COUPLING TO ELECTRON BERNSTEIN WAVES IN TOKAMAKS*

A. Bers, A. K. Ram, and S. D. Schultz
Plasma Science and Fusion Center,
Massachusetts Institute of Technology, Cambridge, MA 02139. U.S.A.

Abstract The excitation and localized damping of electron Bernstein waves (EBW) in toroidally confined plasmas presents the possibility of efficient means for plasma heating, current drive, and current profile control with external power in the electron cyclotron range of frequencies (ECRF). Such use of EBW is particularly interesting for high-β plasmas (e.g., START, MAST, NSTX) but is also applicable to moderate and low-β tokamaks on which it can be tested.

In this short paper, we focus on the excitation of EBWs from the outboard side by either mode conversion of an X-mode or its direct coupling in an NSTX-type of plasma. Fig. 1 shows the distribution of critical frequencies in propagation across the magnetic field along the equatorial plane, and Fig. 2 gives the local kinetic dispersion relation for the same direction; both use the plasma (density and temperature) and magnetic field profiles (Fig. 3) for NSTX. From Figs. 1 and 2, we note that the triplet of R-cutoff—upper-hybrid-resonance (UHR)—L-cutoff forms a mode conversion resonator (i.e., a resonator containing mode conversion to EBW as an effective dissipation) [1]–[3]. In such a triplet resonator, one can in principle obtain complete mode conversion of the incident power to EBW [4].

We consider the simplest cold-plasma slab model in the equatorial plane, inhomogeneous in x (the radial direction), and with arbitrary x-variations for both the (toroidal) z-directed and the (poloidal) y-directed magnetic fields: $\vec{B}_0(x) = \hat{y}B_p(x) + \hat{z}B_T(x)$. The detailed linearized field analysis is straightforward. For numerically integrating the field equations, it is found convenient to formulate them as a set of four coupled first-order differential equations:

$$\frac{d\vec{F}_c}{d\xi} = i \overleftrightarrow{A}_c \cdot \vec{F}_c \qquad (1)$$

where $\vec{F}_c^T = [E_y \; E_z \; (-cB_y) \; cB_z]$, we have let $(\omega x/c) \equiv \xi$, and the variations in the directions y and z (in which the equilibrium is assumed uniform) have been Fourier analyzed with components, respectively, $\exp(ik_y y)$ and $\exp(ik_z z)$. The tensor (matrix)

$\overleftrightarrow{A}_c$ is found to be given by

$$\overleftrightarrow{A}_c = \frac{1}{K_{xx}} \begin{bmatrix} -n_y\chi_{xy} & -n_y\chi_{xz} & -n_yn_z & K_{xx}-n_y^2 \\ -n_z\chi_{xy} & -n_z\chi_{xz} & K_{xx}-n_z^2 & -n_yn_z \\ \begin{array}{c} K_{xx}(\chi_{yz}+n_yn_z) \\ +\chi_{xy}\chi_{xz} \end{array} & \begin{array}{c} K_{xx}(K_{zz}-n_y^2) \\ +\chi_{xz}^2 \end{array} & n_z\chi_{xz} & n_y\chi_{xz} \\ \begin{array}{c} K_{xx}(K_{yy}-n_z^2) \\ +\chi_{xy}^2 \end{array} & \begin{array}{c} K_{xx}(\chi_{yz}+n_yn_z) \\ +\chi_{xy}\chi_{xz} \end{array} & n_z\chi_{xy} & n_y\chi_{xy} \end{bmatrix} \tag{2}$$

where $n_y = (ck_y/\omega)$, $n_z = (ck_z/\omega)$, and the susceptibility χ_{ij} and permitivity $K_{ij} = \delta_{ij} + \chi_{ij}$ elements are as found from standard cold plasma perturbation theory for the considered equilibrium $\vec{B}_0(x)$ and $n_0(x)$. The solid line curve in Fig. 4 shows the mode conversion/resonant absorption as a function of frequency, obtained from a numerical integration of (1) for NSTX plasma and magnetic field profiles, assuming $n_y = 0$ and $n_z = 0.1$. We note that high mode conversion efficiencies, $> 80\%$, are obtained over a broad range of frequencies of about 4 GHz around the peak in $C \approx 0.97$ at $f = 16$ GHz.

