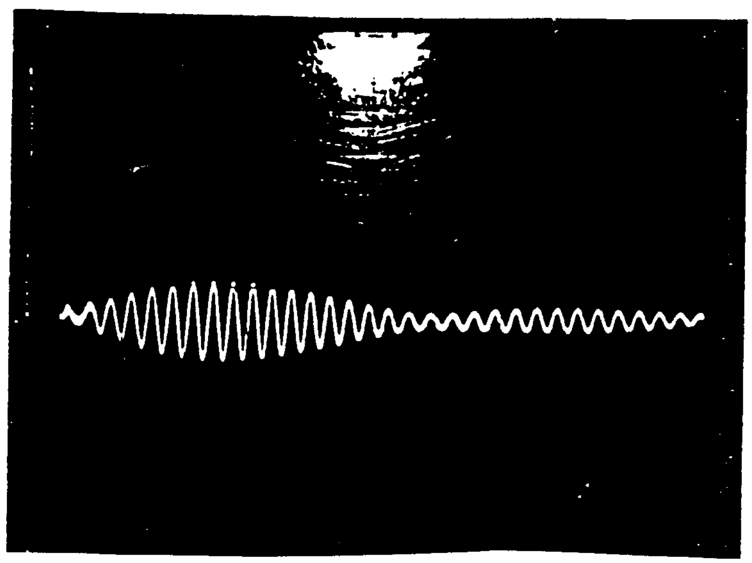


Figure 9. Waveform due to the beat of the two pendulums.

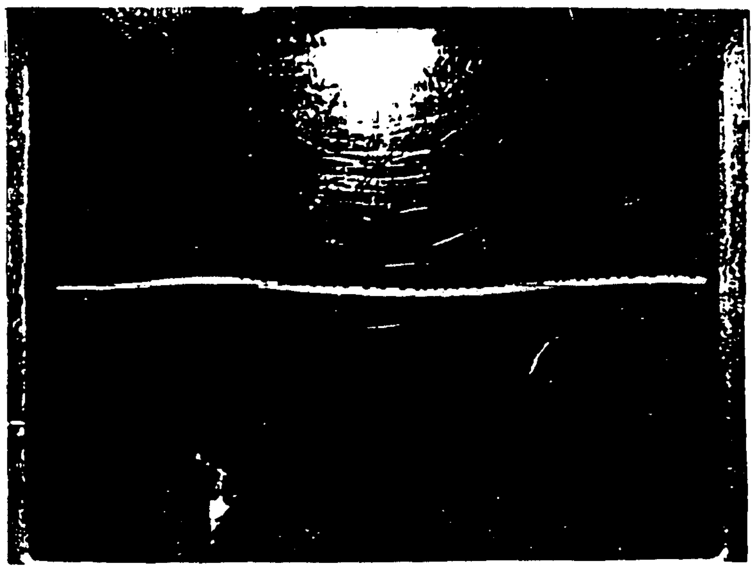


Figure 10. Expanded trace of the first half cycle of the natural oscillation of each pendulum.

