2.29 Numerical Fluid Mechanics
Fall 2011

Units: (3-0-9, 4-0-8)

Lectures and Recitations:

Lectures: **2 sessions/week, 1.5 hours/session**
Recitations/Reviews: **1 session/week, 1 hour/session**
These recitations and review sessions will not be held every week. They will be used in response to student's requests or the needs of the course and teaching staff, including make-up lectures. Students will be informed in advance when these sessions are planned.

Prerequisite: 2.006 or 2.016 or 2.20 or 2.25, 18.075

Subject Summary and Objectives:
Introduction to numerical methods and MATLAB: errors, condition numbers and roots of equations. Navier-Stokes. Direct and iterative methods for linear systems. Finite differences for elliptic, parabolic and hyperbolic equations. Fourier decomposition, error analysis and stability. High-order and compact finite-differences. Finite volume methods. Time marching methods. Navier-Stokes solvers. Grid generation. Finite volumes on complex geometries. Finite element methods. Spectral methods. Boundary element and panel methods. Turbulent flows. Boundary layers. Lagrangian Coherent Structures. Subject includes a final research project.

***Specific Objectives*:**

- To introduce and develop the main approaches and techniques that constitute the basis of numerical fluid mechanics for engineers and applied scientists.
- To familiarize students with the numerical implementation of these techniques and numerical schemes, so as provide them with the means to write their own codes and software, and so acquire the knowledge necessary for the skillful utilization of CFD packages or other more complex software.
- To cover a range of modern approaches for numerical and computational fluid dynamics, without entering all these topics in detail, but aiming to provide students with a general knowledge and understanding of the subject, including recommendations for further studies.

This course continues to be a work in progress. New curricular materials are being developed for this course, and feedback from students is always welcome and appreciated during the term. For example, recitations and reviews on specific topics can be provided based on requests from students.

Students are strongly encouraged to attend classes and recitations/reviews. The instructor and teaching assistant are also available for consultation during office hours. Appointments can also be scheduled by emails and/or phone.

Evaluation and Grading:

The final course grade will be weighted as follows:

Homework	**(8 in total, 5.5% each)**	**44 %**
Quizzes	**(2)**	**30 %**
Final project	**(1)**	**26 %**

Project:

There will be a final project for this class. Students can select the topic of their project in consultation with the instructor and TA. Possible projects include:

i) Comprehensive reviews of material not covered in detail in class, with some numerical examples;

ii) Specific fluid-related problems or questions that are numerically studied or solved by the applications of approaches, methods or schemes covered in class;

iii) A combination of i) and ii).

Projects will be due at the end of term. If possible, we plan to have a final session where all students will make a presentation of their projects to the whole class and staff. We find that such presentations provide an excellent means for additional learning and sharing.

2.29 Numerical Fluid Mechanics
Sample Project Titles (26% of grade)

i) "Comprehensive" Methodological Reviews and Comparisons

- Review of autonomous/adaptive generation of computational grids in complex geometries
- Advanced unstructured grids schemes for numerical fluid mechanics applications in
  - Heat transfer/thermodynamics, Ocean Eng./Science, Civil Engineering, etc.
- Review of Multigrid methods and comparisons of schemes in idealized examples
- Comparisons of solvers for banded/sparse linear systems: theory and idealized examples
- The use of spectral methods for turbulent flows
- Novel advanced computational schemes for reactive/combustion flows: reviews and examples
- Numerical dissipation and dispersion: review and examples of artificial viscosity
- etc.

2.29 Numerical Fluid Mechanics
Sample Project Titles (26% of grade), Cont'd

ii) Computational Fluid Studies and Applications

- Idealized simulations of compressible air flows through pipe systems
- Computational simulations of idealized physical and biogeochemical dynamics in oceanic straits
- Simulations of flow fields around a propeller using a (commercial) CFD software: sensitivity to numerical parameters
  - e.g. sensitivity to numerical scheme, grid resolution, etc
- Simulation of flow dynamics in an idealized porous medium
- Pressure distribution on idealized ship structures: sensitivity to ship shapes and to flow field conditions
- Finite element (or Finite difference) simulations of flows for
  - Idealized capillaries, Laminar duct flows, idealized heat exchangers, etc
- etc.

Sample Project Titles (26% of grade), Cont'd

iii) Combination of i) Reviews and ii) Specific computational fluid studies

- Review of Panel methods for fluid-flow/structure interactions and preliminary applications to idealized oceanic wind-turbine examples
- Comparisons of finite volume methods of different accuracies in 1D convective problems
- A study of the accuracy of finite volume (or difference or element) methods for two-dimensional fluid mechanics problems over simple domains
- Computational schemes and simulations for chaotic dynamics in nonlinear ODEs
- Stiff ODEs: recent advanced schemes and fluid examples
- High-order schemes for the discretization of the pressure gradient term and their applications to idealized oceanic/atmospheric flows
- etc.

2.29 Numerical Fluid Mechanics
Projects completed in Spring 2008

- Analysis of Simple Walking Models: Existence and Stability of Periodic Gaits
- Simulations of Coupled Physics-Biology in Idealized Ocean Straits
- High-resolution Conservative Schemes for Incompressible Advections: The Magic Swirl
- Multigrid Method for Poisson Equations: Towards atom motion simulations
- Stability Analysis for a Two-Phase Flow system at Low Pressure Conditions
- Particle Image Velocimetry and Computations: A Review
- Real-time Updates of Coastal Bathymetry and Flows for Naval Applications

- Simulation of Particles in 2D Incompressile Flows around a Square Block
- Panel Method Simulations for Cylindrical Ocean Structures
- 2D viscous Flow Past Rectangular Shaped Obstacles on Solid Surfaces
- Three-dimensional Acoustic Propagation Modeling: A Review
- Immersed Boundary Methods and Fish Flow Simulations: A Review

2.29 Numerical Fluid Mechanics
Projects completed in Fall 2009

- Lagrangian Coherent Structures and Biological Propulsion
- Fluid Flows and Heat Transfer in Fin Geometries
- Boundary Integral Element Methods and Earthquake Simulations
- Effects of Wind Direction on Street Transports in Cities simulated with FLUENT
- Stochastic Viscid Burgers Equations: Polynomial Chaos and DO equations

- Modeling of Alexandrium fundyense bloom dynamics in the Eastern Maine Coastal Current: Eulerian vs. Lagrangian Approach
- Coupled Neutron Diffusion Studies: Extending Bond Graphs to Field Problems
- CFD Investigation of Air Flow through a Tube-and-Fin Heat Exchanger
- Towards the use of Level-Set Methods for 2D Bubble Dynamics
- Mesh-Free Schemes for Reactive Gas Dynamics Studies
- A review of CFD usage at Bosch Automotive USA

Numerical Fluid Mechanics – Outline Lectures 1-2

- Introduction to Computational Fluid Dynamics
- Introduction to Numerical Methods in Engineering
  - Digital Computer Models
  - Continuum and Discrete Representation
  - Number representations
  - Arithmetic operations
  - Errors of numerical operations. Recursion algorithms

- Error Analysis
  - Error propagation – numerical stability
  - Error estimation
  - Error cancellation
  - Condition numbers

What is CFD?

