Limited Stock Market Participation

by

Annette Vissing-Jørgensen

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Signature of Author ....
Department of Economics
May 4, 1998

Certified by ....
Ricardo J. Caballero
Professor of Economics
Thesis Supervisor

Certified by ....
Olivier J. Blanchard
Professor of Economics
Thesis Supervisor

Accepted by ....
Peter Temin
Chairperson, Department Committee on Graduate Students

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Abstract

This thesis analyzes the effects of limited stock market participation on asset returns.

In Chapter 1, I argue that limited stock market participation should be considered an important part of the solution to the equity premium puzzle. A simple condition is given under which the equity premium predicted by the standard CCAPM is only a fraction λ of the true equity premium generated by the process for consumption, where λ is the fraction of stockholders in the population. Correspondingly, estimating Euler equations involving stock returns without excluding nonstockholders will result in an estimate of relative risk aversion which is too high by factor $1/\lambda$. With an average value of λ of about 20 percent in postwar US data, this suggests limited stock market participation as a plausible explanation of the equity premium puzzle. I test this hypothesis using micro consumption data from the Consumer Expenditure Survey. The empirical results based on estimation of Euler equations show that accounting for differences in consumption patterns of stockholders and nonstockholders should be considered a major part of the solution to the puzzle.

To support this finding Chapter 2 uses micro data on income and asset holdings from the Panel Study of Income Dynamics to analyze reasons for nonparticipation and for heterogeneity in portfolio choice within the set of stock market participants. The focus of the chapter is on non-financial income. I find evidence of a strong positive effect of mean non-financial income on the probability of stock market participation and on the proportion of wealth invested in stocks conditional on being a participant. The volatility of non-financial income is found to have a strong negative impact on these two quantities. Both these results are consistent with the theoretical literature on portfolio choice in the presence of non-financial income. However, only a small or insignificant effect of the covariance of non-financial income with the stock market return on portfolio choice is present. This finding supports the results of Chapter 1 in the sense that a such effect (along with a short sales constraint on stocks) would tend to increase the covariance of nonstockholder consumption growth with the stock return relative to that of stockholders. Using three observations of portfolio choice, Chapter 2 furthermore provides new evidence of the importance of fixed participation costs on portfolio choice.

Understanding the main reasons for nonparticipation is crucial not only for having confidence in the results of Chapter 1. Stock market participation has increased dramatically in the postwar period with large potential effects on returns. Chapter 3 contains a general equilibrium analysis of the effect of limited stock market participation on asset returns. Using an OLG model, I first analyze the benchmark case of identical agents, no cost of entry, and an exogenous restriction on participation. The presence of a fixed cost of stock market entry is then assumed and different reasons for entry, corresponding to households being distributed along different characteristics, are analyzed. The model shows that the effect of higher participation is likely to be a stock market boom and a decrease in the equity premium if risk aversion is sufficiently low.

Thesis Supervisor: Ricardo J. Caballero
Title: Professor of Economics

Thesis Supervisor: Olivier J. Blanchard
Title: Professor of Economics

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Chapter 1

Limited Stock Market Participation and the Equity Premium Puzzle

1.1 Introduction

Empirical tests of the consumption capital asset pricing model with constant relative risk aversion have rejected the model in several ways. In a general equilibrium model Mehra and Prescott (1985) showed that the covariance of US per capita consumption growth and stock returns is too low to explain the observed equity premium unless risk aversion is assumed to be implausibly high. Hansen and Jagannathan (1991) derived a lower bound for the standard deviation of any valid stochastic discount factor and found that the stochastic discount factor implied by the CCAPM only satisfies the bound if risk aversion is very large. Hansen and Singleton (1982, 1983) found that when Euler equations for an assumed representative agent were estimated jointly for aggregate stock returns and a nominally riskfree rate, the overidentifying restrictions strongly rejected the model. This indicates predictable deviations from the Euler equations. Mehra and Prescott's finding is the well known equity premium puzzle. As Weil (1989) emphasized it implies a risk free rate puzzle. If consumers with CRRA utility are very averse to differences in consumption across states, they are also very averse to differences in consumption across time. Since the low observed riskfree rate offers little incentive to save, the observed average growth rate of per capita consumption is inconsistent with the large estimate of risk aversion unless the representative agent has a negative discount rate. In Hansen and

Jagannathan's test this puzzle takes the form of the CCAPM stochastic discount factor entering the bound at a mean implying a very large riskfree rate.

This chapter proposes limited stock market participation as a unified framework for explaining these rejections of the model. If the consumption growth of nonstockholders covaries less with stock returns than that of stockholders, including the consumption of nonstockholders in the consumption measure used to test the CCAPM will lead to an upward biased estimate of risk aversion. Furthermore, since Euler equations involving stock returns will not hold for nonstockholders, limited stock market participation has the potential of explaining the rejections of the model based on tests of overidentifying restrictions.

The first section of the chapter thus asks the following question. Suppose observed consumption and asset return data are generated by an economy characterized by limited stock market participation. How large will the upward bias in the estimate of risk aversion be if limited participation is not taken into account in the estimation? This question is answered based on aggregation of Euler equations without imposing any general equilibrium structure. The condition for the results therefore takes the form of a condition directly on the consumption of nonstockholders.

Mankiw and Zeldes (1991) first emphasized that nonstockholders should be excluded in tests of the CCAPM, and estimated Euler equations for stockholders and nonstockholders separately using data from the PSID. They found large differences between the two groups but the estimate of risk aversion remained as high as 35.2 for the richest group of stockholders. However, the PSID only contains data on food consumption which is likely to be one of the most stable consumption components. In this chapter I therefore use data on consumption of all nondurables and services from the Consumer Expenditure Survey to test the theory. The data frequency in the CEX is furthermore higher than in the PSID, attenuating problems of time aggregation. Furthermore, the CEX is designed with the purpose of collecting consumption data. Thus, although still important, measurement error in consumption is likely to be smaller for CEX consumption data than for the PSID consumption data. Brav and Geczy (1996) conducts a study much like that of Mankiw and Zeldes but using CEX data. They focus mainly on stockholders and find that risk aversion estimates decline as they look at still wealthier layers of stockholders. The present chapter analyzes differences between stockholders and nonstockholders. Euler equations are estimated for each of the two groups to show how lower correlation of consumption growth and stock returns for nonstockholders implies large risk aversion estimates for the total set of households as found on aggregate data. The results are overall positive. Differences in risk aversion estimates obtained for stockholders and nonstockholders are large, although the results differ somewhat across estimation methods. As part of the empirical analysis I estimate Euler equations for stock and bond returns separately. The purpose is to determine firstly whether risk aversion estimates for stockholders are consistent across different assets, and secondly whether the CCAPM is valid for nonstockholders as long as attention is restricted to relatively riskless assets. If the latter is not the case, then fundamentally different factors are at play for nonstockholders causing violation of Euler equations for unconstrained optimization. The results provide evidence of such violations. Possible reasons could be frequently binding borrowing constraints or irrationality, but I do not attempt to determine this.

The concept of limited stock market participation is related to the large literature on asset pricing with incomplete markets and uninsurable labor income shocks. The main idea behind that approach is that individual consumption growth may have a higher covariance with stock returns than aggregate consumption if uninsurable idiosyncratic risk is higher when stock returns are low. The size of the effects on the equity premium in general equilibrium depends on how persistent the idiosyncratic shocks are, with the paper by Constantinides and Duffie (1996) as the limiting case of random walk shocks. Other important factors are the size of transaction costs for stocks and bonds, the extent of borrowing constraint, and the size the outside supply of bonds. As emphasized in the survey by Kocherlakota (1996), this literature has had some success in explaining low riskless rates but less in explaining the equity premium puzzle. The empirical work on the correlation of stock returns and idiosyncratic shocks is still at an early stage. I will briefly return to this issue later with some negative results based on the CEX data used in my empirical analysis of limited stock market participation[1].

[1] It should be pointed out that individual consumption growth rates and the growth rate of per capita consumption can differ even with complete markets. A simple example is that of an OLG economy in which new generations are born with higher lifetime incomes than existing generations due to productivity growth. In this setup all individuals can have flat consumption profiles over their lifetime at the same time as aggregate consumption growth is positive. Using aggregate consumption data in the CCAPM then implies overprediction of the riskless rate.

With limited participation markets are incomplete in the extreme sense of one group of people not trading in the stock market at all. This has the potential of generating large differences in consumption growth across the two types of agents. In general equilibrium the fact that only a fraction of the population must hold all stock market risk endogenously causes their consumption to have a higher correlation with dividends than it would have had if all agents shared the risk. As a result the equilibrium market price of risk is increased. In a recent paper, Basak and Cuoco (1997) building on work by Saito (1992), elegantly derive this effect in a continuous time model. A drawback of their paper is that the separation between stockholders and nonstockholders is exogenous. It is clear that the reason for nonparticipation matters for the general equilibrium effects of limited stock market participation. Suppose for example that stockholders have much lower risk aversion than nonstockholders. Then as more agents enter, the 'average' risk aversion of stockholders increase and it is not a priori clear whether increased participation should decrease the market price of risk and the equity premium in this case.

Understanding the general equilibrium effects of limited stock market participation is important not only for confirming that this type of model can generate a higher equity premium. There has been a dramatic increase in participation and risk sharing in the US since the 1950s. A general equilibrium model of limited participation will enable us to evaluate how this may have contributed to the path of asset prices and returns which has been observed, and to consider the potential effects of continued entry in the future. This will be the focus of Chapter 3, where I first consider the benchmark case of identical agents, no cost of entry, and an exogenous restriction on how large a fraction of agents are allowed to hold stocks. The presence of a fixed cost of stock market entry is then assumed and I analyze different reasons for entry corresponding to households being distributed along different characteristics. The analysis furthermore considers the question of expected versus unexpected entry. If stock market entry is expected ahead of time, should asset prices change when the realization of future entry is made or when entry actually takes place? If markets are efficient and thus incorporate the information about future entry, one might think that asset prices should move at the time the realization about future entry is made. On the other hand one could argue that since risk sharing does not increase until entry actually takes place, actual and expected returns in the periods before entry occurs should be unaffected. A formal model will make it possible to determine which of these arguments are correct.

1.2 Euler equations and limited stock market participation

Suppose the observed consumption and asset return data are generated by an economy characterized by limited stock market participation. Given the consumption data, how much smaller is the equity premium predicted by the model which ignores limited participation compared to the equity premium predicted using only the consumption of stockholders? In addition, how different is the estimate of relative risk aversion which is obtained based on each of the models? These are the questions in focus in the analysis which follows. The analysis is based on aggregation of Euler equations with no general equilibrium structure explicitly imposed. This is the most relevant setup if the purpose is to derive the bias in risk aversion estimates from estimations of Euler equations which do not account for limited stock market participation. It does, however, imply that the condition needed for the results must be stated directly as a condition on the consumption of nonstockholders and not as conditions on the underlying fundamentals. Such more fundamental conditions can only be derived by imposing more structure on the problem. It is clear that differences in consumption patterns between stockholders and nonstockholders depend in the reason why nonparticipants have chosen to stay out of the stock market. I will return to this issue after deriving the condition on nonstockholder consumption which is needed for limited stock market participation to be the explanation of implausible risk aversion estimates based on aggregate data. A general equilibrium framework will also be needed to derive predictions about the volatility of stochastic discount factors based on the consumption of stockholders or nonstockholders. This will be addressed in section 4.

To separate the effects of limited participation from those of incomplete markets discussed previously, this section will focus on Euler equations which take cross-sectional heterogeneity in consumption into account. The results are similar if one assumes a representative agent within each of the groups of agents in focus, as long as the cross-sectional variance of consumption growth is uncorrelated with asset returns.

1.2.1 Upward biased risk aversion estimates

An analysis which ignores limited stock market participation will assume that Euler equations for stocks and bonds hold for all households[2]. This would lead to an aggregated Euler equation of the form

$$E_t\left[M_{t+1}^T R_{i,t+1}\right] = 1, \qquad M_{t+1}^T \equiv E_h\left[\delta\left(\frac{C_{t+1}^h}{C_t^h}\right)^{-\gamma}\right] \tag{1.1}$$

$R_{i,t+1}$ denotes the gross return to holding asset i from date t to t+1. i=s will be used to refer to stocks, and i=f to refer to one period bonds. E_h denotes the cross-sectional mean across households at time t. E_t refers to the conditional expectation given all information known at time t. The parameter δ is the discount factor equal to $\frac{1}{1+\beta}$ where β is the discount rate. γ is the coefficient of relative risk aversion. Preferences are assumed identical for all agents. T denotes that the total set of households are used. I will use a to refer to stockholders and na to refer to nonstockholders. For stocks 1.1 is not valid, since nonstockholders are included. In other words, $M_{t+1}^T$is not a valid stochastic discount factor for pricing stocks, since nonstockholders are at a corner. Thus their consumption does not satisfy the Euler equation for $R_{s,t+1}$. M_{t+1}^T is a valid stochastic discount factor for pricing riskless assets under the maintained assumption that all households are at an interior solution with respect to riskless assets (borrowing is allowed).

The true aggregated Euler equation for the stock return only includes stockholders in the cross-sectional aggregation:

$$E_t\left[M_{t+1}^a R_{s,t+1}\right] = 1, \qquad M_{t+1}^a \equiv E_{h,a}\left[\delta\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)^{-\gamma}\right] \tag{1.2}$$

The stochastic discount factor M_{t+1}^T can be rewritten as:

$$M_{t+1}^T \equiv E_h\left[\delta\left(\frac{C_{t+1}^h}{C_t^h}\right)^{-\gamma}\right] \tag{1.3}$$

[2]The Euler equations are the first order conditions for intertemporal optimization. I assume absence of market frictions throughout the paper. Therefore the Euler equations hold with equality for households who are at an interior solution with respect to the asset in focus.

$$= \left(\lambda E_{h,a} \left[\delta \left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}} \right)^{-\gamma} \right] + (1-\lambda) E_{h,na} \left[\delta \left(\frac{C_{t+1}^{h,na}}{C_t^{h,na}} \right)^{-\gamma} \right] \right) = \left[\lambda M_{t+1}^{a} + (1-\lambda) M_{t+1}^{na} \right] \quad (1.4)$$

λ is the proportion of stockholders in the population. As $M_{t+1}^{T}, M_{t+1}^{na}$ is not a valid stochastic discount factor for pricing stocks. Using 1.1 for stocks and bonds we obtain the equity premium predicted by the model which ignores limited participation:

$$\begin{aligned} (E_t R_{s,t+1} - R_{f,t+1})^T &= -\frac{cov_t \left(R_{s,t+1}, \lambda M_{t+1}^{a} + (1-\lambda) M_{t+1}^{na} \right)}{E_t \left(\lambda M_{t+1}^{a} + (1-\lambda) M_{t+1}^{na} \right)} & (1.5) \\ &= -R_{f,t+1} cov_t \left(R_{s,t+1}, \lambda M_{t+1}^{a} + (1-\lambda) M_{t+1}^{na} \right) & (1.6) \end{aligned}$$

Using 1.2 we get the equity premium which would in fact be consistent with the consumption processes observed [3]. If the CCAPM holds for the set of stockholders this will equal the observed equity premium:

$$E_t R_{s,t+1} - R_{f,t+1} = -\frac{cov_t \left(R_{s,t+1}, M_{t+1}^{a} \right)}{E_t \left(M_{t+1}^{a} \right)} = -R_{f,t+1} cov_t \left(R_{s,t+1}, M_{t+1}^{a} \right) \quad (1.7)$$

For limited stock market participation to explain the equity premium puzzle it must be the case that $(E_t R_{s,t+1} - R_{f,t+1})^T < E_t R_{s,t+1} - R_{f,t+1}$. Faced with actual asset returns, estimations based on the total set of households will then lead to an upward biased estimate of risk aversion[4]. Comparing 1.6 and 1.7 we immediately get the following result.

RESULT 1:

a) $cov_t \left(R_{s,t+1}, M_{t+1}^{na} - M_{t+1}^{a} \right) = 0 \Rightarrow (E_t R_{s,t+1} - R_{f,t+1})^T = E_t R_{s,t+1} - R_{f,t+1}$

b) $cov_t \left(R_{s,t+1}, M_{t+1}^{na} \right) = 0 \Rightarrow (E_t R_{s,t+1} - R_{f,t+1})^T = \lambda \left(E_t R_{s,t+1} - R_{f,t+1} \right)$

Proof: The result follows directly from comparison of 1.6 and 1.7.

[3] We could equivalently use the stochastic discount factor based on the consumption of any of the stockholders in this relation. With the Euler equation for the stock return holding for each stockholder, the covariance of stock returns with the consumption growth of each stockholder is identical (for identical stockholder preferences as assumed here). Note that this, along with the restriction on consumption growth for each agent implied by the Euler equation for the riskless asset, does not imply a degenerate cross-sectional distribution of consumption growth for stockholders as long as markets are incomplete. Only with complete markets will the consumption growth rates of any two stockholders be equalized state by state.

[4] This will be derived explicitly below.

In words, if the assumed stochastic discount factor differs from the true one only by a quantity which is uncorrelated with the stock return, the two models will predict the same equity premium for a given consumption pattern. The intuition is that under this condition the consumption growth of nonstockholders covaries with the stock return in the same way as the consumption growth of stockholders. Therefore, including nonstockholder consumption in the stochastic discount factor will not lead to a different prediction for the equity premium. However, as discussed in more detail below, we would expect consumption growth of nonstockholders to be less correlated with stock returns than that of stockholders. Result 1b) shows that if 'on average' the consumption growth of nonstockholders does not covary with the stock return then including nonstockholders in the CCAPM will lead to a predicted equity premium which is too low by factor λ. **Thus if only 20 percent of the population are stockholders and the true equity premium is 6 percent, the CCAPM with nonstockholders included in the consumption measure will predict an equity premium of only 1.2 percent**[5]. By 'on average' I mean that the average (across nonstockholders) of the covariances of $\left(\frac{C_{t+1}^{h,na}}{C_t^{h,na}}\right)^{-\gamma}$ and $R_{s,t+1}$ must equal zero. Thus, the covariance for each non-stockholder need not be zero. Result 1 is quite general. All that has been assumed is that the Euler equations for stocks and bonds hold for stockholders and the Euler equation for bonds hold for nonstockholders. No specific distributional assumptions have been imposed, neither cross-sectionally nor in the time series dimension.

How does the difference in the predicted equity premium translate into bias in estimates of the coefficient of relative risk aversion when the model ignoring limited participation is estimated on a set of consumption and asset return data? To get a closed form answer to this question more structure must be imposed. Let I^a denote the set of stockholders and I^{na} the set of nonstockholders. I make the following distributional assumptions:

ASSUMPTION:

a) $\frac{C_{t+1}^{h,a}}{C_t^{h,a}}$, $R_{s,t+1} \sim$ joint log-normal, $\forall h \in I^a$, conditional on information known at t

b) $\frac{C_{t+1}^{h,na}}{C_t^{h,na}}$, $R_{s,t+1} \sim$ joint log-normal, $\forall h \in I^{na}$, conditional on information known at t

[5]The fraction of stockholders in the adult US population was around 6-8 percent in the 1950s. It has been gradually increasing since then. The latest available estimate is 41 percent in 1995. Fig.1, 2a and 2b of Chapter 3 document these numbers.

For stockholders the Euler equation for stocks imposes that the covariance of $\ln \frac{C_{t+1}^{h,a}}{C_t^{h,a}}$ and $\ln R_{s,t+1}$ must be the same for all stockholders, but this need not be the case for nonstockholders.

Using this assumption the Euler equation for each h$\in I^a$ can be log-linearized as shown by Hansen and Singleton (1983):

$$
\begin{aligned}
0 &= \ln\delta + E_t \ln R_{s,t+1} - \gamma E_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right) + \frac{1}{2} V_t \ln R_{s,t+1} && (1.8)\\
&+ \frac{1}{2}\gamma^2 V_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right) - \gamma\, cov_t\left(\ln R_{s,t+1}, \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right) && (1.9)
\end{aligned}
$$

Summing 1.8 over stockholders implies:

$$
\begin{aligned}
0 &= \ln\delta + E_t \ln R_{s,t+1} - \gamma E_{h,a}\left[E_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right] + \frac{1}{2} V_t \ln R_{s,t+1} && (1.10)\\
&+ \frac{1}{2}\gamma^2 E_{h,a}\left(V_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right) - \gamma\, E_{h,a}\left[cov_t\left(\ln R_{s,t+1}, \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)\right]
\end{aligned}
$$

Similarly for the riskless rate:

$$
0 = \ln\delta + \ln R_{f,t+1} - \gamma E_{h,a}\left[E_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right] + \frac{1}{2}\gamma^2 E_{h,a}\left(V_t \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right) \qquad (1.11)
$$

The cross-sectional expectation can be moved inside $\mathrm{E}_t(\cdot)$ and $\mathrm{cov}_t(\cdot)$. One method of estimating δ and γ is then to replace expectations by actual values plus an expectational error, isolate either the consumption measure or the interest rate as the dependent variable and use two stage least squares or instrumental variables estimation[6][7]. 1.10 and 1.11 can be estimated separately or jointly using three stage least squares[8]. Alternatively, one can subtract 1.11 from 1.10, assume constant conditional variances and covariances and use the law of iterated expectations

[6] Ordinary least squares would lead to inconsistent estimates since asset returns and consumption are jointly determined.

