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Approaching Gaussian Relay Network Capacity in the High SNR Regime: End-to-End Lattice Codes

Yun Xu

Dept. of Electrical Eng.
Yale University
New Haven, CT 06511
Email: yun.xu@yale.edu

Edmund Yeh

Dept. of Electrical & Computer Eng.
Northeastern University
Boston, MA 02115
Email: eyeh@ece.neu.edu

Muriel Médard

Dept. of Electrical Eng. & Computer Sci.
Massachusetts Institute of Technology
Cambridge, MA 02139
Email: medard@mit.edu

Abstract—We present a natural and low-complexity technique for achieving the capacity of the Gaussian relay network in the high SNR regime. Specifically, we propose the use of end-to-end structured lattice codes with the amplify-and-forward strategy, where the source uses a nested lattice code to encode the messages and the destination decodes the messages by lattice decoding. All intermediate relays simply amplify and forward the received signals over the network to the destination. We show that the end-to-end lattice-coded amplify-and-forward scheme approaches the capacity of the layered Gaussian relay network in the high SNR regime. Next, we extend our scheme to non-layered Gaussian relay networks under the amplify-and-forward scheme, which can be viewed as a Gaussian intersymbol interference (ISI) channel. Compared with other schemes, our approach is significantly simpler and requires only the end-to-end design of the lattice precoding and decoding. It requires little knowledge of the network topology or the individual channel gains.

I. INTRODUCTION

Finding the capacity of Gaussian relay networks with one source, one destination, and a set of relays, has been a long-standing open problem in network information theory. The relay channel was first investigated in the seminal work of Cover and El Gamal [1]. More recently, Kramer *et al.* considered transmission techniques for larger Gaussian relay networks, *e.g.*, the amplify-and-forward, decode-and-forward, and compressed-and-forward schemes [2]. Avestimeher *et al.* presented a deterministic model for Gaussian relay networks and proposed the quantize-map-and-forward strategy [3]. It has been shown that the quantize-map-and-forward strategy can achieve a rate within a constant number of bits from the information-theoretic cut-set bound for Gaussian relay networks, which is independent of the channel gains and the operating SNR [3]. In [4], Ozgur and Diggavi incorporated lattice codes, lattice quantization, and lattice-to-lattice mapping into the quantize-map-and-forward scheme. It was shown that the lattice-based quantize-map-and-forward scheme can still achieve the capacity of Gaussian relay networks within a constant gap. While offering strong performance in terms of achievable rates, the schemes presented in [3], [4], involve considerable operational complexity at intermediate relays.

As pointed out in [5], in wireless communication settings, signals simultaneously transmitted from different sources add, leading to the receiver obtaining a superposition of these signals, scaled by the channel gains. Since the relays are

not interested in the messages sent by the source, they do not necessarily have to decode, compress or quantize the messages. Since the relays have already observed the sum of the signals, in some settings a natural strategy would be to simply amplify and forward without explicitly dealing with the noise.

A multihop amplify-and-forward scheme with random encoding and decoding was proposed in [5]. In this strategy, the message sent by the source is propagated over many intermediate nodes (relays) and possibly over multiple hops. All relays exploit the interference and forward the received signals over the network to the destination. It has been shown in [5] that the achievable rate of the multihop amplify-and-forward scheme approaches the capacity of the Gaussian relay network when the SNR at the destination is sufficiently high.

In this paper, we propose the use of end-to-end structured lattice codes with the amplify-and-forward strategy, where the source uses a nested lattice code to encode the messages and the destination decodes the messages by lattice decoding. All intermediate relays simply amplify and forward the received signals over the network to the destination. Relative to the random coding approach of [5], the use of structured lattice codes significantly reduces system complexity by making possible computationally tractable encoding and decoding. Furthermore, the use of end-to-end lattice codes implies that we require little information concerning the network topology, or individual channel gains. Instead, we require only the end-to-end channel response, which can be obtained by using probing signals.