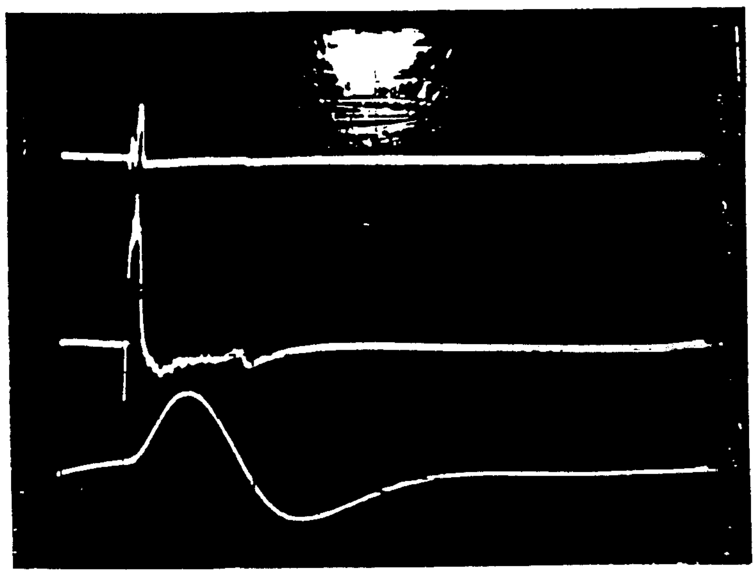


Figure 11. Relationship of the plasma pulses and signal of the thrust balance shown in Figure 6.

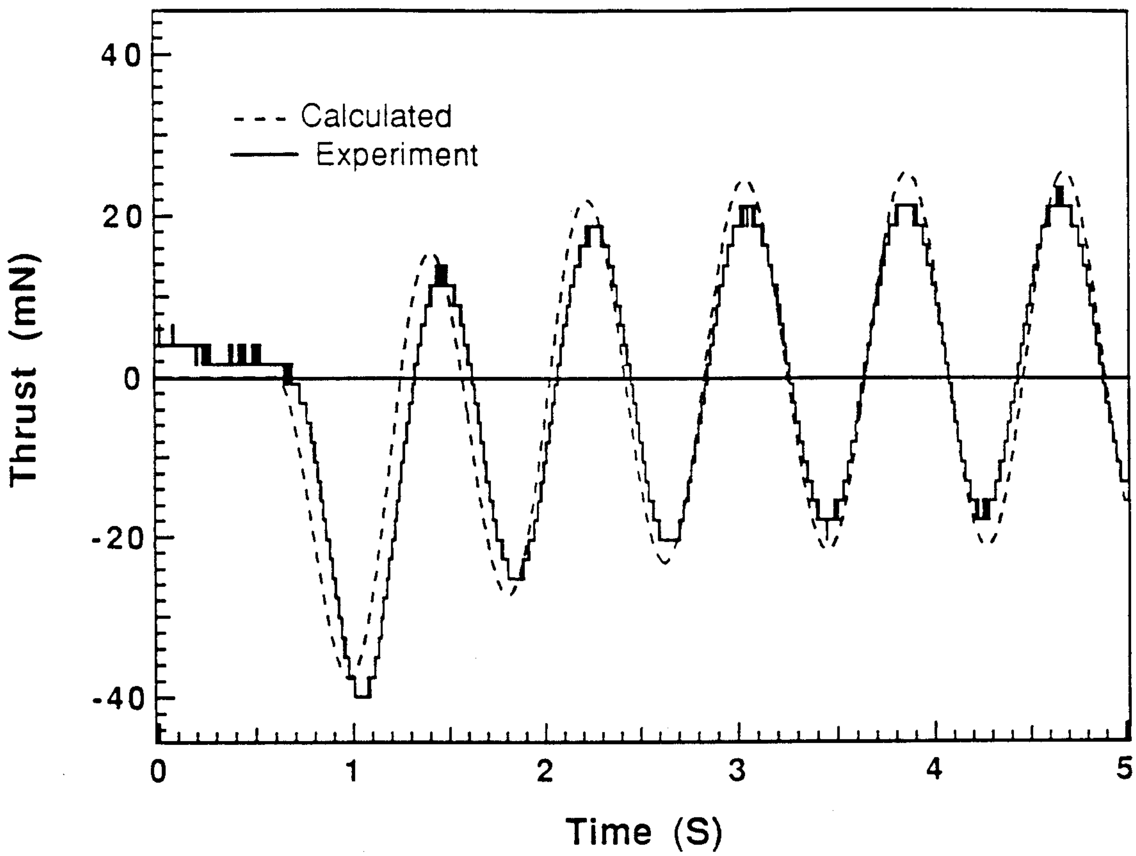


Figure 12. The thrust signal waveforms: Solid curve is the experimental result produced by the working balance shown in Figure 7 and the dashed curve is the theoretical results.