[7] It must be assumed that conditional variances and covariances are constant over time or uncorrelated with left hand side variables (and instruments).

[8] In the asset pricing literature aggregate data are most often used and thus careful aggregation is not possible. In the consumption literature using microdata, an equation like 1.11 is estimated by Attanasio and Weber (1995).

to do a calibration like that of Mankiw and Zeldes (1991):

$$\gamma = \frac{E\left(\ln R_{s,t+1} - \ln R_{f,t+1}\right) + \frac{1}{2} V \ln R_{s,t+1}}{cov\left(\ln R_{s,t+1}, E_{h,a} \ln\left(\frac{C^{h,a}_{t+1}}{C^{h,a}_{t}}\right)\right)} \tag{1.12}$$

The corresponding approach which ignores limited stock market participation will include all households in the summations. Let γ^* denote the estimate of relative risk aversion when nonstockholders are included in the calibration:

$$\gamma^* = \frac{E\left(\ln R_{s,t+1} - \ln R_{f,t+1}\right) + \frac{1}{2} V \ln R_{s,t+1}}{cov\left(\ln R_{s,t+1}, E_{h} \ln\left(\frac{C^{h}_{t+1}}{C^{h}_{t}}\right)\right)} \tag{1.13}$$

When nonstockholder consumption growth have a lower covariance with stock returns than stockholder consumption growth, γ^* is an upward biased estimate of γ. The contribution of this section is to quantify the bias.

RESULT 2: $\text{cov}\left(lnR_{s,t+1}, E_{h,na} \ln\left(\frac{C^{h,na}_{t+1}}{C^{h,na}_{t}}\right)\right) = 0 \Rightarrow \gamma^* = \frac{1}{\lambda}\gamma$

Proof: Rewrite the denominator of 1.13

$$\begin{aligned} & cov\left(\ln R_{s,t+1}, E_{h} \ln\left(\frac{C^{h}_{t+1}}{C^{h}_{t}}\right)\right) \\ = \; & \lambda cov\left(\ln R_{s,t+1}, E_{h,a} \ln\left(\frac{C^{h,a}_{t+1}}{C^{h,a}_{t}}\right)\right) + (1-\lambda) cov\left(\ln R_{s,t+1}, E_{h,na} \ln\left(\frac{C^{h,na}_{t+1}}{C^{h,na}_{t}}\right)\right) \end{aligned}$$

Under the condition assumed, the second term is zero. Comparing 1.12 and 1.13 then implies:

$$\gamma\, cov\left(\ln R_{s,t+1}, E_{h,a} \ln\left(\frac{C^{h,a}_{t+1}}{C^{h,a}_{t}}\right)\right) = \gamma^* \lambda cov\left(\ln R_{s,t+1}, E_{h,a} \ln\left(\frac{C^{h,a}_{t+1}}{C^{h,a}_{t}}\right)\right)$$

The result follows by rearrangement.

The estimates of γ based on instrumental variables estimation will be related by the same formula under the additional assumption that $E_{h,na} \ln\left(\frac{C^{h,na}_{t+1}}{C^{h,na}_{t}}\right)$ be uncorrelated with all instruments used.

Result 2 says that if 'on average' the consumption growth of nonstockholders is uncorrelated with stock returns then a calibration (or instrumental variables estimation) based on the CCAPM but including the consumption of nonstockholders in the consumption measure, will lead to an upward biased estimate of risk aversion. The factor determining the bias is again λ, the proportion of stockholders in the population. **Thus if only 20 percent of the population are stockholders and the true coefficient of relative risk aversion is 4, the CCAPM with nonstockholders included in the consumption measure will result in an estimate of 20** For λ as low as in the 1950s, i.e. around 7 percent, γ^* will be biased upward by a factor of 14. It is important to note that it is the fraction of the population who are stockholders which matter for the bias and not the fraction of consumption accounted for by stockholders. This is due to the fact that I have aggregated taking the cross-sectional heterogeneity in consumption growth rates into account. If some households have higher levels of consumption than others, they do not get a higher 'weight' in the calculations since what matters is consumption growth rates. Had I assumed a representative agent and thus used the growth rate of average consumption, the bias in the estimate of γ would be determined by the fraction of consumption accounted for by nonstockholders. This fraction is smaller than 1-λ but still large.

The condition in Result 2 is the same as the one in Result 1b but with the log-normality assumptions imposed. It is written here as an unconditional covariance, but since the derivation of 1.12 relies on constant conditional variances and covariances (conditional on information known at time t), the condition for Result 2 is in fact the same as that for Result 1b. Alternatively one could allow time-variation in variances and covariances and impose unconditional joint log-normality of stock returns, bond returns and consumption growth rates (for each household h). The numerator of 1.12 and 1.13 would then be modified by a term involving the unconditional variance of the bond rate, and the covariance of consumption growth and the bond rate would enter the denominator. Both of these extra terms are small for aggregate consumption data and for all groups of households in the CEX which we consider. Therefore, empirically, the assumptions of conditional joint log-normality and unconditional joint log-normality lead to similar results.

Summing up, this section has shown that the condition that nonstockholder consumption growth be conditionally uncorrelated with stock returns implies two results. Firstly, for given consumption processes the equity premium predicted by a relation which ignores limited stock market participation is only a fraction λ of the true equity premium generated by those consumption processes, where λ is the fraction of stockholders in the population. Secondly, estimating Euler equations involving the stock return without taking limited stock market participation into account will result in an upward biased estimate of the coefficient of relative risk aversion γ. The bias can be large as the examples showed. Admittedly, the distributional assumptions (log-normality in the time series dimension) needed to obtain closed form solutions for the bias are strong. I will return to the possibility of using GMM estimation to avoid distributional restrictions in the empirical section.

1.2.2 Plausibility of the covariance condition on nonstockholder consumption growth

Whether limited participation based on these results should be considered a promising explanation of the equity premium puzzle obviously depends on the plausibility of the condition on nonstockholder consumption growth. Suppose, as an extreme case of the literature on uninsurable idiosyncratic income shocks, that nonparticipants have chosen to stay out of the stock market because their idiosyncratic labor income is strongly positively correlated with stock returns and they face a short sales constraint on stocks[9]. Then consumption growth of nonstockholders could have a higher covariance with stock returns than consumption growth of stockholders. To preview the empirical results, I find that in the Consumer Expenditure Survey the covariance of consumption growth with stock returns is much lower for nonstockholders than for stockholders. This implies that labor income shocks of the above type are unlikely to be an important reason for nonparticipation.

Empirical papers on the determinants of stockownership find that the probability of stockownership is increasing in wealth, income and education[10]. The dependence of wealth and in-

[9] By definition, the idiosyncratic risk of all agents cannot be correlated with stock returns. It is possible, however, that the correlation is positive for one set of agents, and correspondingly negative for another set of agents.

[10] References include Mankiw and Zeldes (1991), Blume and Zeldes (1994) and Bertaut and Haliassos (1995) all using US data, and Arrondel and Masson (1986) using French data.

There is some evidence that uncertainty in income from nonfinancial sources decreases the probability of come is consistent with a fixed information cost of stock market entry which is identical across agents. A higher probability of stockownership for more educated individuals is consistent with education lowering the fixed cost. The results of Blume and Zeldes (1994) and Bertaut and Haliassos (1995) furthermore show that people who indicate they are willing to take average or above average risk are more likely to be stockholders. This points toward heterogeneous risk aversion as another dimension of heterogeneity which causes some but not others to enter the stock market (a fixed entry cost still needs to be present to explain zero and not just small stockholdings). Furthermore, the significance of wealth may be a reflection of different discount rates to the extent that more patient individuals will tend to have higher wealth at a given age.

Neither of these separating factors (heterogeneity in wealth, income, risk aversion or discount rates, each combined with the presence of a fixed cost of entry) invalidate the claim that nonstockholder consumption growth is likely to have a low conditional covariance with stock returns. In any economy in which stock market risk is the only type of uncertainty and in which this risk is not shared by labor, consumption growth of households holding only riskless bonds is conditionally riskless. It therefore has a conditional covariance with stock returns of zero, as needed for Result 1b and 2. For nonstockholder consumption growth to have a positive correlation with stock returns, it must be the case that labor income of nonstockholders is positively correlated with the stock market and nonstockholders face a short-sales constraint for stocks. Alternatively, additional sources of uncertainty must be present. Chapter 2 of this thesis provides an empirical analysis of the effect of nonfinancial income of portfolio choice using data from the PSID. I find only weak evidence of a negative effect of the covariance of nonfinancial income with stock returns on the optimal proportion of financial wealth invested in stocks. Strong effects of the mean and variance of nonfinancial income on portfolio choice is found but these effects do not invalidate the hypothesis of lower covariance of nonstockholder consumption growth with stock returns.

In the end it is an empirical question whether nonstockholder consumption growth covaries more or less with stock returns than stockholder consumption growth. This raises the question of whether the reason for nonparticipation matters for the validity of Euler equation estimations for stockholders and nonstockholders. For nonstockholders the Euler equation involving the

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stockownership.

stock return is by assumption not valid. The issue of validity concerns, for stockholders, the Euler equations for stocks and for bonds, and for nonstockholders the Euler equation for bonds.

The Euler equation for stocks (as well as all Euler equations involving the bond return) still holds with equality for agents who are stockholders in the presence of a fixed cost of stock market entry. Furthermore, if wealth heterogeneity is the main factor behind the choice to pay the fixed cost and enter the stock market or not, then all estimations of relative risk aversion coefficients outlined above are still valid. One of the attractive features of estimating Euler equations is precisely that one only need to assume that observed consumption data are generated by optimizing consumers facing frictionless markets. Information about endowments and income processes is not needed. This also implies that the Euler equation for bonds remain valid for nonstockholders if nonparticipation is due to particular income processes for nonstockholders.

If heterogeneity in discount rates is important, estimates of relative risk aversion based on the log-linearized model are still valid. Estimates of risk aversion based on the simple calibrations in 1.12 and 1.13 do not involve estimation of discount rates and are thus unaffected by heterogeneity in discount rates. Instrumental variables estimation of γ based on equations 1.10 and 1.11 is also essentially unaffected by heterogeneity in discount rates. This is because the discount factor is isolated in the constant term. When individual log-linearized Euler equations are summed across agents, as shown in 1.10 and 1.11 for the set of stockholders, the term $\ln\delta$ is replaced by an average of agents' log discount factors. This does not affect estimates of γ.

If risk aversion is heterogeneous, leading households with low risk aversion to be those who pay the fixed cost and enter the stock market, then complications arise. If long time series of data were available for each agent, Euler equations could be estimated for each stockholder and each nonstockholder to get consistent estimates of each agent's coefficient of relative risk aversion. However, in the Consumer Expenditure Survey used in the empirical section of the present chapter, each household is not observed for a sufficiently long period for this to be feasible. As will be explained below, a simple cohort technique must be applied. This involves using consumption growth observations for similar agents from earlier and later periods to obtain an estimate of risk aversion for a given type of agents. If risk aversion differs within the set of agents, the resulting estimate will be a (complicated) function of the risk aversion coefficients of the agents involved. This implies that to get a precise estimate of risk aversion, it is desirable to be able to identify groups of agents with similar values of risk aversion. Splitting agents into stockholders and nonstockholders by itself goes some way towards solving this problem. In addition, I split the set of stockholders into three layers by size of stockholdings. Aside from providing more precise estimates of risk aversion in the presence of risk aversion heterogeneity, this is of interest with respect to the issue of diversification. Suppose risk aversion is homogeneous but that an Euler equation involving the return on an aggregate stock market index is estimated for stockholders whose stock portfolio is not as diversified as the index. The result is likely to be an upward biased estimate of risk aversion. To the extent that wealthier stockholders are more diversified than less wealthy stockholders, risk aversion estimates will be less biased for richer layers of stockholders[11].

1.3 Empirical results based on the Consumer Expenditure Survey

1.3.1 Empirical strategy

The empirical analysis contains three parts. First, a calibration exercise corresponding to equation 1.12. Second, estimation of the linearized model in equations 1.10 and 1.11, using a linear GMM estimation approach. Third, a Hansen-Jagannathan bound analysis. Each of these parts emphasizes different features of the data. The calibration focuses on the unconditional equity premium. Instrumental variables estimation uses information about time-variation in expected consumption growth and asset returns. The Hansen-Jagannathan analysis focuses on the volatility of the stochastic discount factor proposed by the CCAPM. The null hypothesis is that the CCAPM is a satisfactory description of the equilibrium relation between consumption and asset return data once we focus on the consumption of stockholders. Under this hypothesis, the risk aversion estimates for stockholders obtained from each part should be identical.

For each of the three parts, the model is estimated/tested first for the set of all households in the sample, then for stockholders and nonstockholders separately and finally for the bottom, middle and top layer of stockholders ranked by size of stockholdings. I first perform

[11] Avery and Elliehausen (1986) and Blume and Zeldes (1994) find large differences in stock portfolio diversification across households and it is likely that richer households are the most diversified.

the analysis taking idiosyncratic consumption components (incomplete markets) into account and then repeat it under the assumption of a representative agent within each of the groups in focus. In the present data set I do not find any systematic correlations between cross-sectional variances of consumption growth rates and the stock return or the equity premium. Thus the calibration and the IV estimates should not be systematically affected if we ignore idiosyncratic components of consumption. The Hansen-Jagannathan bound (HJ bound) analysis is affected more dramatically by the presence of idiosyncratic consumption components and will depend crucially on whether or not we assume a representative agent within each group. The purpose of that part of the empirical analysis is therefore twofold. Firstly, conditional on acknowledging incomplete markets, does the stochastic discount factor based on the consumption of all agents enter the bound at a plausible value of risk aversion? If it does, then the rejection of the CCAPM based on HJ bound analysis is resolved. If not, then we must consider whether the stochastic discount factor based on stockholder consumption is more volatile, for given γ, than that based on nonstockholder consumption.

1.3.2 Data

The CEX data available cover the period 1980:1-1994:4. In each quarter approximately 5000 households are interviewed. Each household is interviewed five times, the first time is practice and the results are not in the data files. The interviews are three months apart and when interviewed households are asked to report consumption for the previous three months separately. Information about other variables is reported on a quarterly basis. Financial information is gathered in the fifth quarter only. The sample is representative of the US. population.

I define a household as a stockholder if an answer of more than one dollar was given to the question: 'Estimated market value of all stocks, bonds, mutual funds and other such securities held by CU on the last day of the previous month'. Since many households who do not hold stocks may hold positive amounts of bonds and non-equity mutual funds, some nonstockholders will unavoidably be classified as stockholders. In general, inability to perfectly identify stockholders and nonstockholders biases against finding differences in risk aversion estimates for the two groups. Attrition is quite substantial with only about half the households making it through all five quarters. I drop households who do not have a fifth interview. Otherwise stockholders who drop out of the sample before the fifth interview would implicitly be classified as nonstockholders.

As will be explained below I choose to match households to the previous quarter to define consumption growth by household. For a given quarter consumption growth observations will be available for households in their third, fourth, or fifth interview. To decide whether a consumption growth observation for a given month belongs to a stockholding household or not, households must be matched forward in time. This creates problems around 1985-86 since sample design and household identification numbers were changed with no records being kept of which new household identification numbers correspond to which old ones. For the first quarter of 1986 two files were created by the BLS. One based on the old sample design and one based on the new design. It is only possible to identify 49 percent of households in both files[12]. This implies, that the number of agents who can be matched forward to their fifth interview is lower for the fourth quarter of 1985 and the first quarter of 1986. As a result the number of observations of consumption growth is lower for these two quarters, since only households who can be identified as stockholders or nonstockholders are kept. In general, the number of observations per quarter varies throughout the sample which leads to heteroscedasticity. The estimations of the log-linearized Euler equations therefore accounts for arbitrary heteroscedasticity.

In addition to the split between stockholders and nonstockholders, the set of stockholders is split into three layers of approximately equal size based on dollar amounts reported. The bottom layer consists of those reporting stockholdings of \$2-\$3500 in real 1982-1984 dollars, using the CPI to deflate the nominal values[13]. The middle and top layers are those with real stockholdings of \$3500-\$20000 and above \$20000, respectively.

The consumption measure used is nondurables and services aggregated as carefully as possible from the disaggregate CEX consumption categories to match the definitions of nondurables and services in the NIPA. Nominal consumption values are deflated by the BLS deflator for nondurables. In leaving out durables, it is implicitly assumed that utility is separable in durables

[12]This is done by determining a number of variables which together uniquely identify a household within a given file and then merging the files by these variables. The precise list of variables used is available on request.

[13]For interviews conducted in the period 1991-1994, about 5 percent of households report stockholdings of \$1. I contacted the BLS regarding this problem. The reports of \$1 are occurances where the household reported owning securities, but did not report the value of these. Since these households cannot be classified by layer of stockholding, I chose to classify them as nonstockholders. Otherwise the results for the set of all stockholders and for the three layers of stockholders would not be comparable.

and nondurables/services. Following Dynarski and Gruber (1997) and Zeldes (1989) extreme outliers were dropped under the assumption that these reflected reporting or coding errors. Specifically, I dropped the bottom and the top percent of consumption growth observations for each month. Remaining consumption growth observations ranged from $C_{t+1}/C_t \simeq 0.2$ to $C_{t+1}/C_t \simeq 5$ within each month[14]. In addition, nonurban households and households residing in student housing were dropped as were households with incomplete income responses. The age group used was households whose head was at least 19 and at most 75 years. These exclusions are standard. More drastically, I drop all consumption observations involving 1980 and 1981, since several measures indicated low data quality in this first part of the survey. Attanasio and Weber (1995) discuss potential problems with the consumption data in the early years of the survey. Finally, I drop the last quarter of data, since for this quarter it is not possible to classify households who are not in their last interview into stockholders and nonstockholders. The final sample consists of 317492 monthly consumption growth observations and 285982 3-monthly consumption growth observations. The proportion of stockholders is 17.30 for the monthly data, and 17.74 for the 3-monthly data.

Monthly NYSE value weighted returns were used as the stock return measure and monthly T-bill returns as the measure of nominally riskless returns. The CPI for total urban consumption was used to calculate real returns. Returns for frequencies lower than a month were aggregated up from the real monthly returns. When considering relations between consumption and asset returns I assume consumption for a period takes place at the end of the period and use the corresponding asset return. For example, when considering consumption growth between March and April the asset return during April is used.

1.3.3 Econometric issues

Matching

With multiple consumption observations per household, household identification numbers can be used to match households across interviews thus exploiting the panel dimension of the CEX.

[14]For each quarter a small number of households reports positive consumption for the interview mouth. These expenditures were not used. Furthermore, between 0 and 5 percent of households report consumption for only 1 or 2 reference months. These households were dropped.

This raises two questions. First, since the log-linarized Euler equations do not contain any individual specific right hand side variables, it is not obvious that matching households across interviews to obtain household level consumption growth observations is beneficial. One could alternatively just sum cross-sectionally within each time period, effectively considering the set of all agents as one large cohort. Specifically, suppose the set of households in the sample was constant over time (i.e. that the data set was a long panel). Then for estimation of the log-linearized model and the calibration, it would not matter whether we matched across households. This is due to the fact that $\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right) = \left[\frac{1}{H^a}\sum_{h=1}^{H^a}\ln C_{t+1}^{h,a}\right] - \left[\frac{1}{H^a}\sum_{h=1}^{H^a}\ln C_t^{h,a}\right]$. However, with 25 percent of the sample being replaced each quarter, matching becomes important. Without matching, the extent to which the consumption of the new households differs from that of those who are no longer in the sample enters the estimation as an additional element of noise. I therefore prefer to match households across quarters. This leads to the second question. Given that four interviews are available for each household (aside from attrition), should a panel data strategy be pursued, or should the time series for $\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)$ (using households who can be matched between period t and t+1) be used? I prefer the latter for two reasons. Expectational errors are likely to be large in Euler equations involving the stock return. Since expectational errors have a large part which is common across agents, using the time series for $\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)$ instead of short panels is beneficial in that expectational errors are more likely to average out (Attanasio and Weber (1995) discuss this issue in more detail). Furthermore, the time series for $\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)$ has the advantage that multiplicative measurement errors tend to average out across agents. The measurement error issue is discussed in more detail in section 3.3.3.

Timing

The fact that households are interviewed every three months for a year, but in each interview report consumption for the previous three months separately leaves open a choice of data frequency for defining consumption growth rates. Let subscript t refer to month t. I consider two alternatives.

a) Monthly consumption growth rates: C_{t+1}/C_t

b) '3-monthly' consumption growth rates: C_{t+3}/C_t

For both alternatives track must be kept not only of when a household was interviewed but also to which month an expenditure pertains. Both methods generate three consumption growth observations per household interview. Using quarterly consumption growth rates, $(C_{t+3} + C_{t+2} + C_{t+1}) / (C_t + C_{t-1} + C_{t-2})$, was also considered. However, since this definition makes it harder to match a given expenditure with asset returns I decided to focus on case a) and b). Initial explorations for quarterly consumption growth rates, with family size controls, gave results fairly similar to alternative b)[15]. Since case b) involves overlapping time periods and thus overlapping expectational errors, the model implies MA(3) error terms in the regressions. In addition, measurement error generates an MA(3) component for case b), and an MA(1) component for case a).

