Incorporating Sell-up in Airline Revenue Management

by

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Abstract

The decision to buy a fare that is higher than the desired fare, under the situation when the desired fare is not available, is known as *sell-up*. Passengers' willingness to sell-up can have considerable impact on airline revenues. The extent of this impact is dependent upon the method used to control booking limits and other parameters associated with passenger demand and fare structure.

In this thesis we demonstrate the importance of incorporating sell-up in airline revenue management. The improvement in revenue, under various scenarios, and under various seat inventory control algorithms, is discussed. We also analyze the influence of demand factor, spill, sell-up rate and fare ratio on these improvements.

A modification of the EMSRb heuristic is proposed to capture the revenue potential associated with passenger sell-up. The proposed rule increases the protection levels, obtained from the EMSRb algorithm, as long as the expected gain, from every additional seat protected, is greater than the expected loss. Unlike the existing models, the proposed rule uses expected spill to determine the expected number of passengers that would sell-up at a given demand level and sell-up rate, and then adjusts the protection levels accordingly. This makes it robust to variations in demand levels.

We have also developed a simulation to compare the performance of the existing rules with that of the proposed heuristic. The simulation has the ability to account for errors in sell-up estimation and variability in demands. It is shown that the revenue gains under the proposed rule may not exist under all situations. In the tests performed in this thesis, the improvements over the original EMSRb algorithm vary from 0% to over 2.5%. Although the gains are not consistent, the proposed rule does not cause any negative impact on overall revenues and hence is unlikely to pose any risk when implemented over the original EMSRb algorithm.

Thesis Advisor: Dr. Peter P. Belobaba
Title: Associate Professor of Aeronautics and Astronautics

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Chapter 1

Introduction

This chapter gives a brief overview of the field of Revenue Management. The two most important components, Differential Pricing and Seat Inventory Control, are discussed. We then introduce the concept of sell-up in Airline Revenue Management. Finally, the goal and structure of this thesis is presented.

1.1 Revenue Management

Revenue Management, Yield Management or Perishable Asset Revenue Management is a set of techniques used by airlines to maximize revenues through differential pricing and seat inventory control. *Differential Pricing* is the practice of offering a variety of fare products differentiated in terms of service amenities and/or travel restrictions, at a variety of price levels. *Seat inventory control* is the practice of determining the number of seats on a flight to be made available for sale to a particular fare product[1].

[1] P.P. Belobaba, "*Airline O-D Seat Inventory Control without Network Optimization*", June 1995.

1.1.1 Differential Pricing

During the period of U.S. airline regulation, the fare structure of airlines was relatively simple and static. Civil Aeronautics Board (CAB) controlled all fares. The fare for each market was established on the basis of average industry cost and distance. All airlines offered similar fare structure and thus there was no competition on the basis of price. Moreover, the CAB fare formula guaranteed a reasonable return on investment for all carriers. Since the airlines could not compete on fares, improving service was the only available option. In an attempt to increase market share, the airlines provided high quality in-flight service and increased their frequency. The increased frequency resulted in lower load factors and a significant number of surplus seats. The airlines then faced the problem of utilizing the available capacity, which was possible by stimulating demand through lower fares. Although the regulations did not allow much flexibility in fares, nevertheless, a few attempts were made to generate new passengers through discount pricing schemes. Among them the most popular ones were student fares, night coach fares and "supersaver" fares introduced by American Airlines. Thus, the use differential pricing, though extremely limited, started even prior to deregulation in the U.S. airline industry.

After deregulation, airlines were free to come up with their own fare structure and change their fares as often as they desired. The most common approach was to differentiate the market into segments based upon consumer behavior and charge each segment according to its willingness to pay. This led to a very dynamic and complex fare structure. Airlines came up with a number of fares in each origin destination (OD) market.

Successful differential pricing requires effective market segmentation. For this purpose, airlines used "fences", which took the form of various restrictions associated with each "fare product". In airline terminology, the term fare product, refers to the fare and the restrictions associated with it. Some common restrictions used by the airlines are

- Saturday night stay
- Advance purchase requirements
- Round trip
- Non-refundability or partial refundability
- Flight validity (only for special holiday seasons, off-peak periods etc.)

Each of the above restriction is designed according to certain characteristics of passenger behavior and usually has different impact on different types of passengers. For example, the *Saturday night stay* is considered to be the most powerful restriction to restrict business passengers from buying a fare associated with it. This same restriction, however, has negligible impact on leisure passengers. In airline terminology, a business passenger is the one who is traveling to fulfill his business obligations and whose traveling expense is borne by the business. He has a tight schedule and is usually unable to purchase the ticket well in advance. Leisure traveler, on the other hand, is the one who is travelling for the purpose of pleasure and is usually paying from his own pocket. He is comparatively much flexible in his travel plans and can purchase tickets well in advance. Note that there might also be people who would have partial characteristics of both these categories. In his Ph. D. thesis, Belobaba has developed a well structured definition of various types of passengers.

The fare structures essentially depend upon market conditions, passenger types and above all, the competition. Airlines serving in the same market usually offer similar fare products. However, the number of seats made available for each of the fare products may vary dramatically.

1.1.2 Seat Inventory Control

In today's competition, the decisions regarding the fare structure of an airline are very much dependent on its competitors. Most airlines match the fares offered by other airlines. However, the actual number of seats offered in each fare class is comparatively independent. This makes leaves Seat Inventory Control a very important component of Airline Revenue Management that can influence the revenue earning capability of an airline in a competitive environment. However controlling as airlines' inventory is much complex than that for other industries.

Consider the example of the music industry. It does not offer discounts on a newly released album that is initially bought by the real big fans or consumers willing to pay the high price. As time passes, most of the big fans have bought the album. At this time the industry starts giving various discount or incentives on the album. The purpose is to capture the consumers who instead of paying the full price, would rather borrow it form a friend or not listen to it at all. Note that the whole process is quite straightforward because of the fact that the customer with the highest willingness to pay comes first. Airlines do not enjoy this luxury. The passengers willing to pay the full or higher fares are essentially the business people who request seats close to the departure date. On the other hand the low fare consumers are the leisure passengers who is ready to book his seat well in advance. This requires airlines to first forecast the number of high fare passengers and protect seats for them. The number of seats protected should be such that only the seats that are not likely to be sold to higher fare passengers are made available to the lower fare passengers. Figure 1.1 shows a typical booking pattern observed in the U.S. airline industry[1].

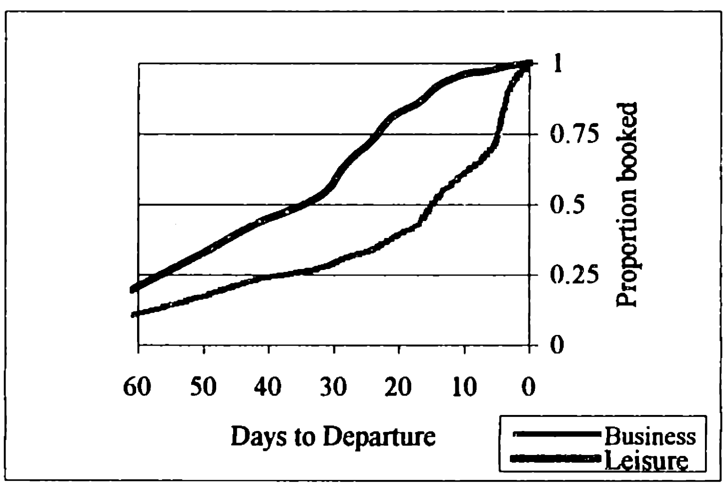

Figure 1.1: Booking pattern, Business versus Leisure passengers

Due to the stochastic nature of demand, the problem of determining the optimal protection level is a complex one. Many attempts have been made to solve the problem.

[1] Based upon experimental scenarios used in PODS, Passenger Origin Destination Simulator, a joint project of MIT Flight Transportation Laboratory and the Boeing Commercial Airplane Group.

Among them, the method of Expected Marginal Seat Revenue (EMSR)[2] and its modifications have been widely adopted by the industry. The method is further discussed in chapter 2.

1.2 Sell-up

Consider the example of a passenger who wishes to travel in a particular OD market. He prefers to buy a cheap fare and requests a particular discounted fare. There are no seats available in his desired fare class and his request is thus refused. At this point the passengers could do one of the following:

- Try another flight for the same OD market, in his desired fare class, with the same airline.
- Try another flight for the same OD market, in his desired fare class, with a different airline.
- Try another flight for a different OD market, with the same or different airline (rare case; only true for some leisure passengers)
- Not travel at all
- Try the next higher fare class on the flight he initially requested.

[2] P. P. Belobaba, "*Air Travel Demand and Airline Seat Inventory Management*", Ph.D. thesis, MIT, May 1987.

The last phenomenon is known as passenger sell-up. Note that all the above options, except for the one when the passenger cancels his travel plans, come under the category of passenger diversion. Sell-up is a special type of diversion in which the passenger accepts a higher fare without changing the flight. It is also known as *vertical shift* or *trade-up*[3].

1.3 Goal of Thesis

According to Robert Crandall, Chairman and CEO of AMR and American Airlines, yield management is "the single most important technical development in transportation management since we entered the era of deregulation in 1979"[4]. The reason is the enormous gains in revenue without any significant costs. Since the development of the Belobaba's EMSR algorithm in 1987, various modifications have been made. The success of each modification is measured by its capability to improve the revenue gains as against the costs or risks associated with it. The purpose of this thesis is to evaluate yet another modification of the EMSR algorithm; the incorporation of passenger sell-up.

The thesis proposes a new approach to incorporate passenger sell-up in the EMSR algorithm. The method considers the expected spill at any demand and uses this value

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[3] Bohutinsky, C.H., "*The Sell-Up Potential of Airline Demand*", MS thesis, MIT Flight Transportation Laboratory, June 1990.
[4] Darrow, Leimkuhler and B. Smith. "*Yield Management at American Airlines*". Interfaces January-February 1992, p.31.

(along with the expected sell-up rate), to compute the expected number of passengers that would sell-up at any protection level. The protection levels are then readjusted to accommodate passenger sell-up. A new simulation is developed to evaluate the performance of this technique. The simulation has the provision of defining different sell-up rates for the EMSR algorithm and the passenger choice. The former can be considered as the sell-up rate assumed by an airline and the later can be considered as the true sell-up rate. By changing he values for the two sell-up rates, under various scenarios, the risk involved in incorporating sell-up is also evaluated.

1.4 Structure of Thesis

The remaining part of this thesis is divided into five chapters. Chapter 2 gives an overview of previous studies done in the area of passenger sell-up. The original EMSRb algorithm and its modifications are briefly described. The study performed by Bohuntinsky is also discussed.

In Chapter 3, we discuss the influence of sell-up on optimal protection levels. We start with a simple deterministic example to show the change in overall revenues by considering passenger sell-up. Then we derive the expressions for optimal protection level under passenger sell-up for simple cases involving two fare classes. By the end of this chapter, we develop our proposed heuristic to incorporate passenger sell-up in the EMSRb algorithm.

In Chapter 4, we present a simulation to analyze the performance of the proposed heuristic. The initial part of the chapter describes the structure and environment of the simulation. In the later part, we simulate two scenarios. The performance of the original EMSRb heuristic and proposed rule is discussed. We primarily focus on the ability of the heuristics to capture the revenue associated with passenger sell-up. The parameters that influence the relative performance of the original and the modified rules are also discussed.

In Chapter 5, we perform sensitivity analysis. We analyze the performance of the modified rules under situations when we do not have perfect knowledge of sell-up rates existing among passengers. We also test situations when there is high variability in demand. The focus is on the changes in relative performance of the modified rules and their robustness to errors on sell-up rate and variability in demand.

Finally, in Chapter 6, we present an evaluation of the proposed rule, summarize our findings and propose future research directions.

Chapter 2

Previous Studies

This chapter gives an overview of the previous work done in the area of passenger sell-up and then explains the difference between the proposed approach and the previous models. It starts with a brief overview of the EMSR algorithm and then goes to the sell-up models and studies done in the past.

2.1 Belobaba's EMSR algorithm

In his doctoral dissertation[1], Belobaba developed a powerful heuristic, which he named as the EMSR heuristic. The heuristic, either in its true form or as a modified version, is being used by a number of airlines to practice Seat Inventory Control. The algorithm considers expected revenues from the stochastic demand for each fare class and determines the protection levels that would maximize the total expected revenue.

The expected number of bookings in any class i, given a seat allocation of S_i, can be defined in the form of the following integral:

[1] Peter. P. Belobaba, *"Air Travel Demand and Airline Seat Inventory Management"*, Ph.D. thesis, Massachusetts Institute of Technology, May 1987.

$$b_i(S_i) = \int_0^{S_i} r_i * p_i(r_i)dr_i + \int_{S_i}^{\infty} p_i(r_i)dr_i * S_i$$

$$b_i(S_i) = \int_0^{S_i} r_i * p_i(r_i)dr_i + P(r_i > S_i) * S_i$$

[2.1]

Where

b_i is the expected number of bookings in class i, at a seat allocation of S_i.

r_i is the random variable for demand in class i

$p_i(r_i)$ is the probability distribution function for the demand in class i

$P_i(r_i > S_i)$ is the probability that the demand for class i would be greater than S_i

Note that the first term in the above equation (integral from 0 to S_i) represents the expected bookings when demand is less then seat allocation and the second term represents the expected bookings when the demand is more than the seat allocation. The expected revenue from class i, can thus be defined as

$$R_i(S_i) = f_i * b_i(S_i)$$

Here f_i is the average fare from class i. The total expected revenue would be the sum of expected revenue from all classes.

Under distinct fare class inventories, a seat can only be sold to a class for which it was initially allocated. Belobaba argued that for distinct fare class inventories, the total revenue would be maximized if we protect seats for the higher class till the following condition holds true.

$$EMSR_i(\pi_i) = EMSR_j(\pi_j)$$

Where

i,j are any two fare classes

π_i is the optimal number of seats allocated for class i

$EMSR_i(\pi_i)$ is the expected marginal seat revenue from the π^{th} seat in class i.

$EMSR_i(\pi_i) = f_i * P(\pi_i)$

Unlike distinct fare class inventories, under nested fare classes, a seat protected for any class can be sold to that class or any higher valued class. For the two-class nested case, Belobaba argued that the revenue would be maximized if the following condition holds true:

$$EMSR_i(\pi_i) \geq f_j \quad [2.2]$$

Where class i is the higher class. In other words, the revenue would be maximized if we protect seats for the higher class till the expected revenue from protecting that seat is more than the next lower fare.

The same concept was used for more than two classes. The total protection level for the higher class is then the sum of the protections against each of the lower classes. In order to reduce computational effort, this approach was later modified. Aggregate values were used to represent combined fare and demand density for all higher classes. The algorithm is known as EMSRb. According to the EMSRb algorithm, for any class i, we keep on protecting seats (for class i and higher), till the following condition holds true:

$$EMSR_i(\pi_i) \geq f_{i+1}$$

[2.3]

Here

$EMSR_i(\pi_i)$ = expected marginal seat revenue from class i and higher

$= f_{1,i} * P(r_{1,i} > S)$

$f_{1,i}$ represents the aggregated fare of class i and higher. It is actually a weighted fare for the higher classes. It is determined as:

$$f_{1,i} = \frac{\sum_{x=1}^{i} f_x * D_x}{\sum_{x=1}^{i} D_x}$$

Where class 1 is the highest valued class D_x is mean demand for any class x.

In the above formulation, $r_{i,n}$ is the aggregated demand for class i and higher. It is the sum of mean demands for class i and all classes above class i. $P(r_{1,i}>S)$ is the probability that the combined demand for class i and higher would be greater than S seats. This probability would depend upon the aggregated standard deviation. The aggregated standard deviation is computed as the square root of the sum of variance of the individual standard deviations. Thus the aggregated standard deviation for the demands for class i and higher, $\sigma_{1,i}$ is computed as

$$\sigma_{1,i} = \sqrt{\sum_{x=1}^{x=i} \sigma_x^{2}}$$

The results achieved through the EMSRb algorithm are even closer to the optimal solution[2].

In his dissertation, Belobaba had also mentioned the possibility of passenger shifts under the case when the desired fare class is not available. He suggested that, given that the lower class demand is reached, the expected marginal seat revenue should be increased considering the probability of sell-up or vertical shift from the lower class passenger. For a two class case, he suggested that given the booking limit for the lower class has been reached, additional seats should be protected for the higher class till the following condition holds true

$$EMSR_1(\pi_{12} + V_{21})*[1-P_2(v)] + P_2(v)*f_1 = f_2 \qquad [2.4]$$

Where

π_{12} = protection level for class 1 from the basic EMSRa formulation

V_{12} = additional seats protected for class 1 considering sell-up.

$P_2(v)$ = Probability that class 2 request would sell-up to class 1

f_1 = class 1 fare

f_2 = class 2 fare

A similar approach was suggested for more than two classes. This was the first attempt to incorporate passenger sell-up in the EMSR algorithm. It recognized the importance of passenger sell-up and suggested that the protection levels should be

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[2] Peter P. Belobaba, *"EMSRa vs. EMSRb Explained: Does it make a Difference?"*, Presentation to the PROS Users Conference, September 24-26, 1996.

increased to accommodate passengers willing to sell-up. This is the fundamental and most important concept behind incorporating passenger sell-up in airline revenue management.

2.2 Belobaba & Weatherford[3]

In this model, Belobaba & Weatherford use the concept of a decision tree to model the impact of passengers' decision (regarding sell-up) on overall revenue. To better understand the model, consider a simple two-class case. We shall use the following notation:

π = Protection level for class 1

B_1: Probability that the next passenger is an original class-1 passenger (i.e. he would not request for class-2)

B_2: Probability that the next passenger would initially make a class-2 request but he may sell-up to class-2, if class-2 is closed.

p_1: Probability that the remaining seats would be enough for a all original class-1 passengers.

R_1 = Revenue from class 1

R_2 = Revenue from class 2

[3] Peter P. Belobaba & Lawrence R. Weatherford, *"Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations"*, Decision Sciences, Volume 27, Spring 1996.

The problem is to determine if we should increase the protection level for class-1 from π to $(\pi+1)$. The possible events along with their probabilities and the expected incremental revenues are shown in the form of a decision tree in Figure 2.1.

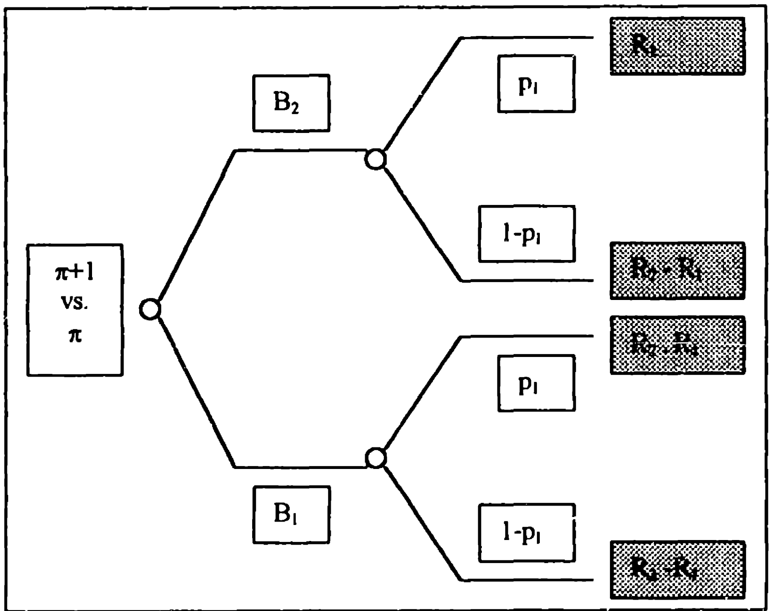

Figure 2.1: Decision Tree for two class case[4]

In Figure 2.1, the white boxes show the probability of each event, while the shaded boxes show the incremental revenue under each event. From the decision tree, we see that expected incremental revenue achieved by increasing the protection level is

$$B_2*[p_1*R_2 + (1-p_1)*(R_2 - R_1)] \ + B_1*[p_1*(R_2 - R_1)] + (1-p_1)*(R_2 - R_1)]$$

Or simply

$$B_2*p_1*R_1 + (R_1 - R_2)$$

To maximize revenue, we should increase the protection level till the incremental revenue achieved is greater than zero. Thus we increase the protection level till

$$B_2*p_1 > \ (R_1 - R_2)/R_1$$

[4] Peter P. Belobaba & Lawrence R. Weatherford, *"Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations"*, Decision Sciences, Volume 27, Spring 1996.

By extending the above rule for n classes and combining it with the EMSRb algorithm, the following optimality condition was developed.

$$P(\pi_n) = \frac{R_{n+1} - R_{1,n}(SU_{n+1,n})}{R_{1,n} - R_{1,n}(1 - SU_{n+1,n})}$$

[2.5]

Where

$P(\pi_n)$ = combined probability of selling the π^{th} seat in class-n or higher (including that through sell-up

π_n= protection level for class-n or higher

$R_{1,n}$ = weighted average revenue from class-1 through n

$R_{1,n+1}$ = revenue from the class immediately lower than n

$SU_{n+1,n}$ = probability of sell-up from class n+1 to n

Belobaba and Weatherford tested the above model under two different scenarios. They found that the modified heuristic could result in additional gains up to 2% in overall revenue when compared to that under the original EMSRb algorithm. They also concluded that the extent of revenue gains was dependent upon the demand factor, the number of fare classes or the fare ratio, and the sell-up rates. The largest gains were observed under situations with high demand factors, high sell-up rates and large fare ratios.

2.3 The missing variable

Both the above models address the issue of sell-up and make good intuitive sense. However note that in both the approaches, the protection level is dependent upon the following variables:

- The probability density of the higher class demand
- The fare-ratio
- The sell-up rate

To capture the sell-up potential, another important variable is the lower class demand. The protection obtained from the above models would be the same irrespective of the lower class demand. At any given protection level, sell-up rate, fare ratio and expected class-1 demand, the higher the expected class-2 demand, the more would be the expected spill. The higher the expected spill, the more would be the expected sell-up. The influence of lower class demand and spill, on passenger sell-up and overall revenue, is described in detail in the next chapter.

2.4 Brumelle et al.[5]

Brumelle et al. developed a decision rule for a two class nested example that maximizes total revenue. It was assumed that the demands for the two fare classes are not

[5] Brumelle, S.L., McGill, J.I., Oum, T.H., Sawaki, K., & Tretheway, M.W., *"Allocation of Airline Seats between Stochastically Dependent Demands"*, Transportation Science, 1990, Vol. 24.

independent of each other. They recognized that a portion of the passengers that are refused bookings in lower fare classes would sell-up or upgrade to the higher fare class, thereby increasing the higher class demand.

They started with a simple seat allocation model with dependent demand and then incorporated two important variants into it. The first one is related to the spill rate and goodwill of full fare passengers. The second one is concerned with the possibility of passenger up-grades or sell-up. For the simple seat allocation model with dependent demands, they proposed the following condition to maximize revenue:

$$BL_2 = \text{Max}\,[\{0 \le S \le \pi : P(D_1 > C - BL_2) \mid D_2 \ge BL_2\} < \frac{R_2}{R_1}] \qquad [2.6]$$

Here

BL_2 = Booking limit for class 2

C = Capacity

S = any seat such that $1 \le S \le C$

R_1 = Revenue from class 1

R_2 = Revenue from class 2

π = Protection for class 1

The above model states that the optimal booking limit for class 2 is the maximum number of seats under which the probability that class 1 demand exceeds the remaining seats, *given that class 2 demand is greater that the booking limit for class 2*, is less than the ratio of fares (R_1/R_2). This model is the similar to the ones developed by Belobaba[6], Mayer[7], Richter[8], Titze and Greisshaber[9], except that the demands of the two fare classes are allowed to be stochastically dependent.

Brumelle et al. argued that airlines are very much concerned about the goodwill of full fare passengers and the monetary loss of turning away a full fare passenger should be more than the difference in fares. They suggested that a goodwill premium, R_G should be included with the revenue from the higher class. To incorporate this variant, They suggested that the basic model be modified as follows:

$$BL_2 = \text{Max}\,[\{0 \leq S \leq \pi : P(D_1 > C - BL_2) \mid D_2 \geq BL_2\} < \frac{R_2}{(R_1 + R_G)}] \qquad [2.7]$$

Brumelle et al. also recognized the possibility that a fraction of the passengers that are refused seats in class 2 would sell-up to class 1. This translates to additional revenue for airlines provided enough seats are protected for class 1 demand and the passengers who might upgrade or sell-up. For the two class example, They proposed the following modification of the basic model:

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[6] P.P. Belobaba, *"Airline Yield Management: An Overview of Seat Inventory Control"*, Transportation Science, 1987, Vol. 21.

[7] M. Mayer, *"Seat Allocation, or a Simple Model of Seat Allocation Via Sophisticated Ones"*, AGIFORS Symp., Proc. 16, 1976.

[8] H. Richter, *"The Differential Revenue Method to Determine Optimal Seat Allotments by Fare Type"*, AGIFORS Symp., Proc 22, 1982.

[9] B. Titze and R. Greisshaber, *"Realistic Passenger Booking Behaviors and the Simple Low Fare/High Fare Seat Allotment Model"*, AGIFORS Symp., Proc. 23, 1983.

$$BL_2 = \text{Max}\,[\{0 \le S \le \pi : P(D_1 + U_{(BL_2)} > C - BL_2) \mid D_2 \ge BL_2\} < \frac{R_2 - SU * R_1}{(1 - SU) * R_1}] \qquad [2.8]$$

Here $U_{(BL_2)}$ is the number of passengers that would upgrade or sell-up to the higher class at booking limit BL_2. The expected value is the number of passengers spilled multiplied by the sell-up rate. $P(D1 + U_{(BL_2)})$ is the joint probability distribution for class 1 demand and number of passengers selling up from class 2 to class 1.

Finally They suggested that the both the upgrade possibility and the goodwill premium can be incorporated in the following model:

$$BL_2 = \text{Max}\,[\{0 \le S \le \pi : P(D_1 + U_{(BL_2)} > C - BL_2) \mid D_2 \ge BL_2\} < \frac{R_2 - SU * R_1}{(1 - SU) * (R_1 + R_G)}] \qquad [2.8]$$

The above model incorporates the sell-up potential into the optimality condition for the two class case. Unlike previous models, it also considers lower class demand. However, it is restricted to a two class case. It requires a joint probability distribution for class 1 demand (D_1) and the number of passengers selling up from class 2 to class 1 ($U_{(BL_2)}$). For more than two classes, it would be too complex to have this joint probability distribution. Suitable heuristics, which incorporate both sell-up rate and lower class demand, might be more practical to implement.

2.5 Bohutinsky[10]

Bohuntinsky developed a strategy to study the existence of passenger sell-up and its impact on overall revenue. Her study involved premature closure of certain fare classes on specific flights. By premature closure, we mean closing a fare class before its usual closing date. The resulting booking patterns (in flights that had premature closure) were then compared with those that had no premature closing. Using these comparisons, she was able to reach certain useful conclusions regarding passenger sell-up.

It was suggested that sell-up is dependent upon demand levels. An airline can expect more instances of sell-up at higher demand levels. At lower demand levels, sell-up is almost non-existent. This is in accordance with the fact that at higher demand levels, we have more spill. Passengers would sell-up only when they are refused bookings in their desired fare classes. At low demand factors, there is not much spill and most of the passengers are able to secure reservations in their desired fare classes.

It was observed that sell-up rate varied with fare classes. Sell-up rate among passengers of higher valued classes was more that that for low valued classes. Also, the market dominance played an important role on the impact of sell-up in revenues. If a market were heavily dominated by an airline, then passengers would be more willing to sell-up. In case of competitive markets, many passengers would fly the competition instead of selling up.

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[10] Bohutinsky, Catherine H., *"The Sell-up Potential of Airline Demand"*, MS thesis, Massachusetts Institute of Technology", June 1990.

Bohuntinsky's study is very important as it verifies the existence of sell-up among passengers and proves its impact on revenue. Premature closing of fare classes is a judgmental method and might be suitable to observe the extent of passengers' willingness to sell-up under various market conditions. It should be noted that the approach tends to be too aggressive to force sell-up among passengers. Premature closing of a fare class, without a more explicit decision rule, may cause heavy spill and the revenue lost due to spill may be more than the gains attributed to passenger sell-up. In order to incorporate the effect of passenger sell-up in the seat inventory control process, it would be desirable to have a mathematical model that closes a fare class after comparing the expected gain with the expected loss.

In this chapter, we discussed past studies and attempts to incorporate passenger sell-up in airline revenue management. A brief description of each of the strategies was presented and relevant conclusions were discussed. The importance of various parameters that effect passenger sell-up was also stressed. In the next chapter, we discuss the impact of sell-up on optimal protection levels and propose another modified heuristic to incorporate passenger sell-up in seat inventory control.

Chapter 3

Influence of Sell-up on Protection Levels

In this chapter, we analyze the effect of sell-up on protection levels. We start by evaluating the influence of sell-up on overall revenues. This gives us some useful insights. Then we move to the deterministic example and establish an expression for the optimal protection level under a two-class case. By deterministic, we mean that the demand for each fare class is know with certainty. After the deterministic case, we move to the stochastic case, dealing first with the distinct fare classes and then with the nested fare classes. Under the stochastic case, the demand is expressed in terms of probability distribution with a mean (average) and some standard deviation (measure of variance). Recall that under the distinct or non-nested fare classes, a seat can only be sold to a class for which it was initially allocated. However under the nested fare classes, a seat protected for any class, can be sold to that class or any higher valued class. Finally, we propose a modified heuristic to incorporate passenger sell-up.

3.1 Influence of Sell-up and Associated Variables on Revenue

In this section, we illustrate the influence of sell-up rate, fare-ratios, demands and protection levels on revenue. We take a simple example with two fare classes '1' and '2'; with 1 being the higher valued class. For simplicity, we shall assume deterministic demands for each class. The total revenue under various values of protection, sell-up rates, fare ratios and demand levels are determined. The base values of relevant variables are assumed as follows:

Capacity, C = 100

Mean demand for class 1, $D_1 = 50$

Mean demand for class 2, $D_2 = 70$

Class 1 fare, f_1 = \$ 200

Class 2 fare, f_2 = \$ 100

Mean Sell-up rate, SU= 0.2

If 'π' is the number of seats allocated to class 1 then at any value of π, the total revenue earned, 'R_T', can be defined as

$R_T = R_{11} + R_{21} + R_2$

= (revenue earned from original class 1 passengers)

+ (revenue earned from sell-up of passengers from class 2 to class 1)

+ (revenue earned from class 2 passengers that did not sell-up)

$$R_T = f_1 * \min[\pi, D_1] + f_1 * \min[(\pi - D_1), \max\{0, (D_2 - C + \pi) * SU\}] + f_2 * \min[(C - \pi), D_2]$$

[3.1]

From the above expression, we observe that total revenue is influenced by allocation level (π), fare ratio (f_2/f_1), sell-up rate (SU), Demand levels (D_1,D_2) and capacity (C). However, even for a simple two class deterministic case, the effect of each of these variables is not straightforward. For each seat allocated to class 1, there is guaranteed revenue of f_1, provided the allocation does not exceed class 1 demand. In addition to this, there also exists a revenue potential due to the sell-up of class 2 passengers, provided the class 2 demand is more than the seats allocated to it and there exists some sell-up. Finally a seat allocated to class 2 guarantees a revenue of f_2, provided the class 2 demand is not less than the seats allocated to it.

The following graphs show the relationship between the relevant variables and total revenue. By comparing the revenue at various protection levels, we also get an idea of the optimal protection level (the protection level for class 1 when the total revenue is maximized). Note that the following graphs only illustrate the effect of sell-up rates, demands and fare-ratio on the optimal protection level. In the next section, we derive an expression for the optimal protection level.