Accounting for kinetic effects in a nonzero temperature plasma removes the resonant absorption at the UHR and replaces it with the kinetic EBW, which propagates the energy away from the mode conversion region. An approximate description that includes the kinetic EBW and the cold plasma modes, coupled near the UHR, follows from general WKB analysis [5], with due attention to conservation of kinetic energy flow density. Thus to include the EBW, we set

$$K_{xx}^K E_x \rightarrow K_{xx}E_x - \frac{d}{d\xi}\left(T\frac{dE_x}{d\xi}\right) \ . \tag{3}$$

$$T = \frac{-3\omega_p^2\omega^2(v_T/c)^2}{(\omega^2-\omega_{ce}^2)(\omega^2-4\omega_{ce}^2)} \tag{4}$$

is obtained from expanding the kinetic (Vlasov) susceptibility χ_{xx}^K to second-order in $(k_\perp v_{Te}/\omega_{ce})$ — appropriate for representing EBW between the first and second electron cyclotron harmonics, and where damping can be neglected. The resulting set of coupled first-order differential equations are

$$\frac{d\vec{F}_K}{d\xi} = i \overleftrightarrow{A}_K \cdot \vec{F}_K \tag{5}$$

where $\vec{F}_K^T = [E_x\ E_y\ E_z\ (iTE_x')\ cB_z\ (-cB_y)]$, we have let $(\omega x/c) \equiv \xi$,

$$\overleftrightarrow{A}_K = \begin{bmatrix} 0 & 0 & 0 & -T^{-1} & 0 & 0 \\ n_y & 0 & 0 & 0 & 1 & 0 \\ n_z & 0 & 0 & 0 & 0 & 1 \\ K_{xx} & \chi_{xy} & \chi_{xz} & 0 & n_y & n_z \\ -\chi_{xy} & K_{yy} - n_z^2 & \chi_{yz} + n_y n_z & 0 & 0 & 0 \\ -\chi_{xz} & \chi_{yz} + n_y n_z & K_{zz} - n_y^2 & 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

The dashed curve in Fig. 4 shows the results from a numerical integration of (6) for the same NSTX parameters used in the mode-conversion/resonance absorption calculations given above. Up to about 16 GHz, the (approximate) kinetic mode-conversion calculation verifies very well the results from the exact mode-conversion/resonance absorption calculation. For frequencies above 16 GHz, the UHR is above $2\omega_{ce}$ and, unless the plasma extends sufficiently out, the power incident on the fast X-mode will only encounter a forward travelling wave (modified slow X-mode/EBW) propagating out of the plasma.

Turning to direct coupling to EBW, we note from Figs. 1 and 2 that for plasma parameters of interest and frequencies not too far above f_{ce}, the distances at the plasma edge over which coupling takes place are shorter than a free-space wavelength. This suggests that effective direct coupling to EBW should be possible from an external slow wave structure that can be placed just inside of the confluence point of the EBW with the slow X-mode. This coupling problem, which is similar to slow-wave coupling for lower-hybrid and ion-Bernstein waves, will be presented elsewhere.

* Work supported by DoE Contract DE-FG02-91ER-54109 and by NSTX.

[1] V. Fuchs et. al., *Phys. Plas.,* **2**, 1637 (1995).

[2] A. K. Ram, A. Bers, S. D. Schultz, and V. Fuchs, *Phys. Plas.,* **3**, 1976 (1996).

[3] A. Bers, A. K. Ram, A. Bécoulet, and B. Saoutic, to appear in *Phys. Plas.*

[4] K. C. Wu, A. K. Ram, A. Bers, and S. D. Schultz, in *Proceedings of the 12th Topical Conference on RF Power in Plasmas,* AIP, New York, 1997, pp. 207–210.

[5] H. Berk and D. L. Book, *Phys. Fluids,* **12**, 649 (1969).