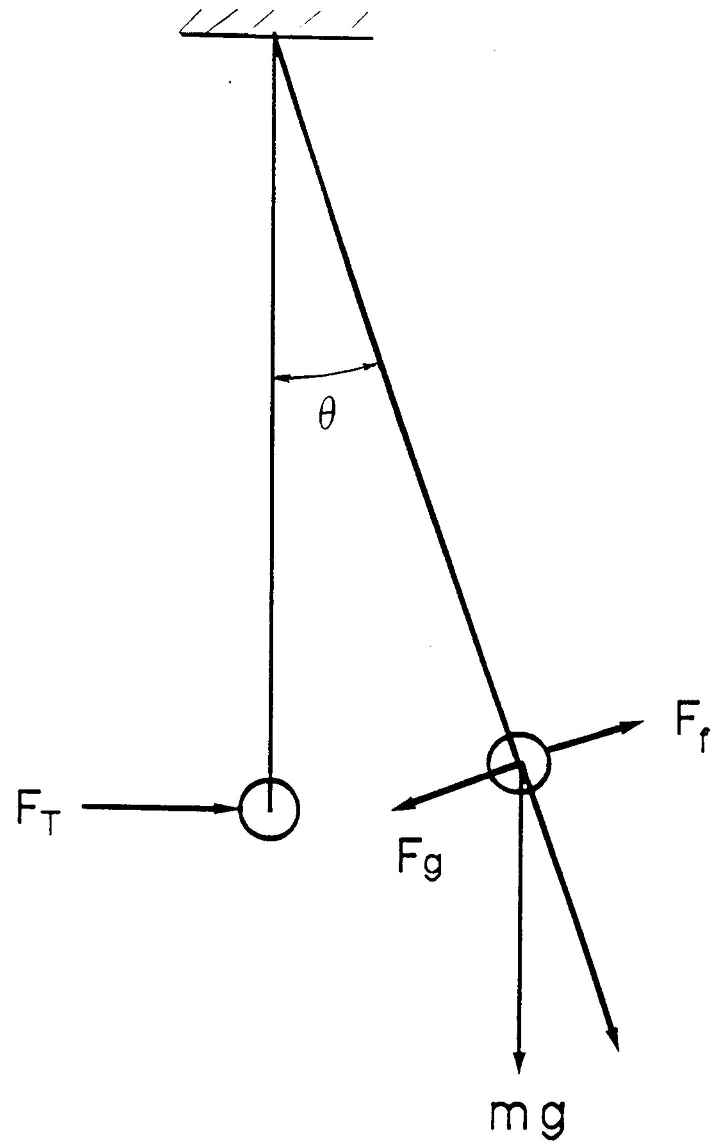


Figure 13. Simple pendulum.