3.1.1 Sell-up Rate

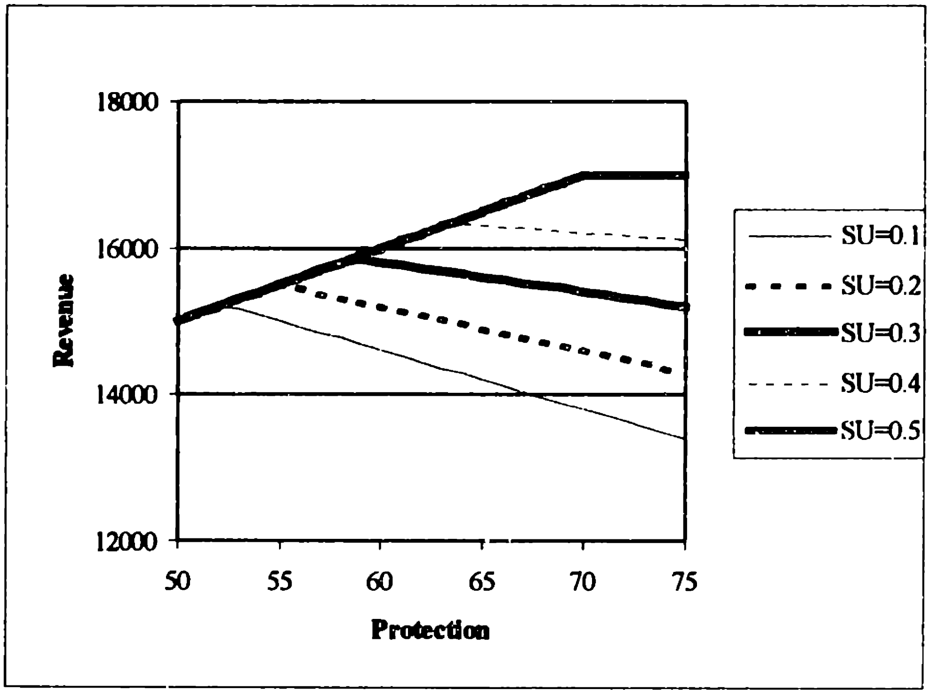

Figure 3.1: Impact of sell-up rate on revenue.

The optimal protection level increases with the increase in sell-up rate. This is because of increase in number of people who would sell-up to the higher class. Also note that under any sell-up rate greater than zero, the optimal protection level is greater than the mean class 1 demand. Under a deterministic case the optimal protection level will never be less than class 1 demand. Moreover, the higher the number of people who sell-up, either due to spill or sell-up rate, the higher would be the optimal protection. Recall that spill is the difference between the mean demand and seat allocation. It is the number of requests that are refused.

An interesting phenomenon is observed when the sell-up rate becomes equal to the fare ratio. In the above example when sell-up rate becomes equal to the fare-ratio (i.e. 0.5), the optimal protection level lies anywhere from 70 to the total capacity. The reason of this is that within this range, the additional revenue achieved by protecting an additional seat is equal to the loss due to additional spill.

3.1.2 Fare-ratio

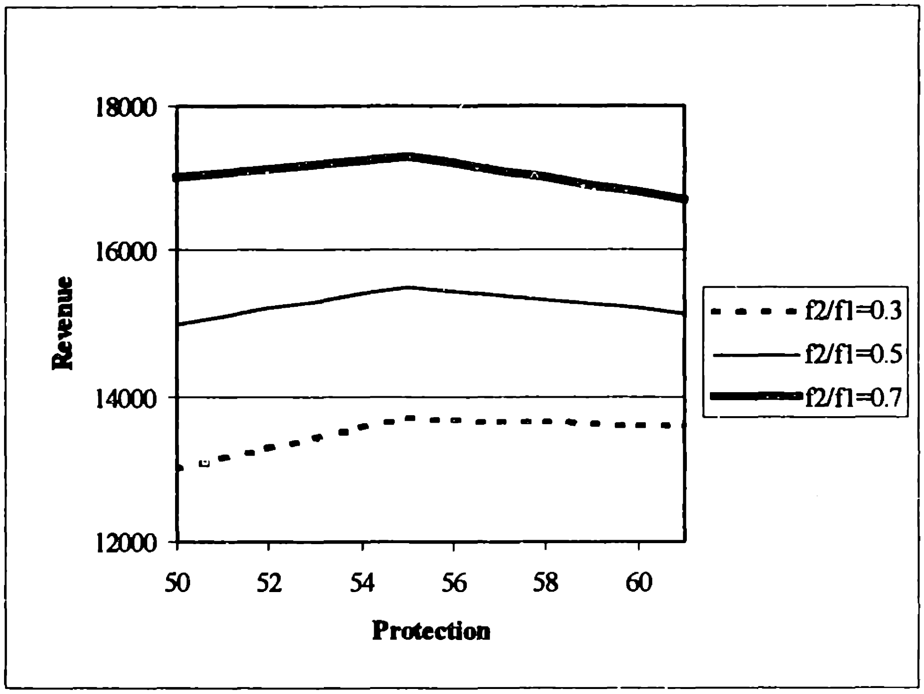

Figure 3.2: Impact of fare ratio on revenue.

As the fare ratio decreases, i.e. the difference between the higher and lower class fare increases, the total revenue at optimal protection level also increases. However, surprisingly the optimal protection level remains the same and is independent of the fare ratio. This is true for a deterministic case under the condition that the sell-up rate is less than the fare ratio. If the sell-up rate is greater then the fare ratio, then the optimal protection level goes to the maximum value i.e. capacity. The reason is that for each incremental protection the revenue achieved through sell-up is greater than the class 2 fare.

3.1.3 Lower class demand

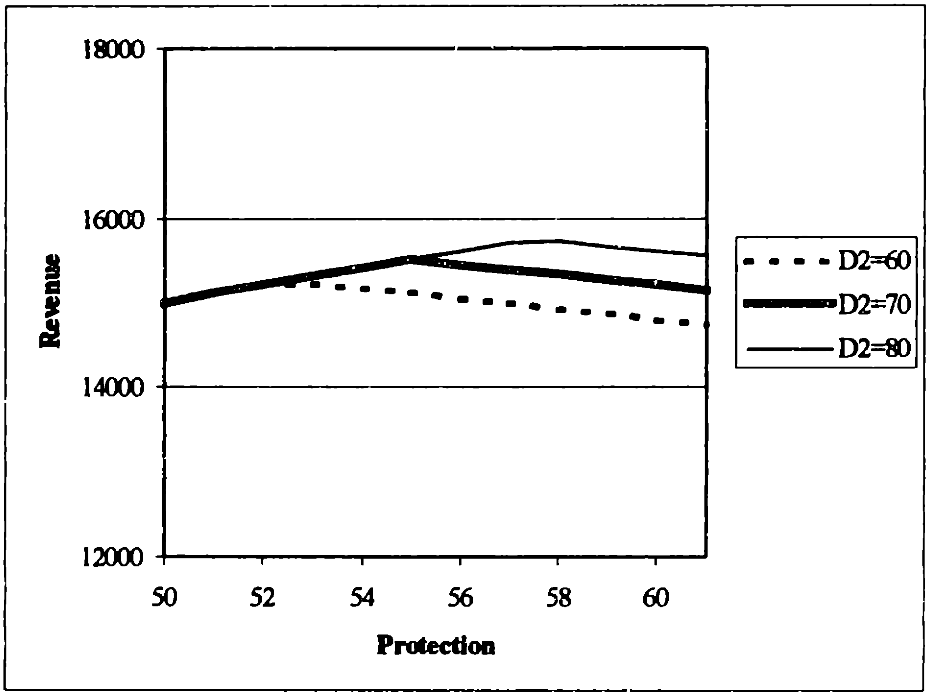

Figure 3.3: Impact of demand factor on revenue.

The optimal protection level increases with the increase in class 2 demand. This is because of higher spill, at a given π, that causes increased quantity of sell-up. Under a scenario when there is no spill (i.e. the sum of demand for both classes is less than capacity), variations in class 2 demand would not alter the optimal protection level.

3.1.4 Upper class demand

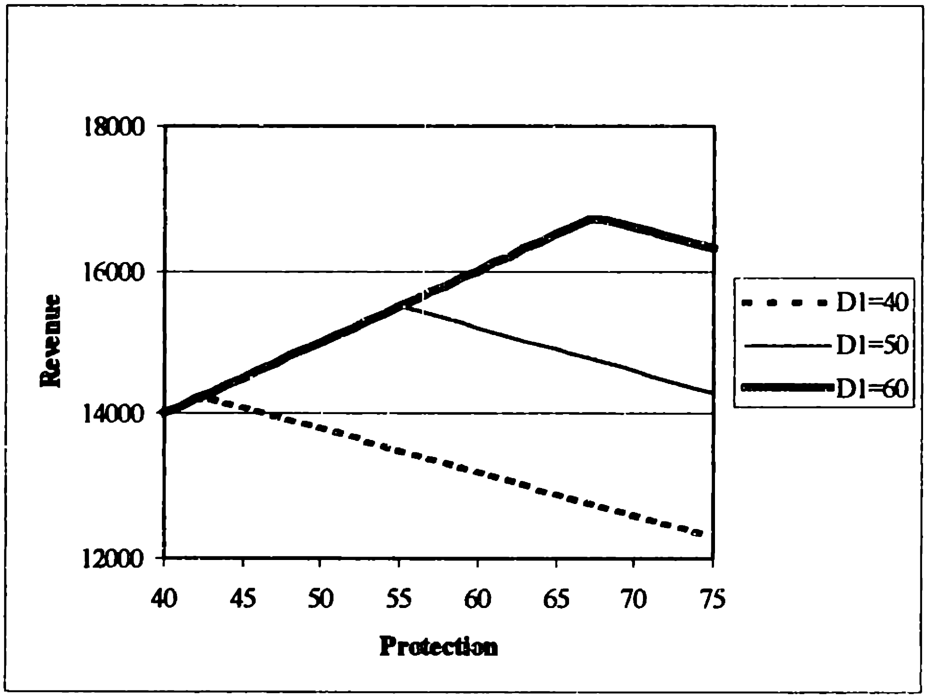

Figure 3.4: Impact of higher class demand on revenue.

As class 1 demand increases, the optimal protection level also increases. Apart from increased class 1 demand, the increase in protection level is also due to the higher spill caused by rejecting more class 2 passengers. This can also be confirmed from the above graph. When class 1 demand increases by 10 (from 50 to 60), the protection level increases by around 13 (55 to 68).

3.2 Deterministic Case (two class)

Consider a case with two classes '1' and '2', with 1 being the higher valued class. The goal is to develop a model for determining the optimal protection level for class 1 (which determines the booking limit for class two). We shall use the following notation:

f_i: Fare value of class i.

π: Protection for the class 1

C: Total capacity

D_i: mean demand for class i.

SU: Sell up rate from class 2 to 1 (effective only when class 2 is closed)

SP_i: Spill from class i

SC_i: Spill cost for class i = $f_i * SP_i$

R_i: Revenues from class i bookings.

3.2.1 Without considering sell up

In this section, we develop an expression for optimal protection level under the assumption that there is no sell-up. In other words, none of the class 2 passengers that are refused bookings (spilled), sell-up to class 1.

R_1 = Revenues from original class 1 pax.

$= f_1 * \min[\pi, D_1]$

R_2 = Revenues from class 2 pax.

$= f_2 * \min[(C - \pi), D_2]$ (where $C-\pi$ = remaining capacity)

It is obvious that under this deterministic case, without considering sell up, the optimal protection level is

$$\pi = \min [C, D_1]$$

The reason is that class 1 fare is higher than class 2. The optimal protection level will never be less than class 1 demand. The upper limit would be the capacity. The spill for the lower class can be simply computed as the difference between class 2 demand and the seats that are not protected for class 1. Note that there is no spill if class 2 demand is less than the seats available to it. Mathematically

$$SP_2 = \max[0, \{D_2 - (C - \pi)\}]$$

And spill cost

$$SC_2 = SP_2 * f_2$$

$$= f_2 * \max[0, \{D_2 - (C - \pi)\}]$$

3.2.2 Considering sell up

When we consider sell up, we would be protecting additional seats for the higher class. By "additional", we mean the number of seats in addition to the ones that would be protected without considering sell up under the optimal conditions.

Now:

R_1 = Revenues from original class 1 pax. + Revenues through sell-up

$= R_{11} + R_{21}$

We can have the following three cases with respect to the protection level, π:

1) $\pi \leq D_1$

If protection level is less than class 1 demand, the number of seats sold in class 1 would be equal to protection level. There would not be any additional revenue gain from passengers who sell-up as the number of seats allocated to class 1 is not enough to accommodate even the original class 1 passengers (the ones who do not buy class 1 as a result of sell-up). Thus,

$R_1 = f_1{*}(\pi) + 0$

$= f_1{*}(\pi)$

2) $D_1 + SP_2 {*}SU > \pi > D_1$

Under this case, the protection level is greater than original class 1 demand but less than the sum of class 1 demand and the number of passengers who sell-up from class 2 to class 1. Since the protection level is less than the total class 1 requests, i.e. the sum of class 1 demand and the number of passengers who sell-up, the total number of bookings in class 1 would be equal to the protection level. The total revenue from class 1 would thus be:

$R_1 = f_1{*}D_1 + f_1{*}\,(\pi - D_1)$

$= f_1{*}(\pi)$

3) $\pi \geq D_1 + SP_2{*}SU$

Under this case the protection level is greater than the sum of class 1 demand and number of passengers who sell-up to class 1. The total number of bookings in class 1 would thus be the sum of class 1 demand and the number of passengers who sell-up from class 1 to class 2. In other words, all requests for class 1 (including those due to sell-up) would generate revenue; however, there might be seats that are protected for class 1 but remain empty. The revenue from class 1 can be computed as:

$R_1 = f_1{*}D_1 + f_1{*}\ SP_2{*}SU$

$= f_1{*}D_1 + f_1{*}\ \max\{0, (D_2 - C + \pi){*}SU\}$

For all the above cases, the revenue from class 2 remains the same. It is the fare times the number of seats that are not protected for class 1, provided class 2 demand is higher than the number of seats available. If the number of seats available is more than class 2 demand, the revenue from class 2 is the demand times class 2 fare.

R_2 = Revenues from class 2

$= f_2{*}\min[(C - \pi), D_2]$

Also Spill cost, at protection level π

$SC_2 = f_2{*}\ SP_2$

$= f_2{*}\ \max[0, \{D_2 - (C - \pi)\}]$

For revenue maximization, we set the protection level such that the spill cost caused by protecting additional class 1 seats is less than the additional revenue achieved.

As mentioned above, by additional seats we mean all the seats protected in addition to that under no sell up or in addition to D_1. Now from above expressions for R_1, R_2 and Spill Cost, we see that:

Additional revenue from protecting (π- D_1) seats is:

1)	$= 0$	*(if $\pi <= D_1$)*
2)	$= f_1 * (\pi - D_1)$	*(if $(D_1 + SP_2 * SU) > \pi > D_1$)*
3)	$= f_1 * \max[0, (D_2 - C + \pi)] * SU$	*(if $\pi \geq D_1 + SP_2 * SU$)*

Note that by additional spill cost, we mean the increase in spill cost due to the increase in protection level by considering sell-up, i.e.

Additional Spill cost = (Total Spill Cost) – (Spill Cost without considering sell up)

There could be two cases:

1) If class 2 demand is greater than the seats that are not protected for class 1.

Additional Spill cost = $f_2 * \max[0, \{D_2 - (C - \pi)\}] - f_2 * \max[0, \{D_2 - (C - D_1)\}]$

$= f_2 * (\pi - D_1)$ $\{\text{if } D_2 > (C - \pi)\}$

2) If class 2 demand is less than the seats that are not protected for class 1 then there would be no spill and thus no spill cost.

Optimal π is the value at which additional revenue is greater than or equal to the additional spill cost. There would be two main scenarios:

1) Sell-up rate is greater than fare ratio

Under this case the protection level is the maximum possible value which is the capacity. For every seat protected there is a revenue gain of f_1*SU against a loss of f_2 (assuming one class 2 passenger is spilled). Since the sell-up rate is greater than the fare ratio

$$SU > f_2/f_1$$

$$f_1 * SU > f_2$$

Hence the revenue gain would always be greater than the loss and the optimal protection level would simply be the capacity. However, this is an unrealistic case as the sell-up rate is usually less than the fare-ratio.

2) Sell-up rate is less than fare ratio

Under this case, the optimal protection level would never be less than class 1 demand plus the number of people that would sell-up at that protection level. The sum of class 1 demand and sell-up, $(D_1 + SP_2 * SU)$, is the deterministic number of passengers that would buy the higher fare, once the booking limit on the lower fare is reached. From now onwards, we shall refer to this value as "combined" class 1 requests. If we are protecting one less seat than the final class 1 requests, then there is a guaranteed loss of f_1 versus a gain of f_2 or less. Since $f_1 > f_2$, the protection level would always be below this threshold value.

Again protecting one more seat than the final class 1 requests means a loss of f_2 against a gain of f_1*SU. Since the sell-up rate is less than the fare ratio

$SU<f_2/f_1$

$f_1*SU< f_2$

Hence we conclude that for revenue maximization under the two-class deterministic case, the optimal protection level is such that

$$\pi = D_1 + SP_2*SU$$

$$\pi = D_1 + \max[0, \{D_2 - (C - D_1)\}*SU]$$

[3.2]

From the above relationship, we observe that the optimal protection level is dependent upon the sell-up rate, demand levels and capacity. It is independent of the fare ratio if sell-up rate is less than the fare ratio. Recall that if sell up rate is greater than the fare ratio than the optimai protection level is equal to capacity. Note that these conditions are true only for the deterministic case where we have a guaranteed final class 1 requests (including the number of sell-ups). For the stochastic case, we have to follow an iterative procedure and set the protection level to the point where additional revenue obtained through sell-up is greater than or equal to the additional spill cost. Under stochasticity, we calculate the additional revenue in terms of expected values. At this point the fare ratio pays a very important role as the difference between the additional revenue through protection and spill cost would be highly dependent upon it.

The following graphs show the effect of sell-up rates and demand levels on optimal protection under the two-class deterministic case. By comparing the optimal protection level at the demand levels assumed in the previous section, we can also check the validity of the above expression for optimal protection level. Unless stated otherwise, we shall use the following base values.

f_1: Fare value of class 1 = \$200

f_2: Fare value of class 2 = \$100

f_2/f_1: Fare Ratio = 0.5

C: Total capacity = 100

D_1: mean demand for class 1 = 50

D_2: mean demand for class 2 = 70

SU: Sell up rate from class 2 to class 1 = 0.2

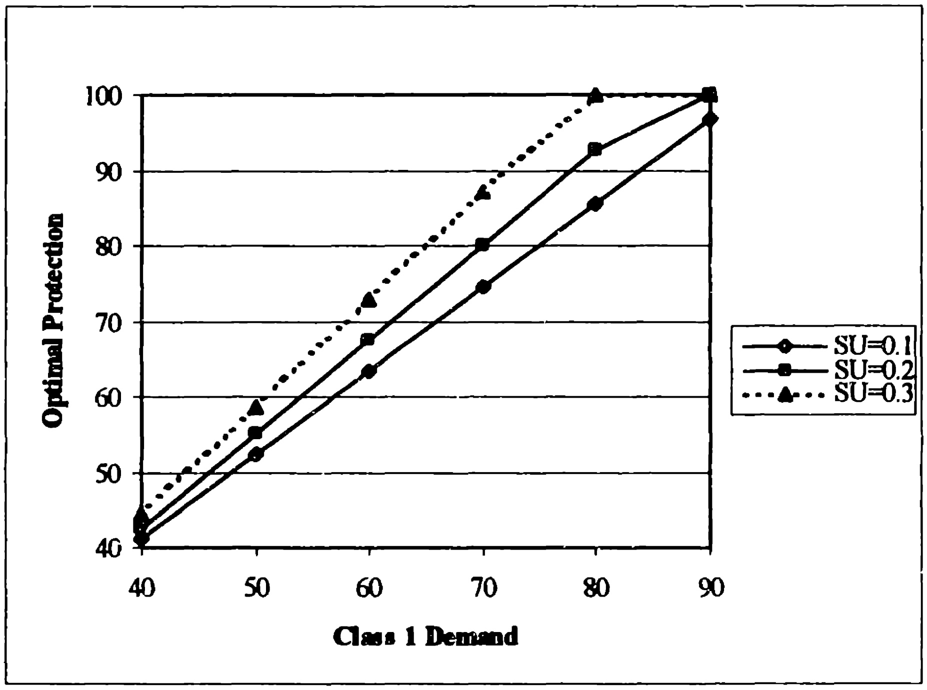

Figure 3.5: Impact of higher class demand on optimal protection.

The above graph shows the relationship between optimal protection levels and class 1 demand. As mentioned in the previous section, for any increase in class 1 demand, the increase in protection level is higher than that of class 1 demand (provided there is some sell-up). The reason is increased spill from class 2 that causes more passengers to buy the higher fare. The upper limit for the optimal protection level is the capacity.

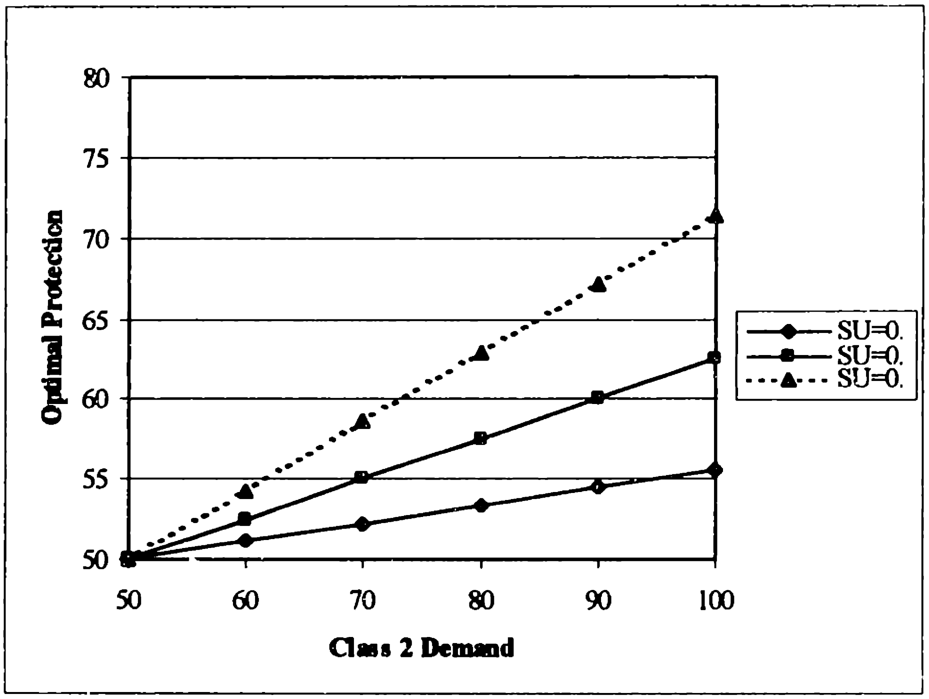

Figure 3.6: Impact of lower class demand on optimal protection

The above graph shows the relationship between optimal protection level and class 2 demand, holding class 1 demand equal to 50. As class 2 demand increases, the optimal protection level for class 1 also increases (provided there is spill). The increase in optimal protection level is highly dependent upon the sell-up rate. At any spill, the higher the sell-up rate, the more would be the number of passengers willing to buy the higher fare and thus greater would be the optimal protection level. In fact, for a deterministic case, the optimal protection level rises by sell-up rate for every unit increase in class 2 demand. The above graphs also confirms that the value of optimal protection level determined by our expression matches the ones determined indirectly through the revenue graphs in the previous section.

3.3 Stochastic Case (two class)

In this section, we shall extend the previous approach for stochastic cases. The basic approach remains the same: Protect seats for the higher class as long as the additional revenue gain is greater than the additional spill cost. We shall first consider distinct fare classes and establish a condition for revenue maximization. Then we shall move to nested fare classes.

The following notation will be used

f_i: Fare value of class i.

S_i: Seat allocation for class i (under distinct fare classes)

π_i: Protection for class 1 to i (under nested fare classes)

C: Total capacity

D_i: Expected (mean) demand for class i.

r_i: a random variable representing the stochastic demand for class-i

$p(r_i)$: probability distribution function for class-i demand.

$F_i(x)$: cumulative probability of having at most x bookings in class-i

$P(r_i>x)$ = Probability that class-i demand is greater than x, $= 1 - F_i(x)$

σ_i : Standard deviation for class-i demand.

SU_{ji}: Expected sell-up rate from class j to i (effective only when class-j is closed)

SP_i: Expected Spill from class-i

E_i: Expected revenues from final class-i bookings.

E_T: Total expected revenue from all classes.

3.3.1 Distinct fare classes

Under distinct fare classes, the aim is to determine the seat allocation that would maximize the overall revenue. For distinct fare classes, we assume that the seats allocated for a fare class cannot be purchased by any other class, even if the fare value of the other class is more than that for the class for which the seat was allocated. In terms of mathematical modeling it means that we do no need to consider the probability that seats allocated for lower classes might be purchased by a higher class even if the higher class demand exceeds the allocation level. Consider a simple case with two classes '1' and '2'; with class 1 being the higher valued class.

a) Without considering sell-up

If S_1 seats are allocated for class 1, expected revenue from class 1 bookings is

$$E_1 = \sum_{x=1}^{S1} f_1 * P(r_1 \geq x)$$

Similarly, expected revenue from class 2 booking is

$$E_2 = \sum_{x=1}^{C-S1-1} f_2 * P(r_2 \geq x)$$

Total expected revenue at any allocation level S_1 is

$$E_T = f_1 * \sum_{x=1}^{x=S1} P(r_1 \geq x) + f_2 * \sum_{x=1}^{C-S1-1} P(r_2 \geq x)$$

[3.3]

b) Considering sell-up

At any allocation level S_1, expected revenue from class 1 bookings

= expected revenue from original class 1 passengers

+ expected revenue from sell-up (due to spill)

$E_1 = E_{11} + E_{21}$

As done in the above section, expected revenue from the original class 1 passengers

$$E_{11} = f_1 * \sum_{x=1}^{x=S1} P(r_1 \geq x)$$

Note that by original class 1 passengers, we mean those who directly make a request for class 1 (excluding the ones who request it only if class 2 is closed)

Expected revenue from sell-up = (Spill from class 2) * (Sell-up rate) *

(Probability that there are enough seats to accommodate sell-up)

$$E_{21} = f_1 * \sum_{x=C-S}^{x=\infty} \sum_{y=1}^{x} P(r_2 > x) * SU_{21} * P(C - y - 1 < r_1 < C - y)$$

$$= f_1 * \sum_{x=C-S}^{x=\infty} P(r_2 > x) * SU_{21} * P(r_1 < C - x)$$

Here 'y' represents the seats that are not protected for class 1. The term $P(C\text{-}y\text{-}1 < r_1 < C\text{-}y)$ refers to the probability that the demand for class 1, a random variable r_1, would be equal to (C-y) seats. The summation of these probabilities, with 'y' varying from 1 to 'x', would actually give us the probability that class 1 demand is less than (C-x). Thus we substitute the internal summation by a single term $P(r_i < C\text{-}y)$. Intuitively, we are summing the probability of all events under which there is sell-up and the passengers who sell-up, occupy those seats which would not be sold to an original class 1 passenger.

Now expected revenue from final class 1 bookings (including sell-up)

$$E_1 = f_1 * \sum_{x=1}^{x=S1} \{P(r_1 \geq x) + f_1 * \sum_{x=C-S1}^{x=\infty} P(r_2 \geq x) * SU_{21} * P\{r_1 < (C-x)\}$$

[3.4]

As before, expected revenue from class 2 bookings

$$E_2 = \sum_{x=1}^{x=C-S1-1} f_2 * P(r_2 \geq x)$$

[3.5]

And total expected revenue at any allocation level S_1 is

$$E_T = E_1 + E_2$$

$$= f_1 * \sum_{x=1}^{x=S1} \{P(r_1 \geq x) + f_1 * \sum_{x=C-S1}^{x=\infty} P(r_2 \geq x) * SU_{21} * P\{r_1 < (C-x)\} + f_2 * \sum_{x=1}^{x=C-S1-1} P(r_2 \geq x)$$

[3.6]

The following graph shows the change in total expected revenue under various levels of allocation (S_1). It is assumed that demands in both classes follow a normal distribution.

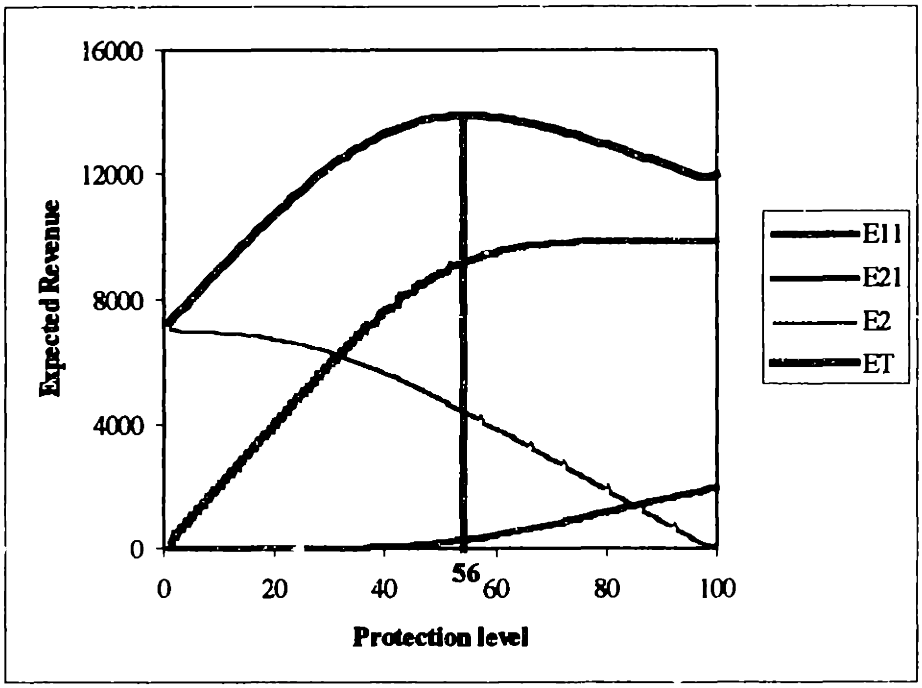

Figure 3.7: Total expected revenues under distinct fare classes.

From Figure 7, we observe that the optimal protection level, under the given fare ratio and sell-up rate, is 56.

Condition for Revenue Maximization (distinct fare classes)

To maximize the total revenue under stochastic case, for distinct fare classes, we protect seats for the higher class till the expected revenue of each incremental seat protected for the higher class is greater than the expected loss due spill.

We introduce some new notations

R_{11}= Additional expected revenue from original class 1 passengers (excluding sell-up) due to incremental protection

R_{21}= Additional expected revenue from sell-up by incremental protection

R_1= The total additional expected revenue from final class 1 bookings due to incremental protection

R_{22} = Expected revenue from class 2 without incremental protection.

Now if the protection level is increased from (S-1) to S,

$R_{11} = f_1$ * {(Probability of selling the S^{th} seat to an original class 1 passenger)

$= f_1 * P(r_1>S)$

R_{21} = change in total expected revenue through sell-up

= (expected rev. through sell-up at allocation level S)

- (expected rev. through sell-up at allocation level S-1)

$$= f_1 * \sum_{x=C-S1}^{x=\infty} P(r_2 \geq x) * SU_{21} * P\{r_1 < (C-x)\} - f_1 * \sum_{x=C-S1+1}^{x=\infty} P(r_2 \geq x) * SU_{21} * P\{r_1 < (C-x)\}$$
$$= f_1 * [P\{r_2 \geq (C-S)\} * SU_{21} * P\{r_1 < (C-x)\}]$$

Now total expected gain is

$R_1 = R_{11} + R_{21}$

$= f_1 * P(r_1 \geq S) + f_1 * [P\{r_2 \geq (C-S)\} * SU_{21}]$

[3.7]

and the expected loss (i.e. the expected revenue if the seat is allocated to class 2)

$R_{22} = f_2 * P(r_2 > C-S)$

$= f_2 * \{1 - F_2(C-S)\}$

[3.8]

We protect the maximum number of seats as long as the following condition holds true

$$f_1 *[P(r_1 \geq S)] + f_1*[P\{r_2 \geq (C-S)\}*SU_{21}] \geq f_2*[P\{r_2 \geq (C-S)\}]$$

[3.9]

Hence for our two class stochastic case, under the assumption of distinct fare classes, the optimal protection level depends upon

- Probability of selling the fare to a original class 1 passenger which in turn depends upon class 1 demand
- Sell-up rate
- Fare ratio
- Incremental Spill, which in turn depends upon class 2 demand

Note that in the above condition, for each incremental protection, we consider the incremental expected spill, which in turn depends upon total class 2 demand. The higher the class 2 demand, the more is the spill, and more would be the protection. Figure 3.8 shows the change in incremental expected revenues as we increase the allocation level.