NSTX: $R = 1.05$ m, $a = 0.44$ m; $n_{e0} = n_E + (n_0 - n_E)(1 - x^2/a^2)^{1/2}$; $n_0 = 4 \times 10^{19}/\mathrm{m}^3$; $T_{e0} = T_E + (T_0 - T_E)(1 - x^2/a^2)^2$; $T_0 = 3$ keV; $(n_E, T_E) = 0.02(n_0, T_0)$

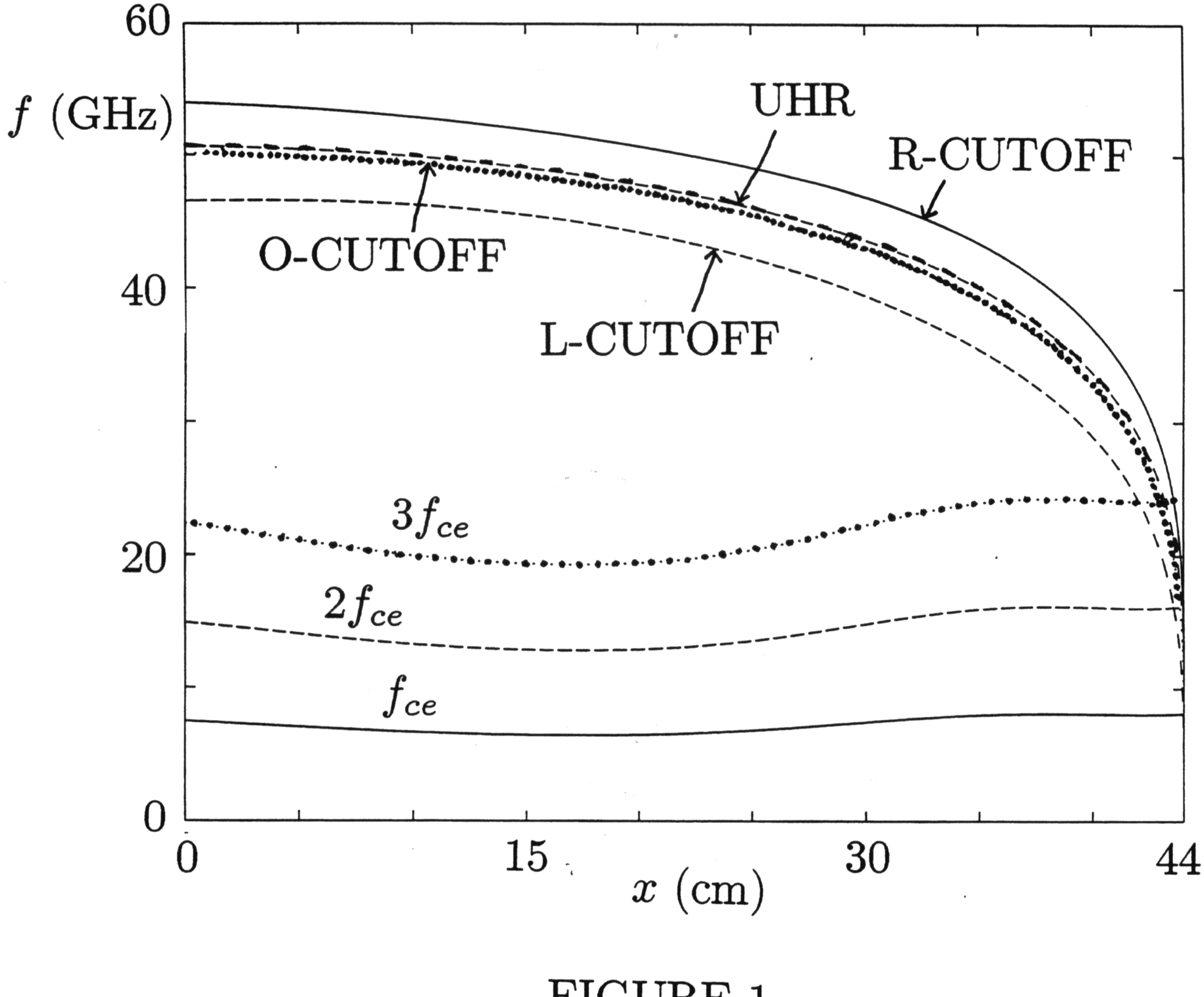
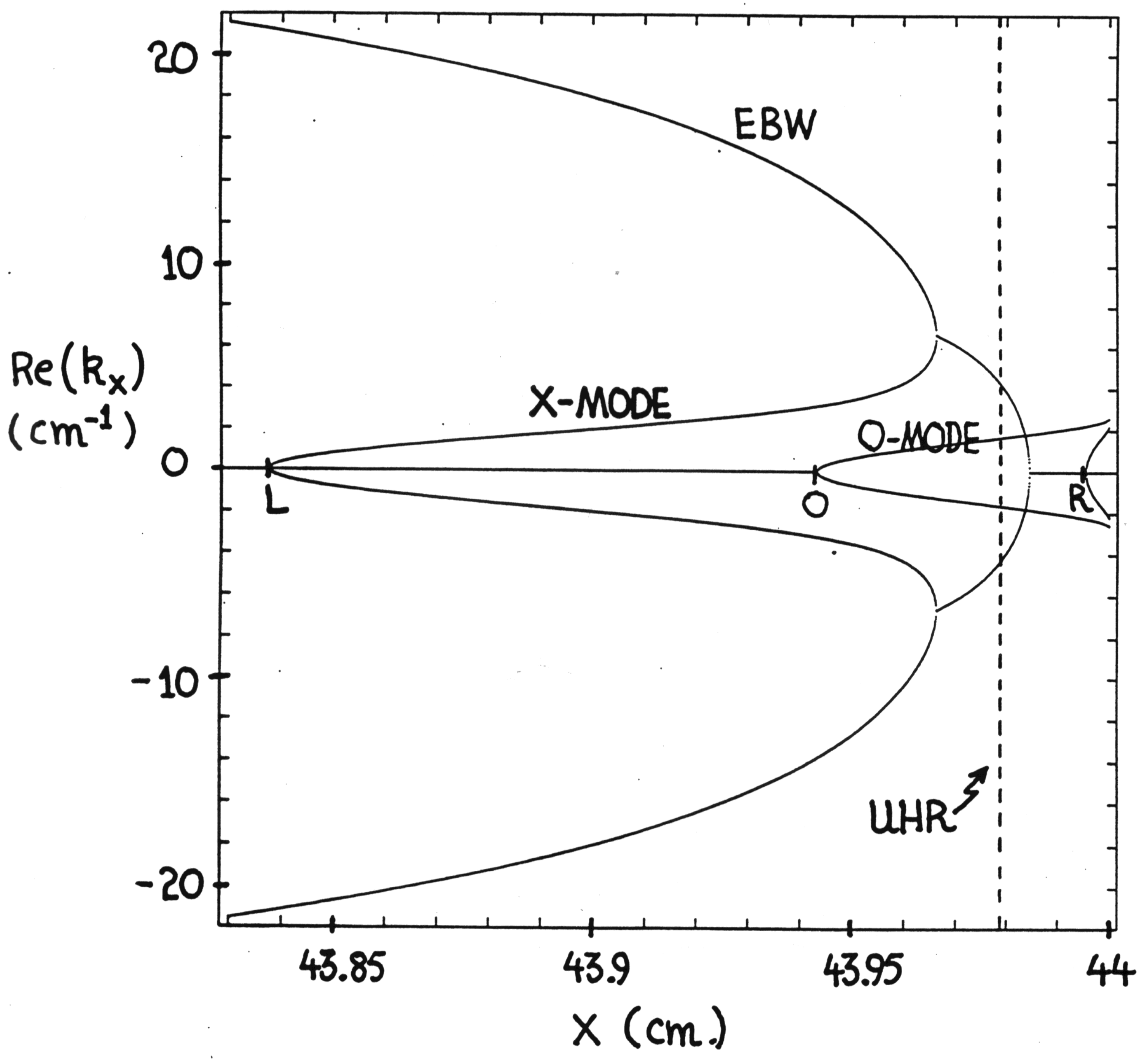