We show that the end-to-end lattice-coded amplify-and-forward scheme approaches the capacity of the layered Gaussian relay network under the high-SNR condition presented in [5]. This result is facilitated by the key observation that a Gaussian layered relay network under amplify-and-forward is equivalent to a point-to-point Gaussian channel. Next, we extend our scheme to non-layered Gaussian relay networks under the amplify-and-forward scheme, which can crucially be viewed as a Gaussian intersymbol interference (ISI) channel. Our lattice-coded amplify-and-forward scheme is simpler than the lattice-based quantize-map-and-forward scheme proposed in [4], since it does not require lattice quantization and lattice-to-lattice mapping at relay nodes. For layered networks, only

end-to-end design of the precoding and decoding with nested lattice codes is required. Thus, the end-to-end lattice-coded amplify-and-forward scheme is a natural and low-complexity technique for achieving the capacity of the Gaussian relay network in the high SNR regime.

The nested lattice code was originally proposed by Erez *et al.* in [6], [7] and [8]. In [7], it was shown that the nested lattice code with lattice decoding can achieve the capacity of the additive white Gaussian noise (AWGN) channel at any SNR. Erez *et al.* showed that the power-constrained AWGN channel can be transformed into the modulo-lattice additive noise (MLAN) channel by minimum mean-square error (MMSE) scaling along with dithering. The capacity of the MLAN channel, achieved by uniform inputs, becomes the capacity of the AWGN channel in the limit of large lattice dimension.

In [6] and [8], Erez *et al.* extended their techniques to the AWGN dirty-paper channel. They obtained the achievable rate at any SNR by incorporating MMSE scaling. It was shown that with an appropriate choice of the lattice, the achievable rate approaches the capacity of the AWGN dirty-paper channel as the dimension of the lattice goes to infinity. These results provide an information-theoretic framework used in [6] to study precoding for the Gaussian ISI channel. Erez *et al.* showed that when combined with the techniques of interleaving/deinterleaving and water-filling, nested lattice precoding and decoding can achieve the capacity of the Gaussian ISI channel [6].

The remainder of this paper is organized as follows. The network model is presented in Section II. The fundamental properties of lattices and nested lattice codes are summarized in Section III. The main result on the lattice-coded multihop amplify-and-forward strategy is presented in Section IV. Section V extends the analysis to non-layered networks. Section VI concludes the paper.

II. NETWORK MODEL

A. Layered Network

We first focus on the layered network in which each path from the source to the destination has the same number of hops. We denote the layer l by \mathcal{L}_l , $l = 0, 1, \dots, L$. Assume that the source s is located at layer \mathcal{L}_0 , and the destination d at layer \mathcal{L}_L . We denote the number of relays at layer \mathcal{L}_l by n_l , and thus $\sum_{l=1}^{L-1} n_l = N$. In a layered network, the input-output relationship is simple due to the fact that all copies of a source message transmitted on different paths arrive at the destination simultaneously. An example of a layered network is shown in Figure 1.

Now consider a layered Gaussian relay network consisting of a single source s , a single destination d , and a set of N relays. The communication link from node i to node j has a nonnegative real channel gain, represented by $h_{ij} \in \mathbb{R}_+$. The channel output at node $j \neq s$ is

$$y_j = \sum_{i \in \mathcal{N}(j)} h_{ij} x_i + z_j. \quad (1)$$

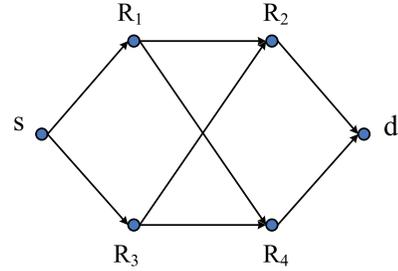


Fig. 1. Example of a Layered Network

where x_i is the channel input at node i , z_j is the real Gaussian noise with zero mean and unit variance, and $\mathcal{N}(j)$ denotes the set of nodes that can transmit to node j with a direct link, i.e., $\mathcal{N}(j) = \{i : h_{ij} > 0\}$. Note that the links are assumed to be directed so that $i \in \mathcal{N}(j)$ does not imply $j \in \mathcal{N}(i)$. We assume that there is an average power constraint at each node:

$$E[X_i^2] \leq P_i. \quad (2)$$

The source s wishes to send a message from a message set $\mathcal{W} = \{1, \dots, 2^{nR}\}$ to the destination d with transmission rate R . The encoding function at the source is given by $X_s^n = f(W)$, $W \in \mathcal{W}$, and a decoding function at the destination d is given by $\hat{W} = g(Y_d^n)$. A (R, n) code consists of a message set \mathcal{W} , an encoding function at the source, and a decoding function at the destination. The average error probability of the (R, n) code is given by $P_e = \Pr[\hat{W} \neq W]$. A rate R is said to be achievable if for any $\epsilon > 0$, there exists a (R, n) code such that $P_e \leq \epsilon$ for a sufficiently large n .

B. High SNR Regime

As in [5], we are interested in the scenario in which all relays forward the data with large enough power to guarantee that the total propagated noise at the destination by multihop amplify-and-forward is low enough. Assume that each node i transmits with the average power P_i given by (2). The power received at relay $j \in \mathcal{L}_l$, $l = 1, \dots, L-1$, is then determined by

$$P_{R,j} = \left(\sum_{i \in \mathcal{L}_{l-1}} h_{ij} \sqrt{P_i} \right)^2, \quad j \in \mathcal{L}_l$$

and the power received at the destination d is given by

$$P_d = \left(\sum_{i \in \mathcal{L}_{L-1}} h_{id} \sqrt{P_i} \right)^2. \quad (3)$$

As in [5], we consider a high SNR regime where for some small $\delta > 0$, the transmit powers of the relays satisfy

$$\min_{j \in \mathcal{L}_l} P_{R,j} \geq \frac{1}{\delta}, \quad l = 1, \dots, L-1. \quad (4)$$

We then assume that P_d remains a constant as $\delta \rightarrow 0$, so that the Multiple-Access Channel (MAC) at the destination

is a bottleneck for the data transmission.¹ The MAC cut-set bound is given by

$$C_{MAC} = \frac{1}{2} \log(1 + P_d). \quad (5)$$

III. LATTICES AND NESTED LATTICE CODES

In this section, we briefly review some basic properties of lattices and nested lattice codes. A more extensive discussion can be found in references such as [6] and [7]. A lattice Λ is a discrete subgroup of the Euclidean space \mathbb{R}^n . If λ_1 and λ_2 are two elements of a lattice Λ , then the sum $\lambda_1 + \lambda_2$ and the additive inverse $-\lambda_1$ are also elements of Λ . Any lattice can be written in terms of its generating matrix G :

$$\Lambda = \{\lambda = G\mathbf{x} : \mathbf{x} \in \mathbb{Z}^n\}.$$

We can then define the nearest neighbor quantizer associated with Λ by

$$Q_\Lambda(\mathbf{x}) = \operatorname{argmin}_{\lambda \in \Lambda} \|\mathbf{x} - \lambda\|.$$

The Voronoi region of a lattice point $\lambda \in \Lambda$ is the set of all points that quantize to it. The fundamental Voronoi region \mathcal{V} is the set of all points that quantize to the origin, i.e.,

$$\mathcal{V} = \{\mathbf{x} : Q_\Lambda(\mathbf{x}) = \mathbf{0}\}.$$

Define the modulo- Λ operation corresponding to \mathcal{V} as

$$\mathbf{x} \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x}).$$

The second moment of a lattice Λ is defined by

$$\sigma_\Lambda^2 = \frac{1}{n \operatorname{Vol}(\mathcal{V})} \int_{\mathcal{V}} \|\mathbf{x}\|^2 d\mathbf{x},$$

and the normalized second moment of a lattice Λ is defined by

$$G(\Lambda) = \frac{1}{n [\operatorname{Vol}(\mathcal{V})]^{1+2/n}} \int_{\mathcal{V}} \|\mathbf{x}\|^2 d\mathbf{x},$$

where $\operatorname{Vol}(\mathcal{V})$ is the volume of \mathcal{V} .