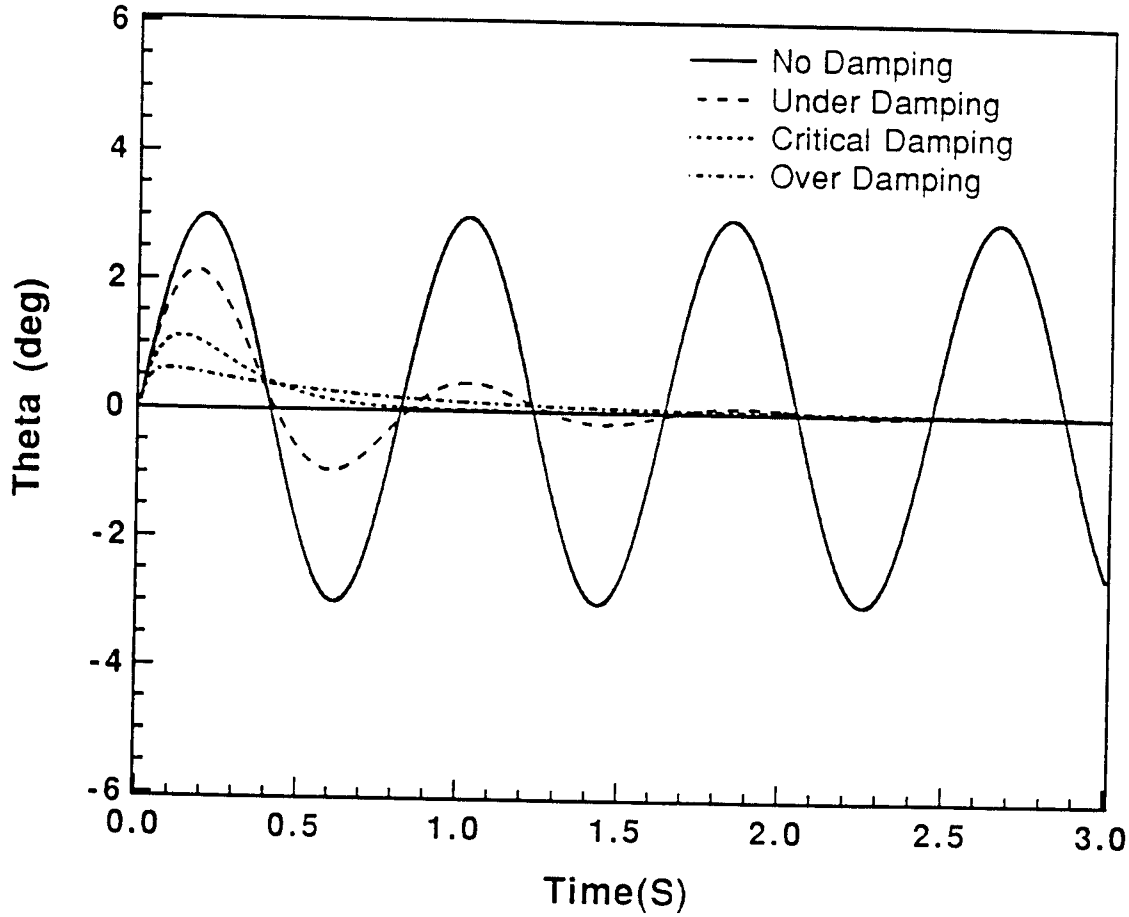


Figure 14. The waveforms of a simple pendulum for $\tau << T_o$.

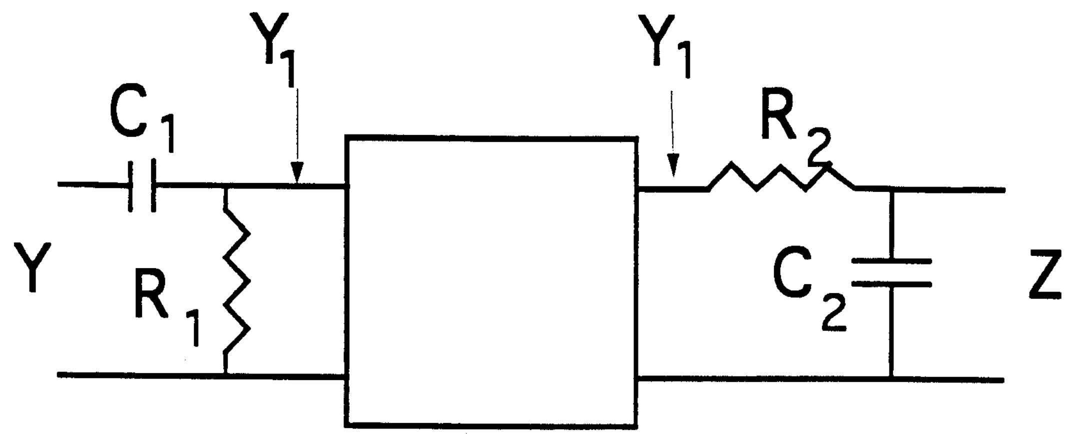


Figure 15. Simplified circuit diagram.