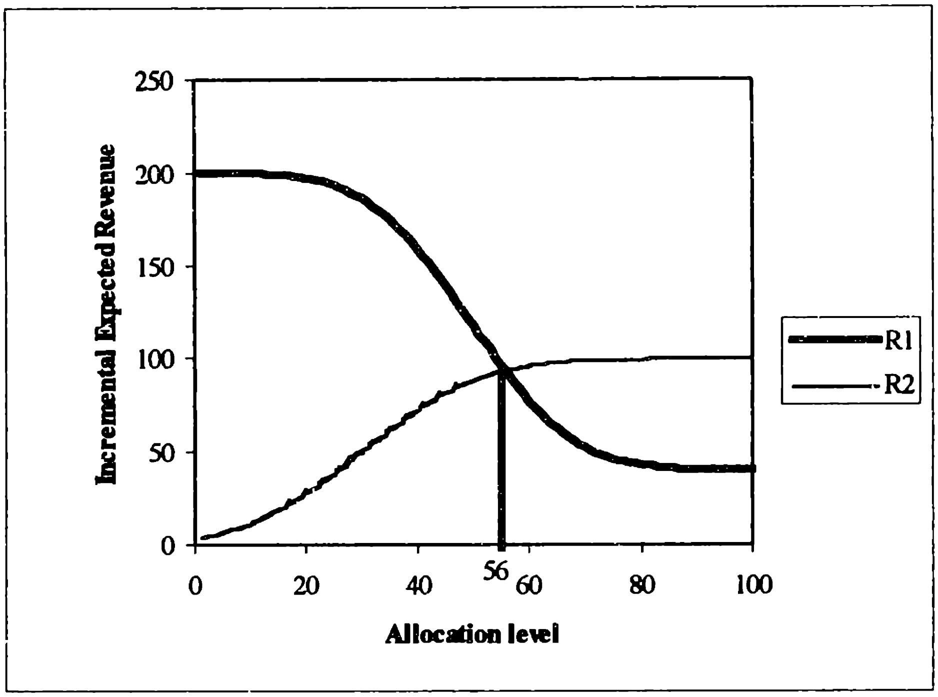

Figure 3.8: Incremental expected revenues under distinct fare classes

The optimal protection level is the seat at which the incremental expected revenue becomes equal to expected revenue of that seat from class 2. The optimal protection level is the 56th seat. This value matches the one obtained through the total revenue curve. Note that without considering sell-up, the optimal protection level would have been the 50th seat.

3.3.2 Nested fare classes

Under nested fare classes, the aim is to determine the protection level for upper classes that would maximize the revenue. The mathematical modeling is more complex than that for the distinct case. We have to consider the probability that the demand for the higher class may exceed the protection level and may take up seats that are not sold to the lower class passengers. As before, consider a case with two classes '1' and '2'; with class 1 being the higher valued class. From Curry's[1] approach, the total expected revenue, ignoring sell-up, can be expressed as:

$$\int_0^{C-\pi} dr_2 p(r_2)[f_2 r_2 + \int_0^{C-r_1} dr_1 p(r_1)\{f_1 r_1\} + f_1 \pi \int_{\pi}^{\infty} dr_1 p(r_1)]$$
$$+[f_2(C-\pi) + \int_0^{\pi} dr_1 p(r_1)(f_1 r_1)] * \int_{C-\pi}^{\infty} dr_2 p(r_2)$$

The above expression is very complex. Even for a two class case, it involves double integrals. As the number of classes increases, so do the number of integrals. Nevertheless, it gives us a very good insight regarding the calculation of expected revenues from each class under a nested case. We have to consider the probability that the higher class demand might increase the protection level and may take up seats that are not sold to the lower class. Considering this fact, the expected revenues E_{11}, E_{21} and E_2, can be redefined for the nested case as follows:

E_{11} = Expected revenues from original class 1 passenger (excluding sell-up)

$$= f_1 * [\sum_{x=1}^{\pi 1}\{P(r_1 > x\} + \sum_{x=\pi 1}^{C}\sum_{y=1}^{x} \{P(r_1 > x)\} * \{P(C-y-1 < r_2 < C-y)\}]$$
$$= f_1 * [\sum_{x=1}^{\pi 1}\{P(r_1 > x\} + \sum_{x=\pi 1}^{C}\{P(r_1 > x)\} * \{P(r_2 < C-x)\}]$$

[3.10]

Where π_1 is the protection level for class 1

E_2 = Expected revenues from original class 2 passengers (excluding sell-up)

$$= f_2 * [\sum_{x=1}^{C-\pi 1} P(r_2 > x)]$$

[3.11]

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[1] Renwick E. Curry, *"Optimal Airline Seat Allocation with Fare Classes Nested by Origins and Destinations"*, Transportation Science, Vol. 24, No. 3, August 1990.

E_{21} = Expected revenues from sell-up of class 2 passengers to class 1

$$= \sum_{x=C-S}^{x=\infty} P(r_2 > x) * SU_{21} * P(r_1 < C - x)$$

[3.12]

Note that, in the above expressions we have assumed that class 2 passengers book first. As for the distinct case, the total expected revenue at any protection level could be expressed as the sum of the above expected revenues.

$$E_T = E_{11} + E_{21} + E_{22}$$
$$= E_1 + E_{22}$$

[3.13]

The following graph shows the change in total expected revenue under various levels of protection. As before, it is assumed that demands in both classes follow normal distribution.

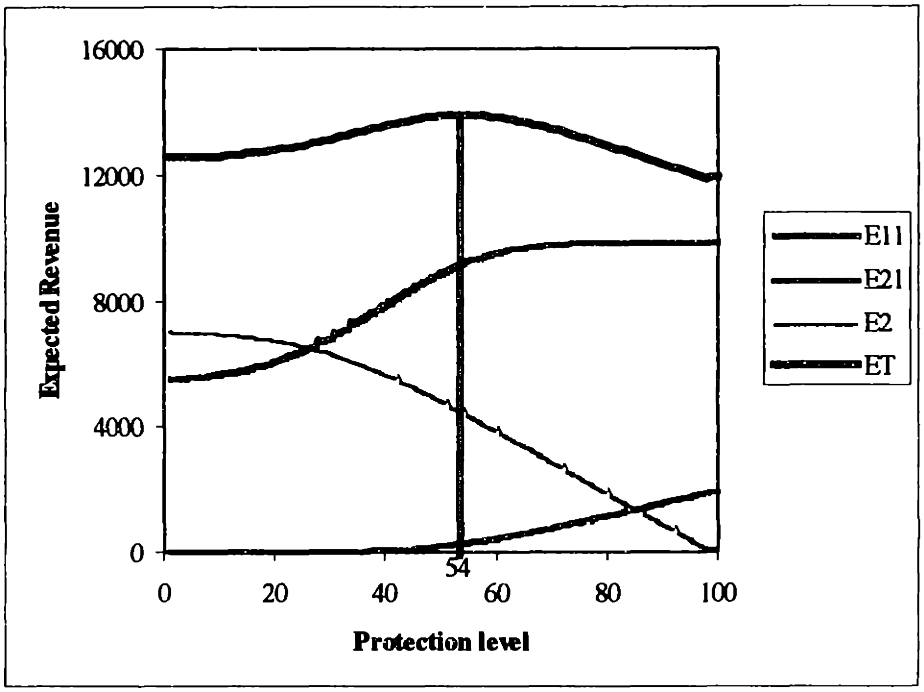

Figure 3.9: Total expected revenues under nested fare classes.

Note the difference between the above curves and those under the case of distinct fare classes. At low protection levels, the expected revenue from the higher class (and subsequently the total revenue) is much higher than that under the distinct case. The reason is the ability to sell unprotected seats to the higher class (if they are not sold to the lower class). The optimal protection level is the 54^{th} seat. It is below that under the distinct case. The reason is the increase in expected revenue from an unprotected seat due to the possibility of selling it to a higher class. Recall that under the distinct fare class case, only those seats could be sold to the higher class that are allocated to that class.

Condition for Revenue Maximization (nested fare classes)

For revenue maximization, we protect seats for the higher class as long as the incremental expected revenue for any seat, under protection, is more than the incremental expected revenue without protecting the seat for the higher class. The incremental expected revenue by protecting the seat for the higher class can be expressed as

R_1= Additional expected revenue through incremental protection.

= (Expected revenue E_1 at protection level S)

- (Expected revenue E_1 at protection level S-1)

From equations 3.7 & 3.9, we get

$$R_1 = f_1 * P(r_1 > S) - f_1 * P(r_1 > S) * P(r_2 < C-S) + f_1 * P(r_2 > C-S) * SU_{21} * P(r_1 < S)$$

[3.14]

In the above expression,

$f_1 P(r_1 > S)$ refers to the incremental expected revenue if the protected seat is sold to an original class 1 passenger.

$f_1 * P(r_1 > S) * P(r_2 < C-S)$ refers to the expected revenue if the seat is not protected for class 1 but the S^{th} class 1 request is still fulfilled by unprotected seat (the term is subtracted to get the net effect of incremental protection).

$f_1 * P(r_2 > C-S) * SU_{21} * P(r_1 < S)$ refers to the incremental expected revenue through sell-up

Again, the loss under incremental protection (or the expected revenue if the seat is sold to a class 2 passenger) is

$$R_2 = f_2 * P(r_2 > C-S)$$

Hence we protect the maximum number of seats as long as the following condition holds true

$$R_1 \geq R_2$$

i.e

$$f_1 * P(r_1 > S) - f_1 * P(r_1 > S) * P(r_2 < C-S) + f_1 * P(r_2 > C-S) * SU_{21} * P(r_1 < S) \geq f_2 * P(r_2 > C-S)$$

[3.15]

Note the difference between equation 3.15 (nested fare classes) and equation 3.9 (distinct fare classes). The incremental expected revenue by protecting an additional seat under nested fare classes is reduced by two possibilities. The first one is the possibility that even an unprotected seat could be sold to class 1, if it has not been sold to class 2. The second factor is related to sell-up. It is possible that class 1 demand is such that all protected seats would be sold to class 1, even without sell-up. In other words there is no additional revenue gain through sell-up if the passengers who sell-up occupy those seats that would have been otherwise sold to class 1.

The following graph shows the change in incremental expected revenues as we increase the protection level. The optimal protection level is the 54th seat. This matches with the value obtained through the curve for total expected revenues.

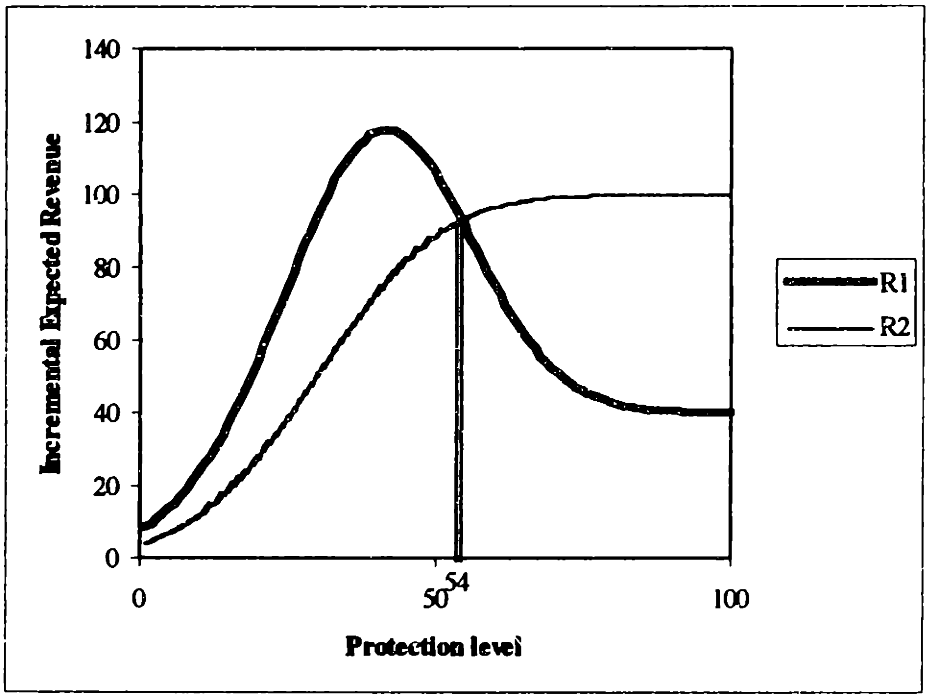

Figure 3.10: Incremental expected revenues under nested fare classes.

Again the curve for R_1 (expected additional revenue from incremental protection) is much different from that under the case of distinct classes, particularly under low protection levels. The reason is the ability to sell an unprotected seat to the higher class. For example the incremental expected revenue from protecting 6 seats instead of 5, out of a total of 100 seats, is not significant. Considering the mean class 2 demand (70), it is very likely that even if the 6th seat is not protected, it would not be sold to class 2 and would be available when class 1 passenger appears. The incremental expected revenue increases with the increase in protection level, reaches a maximum value and then again diminishes. For example the incremental expected revenue by protecting 41 seats instead of 40 is quite significant. The reason is that if the 41st seat is not protected, then based on mean demands of class 1 (50) and class 2 (70) it is likely that the seat would be sold to class 2 and the 41st class 1 passenger turns up but is refused. The value of this incremental expected revenue is further enhanced by the fact that additional protection causes additional spill from class 2, which in turn causes additional number of people to sell-up to the higher class. Again the incremental spill at higher value of total protection is more than that under a lower value of total protection. This leads us to a very important point; the incremental expected revenue for protecting an additional seat is not only dependent upon the demand distribution and sell-up rate, but also on total protection level or booking limits.

3.4 Generalized heuristic for multiple classes

Consider the case with n classes, with class 1 being the highest valued class and class n being the lowest valued class. Let class i be any intermediate class. According to the EMSRb heuristic, the overall revenue would be maximized if we protect maximum number of seats for class i (and higher) as long as the following condition holds true

$$f_{1,i} * P(r_{1,i} > S) \geq f_j$$

Here

$j = i + 1$ (next lower valued class after class i)

$f_{1,i}$ = aggregated fare for classes 1 to i

$$= \frac{\sum_{x=1}^{i} f_x * D_x}{\sum_{x=1}^{i} D_x}$$

D_x = mean demand for class x

$r_{i,n}$ = aggregated demand from class 1 to i

The above heuristic is very practical and has been successfully used by the airline industry. Here, we shall attempt to incorporate the sell-up potential in this heuristic. From our analysis in Section 3.2, we have seen that it is desirable to increase the protection level when there exists sell-up. The question is how much additional protection should we have? When we protect an additional seat in the hope of sell-up there are two possibilities:

- A lower class passenger, who is spilled out, decides to sell-up and buys the higher fare. In this case, the airline realizes revenue equal to the higher fare.
- None of the spilled out passengers, if any, decide to buy the higher fare and the seat remains unsold. In this case the airline loses revenue equal to the lower fare, assuming that the seat would have been sold to the lower valued fare class. Note that this assumption is consistent with the assumptions associated with the EMSRb heuristic.

The revenue would be maximized if we protect seats till the expected gain in revenue through additional protection is more than the expected loss. Lets assume that we have determined the protection levels using the EMSRb heuristic. For any class i (and higher), the protection level is π_i. The expected gain for protecting an additional seat (in addition to the protection level obtained from the EMSRb algorithm) is the product of the probability of passenger sell-up from the next lower class and the fare for class i. Assuming that the seat, if not protected, would be sold to the next lower class, the expected loss is the fare of the next lower class. Thus, to maximize revenue, we continue to protect additional seats till the following condition holds true:

$$f_i * P(\text{sell-up}, \pi_i) \geq f_j$$

Here 'j' is the next lower class after class i. *P(sell-up, π_i)* is the probability that the additional seats protected in the hope of sell-up would be sold to passengers that are spilled out from the next lower class and are willing to sell-up. This probability would depend upon the mean or the expected number of passengers that would sell-up from class j to class i. The expected number of passengers willing to sell-up is the product of the number of passengers spilled and the mean sell-up rate. Note that for a given number of passengers willing to sell-up, the probability of sell-up reduces as we increase the number of passengers. In other words the expected revenue associated with passenger sell-up reduces as we protect more and more additional seats. The idea is very much similar to the original EMSRb heuristic. We are protecting additional seats for a certain demand. The protection levels are determined on the basis of expected revenues with and without protection. It is interesting that the demand here is the expected sell-up and depends upon the booking limits determined by the EMSRb algorithm itself.

The proposed heuristic rule can now be summarized as follows:

- Use the original EMSRb algorithm to determine the protection levels for each class.
- Determine the expected spill from each class. The expected spill is the difference between the mean demand and the seats available to that class.

Note that under the nested fare structure, the higher class passengers have access to all lower fare seats. However, it is assumed that lower class book before the higher class and only the seats protected for the higher class would be available to them.

- For each fare class,
  - Determine the expected number of passengers willing to sell-up. It is the product of expected spill and mean sell-up rate. Assume that the expected number of passengers willing to sell-up has a normal distribution with standard deviation equal to

    $$Z^*\sqrt{(\text{mean})}$$

    Here 'Z' is the coefficient of standard deviation. It is a measure of variability.
  - For passenger = 1 to (expected number of sell-up)
    - Increase protection level if

      $$f_{i+1}{}^*P(\text{sell-up}) \geq f_j$$

      Here

      f_{i+1} is the fare of the next higher class.

      P(sell-up) is the probability that the number of passengers selling up would be more than the additional seats protected.
    - Continue to increase the protection level as long as the above expression is true.

It must be mentioned here that this is a heuristic and would not give optimal values of protection levels. However, we do expect that the protection levels achieved through the above approach would capture the sell-up potential and increase the revenue potential by adjusting the protection levels in accordance with changes in sell-up rate and expected demands.

In this chapter, we explained the impact of sell-up on protection levels that maximize revenue. We started with a simple deterministic example and used it to illustrate the influence of sell-up on overall revenue. Then we derived the expression for optimal protection level under a two class deterministic case and showed the effects of various variables on the value of optimal protection level. After getting some insight into the influence of sell-up on optimal protection levels, we moved to the stochastic case and dealt with the nested and the non-nested cases. For the stochastic case, we developed a heuristic that helps us to determine the protection levels that would maximize revenue. It was emphasized that although the heuristic does not give optimal protection levels, however, it is a good approximation to capture the effect of sell-up. In the next chapter, we describe a simulation that uses this heuristic and discuss the results.

Chapter 4

Simulation

Simulation is the procedure in which a computer-based mathematical model of a physical system is used to perform experiments on that system by generating external demands and observing how the system reacts to the demands over a period of time[2]. Simulation is a very effective tool in measuring the performance of heuristics, particularly when we are dealing with complexities like uncertainties in demands and passenger behavior. In this chapter, we use simulation to observe the performance of the EMSRb heuristic and its modifications under various scenarios. We will primarily be concerned with the abilities of the heuristics to capture the revenue associated with passenger sell-up. A simulation software is developed for this purpose. The initial part of the chapter describes the simulation environment and the assumptions associated with it. In the latter part, we simulate different cases and discuss the results.

4.1 Why Simulation?

The modified heuristic described in the previous chapter seems promising in capturing the revenue potential associated with the sell-up phenomenon. It adjusts the

[2] R. C. Larson and A. R. Odoni, *"Urban Operations Research"*, Prentice-Hill, Inc., 1967.

protection levels obtained through the original EMSRb algorithm by considering the expected revenue associated with sell-up. It does not protect additional seats if the expected revenue through sell-up is less than the next lower fare. Theoretically, it ensures that there is little risk of overprotection and the overall revenue under the modified rule should not be less than the one under the original EMSRb algorithm. However, there is no guarantee that the booking limits obtained through the proposed rule would always result in more revenue as compared to that under the original EMSRb heuristic. As mentioned before, the proposed rule is a heuristic attempt to capture the revenue associated with passenger sell-up. The stochastic demand, multiple fare classes and most importantly, the nested fare structure makes the problem too complex to analytically measure the effectiveness of the proposed rule under various conditions. Simulation is a suitable solution to this problem. By simulating the booking process under various scenarios, we can analyze the performance of the original as well as the proposed heuristic rules.

4.2 Overview of Simulation Program

Figure 4.1 shows the overall simulation process. Here we give a brief description of each step.

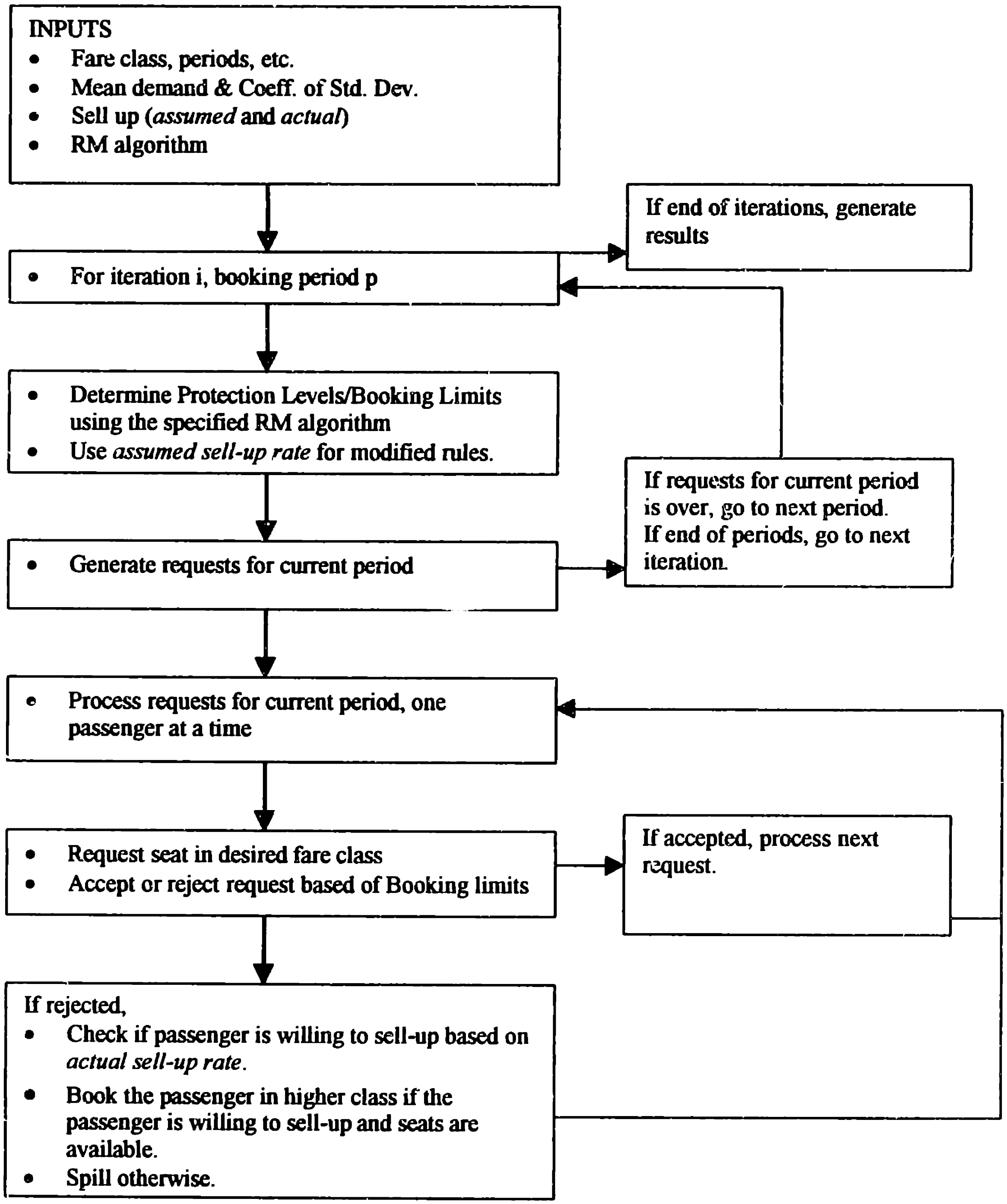

Figure 4.1: The Simulation process.

4.2.1 Inputs

The first step is to gather the input data. The simulation requires information regarding the fare classes, booking periods, mean demands for each fare class under each period, sell-up rates, demand factors and number of iterations to be simulated. The beginning of each booking period is also the revision point for determining the booking limits. By revision, we mean the recalculation of booking limits through the revenue management algorithm considering the expected future demands. The higher the number of booking periods, the greater would be the number of booking limit revisions done by the simulation and the better would be the resulting booking limits. However, for each booking period, we need to have an estimate of mean demand for each fare class for that period. Since forecasting for very small time periods is difficult and inaccurate, the number of booking periods should be such that we have a reasonable estimate of mean demands during that period.

For each fare class, two sell-up rates are defined. One is considered as the *"assumed sell-up rate"* while the other one is considered as the *"actual sell-up rate"*. The assumed sell-up rate is used by the revenue management algorithm in determining the protection levels whereas the actual sell-up rate is used during the processing of each request. The idea is to analyze the situation when there are differences in the sell-up rates assumed by the airline and actual sell-up rates existing in the market. The mean sell-up rate supplied as input refers to buying the next higher valued fare. It is assumed that sell-up will occur between adjacent classes only.

Finally, the number of iterations determine the number of times we want to simulate the same departure case. The end results are generated as the mean of all iterations. The higher the number of iterations, the higher would be the reliability of the results.

4.2.2 Protection Levels

For determining the protection levels, we need to have an estimate of the expected demands in the future. We assume that the demand for each fare class, for each period, is normally distributed with a mean "μ" and standard deviation "σ". The mean demands for each period are given as input. The standard deviation is estimated as

$$\sigma = Z * \sqrt{\mu}$$

Here "Z" is the Z-factor. It is the coefficient that refers to the variability in demands. For our examples, we would assume a value of 1.0. The reason is that we will be using the Poisson distribution to generate the demand. In a Poisson distribution, the standard deviation is equal to the square root of the mean.

Once we have an estimate of mean demands and standard deviations, we can use a revenue management algorithm for determining the protection levels. In our simulation, we have a choice of three heuristics:

1. **EMSRb**: This is the original EMSRb algorithm as proposed by Belobaba[1]. We start with the highest valued fare class and keep on protecting seats as long as the expected marginal seat revenue (EMSR) from the combined higher classes is greater than or equal to the fare for the next lower class. Except for the highest valued class, we consider aggregate values of demand and fare. For example if there are 'n' classes and 'i' and 'j' are two intermediate classes with 'j' being the next immediate lower valued class after class 'i', then according to this heuristic rule, we keep on protecting seats for the classes 'i' and higher till the aggregated EMSR value for any seat 'S' is higher than the next lower class fare. Mathematically this condition is written as:

$$EMSR(S_i) \geq R_j$$

Here

$EMSR(S_i) = R_{1,i} * P(S_i)$

$R_{1,i}$ is the aggregated fare for classes i and higher

$P(S_i)$ is the probability that the sum of requests for class i and higher would be more than the S seats.

R_j is the fare for class j

The approach used to determine the aggregated fares has been explained in Section 2.1.

[1] Peter P. Belobaba, *"Optimal vs. Heuristic Methods for Nested Seat Allocation"*, Presentation to the AGIFORS Yield Management Study Group, Brussels, May 1992.

2. **EMSRb2**: This is the modified rule proposed by Belobaba and Weatherford[2]. As with the original EMSRb algorithm, it starts with the highest valued class and keeps on protecting seats as long as the expected marginal seat revenue from the higher classes is greater than or equal to the fare for the next lower class. However, the EMSR value for the higher class is modified to incorporate sell-up. For any two classes 'i' and 'j', with class i being the higher valued class, the modified rule is:

$$R_{1,i}(1-SU_{j,i})*P(S_i) - R_{1,i}\,(SU_{j,i}) \geq R_j$$

Here

$SU_{j,i}$ is the mean *assumed* sell-up rate from class j to class i.

This rule has been discussed in detail in Section 2.2

3. **EMSRb3**: This is the new proposed rule. Under this rule, we first determine the protection levels and booking limits using the original EMSRb algorithm. Using the booking limits and expected demands, we determine the expected spill from each class. The expected number of passengers willing to sell-up is the product of expected spill and mean sell-up rate (assumed sell-up rate). We now adjust the protection levels considering the expected number of passengers who would be willing to sell-up. For each class, we increase the protection level as long as the expected revenue from sell-up is more than the next lower fare. It is assumed that

[2] Peter P. Belobaba & Lawrence R. Weatherford, *"Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations"*, Decision Sciences, Volume 27, Spring 1996.

the number of passengers willing to sell-up has a normal distribution with a mean μ_{su} and a standard deviation σ_{su}.

where $\sigma_{su} = Z^* \sqrt{\mu_{su}}$

Here we use the same Z-factor value as that for the mean input demands. This implies that the relationship between standard deviation and mean is same for both mean input demands and the number of passengers willing to sell-up. The proposed rule has been described in detail Section 3.4.

4.2.3 Generation of Requests

For each fare class, we assume that the passengers arrive as a Poisson process and the requests are generated as a Poisson distribution. Based on previous studies done by Williamson[3] and Lee[4], the assumption of Poisson distribution is reasonable. After the booking limits have been revised, we generate the requests for all fare classes. The values are stored as the *actual* requests received during the period. Note that the requests generated for each period are isolated from the process of booking limit revision, which is solely based upon the mean demands supplied as inputs.

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[3] Elizabeth L. Williamson, *"Airline Network Seat Inventory Control: Methodologies and Revenue Impacts"*, Ph. D. thesis, Massachusetts Institute of Technology, Cambridge, MA, June 1992.
[4] A. O. Lee, *"Airline Reservations Forecasting: Probabilistic and Statistical Models of the Booking Process"*, Ph. D. thesis, Massachusetts Institute of Technology, Cambridge, MA, Sept. 1990.

4.2.4 Processing of Requests

Once we have generated the *actual* requests during a period, we process each request, one at a time. It is assumed that during each period, the lower class requests always arrive before the upper class ones. This assumption may not be always be true, however, if we have a large number of booking periods, the error if any, is not significant. The reason is that on one hand, the number of requests for each booking period reduces as we increase the number of booking periods, and on the other hand, the booking limits are reevaluated at the beginning of each period.

In our simulation, for each period, we start with the processing of requests for the lowest valued class and move to the higher valued classes. Each request is handled according to the following process:

1. Check if seat is available in the desired class.
2. If seat is available, book the passenger and reduce the availability and booking limits.
3. If seat is not available, check if the current passenger is willing to sell-up. To decide whether the current passenger is willing to sell-up or not, we perform a Bernoulli trial using the mean sell-up rate for the class (as supplied in inputs). The sell-up rate used here is the "*actual*" sell-up rate and may differ from the one assumed during the determination of protection levels.
4. If the current passenger is willing to sell-up, book him in the next higher class and reduce the availability and booking limits accordingly.

5. If the current passenger is not willing to sell-up, he is spilled out of the system. As mentioned before, we are only considering sell-up between adjacent fare classes.

4.2.5 Results

The results are recorded in three different files. The first one presents a summary for all iterations. It records the mean values for overall revenue, loads, load factor, requests for each fare class, spill and number of passengers who sold up to higher classes. The second one records these parameters for each iteration. The third one keeps track of the revision of protection levels and booking limits under each period. This information helps in comparing the difference in protection levels under the different types of revenue management techniques used.

4.3 Test Cases

In this section we study the performance of the three heuristics. Two test scenarios have been built for this purpose. Both scenarios consider a single leg example.

4.3.1 Scenario 1

This is a small three-class example. Table 4.1 presents the mean demand and fare for each class. These values are valid for all cases tested under Scenario 1.

Class	Mean Demand	Fare ($)
1	45.04	600
2	48.05	300
3	57.06	150
Total Demand	150.15	

Table 4.1: Scenario 1 parameters.

There are 18 booking periods and the demand factors (demand/capacity) range from 0.8 to 1.5. The capacity of the aircraft is 150. In order to test the performance of the original EMSRb algorithm and the modified rules, we tested the above scenario under various values of sell-up rates. The following cases were developed:

Base Case: Sell-up does not exist. Both assumed as well as actual sell-up rates are given as 0.