60
40
20
0
f (GHz)
UHR
R-CUTOFF
O-CUTOFF
L-CUTOFF
$3f_{ce}$
$2f_{ce}$
f_{ce}
0
15
30
44
x (cm)

FIGURE 1



20
10
0
-10
-20
Re(k_x)
(cm^{-1})
EBW
X-MODE
O-MODE
L
O
R
UHR
43.85
43.9
43.95
44
X (cm.)

FIGURE 2

Courtesy of
R. Majeski

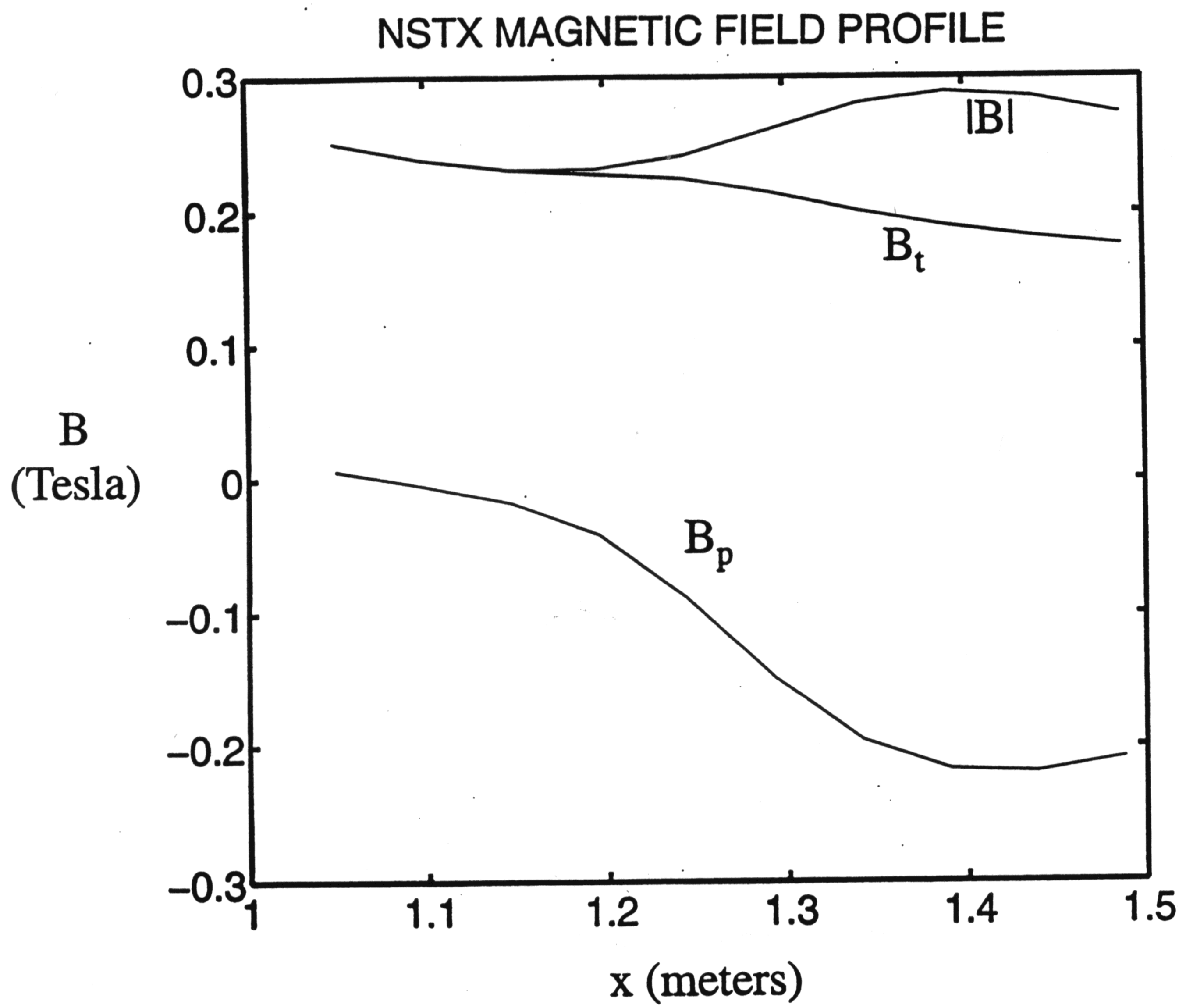