Two lattices Λ_1 and Λ_2 are said to be nested if $\Lambda_1 \subseteq \Lambda_2$, where Λ_1 is called the coarse lattice and Λ_2 the fine lattice. Denote by \mathcal{V}_1 and \mathcal{V}_2 the fundamental Voronoi regions of Λ_1 and Λ_2 , respectively. The coding rate is defined by

$$R = \frac{1}{n} \log \left[\frac{\operatorname{Vol}(\mathcal{V}_1)}{\operatorname{Vol}(\mathcal{V}_2)} \right].$$

The points in the set

$$\mathcal{C} = \Lambda_2 \cap \mathcal{V}_1$$

are called the coset leaders of Λ_1 relative to Λ_2 . For each $\mathbf{c} \in \mathcal{C}$, the shifted coarse lattice $\Lambda_{1,\mathbf{c}} = \mathbf{c} + \Lambda_1$ is called a coset of Λ_1 relative to Λ_2 .

The use of high-dimensional nested lattice code is justified by the existence of asymptotically good lattices. We consider two types of goodness as introduced in [6] and [7].

¹Note that for network capacity, the worst case occurs when the bottleneck is at the MAC at the destination. In this case, the noise is propagated over more hops than in any other case.

(1) *Good for AWGN Channel Coding*: For any $\epsilon > 0$ and sufficiently large n , there exists an n -dimensional lattice Λ with the volume of the fundamental Voronoi region $\operatorname{Vol}(\mathcal{V}) < 2^{n[h(Z)+\epsilon]}$, where Z is Gaussian noise with variance σ_Z^2 , and $h(Z) = \frac{1}{2} \log(2\pi e \sigma_Z^2)$ is the differential entropy of Z , such that

$$P_e = \Pr[Z \notin \mathcal{V}] < \epsilon.$$

(2) *Good for Source Coding*: For any $\epsilon > 0$ and sufficiently large n , there exists an n -dimensional lattice Λ whose normalized second moment $G(\Lambda)$ satisfies

$$\log(2\pi e G(\Lambda)) < \epsilon.$$

IV. LATTICE-CODED MULTIHOP AMPLIFY-AND-FORWARD

In [5], it is shown that the simple multihop amplify-and-forward scheme with random coding can approximately achieve the unicast capacity of a Gaussian relay network in the high SNR regime. In this paper, we propose the use of structured nested lattice codes in conjunction with the multihop amplify-and-forward scheme. Choose a pair of nested lattices (Λ_1, Λ_2) , $\Lambda_1 \subseteq \Lambda_2$, with the coding rate

$$R = \frac{1}{n} \log \left[\frac{\operatorname{Vol}(\mathcal{V}_1)}{\operatorname{Vol}(\mathcal{V}_2)} \right] \geq R_{LAF},$$

where R_{LAF} is the rate achieved by the lattice-coded amplify-and-forward scheme defined by (10). We choose the coarse lattice Λ_1 to be good for source coding, with the second moment $\sigma_{\Lambda_1}^2 = P_s$, where P_s is the average power constraint of the source node s . We choose the fine lattice Λ_2 to be good for AWGN channel coding. Let Q_{Λ_2} denote the nearest neighbor quantizer of the fine lattice Λ_2 . We apply the scheme proposed in [6], [7] and [8] to the layered Gaussian relay network as follows:

(1) *Source*: the source s maps the message $W \in \mathcal{W}$ uniformly at random to a coset leader of Λ_1 relative to Λ_2 :

$$\mathbf{t} \in \mathcal{C} = \Lambda_2 \cap \mathcal{V}_1.$$

Then a random dither vector \mathbf{u} is generated uniformly over \mathcal{V}_1 , i.e., $\mathbf{u} \sim \operatorname{Unif}(\mathcal{V}_1)$. Given the message W , the source encoder sends

$$\mathbf{x}_s = [\mathbf{t} + \mathbf{u}] \bmod \Lambda_1. \quad (6)$$

(2) *Relay*: each relay $i \in \mathcal{L}_l, l = 1, \dots, L-1$, performs the multihop amplify-and-forward scheme

$$\mathbf{x}_{l,i} = \beta_i \mathbf{y}_{l,i}, \quad i \in \mathcal{L}_l \quad (7)$$

where the amplification gain is chosen as

$$\beta_i = \frac{\sqrt{P_i}}{\sqrt{(1+\delta)P_{R,i}}}, \quad i \in \mathcal{L}_l. \quad (8)$$

In [5], it is shown that the power constraint (2) at each node i is satisfied by choosing the amplification gain in (8). Also shown in [5] are the following two results on the propagated noise.