Case 1: There is moderate sell-up and the actual sell-up rate among passengers is equal to the assumed sell-up rate in the EMSRb decision rules.

Case 2: There is heavy sell-up. Again, the actual sell-up rate among passengers is equal to the assumed sell-up rate.

For each case, we simulated 500 iterations. The results presented here are the mean values over 500 iterations.

4.3.1.1 Base Case

Under the base case, we assume that there is no sell-up. We use the original EMSRb algorithm to control the booking limits and observe its performance. Note that when there is no sell-up, both the modified rules are reduced to the original EMSRb algorithm and we get the same protection levels and thus same results. The results can be observed in Table 4.2.

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39908	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.53	0.01	0
0.9	89.51	44705	1	40.54	40.38	40.28	0.1	0
			2	43.24	43.11	42.92	0.19	0
			3	51.35	51.7	51.07	0.63	0
1	95.86	48700	1	45.04	45.14	44.48	0.66	0
			2	48.05	48.29	47.44	0.86	0
			3	57.06	56.97	51.87	5.1	0
1.1	97.42	51398	1	49.54	49.51	48.4	1.12	0
			2	52.86	52.91	51.33	1.57	0
			3	62.77	63.15	46.4	16.75	0
1.2	98.04	54401	1	54.05	54.77	53.35	1.42	0
			2	57.66	57.66	55.56	2.11	0
			3	68.47	68.52	38.15	30.36	0
1.3	97.82	56660	1	58.55	58.55	57.07	1.48	0
			2	62.47	62.65	59.78	2.87	0
			3	74.18	73.7	29.88	43.82	0
1.4	97.53	59149	1	63.06	62.54	61.19	1.36	0
			2	67.27	67.42	64.47	2.95	0
			3	79.88	80.35	20.63	59.72	0
1.5	97.69	62088	1	67.56	67.89	66.15	1.73	0
			2	72.08	72.3	68.92	3.37	0
			3	85.59	85.25	11.46	73.79	0

Table 4.2: Scenario 1, Base Case.

As expected, the EMSRb heuristic does a very good job in limiting the lower valued class passengers and protecting seats for the higher valued class passengers. At lower demand factors, there are enough seats and there is no benefit of using EMSRb to impose the booking limits. However, as the demand factor increases, the overall revenue is highly influenced by the booking limits. If enough seats are not protected for the higher valued class passengers then there is a risk of spilling too many of them. On the other hand, if too many seats are protected for the higher class passengers, then there is a risk of having too many empty seats. As evident from Table 4.2, the EMSRb heuristic adjusts the booking limits such that the most of the passengers spilled are those requesting lower fares.

Table 4.2 also shows that at higher demand factors, we can have a significant spill. Among those spilled, there may be passengers who are willing to sell-up. Even at low sell-up rates, the number of passengers willing to sell-up could be high if there is considerable spill. This could have a significant impact on revenue. It would be interesting to see the performance of the original EMSRb algorithm and its modifications when we consider sell-up. In the next three cases, we consider sell-up under the same scenario.

4.3.1.2 Case 1

Under this case, we assume that there exists moderate sell-up among passengers of class 2 and 3. Specifically, we assume that the sell-up rate among passengers desiring to travel in class 2 is 0.3 and that for class 3 is 0.2. This is in accordance with real life scenarios. The sell-up rate among higher valued classes is usually more than that in lower valued classes[5]. Also, our intuition would agree with the fact that the class 2 passenger, who is not buying the deep discounted fare (class 3), is more likely to go for the next higher fare as compared to the class 3 passenger who is shopping for the deep discounted fare.

It must also be mentioned here that the sell-up rates are only applicable to passengers who are spilled. Thus a sell-up rate of 0.3 would translate to the fact the 3 out of every 10 passengers that are spilled, would go for the next higher fare. It is stressed that the sell-up rate should not be tied with the individual demand for any class. It is the spill that causes passengers to sell-up and spill is dependent upon the total demand, booking limits and capacity, rather than the individual demand for any class.

Tables 4.3, 4.4 and 4.5 present the results under the three heuristic rules used to control the booking limits. Note that under this case, we are assuming that the *assumed* sell-up rate is equal to the *actual* sell-up rate. The validity of this statement is discussed in Chapter 5.

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[5] Bohutinsky, Catherine H., *"The Sell-up Potential of Airline Demand"*, MS thesis, Massachusetts Institute of Technology", May 1987.

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39909	1	36.03	35.86	35.85	0.02	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.53	0.01	0
0.9	89.55	44733	1	40.54	40.38	40.31	0.17	0
			2	43.24	43.11	42.98	0.25	0.09
			3	51.35	51.7	51.05	0.65	0.12
1	96.1	48865	1	45.04	45.14	44.52	1.03	0
			2	48.05	48.29	48.06	1.27	0.41
			3	57.06	56.97	51.58	5.39	1.03
1.1	97.97	51817	1	49.54	49.51	48.39	1.97	0
			2	52.86	52.91	53.31	3.03	0.84
			3	62.77	63.15	45.26	17.89	3.43
1.2	98.74	55048	1	54.05	54.77	53.27	2.99	0
			2	57.66	57.66	59.06	4.89	1.49
			3	68.47	68.52	35.78	32.74	6.29
1.3	98.82	57713	1	58.55	58.55	57.23	3.36	0
			2	62.47	62.65	64.81	7.03	2.04
			3	74.18	73.7	26.19	47.51	9.19
1.4	98.89	60526	1	63.06	62.54	61.56	3.77	0
			2	67.27	67.42	70.5	9.23	2.79
			3	79.88	80.35	16.28	64.07	12.31
1.5	99.25	63608	1	67.56	67.89	66.52	4.88	0
			2	72.08	72.3	75.63	11.79	3.51
			3	85.59	85.25	6.72	78.53	15.13

Table 4.3: Scenario 1, Case 1, under original EMSRb

Demand Factor	**Load Factor (%)**	**Mean Revenue**	**Class**	**Demand (Input)**	**Requests (Actual)**	**Load**	**Spill**	**Pax. Sell-up**
0.8	79.96	39913	1	36.03	35.86	35.86	0.01	0
			2	38.44	38.57	38.58	0.01	0
			3	45.65	45.54	45.5	0.04	0.01
0.9	89.33	44732	1	40.54	40.38	40.38	0.07	0
			2	43.24	43.11	43.09	0.25	0.06
			3	51.35	51.7	50.53	1.17	0.23
1	95.07	48948	1	45.04	45.14	44.99	0.51	0
			2	48.05	48.29	48.74	1.12	0.36
			3	57.06	56.97	48.88	8.09	1.56
1.1	96.65	52154	1	49.54	49.51	49.36	0.92	0
			2	52.86	52.91	54.65	2.57	0.76
			3	62.77	63.15	40.97	22.18	4.31
1.2	97.42	55624	1	54.05	54.77	54.64	1.36	0
			2	57.66	57.66	60.77	4.32	1.23
			3	68.47	68.52	30.72	37.8	7.42
1.3	97.47	58322	1	58.55	58.55	58.62	1.74	0
			2	62.47	62.65	66.76	6.36	1.81
			3	74.18	73.7	20.82	52.87	10.46
1.4	97.68	61234	1	63.06	62.54	62.78	2	0
			2	67.27	67.42	73.37	7.77	2.23
			3	79.88	80.35	10.38	69.97	13.72
1.5	98.1	64509	1	67.56	67.89	68.41	2.7	0
			2	72.08	72.3	77.68	10.96	3.22
			3	85.59	85.25	1.07	84.19	16.34

Table 4.4: Scenario 1, Case 1, under EMSRb2

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44730	1	40.54	40.38	40.31	0.15	0
			2	43.24	43.11	43	0.25	0.08
			3	51.35	51.7	50.94	0.76	0.14
1	95.81	48927	1	45.04	45.14	44.7	0.77	0
			2	48.05	48.29	48.36	1.17	0.34
			3	57.06	56.97	50.65	6.32	1.23
1.1	97.35	52083	1	49.54	49.51	48.89	1.3	0
			2	52.86	52.91	54.54	2.31	0.67
			3	62.77	63.15	42.61	20.55	3.94
1.2	98.06	55623	1	54.05	54.77	54.05	1.71	0
			2	57.66	57.66	61.59	3.25	0.98
			3	68.47	68.52	31.45	37.06	7.18
1.3	97.9	58432	1	58.55	58.55	57.97	1.86	0
			2	62.47	62.65	68.8	4.45	1.28
			3	74.18	73.7	20.08	53.62	10.6
1.4	97.65	61558	1	63.06	62.54	62.32	1.65	0
			2	67.27	67.42	76.94	4.95	1.43
			3	79.88	80.35	7.2	73.14	14.48
1.5	98.41	64550	1	67.56	67.89	68.13	2.79	0
			2	72.08	72.3	78.31	10.25	3.04
			3	85.59	85.25	1.18	84.08	16.26

Table 4.5: Scenario 1, Case 1, under EMSRb3

As seen from the above tables, there is a significant change in overall revenue as compared to that when we did not consider sell-up. It would be interesting to compare the difference in overall revenues, just by including the possibility of sell-up (both assumed and actual). Figure 4.2 presents the change in overall revenues as compared to that under the base case. Recall that the results under the base case are same for all three heuristics considered here.

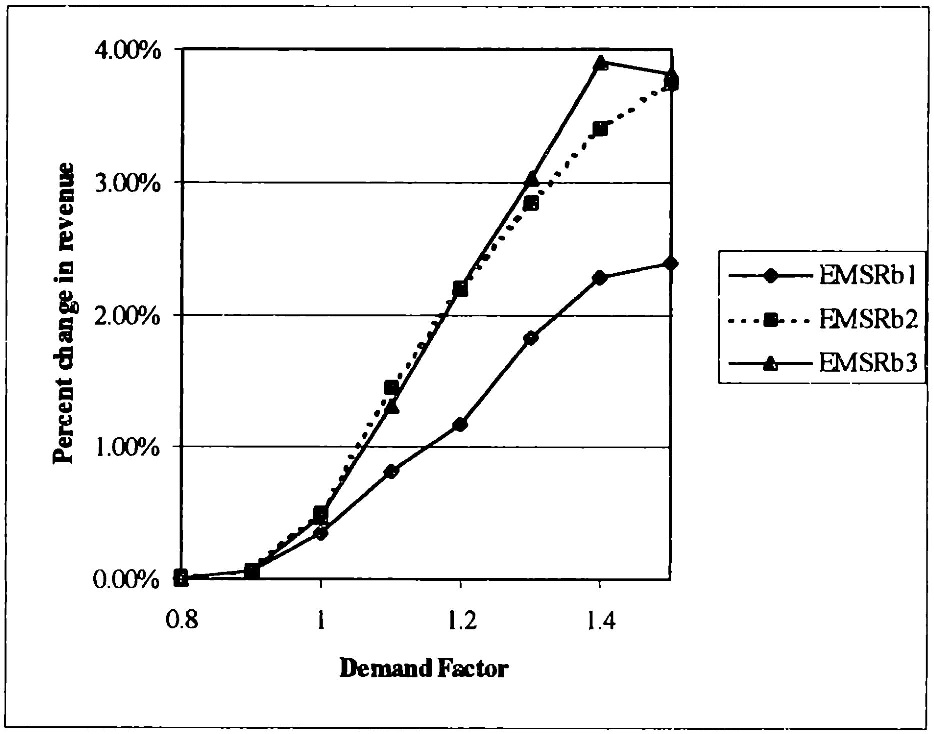

Figure 4.2: Impact of sell-up on revenue, under Scenario 1.

As seen in Figure 4.2, just by including sell-up in our models, there is a considerable change in overall revenues. Even at a demand factor of 1.0, the change in overall revenue is around 0.5%. As the demand factor increases, the change in revenue also increases. At demand factors above 1.4, the change could be as high as 3.5%. The reason being that, under a given sell-up rate, the number of passengers willing to sell-up increases with the increase in spill. It must be stressed that the percentage increase in revenues in Figure 4.2 should not be taken as the ability of a heuristic to increase the overall revenue. It only shows the difference in overall revenues when sell-up is accounted for in the simulation. The comparison is made with the Base Case where there was neither *assumed* nor *actual* sell-up. Even without any modification in the original EMSRb algorithm, there is a significant increase in overall revenues under sell-up, as there are passengers who will purchase a higher fare seat when a lower fare class is not available.

It would be interesting to compare the performance of the modified heuristics, EMSRb2 and EMSRb3, with that of the original EMSRb heuristic under equal values of sell-up parameters. Figure 4.3 presents the difference in overall revenues under the two modified rules as compared to that under the original EMSRb heuristic.

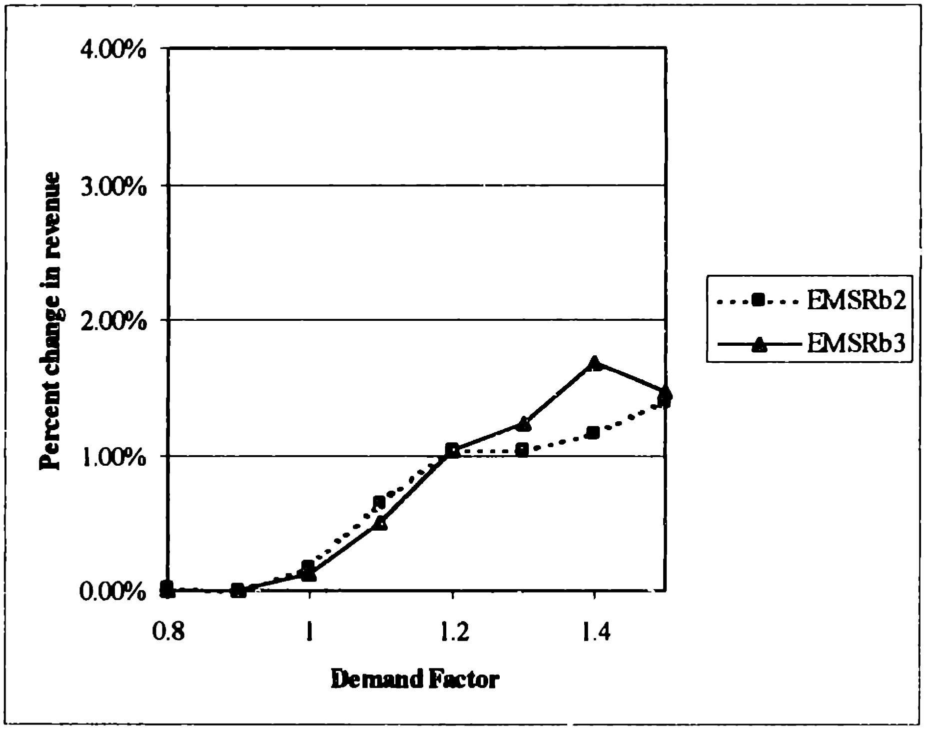

Figure 4.3: Performance of modified rules under Scenario 1, Case 1.

From Figure 4.3, we observe that both the modified EMSRb2 rule and the proposed EMSRb3 heuristic can have a positive impact on the overall revenue when compared to that under the original EMSRb algorithm. At low demand factors, below 1.0, there is no change in overall revenue. The reason is that there is not much spill and hardly any sell-up. However, at high demand factors, around 1.4, the increase in overall revenue can be as high as 1.5%. The reason for the differences in total revenues under the three heuristics can be traced back to tables 4.3, 4.4 and 4.5. Here we are reproducing the results, for a demand factor of 1.2.

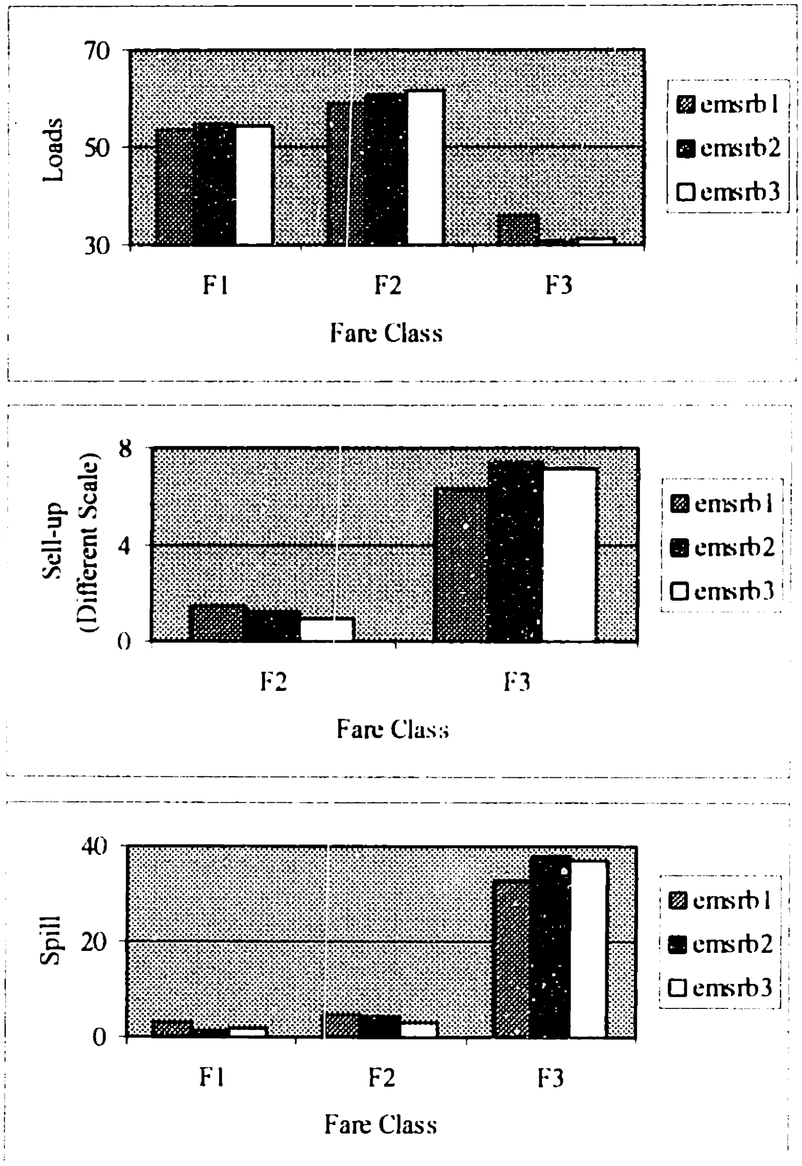

Figure 4.4: Loads, Spill and Sell-up for Scenario 1, Case 1 (DF=1.2).

Figure 4.4 presents the actual load, spill and sell-up, under the three rules, at demand factor of 1.2. The loads under the modified rules have a greater proportion of higher class passengers as compared to that under the original heuristic. Both the modified rules are able to accommodate more of the higher class passengers. It is interesting to see that the actual number of passengers selling up from class 2 to class 1 is more under the original EMSRb heuristic. However, since the original EMSRb heuristic does not protect any additional seats for passenger sell-up, many of the passengers sell-up from class 2 to class 1 are simply replacing the original class 1 passengers. This is evident from Figure 4.4, which shows that the original EMSRb heuristic results in spilling more class 1 passengers as compared to that under the modified rules. Regarding class 3, both sell-up and spill values are higher under the modified rules. Again, the reason is that the modified rules protect additional seats for passengers that might sell-up. This results in lower booking limits for class 3.

At a demand factor of 1.2, the overall revenue gains are similar under both the modified rules, EMSRb2 and MESRb3 is similar. This is not true at all demand factors. At a demand factor of 1.1, the EMSRb2 heuristic outperforms the EMSRb3 rule by a small but significant (with 95% confidence) margin. On the other hand, the EMSRb3 performs considerably better than the EMSRb2 heuristic at demand factors of 1.3 and 1.4. We are 99% confident that the improvements under the EMSRb3 heuristic are significant. From Figure 4.3, it seems that EMSRb2 perform better at demand factors below 1.2 and that EMSRb3 performs better at demand factors above 1.2. However, this trend is not observed under all cases. EMSRb2 is very sensitive to sell-up rates. In the very next case, when we increase the sell-up rates, the trend is almost opposite to that seen in Case 1.

4.3.1.3 Case 2

Under this case, we assume that there exist higher sell-up rates among passengers of class 2 and 3. Specifically, we assume that the sell-up rate among passengers desiring to travel in class 2 is 0.4 and that for class 3 is 0.3. As before, the sell-up among class 2 passengers is more than that among those of class 3. Also, it is assumed that the sell-up is existent only among the adjacent classes. Again, the sell-up rate is only applicable to passengers that are spilled.

Tables 4.6, 4.7 and 4.8 present the results under the three heuristic rules used to control the booking limits. As before, we are assuming that the sell-up rate assumed is equal to the actual sell-up rate.

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39909	1	36.03	35.86	35.85	0.02	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.53	0.01	0
0.9	89.57	44739	1	40.54	40.38	40.29	0.2	0
			2	43.24	43.11	43.03	0.27	0.11
			3	51.35	51.7	51.04	0.66	0.19
1	96.24	48935	1	45.04	45.14	44.49	1.26	0
			2	48.05	48.29	48.41	1.49	0.61
			3	57.06	56.97	51.46	5.51	1.6
1.1	98.34	5[illegible]25	1	49.54	49.51	48.59	2.51	0
			2	52.86	52.91	54.23	3.93	1.59
			3	62.77	63.15	44.69	18.46	5.25
1.2	99.15	55395	1	54.05	54.77	53.48	3.99	0
			2	57.66	57.66	60.14	6.96	2.7
			3	68.47	68.52	35.1	33.41	9.43
1.3	99.22	58127	1	58.55	58.55	57.59	5.06	0
			2	62.47	62.65	65.9	10.39	4.1
			3	74.18	73.7	25.34	48.36	13.65
1.4	99.42	61042	1	63.06	62.54	62.06	5.92	0
			2	67.27	67.42	71.64	13.94	5.44
			3	79.88	80.35	15.43	64.91	18.16
1.5	99.61	64010	1	67.56	67.89	66.93	7.82	0
			2	72.08	72.3	76.52	17.63	6.86
			3	85.59	85.25	5.97	79.28	21.86

Table 4.6: Scenario 1, Case 2, under original EMSRb

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	77.96	39666	1	36.03	35.86	35.86	0	0
			2	38.44	38.57	39.93	0	0
			3	45.65	45.54	41.16	4.38	1.35
0.9	86.76	44487	1	40.54	40.38	40.43	0.02	0
			2	43.24	43.11	45.15	0.16	0.07
			3	51.35	51.7	44.55	7.14	2.2
1	92.12	48938	1	45.04	45.14	45.24	0.2	0
			2	48.05	48.29	52.35	0.83	0.3
			3	57.06	56.97	40.58	16.39	4.89
1.1	94.32	52622	1	49.54	49.51	49.84	0.56	0
			2	52.86	52.91	59.8	2.33	0.89
			3	62.77	63.15	31.85	31.31	9.22
1.2	95.97	56628	1	54.05	54.77	55.7	1.03	0
			2	57.66	57.66	66.48	4.73	1.95
			3	68.47	68.52	21.77	46.74	13.55
1.3	96.59	59677	1	58.55	58.55	59.89	1.51	0
			2	62.47	62.65	73.28	7.43	2.85
			3	74.18	73.7	11.72	61.98	18.06
1.4	97.28	62903	1	63.06	62.54	64.62	2.1	0
			2	67.27	67.42	79.59	10.37	4.17
			3	79.88	80.35	1.71	78.64	22.54
1.5	98.57	65720	1	67.56	67.89	71.22	4.26	0
			2	72.08	72.3	76.64	19.03	7.59
			3	85.59	85.25	0	85.25	23.37

Table 4.7: Scenario 1, Case 2, under EMSRb2

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39910	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.58	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.54	44758	1	40.54	40.38	40.33	0.17	0
			2	43.24	43.11	43.08	0.26	0.12
			3	51.35	51.7	50.89	0.8	0.23
1	95.9	49058	1	45.04	45.14	44.75	0.97	0
			2	48.05	48.29	48.96	1.34	0.58
			3	57.06	56.97	50.14	6.84	2
1.1	97.54	52498	1	49.54	49.51	49.07	1.62	0
			2	52.86	52.91	56.47	2.96	1.17
			3	62.77	63.15	40.78	22.37	6.52
1.2	98.21	56323	1	54.05	54.77	54.4	2.17	0
			2	57.66	57.66	64.98	4.32	1.8
			3	68.47	68.52	27.94	40.58	11.63
1.3	98.12	59503	1	58.55	58.55	58.67	2.28	0
			2	62.47	62.65	73.5	6.26	2.4
			3	74.18	73.7	15.01	58.69	17.11
1.4	98.19	62779	1	63.06	62.54	63.28	2.6	0
			2	67.27	67.42	81.41	8.43	3.33
			3	79.88	80.35	2.6	77.74	22.42
1.5	99.07	65634	1	67.56	67.89	70.35	4.4	0
			2	72.08	72.3	77.91	17.75	6.86
			3	85.59	85.25	0.34	84.91	23.36

Table 4.8: Scenario 1, Case 2, under EMSRb3

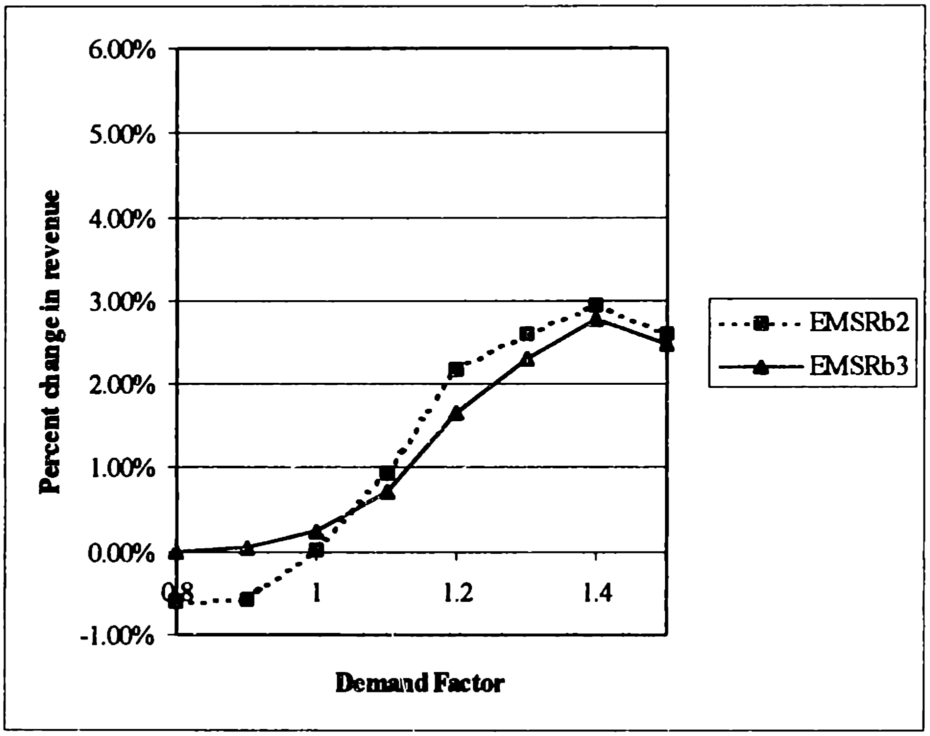

Figure 4.5: Performance of modified rules under Scenario 1, Case 2.

At increased sell-up rates and high demand factors, the relative performance of the modified rules is further increased. At a demand factor of 1.2, the increase in overall revenue under the modified rules, as compared to that under the original EMSRb heuristic, is around 2%. The increase in revenue is primarily due to the fact that under the proposed rule, the protection levels are adjusted to accommodate more of class 1 and class 2 passengers. As explained in the previous case, there are passengers selling up even under the original EMSRb heuristic. However, many of them are simply replacing the original higher class passengers. The end result is that that more higher class passengers are spilled under the original EMSRb heuristic as compared to that under the two modified rules. Comparing the performance of the two modified rules, EMSRb2 and EMSRb3, we observe that EMSRb2 performs even better than EMSRb3 at demand factors above 1.0. However, at lower demand factors, below 1.0, there is a risk of revenues getting than that under the original EMSRb algorithm.

It should be stressed that the above discussion is based upon the example used in Scenario 1. The benefits of using theEMSRb2 or EMSRb3 rules may not exist in all situations. In Scenario 2, we consider an example where the benefits of using the modified EMSRb2 rule or the proposed EMSRb3 rule are not as considerable as under Scenario 1.

4.3.2 Scenario 2

This is a seven-class example. Table 4.1 presents the mean demand and fare for each class. These values are valid for all cases tested under Scenario 1.

Class	Mean Demand	Fare ($)
1	45.04	500
2	11.98	460
3	22.99	380
4	15	300
5	48.05	260
6	57.06	220
7	37.96	180
Total Demand	238.08	

Table 4.9: Scenario 2 parameters

The capacity of the aircraft is 238 seats. As under Scenario 1, there are 16 booking periods and the demand factors range from 0.8 to 1.5. We tested the above scenario under the following cases:

Base Case: Sell-up does not exist. Both *assumed* as well as *actual* sell-up rates are given as 0.

Case 1: Sell-up exists and the *actual* sell-up rate among passengers is equal to the *assumed* sell-up rate.

As before, for each case, we simulated 500 iterations. The results presented here are the mean values.

4.3.2.1 Base Case

As under Scenario 1, the Base Case does not consider sell-up. We neither assume any sell-up rate, nor there exists any actual sell-up among the passengers. We use only the original EMSRb algorithm. Recall that when there is no sell-up, both the modified rules are reduced to the original EMSRb algorithm and we get the same protection levels and the same results. The results can be observed from Table 4.10

Table 4.10: Scenario 2, Base Case

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65574	1	40.54	40.32	40.18	0.14	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.63	0.02	0
			5	43.24	43.18	43.1	0.08	0
			6	51.35	51.74	51.64	0.09	0
			7	34.16	34.17	34.06	0.1	0
1	96.78	71261	1	45.04	45.05	44.27	0.79	0
			2	11.98	11.9	11.79	0.11	0
			3	22.99	22.98	22.85	0.13	0
			4	15	15.11	14.89	0.22	0
			5	48.05	47.58	46.61	0.97	0
			6	57.06	57.45	56.16	1.28	0
			7	37.96	37.79	33.78	4.01	0
1.1	98.33	74479	1	49.54	49.19	47.97	1.21	0
			2	13.18	12.98	12.87	0.11	0
			3	25.29	25.48	25.31	0.18	0
			4	16.5	16.39	15.97	0.42	0
			5	52.86	52.74	50.71	2.03	0
			6	62.77	62.94	59.18	3.76	0
			7	41.76	41.59	22.01	19.59	0
1.2	98.43	77248	1	54.05	53.74	52.34	1.4	0
			2	14.38	13.96	13.82	0.14	0
			3	27.59	27.58	27.31	0.27	0
			4	18	17.94	17.44	0.49	0
			5	57.66	57.58	55.04	2.54	0
			6	68.47	68.37	62.53	5.84	0
			7	45.55	45.46	5.79	39.67	0
1.3	98.9	79927	1	58.55	58.63	56.91	1.72	0
			2	15.57	15.54	15.3	0.24	0
			3	29.89	29.83	29.42	0.4	0

			4	19.5	19.6	18.95	0.65	0
			5	62.47	62.33	58.05	4.28	0
			6	74.18	73.79	56.46	17.33	0
			7	49.35	49.29	0.3	48.99	0
1.4	99.02	82106	1	63.06	62.99	60.96	2.03	0
			2	16.77	17.04	16.83	0.2	0
			3	32.19	32.54	32.01	0.53	0
			4	21	20.97	19.98	0.99	0
			5	67.27	66.56	60.92	5.64	0
			6	79.88	79.02	44.81	34.21	0
			7	53.14	53	0.16	52.84	0
1.5	98.97	84165	1	67.56	67.81	65.66	2.15	0
			2	17.97	17.78	17.53	0.26	0
			3	34.49	34.43	33.9	0.53	0
			4	22.5	22.52	21.53	0.98	0
			5	72.08	71.65	65.33	6.32	0
			6	85.59	85.75	31.48	54.27	0
			7	56.94	56.79	0.11	56.68	0

From the above table we observe that, as the demand factor increases, the spill also increases. However, as mentioned above, we have assumed that none of the passengers are willing to sell-up. In the next case we shall incorporate sell-up and compare the results.