Comparing to other papers using CEX data, Attanasio and Weber (1995) use only the consumption data from the last month before the interview. They do this to avoid complicating the error structure, but at the cost of a substantial loss of consumption information. Brav and Geczy (1996) use all consumption reported by a household for the quarter before the interview but do not consider when within the quarter the household was interviewed. Since the difference between the first and last expenditure reported by households interviewed in a given quarter is 5 months it seems important to try to match consumption and asset returns as closely as possible.

Measurement errors

As in all studies based on micro data the issue of measurement error arises. Although, to my knowledge, no validation study has been done for the Consumer Expenditure Survey we know from the PSID validation study that measurement errors in micro data can be large. Duncan and Hill (1985) find that 15 to 30 percent of the cross-sectional variation in earnings is measurement error. It is likely that people remember their earnings more accurately than their consumption resulting in larger measurement error for consumption. Again for the PSID, Runkle (1991) estimates that 75 percent of the part of consumption growth variation which is unexplained by family specific interest rates is noise. Although designed with the purpose of

[15]Note that a) and b) are both consistent with a decision frequency of one month. If this is a reasonable assumption then one needs to carefully consider time aggregation biases when estimating the model on quarterly data.

collecting consumption data, an indication of large measurement errors in the CEX consumption data is that the cross-sectional standard deviation of consumption growth drops by about 10 to 15 percent after introduction of a new questionnaire in April 1991.

The conditions under which consistent estimates of relative risk aversion can be obtained based on estimation of Euler equations are very strict. The measurement error in individual consumption must be multiplicative and independent of the true consumption level and asset returns[16][17]. This is the case whether we log-linearize the Euler equations or estimate them in the original non-linear form. To be specific, suppose we had a long time series of consumption observations for an agent h and wanted to test the CCAPM. Let $C_t^{h,*}$ be the true consumption of agent h at t and assume observed consumption is given by $C_t^h = C_t^{h,*}\varepsilon_t^h$, where ε_t^h is a measurement error. The true Euler equation is:

$$E_t\left[\delta\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma} R_{i,t+1}\right] = 1 \tag{1.14}$$

However, our estimates $\hat{\delta}$ and $\hat{\gamma}$ solve the empirical equivalent of:

$$E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h}}{C_t^{h}}\right)^{-\hat{\gamma}} R_{i,t+1}\right] = 1 \Leftrightarrow E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\hat{\gamma}} R_{i,t+1}\right] = 1 \tag{1.15}$$

If ε_{t+1}^h and ε_t^h are conditionally independent of C_{t+1}^h and C_t^h and $R_{i,t+1}$ 1.15 implies:

$$E_t\left[\left(\frac{\varepsilon_{t+1}^h}{\varepsilon_t^h}\right)^{-\hat{\gamma}}\right] E_t\left[\hat{\delta}\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\hat{\gamma}} R_{i,t+1}\right] = 1 \tag{1.16}$$

ε_t^h is unobservable and thus not included in the time t information set. ε_{t+1}^h and ε_t^h are bounded from below by zero under the reasonable assumption that no one reports negative consumption. Thus the first expectation is positive. From here observe that the estimate of

[16] Fortunately, introspection suggest that multiplicative measurement errors are more plausible than additive, since people are more likely to misreport their consumption by some (stochastic) fraction than to misreport it by the same dollar amount no matter how large the true level.

[17] For log-linearized Euler equations measurement errors must be uncorrelated wtih the true level of consumption, not necessarily independent.

δ will be inconsistent whereas γ will be consistently estimated. If measurement errors are lognormal, $\ln \varepsilon_t^h \sim N\left(\mu_\varepsilon, \sigma_\varepsilon^2\right) \forall t$, then our estimate of δ is inconsistent by the factor:

$$\frac{1}{E_t\left[\left(\frac{\varepsilon_{t+1}^h}{\varepsilon_t^h}\right)^{-\gamma}\right]} = \frac{1}{\exp\left(\gamma^2\left(\sigma_\varepsilon^2 - \sigma_{\varepsilon,\varepsilon}\right)\right)} \tag{1.17}$$

where $\sigma_{\varepsilon,\varepsilon}$ is the covariance of $\ln\varepsilon_{t+1}^h$ and $\ln\varepsilon_t^h$[18]. It follows that $\ln\delta$ in the log-linearized model will be inconsistent by the quantity $-\gamma^2(\sigma_\varepsilon^2 - \sigma_{\varepsilon,\varepsilon})$. If we do not have a long time series of consumption for each agent and instead aggregate over consumers within each period as in 1.11 the inconsistency remains the same unless measurement errors are correlated across people which is unlikely. If the choice is between a short panel and a time series of large cross-sections, the latter is preferable with respect to estimation of γ, since the effect of measurement errors then mainly affects the estimate of δ (this is a small sample argument unlike the above).

Even in the above case in which γ can be estimated consistently based on Euler equations, inference based on means and standard deviations of stochastic discount factors as in Hansen-Jagannathan bound analysis will be problematic. For the present case in which a long time series of consumption for each agent is not available and I instead aggregate over consumers within each period, the true stochastic discount factor is $M_{t+1}^* = E_h\left[\delta\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}\right]$ (for H$\rightarrow\infty$). With measurement error of the above type, we do not observe M_{t+1}^* but instead $M_{t+1} = E_h\left[\delta\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\gamma}\right]$. Assume again that ε_{t+1}^h and ε_t^h are both conditionally independent of $C_{t+1}^{h,*}$ and $C_t^{h,*}$ and that measurement errors and consumption growth rates are lognormal. Under these assumptions $M_{t+1} = E_h\left[\delta\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^h}{C_t^{h,*}\varepsilon_t^h}\right)^{-\gamma}\right] = E_h\left[\left(\frac{\varepsilon_{t+1}^h}{\varepsilon_t^h}\right)^{-\gamma}\right]M_{t+1}^*$. If furthermore the cross-sectional distribution of measurement errors equals the distribution of measurement errors for each agent, which seems plausible, then $E_h\left(\frac{\varepsilon_{t+1}^h}{\varepsilon_t^h}\right)^{-\gamma} = E\left(\frac{\varepsilon_t^h}{\varepsilon_{t+1}^h}\right)^{-\gamma} = \exp\left(\gamma^2\left(\sigma_\varepsilon^2 - \sigma_{\varepsilon,\varepsilon}\right)\right) \equiv F$. Both the unconditional mean and the unconditional standard deviation of M_{t+1} will be upward biased by this factor.

To sum up, under strong (but plausible) assumptions on measurement errors, γ can still be estimated consistently based on the calibration and the IV approach. Even non-linear GMM

[18] Under the assumption that the time t information set does not contain any in information about ε_t^h or ε_{t+1}^h, the conditional expectation equals the unconditional expectation.

estimation of γ is still consistent under the conditions stated above. Little can be said about the signs and magnitudes of the biases caused by measurement errors with different statistical properties than those assumed here. In addition, measurement errors even of the restrictive type analyzed here causes biases in estimates of means and standard deviations of stochastic discount factors based on micro data. I will attempt a correction for this based on the factor F. Interestingly, if we assume a representative agent within each of the sets of agents analyzed, then asymptotically as the number of agents in each group increases, the measurement errors cancel out (this is true for additive mean zero measurement errors and for multiplicative measurement errors which are independent of the true level of consumption). A correction for aggregation bias can then be considered.

Seasonality

Most of the empirical papers in the equity premium puzzle literature, including the previously mentioned paper by Brav and Geczy, use consumption data which are seasonally adjusted using the X-11 seasonal adjustment program or a similar method. An important exception is Ferson and Harvey (1992) who emphasize that since data adjusted by the X-11 method are weighted averages of past and, in revised data, future expenditures, this type of seasonal adjustment can induce spurious correlation between the error terms of a model and lagged values of the variables (e.g. spurious rejection of a model based on tests of overidentifying restrictions when the instruments include lagged consumption values). In addition the X-11 method changes the mean of the growth rate of a series which in the present context would cause biases in means of stochastic discount factors. More importantly, the economic model of seasonality used by Ferson and Harvey (1992) and several papers in the consumption literature (a multiplicative seasonal component in preferences) implies that seasonal adjustment by dummies is valid in log-linearized Euler equations.

Summary: Estimated relations

Thus the log-linearized Euler equations which are estimated in the present paper take the following form, with the stockholders' Euler equation for stock returns as an example:

$$\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)=\beta_0+\beta_1 D_1+...+\beta_{11}D_{11}+\frac{1}{\gamma}\ln R_{s,t+1}+u_t \tag{1.18}$$

where:

$$\begin{aligned}\beta_0 \equiv\ & \frac{1}{\gamma}\ln\delta+\frac{1}{2\gamma}V_t\ln R_{s,t+1}+\frac{1}{2}\gamma\frac{1}{H^a}\sum_{h=1}^{H^a}\left(V_t\ln\left(\frac{\varepsilon_{t+1}^{h,a}}{\varepsilon_t^{h,a}}\right)\right)\\ & +\frac{1}{2}\gamma\frac{1}{H^a}\sum_{h=1}^{H^a}\left(V_t\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)-cov_t\left(\ln R_{s,t+1},\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)\end{aligned}$$

$$u_t\equiv\frac{1}{\gamma}\left(E_t\ln R_{s,t+1}-\ln R_{s,t+1}\right)-\frac{1}{H^a}\sum_{h=1}^{H^a}\left[\left(E_t\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)-\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)\right] \tag{1.19}$$

If the conditional variances and covariances in the expression for the constant term are not constant, the stochastic components enter the error term. This does not cause problems for the estimation as long as these components are uncorrelated with the asset returns and the instruments used.

For each group of households, the Euler equation for the stock return and the Euler equation for the bond return are first estimated separately. The estimation method used is 2SLS modified to account for autocorrelated error terms of the MA(1) form for monthly data and the MA(3) form for 3-monthly data. Furthermore, I correct for heteroscedasticity of arbitrary form. Heteroscedasticity is likely to be present because of a varying number of observations per quarter. 2SLS estimation with corrections for serially correlated and/or autocorrelated error term is what is commonly referred to as optimal instrumental variables estimation. 2SLS is used rather than OLS because of endogeneity of asset returns. The Euler equations for stock and bond returns are then estimated jointly using 3SLS again modified to account for autocorrelated and heteroscedastic error terms. 3SLS is used to gain efficiency from exploiting the cross-equation correlation in error terms caused by correlated expectational errors. Furthermore, it allows us to impose identical values for $1/\gamma$ and the other parameters across equations and test if this leads to rejection of the model according to overidentification tests[19]. The esti-

[19] For simplicity I impose that all parameters be identical across the log-linearized Euler equations for the stock return and for the T-bill rate. β_0 in (3.5) differs slightly for the stock return and the bond return. The difference is given by $\frac{1}{2\gamma}V_t\ln R_{s,t+1}$-$cov\left(\ln R_{s,t+1},\frac{1}{H^a}\sum_{h=1}^{H^a}\ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)$, which is very close to zero for reasonable values

mators used here are GMM estimators. They simplify to 2SLS and 3SLS (with corrections for autocorrelation and heteroscedasticity) because the model is log-linearized. I will refer to this part of the empirical analysis the instrumental variables estimations.

It is important to emphasize that 1.18 implies that no seasonal adjustment is needed in equations based on the equity premium. Thus the equations for the calibration is, again with the relation for stockholders as the example:

$$\gamma = \frac{E\left(\ln R_{s,t+1} - \ln R_{f,t+1}\right) + \frac{1}{2} V \ln R_{s,t+1}}{cov\left(\ln R_{s,t+1}, E_{h,a} \ln\left(\frac{C_{t+1}^{h,a}}{C_t^{h,a}}\right)\right)}$$

1.3.4 Results

Calibrations

Table 1 shows the results of calibrations based on log-linearized Euler equations, corresponding to equations 1.12 and 1.13. For reference the two first rows show the results for US per capita nondurable consumption data. To avoid potential problems of incorrect seasonal adjustment the next two blocks show the calibration results for annual CEX data[20]. For comparison with Brav and Geczy (1996) these are also calculated with 91-94 left out and the 1980-81 data are included. The results show much lower relative risk aversion estimates for stockholders than for nonstockholders. For annual CEX data 1980-94 the estimate obtained for stockholders is 21.1 compared to 46.7 for nonstockholders. When the period 1991-94 is left out the estimate for stockholders is 13.1 compared to 25.1 for nonstockholders. Although the estimates are smaller than those obtained by Mankiw and Zeldes (1991) for the PSID, they remain large even for stockholders.

For monthly and 3-monthly data the differences between stockholders and nonstockholders are again clear and do not depend on whether a representative agent (complete markets) assumption is imposed. This is consistent with low correlations of stock returns with the cross sectional variance of consumption growth as will be discussed in more detail below. As men-

of γ.

[20] Annual CEX data were created under the assumption of a representative agent within each of the groups of households analyzed. This was done by simply summing all consumption reported for a given year by households in this group.

tioned earlier the multiplicative seasonal factor drops out when we focus on Euler equations for the equity premium. The results for monthly and 3-monthly data were quite similar when I used monthly dummies to seasonally adjust the CEX consumption data before doing the calibration. Most notably, with seasonally adjusted consumption data, the covariance of nonstockholder consumption growth with stock returns was even lower than the numbers shown, and occasionally even negative. For the PSID, Poterba and Samwick (1995) also found negative correlations of nonstockholder consumption growth and stock returns when they used the Skinner consumption index instead of only food consumption.

The pattern of gradually lower risk aversion estimates for richer stockholders apparent in the calibration results of Brav and Geczy is less clear in my results. This remains the case in the instrumental variables results described in the next section, and indicates that the differences in methodology between Brav and Geczy (1996) and the present analysis are important.

The last column in Table 2 shows bootstrap confidence intervals for the risk aversion estimates. The confidence intervals are very large. Essentially, this is because the denominator of the estimator of γ is the covariance of log consumption growth and the log stock return. If this covariance is small and imprecisely estimated the confidence interval for γ becomes large. This is consistent with the large standard errors for the correlation of stock returns and consumption growth rates found by Poterba and Samwick (1995). The problem is that only one of the restriction implied by the CCAPM is tested (the unconditional Euler equation for the equity premium) and that our sample consists of 13 years of data. Given the large measurement errors which are likely to be present in micro consumption data, this is not enough information to get a precise estimate of γ. For annual US data over the period 1889-1978, Kocherlakota (1996) shows that when the Euler equation is not linearized, values of γ less than about 8.5 are rejected[21].

[21] I repeated the test outlined in Kocherlakota (1996) for my data set under the assumption of a representative agent within each group of households. I calculated the test with a correction for MA(1) error terms for monthly data and MA(3) error terms for 3-monthly data. For monthly data, the smallest γ not rejected by the data is 30.6 for nonassetholders and 1.5 for assetholders. For 3-monthly data, the numbers are 26.5 and 14. I hesitate to interpret these numbers as evidence for differences between the two groups. The number of stockholders in my sample is much smaller than the number of nonstockholders which is likely to be important for this test.

Instrumental variables estimations

The results of instrumental variables estimations of the log-linearized model in equation 1.17 are shown in tables 2-7. The estimation methods are as described in section 3.3.5.

Table 2 shows the 2SLS estimation with corrections for heteroscedasticity and for autocorrelation of the MA(1) form. In the first column of the table, the instruments used are 12 seasonal dummies and lag 2-12 of the monthly log gross real riskless rate, the log gross real stock return and inflation (instrument set Z1). With potential autocorrelation of the MA(1) type, lag 1 of the variables are not valid instruments[22]. I considered using lagged consumption growth rates as instruments, but these variables had very low correlations with stock and T-bill returns.

The results are favorable to the limited participation theory. **For nonstockholders, $1/\gamma$ is estimated to be 0.0441 implying a γ estimate of around 20. For stockholders, however, the estimate of $1/\gamma$ is 0.4586 corresponding to a relative risk aversion of about 2.** The estimates of $1/\gamma$ vary a bit across layers of stockholders but are consistently higher than for nonstockholders. For the set of all stockholders the estimate of $1/\gamma$ is significantly different from zero at the 5 percent level. For nonstockholders and for the set of all households the estimates are insignificantly different from zero.

The remaining columns of the table show the effect of varying the instruments. Instrument set Z2 contains only those of the lags in instrument set Z1 which have a correlation of above 0.1 with the T-bill rate (one of the two variables being instrumented). Z3 is as Z1 but without the inflation lags, and Z4 furthermore leaves out the lagged stock returns. Z5 consist of the six lags of the T-bill rate which have the highest correlation with the current T-bill rate. The results for Z2-Z5 are consistent with those for Z1 although with some tendency for all the estimates of $1/\gamma$ to increase as still more instruments are left out[23].

Tables 3 and 4 show separate estimations for the riskless rate and for the stock return. The estimation method used is 2SLS with a correction for first order autocorrelation and for

[22]In their GMM estimation Brav and Geczy (1996) use once lagged values of consumption growth and the excess return on stocks as instruments. If the error terms are autocorrelated as I have argued, this causes potential biases in their results.

[23]This may be a reflection of weak instruments. For the case of a single equation estimated by 2SLS, it is known that if the explanatory power of the first stage is fixed while more instruments are added, the estimator will tend to be biased towards the biased and inconsistent OLS estimator. I am not aware of any paper which has analyzed if a similar result is true for 3SLS.

heteroscedasticity, i.e. optimal IV. The instruments are as for Table 4, except that for Table 4, Z2 and Z5 are based on correlations of instruments with the stock return. Differences between nonstockholders and stockholders are again apparent. The estimates of $1/\gamma$ are generally larger when the T-bill rate is used than when stock returns are used. However, the results for the riskless rate Euler equations are quite different across instrument sets and the standard errors are large. This is less true for the estimations based on stock returns, which most likely reflect the much larger variability of stock returns which better enables us to identify risk aversion.

The differences in estimates of $1/\gamma$ between stockholders and nonstockholders even for estimations based on the T-bill return is important. For one of the instrument sets the estimates of $1/\gamma$ for nonstockholders is negative. For 3-monthly data the estimate of risk aversion for nonstockholders is negative for 4 of 5 instrument choices, cf. Table 6. This indicates that fundamentally different factors are at play for nonstockholders causing the Euler equation for the riskless rate to be violated. Frequently binding borrowing constraints is one possibility. Throughout the theoretical and the empirical analysis I have assumed that no important market frictions in the form of transaction costs, borrowing constraints or short sales constraints are present. The presence of transactions costs will imply that Euler equations do not hold with equality in periods where agents trade. The presence of binding borrowing constraints for bonds (short sales constraints on stocks) would imply that the Euler equation involving the bond return (stock return) only hold with inequality.

The model is not rejected for any of the layers according to tests of the overidentifying restrictions. This could be interpreted as saying that limited stock market participation is not the solution to the previous rejections of the model due to predictable deviations from the Euler equation. The next draft of the paper will include lags of the dividend price ratio in the instrument set. This variable has been found to have strong predictive power for the equity premium (although in particular at longer horizons than monthly or quarterly data). It may also be a good instrument in terms of detecting predictable deviations from the Euler equations involving stock returns if any such are present, as I had expected would be the case for nonstockholders. As an alternative to the 2SLS estimations (with MA-corrections), I will furthermore consider LIML estimation since this is known to have better small sample properties when instruments are weak[24].

All estimations in Tables 2-7 were repeated under the representative agent assumption with results much like those obtained when taking cross-sectional consumption growth heterogeneity (incomplete markets) into account.

Overall the instrumental variables results support the hypothesis that differences between stockholders and nonstockholders in the (conditional) correlation of consumption growth with stock returns is an important part of the solution to the equity premium puzzle.

Why do the instrumental variables results look better for our hypothesis than the results of the calibration? The calibration focuses on the unconditional equity premium. Instrumental variables estimation uses information about time-variation in expected consumption growth and asset returns. Thus the fact that I obtain plausible risk aversion estimates from the IV estimations (for stock returns and for bond returns) when focusing on stockholders means that stockholders do in fact change their consumption decisions in accordance with their conditional expectation of asset returns. In other words, the CCAPM is an accurate description of stockholder optimization in a dynamic sense. The relation on which the calibration is based, was derived by subtracting the Euler equation for the bond return from that for the stock return and using the law of iterated expectations. In effect, the calibration identifies γ from the equity premium and the difference in the intercepts of the two log-linearized Euler equations. This is seen by comparing equations 1.8 and 1.10. In this static (unconditional) sense the CCAPM leads to less plausible risk aversion estimates even for stockholders.