Computational Fluid Dynamics is a branch of computer-based science that provides numerical predictions of fluid flows

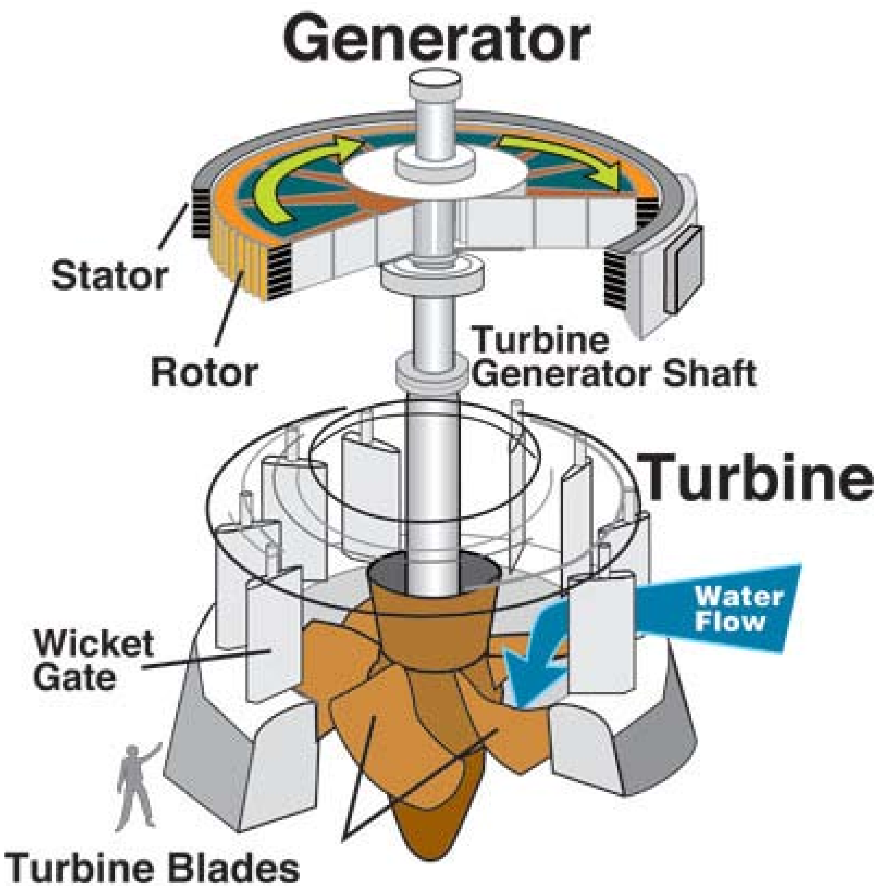
- Mathematical modeling (typically a system of non-linear, coupled PDEs, sometimes linear)
- Numerical methods (discretization and solution techniques)
- Software tools

CFD is used in a growing number of engineering and scientific disciplines

Several CFD software tools are commercially available, but still extensive research and development is ongoing to improve the methods, physical models, etc.

Examples of "Fluid flow" disciplines where CFD is applied

Engineering: aerodynamics, propulsion, Ocean engineering, etc.



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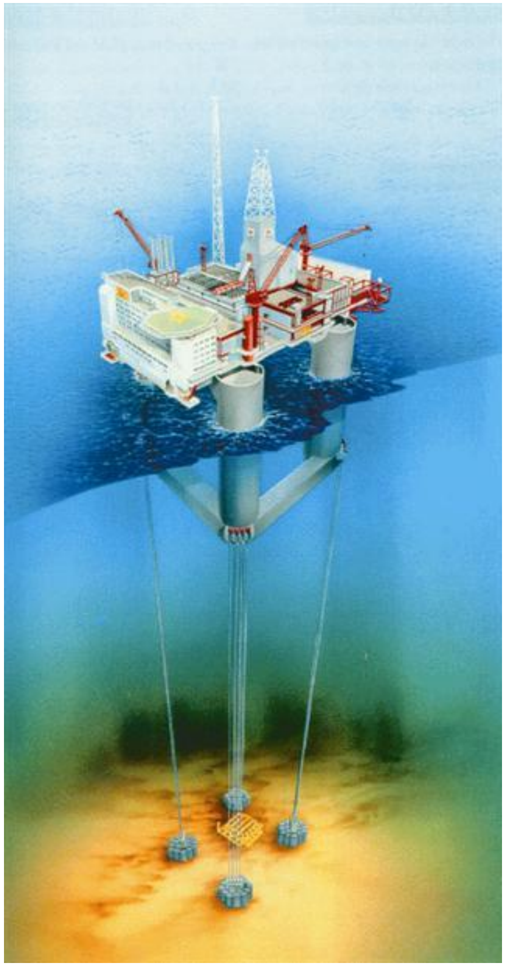