4.3.2.2 Case 1

This is similar to the Case 1 under Scenario 1. We assume moderate sell-up among all the classes. Table 4.10 shows the value of mean sell-up rate assumed for each class.

Class	Mean Sell-up Rate
1	~
2	0.3
3	0.2
4	0.15
5	0.15
6	0.15
7	0.15

Table 4.11: Mean sell-up rates for Case 1, Scenario 2.

It is assumed that the actual sell-up rate is equal to the assumed sell-up rate. As mentioned earlier, this assumption may not be true in real life where we cannot have an accurate estimate of the actual sell-up rate existing among passengers. Nevertheless, this exercise gives us a fair idea of the impact of sell-up on overall revenues. Tables 4.11, 4.12 and 4.13 present the results under the three heuristic rules used to control the booking limits.

Table 4.12: Scenario 2, Case 1, under original EMSRb

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571	1	40.54	40.32	40.16	0.16	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.06	0.11	0.01
1	96.84	71274	1	45.04	45.05	44.19	0.9	0
			2	11.98	11.9	11.77	0.15	0.04
			3	22.99	22.98	22.84	0.17	0.02
			4	15	15.11	14.95	0.27	0.03
			5	48.05	47.58	46.63	1.14	0.1
			6	57.06	57.45	56.45	1.49	0.19
			7	37.96	37.79	33.66	4.13	0.49
1.1	98.51	74563	1	49.54	49.19	47.76	1.47	0
			2	13.18	12.98	12.87	0.16	0.05
			3	25.29	25.48	25.3	0.24	0.05
			4	16.5	16.39	16.03	0.58	0.06
			5	52.86	52.74	50.89	2.55	0.22
			6	62.77	62.94	60.42	5.1	0.71
			7	41.76	41.59	21.19	20.4	2.57
1.2	98.69	77378	1	54.05	53.74	52.05	1.76	0
			2	14.38	13.96	13.81	0.23	0.07
			3	27.59	27.58	27.26	0.4	0.07
			4	18	17.94	17.52	0.76	0.08
			5	57.66	57.58	55.24	3.52	0.35
			6	68.47	68.37	65.07	8.75	1.19
			7	45.55	45.46	3.92	41.55	5.46
1.3	99.13	79982	1	58.55	58.63	56.51	2.22	0
			2	15.57	15.54	15.27	0.37	0.09
			3	29.89	29.83	29.34	0.59	0.09

			4	19.5	19.6	19.17	1.06	0.1
			5	62.47	62.33	59.27	6.23	0.63
			6	74.18	73.79	56.22	22.71	3.17
			7	49.35	49.29	0.15	49.14	5.14
1.4	99.25	82201	1	63.06	62.99	60.5	2.6	0
			2	16.77	17.04	16.83	0.38	0.1
			3	32.19	32.54	31.85	0.89	0.17
			4	21	20.97	20.24	1.57	0.2
			5	67.27	66.56	63.61	8.62	0.83
			6	79.88	79.02	43.1	39.86	5.67
			7	53.14	53	0.1	52.91	3.94
1.5	99.27	84317	1	67.56	67.81	65.09	2.85	0
			2	17.97	17.78	17.49	0.46	0.13
			3	34.49	34.43	33.73	0.89	0.16
			4	22.5	22.52	21.63	1.84	0.19
			5	72.08	71.65	69.69	10.62	0.95
			6	85.59	85.75	28.58	60.2	8.66
			7	56.94	56.79	0.05	56.74	3.04

Table 4.13: Scenario 2, Case 1, under EMSRb2

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.8	65571	1	40.54	40.32	40.2	0.13	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.44	0.02	0
			4	13.5	13.65	13.63	0.02	0
			5	43.24	43.18	43.09	0.1	0
			6	51.35	51.74	51.67	0.1	0.01
			7	34.16	34.17	33.99	0.18	0.03
1	96.6	71287	1	45.04	45.05	44.42	0.66	0
			2	11.98	11.9	11.85	0.07	0.03
			3	22.99	22.98	22.86	0.14	0.02
			4	15	15.11	14.95	0.25	0.02
			5	48.05	47.58	46.78	0.99	0.1
			6	57.06	57.45	56.61	1.5	0.19
			7	37.96	37.79	32.42	5.36	0.67
1.1	98.09	74560	1	49.54	49.19	48.17	1.05	0
			2	13.18	12.98	12.88	0.15	0.04
			3	25.29	25.48	25.28	0.24	0.05
			4	16.5	16.39	16.19	0.45	0.04
			5	52.86	52.74	51.11	2.33	0.25
			6	62.77	62.94	60.69	5.14	0.7
			7	41.76	41.59	19.13	22.46	2.89
1.2	98.33	77408	1	54.05	53.74	52.55	1.25	0
			2	14.38	13.96	13.8	0.23	0.06
			3	27.59	27.58	27.28	0.37	0.07
			4	18	17.94	17.63	0.59	0.06
			5	57.66	57.58	55.63	3.3	0.28
			6	68.47	68.37	64.57	9.41	1.34
			7	45.55	45.46	2.56	42.9	5.61
1.3	98.87	80047	1	58.55	58.63	56.96	1.74	0
			2	15.57	15.54	15.38	0.26	0.07

			3	29.89	29.83	29.37	0.57	0.1
			4	19.5	19.6	19.28	0.88	0.12
			5	62.47	62.33	60.03	5.76	0.55
			6	74.18	73.79	54.16	24.6	3.46
			7	49.35	49.29	0.14	49.16	4.97
1.4	98.97	82299	1	63.06	62.99	61.12	1.97	0
			2	16.77	17.04	16.84	0.33	0.09
			3	32.19	32.54	31.96	0.75	0.14
			4	21	20.97	20.35	1.35	0.16
			5	67.27	66.56	64.7	7.97	0.72
			6	79.88	79.02	40.49	42.44	6.11
			7	53.14	53	0.09	52.91	3.91
1.5	99.02	84403	1	67.56	67.81	65.64	2.28	0
			2	17.97	17.78	17.43	0.47	0.1
			3	34.49	34.43	33.9	0.72	0.12
			4	22.5	22.52	21.99	1.56	0.18
			5	72.08	71.65	70.38	10.15	1.03
			6	85.59	85.75	26.26	62.5	8.87
			7	56.94	56.79	0.06	56.73	3.01

Table 4.14: Scenario 2, Case 1, under EMSRb3

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.76	71289	1	45.04	45.05	44.3	0.78	0
			2	11.98	11.9	11.79	0.13	0.03
			3	22.99	22.98	22.85	0.14	0.02
			4	15	15.11	14.99	0.25	0.02
			5	48.05	47.58	46.62	1.14	0.13
			6	57.06	57.45	56.46	1.53	0.18
			7	37.96	37.79	33.27	4.52	0.55
1.1	98.29	74559	1	49.54	49.19	47.97	1.25	0
			2	13.18	12.98	12.87	0.14	0.03
			3	25.29	25.48	25.31	0.24	0.03
			4	16.5	16.39	16.13	0.5	0.06
			5	52.86	52.74	50.86	2.51	0.24
			6	62.77	62.94	60.82	4.97	0.63
			7	41.76	41.59	19.96	21.64	2.85
1.2	98.5	77417	1	54.05	53.74	52.34	1.46	0
			2	14.38	13.96	13.81	0.21	0.05
			3	27.59	27.58	27.28	0.37	0.07
			4	18	17.94	17.63	0.64	0.07
			5	57.66	57.58	55.37	3.46	0.33
			6	68.47	68.37	65.11	8.86	1.24
			7	45.55	45.46	2.89	42.57	5.6
1.3	98.97	80039	1	58.55	58.63	56.87	1.85	0
			2	15.57	15.54	15.29	0.32	0.08
			3	29.89	29.83	29.34	0.58	0.07

			4	19.5	19.6	19.31	0.91	0.1
			5	62.47	62.33	59.79	5.9	0.61
			6	74.18	73.79	54.77	24.04	3.37
			7	49.35	49.29	0.17	49.13	5.01
1.4	99.08	82328	1	63.06	62.99	60.94	2.14	0
			2	16.77	17.04	16.87	0.32	0.09
			3	32.19	32.54	31.94	0.73	0.14
			4	21	20.97	20.4	1.26	0.12
			5	67.27	66.56	65.06	7.53	0.69
			6	79.88	79.02	40.49	42.64	6.03
			7	53.14	53	0.12	52.88	4.1
1.5	99.07	84491	1	67.56	67.81	65.54	2.38	0
			2	17.97	17.78	17.54	0.37	0.11
			3	34.49	34.43	33.88	0.7	0.13
			4	22.5	22.52	21.95	1.43	0.15
			5	72.08	71.65	72.09	8.83	0.86
			6	85.59	85.75	24.69	64.06	9.26
			7	56.94	56.79	0.1	56.7	3

It is very interesting to see the change in results with the increase in number of classes and the number of seats. The first important observation is the decrease in impact of sell-up on overall revenues. The reason is the relatively smaller difference in fares as compared to that in Scenario 1. Recall that in Scenario 1, for each fare class, the next higher valued fare was twice the value. Thus for each passenger sell-up, the gain in revenue was significant. Under Scenario 2, the increase in fare for adjacent classes varies from as low as 8.7 % (class 2 to class 1) to a moderate 26.7% (class 4 to class 3). Figure 4.6 presents the impact of sell-up on overall revenue. Note that the comparison is made with the Base Case when there is no sell-up i.e. both actual and assumed sell-up rates are considered as zero.

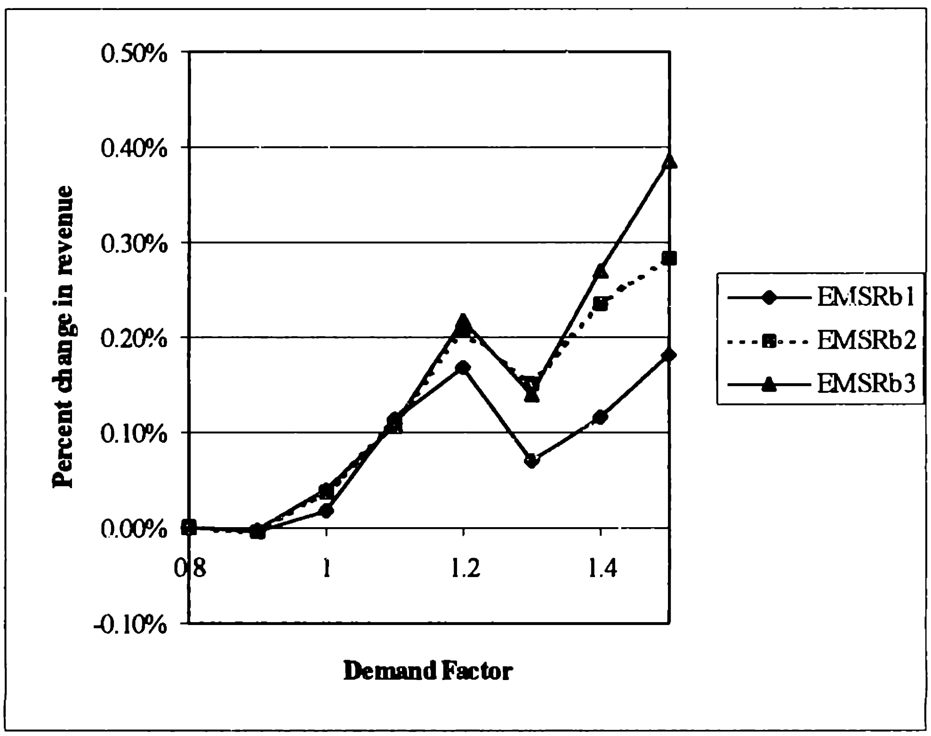

Figure 4.6: Impact of sell-up on revenue, under Scenario 2.

From Figure 4.6, we observe that as demand factor increases beyond 0.9, we begin to see the impact of sell-up. However the increase in revenue is not considerable. Even at a demand factor of 1.5, the difference is less than 0.4%. This is very small as compared to that under Scenario 1. Recall that under Scenario 1, the corresponding value was around 2%. The point here is that the impact of sell-up varies with the relative difference in fares. The higher the difference, the greater the impact.

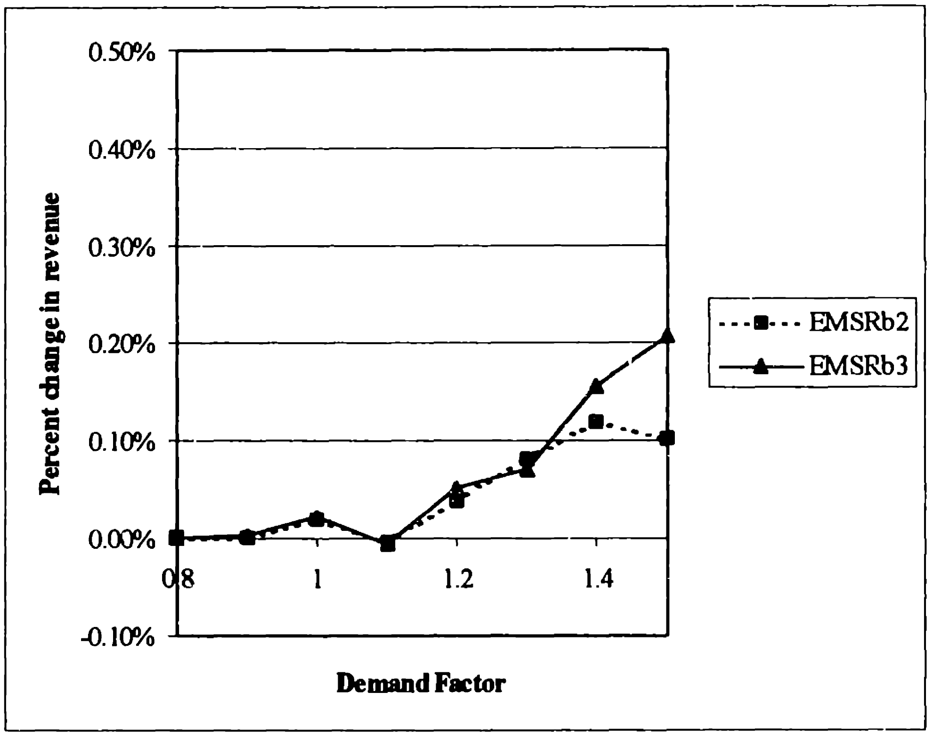

Figure 4.7: Performance of modified rules under Scenario 2, Case 1.

Figure 4.7 compares the overall revenues, under the two modified rules, with that of the original EMSRb algorithm. As discussed earlier, sell-up does not have as big an impact on revenues under Scenario 2. The benefit of using the modified rules, over the original EMSRb heuristic is therefore smaller, compared to that under Scenario 1.

Comparing the performance of the two modified rules, EMSRb2 and EMSRb3, we observe that the overall revenue is similar under both the rules, if the demand factor is less than 1.4. At very high demand factors, above 1.4, there is significant (with 95% confidence interval) improvement in revenue, under EMSRb3.

The two scenarios give us a fair idea of the performance of the proposed EMSRb3 rule. We saw that the benefits of using the proposed EMSRb3 rule are not consistent under all cases. We witnessed a considerable improvement in revenue under Scenario 1. The improvement in Scenario 2 was smaller, compared to that under Scenario 1. The reason can be linked with two important differences in the overall set up of the two Scenarios: The relative difference in fares among adjacent classes and the total number of classes. The revenue gains through sell-up are high under Scenario 1 when the difference in fares among adjacent classes is high. Also, the percentage increase in revenue under the proposed rule is more significant under Scenario 1 when we have fewer classes as compared to that under Scenario 2. With the increase in number of classes, there is a decrease in the number of people expected to sell-up to the next higher class. The reason is lesser expected spill from each class. Note that this statement is based upon the assumption that sell-up is only existent among adjacent classes. This might not be true in the real world where passengers may sell-up to nonadjacent classes. Since the modified rules assume sell-up among adjacent classes only, the protection levels under Scenario 2 are not much different than that under the original EMSRb heuristic. The result is smaller revenue gains over the original heuristic.

A desirable quality of the proposed rule is that for every additional seat protected, it compares the expected gain with the expected loss. This ensures that there is not much risk of losing any revenue when we move to the proposed EMSRb3 rule from the original EMSRb heuristic. This is not the case for EMSRb2 rule. As observed under Scenario 1, the overall revenue, under EMSRb2, could get lower then that under the original heuristic. This is particularly true under high sell-up rates and low demand factors.

In this chapter, we presented a simulation to test the performance of the modified EMSRb2 rule and the proposed EMSRb3 rule. The initial portion of the chapter described the simulation structure, its environment and the assumptions associated with it. In the remaining parts, we tested a number of cases under the original EMSRb heuristic and the modified rules, EMSRb2 and EMSRb3. The results, under each rule were discussed. We primarily concentrated on the performance of the modified rules against that of the original EMSRb rule. We also compared the relative performance of the two modified rules. In the next chapter, we perform sensitivity analysis and discuss the performance of the three rules under high demand variations and errors in sell-up estimation.

Chapter 5

Sensitivity Analysis

In the last chapter, we made two strong assumptions: The actual sell-up rate is equal to the assumed sell-up rate, and the requests arrive as a Poisson process. The first assumption implies that we have perfect knowledge of the sell-up rate existing among passengers. The second assumption implies that the variance of the demand distribution is equal to mean (characteristic of the Poisson distribution). In this chapter, we perform sensitivity analysis to study the performance of the original EMSRb algorithm and the modified rules, under situations which deviate from the above assumptions. We shall use the same scenarios as developed in Chapter 4.

5.1 Sell-up Rate

As mentioned above, our analysis in the previous chapter was based upon the assumption that we have perfect knowledge of the *actual* sell-up rate existing among the spilled passengers. This is rarely the case. In reality is very difficult, if not impossible, to accurately estimate the *actual* sell-up rate. To perform sensitivity analysis, we intentionally introduce some error in the estimation of sell-up rate. This implies that, in our simulation, the *assumed* sell-up rate would be different than the *actual* sell-up rate.

Recall that the simulation uses the *assumed* sell-up rate to control the booking limits and the *actual* sell-up rate to generate requests.

5.1.1 Scenario 1

This is the three class scenario developed in Chapter 4. For sensitivity analysis, we introduce differences of 0.05, 0.1, 0.15 and 0.2 on both negative and positive side, in the assumed vs. actual sell-up rate, and simulate Case 1 under the three heuristics. Recall that under Case 1, the actual sell-up rate for class 2 and class 3 was 0.2 and 0.3 respectively. An error of 0.2 would translate to 100% and 67%, respectively, for the assumed sell-up rates of class 3 and class 2. Detailed results for the simulation runs are included in appendix. Here we present the overall impacts on revenue.

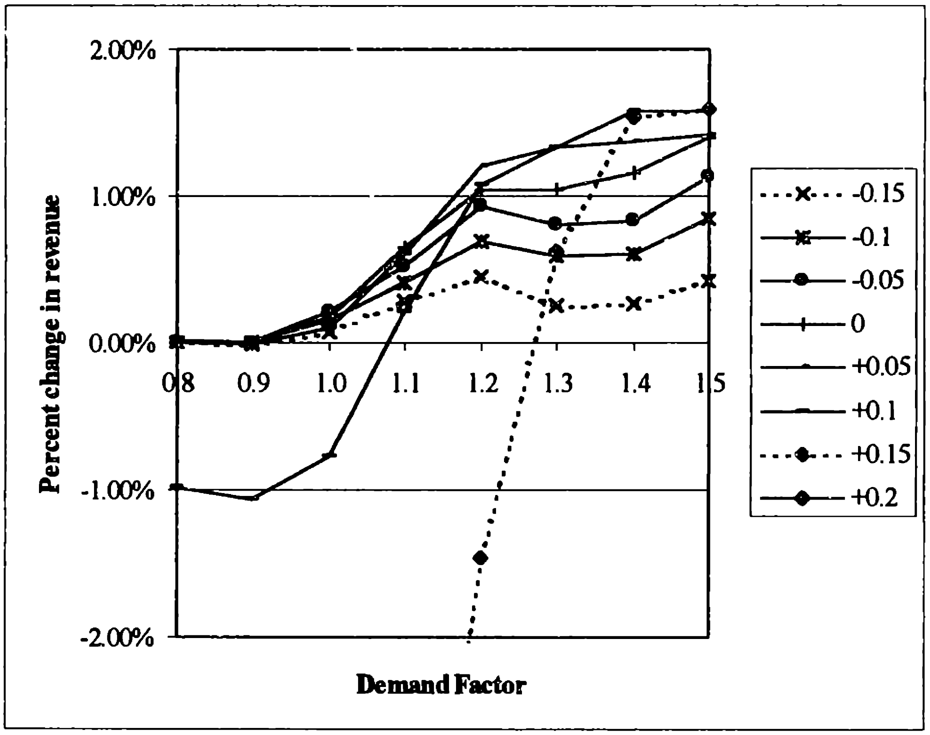

Figure 5.1: Sensitivity of assumed sell-up rate under EMSRb2 (Scenario 1, Case 1).

Figure 5.1 compares the performance of the modified EMSRb2 rule with that of the original EMSRb heuristic, under the various errors in sell-up rates. A '-' indicates underestimation of actual sell-up rate. In the simulation it implies that the *assumed* sell-up rate is less than the *actual* value. Similarly a '+' indicates over estimation, i.e. the *assumed* sell-up rate is greater than the *actual* value. Note that we are only changing the *assumed* sell-up rate in the simulation. The *actual* sell-up rate is not changed. This implies that the results under the original EMSRb algorithm remain unchanged.

When sell-up rate is underestimated (assumed sell-up rate is less than the actual sell-up rate), the benefits of using the modified EMSRb2 rule decrease. The greater the error, the lesser is the improvement in revenue. The reason is that not enough seats are protected to capture sell-up. When sell-up rate is overestimated (assumed sell-up rate is greater than the actual sell-up rate), the results are very sensitive to both the amount of error and the demand factor. If the error is small (+0.05 in the current case), then it does not have any negative impact. In fact, at high demand factor, a slight overestimation (+0.5) can result in some improvements in revenue over the original EMSRb heuristic. However, at high errors or low demand factors, or both, there can be a substantial decrease in overall revenues. When the difference between the assumed sell-up rate and actual sell-up rate is +0.15, the overall revenue gain over original EMSRb is very sensitive to demand factors. It varies from –11.4% at demand factor of 0.8 to +1.6% at demand factor of 1.5. At very higher errors (+0.2) there is a negative impact at all demand factors and the overall revenue decreases by 9 to 12% at all demand factors (not shown in Figure 5.1). This suggests that the sell-up rates assumed under the modified EMSRb2 rule must be on the conservative side. An underestimation may cause a slight decrease in revenue gains but an overestimation could result in substantial losses, particularly at lower demand factors.

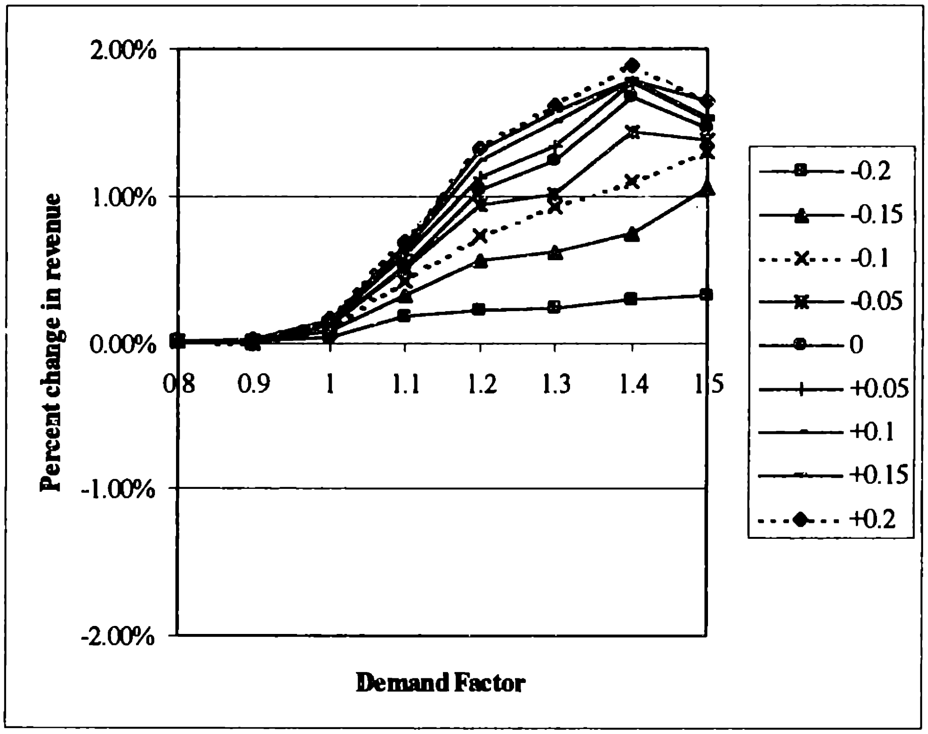

Figure 5.2: Sensitivity of assumed sell-up rate under EMSRb3 (Scenario 1, Case 1).

Figure 5.2 compares the performance of the proposed EMSRb3 rule with that of the original EMSRb heuristic, under the various errors in sell-up rates. Again a '-' indicates under estimation, i.e. the *assumed* sell-up rate is less than the *actual* value and a '+' indicates over estimation. As with EMSRb2, the benefits of using the modified rule decrease when sell-up is underestimated. The reason is that the protection levels are not enough to capture passengers willing to sell-up to high classes or the passengers selling up are only replacing other higher class passengers. The greater the under estimation, the lesser is the improvement in revenue over the original EMSRb algorithm.

An interesting phenomenon is observed under overestimation. The improvements in overall revenues are further increased if the sell-up rate is overestimated at high demand factors. The reason is that the additional seats protected are sold to higher class passengers if the demand factor is high. At lower demand there is no considerable impact of overestimating the sell-up rate. This suggests that, under the proposed EMSRb3 rule, there is not much risk of negative impacts on overall revenues, even under high errors on sell-up estimation.

5.1.2 Scenario 2

This is similar to Case 1, under Scenario 2. We introduce some error in the estimation of sell-up rates. As mentioned before, in reality it is not possible for an airline to accurately measure the true sell-up rate existing among its customers. The result is that the sell-up rate estimated or *assumed* by the airline is different from the *actual* sell-up. Again, an error of 0.05, 0.1, 0.15 and 0.2 on both the negative and positive side, is introduced in the actual sell-up rate. For the actual sell-up rates, among various classes please refer to Table 4.11. The maximum error of 0.2 would thus translate to 67% for class 2 and 100% for all the remaining classes. We expect that, in reality the sell-up rate would be within these errors. Note that the sell-up rate cannot be less than zero. If after deducting the error, the resulting value of assumed sell-up rate becomes less than zero, we consider it as zero in our simulation. Figure 5.3 and 5.4 compare the performance of the modified EMSRb2 rule and the proposed EMSRb3 rule with that of the original EMSRb heuristic. Detailed results are included in appendix.

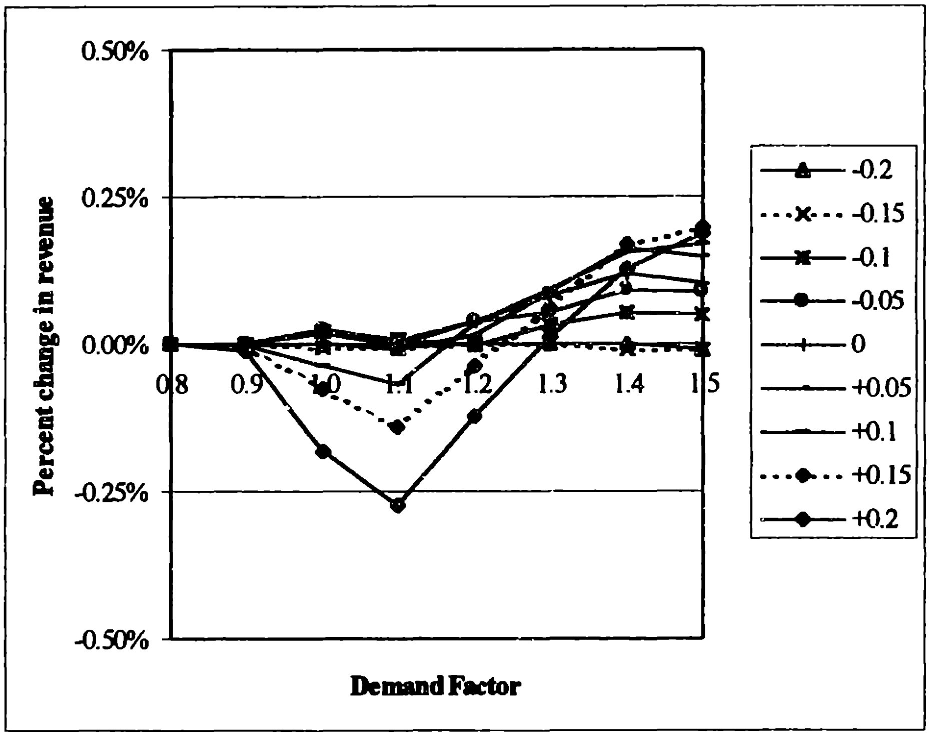

Figure 5.3: Sensitivity of assumed sell-up rate under EMSRb2 (Scenario 2, Case 1).

Figure 5.3 compares the performance of the modified EMSRb2 rule, with the original EMSRb heuristic, under various errors in estimation of sell-up rate. As under Scenario 1, a '-' indicates underestimation whereas a '+' indicates overestimation. When the sell-up rate is underestimated, the benefits of the modified rule over the original EMSRb algorithm are reduced. The greater the underestimation, the lesser is the improvements. When the sell-up rate is overestimated, the improvements in overall revenue are very sensitive to demand factors. At higher demand factors, there is a positive impact on overall revenues and the improvements over the original EMSRb algorithm are further enhanced. However, at lower demand factors, there is a risk of negative impact on revenues. Again, this suggests under low or moderate demand factors, the sell-up rates assumed under the modified EMSRb2 should be conservative. An underestimation is more desirable than overestimation.

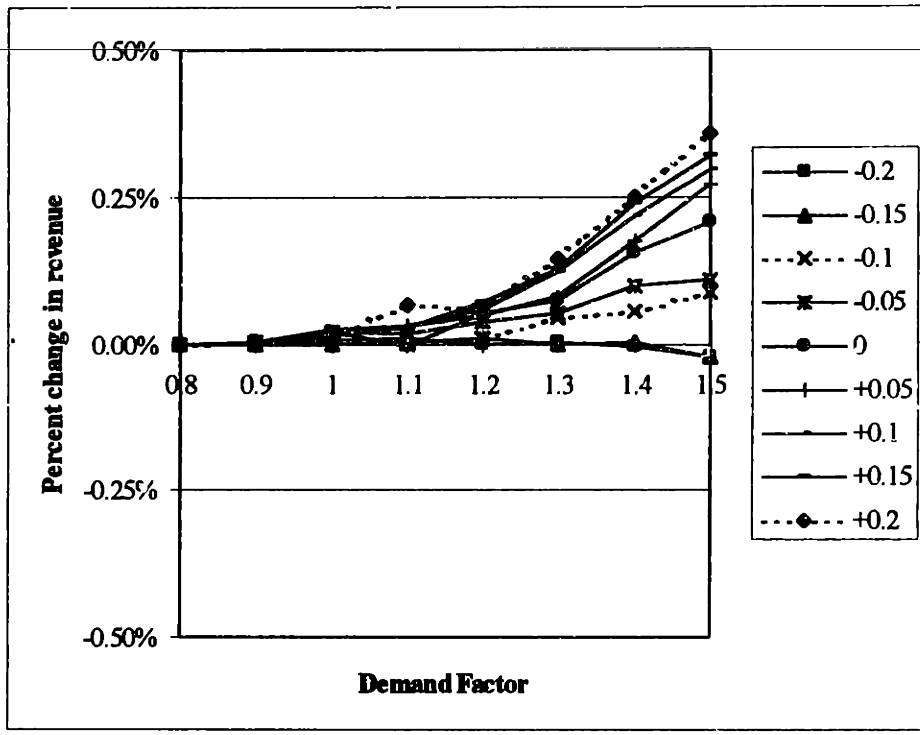

Figure 5.4: Sensitivity of assumed sell-up rate under EMSRb3 (Scenario 2, Case 1).