It is important to determine whether violations of the log-normality assumption is the reason for differences in results between methods. GMM estimation of the original nonlinear Euler equations allows consideration of this. Unfortunately, the results of nonlinear GMM with only 13 years of monthly data are likely to be sensitive to the exact set of instruments chosen. I estimated the conditional Euler equation for the equity premium under the assumption of a

[24]First stage F-tests indicate problems of weak instruments for the stock return. p-values exceeded the 5 percent level for several of the instrument sets considered both for monthly and 3-monthly data.

representative agent within each group of households using nonlinear GMM:

$$E_t\left[\delta\left(\frac{E_h\left[C_{t+1}^h\right]}{E_h\left[C_t^h\right]}\right)^{-\gamma}(R_{s,t+1}-R_{f,t+1})\right]=0$$

The results are shown in Table 8. Consistent with the earlier assumption of multiplicative seasonality I do not seasonally adjust the consumption data. The instruments used are as in Table 2 and 5 except that the 12 monthly dummies in the instrument set is replaced by a constant. For 3-monthly data, risk aversion estimates for stockholders are consistently lower than for nonstockholders but the standard errors are large. For monthly data the results are sensitive to the choice of instruments. No firm conclusions on the importance of lognormality assumptions can be drawn. Interestingly, the pattern of gradually lower risk aversion estimates for richer stockholders apparent in the calibration results of Brav and Geczy is apparent in the nonlinear GMM results with much lower risk aversion estimates for the richest stockholders.

Hansen-Jagannathan bounds

The calibration results in Table 1 showed that assuming a representative agent within each group of households had little effect on the results. The same was the case for the instrumental variables estimations. Risk aversion estimates for nonstockholders and for the set of all agents are large whether a representative agent is assumed or whether cross-sectional differences in consumption growth are carefully accounted for.

This shows that higher variance of idiosyncratic income shocks when stock returns are low is not a plausible explanation of the equity premium puzzle, as had been suggested by Constantinides and Duffie (1996). Table 9 clarifies this point by showing the correlations between the cross-sectional variance of log consumption growth and stock returns for the CEX data set. The correlations are close to zero for all groups of households.

In this section I show that despite its lack of importance for explaining the equity premium puzzle, accounting for cross-sectional heterogeneity in consumption growth is crucial for Hansen-Jagannathan bound analysis. In essence, even if idiosyncratic income risk does not lead to higher correlation of individual consumption growth rates and stock returns, it still makes individual consumption growth rates much more volatile than the growth rate of aggregate consumption. This clearly illustrates that Hansen-Jagannathan bound analysis emphasizes a different feature of consumption and asset return data than Euler equation estimation or the simple calibration. Differences in volatilities of stochastic discount factors based on stockholder and nonstockholder consumption growth are found to be minor compared to the effects of accounting for cross-sectional differences in consumption growth rates within each group.

As shown by Campbell (1997), the stochastic discount factor which is based on an invalid complete markets assumption, $M_{t+1}^{RA} \equiv \delta\left(\frac{E_h[C_{t+1}^h]}{E_h[C_t^h]}\right)^{-\gamma}$, differs from the stochastic discount factor which is based on the cross-sectional average of investors' intertemporal marginal rates of substitution. The latter is identical to the stochastic discount factor M_{t+1}^T described in section 2 of this paper. If all households are stockholders then M_{t+1}^T is a valid stochastic discount factor[25]. Campbell (1997) clarifies the analysis of Constantinides and Duffie (1996) by showing that the two are related by $M_{t+1}^T/M_{t+1}^{RA} = exp\left(\frac{\gamma(\gamma+1)}{2}V_h\left[\Delta lnC_{t+1}^h\right]\right) \equiv K_{t+1}$. $V_h(\Delta \ln C_{t+1}^h)$ is the variance of the cross-sectional distribution of consumption growth[26]. Table 9 showed that the correlation of $V_h(\Delta \ln C_{t+1}^h)$ with the stock market return was close to zero. If $V_h(\Delta \ln C_{t+1}^h)$ and thus K_{t+1} is constant over time then the mean and the standard deviation of M_{t+1}^{RA} are biased down by a factor of 1/K.

One way of determining whether acknowledging incomplete markets is enough to get the stochastic discount factor based on the consumption of the total set of agents into the HJ-bound for reasonable γ, is to use aggregate US data and correct by the factor K. We need an estimate of $V_h(\Delta \ln C_{t+1}^h)$. For the CEX data the estimate is around $0.35^2 = 0.1225$ for both monthly and 3-monthly data. Suppose the number is the same for annual data. A large part of this is likely to be measurement error. To be very conservative assume 95 percent of the observed cross-sectional variance is measurement error. This implies an estimate of $V_h(\Delta \ln C_{t+1}^h)$ of about 0.006. Fig. 3 shows the standard HJ-bound result based on annual US data along with the results after aggregation correction by factor K. Surprisingly, the mean of the stochastic discount factor which acknowledge incomplete markets increases so fast in risk aversion that

[25]With limited stock market participation, one can think of this analysis as a derivation of the conditions under which a representative stockholder can be assumed in empirical analysis.

[26]The conditions which imply this relation are that:

a) The cross-sectional distribution of individual consumption is lognormal

b) The change from time t to t+1 in individual log consumption is cross-sectionally uncorrelated with the level of individual log consumption at t.

this stochastic discount factor does not enter the bound for any γ. For larger estimates of $V_h(\Delta \ln C^h_{t+1})$ the problem becomes even worse. If $V_h(\Delta \ln C^h_{t+1})$ and thus K_{t+1} is not constant over time but independent of the growth rate of average consumption, it can be shown that the correction to the standard deviation of M^{RA}_{t+1} is larger than the correction to the mean. However the correction then requires an estimate of the variance of K. Obtaining a such estimate based on the CEX was considered too ambitious given the importance of measurement error for this quantity.

With micro data it is possible to determine the effect of idiosyncratic consumption components without making any assumptions about the distribution of income shocks, since the aggregation can be controlled directly. Therefore, as an alternative to correcting the aggregate US data, I constructed the 'carefully aggregated' stochastic discount factors based on the CEX micro data. Unfortunately, as outlined in the section on measurement error this is not without problems either, since fairly restrictive log normality assumptions must be imposed to derive a correction for measurement error. I do, however, avoid assumptions about the permanence of income shocks. The results are shown in Fig. 4, 5 for monthly data and Fig. 8, 9 for 3-monthly data. For the measurement error correction, I assume that 80 percent of the cross-sectional variance of consumption growth is measurement error. Even with this correction, the stochastic discount factors remain far to the right of the bound even for low values of γ. This confirms the results based on aggregate US data.

Differences between stockholders and nonstockholders are small. For each month I have approximately 360 consumption growth observations for stockholders and 6 times as many for nonstockholders, implying that measurement errors are more likely to cancel out for nonstockholders. To avoid any such effects, the figures are based on 360 randomly sampled consumption growth observations for nonstockholders for each month.

I conclude that accounting for cross-sectional differences in consumption growth dramatically affects the means and standard deviations of stochastic discount factors. The effect on the mean of the stochastic discount factor was to be expected given the success of the incomplete markets literature in explaining the low level of the riskless rate. It is likely that the restrictive distributional assumptions needed to derive either an aggregation correction for aggregate data or a measurement error correction for micro data are what cause the mean to increase so fast in γ that the stochastic discount factors never enter the bound[27].

The HJ bound analysis clearly shows differences among the three layers of stockholders. The higher the stockholdings, the more volatile the stochastic discount factor, cf. Fig. 5 and 9. This is intuitive. If I assume a representative agent within each layer of stockholders as shown in Fig. 2, 3, 6 and 7, then the stochastic discount factor enter the bound at lower values of γ the richer the layer. However, correction for aggregation based on the κ factor (not shown) again takes us left of the HJ bounds.

1.4 Conclusion

This chapter has demonstrated that limited stock market participation should be considered an important part of the solution to the equity premium puzzle. Section 2 showed that under the condition that the conditional correlation of nonstockholder consumption growth with stock returns is zero, estimation of Euler equations involving stock returns without excluding nonstockholders will result in an upward biased estimate of relative risk aversion. The bias is given by the factor $1/\lambda$, where λ is the fraction of stockholders in the population.

This hypothesis was tested using micro consumption data from the Consumer Expenditure Survey. Differences in risk aversion estimates for stockholders and nonstockholders are large, although the results differ somewhat across estimation methods. A negative result from the estimations is that the test of overidentifying restrictions did not reject the Euler equation involving the stock return for nonstockholders. The empirical section furthermore emphasized that Hansen-Jagannathan bound analysis focuses on a different feature of consumption and asset return data than Euler equation estimation or the simple calibration (consumption growth volatility rather than correlation of consumption growth with stock returns). I showed that despite its lack of importance for explaining the equity premium puzzle, accounting for cross-sectional heterogeneity in consumption growth within the set of stockholders was more important for the HJ bound analysis than the distinction between stockholders and nonstockholders.

[27] I have not considered standard errors for either the bound nor the means and standard deviations of the stochastic discount factor. One could consider a formal statistical test based eg. on the vertical distance from the bound as in Burnside (1994). Imposing positivity constraints on the bound could also be considered.

Table 1. Calibration of γ for log linear model. Bootstrap confidence intervals. Real value weighted NYSE returns and real T-bill returns.

Data	Group	Cov(ln(Rs), Δln(C))	Corr(ln(Rs),Δln(C))	γ	Bootstrap confidence interval
US annual, 1930-96		0.0035	0.573	20.7	[-6.75;48.14]
US quarterly, 1947:2-1996:4		0.00027	0.082	72.3	[-708.6;1221.6]
CEX annual, 1980-94	All	0.00161	0.536	36.5	[-315.6 ; 407.0]
Representative agent assumption	Stockholders	0.00279	0.641	21.1	[-4.3 ; 202.5]
	Nonstockholders	0.00126	0.438	46.7	[-541.1 ; 607.7]
	Bottom	0.00151	0.328	39.0	[-416.9 ; 359.4]
	Middle	0.00500	0.563	11.8	[-0.7 ; 88.3]
	Top	0.00214	0.219	27.5	[-204.9 ; 339.4]
CEX annual, 1980-90	All	0.00240	0.630	21.5	[-269.1 ; 293.5]
Representative agent assumption	Stockholders	0.00394	0.724	13.1	[-10.4 ; 187.7]
	Nonstockholders	0.00201	0.573	25.1	[-299.2 ; 487.4]
	Bottom	0.00178	0.368	29.0	[-622.9 ; 865.3]
	Middle	0.00679	0.663	7.6	[-4.33 ; 103.6]
	Top	0.00356	0.285	14.5	[-195.6 ; 138.8]
CEX monthly, 1982-94	All	0.00014	0.072	54.1	[-784.3 ; 711.7]
No representative agent assumption	Stockholders	0.00033	0.130	22.9	[-136.3 ; 253.4]
	Nonstockholders	0.00010	0.054	74.9	[-882.4 ; 619.7]
	Bottom	0.00032	0.111	23.6	[-214.3 ; 265.8]
	Middle	0.00038	0.150	19.7	[-662.1 ; 235.7]
	Top	0.00029	0.084	25.8	[-177.9 ; 538.1]
CEX monthly, 1982-94	All	0.00017	0.007	42.9	[-518.5 ; 492.5]
Representative agent assumption	Stockholders	0.00043	0.144	17.3	[-59.6 ; 153.2]
	Nonstockholders	0.00010	0.042	77.2	[-683.8 ; 512.2]
	Bottom	0.00032	0.094	23.5	[-225.8 ; 241.3]
	Middle	0.00052	0.169	14.4	[-77.9 ; 100.6]
	Top	0.00041	0.095	18.3	[-167.4 ; 191.8]

Formula: $\gamma=(E[\ln(R_{s,t+1})-\ln(R_{f,t+1})]+0.5V(\ln(R_{s,t+1})))/Cov(\ln(R_{s,t+1}),\ln(C_{t+1})-\ln(C_t))$.

US quarterly data seasonally adjusted by the BEA.

CEX quarterly and monthly data not seasonally adjusted (seasonal adjustment with 12 seasonal dummies gives similar results).

The bootstrap results are based on 1000 replications. The bootstrap does not take serial correlation of observations into account.

Table 1, continued

Data	Group	Cov(ln(Rs),Δln(C))	Corr(ln(Rs),Δln(C))	γ	Confidence interval
CEX 3-monthly, 1982-94	All	0.00028	0.074	81.4	[-1297.0 ; 705.9]
No representative agent assumption	Stockholders	0.00048	0.101	48.1	[-721.9 ; 548.1]
	Nonstockholders	0.00024	0.065	95.1	[-857.6 ; 675.4]
	Bottom	0.00032	0.066	72.1	[-524.9 ; 615.3]
	Middle	0.00065	0.121	35.7	[-349.0 ; 314.3]
	Top	0.00038	0.063	60.1	[-388.2 ; 544.3]
CEX 3-monthly, 1982-94	All	0.00025	0.051	93.6	[-686.7 ; 688.2]
Representative agent assumption	Stockholders	0.00059	0.101	39.0	[-260.8 ; 533.7]
	Nonstockholders	0.00014	0.031	160.4	[-1234.7 ; 1008.5]
	Bottom	0.00033	0.058	69.7	[-611.7 ; 386.1]
	Middle	0.00089	0.141	25.8	[7.5; 177.8]
	Top	0.00032	0.041	71.6	[-481.3 ; 349.9]

Formula: $\gamma=(E[\ln(R_{s,t+1})-\ln(R_{f,t+1})]+0.5V(\ln(R_{s,t+1})))/Cov(\ln(R_{s,t+1}),\ln(C_{t+1})-\ln(C_t))$.
US quarterly data seasonally adjusted by the BEA.
CEX quarterly and monthly data not seasonally adjusted (seasonal adjustment with 12 seasonal dummies gives similar results).
The bootstrap results are based on 1000 replications. The bootstrap does not take serial correlation of observations into account.

Table 2. 3SLS with MA(1) correction, monthly data, 1982-94, real value weighted NYSE return and real T-bill return. No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 47.12)	1/γ (std.)	Over-ident. test (crit.value 31.41)	1/γ (std.)	Over-ident. test (crit.value 33.93)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 12.59)
All	0.0933 (0.0784)	22.67	0.1611 (0.1003)	15.69	0.1580 (0.0924)	14.18	0.2002 (0.1365)	8.09	0.1800 (0.2112)	1.49
Stock-holders	0.4586 (0.1540)	21.73	0.6248 (0.2023)	13.16	0.5038 (0.1888)	14.79	0.8270 (0.2904)	5.87	0.9278 (0.4681)	3.32
Non-stock-holders	0.0441 (0.0751)	21.77	0.0936 (0.0993)	17.60	0.1006 (0.0851)	14.29	0.1073 (0.1186)	9.48	0.0817 (0.1931)	1.15
Bottom layer	0.4693 (0.2370)	35.59	0.7111 (0.2938)	28.17	0.4420 (0.2580)	27.77	1.1670 (0.4478)	14.97	1.5281 (0.6444)	5.50
Middle layer	0.7366 (0.2203)	25.36	0.8851 (0.2824)	18.95	0.6927 (0.2629)	15.04	0.9562 (0.4112)	3.67	0.9645 (0.6473)	1.74
Top layer	0.1353 (0.3328)	25.70	0.5047 (0.3945)	19.24	0.2246 (0.4130)	16.21	0.4942 (0.5538)	3.61	0.6770 (0.8832)	1.90

Note: Z1: 12 monthly dummies and lags 2-12 of monthly log gross real T-bill returns, log gross real value weighted NYSE stock returns and inflation.
Z2: 12 monthly dummies and lags 2-12 of monthly log gross real T-bill returns, lags 4-7 of monthly log gross real stock returns, and lags 3-9 and 11-12 of monthly inflation. All instruments in Z2 have correlations of 0.1 or more with the seasonally adjusted log gross T-bill return (seasonal adjustment by 12 monthly dummies).
Z3: As Z1 but with no inflation instruments. Z4: As Z1 but with no inflation or stock return instruments.
Z5: 12 monthly dummies and the six lags of log gross real T-bill return which have the highest correlation with the seasonally adjusted log gross real T-bill return, i.e. lags 2, 7, 8, 9, 10, and 11.
Standard errors and overidentification tests calculated with adjustment for MA(1) error terms. 5 percent critical values used.

Table 3. 2SLS with MA(1) correction, monthly data, 1982-94, real T-bill return. No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 47.12)	1/γ (std.)	Over-ident. test (crit.value 31.41)	1/γ (std.)	Over-ident. test (crit.value 33.93)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 12.59)
All	0.3228 (0.3450)	22.89	0.2829 (0.3690)	17.81	0.2783 (0.4676)	16.78	-0.0469 (0.6124)	10.35	0.3868 (0.6273)	2.00
Stock-holders	0.6170 (0.6987)	24.61	0.3052 (0.7609)	17.32	0.7757 (0.9646)	18.77	0.8164 (1.1397)	11.45	1.2025 (1.1418)	6.29
Non-stock-holders	0.2458 (0.3240)	21.45	0.1648 (0.3469)	18.71	0.1975 (0.4496)	15.25	-0.2501 (0.5951)	10.25	0.2744 (0.6238)	1.17
Bottom layer	0.6542 (0.9657)	34.22	0.2743 (1.0436)	28.61	0.9308 (1.1870)	26.80	0.5923 (1.4737)	17.24	0.9719 (1.4638)	7.33
Middle layer	1.1952 (1.2641)	27.55	0.5035 (1.3742)	22.30	0.3583 (1.5371)	18.06	0.0289 (1.8287)	7.82	0.9529 (1.9140)	3.66
Top layer	0.0286 (1.4669)	24.09	0.9384 (1.6194)	17.24	1.0132 (1.9249)	15.66	2.0893 (2.2286)	3.59	2.4695 (2.4053)	1.39

Note: As for Table 2.

Table 4. 2SLS with MA(1) correction, monthly data, 1982-94, real value weighted NYSE return. No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 47.12)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 33.93)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 12.59)
All	0.0412 (0.0378)	22.59	0.0886 (0.0554)	7.78	0.0812 (0.0463)	13.92	0.1161 (0.0706)	8.00	0.1489 (0.0881)	4.61
Stock-holders	0.2199 (0.0701)	21.16	0.1767 (0.0996)	9.01	0.2354 (0.0935)	14.73	0.4228 (0.1488)	5.59	0.3533 (0.1570)	3.75
Non-stock-holders	0.0235 (0.0366)	21.41	0.0810 (0.0551)	5.94	0.0563 (0.0423)	13.98	0.0684 (0.0614)	9.46	0.1204 (0.0827)	4.57
Bottom layer	0.2167 (0.1118)	32.46	-0.0485 (0.1491)	16.61	0.2079 (0.1260)	24.76	0.5711 (0.2238)	13.68	0.2947 (0.2265)	9.61
Middle layer	0.3341 (0.1071)	25.23	0.3382 (0.1816)	3.38	0.3217 (0.1311)	14.98	0.5232 (0.2156)	3.46	0.5545 (0.2728)	0.99
Top layer	0.0646 (0.1517)	24.24	0.1800 (0.2313)	7.62	0.1093 (0.1976)	14.94	0.2630 (0.2678)	3.64	0.2866 (0.2785)	2.17

Note: Z1: 12 monthly dummies and lags 2-12 of monthly log gross real T-bill returns, log gross real value weighted NYSE stock returns and inflation.

Z2: 12 monthy dummies and lags 4, 6-7 and 10 of monthly log gross real T-bill returns, lags 4, 5, 8, 10, 12 of monthly log gross real stock returns, and lags 4 and 7 of monthly inflation. All instruments in Z2 have correlations of 0.1 or more with the seasonally adjusted log gross real value weighted NYSE return (seasonal adjustment by 12 monthly dummies).

Z3: As Z1 but with no inflation instruments

Z4: As Z1 but with no inflation or stock return instruments.

Z5: 12 monthly dummies and the six lags of log gross real T-bill return which have the highest correlation with the seasonally adjusted log gross real value weighted NYSE return, ie. lags 4, 6-10.

Standard errors and overidentification tests calculated with adjustment for MA(1) error terms.

Table 5. 3SLS with MA(3) correction, 3-monthly data, at monthly frequency, 1982-94. Real value weighted NYSE return real T-bill return. No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 21.03)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 15.51)	1/γ (std.)	Over-ident. test (crit.value 9.49)	1/γ (std.)	Over-ident. test (crit.value 5.99)
All	0.2197 (0.0797)	14.09	0.2321 (0.0770)	14.02	0.1836 (0.0909)	12.59	0.3154 (0.1140)	9.55	0.0362 (0.1557)	1.74
Stock-holders	0.3785 (0.1229)	12.59	0.3937 (0.1347)	12.48	0.3338 (0.1218)	11.78	0.5378 (0.1952)	4.91	0.2713 (0.2028)	1.62
Nonstock-holders	0.1658 (0.0899)	12.85	0.1806 (0.0877)	12.73	0.1422 (0.0986)	11.28	0.2600 (0.1248)	9.47	0.0007 (0.1622)	1.54
Bottom layer	0.4554 (0.1100)	13.06	0.4677 (0.1115)	12.52	0.4328 (0.1303)	11.75	0.5710 (0.2281)	3.03	0.3608 (0.2543)	2.02
Middle layer	0.1829 (0.1814)	11.34	0.1847 (0.1856)	11.36	0.1935 (0.2178)	7.52	0.0873 (0.2396)	5.86	-0.1957 (0.3055)	0.42
Top layer	0.3718 (0.1834)	10.32	0.4578 (0.2345)	9.52	0.3773 (0.1703)	8.48	0.9655 (0.4087)	2.33	0.6584 (0.4577)	0.14

Note: Z1: 12 monthly dummies and lags 4,7,10,13 of quarterly log gross real T-bill returns, quarterly log gross real quarterly NYSE stock returns and quarterly inflation.
Z2: As Z1 except for dropping lag 10 of the stock return. All instruments in Z2 have correlations of 0.1or more with the log gross real quarterly T-bill return.
Z3: As Z1 but with no inflation instruments.
Z4: As Z1 but with no inflation or stock return instruments.
Z5: 12 monthly dummies and the two lags of quarterly log gross real T-bill return which have the highest correlation with the seasonally adjusted quarterly log gross real T-bill return, i.e. lags 7 and 10.
Standard errors and overidentification tests calculated with adjustment for MA(3) error terms.