Lemma 1: [5] At any node $i \in \mathcal{L}_l$, the noise propagated from all nodes in layer $\mathcal{L}_{l-k}, k = 1, \dots, l-1$, via the multihop

amplify-and-forward scheme in the high SNR regime has the power

$$P_{z,i}^{l-k} \leq \frac{\delta P_{R,i}}{(1+\delta)^k}.$$

Corollary 2: [5] The total noise propagated to the destination $d \in \mathcal{L}_L$ has the power

$$P_{z,d} = \sum_{k=1}^{L-1} P_{z,d}^{L-k} = \delta P_d \sum_{k=1}^{L-1} \frac{1}{(1+\delta)^k} \leq L\delta P_d. \quad (9)$$

(3) *Destination:* the destination computes

$$\hat{\mathbf{y}}_d = Q_{\Lambda_2}(\alpha \mathbf{y}_d + \mathbf{u}) \bmod \Lambda_1,$$

where

$$\alpha = \frac{\gamma}{1+\gamma}, \quad \gamma = \frac{P_d}{(1+\delta)^{L-1} P_{z,d}}.$$

The following theorem is the main result for the lattice-coded amplify-and-forward scheme.

Theorem 3: In a layered relay network (1) in the high SNR regime defined by (4), the lattice-coded multihop amplify-and-forward achieves the rate

$$R_{LAF} \geq \frac{1}{2} \log \left[1 + \frac{1}{(1+\delta)^{L-1}} \frac{P_d}{1+L\delta P_d} \right]. \quad (10)$$

Proof. As in [5], if the amplification gain at each relay is chosen as (8), the received signal at the destination can be written as

$$\mathbf{y}_d = \hat{h}_d x_s + \hat{z}_d + z_d, \quad (11)$$

where \hat{z}_d the total propagated noise, z_d is the noise at the destination, and

$$\hat{h}_d = \frac{\sqrt{P_d}}{\sqrt{P_s(1+\delta)^{L-1}}}. \quad (12)$$

By (11) and (12), the received signal power at the destination is

$$\hat{P}_d = \frac{P_d}{(1+\delta)^{L-1}}.$$

By Corollary 2, the power of the total propagated noise \hat{z}_d is

$$P_{z,d} = \delta P_d \sum_{k=1}^{L-1} \frac{1}{(1+\delta)^k} \leq L\delta P_d.$$

Therefore the SNR at the destination satisfies

$$SNR \geq \frac{1}{(1+\delta)^{L-1}} \frac{P_d}{1+L\delta P_d}. \quad (13)$$

In other words, the received signal y_d can be viewed as the output of the AWGN channel characterized by (11) with the SNR given by (13). The capacity, or equivalently the achievable rate via amplify-and-forward, is then given by

$$\begin{aligned} R_{LAF} &= \frac{1}{2} \log(1+SNR) \\ &\geq \frac{1}{2} \log \left[1 + \frac{1}{(1+\delta)^{L-1}} \frac{P_d}{1+L\delta P_d} \right]. \end{aligned}$$

It is shown in [6] and [7] that if we choose the coarse lattice to be good for source coding and the fine lattice Λ_2 to be good for AWGN channel coding, nested lattice codes can achieve the capacity of the AWGN Gaussian channel when the dimension, or equivalently, the length of the codewords n , tends to infinity. Hence, the above lattice-coded amplify-and-forward scheme can achieve the rate R_{LAF} . ■

Note that as $\delta \rightarrow 0$, the rate achieved by the lattice-coded multihop amplify-and-forward scheme in (10) approaches the MAC cut-set bound (5), and the unicast capacity of the Gaussian relay network.