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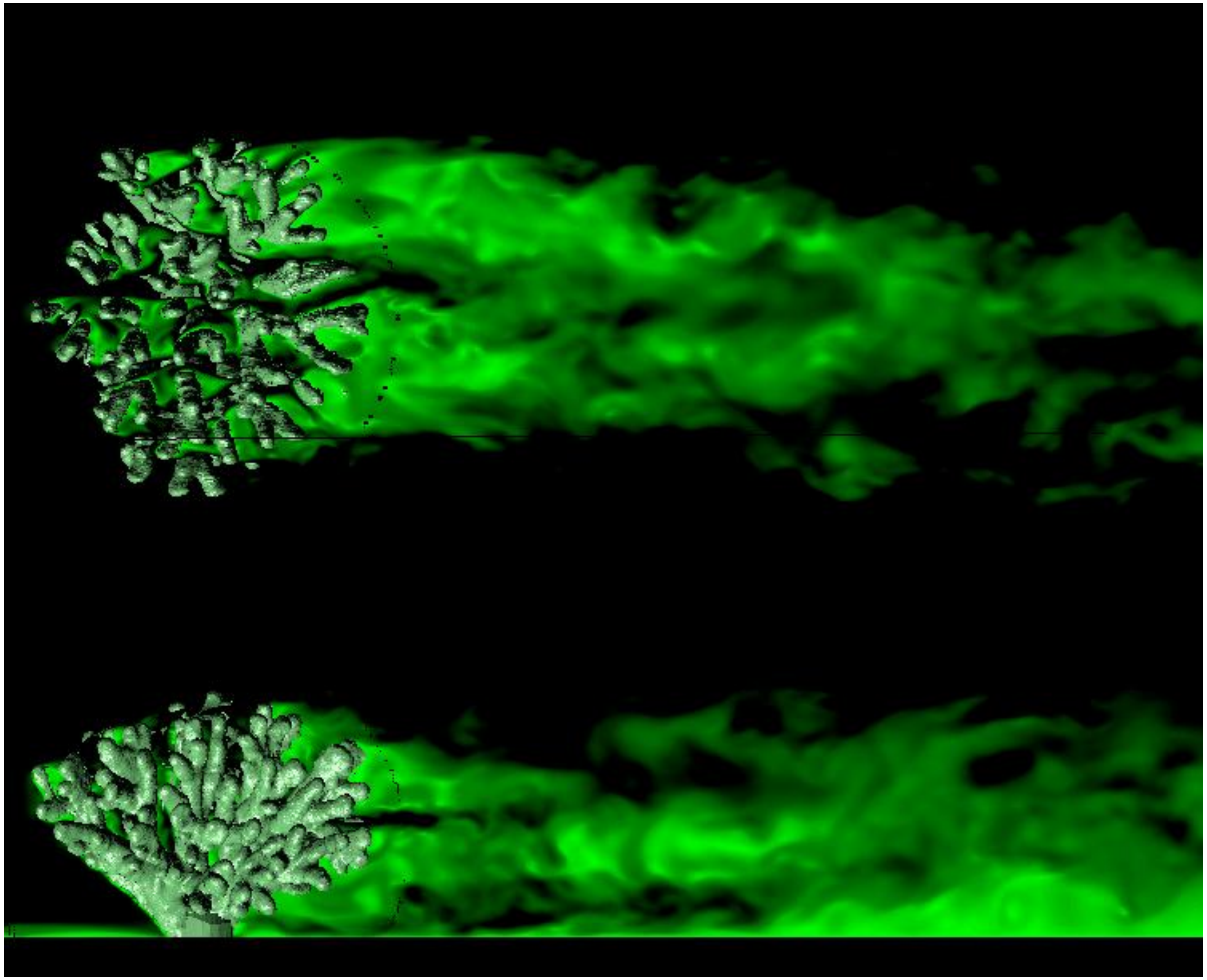
Examples of “Fluid flow” disciplines where CFD is applied

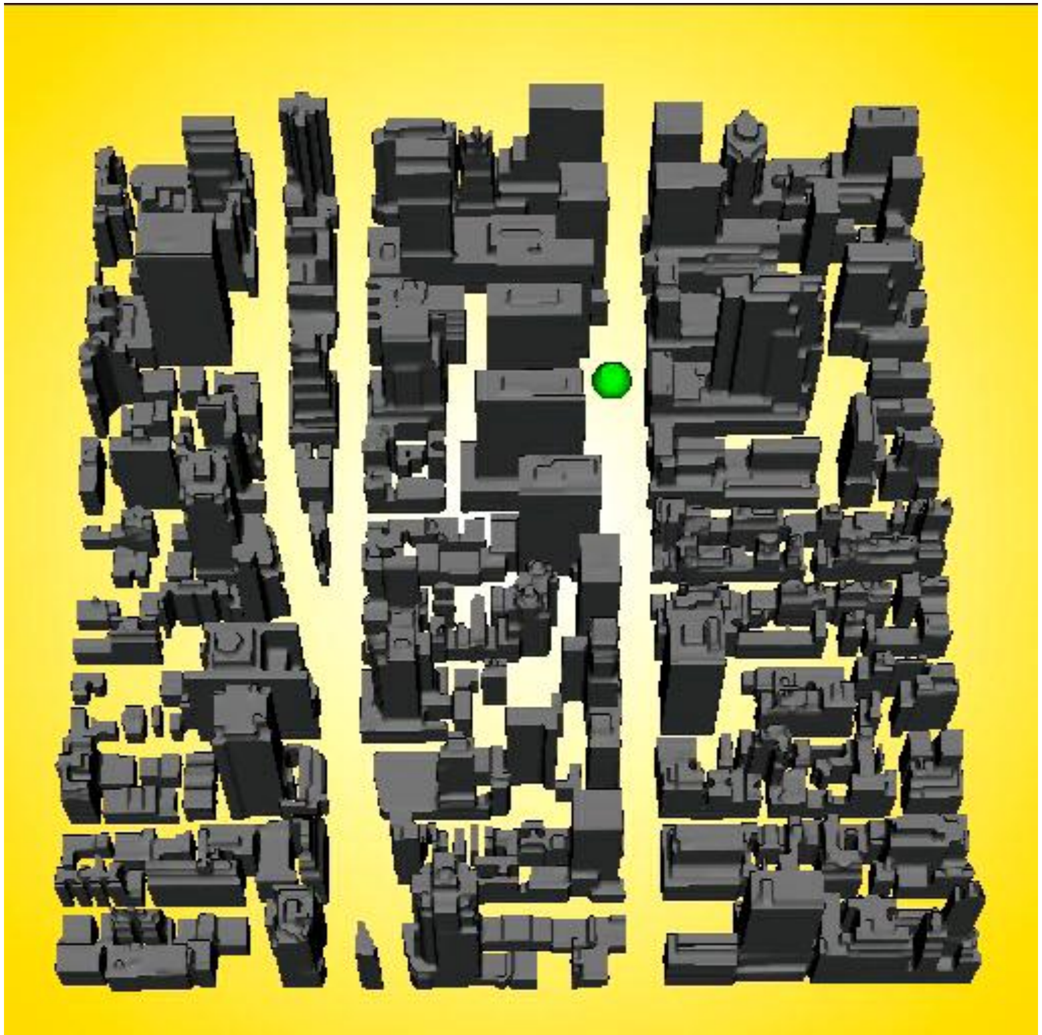
Biological systems: nutrient transport, pollution etc.