Table 6. 2SLS with MA(3) correction, 3-monthly data, at monthly frequency, 1982-94, real T-bill return. No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 21.03)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 15.51)	1/γ (std.)	Over-ident. test (crit.value 9.49)	1/γ (std.)	Over-ident. test (crit.value 5.99)
All	-0.0525 (0.3399)	14.42	-0.0528 (0.3399)	14.37	0.0408 (0.3990)	12.31	-0.3223 (0.4984)	10.30	0.1790 (0.5046)	1.71
Stock-holders	0.5880 (0.5147)	14.42	0.5912 (0.5143)	14.40	1.0506 (0.5214)	12.40	0.8079 (0.6494)	9.62	1.0525 (0.6472)	1.22
Nonstock-holders	-0.1438 (0.3275)	13.44	-0.1427 (0.3276)	13.40	-0.1555 (0.4156)	11.24	-0.4805 (0.5093)	9.62	0.0263 (0.5269)	1.65
Bottom layer	1.0718 (0.5812)	16.73	0.9669 (0.5905)	15.55	1.6240 (0.6699)	12.86	0.7548 (0.8183)	8.00	1..3588 (0.8576)	1.26
Middle layer	0.0805 (0.7171)	11.54	0.0882 (0.7222)	11.54	0.0010 (0.8562	8.45	-0.7901 (0.9726)	5.04	-0.6199 (0.9730)	0.54
Top layer	0.4638 (0.7983)	14.96	0.3836 (0.8735)	14.91	1.0629 (0.8484)	12.24	2.0482 (1.2176)	8.64	2.3043 (1.2302)	0.04

Note: As for Table 5.

Table 7. 2SLS with MA(3) correction, 3-monthly data, at monthly frequency, 1982-94, real value weighted NYSE return No representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	1/γ (std.)	Over-ident. test (crit.value 21.03)	1/γ (std.)	Over-ident. test (crit.value 19.68)	1/γ (std.)	Over-ident. test (crit.value 15.51)	1/γ (std.)	Over-ident. test (crit.value 9.49)	1/γ (std.)	Over-ident. test (crit.value 5.99)
All	0.1227 (0.0405)	13.53	0.1126 (0.0470)	10.13	0.1032 (0.0472)	12.37	0.1900 (0.0603)	8.56	0.01946 (0.0906)	1.73
Stock-holders	0.2083 (0.0674)	12.41	0.1947 (0.0755)	8.16	0.1811 (0.0666)	11.62	0.3201 (0.1146)	4.17	0.1562 (0.1173)	1.61
Nonstock-holders	0.0936 (0.0458)	12.25	0.0929 (0.0529)	8.94	0.0821 (0.0513)	11.01	0.1619 (0.0652)	8.63	-0.0006 (0.0946)	1.50
Bottom layer	0.2469 (0.0600)	12.10	0.2646 (0.0834)	4.79	0.2394 (0.0701)	10.87	0.3300 (0.1291)	2.48	0.2150 (0.1504)	2.01
Middle layer	0.0976 (0.1017)	11.55	0.0100 (0.1175)	2.81	0.1120 (0.1209)	7.46	0.0634 (0.1331)	5.90	-0.1148 (0.1771)	0.39
Top layer	0.2079 (0.0991)	10.20	0.2940 (0.1386)	6.02	0.2081 (0.0892)	8.10	0.5635 (0.2309)	1.76	0.3842 (0.2798)	0.15

Note: Z1: 12 monthly dummies and lags 4,7,10,13 of quarterly log gross real T-bill returns, quarterly log gross real quarterly NYSE stock returns and quarterly inflation.

Z2: As Z1 except for dropping lag 10 of the stock return. All instruments in Z2 have correlations of 0.1 or more with the seasonally adjusted quarterly log gross real value weighted NYSE return (seasonal adjustment by 12 monthly dummies).

Z3: As Z1 but with no inflation instruments.

Z4: As Z1 but with no inflation or stock return instruments.

Z5: 12 monthly dummies and the two lags of quarterly log gross T-bill return which have the highest correlation with the seasonally adjusted quarterly log gross real value weighted NYSE return, ie. lags 7 and 10.

Standard errors and overidentification tests calculated with adjustment for MA(3) error terms.

Table 8a. GMM estimation with MA(1) correction, monthly data, 1982-94. Representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	γ (std.)	Over-ident. test (crit.value 47.12)	γ (std.)	Over-ident. test (crit.value 31.41)	γ (std.)	Over-ident. test (crit.value 33.93)	γ (std.)	Over-ident. test (crit.value 19.68)	γ (std.)	Over-ident. test (crit.value 12.59)
All	22.97 (8.50)	31.23	14.87 (10.56)	23.55	30.66 (12.69)	22.72	24.77 (18.17)	14.62	33.52 (21.64)	7..67
Stock-holders	8.70 (4.05)	31.08	13.74 (4.64)	22.21	11.46 (5.98)	23.96	15.62 (7.20)	14.77	19.24 (7.52)	4.05
Non-stock-holders	19.86 (8.39)	31.64	5.24 (9.91)	23.91	26.14 (11.36)	21.97	13.43 (19.53)	14.56	16.31 (22.89)	8.62
Bottom layer	8.03 (4.05)	30.76	6.48 (4.51)	24.52	10.34 (5.09)	22.80	8.55 (5.71)	13.18	9.88 (6.27)	8.29
Middle layer	4.97 (2.36)	31.52	4.22 (2.62)	23.66	5.87 (3.30)	23.77	6.62 (4.33)	15.54	12.59 (5.00)	3.26
Top layer	0.43 (1.92)	31.15	2.11 (2.32)	23.33	2.05 (3.72)	24.02	1.64 (6.01)	15.47	2.59 (6.08)	7.25

Note: Z1: Constant and lags 2-12 of monthly log gross real T-bill returns, monthly log gross real NYSE stock returns and inflation.

Z2: Constant and lags 2-12 of monthly log gross real T-bill returns, lags 4-7 of monthly log gross real stock returns, and lags 3-9 an 11-12 of monthly inflation. All instruments in Z2 have correlations of 0.1 or more with the seasonally adjusted monthly log gros real T-bill return (seasonal adjustment by 12 monthly dummies).

Z3: As Z1 but with no inflation instruments. Z4: As Z1 but with no inflation or stock return instruments.

Z5: Constant and the six lags of log gross real T-bill return which have the highest correlation with the seasonally adjusted monthly log gross real T-bill return, i.e. lags 2, 7, 8, 9, 10, and 11.

Standard errors and overidentification tests calculated with adjustment for MA(1) error terms.

Consumption data not seasonally adjusted. No seasonal dummies in the instrument sets.

Table 8b. GMM estimation with MA(3) correction, 3-monthly data, 1982-94. Representative agent assumption.

	Instrument set Z1		Instrument set Z2		Instrument set Z3		Instrument set Z4		Instrument set Z5	
	γ (std.)	Over-ident. test (crit.value 47.12)	γ (std.)	Over-ident. test (crit.value 31.41)	γ (std.)	Over-ident. test (crit.value 33.93)	γ (std.)	Over-ident. test (crit.value 19.68)	γ (std.)	Over-ident. test (crit.value 12.59)
All	37.48 (11.65)	18.29	36.71 (11.61)	17.19	37.77 (11.97)	11.99	56.00 (13.94)	5.62	54.52 (15.81)	0.06
Stock-holders	18.13 (6.78)	17.37	19.34 (6.89)	16.05	23.18 (7.67)	9.06	31.45 (9.58)	5.06	32.85 (9.74)	0.14
Non-stock-holders	34.93 (12.52)	19.09	33.11 (12.49)	18.19	35.29 (13.01)	12.87	59.36 (15.24)	6.34	58.12 (16.05)	0.39
Bottom layer	13.81 (8.22)	18.03	12.99 (8.34)	16.80	17.32 (8.63)	8.54	16.78 (11.33)	8.16	20.81 (12.05)	3.95
Middle layer	10.67 (5.05)	15.60	11.68 (5.07)	14.07	14.70 (5.33)	8.18	25.94 (6.07)	0.67	23.37 (7.37)	0.04
Top layer	2.20 (5.22)	18.09	2.93 (5.28)	16.06	8.06 (6.40)	9.29	8.82 (9.54)	9.07	6.55 (11.55)	7.23

Note: Z1: Constant and lags 4, 7, 10, 13 of quarterly log gross real T-bill returns, quarterly log gross real NYSE stock returns and quarterly inflation

Z2: As Z1 except for dropping lag 10 of the real stock return. All instruments in Z2 have correlations of 0.1 or more with the seasonally adjusted quarterly log gross real T-bill return (seasonal adjustment by 12 monthly dummies).

Z3: As Z1 but with no inflation instruments. Z4: As Z1 but with no inflation or stock return instruments.

Z5: Constant and the two lags of quarterly log gross real T-bill return which have the highest correlation with the seasonally adjusted quarterly log gross real T-bill return, i.e. lags 7 and 10.

Standard errors and overidentification tests calculated with adjustment for MA(3) error terms.

Consumption data not seasonally adjusted. No seasonal dummies in the instrument sets.

Table 9. Correlations of log asset returns with the cross-sectional standard deviation of log consumption growth, CEX data.

Period and data frequency	Asset return	Category of households for which the cross-sectional standard deviation is calculated					
		All	Stockholders	Nonstockholders	Bottom layer	Middle layer	Top layer
1982-1994, monthly data	Stock return	0.024	0.043	0.019	0.072	0.033	0.013
	Equity premium	0.013	0.030	0.008	0.064	0.016	0.000
1982-1994, 3-monthly data	Stock return	-0.054	-0.028	-0.056	-0.139	-0.040	0.056
	Equity premium	-0.084	-0.057	-0.086	-0.159	-0.064	0.022

Note: The stock return is the real NYSE value weighted return, the nominally riskless return is the real return on T-bills. The returns are logs of gross returns and the equity premium is the difference between the log stock and the log bond return. For the CEX data, refer to section 3 of the paper. 3-monthly data means C_{t+3}/C_t where t denotes month t.

Fig.1. H–J bound. Annual US data, NYSE and T-bill returns, 1930–1996, Vh=0.006

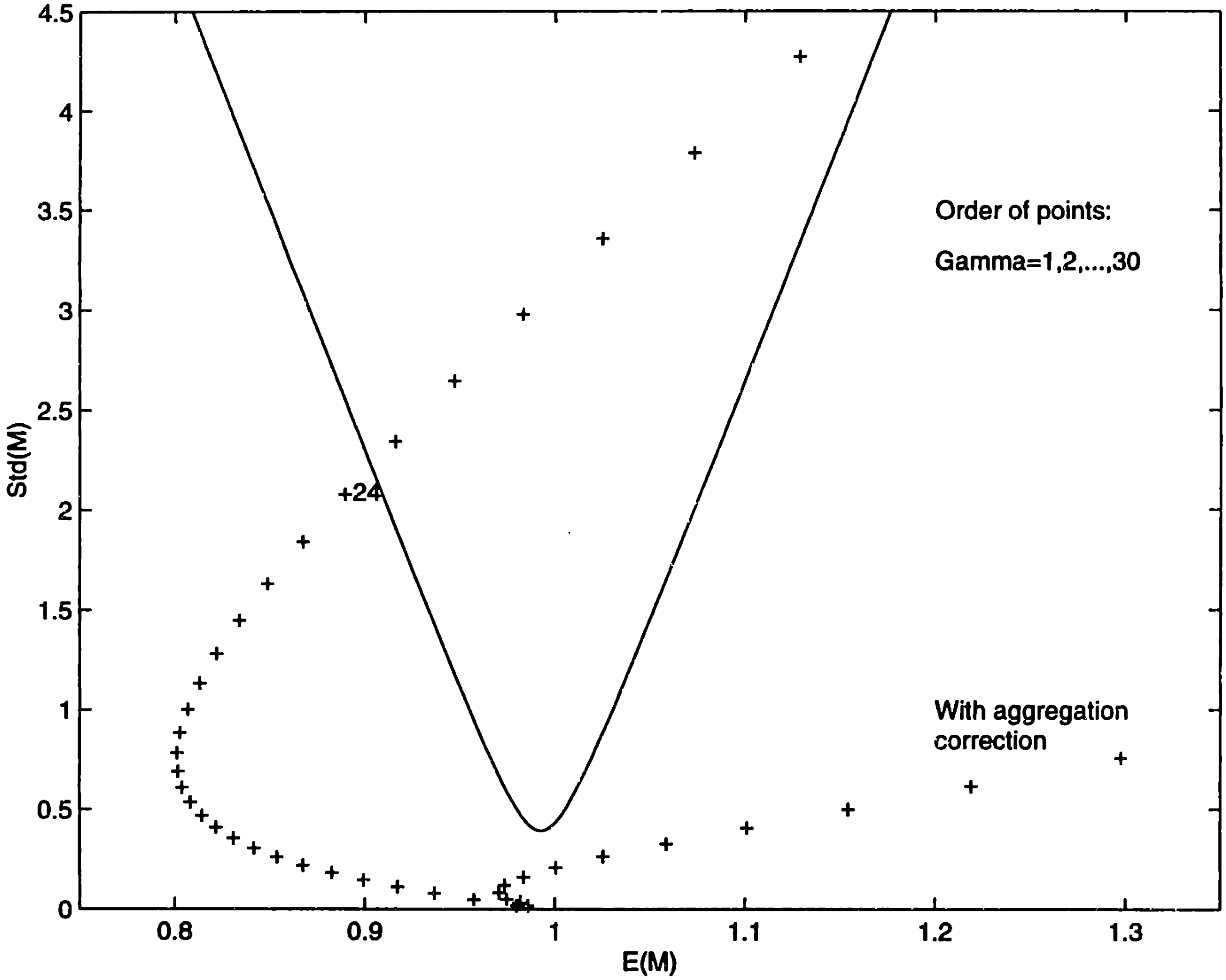


Fig.2. H-J bound. CEX monthly data, NYSE and T-bill returns, 1982-1994. Repr. agent assumed

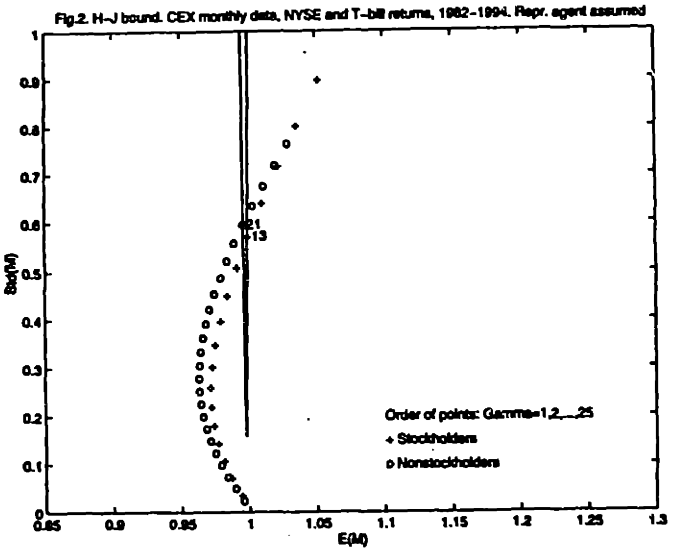


Fig.3. As Fig.2 but for 3 layers of stockholders

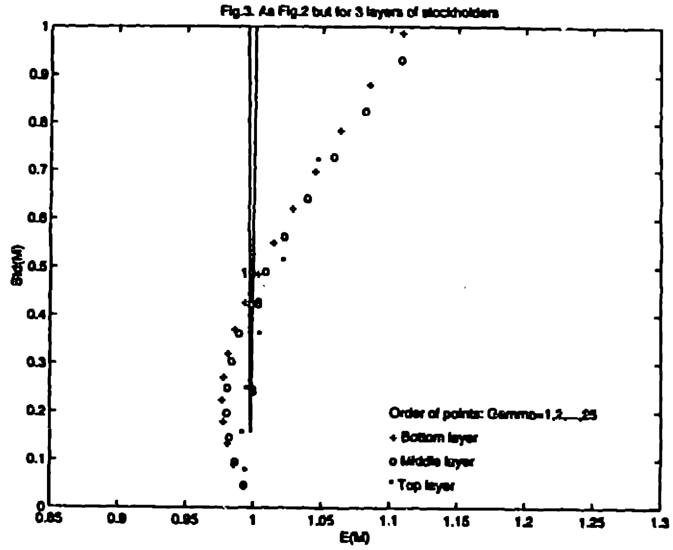


Fig.4. H-J bound. CEX monthly data, NYSE and T-bill returns, 1982-1994. No repr. agent assumed

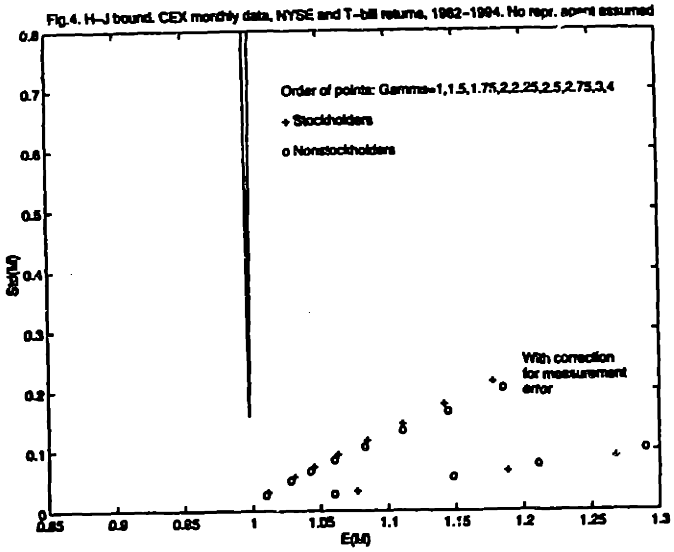


Fig.5. As Fig.4 but for 3 layers of stockholders

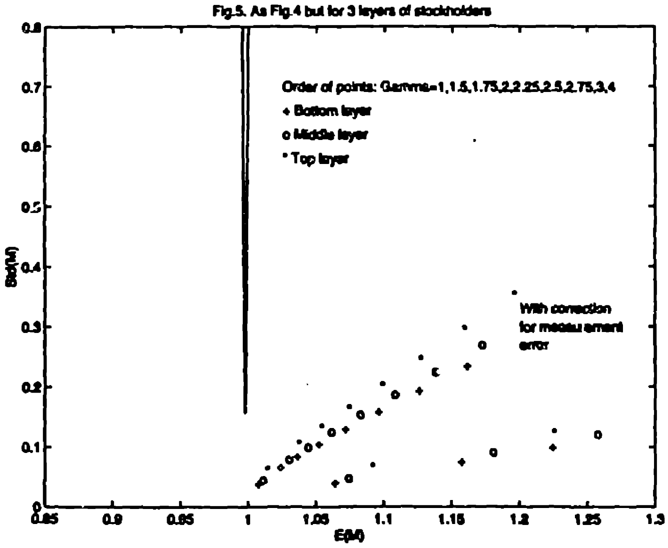


Fig.6. H-J bound. CEX 3-monthly data, NYSE and T-bill returns, 1982-1994. Repr. agent assumed

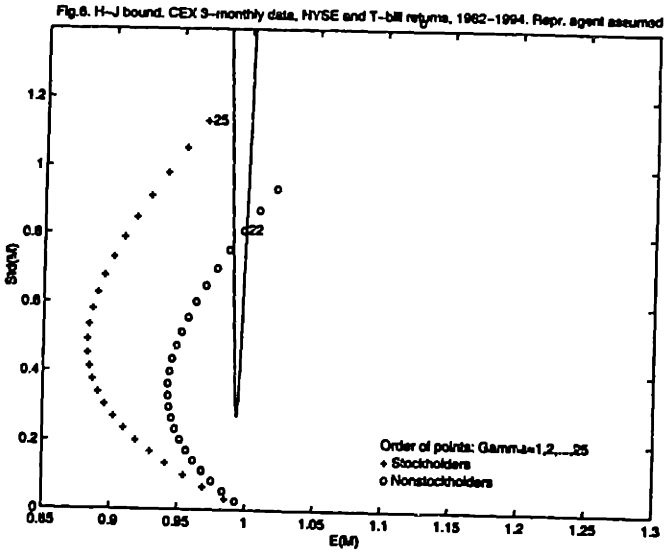


Fig.7. As Fig.6 but for 3 layers of stockholders

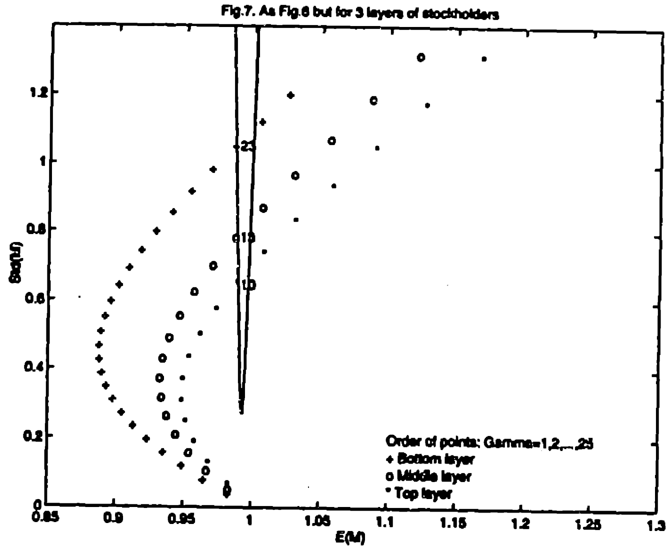


Fig.8. H-J bound. CEX 3-monthly data, NYSE and T-bill returns, 1982-1994. No repr. agent assumed