Figure 5.4 compares the performance of the proposed EMSRb3 rule with that of the original EMSRb heuristic, under various errors in sell-up rates. Again a '-' indicates underestimation whereas a '+' indicates overestimation of actual sell-up rates. When the sell-up rate is underestimated the trend is similar to that under the EMSRb2 rule. The higher the error, the lesser are the benefits of the modified rule over the original EMSRb algorithm. Again, there is not much risk of revenues getting below that under the original heuristic, if the sell-up rates are underestimated.

When the sell-up rate is overestimated, the impact is dependent upon demand factor. At low demand factors, the impact is not considerable. At high demand factors, there is positive impact on overall revenues. The overestimation results in overprotection of seats for higher class passengers. At high demand factors, this is desirable and enhances the improvements of the modified EMSRb3 rule. This suggests that it is more desirable to have an overestimated sell-up rate as compared to an underestimated sell-up rate when using the modified EMSRb3 rule.

5.2 Variability in Requests

In our previous analysis, we assumed that the requests arrive as a Poisson process and the variance is equal to the mean forecasted demand. Recall the demand is forecasted for each fare class, under each booking period. Since the demand is broken into many booking periods, the variability could be high and our assumption, that variance is equal to mean, may no longer be valid. To perform sensitivity analysis in this area, we increase the variability in demands. Specifically we target for a Z-factor of 2. Recall that the relationship between Z-factor, Standard Deviation (σ) and mean demand (μ) is as follows:

$$\sigma = Z^* \sqrt{\mu}$$

A Z-factor of 2 implies that the standard deviation is equal to twice the square root of mean or the variance is four times the mean demand. This seems very large, however, it is not unrealistic as the demand for each fare class is divided into 18 booking periods and the number of requests for a specific fare class, under a specific booking period, can be highly variable. We simulated both Scenario 1 and Scenario 2, of Chapter 4, under high variability.

To increase variability, we increase demands that are above mean and decrease those that are below mean values. Note that these changes are made within the demands generated for various iterations, for the same fare class and the same booking period. The generated demands are adjusted such that the mean value of the demands generated over iterations remains unchanged. Specifically, for every reduction of demand (for any iteration that has demand below mean), there is a corresponding increase in demand (under an iteration that has demand above mean).[1]

5.2.1 Scenario 1

This is the same three class scenario developed in Chapter 4. Here we shall consider Case 1, under which there exists moderate sell-up. Recall that the sell-up rate for class 2 and class 3 is 0.3 and 0.2, respectively. Under increased variability, the performance of all the booking control algorithms suffers. Detailed results under the three booking control rules, original EMSRb, EMSRb2 and EMSRb3 are included in appendix. Here we shall focus on the changes in comparative performance of the two modified rules under increased variability.

[1] The method is adopted from Bratu, Stephane, *"Network Value Concept in Airline Revenue Management"*, MS thesis, MIT Flight Transportation Laboratory, May 1998.

The performance of all the three heuristics is affected at increased variance. However, the impact is not same under all rules. It would be interesting to compare the performance of the modified rules with that of the original EMSRb algorithm under increased variance.

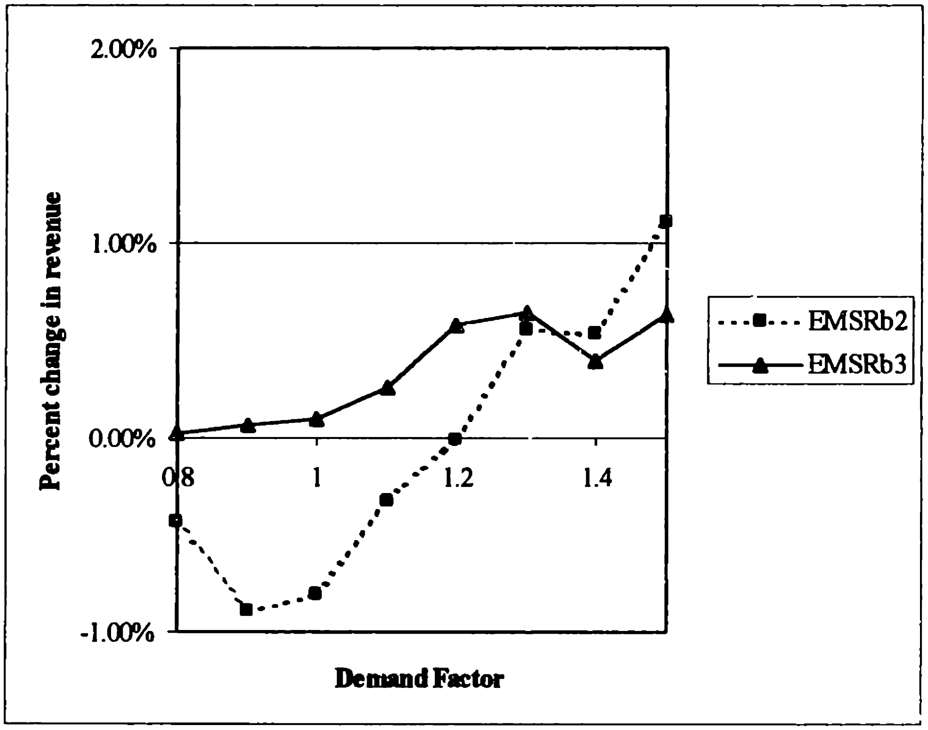

Figure 5.5: Performance of modified rules under high variance (Scenario 1, Case 1).

The comparative performance of the modified rules has changed considerably under increased variance. Under increased variance, EMSRb2 is very sensitive to demand factor. At higher demand factors, above 1.2, there is a positive impact on overall revenue, however, at low demand factors, there is a risk of negative impact. At a demand factor of 0.9, the overall revenue could be 0.9% less than that under the original algorithm. The reason is that the additional protection is not desirable under increased variance. There is a risk that the demand would be much less than that expected and the seats may remain unsold.

The benefits of EMSRb3, over the original EMSRb algorithm, are also reduced. Again, the reason is the high variability in demand. The additional protection (for passengers that might sell-up) is beneficial when the demand is equal to or higher than that expected. At low demands, there is not much spill and not many passengers would be willing to sell-up. However, unlike the case of EMSRb2, there is not much risk of revenues getting lower than that under the original EMSRb algorithm.

5.2.2 Scenario 2

This is the 7 class scenario, developed in Chapter 4. For sell-up rates under various fare classes, please refer to Table 4.9. We simulated this scenario under the original EMSRb algorithm as well as the modified rules, EMSRb2 and EMSRb3. Detailed results are included in appendix. Here we shall concentrate on the comparative performance of the modified rules against the original EMSRb algorithm.

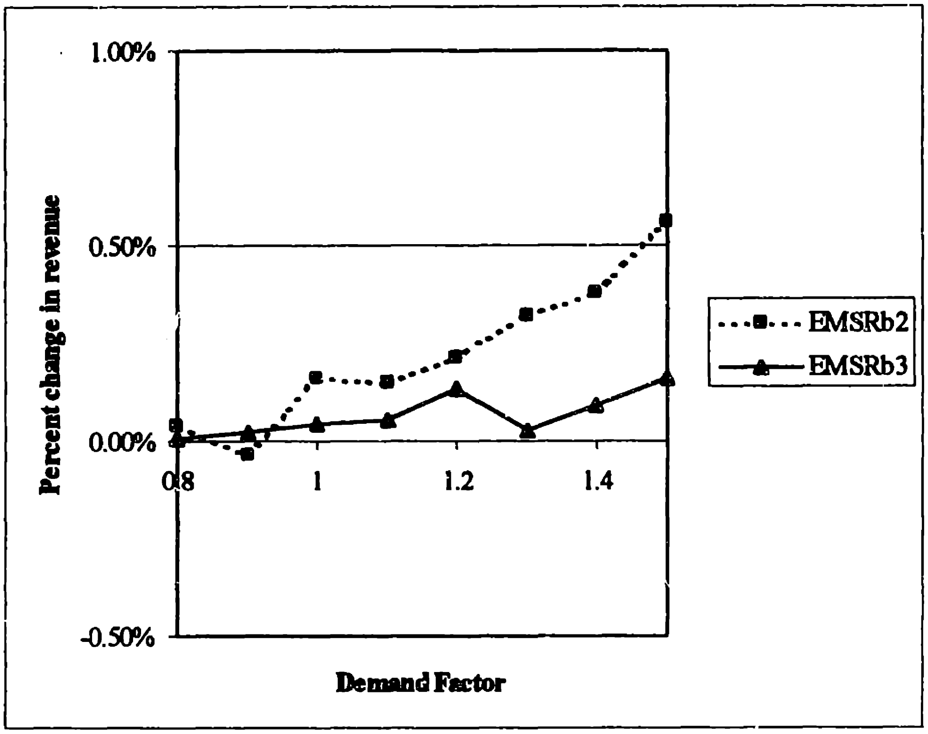

Figure 5.6: Performance of modified rules under high variance (Scenario 2, Case 1).

Figure 5.6 shows the performance of the modified rules under high variance. The performance of EMSRb3 as compared to that of the original EMSRb is not much different than that under low variance. Recall the even under low variance, the improvement in revenue under EMSRb3 was not considerable in Scenario 2. The high variance does not make much difference. The performance of EMSRb2 is, however, enhanced under increased variance. This is particularly observed at higher demand factors. At a demand factor of 1.5, the improvement in revenue over the original EMSRb algorithm, could be above 0.5%. Recall that under low variance, this figure was merely 0.1%

Comparing the relative performance of the two modified rules, we observe that the modified rule EMSRb2 is very sensitive to the fare structure and variations in demand. In Scenario 2, we observed that the benefits of EMSRb2, over the original algorithm are greater than that under lesser variance. However, in Scenario 1, the improvements in revenue, over the original EMSRb algorithm, are highly sensitive to the demand factor. At high demand factors (above 1.2), the revenue gains are similar to that under lesser variance but at low demand factors, the overall revenue is less than that under the original algorithm. The proposed EMSRb3 rule is more robust to variations in demand. Although the improvements over the original algorithm are reduced under high variance, however, there is not much risk of revenues going lower than that under the original algorithm.

In this chapter we performed sensitivity analysis on the performance of the modified rules. In the real world, neither the estimated sell-up rates are accurate, nor does the demand follows a true Poisson distribution. We observed the effects of misestimating the sell-up rates and high variability in demands. The focus was in the changes in comparative performance of the two modified rules, against the original EMSRb algorithm. In the next chapter, we conclude our findings and suggest future research directions.

Chapter 6

Conclusions

6.1 Research Findings

In this thesis we were have emphasized the importance of passenger sell-up. It was shown that passenger sell-up can account for considerable improvements in airline revenues. The extent of improvement varies under the different types of booking control algorithms. The parameters like sell-up rate, demand factor, spill, and fare ratios, also effect the comparative performance of various booking control methods.

At low demand factors, there is not much spill. Most of the passengers are able to obtain seats in their desired fare class and hardly any one is willing to sell-up. Under these circumstances, no improvements in revenue are observed. However, at high demand factors, there is considerable spill. Depending on the sell-up rate, there might be a number of passengers willing to sell-up. This can result in huge revenue gains for airlines. The higher the sell-up rates, the greater are the revenue gains.

Fare ratio also plays an important role in determining the revenue gains. If the fare ratio is low, i.e., if the difference between adjacent fare classes is high, the additional revenue gained through sell-up is more. The lower the fare ratio, the more would be the revenue gains.

We proposed a modified heuristic (referred as EMSRb3) to incorporate passenger sell-up. Unlike most of the previous models, this heuristic also incorporates the lower class demand in determining the protection levels for higher classes, given that sell-up is expected. Specifically, it is an extension of the original EMSRb algorithm. The EMSRb algorithm is first used to determine the protection levels. Based on these protection levels, expected spill is computed for each fare class. Using the expected spill values and the sell-up rates, the new heuristic estimates the expected number of passengers that would be willing to sell-up. The booking limits are then readjusted based upon the expected revenues from passengers willing to sell-up. The heuristic continues to protect seats, in addition to the protection levels obtained from the original EMSRb algorithm, as long as the expected revenue from passengers selling up is higher than the lower class fare.

A simulation was developed to analyze the revenue performance of the proposed rule and that of the existing rules. We focussed on the relative performance of the two modified rules, the one developed by Belobaba and Weatherford[1] (referred as EMSRb2), and the proposed EMSRb3 rule, against that of the original EMSRb algorithm. The proposed rule results in revenue gains over the original EMSRb algorithm. However, it is

[1] Peter P. Belobaba & Lawrence R. Weatherford, *"Comparing Decision Rules that Incorporate Customer Diversion in Perishable Asset Revenue Management Situations"*, Decision Sciences, Volume 27, Spring 1996.

stressed that the revenue gains vary under different scenarios. In our test cases, the proposed rule showed considerable improvements under Scenario 1. But in Scenario 2, the gains were minimal. Scenario 1 is characterized by few fare classes and low fare ratios. Scenario 2, on the other hand, is characterized by many fare classes and higher fare ratios. Nevertheless, the proposed rule never resulted in losses in overall revenues when compared to that under the original algorithm. This was not the case with EMSRb2. Under low demand factors, it can result in revenues lower than that under the original algorithm.

. Our analysis recognizes the fact that the sell-up rates assumed by an airline are not likely to be accurate. The simulation, we developed, has the provision of specifying two different values of sell-up rates: *assumed* sell-up rate and *actual* sell-up rate. The *assumed* sell-up rate is used for computing the protection levels whereas the *actual* sell-up rate is used for deciding if a simulated passenger would actually sell-up to the next higher class. We also analyzed the performance of the two modified rules under situations when the assumed sell-up rate is not equal to the actual value (i.e., under errors in sell-up estimation). It was shown that the proposed EMSRb3 rule is more robust to errors in sell-up estimation than the EMSRb2 rule. There may be situations under which EMSRb2 outperforms the proposed rule, however, it is more sensitive to errors in sell-up estimation. This is particularly true for overestimated sell-up rates, i.e., when the assumed sell-up rates are greater then the actual sell-up rates. If the sell-up rates are seriously overestimated, then there are risks of extreme negative impacts on overall revenue, under the EMSRb2 heuristic. On the contrary, the proposed EMSRb3 rule is not adversely affected by overestimation of sell-up rates.

With under estimated sell-up rates, the improvements in revenue, under both the modified rules, EMSRb2 and EMSRb3, are reduced. The more the underestimation, the lesser are the improvements in revenue. The reason is that with underestimated sell-up rates, the modified rules have protection levels that are close to the original algorithm and the resulting performance is close to that under the original algorithm.

We also analyzed the performance of the two modified rules under increased variability in demands. Again it was shown that the proposed EMSRb3 rule is more robust. Although the improvements in revenue, as compared to the original EMSRb algorithm, were reduced under higher variability, the decrease in revenue was consistent under both the scenarios. Most importantly, there was not much risk of losing revenue against the original EMSRb algorithm. The performance of EMSRb2 rule, under increased variability was not consistent over the two scenarios tested. In the first scenario, under low demand factors, it resulted in revenue losses over the original EMSRb algorithm. Surprisingly, in the second scenario, under high demand factors, the performance of the EMSRb2 rule was further enhanced and the revenue gains were even greater than that under lesser variability.

Based on our analysis we cannot guarantee which of the two modified rules will outperform the other one in the real world. However, do we believe that the proposed

EMSRb3 rule is more robust to uncertainties in demands and sell-up rates and is less likely to cause a negative impact on overall revenues.

6.2 Future Research Directions

We believe that there is a lot of room for further research in the area of passenger sell-up. Some of the research directions are mentioned below:

- *Sell-up among non-adjacent fare classes:* Throughout our analysis, we assumed that sell-up is only existent among adjacent fare classes. This might not be true under large number of fare classes. When the fare ratio is high, i.e. the difference between fares is less, sell-up would also exist among non-adjacent fare classes. It would be interesting to incorporate this fact into the decision rule.

- *Combination of EMSRb2 and EMSRb3:* EMSRb2 uses decision tree concept to incorporate passenger sell-up in the original heuristic. On the other hand, EMSRb3 uses expected spill and expected sell-up to readjust the protection levels obtained from the original algorithm. It might be beneficial to combine both these modified rules.

- *Estimation of Sell-up rates:* Not much work had been done to estimate passenger sell-up. Estimation of sell-up rate is the prerequisite for implementing any rule that incorporates passenger sell-up in airline revenue management.

- *Competitive Environment:* Our analysis is based upon the simplified assumption that there exists only one airline in the market. It would be beneficial to analyze the performance of the proposed rule under a competitive environment with more than one airline.

- *Network Effects:* Our analysis is based upon a single leg case. It would be interesting to study the impact of passenger sell-up on the whole network.

Appendix

A1: Sensitivity of sell-up rate

Table A1.1: Scenario 1, Case 1, under EMSRb2, with –0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.58	0.01	0
			3	45.65	45.54	45.51	0.03	0.01
0.9	89.43	44734	1	40.54	40.38	40.36	0.11	0
			2	43.24	43.11	43.01	0.26	0.09
			3	51.35	51.7	50.78	0.92	0.16
1	95.51	48968	1	45.04	45.14	44.94	0.62	0
			2	48.05	48.29	48.36	1.28	0.42
			3	57.06	56.97	49.96	7.01	1.34
1.1	97.08	52088	1	49.54	49.51	49.16	1.15	0
			2	52.86	52.91	54.16	2.78	0.8
			3	62.77	63.15	42.3	20.85	4.04
1.2	97.95	55563	1	54.05	54.77	54.57	1.66	0
			2	57.66	57.66	59.78	4.78	1.46
			3	68.47	68.52	32.57	35.95	6.9
1.3	97.9	58184	1	58.55	58.55	58.38	2.15	0
			2	62.47	62.65	65.9	6.64	1.98
			3	74.18	73.7	22.56	51.13	9.89
1.4	98.05	61035	1	63.06	62.54	62.57	2.48	0
			2	67.27	67.42	72.14	8.53	2.51
			3	79.88	80.35	12.37	67.98	13.25
1.5	98.5	64333	1	67.56	67.89	67.93	3.27	0
			2	72.08	72.3	77.34	10.95	3.31
			3	85.59	85.25	2.48	82.77	16

Table A1.2: Scenario 1, Case 1, under EMSRb2, with –0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0.01	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.48	44734	1	40.54	40.38	40.34	0.13	0
			2	43.24	43.11	42.97	0.27	0.09
			3	51.35	51.7	50.91	0.79	0.14
1	95.74	48938	1	45.04	45.14	44.8	0.73	0
			2	48.05	48.29	48.24	1.27	0.39
			3	57.06	56.97	50.56	6.41	1.21
1.1	97.48	52029	1	49.54	49.51	49.01	1.41	0
			2	52.86	52.91	53.61	3.03	0.91
			3	62.77	63.15	43.6	19.55	3.73
1.2	98.26	55434	1	54.05	54.77	54.31	1.98	0
			2	57.66	57.66	59.25	5.09	1.51
			3	68.47	68.52	33.84	34.68	6.67
1.3	98.32	58056	1	58.55	58.55	58.03	2.51	0
			2	62.47	62.65	65.48	6.85	1.99
			3	74.18	73.7	23.97	49.73	9.69
1.4	98.48	60898	1	63.06	62.54	62.43	2.85	0
			2	67.27	67.42	70.97	9.2	2.74
			3	79.88	80.35	14.33	66.02	12.74
1.5	98.71	64152	1	67.56	67.89	67.81	3.61	0
			2	72.08	72.3	76.18	11.78	3.53
			3	85.59	85.25	4.08	81.17	15.66

Table A1.3: Scenario 1, Case 1, under EMSRb2, with –0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0.01	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44729	1	40.54	40.38	40.32	0.15	0
			2	43.24	43.11	42.98	0.27	0.08
			3	51.35	51.7	50.96	0.74	0.14
1	95.93	48900	1	45.04	45.14	44.66	0.84	0
			2	48.05	48.29	48.12	1.33	0.36
			3	57.06	56.97	51.11	5.86	1.15
1.1	97.77	51964	1	49.54	49.51	48.8	1.59	0
			2	52.86	52.91	53.36	3.14	0.88
			3	62.77	63.15	44.49	18.67	3.59
1.2	98.45	55294	1	54.05	54.77	54.08	2.27	0
			2	57.66	57.66	58.73	5.4	1.57
			3	68.47	68.52	34.86	33.65	6.47
1.3	98.5	57859	1	58.55	58.55	57.8	2.91	0
			2	62.47	62.65	64.57	7.42	2.17
			3	74.18	73.7	25.38	48.32	9.34
1.4	98.57	60688	1	63.06	62.54	62.06	3.22	0
			2	67.27	67.42	70.55	9.48	2.74
			3	79.88	80.35	15.24	65.11	12.61
1.5	98.98	63878	1	67.56	67.89	67.33	4.23	0
			2	72.08	72.3	75.38	12.22	3.68
			3	85.59	85.25	5.75	79.5	15.3

Table A1.4: Scenario 1, Case 1, under EMSRb2, with –0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39909	1	36.03	35.86	35.85	0.02	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.53	0.01	0
0.9	89.55	44736	1	40.54	40.38	40.32	0.16	0
			2	43.24	43.11	42.97	0.27	0.09
			3	51.35	51.7	51.04	0.66	0.13
1	96.04	48881	1	45.04	45.14	44.65	0.93	0
			2	48.05	48.29	47.87	1.45	0.44
			3	57.06	56.97	51.55	5.42	1.03
1.1	97.97	51891	1	49.54	49.51	48.63	1.85	0
			2	52.86	52.91	53.1	3.16	0.97
			3	62.77	63.15	45.22	17.93	3.36
1.2	98.69	55167	1	54.05	54.77	53.79	2.66	0
			2	57.66	57.66	58.37	5.55	1.68
			3	68.47	68.52	35.88	32.63	6.26
1.3	98.76	57824	1	58.55	58.55	57.73	3.11	0
			2	62.47	62.65	64.16	7.57	2.3
			3	74.18	73.7	26.24	47.45	9.08
1.4	98.86	60595	1	63.06	62.54	61.82	3.58	0
			2	67.27	67.42	70.22	9.6	2.86
			3	79.88	80.35	16.26	64.09	12.4
1.5	99.09	63680	1	67.56	67.89	67.03	4.56	0
			2	72.08	72.3	74.82	12.53	3.7
			3	85.59	85.25	6.8	78.46	15.05

Table A1.5: Scenario 1, Case 1, under EMSRb2 with +0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.95	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.59	0	0
			3	45.65	45.54	45.49	0.05	0.02
0.9	89.13	44724	1	40.54	40.38	40.41	0.04	0
			2	43.24	43.11	43.23	0.2	0.07
			3	51.35	51.7	50.05	1.65	0.33
1	94.35	48915	1	45.04	45.14	45.08	0.35	0
			2	48.05	48.29	49.33	0.91	0.29
			3	57.06	56.97	47.11	9.86	1.94
1.1	95.66	52135	1	49.54	49.51	49.48	0.68	0
			2	52.86	52.91	55.64	2.1	0.64
			3	62.77	63.15	38.38	24.77	4.84
1.2	96.56	55717	1	54.05	54.77	54.94	0.93	0
			2	57.66	57.66	61.8	3.75	1.09
			3	68.47	68.52	28.1	40.41	7.89
1.3	96.72	58486	1	58.55	58.55	58.77	1.32	0
			2	62.47	62.65	68.52	5.12	1.53
			3	74.18	73.7	17.79	55.9	10.99
1.4	96.92	61361	1	63.06	62.54	63.06	1.52	0
			2	67.27	67.42	74.53	7.06	2.03
			3	79.88	80.35	7.8	72.55	14.17
1.5	97.68	64520	1	67.56	67.89	68.76	2.47	0
			2	72.08	72.3	77.33	11.35	3.34
			3	85.59	85.25	0.43	84.83	16.39

Table A1.6: Scenario 1, Case 1, under EMSRb2, with +0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	77.64	39521	1	36.03	35.86	35.86	0	0
			2	38.44	38.57	39.44	0	0
			3	45.65	45.54	41.16	4.38	0.87
0.9	86.28	44257	1	40.54	40.38	40.42	0	0
			2	43.24	43.11	44.38	0.14	0.04
			3	51.35	51.7	44.62	7.07	1.41
1	91.22	48486	1	45.04	45.14	45.16	0.14	0
			2	48.05	48.29	50.94	0.49	0.16
			3	57.06	56.97	40.73	16.24	3.14
1.1	93.07	51926	1	49.54	49.51	49.61	0.3	0
			2	52.86	52.91	57.73	1.31	0.4
			3	62.77	63.15	32.27	30.89	6.13
1.2	94.32	55643	1	54.05	54.77	55.07	0.46	0
			2	57.66	57.66	64.25	2.57	0.75
			3	68.47	68.52	22.15	46.36	9.16
1.3	94.74	58491	1	58.55	58.55	58.98	0.77	0
			2	62.47	62.65	70.89	3.9	1.2
			3	74.18	73.7	12.23	61.46	12.14
1.4	95.25	61499	1	63.06	62.54	63.19	0.88	0
			2	67.27	67.42	77.54	5.32	1.53
			3	79.88	80.35	2.15	78.19	15.44
1.5	97.32	64626	1	67.56	67.89	69.46	2.05	0
			2	72.08	72.3	76.48	12.2	3.62
			3	85.59	85.25	0.04	85.21	16.38

Table A1.7: Scenario 1, Case 1, under EMSRb2, with +0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	55.7	35824	1	36.03	35.86	35.86	0	0
			2	38.44	38.57	47.69	0	0
			3	45.65	45.54	0	45.54	9.12
0.9	62.55	40261	1	40.54	40.38	40.38	0	0
			2	43.24	43.11	53.44	0	0
			3	51.35	51.7	0	51.7	10.33
1	69.91	45001	1	45.04	45.14	45.14	0	0
			2	48.05	48.29	59.72	0	0
			3	57.06	56.97	0	56.97	11.43
1.1	76.75	49393	1	49.54	49.51	49.51	0	0
			2	52.86	52.91	65.61	0	0
			3	62.77	63.15	0	63.15	12.71
1.2	84.02	54254	1	54.05	54.77	54.81	0	0
			2	57.66	57.66	71.22	0.15	0.04
			3	68.47	68.52	0	68.52	13.7
1.3	89.81	58073	1	58.55	58.55	58.86	0.13	0
			2	62.47	62.65	75.85	1.53	0.44
			3	74.18	73.7	0	73.7	14.73
1.4	94.12	61471	1	63.06	62.54	63.73	0.58	0
			2	67.27	67.42	77.45	5.8	1.77
			3	79.88	80.35	0	80.35	15.83
1.5	96.66	64641	1	67.56	67.89	70.49	1.64	0
			2	72.08	72.3	74.5	13.99	4.24
			3	85.59	85.25	0	85.25	16.19

Table A1.8: Scenario 1, Case 1, under EMSRb2, with +0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	54.56	35441	1	36.03	35.86	36.29	0	0
			2	38.44	38.57	45.55	1.48	0.43
			3	45.65	45.54	0	45.54	8.46
0.9	62.51	40253	1	40.54	40.38	40.41	0	0
			2	43.24	43.11	53.35	0.08	0.03
			3	51.35	51.7	0	51.7	10.33
1	69.5	44903	1	45.04	45.14	45.42	0	0
			2	48.05	48.29	58.83	0.9	0.28
			3	57.06	56.97	0	56.97	11.44
1.1	73.33	48466	1	49.54	49.51	51.56	0	0
			2	52.86	52.91	58.44	6.65	2.04
			3	62.77	63.15	0	63.15	12.18
1.2	74.39	51562	1	54.05	54.77	60.29	0	0
			2	57.66	57.66	51.3	18.27	5.51
			3	68.47	68.52	0	68.52	11.91
1.3	73.87	53486	1	58.55	58.55	67.48	0	0
			2	62.47	62.65	43.33	30.07	8.93
			3	74.18	73.7	0	73.7	10.76
1.4	72.82	55297	1	63.06	62.54	75.09	0	0
			2	67.27	67.42	34.15	41.94	12.54
			3	79.88	80.35	0	80.35	8.67
1.5	72.6	57936	1	67.56	67.89	84.22	0	0
			2	72.08	72.3	24.68	54.3	16.33
			3	85.59	85.25	0	85.25	6.68

Table A1.9: Scenario 1, Case 1, under EMSRb3, with –0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44734	1	40.54	40.38	40.32	0.15	0
			2	43.24	43.11	42.99	0.25	0.09
			3	51.35	51.7	50.95	0.75	0.14
1	95.86	48923	1	45.04	45.14	44.69	0.81	0
			2	48.05	48.29	48.31	1.18	0.35
			3	57.06	56.97	50.79	6.19	1.2
1.1	97.49	52075	1	49.54	49.51	48.92	1.38	0
			2	52.86	52.91	54.17	2.52	0.78
			3	62.77	63.15	43.15	20	3.79
1.2	98.19	55567	1	54.05	54.77	54.06	1.83	0
			2	57.66	57.66	60.99	3.74	1.11
			3	68.47	68.52	32.24	36.28	7.07
1.3	98.01	58301	1	58.55	58.55	57.99	2.07	0
			2	62.47	62.65	67.68	5.28	1.51
			3	74.18	73.7	21.34	52.36	10.32
1.4	97.91	61405	1	63.06	52.54	62.33	1.99	0
			2	67.27	67.42	75.52	5.95	1.78
			3	79.88	80.35	9.01	71.34	14.05
1.5	98.49	64493	1	67.56	67.89	67.9	2.94	0
			2	72.08	72.3	78.5	10.13	2.95
			3	85.59	85.25	1.33	83.92	16.33

Table A1.10: Scenario 1, Case 1, under EMSRb3, with –0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44732	1	40.54	40.38	40.32	0.15	0
			2	43.24	43.11	43	0.25	0.09
			3	51.35	51.7	50.96	0.74	0.14
1	95.9	48909	1	45.04	45.14	44.67	0.86	0
			2	48.05	48.29	48.2	1.24	0.38
			3	57.06	56.97	50.99	5.99	1.15
1.1	97.59	52031	1	49.54	49.51	48.85	1.5	0
			2	52.86	52.91	53.94	2.7	0.83
			3	62.77	63.15	43.59	19.57	3.74
1.2	98.33	55455	1	54.05	54.77	53.97	2.04	0
			2	57.66	57.66	60.28	4.23	1.24
			3	68.47	68.52	33.24	35.28	6.84
1.3	98.27	58251	1	58.55	58.55	58.07	2.27	0
			2	62.47	62.65	66.72	5.87	1.79
			3	74.18	73.7	22.62	51.08	9.95
1.4	98.17	61200	1	63.06	62.54	62.3	2.34	0
			2	67.27	67.42	73.85	7.1	2.1
			3	79.88	80.35	11.1	69.25	13.53
1.5	98.59	64442	1	67.56	67.89	67.7	3.09	0
			2	72.08	72.3	78.63	9.95	2.9
			3	85.59	85.25	1.55	83.7	16.28

Table A1.11: Scenario 1, Case 1, under EMSRb3, with –0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44730	1	40.54	40.38	40.31	0.16	0
			2	43.24	43.11	42.99	0.26	0.09
			3	51.35	51.7	50.97	0.73	0.14
1	95.93	48905	1	45.04	45.14	44.67	0.86	0
			2	48.05	48.29	48.14	1.28	0.39
			3	57.06	56.97	51.08	5.89	1.13
1.1	97.7	51980	1	49.54	49.51	48.78	1.57	0
			2	52.86	52.91	53.63	2.96	0.84
			3	62.77	63.15	44.13	19.02	3.68
1.2	98.48	55356	1	54.05	54.77	53.9	2.26	0
			2	57.66	57.66	59.61	4.71	1.39
			3	68.47	68.52	34.21	34.3	6.66
1.3	98.43	58074	1	58.55	58.55	57.98	2.57	0
			2	62.47	62.65	65.58	6.69	2
			3	74.18	73.7	24.08	49.61	9.63
1.4	98.39	60982	1	63.06	62.54	62.33	2.69	0
			2	67.27	67.42	71.96	8.52	2.48
			3	79.88	80.35	13.3	67.05	13.06
1.5	98.67	64288	1	67.56	67.89	67.58	3.44	0
			2	72.08	72.3	77.85	10.44	3.13
			3	85.59	85.25	2.57	82.69	16