Examples of “Fluid flow” disciplines where CFD is applied

Building, City and Homeland security: hazard dispersion,etc.

https://computation.llnl.gov/casc/audim/images/manhattan-new-2.mpeg

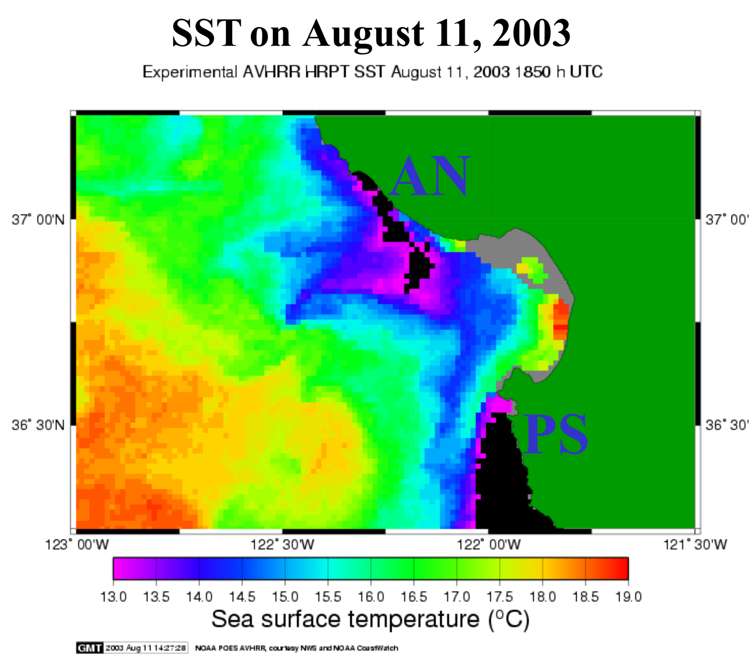


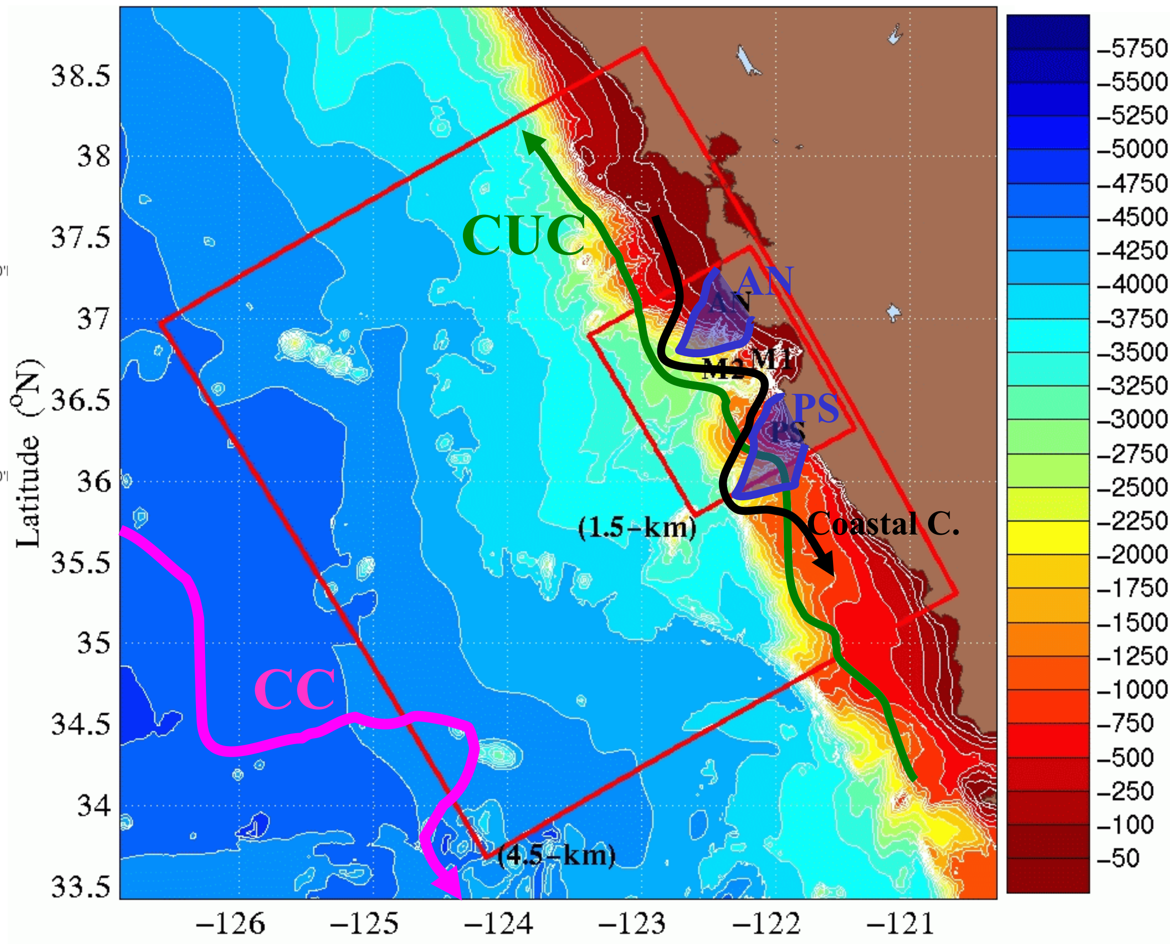
Examples of "Fluid flow" disciplines where CFD is applied

Meteorology, Oceanography and Climate:

hurricanes, tsunamis, coastal management, climate change, etc.