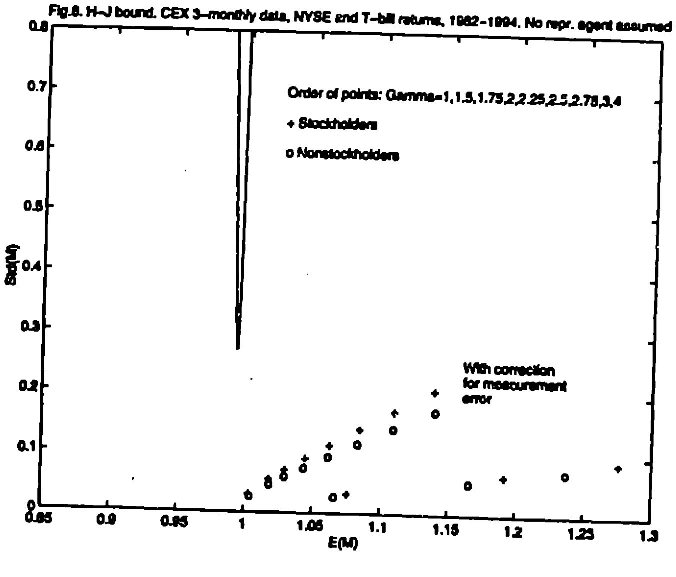
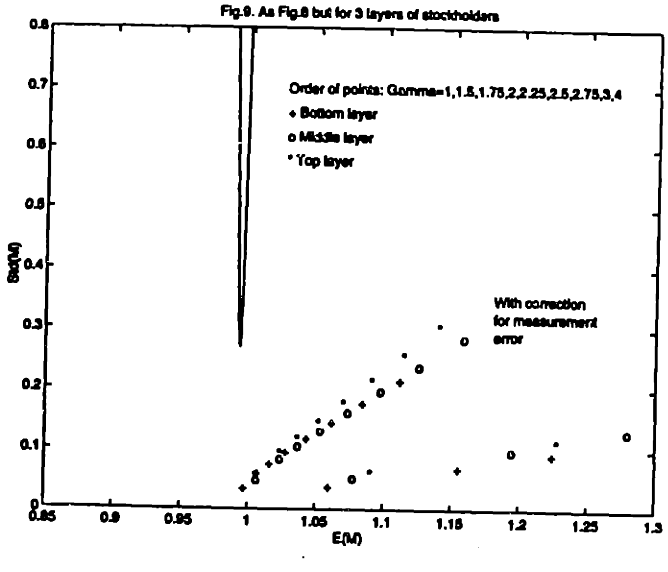


Fig.9. As Fig.8 but for 3 layers of stockholders



Chapter 2

An Empirical Investigation of the Effect of Non-Financial Income on Portfolio Choice

2.1 Introduction

Households differ dramatically in their portfolio choices. Among households surveyed in the wealth supplements to the Panel Study of Income Dynamics, 49.66 percent of those with positive financial wealth do not hold positions in either stocks nor bonds[1]. An additional 23.04 percent hold stocks but no bonds, whereas 14.79 percent hold bonds but no stocks. Only 12.50 percent hold both stocks and bonds. Furthermore, even within the set of households who hold both stocks and bonds, observed portfolio compositions differ substantially. These facts are not consistent with predictions from standard models of agents maximizing expected lifetime utility subject to initial wealth and the possibility of investing in all existing assets at zero transaction costs.

In the standard model of lifetime consumption and portfolio choice of Samuelson (1969) and Merton (1969), (1971), agents live off income generated by their invested wealth. The set of

[1] Averages of numbers from 1984, 1989 and 1994. Stockholdings include stocks held through mutual funds and in IRAs. The bond measure does not include money market bonds, or Treasury bills. Some non-bond categories of wealth are included in the bonds measure (life insurance policies, collections, and rights in trusts or estates).

available assets includes a conditionally riskless asset and N risky assets. Without transaction costs this model predicts that agents should take positions in all existing assets counter to the frequently observed zero holdings. The optimal portfolio of risky assets and the split between risky and riskless assets will vary across agents with different preferences, wealth and investment horizon. Conditions on return distributions/utility functions have been derived, under which differences in wealth and investment horizon across agents should not lead to differences in portfolio choice. Investment horizons are irrelevant if agents face a constant investment opportunity set (i.i.d returns). CRRA preferences are sufficient for wealth not to matter. Heterogeneity in risk aversion always implies differences in portfolio choice.

It is well known that actual returns are not i.i.d implying potential heterogeneity in portfolio choices across age groups. There is less agreement as to whether CRRA utility is a reasonable approximation. Conditional on this assumption, evidence is starting to accumulate documenting heterogeneity in the (constant) coefficient of relative risk aversion. In a very interesting paper, Barsky et al (1997) document heterogeneity in risk aversion based on micro data from the Health and Retirement Study. About 12000 respondents answered questions concerning gambles over lifetime income. The answers reveal considerable heterogeneity in risk tolerance and the survey measure of risk tolerance significantly predicts portfolio shares in stocks, bonds, treasury bills and checking and savings accounts with the expected signs. However, the incremental predictive power of risk tolerance is low and the remaining unexplained variation in portfolio choice large, even after including wealth, income and demographic controls (including age).

The purpose of this paper is to determine empirically whether accounting for differences in nonfinancial income patterns across households can help explain the remaining heterogeneity in observed portfolio choices. The theoretical literature on non-financial income and background risk predicts three effects. A larger mean of non-financial income should lead agents to invest a larger fraction of financial wealth in stocks, since agents with alternative sources of income can rely on this for consumption purposes should their financial investments fail. The variance of non-financial income should have a negative effect on the proportion invested in stocks due to background risk inducing more risk averse behavior. A non-zero covariance of non-financial income with stock returns should cause a hedging component of asset demand of the opposite sign of the covariance.

I focus on the two largest sources on nonfinancial income, namely labor income and income from privately held businesses. I concentrate mainly on the decision to hold stocks and the proportion of financial wealth held in stocks. Future work will also consider bonds. I use all available years of income data from the PSID, 1967-1992, and three observations of portfolio choice from the 1984, 1989 and 1994 wealth supplements. A two step procedure is used, similar to the one followed in previous papers, in which the first step consist of estimating the relevant moment of income processes which are then used as regressors in the second step focusing on portfolio choice.

My results based on probit and tobit regressions document economically important and statistically significant mean and variance effects of non-financial income on portfolio choice. Weaker evidence is found for a covariance effect. I then split non-financial income into labor and business (including farm) income. The mean and standard deviations of both types on non-financial income enter the regressions with the expected signs and are significant. Some evidence of a covariance effect remains for labor income, but not for business income. The results concerning the mean and variance effect of labor income confirms the findings of Guiso et al. (1996) using a somewhat different methodology and a different data set. The results for business income are also encouraging and more conclusive than the findings of Heaton and Lucas (1997).

Using three observations of portfolio choice for the same set of household allows me to provide new evidence concerning the importance of fixed costs of participating in the stock market and the bond market. Many papers have emphasized widespread non-participation in on or both of these markets based on a cross-section of households. By following households over time it is possible to determine whether households either stay in/out of a given market or whether there is widespread movements in and out of markets. Somewhat surprisingly the data show large movements in and out of the stock and the bond market over time. Of 1855 households with positive financial wealth in both 1984 and 1989, 24.58 percent had positive stockholdings in one year but not the other. 31.75 percent had positive bondholdings in one year but not the other (results are similar using the 1989-94 panel). Equally interesting, the three year panel 1984-89-94 shows that many households were out of the stock market in 1984, in the market in 1989 and then out again in 1994. This was the case for bonds as well as for stocks, and is therefore probably not due to people getting scared by the October 1987 stock market crash.

The availability of a panel of observations of portfolio choice furthermore makes it possible to partially address the issue of individual effects arising in cross-sectional regressions concerning the decision to hold stock and/or the proportion of financial wealth invested in stocks. Individual (household) effects are likely to be highly correlated with the income regressors if the fixed effect represents heterogeneity in preference parameters. Consider a case in which we are trying to determine the effect of the mean and standard deviation of labor and business income on the proportion of financial assets allocated to stocks. Suppose that households are heterogeneous in terms of their coefficient of relative risk aversion but that we do not have household level risk aversion measures. Less risk averse households are likely to invest more in stocks. However, they are also more likely to self-select into riskier jobs or to become business owners and will therefore tend to have higher standard deviation of labor (business) income, and also higher mean labor (business) income to the extent the risk is compensated by a higher mean. Our regressors will be endogenous and the coefficient estimates on both the means and the standard deviations will be upward biased. The effect of mean income will be exaggerated and we may get an unexpected positive or insignificant coefficient for labor (business) income risk on stockholdings (similar problems arise for the covariance-effect as for the variance-effect). The Survey of Consumer Finances contains a self-reported measure of risk aversion which several papers have found significant in regressions involving stockholdings[2], confirming the findings of Barsky et al. (1997). The latter also documented an economically large although not statistically significant of risk tolerance on the probability of being self-employed.

This problem can be addressed using a panel of portfolio choices if the three moments vary over time at the household level. With noisy micro data and portfolio choices only five years apart, it is probably too ambitious to try to detect time variation in covariances. I therefore restrict attention for this part of the paper to the mean and standard deviation of non-financial

[2]See Blume and Zeldes (1994) and Bertaut and Haliassos (1995). Due to the lack of time dimension the SCF by itself cannot be used to test for the three effects of non-financial income on portfolio choice. One could consider estimating the moments of income by demographic groups using the PSID and then use these as regressors in an analysis based on SCF asset data and the SCF risk aversion measure.

The 1996 PSID for the first time includes a measure of risk aversion similar to the one from the Health and Retirement Study used by Barsky et al. I will return to the potential use of this measure.

income and use five year windows around 1984 and 1989. This results in a two year panel which is analyzed using conditional logit estimation and the trimmed least squares estimator of Honore (1992) which is a fixed effect estimator of the tobit model. The results which rely purely on time-variation in portfolio choice for each household confirm the importance of the mean effect of labor income but not of business income.

Aside from its importance for understanding portfolio choice and thus the determination of prices of financial assets, the question of whether observed heterogeneity is consistent with optimizing behavior has important policy implications. One set of implications concerns the optimal portfolio composition of a social security trust fund. Firstly, if people are not making rational portfolio choices, we should not base portfolio decisions for a social security trust fund on an 'average' of observed household portfolio choice. Secondly, if people are rational but have heterogeneous optimal portfolio composition, then imposing the same portfolio composition on all households could imply large welfare losses.

Another policy implication concerns the effect of taxation of labor/business income on agents' lifetime utility. A proportional tax will have the effect of decreasing the mean and standard deviation of after-tax labor/business income which will affect agents' consumption and portfolio choice. This effect of taxation has been analyzed by Elmendorph and Kimball (1991). The results documenting strong mean- and variance-effects of non-financial income on portfolio choice, emphasize the importance of this issue.

2.2 Related literature

The recent asset pricing literature has paid much attention to the effects of labor income on portfolio choice and general equilibrium asset pricing, especially focusing on the effects of uninsurable idiosyncratic shocks. It has been known since Merton(1971) that the existence of certain non-financial income should cause agents with HARA utility to invest a larger fraction of their financial wealth in risky assets. This is the 'mean-effect' mentioned above. However, labor income is generally risky. With an incomplete set of financial markets agents cannot rely solely on financial markets to insure themselves. Furthermore, moral hazard problems prevent insurance contracts between labor income earners and potential insurers. The uninsurable part of nonfinancial income implies a 'variance-effect' on portfolio choice. Gollier and Pratt (1996) consider the effect of unfair background risks, i.e. risks with nonpositive expectations. In a one period model they show that all familiar DARA utility functions are risk vulnerable, meaning that any unfair background risk makes risk-averse agents behave in a more risk averse way. Viceira (1997) extends this result to a multi-period model in which wealth accumulation is endogenous. Viceira (1997) furthermore clearly shows the effect on portfolio choice of the sign of the covariance of labor income innovations with the stock return. Positive covariance of the stock return and innovations to permanent or transitory income generates a negative hedging component of asset demands and vice versa for negative covariance[3].

The theory of background risk can be applied to uninsurable income from privately held businesses as well as to labor income. In my sample from the PSID of households with positive financial wealth and for which either labor income or business and farm income was positive, about 7 percent of households earned more from business income than from labor income[4]. To the extent that business income risk is undiversifiable it may have strong negative effects on the optimal share of financial wealth invested in stocks, especially given the much larger standard deviation in percentage terms of business income than labor income. However, careful analysis of the total effect of business income on portfolio choice is needed since households with a large share of their income from businesses tend to have higher mean income, which tend to increase the optimal share of financial wealth in stocks.

Empirical work testing these predictions are still at a quite early stage. Let me briefly mention three papers, two which focuses on labor income risk and one which also considers business risk. To estimate household level income processes a (long) panel data set of income observations and at least one observation of portfolio choice is needed. Guiso, Japelli et al. circumvent the need for a panel data set by using the 1989 Bank of Italy Survey of Household Income and Wealth. In this survey respondents are asked to distribute probability weights to given intervals of inflations and nominal labor and pension income changes one year ahead.

[3]Several papers analyze the general equilibrium effects of uninsurable income risk on asset prices. See, for example, Mankiw (1986), Heaton and Lucas (1996), Constantinides and Duffie (1996), Krusell and Smith (1997), and Telmer et al. (1998).

[4]This number is based on the 5-year income window around 1989. See data section for description of data and income windows. The 7 percent number is a downward biased estimate since wife's labor part of business income is included in labor income.

Based on the answers an estimate of expected income variance can be constructed (but the covariance-effect cannot be tested). This variable is an economically and statistically significant predictor of the proportion of wealth in risky assets. The level of income enters positively, which could be interpreted as support for the mean-effect predicted by theory. Gakidis (1997) uses 7 years of income data from the PSID and the 1984 wealth supplement. He estimates labor income processes by demographic groups defined by occupation, age, and education as an alternative to estimating income processes by household. The most important finding is a significantly negative effect of the probability of zero income events on the probability of being a stockholder and on the proportion held in stocks conditional on being a stockholder. He finds no evidence of a variance-effect aside from the zero-income probabilities, but some evidence of a mean effect. The covariance effect is not considered, probably due to the short sample. Heaton and Lucas (1997) use the 1979-1990 Panel of Individual Tax Return Data and exploits the panel dimension to calculate, by household, the standard deviation of labor income and business income and the covariance of these two income components with the S&P500 stock return. These are then used as regressors in a random effects regression with the proportion of financial wealth invested in stocks as the dependent variable. The results are inconclusive most likely due to the poor quality of asset data in this data set[5]. When the sum of labor and business income is used they find evidence of a positive variance-effect (counter to the prediction from theory). However, when labor and business income are included separately, and the sample restricted to those with average business income above $500 the standard deviation of business income has the expected negative sign and is significant. None of the covariance variables are significant.

2.3 Empirical framework

2.3.1 Basic relation and null hypothesis

The theory on portfolio choice in the presence of non-financial income does not allow a closed form solution to be derived when non-financial income is risky (and less than perfectly correlated with the stock return) and/or investment opportunities vary over time. Progress has been

[5] Asset holdings must be estimated based on information on dividends, interest income and capital gains.

made by Campbell and Viceira (1996) for the case of time varying investment opportunities and Viceira (1997) for the model with uninsurable labor income. By log-linearizing the Euler-equations and the budget constraints they obtain analytical solutions to approximate problems. Unfortunately, the log-linearization constants are complicated functions of the underlying parameters and numerical solutions must still be used to determine, for example, the effect of an increase in the standard deviation of permanent income growth on the optimal portfolio share of the risky asset. Given this, I chose to focus on a simple linear model. The basic relation used for estimations is the following:

$$\begin{aligned}
\alpha_{it}^{*} &= c_i + \delta_0 D_t + \delta_1 A_{it} + \delta_2 A_{it}^2 + Z_{it}\gamma + \beta_1 W_{it}^f + \beta_2 \left(W_{it}^f\right)^2 \\
&+\beta_3 E_t\left(Y_{i,t+1}\right) + \beta_4 \left(V_t(Y_{i,t+1})\right)^{1/2} + \beta_5 cov_t(Y_{it+1}, R_{t+1}^s) + u_{it} \\
&= X_{it}\xi + u_{it}
\end{aligned} \tag{2.1}$$

Subscript t refers to years in which portfolio data are available. α_{it}^{*} denotes the optimal proportion of time t financial wealth invested in stocks by household i. c_i is an unknown individual effect for household i. D_t is a time dummy intended to capture hedging demands caused by time varying investment opportunities affecting all households (e.g. predictability of the equity premium based on the dividend-price ratio). A_{it} is the age of the household head which will matter in the presence of nonfinancial income or under time-varying investment opportunities (for example, negatively autocorrelated stock returns). The shape of the age-dependence will depend on the specific processes for non-financial income and asset returns, so age squared is also included in the relation to allow a more flexible functional form. Age could also matter for other reasons, for example because of health risks being age dependent. Z_{it} is a vector $(1\times K)$ vector of demographic variables intended to capture heterogeneity in tastes. Z_{it} includes education dummies, a race dummy and a variable equal to the number of children 18 years or younger. The significance of education may have alternative interpretations than education dependent tastes, an issue I shall return to. Financial wealth W_{it}^f could enter because of wealth dependent relative risk aversion (with a negative sign if relative risk aversion is decreasing in wealth) or because of the presence of non-financial wealth. If labor income is constant throughout life, the relation between α and W^f is negative in the

CRRA case[6]. For now, assume that c_i is uncorrelated with the other regressors and include the stochastic part of c_i in u_{it}, leaving a constant term c which is the same across individuals. Thus $X_{it}=\left[1\ D_t\ A_{it}\ A_{it}^2\ Z_{it}\ W_{it}^f\ \left(W_{it}^f\right)^2\ E_t\left(Y_{i,t+1}\right)\ \left(V_t(Y_{i,t+1})\right)^{1/2}\ cov_t(Y_{it+1,}R_{t+1}^s)\right]$ and $\xi' = [c\ \delta_0\ \delta_1\ \delta_2\ \gamma'\ \beta_1\ \beta_2\ \beta_3\ \beta_4\ \beta_5]$

The variables of most interest are $E_t\left(Y_{i,t+1}\right)$, $\left(V_t(Y_{it+1})\right)^{1/2}$, and $cov_t(Y_{it+1,}R_{t+1}^s)$. $Y_{i,t+1}$ refers to non-financial income of household i in period t. R_{t+1}^s is the return on a properly defined stock market index. All three moments are potentially time-varying. The null hypothesis, as predicted by the theory of portfolio choice in the presence of non-financial income, is:

$H_0 : \beta_3 > 0, \beta_4 < 0, \beta_5 < 0.$

2.3.2 Accounting for nonparticipation

The large number of households who are not in the stock market despite having positive financial wealth suggests the importance of a fixed cost of entering the market and/or a fixed per period cost of following the market. Suppose the latter is the case. Let $V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)$ denote the value function of household i at time t if the household chooses to invest in the stock market in period t and act optimally from time t+1 onwards. F_t is the fixed cost. The argument S_{it} refers to the set of individual specific state variables, which in the present setup consists of A_{it}, Z_{it}, $E_t\left(Y_{i,t+1}\right)$, $\left(V_t(Y_{i,t+1})\right)^{1/2}$, and $cov_t(Y_{it+1,}R_{t+1}^s)$. S_t refers to state variables common to all agents. The dividend-price ratio is an example of this. Denote by $V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right)$ the value function of the same household if it chooses to stay out of the stock market in period t+1. The household will invest in the stock market if $V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right) > V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right)$. Thus we will observe:

$$\alpha_{it} = \left\{ \begin{array}{l} \alpha_{it}^* \text{ if } V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right) / V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right) > 1 \\ 0 \text{ otherwise} \end{array} \right\} \tag{2.2}$$

In this setup, the variables determining the participation decision are, aside from F_t, the same as those determining the optimal proportion invested in stocks once in the market. The larger α_{it}^*, the more likely the household is to participate. To a first order approximation

[6] See Merton (1990) equation (5.71).