V. EXTENSIONS

A. Non-layered Networks

In layered networks, each path from the source to the destination has the same number of hops, so that all copies of the source message transmitted on different paths arrive at the destination with the same delay. In non-layered networks, however, copies of the source message may arrive at the destination with different delays through different paths. Assume that the number of hops (length) in the longest path is $L \geq 1$. We can then classify all paths from the source to the destination according to the path length

$$P_l = \{\text{paths of length } l\}$$

Assume that the number of paths of length l is $K_l, l = 1, \dots, L$. As shown in [5], the received signal at the destination d at time t is then given by

$$\begin{aligned} y_d(t) &= h_0 x_s(t) + \sum_{j \in P_1} h_{j,1} x_s(t-1) + \dots \\ &\quad + \sum_{j \in P_L} h_{j,L} x_s(t-L) + z_e(t), \end{aligned}$$

where h_0 is the channel gain on the direct link from the source to the destination, and $h_{j,l}$ is the equivalent channel gain of path j in the set P_l . Note that $h_{j,l}$ depends on the network topology, and contains the accumulated channel gains and amplification gains on the source-destination path j . Finally, $z_e(t)$ denotes the total propagated noise at the destination.

From (14), we see that under the amplify-and-forward scheme, the non-layered Gaussian relay network is equivalent to a Gaussian ISI channel:

$$y_d(t) = \sum_{l=0}^L h_l x_s(t-l) + z_e(t), \quad (14)$$

where $x_s(t-l), l = 0, \dots, L$, are the inputs to the Gaussian ISI channel, $h_l = \sum_{j \in P_l} h_{j,l}$ represents the ISI coefficient, and $y_d(t)$ stands for the received samples. The additive Gaussian noise is denoted by $z_e(t)$.

We now focus on the feedforward MMSE decision feedback (MMSE-DFE) equalizing filter for the Gaussian ISI channel [9], [10]. The output of the MMSE-DFE feedforward filter can be written as

$$r(t) = x_s(t) + s(t) + n(t), \quad (15)$$

where $s(t) = \sum_{l=1}^L \hat{h}_l x_s(t-l)$ is the post-cursor intersymbol interference, and \hat{h}_l and $n(t)$ represent the ISI coefficients and the sampled noise at the output of the MMSE-DFE feedforward filter, respectively.

The SNR associated with the MMSE-DFE filter is defined by [9]

$$SNR_{MMSE-DFE} = \frac{E[X_s^2(t)]}{E[N^2(t)]},$$

and the capacity of the Gaussian ISI channel is given by [10]

$$C_{ISI} = \frac{1}{2} \log(1 + SNR_{MMSE-DFE}). \quad (16)$$

If the encoder knows the entire post-cursor intersymbol interference vector before transmission, as mentioned in [6], the input-output relationship given by (15) can be viewed as the Gaussian dirty-paper channel whose capacity is given by (16). Based on that observation, [6] proposed a coding strategy for the Gaussian ISI channel, in which the MMSE-DFE feedback equalizing filter is replaced by nested lattice precoding, as described in the dirty paper case. In the interleaver, the messages are encoded row by row and are transmitted column by column. When a message which comprises the j th row of the interleaver is to be encoded, the post-cursor interfering symbols belong to the codewords for messages which have been already encoded, similar to the dirty paper scenario [6]. In [6], it is shown that nested lattice precoding with interleaving/deinterleaving and waterfilling can achieve the capacity of the Gaussian ISI channel given by (16). Waterfilling is required because the sampled noise $n(t)$ in (15) may not be white Gaussian. If we incorporate nested lattice precoding into the multihop amplify-and-forward scheme, the achievable rate can thus be obtained as the capacity of the corresponding Gaussian ISI channel. The lattice-coded multihop amplify-and-forward scheme is shown in Figure 2.

Unlike the case for layered networks, in order to implement nested lattice precoding with interleaving for non-layered Gaussian wireless relay networks, it is necessary to know the channel gains as manifested in the ‘‘ISI coefficients.’’ In the absence of such knowledge, techniques such as blind equalization may be needed to preserve the performance of our scheme.

If the encoder does not know the entire post-cursor ISI vector before transmission, we have to use the original MMSE-DFE technique proposed in [9] with nested lattice encoding and decoding. In that case, however, the decoded messages must be fed back. Thus, any decoding error will affect the performance of the MMSE-DFE feedback filter, and hence the subsequent decoding process. This may lead to the decoding error propagating over multiple symbols.