REGIONAL FEATURES of Monterey Bay and California Current System and Real-time Modeling Domains (AOSN2, 4 Aug. – 3 Sep., 2003)





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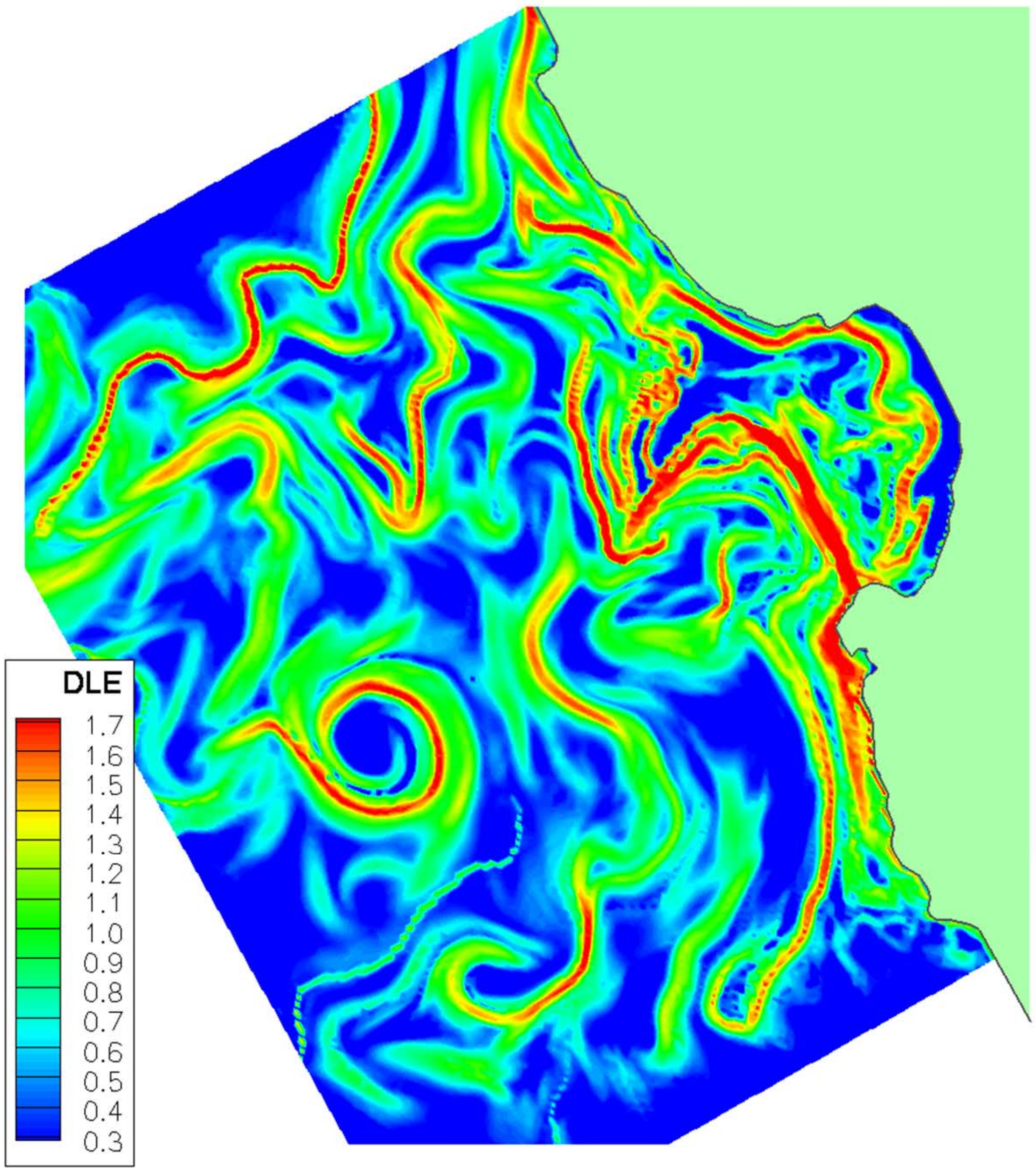
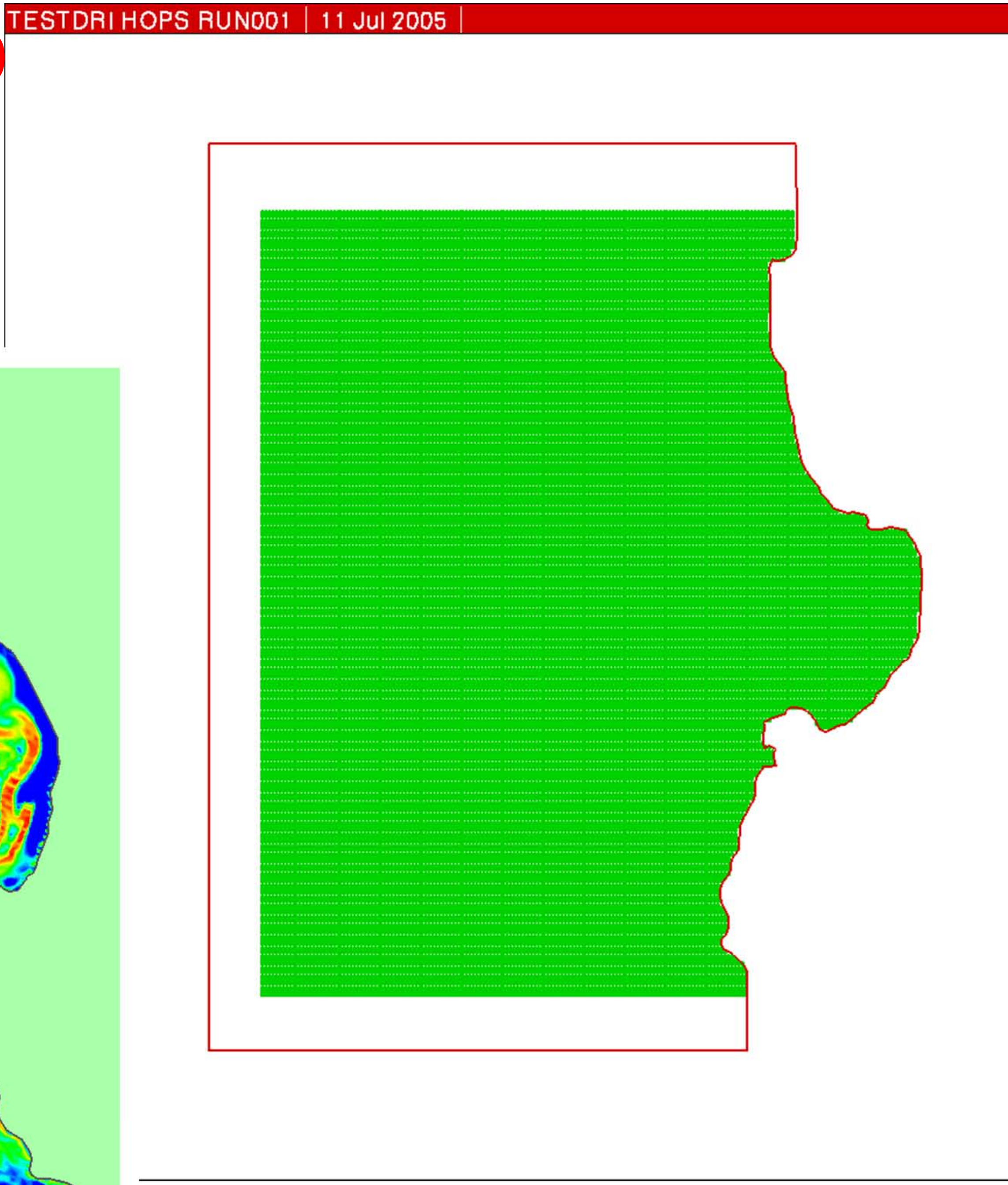
REGIONAL FEATURES