$V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)/V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right)$ will be linear in the same variables as α_{it}^* :

$$\begin{aligned}
&V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)/V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right) \simeq \\
&\overline{c} + \overline{\delta}_0 D_t + \overline{\delta}_1 A_{it} + \overline{\delta}_2 A_{it}^2 + Z_{it}\overline{\gamma} + \overline{\beta}_1 W_{it}^f + \overline{\beta}_2\left(W_{it}^f\right)^2 \\
&+\overline{\beta}_3 E_t\left(Y_{i,t+1}\right) + \overline{\beta}_4\left(V_t(Y_{i,t+1})\right)^{1/2} + \overline{\beta}_5 cov_t(Y_{it+1}, R_{t+1}^s) + \overline{u}_{it} \\
&= X_{it}\overline{\xi} + \overline{u}_{it}
\end{aligned} \tag{2.3}$$

Bars denote that this parameter is a constant times the corresponding parameter in 2.1 (the constants will differ from parameter to parameter). $\overline{u}_{it}$ is a constant times u_{it}. We observe only whether $V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)/V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right)$ is larger than one or not. Assuming that u_{it} is normally distributed we have:

$$\begin{aligned}
&E\left[\alpha_{it}|V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)/V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right) > 1\right] && (2.4)\\
&= \alpha_{it}^* + \sigma_u\lambda\left(\left(X_{it}\overline{\xi} - 1\right)/\sigma_u\right) = X_{it}\xi + \sigma_u\lambda\left(\left(X_{it}\overline{\xi} - 1\right)/\sigma_u\right) && (2.5)
\end{aligned}$$

$\lambda(\cdot) \equiv \frac{\phi(\cdot)}{\Phi(\cdot)}$ where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the density function and the cumulative density function of the standard normal. In principle one could therefore estimate the relation:

$$\alpha_{it}|\left[V_t^s\left(W_{it}^f - F_t, S_{it}, S_t\right)/V_t^{ns}\left(W_{it}^f, S_{it}, S_t\right) > 1\right] = X_{it}\xi + \sigma_u\lambda\left(\left(X_{it}\overline{\xi} - 1\right)/\sigma_u\right) + v_{it} \tag{2.6}$$

This could be done using maximum-likelihood or using a Heckman two-step estimation procedure. Doing this the parameters would be identified off the different functional form of the two parts of the equation. However, since the assumption of a linear model for the optimal share, $\alpha_{it}^* = X_{it}\xi + u_{it}$, is only an approximation to theory, and since a first order approximation was used for the ratio of value functions, I do not consider this a reliable approach. ξ and $\overline{\xi}$ could be separately identified if variables could be found which strongly affect the probability of being a stockholder without affecting the optimal portfolio share for stocks conditional on participation. I do not think this is possible[7].

Based on these considerations I do not attempt to separately identify ξ and $\overline{\xi}$ and therefore

[7]Gakidis (1997) assumes education can play this role. If one, caveats aside, runs 2.6 using a Heckman two-step procedure, education dummies are highly significant in both the regression and the selection equation.

use a tobit model. This model assumes that

$$\alpha_{it} = \left\{ \begin{array}{l} \alpha_{it}^{*} \text{ if } \alpha_{it}^{*} > 0 \\ 0 \text{ otherwise} \end{array} \right\} \tag{2.7}$$

corresponding to the restriction $\overline{\xi}=\xi$ and $\overline{u}_{it}=u_{it}$. The resulting estimate of ξ will be an average of ξ and $\overline{\xi}$. The variable for which this is most important is wealth. Given a fixed cost of participating in the stock market wealthier households will be more likely to participate than less wealthy households. This implies that a positive coefficient on wealth does not necessarily have the interpretation that relative risk aversion is declining in wealth, a point emphasized by Guiso et al. (1996). In addition to estimating tobit models I estimate probit models to show that most of the predictive power in the tobit models is likely to come from the ability to predict which households will participate rather than an ability to closely predict differences in holdings within the set of participants.

Under the assumption that the individual effects are uncorrelated with the right hand side variables the parameters can be consistently estimated using the cross-section of households for any of the three years with portfolio data. For efficiency, data for the three years can be pooled and the probit and tobit regression run on the pooled data set.

As I will argue later, it is likely that part of the participation cost takes as form of a fixed entry cost rather than a fixed per period participation costs. This may lead to correlation of the error term for a given household across years. I will address this issue in future versions of this paper.

2.4 Data

For the purpose of estimating household level income processes a fairly long panel of income information is need. This motivates the use of the Panel Study of Income Dynamics for my analysis, along with the availability of several years of wealth and portfolio information in this data set. I use the Survey Research Center sample of the PSID which was representative of the civilian noninstitutional population of the US when the study was started in 1968. The PSID has tracked all original family units and their adult offspring over time, so with low attrition rates the sample remains representative as long as offsprings are included. I excluded the poverty sample and the Latino sample.

The last year for which final release data are available is 1993. From the 1968-93 family files I construct a data set containing information for each of the households ever in the sample during this period. I use the family files rather than the individual files since wealth information is available at the household level. There are 6322 such households (after excluding the poverty and latino samples). For split-offs, information for years prior to the split-off was coded as missing.

Wealth information from the 1984, 1989 and 1994 supplements is used to calculate net financial wealth, defined as the sum of cash (checking and savings accounts, money market bonds, Treasury bills, including such assets held in IRA's), bonds (bond funds, cash value in life insurance policies, collections, rights in trusts or estates), and stocks (shares of stock in publicly held corporations, mutual funds, or investment trusts, including stocks in IRA's). To identify entries for which imputations were used, I use the wealth information as given in the family files instead of the wealth supplement files. Imputed values for cash, bonds or stocks can then be coded as missing. Topcoding of wealth or income variables is very rare in the PSID and topcoded variables were left at their topcodes.

I define labor income as the sum of head's labor income and spouse's labor income. Business income is defined as head's and spouse's asset part of unincorporated business income and farm income. Thus the labor part of business and farm income is included in labor income. This is done because the wage and business/farm components of spouse's labor income is not available separately. Household years in which the head is a student are dropped. Three estimates of each of the three income moments are then constructed at the household level. One based on the 5-year window around 1984[8], one based on the 5-year window around 1989, and one based on the 15-year period 1978-1992. For each time interval only households with no changes in head or spouse within that period are used. For the 5-year windows, households with 3 or more non-zero observations or more of labor plus business/farm income are used. For the 15-year window households with 10 or more observations are used. If an income component is zero in a

[8]This correspond to interview years 1983-87, since income for the previous year is reported when a household is interviewed.

particular year, that value is not used to calculate the moments. Income variables are deflated by the consumer price index, with 1982-84 as basis year.

The use of windows is motivated by potential time-variation in nonfinancial income. It is this time-variation which, if present, can be used to construct fixed-effect estimators when exploiting the panel of portfolio information. Since the latest available income information refers to 1992, it is not possible to construct a window around 1994. The 1994 portfolio information is therefore only used for descriptive statistics and for the regressions with the 15 year window. I do not calculate the covariance of income components with the real stock market return for the 5-year windows. It is unlikely that the covariance of an income component and the stock market return can be estimated to any level of precision with 5 years of data, and aside from that, it is not clear that this covariance would change much over time should we be able to estimate it precisely. The reason for using a 15 year period in stead of the entire sample for each household for the covariance estimation is that many households change composition over time, even if only changes in heads and spouses are considered. Therefore, restricting the sample to households with the same head and spouse for all years would imply a very small (and far from representative) sample.

The stock return used for calculating the covariance of stocks and nonfinancial income is the real value weighted NYSE index.

2.5 Results

2.5.1 New evidence on the importance of fixed participation costs

For comparison with previous studies, Table 1 confirms for the present data set, the well known fact that in any given year only a fraction of households with positive financial wealth participate in the stock market or in the bond market. An upward trend in stock market participation which will be the focus of chapter 3 is clear from the PSID data. Of households with positive financial wealth 44.06 percent participated in the stock market in 1994, up from 34.12 percent in 1989 and 28.47 percent in 1984. Within the set of stockholders, both the median and mean of stockholdings in dollars and then mean percentage of financial wealth held in stocks increases strongly between 1989 and 1994[9]. To give a representative picture which can provide information about the US population as a whole, these numbers are based on all household in my sample from the PSID[10].

With three observations of portfolio choice for a group of households, it is possible to analyze patterns of participation and trading over time. The results are shown in Fig. 1-3 and Table 2, all based on households with positive financial wealth[11]. Fig. 1a. focuses on the set of households for which portfolio information is available for both 1989 and 1994. The figure plots the 1994 share of financial wealth held in stocks against the 1989 share. In the absence of a fixed cost of participating in the stock market, and with no nonfinancial income and i.i.d asset returns, standard finance theory predicts that all households should be at a point along the 45 degree line in this figure (the origin not included). With nonfinancial income and/or returns which are not i.i.d points off the 45 degree line but in the interior of the first quadrant are potentially consistent with theory. Only if fixed costs are important can we explain the large number of households at the origin or along one of the axes (71.72 percent). Previous evidence based on cross sections of households would lead us to expect many observations of zero stockholdings in each year. Somewhat surprisingly, the figure shows that many households participate in the stock market in one year but not the other. These are the points along the axes forming an angle in the graph. 28.10 percent of households are on this angle, not including the origin. Fig. 1b. shows similar results based on the 1984-89 panel. Notice that many of the points on the angles are far from zero. This reflects households who move from a zero to a substantially positive fraction of wealth in stocks or the other way around and for which the entry/exit thus does not correspond to 'marginal' changes in stockholdings as a percentage of financial wealth.

Fig. 1c. focuses on households with positive financial wealth for which three observations of portfolio choice are available. The change in the share of financial wealth held in stocks between 1989 and 1994 is plotted against the 1984-89 change. A 'triplet' of lines is apparent. The vertical line corresponds to households who did not participate in the stock market in 1984

[9]This is not a necessary consequence of the stock market boom, since with more participants each participant could in theory hold the same amount or the same percent of financial wealth in stocks in 1994 as in 1989.

[10]Thus not all of the households used for the tabulations are the same for all three years due to split-offs etc.

[11]As for Table 1 I do not drop households with changes in household composition.

or 1989 but did participate in 1994 (points showing zero change between 1984 and 1989 are all for non-participant who had a zero share in both years). The horizontal line corresponds to households who participated in 1984 but not in 1989 or 1994. The most interesting line is the downward sloping one, which shows that many households entered the stock market some time between 1984 and 1989, but left the market again some time between 1989 and 1994[12]. It is tempting to interpret this as households who entered but got scared by the market crash in October 1987. However, Fig. 2c. shows a similar pattern for bond holdings. A more plausible explanation for this pattern is large changes in optimal portfolio shares combined with a fixed per period cost of participating in the market. The graphs for the remaining component of financial wealth, cash, are shown in Fig. 3. The lines are the 'reverse' of those shown in Fig.1 and 2, which is intuitive.

What is the nature of these fixed costs? The fact that many households in the 1984-89 panel or the 1989-94 panel are out of the stock market in both years, or in the first year but not the second, points towards a fixed entry cost. Households who participate in the market the first year but not the second indicate a fixed cost per period cost of being in the market (a 'continuation' cost). Both the entry cost and the 'continuation' cost could be interpreted as information costs. This is consistent with the significance of education in probit/tobit models of stockholding as has been documented by many authors (and as shown in the regressions to follow). An alternative interpretation is a fixed trading cost which makes it optimal to trade only when large changes in the proportion of wealth invested in a particular asset occurs. As regards trading costs of entry, they could take the form of minimum investment requirements for investment in mutual funds.

It is hard to determine whether information costs or trading costs are more important. The recent surge in stock market participation has happened during a period in which both costs have been falling. The availability of low cost index funds makes it easy to invest in the stock market. This is the case both in terms of the amount of information needed, in terms of the initial transactions cost one has to incur to obtain a diversified portfolio, and in terms of the cost of changing the share of financial wealth invested in stocks.

[12] From the available information it is not possible to determine if households entered and then left the market again, or left and then reentered, in years in between 1984 and 1989 or in between 1989 and 1994.

2.5.2 Estimation results

Cross-sectional regressions on pooled data for 1984, 1989, and 1994

The results from the cross-sectional regression with pooled data are overall very encouraging in terms of documenting a mean-effect and a variance-effect of nonfinancial income on the fraction of financial wealth invested in stocks. Some, but weaker, evidence is found of a covariance-effect.

Table 3 and 4 show the probit and tobit estimations when labor and business income are not separated. The results with separate variables for these two sources of nonfinancial income are given in Table 5 and 6. In each table the heading 'two windows' refers to the estimation with pooled data for 1984 and 1989 and 5 year windows around each of these years to calculate the income moments. Within each window only households with no change in household composition are used. Changes across the windows are allowed to preserve observations and a household need not be present in both years[13]. The columns under the heading 'one window' show the results when using a 15 year window in order to be able to estimate the covariance of nonfinancial income with the stock return. Only households with positive financial wealth are included. For the tobit models censoring at zero and one is used. I account for censoring at one because although nothing prevents households having a portfolio share for a given asset above one, I am not able to observe this because negative values of stocks, bonds and cash are coded as zeros in the PSID.

Focus first on the two-window regressions in Table 3. The results show clear evidence of a positive mean-effect and a negative variance-effect of nonfinancial income on the probability of stock market participation. The effects are economically important as well as statistically significant. The marginal effects (evaluated at the means of the right hand side variables) show that an increase in mean real nonfinancial income of 10000 dollars in 1982-84 prices increases the probability of participation by about 6 percent[14]. The effect of a change in the standard deviation of real nonfinancial income is of a similar magnitude but negative. In the tobit model in Table 4, a strong mean- and variance-effect is again present. An increase in mean

[13] In using pooled estimation with no corrections to the usual probit and tobit standard errors, I am implicitly assuming that the error terms for a given household is uncorrelated across the two windows. As mentioned earlier, in the presence of a fixed entry cost this assumption may be violated an issue which will be addressed in future versions.

[14] The price was set to 100 in the basis year, not 1.

real nonfinancial income of 10000 dollars in 1982-84 prices increases the optimal portfolio share for stocks by about 4 percentage points (again evaluated at the mean of the right hand side variables).

For the 15-year window the tobit regression in Table 4 documents a statistically significant negative effect of the covariance of nonfinancial income with the stock market index on the optimal share of financial wealth invested in stocks. Again this is as predicted by theory, but unlike the mean-effect and the variance-effect, the covariance-effect is fairly small in economic terms. The 25th and 75th percentiles for the covariance are -1.01 and 1.50. Thus even a move from the 25th to the 75th percentile changes the optimal portfolio share by as little as little as 2.5 percentage points[15]. Based on the probit results the covariance effect is insignificant.

Demographic variables enter the regressions with the same signs as has been found in other studies. Increases in real financial wealth lead to a higher probability of stock market participation up to about \$1.2 million dollars in 1982-84 prices. Based on the two-window tobit results, higher financial wealth increased the optimal fraction of financial wealth invested in stocks up to a level of financial wealth of \$5.4 million dollars in 1982-84 prices. Thus for nearly all households higher wealth leads to larger participation, again consistent with fixed entry/fixed participation costs but also consistent with relative risk aversion being decreasing in wealth.

Splitting nonfinancial income in business and labor income shows that both components have a significant positive mean-effect and a significant negative variance-effect, see Table 5 and 6. The size of the two effects are similar for business income and labor income. Thus once the effect of income is decomposed into the mean- and the variance effect labor and business income have essentially the same effect on portfolio choice. However given the different means and variances of the two income components the total effect of nonfinancial income on portfolio choice is smaller (less positive/more negative) for the typical business owner than the typical wage earner. Considering the standard deviation as a percent of the mean for each of the two income components, the cross-sectional median is 38 percent for business and farm income, compared to 16 percent for labor income[16]. Regarding the covariance effect, the split shows

[15]This number should be interpreted with some caution since I am using the marginal effect to evaluate a large change in the covariance.

[16]These numbers are based on the 5-year income window around 1989. For each of the two categories all households with nonzero income from that source are used for the calculation. The corresponding numbers for

that the results for total nonfinancial income are driven by a significantly negative covariance-effect for labor income in the tobit regression. For business income the coefficient estimate on the covariance-variable in the tobit model is slightly larger than that for labor income. The fairly small number of households with non-zero business income (about 80 per year for the 15-year window) could be the reasons that the coefficient is insignificant.

In sum, the cross-sectional regressions strongly support the theoretical prediction of a positive effect of mean nonfinancial income on the optimal share of financial wealth invested in stocks, and of a negative effect of the variance of nonfinancial income. Some evidence is found of a negative covariance effect but this effect is both economically and statistically weaker.

As a caveat, let me briefly mention an issue which to my knowledge has not been considered in the empirical literature on nonfinancial income and portfolio choice. The empirical procedure followed in this literature, for example by Guiso et al. and Heaton and Lucas (1997), is a two step procedure. Estimated moments of income are used as regressors in the second step. These moments are estimated, causing a generated regressor problem. This does not affect consistency of the second step estimates but does imply that the standard errors should be corrected. If the first step had been a parametric model this can be done following well known methods, as described in Newey and McFadden (1994). However since the first step in the present setup estimates one or several moments for each of several thousand households, standard methods cannot be applied. Future work will consider the possibility of a correction to the standard errors based on alternative methods.

Accounting for fixed effects

The above pooled cross-section estimations have not exploited the panel dimension of the asset information in the PSID. With a panel it is possible to address the potentially serious issue of individual effects. As discussed in the introduction the most important individual effect in the present context is likely to be heterogeneity in risk aversion. Since risk aversion could be correlated with the regressors (indeed, theory predicts that they should be), the random effects assumption underlying the above estimations may be invalid. If so this leads to inconsistency of parameters estimates using a random effect assumption. To determine whether this is the case,

the 5-year window around 1984 is 50 percent for business income and 17 percent for labor income.

two approaches are possible. The first one assumes that the distribution of the individual effects conditional on $\{X_{it}\}_{t=1}^{T}$ is a linear function of these T observations of the regressors. This is the 'correlated random-effects' approach developed for the probit model by Chamberlain (1984) and generalized to the tobit model by Jakubson (1988). To avoid restrictive assumptions of this type, I instead use the conditional logit model for the participation decision and the trimmed least squares estimator from Honore (1992) for the panel tobit model.

The results of the conditional logit estimation for the stockholding decision are given in Table 7 and 8. The idea of this fixed-effects model is to condition on the sum of the discrete dependent variable over the sample (at the household level). In the resulting conditional likelihood the individual effects cancel, see Chamberlain (1980). The sample is the same as for the probit regressions in Table 3 and 5. However, households who do not enter or exit the stock market in one of the two years contributes zeros to the conditional likelihood and are dropped. This leaves less than 500 households. As an alternative sample I therefore drop the restriction on no household composition changes within windows. This results in a larger sample of around 750 households, but with lower quality of the income data in the sense that a time series of income observation for a given household may not refer to the same head and spouse for all years. I furthermore consider a cutoff of 5 percent of financial wealth in stocks for being considered a stockholder. For all four resulting cases the mean-effect of nonfinancial wealth is again present, now for the probability of entering or exiting the stock market. There is no evidence of a variance effect. Financial wealth is significant only when the larger sample is used[17]. Given the assumed lack of time-variation in the covariance of nonfinancial wealth with the stock return, it is not possible to test whether the (weak) evidence of a covariance effect is robust to controlling for individual effects using fixed-effects estimation. Splitting income into labor and business income as shown in Table 8 reveals that the effects for total nonfinancial income are driven by labor income (the comparatively small number of households with positive business income may be a factor behind the insignificant results for business income). Overall the conditional logit results show that the mean-effect is robust to controlling for individual effects by identifying the effect off time variation in stock market participation for each households, rather than from

[17]There are no demographic variables in the regression aside from the number of children under 19 years of age since the remaining variables aside from age do not vary over time, and thus drop out of any fixed effect estimation. Age effects are captured in the time dummy since the two cannot be distinguished in this setup.

cross-sectional differences as done in the (pooled) cross-sectional estimations above. The lack of evidence of a variance effect based on the present estimates does not necessarily mean that one is not present once controlling for individual effects. It is possible that there is insufficient time-variation in the standard deviation of nonfinancial income or that the time-variations are too small to be precisely estimated.

When I exploit the information about the level of stockholdings for participants, by using Honore's the trimmed least squares estimatoı, results were extremely noisy[18]. No variables were significant aside from the time dummy. This may not just be a reflection of noisy data. With fixed costs of trading households will not adjust the number of shares held each period. Only when state variable changes have caused a sufficiently large misalignment of their actual portfolio from their optimal portfolio will it be worthwhile to trade. Methodologically the resulting framework becomes similar to the one used in the investment literature on adjustment costs. If more years of portfolio observations were available a model along the lines of Caballero, Engel and Haltiwanger (1995) could be estimated. Unfortunately this is not feasible with the PSID data set.

An alternative approach to controlling for individual effects is possible if one is willing to make the assumption that risk aversion (and no other preference parameters) is the main component of the individual effect. For the first time the 1996 questionnaire of the PSID includes a series of questions designed to provided an estimate of relative risk aversion (or equivalently the inverse of relative risk aversion called risk tolerance). The methodology used is the same as the one described in the paper by Barsky et al. (1997). The risk aversion questions are only asked to employed respondents. By using the household identification numbers in the 1994-96 early release files the estimates of risk tolerance can be merged back into the sample. I only merge backwards to 1989 to avoid serious selection problems due to fact that a household has to remain in the sample until 1996 and the head has to be employed for risk tolerance estimates to be available.