VI. CONCLUSION

In this paper, we considered an end-to-end lattice-coded multihop amplify-and-forward strategy for Gaussian wireless relay networks in the high SNR regime. When the power received at all relays are large enough, our strategy performs well for both layered and non-layered Gaussian relay networks. In

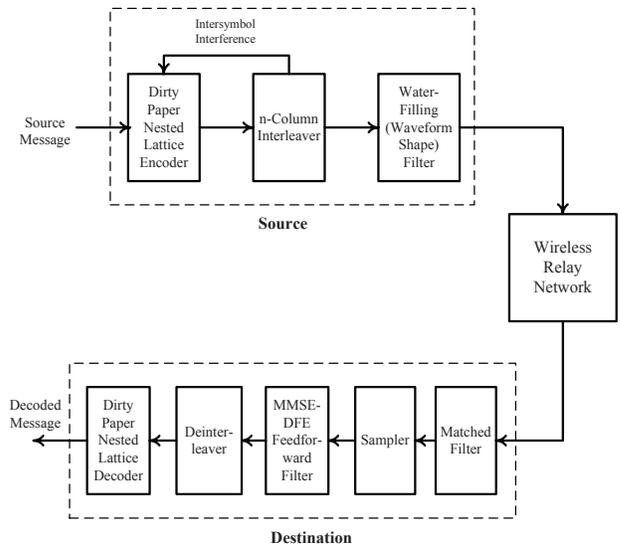


Fig. 2. Lattice-Coded Multihop Amplify-and-Forward Scheme

the worst case, the bottleneck of the multihop transmission is at the multi-access channel (MAC) at the destination. We showed that our strategy approaches the MAC cut-set bound as the received powers at the relays increase. The lattice-coded multihop amplify-and-forward scheme is simpler than the decode-and-forward scheme and the quantize-map-and-forward scheme. Our scheme requires only end-to-end design: lattice precoding at the source and decoding at the destination. It does not require any knowledge of the network topology or the individual channel gains.

REFERENCES

- [1] T. M. Cover and A. E. Gamal, ‘‘Capacity theorems for the relay channel,’’ *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, 1979.
- [2] G. Kramer, M. Gastpar, and P. Gupta, ‘‘Cooperative strategies and capacity theorems for relay networks,’’ *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.
- [3] A. S. Avestimehr, S. N. Diggavi, and D. Tse, ‘‘Wireless network information flow: A deterministic approach,’’ *IEEE Transactions on Information Theory*, vol. 57, no. 4, pp. 1872–1905, 2011.
- [4] A. Ozgur and S. Diggavi, ‘‘Approximately achieving Gaussian relay network capacity with lattice codes,’’ in *Proceedings of IEEE International Symposium on Information Theory*. Austin, Texas, USA, 2010.
- [5] I. Maric, A. Goldsmith, and M. Médard, ‘‘Multihop analog network coding via amplify-and-forward: The high SNR regime,’’ *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 793–803, 2012.
- [6] R. Zamir, S. Shamai, and U. Erez, ‘‘Nested linear/lattice codes for structured multiterminal binning,’’ *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1250–1276, 2002.
- [7] U. Erez and R. Zamir, ‘‘Achieving $\frac{1}{2} \log(1 + SNR)$ on the AWGN channel with lattice encoding and decoding,’’ *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2293–2314, 2004.
- [8] U. Erez, S. S. (Shitz), and R. Zamir, ‘‘Capacity and lattice-strategies for cancelling known interference,’’ *IEEE Transactions on Information Theory*, vol. 51, no. 11, pp. 3820–3833, 2005.
- [9] J. Cioffi, G. Dudevoir, M. Eyuboglu, and G. Forney, ‘‘MMSE decision-feedback equalizers and coding-part I: Equalization results,’’ *IEEE Transactions on Communications*, vol. 43, no. 10, pp. 2582–2594, 1995.
- [10] —, ‘‘MMSE decision-feedback equalizers and coding-part II: Coding results,’’ *IEEE Transactions on Communications*, vol. 43, no. 10, pp. 2595–2604, 1995.