- **Upwelling centers at Pt AN/ Pt Sur**:…………Upwelled water advected equatorward and seaward
- **Coastal current, eddies, squirts, filam., etc:**….Upwelling-induced jets and high (sub)-mesoscale var. in CTZ
- **California Undercurrent (CUC)**:……………..Poleward flow/jet, 10-100km offshore, 50-300m depth
- **California Current (CC)**:……………………Broad southward flow, 100-1350km offshore, 0-500m depth

Ocean Realization #1 (hindcast)

Flow field evolution (right) and its DLE for T= 3 days (below)

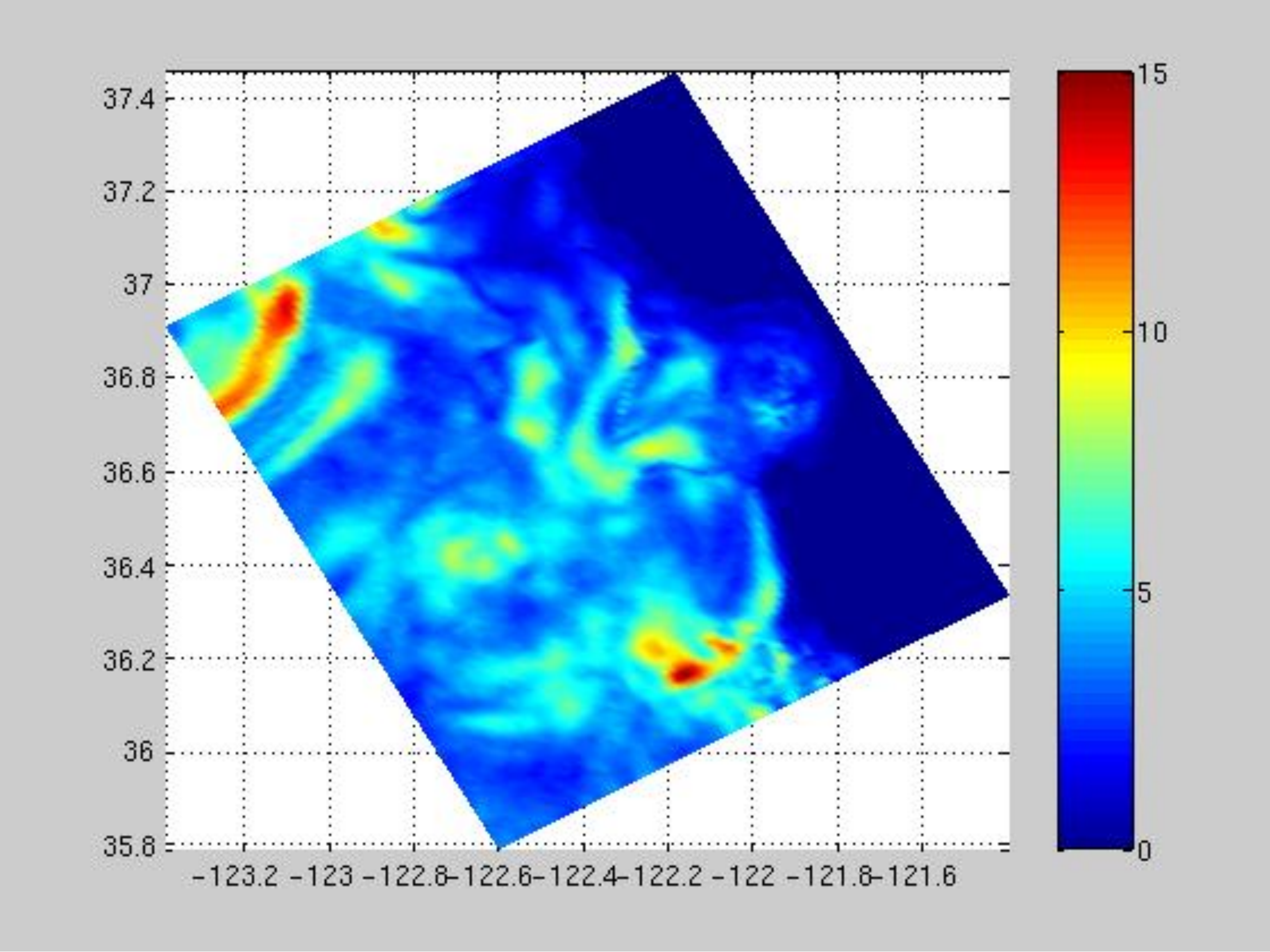




Upwelling (Aug 26-Aug 29, 2006): |u| error Std. estimate

Main features (from north to south)

- Uncertainty ICs:
  - -a function of past dynamics and of past measurement types/locations
- Northward advection of cold eddy field
- Pt AN Upwelling Plume/Squirts formation, position and instabilities
- Offshore eddy
- Pt Sur upwelling frontal position and instabilities
- Daily cycles and wind-driven uncertainty reduction/burst



HD Stereo Theatre

Promotional poster removed due to copyright restrictions; see examples of HD Stereo Theatre simulations at the following URL:

http://gladiator.ncsa.illinois.edu/Images/cox/pics.html

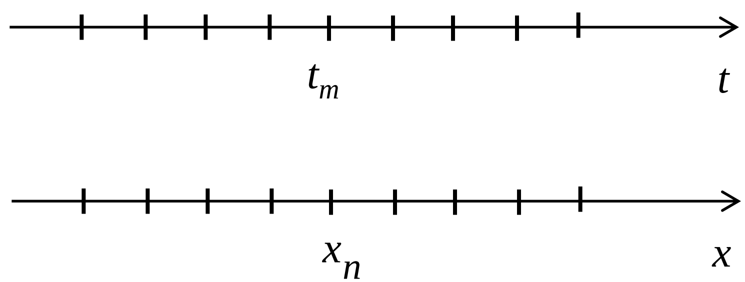
From Mathematical Models to Numerical Simulations

Continuum Model

$$\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial x} = 0$$

Sommerfeld Wave Equation (c= wave speed). This radiation condition is sometimes used at open boundaries of ocean models.