Table A1.12: Scenario 1, Case 1, under EMSRb3, with –0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39909	1	36.03	35.86	35.85	0.02	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.53	0.01	0
0.9	89.55	44736	1	40.54	40.38	40.32	0.16	0
			2	43.24	43.11	42.96	0.27	0.1
			3	51.35	51.7	51.05	0.65	0.12
1	96.05	48882	1	45.04	45.14	44.63	0.94	0
			2	48.05	48.29	47.91	1.43	0.43
			3	57.06	56.97	51.54	5.43	1.05
1.1	97.9	51908	1	49.54	49.51	48.75	1.74	0
			2	52.86	52.91	52.94	3.37	0.98
			3	62.77	63.15	45.17	17.99	3.4
1.2	98.68	55172	1	54.05	54.77	53.81	2.69	0
			2	57.66	57.66	58.35	5.49	1.73
			3	68.47	68.52	35.86	32.66	6.17
1.3	98.7	57851	1	58.55	58.55	57.95	2.97	0
			2	62.47	62.65	63.78	7.93	2.36
			3	74.18	73.7	26.33	47.37	9.06
1.4	98.76	60706	1	63.06	62.54	62.39	3.28	0
			2	67.27	67.42	69.38	10.3	3.13
			3	79.88	80.35	16.36	63.98	12.27
1.5	99.03	63814	1	67.56	67.89	67.6	4.23	0
			2	72.08	72.3	74.1	13.17	3.94
			3	85.59	85.25	6.85	78.41	14.97

Table A1.13: Scenario 1, Case 1, under EMSRb3, with +0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44735	1	40.54	40.38	40.32	0.14	0
			2	43.24	43.11	43	0.25	0.08
			3	51.35	51.7	50.94	0.76	0.14
1	95.78	48934	1	45.04	45.14	44.72	0.74	0
			2	48.05	48.29	48.39	1.16	0.32
			3	57.06	56.97	50.56	6.41	1.25
1.1	97.31	52128	1	49.54	49.51	48.94	1.31	0
			2	52.86	52.91	54.74	2.15	0.74
			3	62.77	63.15	42.29	20.86	3.98
1.2	97.89	55677	1	54.05	54.77	54.07	1.58	0
			2	57.66	57.66	62.13	2.94	0.88
			3	68.47	68.52	30.64	37.87	7.41
1.3	97.71	58498	1	58.55	58.55	57.98	1.71	0
			2	62.47	62.65	69.49	3.98	1.14
			3	74.18	73.7	19.09	54.6	10.82
1.4	97.49	61620	1	63.06	62.54	62.3	1.54	0
			2	67.27	67.42	77.65	4.4	1.3
			3	79.88	80.35	6.28	74.07	14.63
1.5	98.37	64585	1	67.56	67.89	68.3	2.72	0
			2	72.08	72.3	78.12	10.41	3.13
			3	85.59	85.25	1.15	84.11	16.23

Table A1.14: Scenario 1, Case 1, under EMSRb3, with +0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.51	44741	1	40.54	40.38	40.32	0.13	0
			2	43.24	43.11	43.03	0.23	0.07
			3	51.35	51.7	50.91	0.79	0.15
1	95.74	48936	1	45.04	45.14	44.73	0.74	0
			2	48.05	48.29	48.45	1.09	0.33
			3	57.06	56.97	50.42	6.55	1.25
1.1	97.17	52146	1	49.54	49.51	48.97	1.19	0
			2	52.86	52.91	54.98	2.03	0.65
			3	62.77	63.15	41.82	21.33	4.1
1.2	97.76	55735	1	54.05	54.77	54.1	1.4	0
			2	57.66	57.66	62.63	2.64	0.72
			3	68.47	68.52	29.92	38.59	7.61
1.3	97.57	58593	1	58.55	58.55	58.07	1.54	0
			2	62.47	62.65	70.05	3.56	1.06
			3	74.18	73.7	18.22	55.47	10.96
1.4	97.39	61627	1	63.06	62.54	62.31	1.48	0
			2	67.27	67.42	77.82	4.36	1.25
			3	79.88	80.35	5.96	74.39	14.77
1.5	98.35	64601	1	67.56	67.89	68.38	2.76	0
			2	72.08	72.3	78.02	10.5	3.25
			3	85.59	85.25	1.13	84.13	16.22

Table A1.15: Scenario 1, Case 1, under EMSRb3, with +0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.49	44738	1	40.54	40.38	40.33	0.14	0
			2	43.24	43.11	43.02	0.23	0.09
			3	51.35	51.7	50.88	0.82	0.15
1	95.7	48938	1	45.04	45.14	44.75	0.75	0
			2	48.05	48.29	48.46	1.11	0.36
			3	57.06	56.97	50.35	6.63	1.27
1.1	97.1	52163	1	49.54	49.51	48.96	1.11	0
			2	52.86	52.91	55.22	1.88	0.56
			3	62.77	63.15	41.47	21.68	4.19
1.2	97.65	55776	1	54.05	54.77	54.16	1.32	0
			2	57.66	57.66	62.88	2.52	0.71
			3	68.47	68.52	29.44	39.07	7.73
1.3	97.39	58633	1	58.55	58.55	58.05	1.45	0
			2	62.47	62.65	70.65	3.2	0.95
			3	74.18	73.7	17.38	56.31	11.2
1.4	97.33	61628	1	63.06	62.54	62.26	1.5	0
			2	67.27	67.42	78.07	4.14	1.22
			3	79.88	80.35	5.65	74.69	14.79
1.5	98.31	64674	1	67.56	67.89	68.62	2.52	0
			2	72.08	72.3	77.83	10.63	3.25
			3	85.59	85.25	1.01	84.25	16.16

Table A1.16: Scenario 1, Case 1, under EMSRb3, with +0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.96	39911	1	36.03	35.86	35.85	0.01	0
			2	38.44	38.57	38.57	0	0
			3	45.65	45.54	45.52	0.02	0.01
0.9	89.49	44740	1	40.54	40.38	40.33	0.13	0
			2	43.24	43.11	43.03	0.23	0.08
			3	51.35	51.7	50.87	0.83	0.15
1	95.68	48945	1	45.04	45.14	44.75	0.73	0
			2	48.05	48.29	48.54	1.07	0.34
			3	57.06	56.97	50.24	6.73	1.31
1.1	97	52175	1	49.54	49.51	48.97	1.06	0
			2	52.86	52.91	55.43	1.77	0.51
			3	62.77	63.15	41.1	22.05	4.3
1.2	97.46	55782	1	54.05	54.77	54.14	1.29	0
			2	57.66	57.66	63.25	2.25	0.66
			3	68.47	68.52	28.8	39.72	7.83
1.3	97.26	58666	1	58.55	58.55	58.01	1.38	0
			2	62.47	62.65	71.18	2.8	0.84
			3	74.18	73.7	16.69	57	11.33
1.4	97.24	61688	1	63.06	62.54	62.41	1.36	0
			2	67.27	67.42	78.17	4.12	1.22
			3	79.88	80.35	5.27	75.08	14.88
1.5	98.26	64672	1	67.56	67.89	68.66	2.45	0
			2	72.08	72.3	77.78	10.71	3.22
			3	85.59	85.25	0.96	84.3	16.2

Table A1.17: Scenario 2, Case 1, under EMSRb2, with –0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.81	65570	1	40.54	40.32	40.18	0.14	0
			2	10.78	10.72	10.7	0.02	0
			3	20.69	20.46	20.44	0.02	0
			4	13.5	13.65	13.63	0.03	0
			5	43.24	43.18	43.11	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.02	0.14	0.02
1	96.69	71292	1	45.04	45.05	44.36	0.71	0
			2	11.98	11.9	11.83	0.09	0.02
			3	22.99	22.98	22.86	0.15	0.02
			4	15	15.11	14.95	0.27	0.03
			5	48.05	47.58	46.63	1.13	0.1
			6	57.06	57.45	56.56	1.49	0.18
			7	37.96	37.79	32.93	4.86	0.61
1.1	98.26	74565	1	49.54	49.19	48.08	1.14	0
			2	13.18	12.98	12.87	0.17	0.04
			3	25.29	25.48	25.25	0.3	0.06
			4	16.5	16.39	16.14	0.49	0.06
			5	52.86	52.74	50.89	2.52	0.24
			6	62.77	62.94	60.59	5.13	0.67
			7	41.76	41.59	20.04	21.55	2.78
1.2	98.51	77409	1	54.05	53.74	52.33	1.49	0
			2	14.38	13.96	13.74	0.27	0.08
			3	27.59	27.58	27.33	0.33	0.05
			4	18	17.94	17.53	0.73	0.08
			5	57.66	57.58	55.56	3.25	0.32
			6	68.47	68.37	64.99	8.95	1.23
			7	45.55	45.46	2.95	42.51	5.58
1.3	98.95	80025	1	58.55	58.63	56.83	1.88	0
			2	15.57	15.54	15.33	0.28	0.07
			3	29.89	29.83	29.42	0.51	0.07
			4	19.5	19.6	19.23	0.97	0.11
			5	62.47	62.33	59.6	6.17	0.59
			6	74.18	73.79	54.94	23.81	3.44
			7	49.35	49.29	0.14	49.15	4.95
1.4	99.1	82276	1	63.06	62.99	60.9	2.24	0
			2	16.77	17.04	16.71	0.47	0.15
			3	32.19	32.54	32.01	0.68	0.14
			4	21	20.97	20.41	1.38	0.15
			5	67.27	66.56	64.33	8.17	0.82
			6	79.88	79.02	41.4	41.51	5.94
			7	53.14	53	0.1	52.9	3.89
1.5	99.14	84392	1	67.56	67.81	65.48	2.48	0
			2	17.97	17.78	17.36	0.55	0.15
			3	34.49	34.43	33.83	0.77	0.13
			4	22.5	22.52	21.93	1.6	0.17
			5	72.08	71.65	70.35	10.12	1.01
			6	85.59	85.75	26.95	61.81	8.82
			7	56.94	56.79	0.06	56.74	3.02

Table A1.18: Scenario 2, Case 1, under EMSRb2, with –0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571	1	40.54	40.32	40.17	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.44	0.02	0
			4	13.5	13.65	13.63	0.03	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.66	0.09	0.01
			7	34.16	34.17	34.04	0.13	0.02
1	96.79	71289	1	45.04	45.05	44.26	0.83	0
			2	11.98	11.9	11.81	0.12	0.04
			3	22.99	22.98	22.86	0.16	0.03
			4	15	15.11	14.92	0.29	0.04
			5	48.05	47.58	46.66	1.1	0.09
			6	57.06	57.45	56.49	1.5	0.18
			7	37.96	37.79	33.36	4.43	0.54
1.1	98.38	74568	1	49.54	49.19	47.96	1.28	0
			2	13.18	12.98	12.83	0.19	0.05
			3	25.29	25.48	25.3	0.23	0.03
			4	16.5	16.39	16.18	0.46	0.05
			5	52.86	52.74	50.89	2.58	0.26
			6	62.77	62.94	60.28	5.32	0.74
			7	41.76	41.59	20.69	20.91	2.66
1.2	98.58	77375	1	54.05	53.74	52.16	1.65	0
			2	14.38	13.96	13.79	0.23	0.07
			3	27.59	27.58	27.31	0.37	0.06
			4	18	17.94	17.49	0.77	0.1
			5	57.66	57.58	55.41	3.46	0.33
			6	68.47	68.37	64.93	8.87	1.29
			7	45.55	45.46	3.54	41.92	5.44
1.3	99.05	80006	1	58.55	58.63	56.67	2.07	0
			2	15.57	15.54	15.27	0.36	0.1
			3	29.89	29.83	29.42	0.56	0.09
			4	19.5	19.6	18.95	1.17	0.15
			5	62.47	62.33	59.87	5.8	0.51
			6	74.18	73.79	55.39	23.41	3.34
			7	49.35	49.29	0.16	49.14	5.01
1.4	99.17	82244	1	63.06	62.99	60.85	2.28	0
			2	16.77	17.04	16.68	0.47	0.13
			3	32.19	32.54	31.95	0.74	0.12
			4	21	20.97	20.2	1.5	0.14
			5	67.27	66.56	63.88	8.38	0.73
			6	79.88	79.02	42.36	40.83	5.7
			7	53.14	53	0.11	52.89	4.17
1.5	99.22	84358	1	67.56	67.81	65.25	2.71	0
			2	17.97	17.78	17.48	0.46	0.14
			3	34.49	34.43	33.76	0.86	0.16
			4	22.5	22.52	21.88	1.67	0.19
			5	72.08	71.65	69.8	10.52	1.03
			6	85.59	85.75	27.92	60.86	8.67
			7	56.94	56.79	0.05	56.74	3.03

Table A1.19: Scenario 2, Case 1, under EMSRb2, with –0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571	1	40.54	40.32	40.16	0.16	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.06	0.11	0.01
1	96.84	71270	1	45.04	45.05	44.19	0.91	0
			2	11.98	11.9	11.77	0.15	0.04
			3	22.99	22.98	22.83	0.18	0.02
			4	15	15.11	14.97	0.26	0.03
			5	48.05	47.58	46.63	1.14	0.11
			6	57.06	57.45	56.43	1.5	0.19
			7	37.96	37.79	33.66	4.12	0.48
1.1	98.51	74559	1	49.54	49.19	47.84	1.4	0
			2	13.18	12.98	12.79	0.24	0.06
			3	25.29	25.48	25.28	0.27	0.05
			4	16.5	16.39	16.05	0.57	0.06
			5	52.86	52.74	50.85	2.56	0.24
			6	62.77	62.94	60.41	5.13	0.67
			7	41.76	41.59	21.21	20.38	2.61
1.2	98.69	77378	1	54.05	53.74	52.05	1.76	0
			2	14.38	13.96	13.81	0.23	0.07
			3	27.59	27.58	27.26	0.4	0.07
			4	18	17.94	17.52	0.76	0.08
			5	57.66	57.58	55.24	3.52	0.35
			6	68.47	68.37	65.07	8.75	1.19
			7	45.55	45.46	3.92	41.55	5.46
1.3	99.13	79982	1	58.55	58.63	56.51	2.22	0
			2	15.57	15.54	15.26	0.38	0.1
			3	29.89	29.83	29.34	0.59	0.09
			4	19.5	19.6	19.17	1.06	0.1
			5	62.47	62.33	59.28	6.23	0.63
			6	74.18	73.79	56.21	22.71	3.18
			7	49.35	49.29	0.15	49.14	5.13
1.4	99.24	82193	1	63.06	62.99	60.63	2.54	0
			2	16.77	17.04	16.65	0.55	0.17
			3	32.19	32.54	31.86	0.86	0.16
			4	21	20.97	20.2	1.55	0.18
			5	67.27	66.56	63.71	8.56	0.78
			6	79.88	79.02	43.06	39.92	5.71
			7	53.14	53	0.1	52.91	3.96
1.5	99.27	84309	1	67.56	67.81	65.12	2.84	0
			2	17.97	17.78	17.45	0.51	0.15
			3	34.49	34.43	33.72	0.92	0.17
			4	22.5	22.52	21.67	1.83	0.22
			5	72.08	71.65	69.59	10.6	0.98
			6	85.59	85.75	28.66	60.23	8.54
			7	56.94	56.79	0.06	56.73	3.14

Table A1.20: Scenario 2, Case 1, under EMSRb2, with –0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571	1	40.54	40.32	40.16	0.16	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.06	0.11	0.01
1	96.84	71275	1	45.04	45.05	44.19	0.91	0
			2	11.98	11.9	11.77	0.16	0.05
			3	22.99	22.98	22.83	0.17	0.03
			4	15	15.11	14.96	0.27	0.02
			5	48.05	47.58	46.66	1.13	0.11
			6	57.06	57.45	56.42	1.5	0.21
			7	37.96	37.79	33.65	4.13	0.48
1.1	98.51	74559	1	49.54	49.19	47.84	1.4	0
			2	13.18	12.98	12.79	0.24	0.06
			3	25.29	25.48	25.28	0.27	0.05
			4	16.5	16.39	16.05	0.57	0.06
			5	52.86	52.74	50.85	2.56	0.24
			6	62.77	62.94	60.41	5.13	0.67
			7	41.76	41.59	21.21	20.38	2.61
1.2	98.69	77378	1	54.05	53.74	52.05	1.76	0
			2	14.38	13.96	13.81	0.23	0.07
			3	27.59	27.58	27.26	0.4	0.07
			4	18	17.94	17.52	0.76	0.08
			5	57.66	57.58	55.24	3.52	0.35
			6	68.47	68.37	65.07	8.75	1.19
			7	45.55	45.46	3.92	41.55	5.46
1.3	99.13	79982	1	58.55	58.63	56.51	2.22	0
			2	15.57	15.54	15.26	0.38	0.1
			3	29.89	29.83	29.34	0.59	0.09
			4	19.5	19.6	19.17	1.06	0.1
			5	62.47	62.33	59.28	6.23	0.63
			6	74.18	73.79	56.21	22.71	3.18
			7	49.35	49.29	0.15	49.14	5.13
1.4	99.25	82201	1	63.06	62.99	60.5	2.6	0
			2	16.77	17.04	16.83	0.38	0.1
			3	32.19	32.54	31.85	0.89	0.17
			4	21	20.97	20.24	1.57	0.2
			5	67.27	66.56	63.61	8.62	0.83
			6	79.88	79.02	43.1	39.86	5.67
			7	53.14	53	0.1	52.91	3.94
1.5	99.27	84309	1	67.56	67.81	65.12	2.84	0
			2	17.97	17.78	17.45	0.51	0.15
			3	34.49	34.43	33.72	0.92	0.17
			4	22.5	22.52	21.67	1.83	0.22
			5	72.08	71.65	69.59	10.6	0.98
			6	85.59	85.75	28.66	60.23	8.54
			7	56.94	56.79	0.06	56.73	3.14

Table A1.21: Scenario 2, Case 1, under EMSRb2, with +0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.79	65569	1	40.54	40.32	40.2	0.13	0
			2	10.78	10.72	10.71	0.02	0.01
			3	20.69	20.46	20.45	0.02	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.66	0.11	0.01
			7	34.16	34.17	33.96	0.21	0.03
1	96.49	71275	1	45.04	45.05	44.48	0.6	0
			2	11.98	11.9	11.86	0.07	0.03
			3	22.99	22.98	22.87	0.13	0.03
			4	15	15.11	15	0.2	0.02
			5	48.05	47.58	46.81	0.93	0.09
			6	57.06	57.45	56.65	1.54	0.16
			7	37.95	37.79	31.96	5.83	0.75
1.1	97.91	74554	1	49.54	49.19	48.29	0.92	0
			2	13.18	12.98	12.88	0.12	0.03
			3	25.29	25.48	25.32	0.21	0.02
			4	16.5	16.39	16.28	0.37	0.04
			5	52.86	52.74	51.24	2.19	0.27
			6	62.77	62.94	60.78	5.24	0.69
			7	41.76	41.59	18.22	23.37	3.08
1.2	98.15	77390	1	54.05	53.74	52.65	1.16	0
			2	14.38	13.96	13.81	0.21	0.06
			3	27.59	27.58	27.34	0.32	0.06
			4	18	17.94	17.66	0.55	0.08
			5	57.66	57.58	55.92	3.03	0.28
			6	68.47	68.37	64.18	9.8	1.37
			7	45.55	45.46	2.03	43.43	5.61
1.3	98.7	80052	1	58.55	58.63	57.21	1.49	0
			2	15.57	15.54	15.39	0.22	0.06
			3	29.89	29.83	29.48	0.44	0.06
			4	19.5	19.6	19.23	0.95	0.1
			5	62.47	62.33	60.27	5.88	0.58
			6	74.18	73.79	53.17	25.27	3.82
			7	49.35	49.29	0.15	49.15	4.65
1.4	98.87	82333	1	63.06	62.99	61.3	1.78	0
			2	16.77	17.04	16.87	0.28	0.08
			3	32.19	32.54	32.05	0.66	0.11
			4	21	20.97	20.46	1.34	0.17
			5	67.27	66.56	64.85	7.92	0.83
			6	79.88	79.02	39.68	43.16	6.21
			7	53.14	53	0.11	52.89	3.82
1.5	98.88	84441	1	67.56	67.81	65.87	2.05	0
			2	17.97	17.78	17.53	0.36	0.11
			3	34.49	34.43	33.86	0.73	0.11
			4	22.5	22.52	22.16	1.43	0.16
			5	72.08	71.65	70.75	10.06	1.08
			6	85.59	85.75	25.1	63.49	9.16
			7	56.94	56.79	0.05	56.74	2.84

Table A1.22: Scenario 2, Case 1, under EMSRb2, with +0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.78	65569	1	40.54	40.32	40.22	0.11	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.07	0.11	0.01
			6	51.35	51.74	51.66	0.11	0.01
			7	34.16	34.17	33.92	0.24	0.03
1	96.32	71247	1	45.04	45.05	44.61	0.47	0
			2	11.98	11.9	11.84	0.09	0.03
			3	22.99	22.98	22.9	0.1	0.02
			4	15	15.11	14.97	0.24	0.03
			5	48.05	47.58	46.89	0.9	0.09
			6	57.06	57.45	56.64	1.63	0.21
			7	37.96	37.79	31.38	6.41	0.83
1.1	97.62	74510	1	49.54	49.19	48.44	0.77	0
			2	13.18	12.98	12.91	0.1	0.02
			3	25.29	25.48	25.36	0.16	0.03
			4	16.5	16.39	16.14	0.42	0.04
			5	52.86	52.74	51.49	1.94	0.17
			6	62.77	62.94	61.09	5.18	0.69
			7	41.76	41.59	16.9	24.69	3.32
1.2	97.99	77405	1	54.05	53.74	52.84	0.94	0
			2	14.38	13.96	13.85	0.15	0.04
			3	27.59	27.58	27.38	0.26	0.04
			4	18	17.94	17.72	0.51	0.07
			5	57.66	57.58	56.05	3.03	0.3
			6	68.47	68.37	63.7	10.22	1.49
			7	45.55	45.46	1.67	43.79	5.56
1.3	98.56	80056	1	58.55	58.63	57.42	1.29	0
			2	15.57	15.54	15.37	0.24	0.08
			3	29.89	29.83	29.49	0.43	0.07
			4	19.5	19.6	19.36	0.82	0.09
			5	62.47	62.33	60.57	5.51	0.58
			6	74.18	73.79	52.23	26.33	3.75
			7	49.35	49.29	0.13	49.17	4.76
1.4	98.71	82328	1	63.06	62.99	61.41	1.65	0
			2	16.77	17.04	16.93	0.24	0.06
			3	32.19	32.54	32.12	0.57	0.13
			4	21	20.97	20.49	1.26	0.15
			5	67.27	66.56	65.33	7.51	0.78
			6	79.88	79.02	38.53	44.3	6.28
			7	53.14	53	0.11	52.89	3.81
1.5	98.73	84460	1	67.56	67.81	66.09	1.8	0
			2	17.97	17.78	17.56	0.32	0.08
			3	34.49	34.43	33.98	0.63	0.09
			4	22.5	22.52	22.08	1.49	0.18
			5	72.08	71.65	71.09	9.83	1.05
			6	85.59	85.75	24.09	64.46	9.27
			7	56.94	56.79	0.07	56.72	2.81

Table A1.23: Scenario 2, Case 1, under EMSRb2, with +0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.76	65569	1	40.54	40.32	40.23	0.09	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.09	0.01
			6	51.35	51.74	51.66	0.12	0.01
			7	34.16	34.17	33.85	0.32	0.04
1	96.17	71220	1	45.04	45.05	44.68	0.41	0
			2	11.98	11.9	11.84	0.06	0.03
			3	22.99	22.98	22.91	0.09	0
			4	15	15.11	15	0.19	0.02
			5	48.05	47.58	46.95	0.84	0.08
			6	57.06	57.45	56.76	1.64	0.2
			7	37.96	37.79	30.75	7.04	0.95
1.1	97.3	74457	1	49.54	49.19	48.57	0.63	0
			2	13.18	12.98	12.96	0.07	0.02
			3	25.29	25.48	25.4	0.13	0.05
			4	16.5	16.39	16.3	0.31	0.05
			5	52.86	52.74	51.42	1.97	0.22
			6	62.77	62.94	61.3	5.13	0.65
			7	41.76	41.59	15.61	25.98	3.5
1.2	97.76	77349	1	54.05	53.74	52.94	0.84	0
			2	14.38	13.96	13.88	0.12	0.04
			3	27.59	27.58	27.37	0.27	0.04
			4	18	17.94	17.75	0.46	0.06
			5	57.66	57.58	56.4	2.73	0.28
			6	68.47	68.37	63.13	10.82	1.55
			7	45.55	45.46	1.21	44.26	5.58
1.3	98.33	80029	1	58.55	58.63	57.6	1.09	0
			2	15.57	15.54	15.41	0.19	0.06
			3	29.89	29.83	29.56	0.36	0.06
			4	19.5	19.6	19.4	0.78	0.09
			5	62.47	62.33	60.95	5.2	0.57
			6	74.18	73.79	50.99	27.52	3.82
			7	49.35	49.29	0.12	49.18	4.72
1.4	98.52	82339	1	63.06	62.99	61.65	1.4	0
			2	16.77	17.04	16.97	0.17	0.06
			3	32.19	32.54	32.18	0.5	0.1
			4	21	20.97	20.69	1.08	0.14
			5	67.27	66.56	65.42	7.59	0.8
			6	79.88	79.02	37.5	45.24	6.45
			7	53.14	53	0.08	52.92	3.72
1.5	98.6	84485	1	67.56	67.81	66.19	1.69	0
			2	17.97	17.78	17.64	0.27	0.07
			3	34.49	34.43	34	0.58	0.13
			4	22.5	22.52	22.17	1.28	0.15
			5	72.08	71.65	71.96	9.16	0.94
			6	85.59	85.75	22.64	65.85	9.47
			7	56.94	56.79	0.07	56.73	2.75

Table A1.24: Scenario 2, Case 1, under EMSRb2, with +0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.74	65562	1	40.54	40.32	40.25	0.08	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0.01
			5	43.24	43.18	43.07	0.11	0.02
			6	51.35	51.74	51.66	0.13	0.01
			7	34.16	34.17	33.77	0.39	0.06
1	95.86	71143	1	45.04	45.05	44.76	0.31	0
			2	11.98	11.9	11.87	0.04	0.02
			3	22.99	22.98	22.9	0.1	0.01
			4	15	15.11	15.1	0.12	0.02
			5	48.05	47.58	47.01	0.78	0.1
			6	57.06	57.45	56.84	1.69	0.21
			7	37.96	37.79	29.67	8.12	1.08
1.1	96.89	74359	1	49.54	49.19	48.73	0.48	0
			2	13.18	12.98	12.94	0.06	0.02
			3	25.29	25.48	25.38	0.12	0.02
			4	16.5	16.39	16.33	0.27	0.02
			5	52.86	52.74	51.62	1.78	0.21
			6	62.77	62.94	61.68	5.05	0.66
			7	41.76	41.59	13.91	27.68	3.79
1.2	97.48	77283	1	54.05	53.74	53.1	0.67	0
			2	14.38	13.96	13.93	0.07	0.02
			3	27.59	27.58	27.39	0.23	0.04
			4	18	17.94	17.79	0.42	0.04
			5	57.66	57.58	56.46	2.72	0.28
			6	68.47	68.37	62.52	11.51	1.59
			7	45.55	45.46	0.82	44.64	5.66
1.3	98.1	79991	1	58.55	58.63	57.8	0.88	0
			2	15.57	15.54	15.44	0.16	0.05
			3	29.89	29.83	29.56	0.35	0.06
			4	19.5	19.6	19.42	0.73	0.09
			5	62.47	62.33	61.5	5.03	0.54
			6	74.18	73.79	49.67	28.64	4.2
			7	49.35	49.29	0.09	49.2	4.51
1.4	98.23	82306	1	63.06	62.99	61.94	1.12	0
			2	16.77	17.04	16.92	0.18	0.07
			3	32.19	32.54	32.33	0.33	0.07
			4	21	20.97	20.71	1.02	0.12
			5	67.27	66.56	65.96	7.42	0.77
			6	79.88	79.02	35.85	46.79	6.82
			7	53.14	53	0.08	52.93	3.62
1.5	98.32	84474	1	67.56	67.81	66.52	1.34	0
			2	17.97	17.78	17.67	0.19	0.05
			3	34.49	34.43	34.15	0.44	0.07
			4	22.5	22.52	22.17	1.2	0.16
			5	72.08	71.65	72.27	9.11	0.86
			6	85.59	85.75	21.18	67.32	9.74
			7	56.94	56.79	0.05	56.74	2.75

Table A1.25: Scenario 2, Case 1, under EMSRb3, with –0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.76	71288	1	45.04	45.05	44.3	0.78	0
			2	11.98	11.9	11.79	0.13	0.03
			3	22.99	22.98	22.85	0.14	0.02
			4	15	15.11	14.99	0.25	0.01
			5	48.05	47.58	46.62	1.15	0.13
			6	57.06	57.45	56.45	1.55	0.19
			7	37.96	37.79	33.28	4.51	0.55
1.1	98.33	74575	1	49.54	49.19	47.98	1.25	0
			2	13.18	12.98	12.86	0.16	0.05
			3	25.29	25.48	25.33	0.24	0.03
			4	16.5	16.39	16.15	0.48	0.08
			5	52.86	52.74	50.89	2.55	0.24
			6	62.77	62.94	60.52	5.12	0.71
			7	41.76	41.59	20.29	21.3	2.7
1.2	98.52	77408	1	54.05	53.74	52.32	1.49	0
			2	14.38	13.96	13.82	0.2	0.06
			3	27.59	27.58	27.29	0.36	0.06
			4	18	17.94	17.59	0.67	0.07
			5	57.66	57.58	55.31	3.52	0.33
			6	68.47	68.37	64.96	8.98	1.25
			7	45.55	45.46	3.18	42.28	5.57
1.3	98.97	80024	1	58.55	58.63	56.84	1.87	0
			2	15.57	15.54	15.31	0.35	0.08
			3	29.89	29.83	29.39	0.56	0.11
			4	19.5	19.6	19.22	0.95	0.13
			5	62.47	62.33	59.48	6.2	0.57
			6	74.18	73.79	55.14	23.71	3.35
			7	49.35	49.29	0.17	49.12	5.05
1.4	99.11	82281	1	63.06	62.99	60.85	2.24	0
			2	16.77	17.04	16.86	0.3	0.09
			3	32.19	32.54	31.89	0.77	0.13
			4	21	20.97	20.39	1.34	0.12
			5	67.27	66.56	64.39	8.15	0.76
			6	79.88	79.02	41.37	41.66	5.98
			7	53.14	53	0.12	52.89	4.01
1.5	99.08	84407	1	67.56	67.81	65.43	2.5	0
			2	17.97	17.78	17.52	0.42	0.12
			3	34.49	34.43	33.85	0.79	0.16
			4	22.5	22.52	21.77	1.64	0.21
			5	72.08	71.65	71.18	9.52	0.9
			6	85.59	85.75	25.98	62.81	9.04
			7	56.94	56.79	0.08	56.72	3.04