FIGURE 3

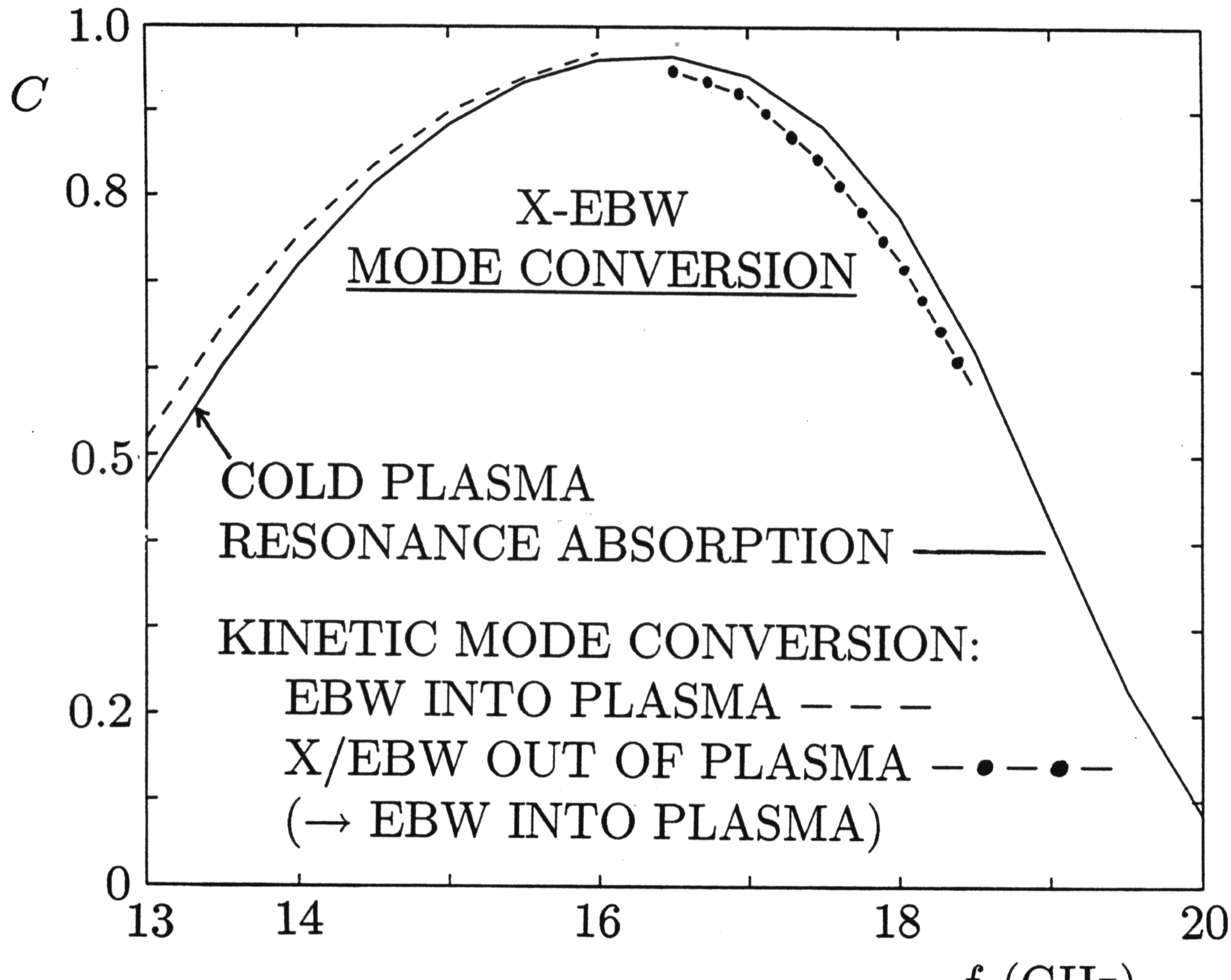

1.0
C
0.8
0.5
0.2
0
13
14
16
18
20
f (GHz)
X-EBW
MODE CONVERSION
COLD PLASMA
RESONANCE ABSORPTION
KINETIC MODE CONVERSION:
EBW INTO PLASMA
X/EBW OUT OF PLASMA
(→ EBW INTO PLASMA)

FIGURE 4