Discrete Model



$$t_m = t_0 + m\Delta t \ , \quad m = 0, 1, \ldots M-1$$
$$x_n = x_0 + n\,\Delta x \ , \quad n = 0, 1, \ldots N-1$$

$$\frac{dw}{dx} \simeq \frac{\Delta w}{\Delta x} \ , \quad \frac{dw}{dt} \simeq \frac{\Delta w}{\Delta t}$$

p parameters

Differential Equation

$$L(p, w, x, t) = 0$$

"Differentiation"
"Integration"

Difference Equation

$$L_{mn}(p_{mn}, w_{mn}, x_n, t_m) = 0$$

System of Equations

$$\sum_{j=0}^{N-1} F_i(w_j) = B_i$$

Linear System of Equations

$$\sum_{j=0}^{N-1} A_{ij} w_j = B_i$$

"Solving linear equations"

Eigenvalue Problems

$$\overline{\overline{\mathbf{A}}}\mathbf{u} = \lambda\mathbf{u} \Leftrightarrow (\overline{\overline{\mathbf{A}}} - \lambda\overline{\overline{\mathbf{I}}})\mathbf{u} = \mathbf{0}$$

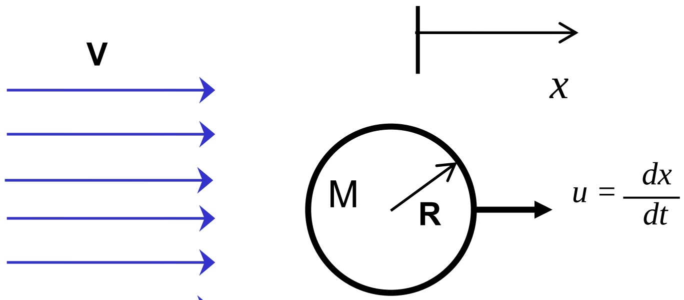
Non-trivial Solutions

$$\det(\overline{\overline{\mathbf{A}}} - \lambda\overline{\overline{\mathbf{I}}}) = 0$$

"Root finding"

Consistency/Accuracy and Stability => Convergence
(Lax equivalence theorem for well-posed linear problems)

Sphere Motion in Fluid Flow



Equation of Motion – 2nd Order Differential Equation

$$M\frac{d^2x}{dt^2} = 1/2\rho C_d \pi R^2 \left(V - \frac{dx}{dt}\right)^2$$

Rewite to 1st Order Differential Equations

$$\frac{dx}{dt} = u$$

$$\frac{du}{dt} = \frac{\rho C_d \pi R^2}{2M}(V^2 - 2uV + u^2)$$

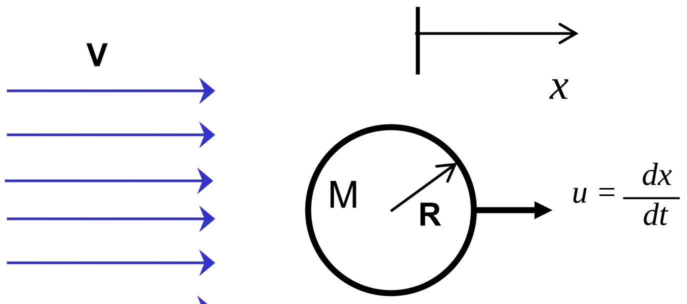
Euler' Method - Difference Equations – First Order scheme

Taylor Series Expansion
(here forward difference)

$$u_{i+1} = u_i + \left(\frac{du}{dt}\right)_i \Delta t, \quad u(0) = 0$$

$$x_{i+1} = x_i + \left(\frac{dx}{dt}\right)_i \Delta t, \quad x(0) = 0$$

Sphere Motion in Fluid Flow
MATLAB Solutions



dudt.m

```
function [f] = dudt(t,u)
% u(1) = u
% u(2) = x
% f(2) = dx/dt = u
% f(1) = du/dt=rho*Cd*pi*r/(2m)*(v^2-2uv+u^2)
rho=1000;
Cd=1;
m=5;
r=0.05;
fac=rho*Cd*pi*r^2/(2*m);
v=1;

f(1)=fac*(v^2-2*u(1)+u(1)^2);
f(2)=u(1);
f=f';
```

sph_drag_2.m

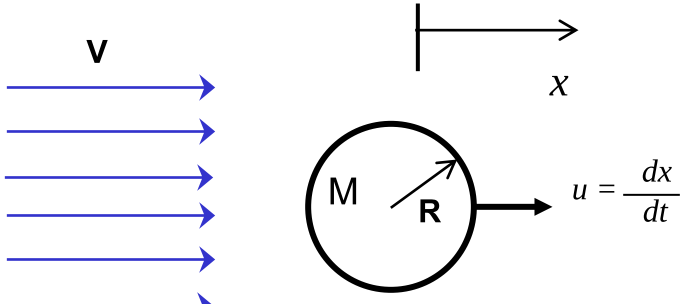
$$u_{i+1} = u_i + \left(\frac{du}{dt}\right)_i \Delta t, \quad u(0) = 0$$

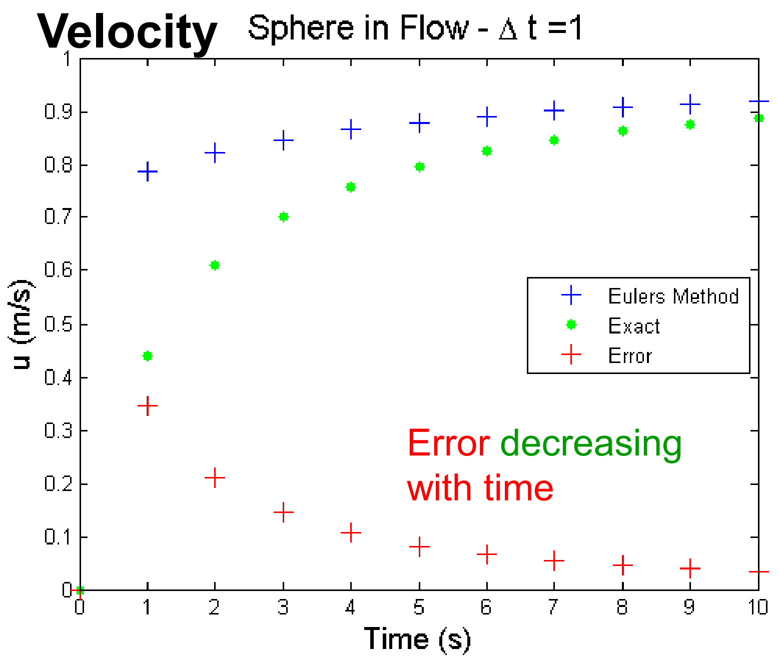
$$x_{i+1} = x_i + \left(\frac{dx}{dt}\right)_i \Delta t, \quad x(0) = 0$$