Table A1.26: Scenario 2, Case 1, under EMSRb3, with –0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.76	71288	1	45.04	45.05	44.3	0.78	0
			2	11.98	11.9	11.79	0.13	0.03
			3	22.99	22.98	22.85	0.14	0.02
			4	15	15.11	14.99	0.25	0.01
			5	48.05	47.58	46.62	1.15	0.13
			6	57.06	57.45	56.45	1.55	0.19
			7	37.96	37.79	33.28	4.51	0.55
1.1	98.33	74563	1	49.54	49.19	47.97	1.26	0
			2	13.18	12.98	12.86	0.16	0.04
			3	25.29	25.48	25.33	0.24	0.03
			4	16.5	16.39	16.12	0.48	0.08
			5	52.86	52.74	50.86	2.54	0.21
			6	62.77	62.94	60.46	5.23	0.67
			7	41.76	41.59	20.41	21.18	2.76
1.2	98.51	77386	1	54.05	53.74	52.29	1.52	0
			2	14.38	13.96	13.82	0.22	0.07
			3	27.59	27.58	27.27	0.38	0.08
			4	18	17.94	17.59	0.67	0.07
			5	57.66	57.58	55.34	3.53	0.33
			6	68.47	68.37	64.75	9.07	1.29
			7	45.55	45.46	3.41	42.05	5.45
1.3	98.99	80017	1	58.55	58.63	56.82	1.94	0
			2	15.57	15.54	15.28	0.35	0.12
			3	29.89	29.83	29.38	0.55	0.08
			4	19.5	19.6	19.22	0.94	0.1
			5	62.47	62.33	59.42	6.2	0.56
			6	74.18	73.79	55.34	23.55	3.29
			7	49.35	49.29	0.15	49.14	5.1
1.4	99.12	82245	1	63.06	62.99	60.87	2.23	0
			2	16.77	17.04	16.82	0.34	0.1
			3	32.19	32.54	31.85	0.79	0.12
			4	21	20.97	20.38	1.4	0.11
			5	67.27	66.56	63.71	8.65	0.8
			6	79.88	79.02	42.15	40.93	5.8
			7	53.14	53	0.14	52.87	4.05
1.5	99.14	84390	1	67.56	67.81	65.46	2.48	0
			2	17.97	17.78	17.49	0.44	0.13
			3	34.49	34.43	33.81	0.83	0.14
			4	22.5	22.52	21.8	1.7	0.21
			5	72.08	71.65	70.04	10.35	0.98
			6	85.59	85.75	27.29	61.53	8.74
			7	56.94	56.79	0.07	56.73	3.08

Table A1.27: Scenario 2, Case 1, under EMSRb3, with –0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308						
			1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571						
			1	40.54	40.32	40.17	0.16	0
			2	10.78	10.72	10.7	0.02	0.01
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.06	0.11	0.01
1	96.84	71274						
			1	45.04	45.05	44.21	0.88	0
			2	11.98	11.9	11.77	0.16	0.04
			3	22.99	22.98	22.83	0.17	0.02
			4	15	15.11	14.94	0.27	0.02
			5	48.05	47.58	46.61	1.15	0.09
			6	57.06	57.45	56.48	1.5	0.18
			7	37.96	37.79	33.64	4.15	0.53
1.1	98.52	74565						
			1	49.54	49.19	47.82	1.42	0
			2	13.18	12.98	12.83	0.21	0.05
			3	25.29	25.48	25.28	0.27	0.05
			4	16.5	16.39	16.05	0.58	0.06
			5	52.86	52.74	50.84	2.56	0.24
			6	62.77	62.94	60.45	5.08	0.67
			7	41.76	41.59	21.2	20.4	2.59
1.2	98.69	77385						
			1	54.05	53.74	52.12	1.71	0
			2	14.38	13.96	13.81	0.24	0.08
			3	27.59	27.58	27.19	0.47	0.09
			4	18	17.94	17.53	0.76	0.08
			5	57.66	57.58	55.23	3.54	0.36
			6	68.47	68.37	65.1	8.72	1.18
			7	45.55	45.46	3.91	41.56	5.45
1.3	99.12	79980						
			1	58.55	58.63	56.64	2.09	0
			2	15.57	15.54	15.21	0.43	0.1
			3	29.89	29.83	29.21	0.7	0.1
			4	19.5	19.6	19.14	1.07	0.08
			5	62.47	62.33	59.34	6.19	0.6
			6	74.18	73.79	56.2	22.74	3.2
			7	49.35	49.29	0.15	49.15	5.14
1.4	99.24	82202						
			1	63.06	62.99	60.66	2.48	0
			2	16.77	17.04	16.76	0.46	0.15
			3	32.19	32.54	31.7	1	0.18
			4	21	20.97	20.23	1.57	0.16
			5	67.27	66.56	63.68	8.61	0.83
			6	79.88	79.02	43.05	39.89	5.73
			7	53.14	53	0.1	52.9	3.92
1.5	99.25	84298						
			1	67.56	67.81	65.22	2.74	0
			2	17.97	17.78	17.42	0.53	0.15
			3	34.49	34.43	33.61	1.03	0.16
			4	22.5	22.52	21.61	1.86	0.21
			5	72.08	71.65	69.61	10.58	0.96
			6	85.59	85.75	28.68	60.19	8.54
			7	56.94	56.79	0.05	56.74	3.12

Table A1.28: Scenario 2, Case 1, under EMSRb3, with –0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65571	1	40.54	40.32	40.17	0.16	0
			2	10.78	10.72	10.7	0.02	0.01
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.64	0.02	0
			5	43.24	43.18	43.1	0.08	0.01
			6	51.35	51.74	51.65	0.1	0.01
			7	34.16	34.17	34.06	0.11	0.01
1	96.84	71277	1	45.04	45.05	44.22	0.88	0
			2	11.98	11.9	11.76	0.16	0.04
			3	22.99	22.98	22.84	0.16	0.02
			4	15	15.11	14.94	0.27	0.02
			5	48.05	47.58	46.61	1.15	0.1
			6	57.06	57.45	56.48	1.5	0.17
			7	37.96	37.79	33.63	4.15	0.53
1.1	98.52	74569	1	49.54	49.19	47.82	1.42	0
			2	13.18	12.98	12.83	0.2	0.05
			3	25.29	25.48	25.29	0.25	0.05
			4	16.5	16.39	16.04	0.58	0.06
			5	52.86	52.74	50.86	2.55	0.23
			6	62.77	62.94	60.43	5.1	0.68
			7	41.76	41.59	21.2	20.4	2.59
1.2	98.69	77377	1	54.05	53.74	52.07	1.75	0
			2	14.38	13.96	13.78	0.26	0.08
			3	27.59	27.58	27.27	0.4	0.08
			4	18	17.94	17.52	0.76	0.09
			5	57.66	57.58	55.25	3.52	0.34
			6	68.47	68.37	65.06	8.75	1.19
			7	45.55	45.46	3.93	41.54	5.44
1.3	99.13	79983	1	58.55	58.63	56.58	2.16	0
			2	15.57	15.54	15.2	0.42	0.11
			3	29.89	29.83	29.32	0.59	0.08
			4	19.5	19.6	19.15	1.07	0.09
			5	62.47	62.33	59.33	6.24	0.62
			6	74.18	73.79	56.18	22.71	3.24
			7	49.35	49.29	0.15	49.14	5.1
1.4	99.25	82197	1	63.06	62.99	60.58	2.58	0
			2	16.77	17.04	16.7	0.49	0.16
			3	32.19	32.54	31.87	0.86	0.15
			4	21	20.97	20.21	1.56	0.19
			5	67.27	66.56	63.67	8.55	0.8
			6	79.88	79.02	43.1	39.92	5.66
			7	53.14	53	0.1	52.9	3.99
1.5	99.26	84299	1	67.56	67.81	65.19	2.79	0
			2	17.97	17.78	17.35	0.57	0.16
			3	34.49	34.43	33.71	0.91	0.14
			4	22.5	22.52	21.67	1.82	0.2
			5	72.08	71.65	69.52	10.61	0.97
			6	85.59	85.75	28.75	60.24	8.47
			7	56.94	56.79	0.06	56.74	3.24

Table A1.29: Scenario 2, Case 1, under EMSRb3, with +0.05 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.33	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.75	71282	1	45.04	45.05	44.31	0.78	0
			2	11.98	11.9	11.79	0.13	0.03
			3	22.99	22.98	22.84	0.15	0.02
			4	15	15.11	14.96	0.24	0.01
			5	48.05	47.58	46.63	1.16	0.09
			6	57.06	57.45	56.5	1.52	0.2
			7	37.96	37.79	33.23	4.56	0.57
1.1	98.29	74586	1	49.54	49.19	48.02	1.22	0
			2	13.18	12.98	12.86	0.14	0.05
			3	25.29	25.48	25.31	0.22	0.02
			4	16.5	16.39	16.11	0.48	0.05
			5	52.86	52.74	50.94	2.46	0.2
			6	62.77	62.94	60.97	4.83	0.67
			7	41.76	41.59	19.72	21.87	2.86
1.2	98.48	77415	1	54.05	53.74	52.34	1.48	0
			2	14.38	13.96	13.81	0.21	0.07
			3	27.59	27.58	27.28	0.37	0.06
			4	18	17.94	17.61	0.63	0.07
			5	57.66	57.58	55.46	3.36	0.3
			6	68.47	68.37	65.12	8.89	1.23
			7	45.55	45.46	2.76	42.7	5.64
1.3	98.95	80047	1	58.55	58.63	56.89	1.83	0
			2	15.57	15.54	15.28	0.33	0.09
			3	29.89	29.83	29.4	0.52	0.07
			4	19.5	19.6	19.29	0.87	0.09
			5	62.47	62.33	60.02	5.75	0.56
			6	74.18	73.79	54.44	24.42	3.44
			7	49.35	49.29	0.19	49.11	5.07
1.4	99.03	82346	1	63.06	62.99	60.99	2.1	0
			2	16.77	17.04	16.86	0.29	0.09
			3	32.19	32.54	31.97	0.69	0.12
			4	21	20.97	20.45	1.17	0.13
			5	67.27	66.56	65.6	7.21	0.64
			6	79.88	79.02	39.67	43.45	6.25
			7	53.14	53	0.15	52.85	4.09
1.5	99.01	84545	1	67.56	67.81	65.65	2.26	0
			2	17.97	17.78	17.56	0.34	0.1
			3	34.49	34.43	33.92	0.65	0.12
			4	22.5	22.52	21.98	1.26	0.14
			5	72.08	71.65	73.05	7.95	0.72
			6	85.59	85.75	23.4	65.57	9.35
			7	56.94	56.79	0.09	56.7	3.22

Table A1.30: Scenario 2, Case 1, under EMSRb3, with +0.10 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.75	71290	1	45.04	45.05	44.3	0.79	0
			2	11.98	11.9	11.82	0.11	0.04
			3	22.99	22.98	22.85	0.15	0.02
			4	15	15.11	14.97	0.24	0.01
			5	48.05	47.58	46.64	1.14	0.1
			6	57.06	57.45	56.51	1.5	0.2
			7	37.96	37.79	33.17	4.62	0.56
1.1	98.25	74585	1	49.54	49.19	47.99	1.24	0
			2	13.18	12.98	12.87	0.15	0.04
			3	25.29	25.48	25.35	0.19	0.03
			4	16.5	16.39	16.13	0.47	0.06
			5	52.86	52.74	51	2.36	0.21
			6	62.77	62.94	61.28	4.62	0.62
			7	41.76	41.59	19.22	22.37	2.96
1.2	98.46	77422	1	54.05	53.74	52.35	1.45	0
			2	14.38	13.96	13.82	0.19	0.06
			3	27.59	27.58	27.3	0.36	0.05
			4	18	17.94	17.65	0.64	0.07
			5	57.66	57.58	55.48	3.3	0.36
			6	68.47	68.37	65.11	8.91	1.2
			7	45.55	45.46	2.64	42.82	5.65
1.3	98.95	80080	1	58.55	58.63	56.92	1.81	0
			2	15.57	15.54	15.3	0.3	0.09
			3	29.89	29.83	29.41	0.51	0.06
			4	19.5	19.6	19.36	0.81	0.09
			5	62.47	62.33	60.34	5.57	0.57
			6	74.18	73.79	53.98	24.75	3.58
			7	49.35	49.29	0.18	49.11	4.94
1.4	99.01	82380	1	63.06	62.99	61.01	2.06	0
			2	16.77	17.04	16.89	0.26	0.07
			3	32.19	32.54	32.03	0.63	0.12
			4	21	20.97	20.42	1.13	0.12
			5	67.27	66.56	66.3	6.8	0.58
			6	79.88	79.02	38.82	44.09	6.54
			7	53.14	53	0.16	52.84	3.89
1.5	98.95	84566	1	67.56	67.81	65.7	2.23	0
			2	17.97	17.78	17.56	0.33	0.12
			3	34.49	34.43	33.94	0.62	0.1
			4	22.5	22.52	22.04	1.19	0.13
			5	72.08	71.65	73.88	7.44	0.71
			6	85.59	85.75	22.28	66.52	9.67
			7	56.94	56.79	0.11	56.69	3.05

Table A1.31: Scenario 2, Case 1, under EMSRb3, with +0.15 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.74	71285	1	45.04	45.05	44.29	0.81	0
			2	11.98	11.9	11.8	0.12	0.05
			3	22.99	22.98	22.85	0.15	0.02
			4	15	15.11	14.99	0.23	0.03
			5	48.05	47.58	46.63	1.13	0.1
			6	57.06	57.45	56.55	1.47	0.18
			7	37.96	37.79	33.13	4.66	0.57
1.1	98.22	74583	1	49.54	49.19	48.02	1.19	0
			2	13.18	12.98	12.85	0.16	0.03
			3	25.29	25.48	25.32	0.2	0.02
			4	16.5	16.39	16.14	0.45	0.04
			5	52.86	52.74	51.03	2.33	0.19
			6	62.77	62.94	61.48	4.5	0.62
			7	41.76	41.59	18.91	22.68	3.04
1.2	98.45	77432	1	54.05	53.74	52.36	1.43	0
			2	14.38	13.96	13.83	0.19	0.05
			3	27.59	27.58	27.3	0.34	0.06
			4	18	17.94	17.68	0.62	0.06
			5	57.66	57.58	55.47	3.28	0.36
			6	68.47	68.37	65.2	8.85	1.17
			7	45.55	45.46	2.46	43	5.69
1.3	98.91	80082	1	58.55	58.63	56.97	1.79	0
			2	15.57	15.54	15.29	0.32	0.13
			3	29.89	29.83	29.41	0.48	0.07
			4	19.5	19.6	19.37	0.78	0.07
			5	62.47	62.33	60.51	5.33	0.55
			6	74.18	73.79	53.66	25.27	3.5
			7	49.35	49.29	0.19	49.1	5.13
1.4	98.97	82397	1	63.06	62.99	61.05	2.01	0
			2	16.77	17.04	16.9	0.26	0.07
			3	32.19	32.54	32.05	0.6	0.12
			4	21	20.97	20.48	1.06	0.11
			5	67.27	66.56	66.68	6.39	0.57
			6	79.88	79.02	38.24	44.77	6.51
			7	53.14	53	0.15	52.85	3.99
1.5	98.89	84587	1	67.56	67.81	65.75	2.15	0
			2	17.97	17.78	17.56	0.31	0.09
			3	34.49	34.43	33.95	0.58	0.09
			4	22.5	22.52	22.17	1.06	0.1
			5	72.08	71.65	74.48	7.05	0.72
			6	85.59	85.75	21.33	67.42	9.88
			7	56.94	56.79	0.13	56.67	3

Table A1.32: Scenario 2, Case 1, under EMSRb3, with +0.20 sell-up error

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.83	58308	1	36.03	35.73	35.73	0	0
			2	9.58	9.56	9.56	0	0
			3	18.39	18.22	18.22	0	0
			4	12	11.96	11.96	0	0
			5	38.44	38.66	38.66	0	0
			6	45.65	45.7	45.7	0	0
			7	30.37	30.15	30.15	0	0
0.9	89.82	65573	1	40.54	40.32	40.18	0.15	0
			2	10.78	10.72	10.71	0.01	0
			3	20.69	20.46	20.45	0.01	0
			4	13.5	13.65	13.65	0.02	0
			5	43.24	43.18	43.09	0.1	0.01
			6	51.35	51.74	51.66	0.1	0
			7	34.16	34.17	34.04	0.13	0.02
1	96.74	71284	1	45.04	45.05	44.29	0.8	0
			2	11.98	11.9	11.8	0.12	0.04
			3	22.99	22.98	22.85	0.14	0.02
			4	15	15.11	14.93	0.22	0.02
			5	48.05	47.58	46.64	1.12	0.1
			6	57.06	57.45	56.57	1.47	0.18
			7	37.96	37.79	33.09	4.69	0.59
1.1	98.2	74611	1	49.54	49.19	48.97	1.17	0
			2	13.18	12.98	12.86	0.15	0.06
			3	25.29	25.48	25.36	0.19	0.02
			4	16.5	16.39	16.15	0.44	0.06
			5	52.86	52.74	51.05	2.25	0.2
			6	62.77	62.94	61.67	4.36	0.56
			7	41.76	41.59	18.56	23.04	3.09
1.2	98.43	77427	1	54.05	53.74	52.37	1.43	0
			2	14.38	13.96	13.84	0.18	0.06
			3	27.59	27.58	27.31	0.34	0.06
			4	18	17.94	17.64	0.6	0.07
			5	57.66	57.58	55.56	3.2	0.31
			6	68.47	68.37	65.13	8.97	1.18
			7	45.55	45.46	2.4	43.06	5.73
1.3	98.89	80096	1	58.55	58.63	56.97	1.74	0
			2	15.57	15.54	15.33	0.28	0.08
			3	29.89	29.83	29.43	0.46	0.07
			4	19.5	19.6	19.4	0.74	0.06
			5	62.47	62.33	60.78	5.13	0.54
			6	74.18	73.79	53.27	25.54	3.58
			7	49.35	49.29	0.18	49.11	5.02
1.4	98.94	82406	1	63.06	62.99	61.06	2.01	0
			2	16.77	17.04	16.9	0.25	0.07
			3	32.19	32.54	32.08	0.59	0.11
			4	21	20.97	20.52	1.04	0.13
			5	67.27	66.56	67.12	6.03	0.59
			6	79.88	79.02	37.62	45.48	6.59
			7	53.14	53	0.18	52.83	4.08
1.5	98.85	84619	1	67.56	67.81	65.78	2.11	0
			2	17.97	17.78	17.63	0.26	0.09
			3	34.49	34.43	33.98	0.56	0.1
			4	22.5	22.52	22.23	0.95	0.11
			5	72.08	71.65	75.12	6.53	0.66
			6	85.59	85.75	20.37	68.42	9.99
			7	56.94	56.79	0.14	56.65	3.04

A2: Sensitivity of Z-factor

Table A2.1: Scenario 1, Case 1, under original EMSRb, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	78.48	39447	1	36.03	35.86	35.52	0.4	0
			2	38.44	38.57	38.69	0.28	0.06
			3	45.65	45.54	43.51	2.03	0.4
0.9	85.26	43607	1	40.54	40.38	39.78	0.86	0
			2	43.24	43.11	43.48	1.03	0.26
			3	51.35	51.7	44.64	7.06	1.4
1	89.96	47422	1	45.04	45.14	43.91	1.75	0
			2	48.05	48.29	49.49	1.74	0.51
			3	57.06	56.97	41.54	15.43	2.93
1.1	92.07	50525	1	49.54	49.51	47.84	2.56	0
			2	52.86	52.91	55.22	3.15	0.89
			3	62.77	63.15	35.05	28.1	5.46
1.2	93.69	54038	1	54.05	54.77	52.82	3.23	0
			2	57.66	57.66	61.26	4.55	1.28
			3	68.47	68.52	26.45	42.06	8.14
1.3	93.86	56553	1	58.55	58.55	56.41	4.08	0
			2	62.47	62.65	66.99	6.65	1.95
			3	74.18	73.7	17.39	56.31	10.99
1.4	94.2	59522	1	63.06	62.54	60.45	4.31	0
			2	67.27	67.42	74.16	7.58	2.21
			3	79.88	80.35	6.69	73.66	14.32
1.5	95.34	62047	1	67.56	67.89	64.89	6.51	0
			2	72.08	72.3	75.97	12.14	3.51
			3	85.59	85.25	2.15	83.11	15.81

Table A2.2: Scenario 1, Case 1, under EMSRb2, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	76.46	39276	1	36.03	35.86	35.88	0.09	0
			2	38.44	38.57	39.51	0.3	0.11
			3	45.65	45.54	39.3	6.24	1.24
0.9	80.84	43222	1	40.54	40.38	40.52	0.13	0
			2	43.24	43.11	45.34	0.91	0.27
			3	51.35	51.7	35.4	16.3	3.14
1	83.39	47040	1	45.04	45.14	45.29	0.35	0
			2	48.05	48.29	52.63	1.5	0.5
			3	57.06	56.97	27.16	29.81	5.84
1.1	84.3	50359	1	49.54	49.51	49.86	0.36	0
			2	52.86	52.91	59.68	2.34	0.71
			3	62.77	63.15	16.91	46.24	9.11
1.2	85.29	54031	1	54.05	54.77	55.29	0.52	0
			2	57.66	57.66	66.39	3.57	1.04
			3	68.47	68.52	6.25	62.26	12.3
1.3	87.3	56868	1	58.55	58.55	59.64	1.09	0
			2	62.47	62.65	69.24	7.31	2.18
			3	74.18	73.7	2.07	71.62	13.91
1.4	89.98	59842	1	63.06	62.54	64.81	1.58	0
			2	67.27	67.42	69.53	12.98	3.85
			3	79.88	80.35	0.64	79.71	15.09
1.5	92.15	62741	1	67.56	67.89	71	2.97	0
			2	72.08	72.3	67.03	20.55	6.09
			3	85.59	85.25	0.2	85.05	15.28

Table A2.3: Scenario 1, Case 1, under EMSRb3, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	78.4	39455	1	36.03	35.86	35.58	0.38	0
			2	38.44	38.57	38.71	0.27	0.1
			3	45.65	45.54	43.32	2.22	0.41
0.9	84.97	43633	1	40.54	40.38	39.93	0.74	0
			2	43.24	43.11	43.63	1.01	0.28
			3	51.35	51.7	43.9	7.8	1.53
1	89.28	47469	1	45.04	45.14	44.17	1.41	0
			2	48.05	48.29	50.03	1.52	0.44
			3	57.06	56.97	39.71	17.26	3.26
1.1	90.89	50652	1	49.54	49.51	48.3	1.98	0
			2	52.86	52.91	56.45	2.63	0.76
			3	62.77	63.15	31.6	31.56	6.17
1.2	92.15	54348	1	54.05	54.77	53.59	2.22	0
			2	57.66	57.66	63.33	3.57	1.03
			3	68.47	68.52	21.31	47.2	9.24
1.3	91.92	56915	1	58.55	58.55	57.12	2.78	0
			2	62.47	62.65	70.18	4.79	1.36
			3	74.18	73.7	10.57	63.13	12.33
1.4	93.19	59756	1	63.06	62.54	61.41	3.42	0
			2	67.27	67.42	74.36	7.77	2.28
			3	79.88	80.35	4.01	76.33	14.71
1.5	94.8	62439	1	67.56	67.89	66.65	5.44	0
			2	72.08	72.3	74.11	13.76	4.2
			3	85.59	85.25	1.45	83.81	15.57

Table A2.4: Scenario 2, Case 1, under original EMSRb, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax Sell-up
0.8	79.1	57775	1	36.03	35.73	35.24	0.49	0
			2	9.58	9.56	9.49	0.07	0
			3	18.39	18.22	18.21	0.01	0
			4	12	11.96	11.93	0.07	0.01
			5	38.44	38.66	38.35	0.35	0.04
			6	45.65	45.7	45.31	0.44	0.04
			7	30.37	30.15	29.72	0.44	0.05
0.9	87.1	63671	1	40.54	40.32	38.5	1.82	0
			2	10.78	10.72	10.53	0.2	0
			3	20.69	20.46	20.38	0.12	0.02
			4	13.5	13.65	13.38	0.38	0.04
			5	43.24	43.18	42.16	1.15	0.1
			6	51.35	51.74	50.79	1.24	0.13
			7	34.16	34.17	31.57	2.6	0.29
1	92.54	68418	1	45.04	45.05	42.03	3.03	0
			2	11.98	11.9	11.51	0.43	0
			3	22.99	22.98	22.71	0.36	0.04
			4	15	15.11	14.7	0.65	0.09
			5	48.05	47.58	45.13	2.86	0.24
			6	57.06	57.45	54.66	3.74	0.41
			7	37.96	37.79	29.5	8.29	0.96
1.1	94.98	71575	1	49.54	49.19	44.72	4.47	0
			2	13.18	12.98	12.47	0.6	0
			3	25.29	25.48	24.99	0.62	0.08
			4	16.5	16.39	15.8	1.06	0.13
			5	52.86	52.74	48.53	5.14	0.47
			6	62.77	62.94	57.67	7.44	0.93
			7	41.76	41.59	21.88	19.72	2.17
1.2	96.43	74569	1	54.05	53.74	48.28	5.46	0
			2	14.38	13.96	13.31	0.75	0
			3	27.59	27.58	26.97	0.76	0.1
			4	18	17.94	17.19	1.29	0.15
			5	57.66	57.58	52.55	6.48	0.54
			6	68.47	68.37	60.52	11.99	1.44
			7	45.55	45.46	10.67	34.79	4.15
1.3	97.34	77501	1	58.55	58.63	52.24	6.39	0
			2	15.57	15.54	14.93	0.73	0
			3	29.89	29.83	29.22	0.85	0.12
			4	19.5	19.6	18.69	1.77	0.24
			5	62.47	62.33	55.81	9.13	0.86
			6	74.18	73.79	58.78	20.11	2.6
			7	49.35	49.29	2.01	47.28	5.09
1.4	97.96	79682	1	63.06	62.99	55.32	7.68	0
			2	16.77	17.04	16.31	0.87	0
			3	32.19	32.54	31.76	1.09	0.14
			4	21	20.97	19.74	2.41	0.31
			5	67.27	66.56	58.52	12.41	1.18
			6	79.88	79.02	51.15	32.68	4.37
			7	53.14	53	0.35	52.65	4.81
1.5	98.43	81786	1	67.56	67.81	59.29	8.52	0
			2	17.97	17.78	16.95	1.03	0
			3	34.49	34.43	33.42	1.37	0.2
			4	22.5	22.52	21.2	2.7	0.36
			5	72.08	71.65	63.49	14.72	1.38
			6	85.59	85.75	39.78	50.05	6.56
			7	56.94	56.79	0.14	56.66	4.08

Table A2.5: Scenario 2, Case 1, under EMSRb2, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.03	57798	1	36.03	35.73	35.4	0.33	0
			2	9.58	9.56	9.52	0.04	0
			3	18.39	18.22	18.21	0.02	0.01
			4	12	11.96	11.93	0.07	0
			5	38.44	38.66	38.31	0.4	0.04
			6	45.65	45.7	45.28	0.5	0.05
			7	30.37	30.15	29.45	0.7	0.08
0.9	86.65	63648	1	40.54	40.32	38.95	1.37	0
			2	10.78	10.72	10.63	0.11	0
			3	20.69	20.46	20.38	0.12	0.02
			4	13.5	13.65	13.43	0.32	0.05
			5	43.24	43.18	42.18	1.19	0.1
			6	51.35	51.74	50.64	1.6	0.19
			7	34.16	34.17	30	4.17	0.5
1	91.68	68526	1	45.04	45.05	43.24	1.81	0
			2	11.98	11.9	11.71	0.22	0
			3	22.99	22.98	22.8	0.24	0.03
			4	15	15.11	14.85	0.51	0.06
			5	48.05	47.58	45.21	2.88	0.25
			6	57.06	57.45	54.44	4.42	0.51
			7	37.96	37.79	25.95	11.84	1.42
1.1	93.69	71682	1	49.54	49.19	46.29	2.9	0
			2	13.18	12.98	12.66	0.38	0
			3	25.29	25.48	25.14	0.43	0.05
			4	16.5	16.39	15.95	0.88	0.09
			5	52.86	52.74	49.25	4.6	0.44
			6	62.77	62.94	57.66	8.35	1.11
			7	41.76	41.59	16.02	25.57	3.06
1.2	94.89	74727	1	54.05	53.74	50.21	3.53	0
			2	14.38	13.96	13.65	0.4	0
			3	27.59	27.58	27.11	0.61	0.09
			4	18	17.94	17.41	1.06	0.13
			5	57.66	57.58	53.53	5.83	0.53
			6	68.47	68.37	59.9	13.38	1.78
			7	45.55	45.46	4.03	41.43	4.92
1.3	96.2	77749	1	58.55	58.63	54.51	4.13	0
			2	15.57	15.54	15.22	0.41	0
			3	29.89	29.83	29.43	0.59	0.09
			4	19.5	19.6	19.01	1.56	0.2
			5	62.47	62.33	56.67	9.07	0.96
			6	74.18	73.79	53.31	25.27	3.41
			7	49.35	49.29	0.79	48.5	4.79
1.4	96.76	79983	1	63.06	62.99	57.8	5.19	0
			2	16.77	17.04	16.73	0.45	0
			3	32.19	32.54	31.96	0.84	0.14
			4	21	20.97	20.04	2.06	0.27
			5	67.27	66.56	60.22	11.79	1.13
			6	79.88	79.02	43.37	39.92	5.45
			7	53.14	53	0.17	52.83	4.27
1.5	97.24	82246	1	67.56	67.81	62.24	5.58	0
			2	17.97	17.78	17.42	0.52	0
			3	34.49	34.43	33.66	1.06	0.16
			4	22.5	22.52	21.7	2.27	0.29
			5	72.08	71.65	65.13	14.35	1.46
			6	85.59	85.75	31.2	57.92	7.82
			7	56.94	56.79	0.09	56.7	3.37

Table A2.6: Scenario 2, Case 1, under EMSRb3, with Z-factor of 2.06

Demand Factor	Load Factor (%)	Mean Revenue	Class	Demand (Input)	Requests (Actual)	Load	Spill	Pax. Sell-up
0.8	79.09	57778	1	36.03	35.73	35.27	0.46	0
			2	9.58	9.56	9.48	0.08	0.01
			3	18.39	18.22	18.21	0.02	0
			4	12	11.96	11.94	0.07	0
			5	38.44	38.66	38.34	0.36	0.04
			6	45.65	45.7	45.3	0.45	0.04
			7	30.37	30.15	29.71	0.45	0.05
0.9	87.08	63683	1	40.54	40.32	38.61	1.74	0
			2	10.78	10.72	10.5	0.23	0.03
			3	20.69	20.46	20.39	0.12	0.01
			4	13.5	13.65	13.38	0.36	0.05
			5	43.24	43.18	42.13	1.18	0.09
			6	51.35	51.74	50.78	1.27	0.13
			7	34.16	34.17	31.46	2.7	0.32
1	92.47	68448	1	45.04	45.05	42.22	2.87	0
			2	11.98	11.9	11.49	0.44	0.03
			3	22.99	22.98	22.73	0.32	0.03
			4	15	15.11	14.72	0.64	0.07
			5	48.05	47.58	45.16	2.86	0.24
			6	57.06	57.45	54.56	3.84	0.44
			7	37.96	37.79	29.22	8.57	0.96
1.1	94.87	71613	1	49.54	49.19	44.98	4.24	0
			2	13.18	12.98	12.44	0.61	0.03
			3	25.29	25.48	25.05	0.55	0.07
			4	16.5	16.39	15.8	1.01	0.11
			5	52.86	52.74	48.55	5.12	0.42
			6	62.77	62.94	57.58	7.71	0.94
			7	41.76	41.59	21.4	20.2	2.35
1.2	96.3	74668	1	54.05	53.74	48.65	5.16	0
			2	14.38	13.96	13.28	0.77	0.07
			3	27.59	27.58	27.02	0.73	0.1
			4	18	17.94	17.31	1.24	0.16
			5	57.66	57.58	52.78	6.38	0.61
			6	68.47	68.37	60.57	11.87	1.57
			7	45.55	45.46	9.59	35.87	4.07
1.3	97.21	77521	1	58.55	58.63	52.52	6.12	0
			2	15.57	15.54	14.94	0.7	0.01
			3	29.89	29.83	29.21	0.82	0.1
			4	19.5	19.6	18.78	1.65	0.2
			5	62.47	62.33	55.75	9.33	0.83
			6	74.18	73.79	58.23	20.63	2.75
			7	49.35	49.29	1.93	47.36	5.07
1.4	97.83	79753	1	63.06	62.99	55.71	7.31	0
			2	16.77	17.04	16.36	0.84	0.03
			3	32.19	32.54	31.78	1.03	0.16
			4	21	20.97	19.78	2.33	0.27
			5	67.27	66.56	58.69	12.44	1.13
			6	79.88	79.02	50.18	33.51	4.57
			7	53.14	53	0.34	52.66	4.67
1.5	98.31	81915	1	67.56	67.81	59.8	8.04	0
			2	17.97	17.78	17	0.93	0.03
			3	34.49	34.43	33.46	1.29	0.15
			4	22.5	22.52	21.29	2.61	0.32
			5	72.08	71.65	64.17	14.38	1.39
			6	85.59	85.75	38.11	51.49	6.89
			7	56.94	56.79	0.16	56.64	3.85

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