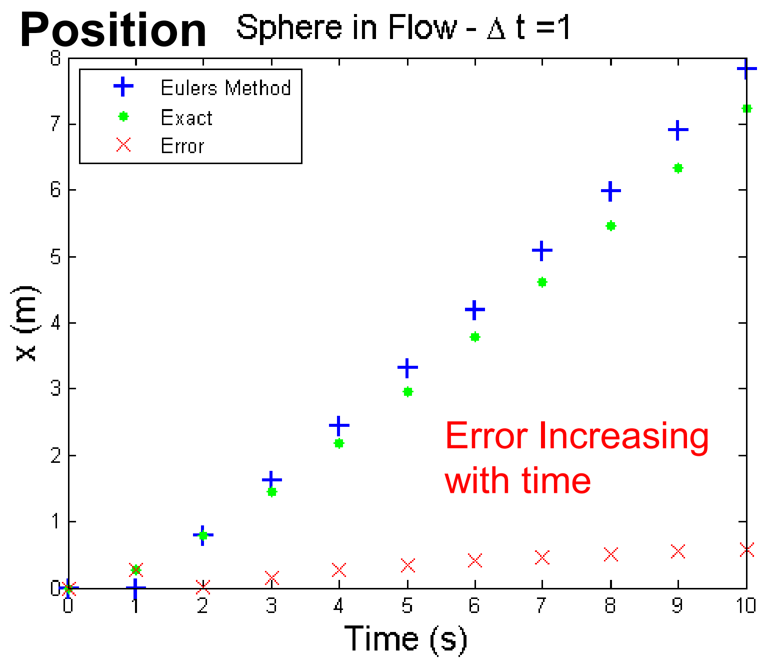
```
x=[0:0.1:10];
%step size
h=1.0;
% Euler's method, forward finite difference
t=[0:h:10];
N=length(t);
u_e=zeros(N,1);
x_e=zeros(N,1);
u_e(1)=0;
x_e(1)=0;
for n=2:N
    u_e(n)=u_e(n-1)+h*fac*(v^2-2*v*u_e(n-1)+u_e(n-1)^2);
    x_e(n)=x_e(n-1)+h*u_e(n-1);
end
% Runge Kutta
u0=[0 0]';
[tt,u]=ode45(@dudt,t,u0);

figure(1)
hold off
a=plot(t,u_e,'+b');
hold on
a=plot(tt,u(:,1),'.g');
a=plot(tt,abs(u(:,1)-u_e),'+r');
...
figure(2)
hold off
a=plot(t,x_e,'+b');
hold on
a=plot(tt,u(:,2),'.g');
a=plot(tt,abs(u(:,2)-x_e),'xr');
...
```

Sphere Motion in Fluid Flow

Error Propagation







2.29 Numerical Fluid Mechanics
Errors

From mathematical models to numerical simulations (e.g 1D Sphere in 1D flow)

Continuum Model – Differential Equations

=> Difference Equations (often uses Taylor expansion and truncation)

=> Linear/Non-linear System of Equations

=> Numerical Solution (matrix inversion, eigenvalue problem, root finding, etc)

Motivation: What are the uncertainties in our computations and are they tolerable? How do we know?

Error Types

- Round-off error: due to representation by computers of numbers with a finite number of digits
- Truncation error: due to approximation/truncation by numerical methods of "exact" mathematical operations/quantities
- Other errors: model errors, data/parameter input errors, human errors.

Numerical Fluid Mechanics - Outline

- Approximation and round-off errors
  - Number representations.
  - Arithmetic operations
  - Errors of numerical operations.
  - Recursion algorithms

- Truncation Errors, Taylor Series and Error Analysis
  - Error propagation – numerical stability
  - Error estimation
  - Error cancellation
  - Condition numbers

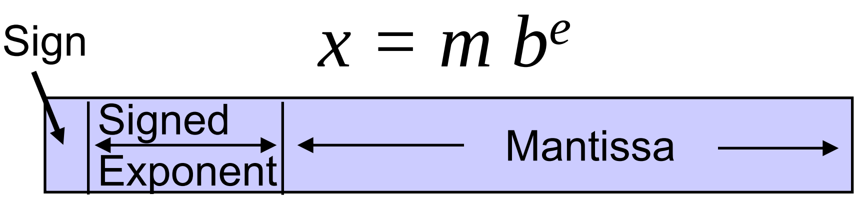
Approximations and Round-off errors

- Significant digits: Numbers that can be used with confidence
  - e.g. 0.001234 and 1.234 $4.56\ 10^3$ and 4,560
  - Omission of significant digits in computers = round-off error
- Accuracy: “how close an estimated value is to the truth”
- Precision: “how closely estimated values agree with each other”
- **True error**: $E_t = \text{Truth - Estimate} = x^t - \hat{x}$
- **True relative error**: $\varepsilon_t = \dfrac{\text{Truth - Estimate}}{\text{Truth}} = \dfrac{x^t - \hat{x}}{x^t}$
- In reality, x^t unknown => use best estimate available $\hat{x}_a$
- Hence, what is used is: $\varepsilon_a = \dfrac{\hat{x}_a - \hat{x}}{\hat{x}_a}$
- Iterative schemes, $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$, stop when $|\varepsilon_a| = \left|\dfrac{\hat{x}_n - \hat{x}_{n-1}}{\hat{x}_n}\right| \le \varepsilon_s$
- For n digits: $\varepsilon_s = \dfrac{1}{2} 10^{-n}$

Number Representations

- **Number Systems:**
  - Base-10: $1{,}234_{10} = 1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$
  - Computers (0/1): base-2 $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10}$
- **Integer Representation (signed magnitude method):**
  - First bit is the sign (0,1), remaining bits used to store the number
  - For a 16-bits computer:
    - Example: -13_{10} = 1 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1
    - Largest range of numbers: $2^{15}-1$ largest number => -32,768 to 32,767 (from 0 to 111111111111111)
- **Floating Number Representation**

$$x = m\, b^e$$



m	Mantissa/Significand = fractional part
b	Base
e	Exponent