The risk tolerance estimate can then be included in a cross-section for 1989 as an estimate of the part of the individual effect which is correlated with the regressors. Table 9 shows the results of a tobit regression for 1989 (using the 5 year income window around this year to

[18] Excellent Gauss programs kindly provided on Bo Honore's home page.

estimate the income moments) with and without the risk tolerance measure included. The risk tolerance measure is significant at the 10 percent level and has the expected positive sign, but the clear evidence of a mean and variance effect of nonfinancial income remains. This is comforting given the mixed results of the fixed effects estimations.

2.6 Conclusion

Observed portfolio choices are not consistent with standard finance theory in the absence of a fixed cost of entering or staying in the stock market. Many households do not participate in the stock market at any point in time even when attention is limited to households with positive financial wealth. In addition, based on three years of wealth information from the PSID, this chapter has furthermore documented that there is a substantial number of households who move in or out of the stock market (and/or the bond market) over time.

I argued in Chapter 1 that the nonparticipation phenomenon should be considered an important part of the solution to the equity premium puzzle. This is the case if the consumption growth of nonstockholders covaries substantially less with the stock return than the consumption growth of stockholders. Empirical evidence based consumption data from the Consumer Expenditure Survey confirmed that this was the indeed the case. This indicates that the primary reason for nonparticipation is not that nonstockholders are faced with nonfinancial income which is highly correlated with the stock market return. The findings of the present chapter confirm this. Only a small or insignificant effect of the covariance of nonfinancial income with stock returns on participation and portfolio choice is found. Rather there is strong evidence of a mean-effect and a variance-effect.

Determining the reason for nonparticipation is crucial, not only for having confidence in the results of chapter 1. Stock market participation has increased dramatically during the postwar period. The positive results concerning the contribution of limited stock market participation to the solution of the equity premium puzzle suggests that this may have had substantial effects on asset prices. To analyze this issue we need to understand the main reasons for nonparticipation. This is the context in which I consider the results of this chapter interesting.

Year	Asset	Pct holding asset	Median (mean) holdings	Conditional median (mean) holdings, dollars	Median (mean) pct of fin. wealth, dollars	Conditional median (mean) pct of fin. wealth	Number of obs. with positive fin. wealth	Total number of obs
1984	Stocks	28.47	0(7715)	6000(27104)	0(12.37)	37.5(43.44)		
	Bonds	25.02	0(10625)	3000(42468)	0(9.98)	31.61(39.87)		
	Cash	97.85	3000(13638)	3000(13938)	100.00(77.66)	100.00(79.37)		
	Total fin. wealth			5000(31978)			2554	3409
1989	Stocks	34.12	0(15062)	10000(44149)	0(14.90)	39.02(43.69)		
	Bonds	28.88	0(6754)	5000(23386)	0(12.28)	35.09(42.34)		
	Cash	98.01	4050(19990)	5000(20395)	100.00(72.87)	100.00(74.35)		
	Total fin. wealth			9000(41806)			2770	3524
1994	Stocks	44.06	0(42409)	21276(96249)	0(24.37)	54.90(55.30)		
	Bonds	27.99	0(9243)	8000(33024)	0(11.72)	33.33(41.88)		
	Cash	96.19	5000(23301)	6000(24223)	80.00(63.91)	83.64(66.44)		
	Total fin. wealth			15000(74952)			2812	3739

Note: All numbers except the last column are for households with positive financial wealth. Conditional median refers to the median asset holding conditional on positive holdings of the asset in question.

Table 1: Summary statistics for households with positive financial wealth, PSID, 1984, 1989 and 1994

Years	Asset	No.(pct) of households with all financial wealth in asset both years (corner of angle)	No.(pct) of households with no financial wealth in asset either year (corner of angle)	No.(pct) of households on angle, except for corner	No.(pct) of households off angle	Total number of house-holds
1984-89 panel	Stocks		987 (53.21)	456 (24.58)	412 (22.21)	1855 (100.00)
	Bonds		1016 (54.77)	589 (31.75)	250 (13.48)	1855 (100.00)
	Cash	615 (33.15)		604 (32.56)	636 (34.29)	1855 (100.00)
1989-94 panel	Stocks		995 (43.62)	641 (28.10)	645 (28.28)	2281 (100.00)
	Bonds		1212 (53.13)	763 (33.45)	306 (13.42)	2281 (100.00)
	Cash	603 (26.44)		760 (33.32)	918 (40.25)	2281 (100.00)
		No.(pct) of households with all financial wealth in asset all three years (center of triplet)	No.(pct) of households with no financial wealth in asset any of the years (center of triplet)	No.(pct) of households on triplet, except for center	No.(pct) of households off triplet	Total
1984 -89-94 panel	Stocks		502 (35.73)	344 (24.48)	559 (39.79)	1405 (100.00)
	Bonds		587 (41.78)	464 (33.02)	354 (25.20)	1405 (100.00)
	Cash	259 (18.43)		374 (26.62)	772 (54.95)	1405 (100.00)

Note: Only households with positive financial wealth are included.

Table 2: Summary statistics documenting the importance of fixed costs

Whether hold stocks or not	Two windows			One window		
	Coefficient	dF/dx	t	Coefficient	dF/dx	t
Age of head	0.0173	0.00554	1.17	0.0152	0.00603	0.77
Age of head squared	-0.000122	-0.0000391	-0.75	-0.000116	-0.000046	-0.60
D(race=white)	0.182	0.0552	1.65	0.295	0.115	2.33
D(education=6-8 years)	0.365	0.129	0.91	0.330	0.131	0.78
D(education=9-12 years)	0.720	0.248	1.93	0.616	0.241	1.54
D(education=13-15 years)	0.885	0.299	2.37	0.847	0.327	2.13
D(education=16+ years)	1.23	0.426	3.28	1.098	0.415	2.75
Number of children<=18	-0.0612	-0.0196	-2.55	-0.049	-0.196	-1.73
Real financial wealth	0.00121	0.000387	16.4	0.000890	0.000354	12.4
Real financial wealth squared	-1.02e-07	-3.27e-08	-69.4	-1.41e-08	-5.59e-09	-10.8
Mean(labor+business income)	0.00184	0.000590	11.1	0.00202	0.000804	8.89
Std(labor+business income)	-0.00201	-0.000644	-6.52	-0.00198	-0.000786	-5.46
Cov(labor+business income, stock return)				-0.00162	-0.000645	-0.20
D(year=89)	0.0767	0.0246	1.53	0.0912	0.0363	1.35
D(year=94)				0.299	0.119	4.07
Constant	-2.703		-5.47	-2.58		-4.04

Note: Column 2-4: Pooled data 1984 and 1989, two 5-year windows for income, n=3254. Column 3-5: Pooled data 1984, 1989 and 1994, one 15 year window for income, n=2582. Only households with positive financial wealth are included. dF/dx refers to the marginal effect evaluated at the mean of the regressors. For dummy variables dF/dx refers to the discrete change in the probability for a change in the dummy variable from 0 to 1.

Table 3: Probit regression for the decision to hold stocks, labor and business income together

Proportion of financial wealth in stocks	Two windows		One window	
	Coefficient	t	Coefficient	t
Age of head	0.0242	3.31	0.00858	0.91
Age of head squared	-0.000197	-2.48	-0.0000371	-0.404
D(race=white)	0.134	2.41	0.130	2.15
D(education=6-8 years)	0.250	1.16	0.157	0.73
D(education=9-12 years)	0.498	2.46	0.372	1.83
D(education=13-15 years)	0.614	3.03	0.523	2.57
D(education=16+ years)	0.814	4.02	0.649	3.20
Number of children<=18	-0.0236	-1.99	-0.00590	-0.443
Real financial wealth	0.000166	8.73	0.000127	8.824
Real financial wealth squared	-3.09e-09	-6.85	-2.05e-09	-7.46
Mean(labor+business income)	0.000442	7.81	0.000832	9.07
Std(labor+business income)	-0.000753	-5.88	-0.000908	-7.21
Cov(labor+business income, stock return)			-0.00987	-3.23
D(year=89)	0.0303	1.23	0.0187	0.58
D(year=94)			0.198	5.82
Constant	-1.75	-6.71	-1.32	-4.21

Note: Column 2-3: Pooled data 1984 and 1989, two 5-year windows for income, n=3245 (2111 censored at 0, 1123 uncensored, 20 censored at 1). Column 4-5: Pooled data 1984, 1989 and 1994, one 15 year window for income, n=2582 (1457 censored at 0, 1087 uncensored, 38 censored at 1). Only households with positive financial wealth are included.

Table 4: Tobit regression for the share of financial wealth invested in stocks, labor and business income together

Whether hold stocks or not	Two windows			One window		
	Coefficient	dF/dx	t	Coefficient	dF/dx	t
Age of head	0.0226	0.00833	1.49	0.143	0.00570	0.68
Age of head squared	-0.000165	-0.0000607	-0.99	-0.000103	-0.0000408	-0.50
D(race=white)	0.203	0.0717	1.84	0.310	0.120	2.37
D(education=6-8 years)	0.398	0.154	0.99	0.408	0.161	0.95
D(education=9-12 years)	0.724	0.274	1.95	0.700	0.273	1.73
D(education=13-15 years)	0.896	0.334	2.41	0.948	0.363	2.34
D(education=16+ years)	1.284	0.474	3.45	1.16	0.436	2.86
Number of children<=18	-0.0519	-0.0191	-2.13	-0.049	-0.0193	-1.62
Real financial wealth	0.000978	0.000360	11.8	0.000952	0.000379	12.2
Real financial wealth squared	-1.98e-08	-7.27e-09	-10.6	-1.51e-08	-5.99e-09	-10.8
Mean(labor income)	0.00181	0.000666	10.2	0.00201	0.000799	7.97
Mean(business income)	0.00222	0.000819	2.92	0.00300	0.00119	2.75
Std(labor income)	-0.00238	-0.000875	-5.59	-0.00161	-0.000641	-3.20
Std(business income)	-0.00308	-0.00113	-2.81	-0.00344	-0.00137	-2.30
Cov(labor income, stock return)				0.00967	0.00384	0.79
Cov(business income, stock return)				-0.0232	-0.00923	-0.90
D(year=89)	0.0929	0.0341	1.82	0.714	0.0284	1.01
D(year=94)				0.296	0.117	3.83
Constant	-2.835		-5.70	-2.69		-4.06

Note: Column 2-4: Pooled data 1984 and 1989, two 5-year windows for income, n=3142. Column 3-5: Pooled data 1984, 1989 and 1994, one 15 year window for income, n=2361. Only households with positive financial wealth are included. dF/dx refers to the marginal effect evaluated at the mean of the regressors. For dummy variables dF/dx refers to the discrete change in the probability for a change in the dummy variable from 0 to 1.

Table 5: Probit regression for the decision to hold stocks, labor and business income separate

Proportion of financial wealth in stocks	Two windows		One window	
	Coefficient	t	Coefficient	t
Age of head	0.0245	3.26	0.00818	0.82
Age of head squared	-0.000199	-2.24	-0.0000285	-0.29
D(race=white)	0.139	2.48	0.137	2.20
D(education=6-8 years)	0.261	1.21	0.174	0.80
D(education=9-12 years)	0.495	2.45	0.387	1.89
D(education=13-15 years)	0.610	3.02	0.544	2.66
D(education=16+ years)	0.819	4.05	0.657	3.21
Number of children<=18	-0.018	-1.51	-0.00328	-0.23
Real financial wealth	0.00017	8.73	0.000129	8.27
Real financial wealth squared	-3.17e-09	-6.86	-2.09e-09	-7.14
Mean(labor income)	0.000447	7.21	0.000910	8.96
Mean(business income)	0.000614	2.12	0.00108	2.36
Std(labor income)	-0.000771	-5.27	-0.00104	-7.03
Std(business income)	-0.000944	-1.94	-0.000754	-1.26
Cov(labor income, stock return)			-0.00896	-2.09
Cov(business income, stock return)			-0.0123	-1.19
D(year=89)	0.0318	1.28	0.0102	0.30
D(year=94)			0.190	5.34
Constant	-1.77	-6.71	-1.37	-4.18

Note: Column 2-3: Pooled data 1984 and 1989, two 5-year windows for income, n=3142 (2034 censored at 0, 1088 uncensored, 20 censored at 1). Column 4-5: Pooled data 1984, 1989 and 1994, one 15 year window for income, n=2361 (1335 censored at 0, 993 uncensored, 33 censored at 1). Only households with positive financial wealth are included.

Table 6: Tobit regression for the share of financial wealth invested in stocks, labor and business income separate

Whether hold stocks or not	Cutoff for stockholding 0 percent of fin. wealth			Cutoff for stockholding 5 percent of fin. wealth		
	Coefficient	t	No. obs	Coefficient	t	No. obs
Only households with no change in household composition			466			474
Number of children<=18	-0.103	-0.54		-0.126	-0.71	
Real financial wealth	0.000628	0.77		-0.0000249	-0.04	
Real financial wealth squared	4.01e-07	0.97		8.62e-08	0.40	
Mean(labor+business income)	0.00260	2.02		0.00250	2.23	
Std(labor+business income)	0.000872	0.49		-0.000803	-0.59	
D(year=89)	0.648	4.05		0.623	4.18	
All households with financial information in both 1984 and 1989			768			754
Number of children<=18	0.0206	0.17		-0.0788	-0.66	
Real financial wealth	0.00162	2.46		0.000985	2.25	
Real financial wealth squared	-4.38e-08	-0.14		-2.25e-07	-1.95	
Mean(labor+business income)	0.00105	1.32		0.0016	2.10	
Std(labor+business income)	0.000676	0.61		-0.0000587	-0.06	
D(year=89)	0.568	4.66		0.567	4.83	

Note: The number of observations used for the conditional logit regressions is lower than for the probit and tobit regressions for two reasons. Firstly, only households for which more than one observation of financial variables is available can be used. Secondly, only households with variation in stock market participation (in/out) over time contributes non-zero values to the likelihood function. Only households with positive financial wealth are included.

Table 7: Conditional logit (fixed effect) regression for the decision to hold stocks, 1984-89 panel, 5 year windows for income, labor and business income together

Whether hold stocks or not	Cutoff for stockholding 0 percent of fin. wealth			Cutoff for stockholding 5 percent of fin. wealth		
	Coefficient	t	No. obs	Coefficient	t	No. obs
Only households with no change in household composition			440			452
Number of children<=18	-0.0601	-0.30		-0.100	-0.54	
Real financial wealth	0.000473	0.55		-0.0000356	-0.05	
Real financial wealth squared	4.10e-07	0.88		8.02e-08	0.36	
Mean(labor income)	0.00228	1.71		0.00188	1.70	
Mean(business income)	0.00325	0.69		0.00737	1.61	
Std(labor income)	0.00045	0.21		-0.00222	-1.22	
Std(business income)	-0.00022	-0.06		-0.00241	-0.71	
D(year=89)	0.654	3.99		0.6038	3.96	
All households with financial information in both 1984 and 1989			728			720
Number of children<=18	0.0660	0.52		-0.0428	-0.35	
Real financial wealth	0.00136	2.01		0.000960	2.15	
Real financial wealth squared	3.82e-08	0.115		-2.15e-07	-1.81	
Mean(labor income)	0.00154	1.78		0.00167	2.11	
Mean(business income)	-0.00239	-0.70		0.000770	0.23	
Std(labor income)	0.00164	1.16		0.00040	0.32	
Std(business income)	0.000664	0.22		-0.00176	-0.55	
D(year=89)	0.592	4.72		0.555	4.64	

Note: The number of observations used for the conditional logit regressions is lower than for the probit and tobit regressions for two reasons. Firstly, only households for which more than one observation of financial variables is available can be used. Secondly, only households with variation in stock market participation (in/out) over time contributes non-zero values to the likelihood function. Only households with positive financial wealth are included.

Table 8: Conditional logit (fixed effect) regression for the decision to hold stocks, 1984-89 panel, 5 year windows for income, labor and business income together

	Risk tolerance measure included		Risk tolerance measure not included	
	Coefficient	t	Coefficient	t
Risk tolerance measure	0.190	1.69		
Age of head	0.0153	1.23	0.0148	1.19
Age of head squared	-0.0000986	-0.70	-0.0000997	-0.71
D(race=white)	0.191	2.05	0.196	2.10
D(education=6-8 years)	0.0175	0.05	0.0196	0.06
D(education=9-12 years)	0.256	0.87	0.255	0.87
D(education=13-15 years)	0.412	1.41	0.417	1.43
D(education=16+ years)	0.534	1.82	0.543	1.86
Number of children<=18	-0.00372	-0.22	-0.00459	-0.28
Real financial wealth	0.000243	5.82	0.000246	5.91
Real financial wealth squared	-2.05e-08	-3.79	-2.11e-08	-3.89
Mean(labor+business income)	0.000421	5.00	0.000422	5.00
Std(labor+business income)	-0.000536	-2.94	-0.000526	-2.88
Constant	-1.40	-3.45	-1.33	-3.31

Table 9: Tobit regression for the share of financial wealth invested in stocks, 1989, 5 year window for income, labor and business income together. Only households with positive financial wealth for which 1996 risk tolerance measure is available are included, n=1188 (721 censored at 0, 462 uncensored, 5 censored at 1)

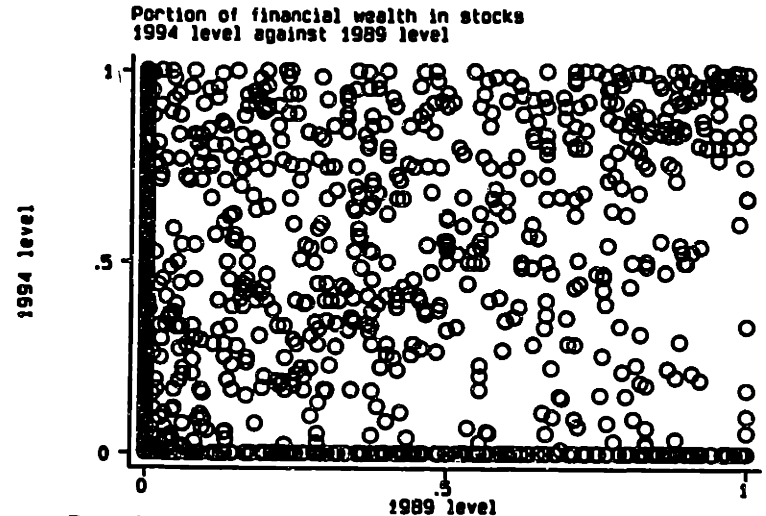


Fig.1a The fixed cost 'stock angle', 1989 and 1994 levels

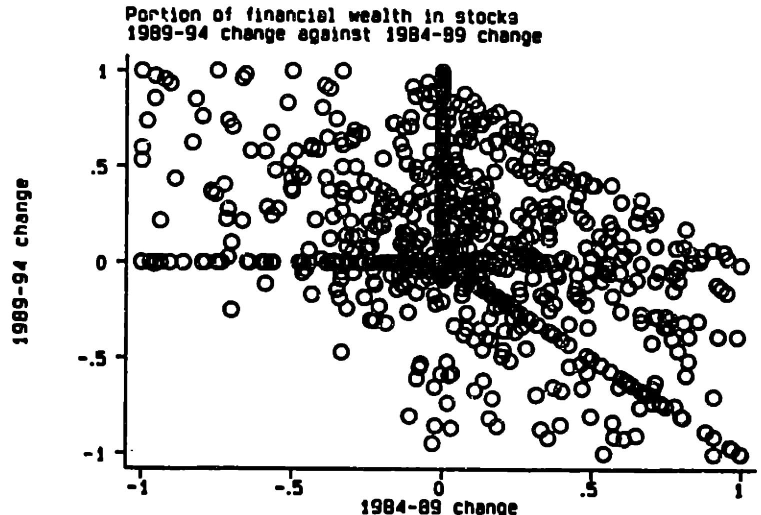


Fig.1c The fixed cost 'stock triplet'

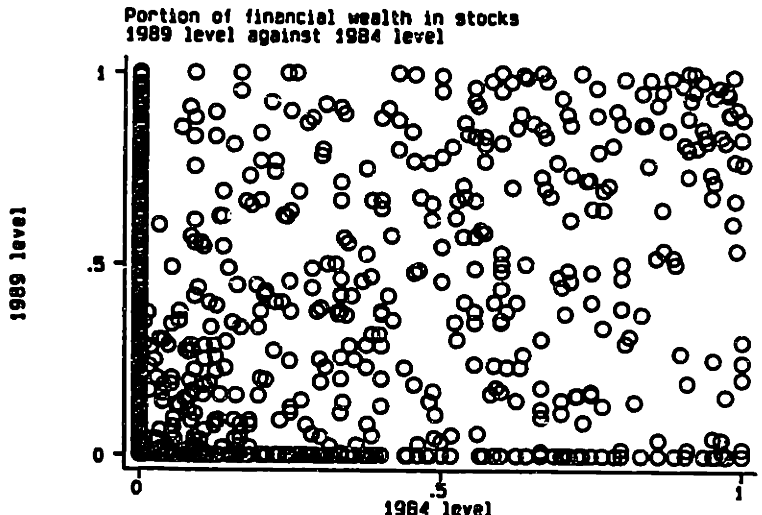


Fig.1b The fixed cost 'stock angle', 1984 and 1989 levels

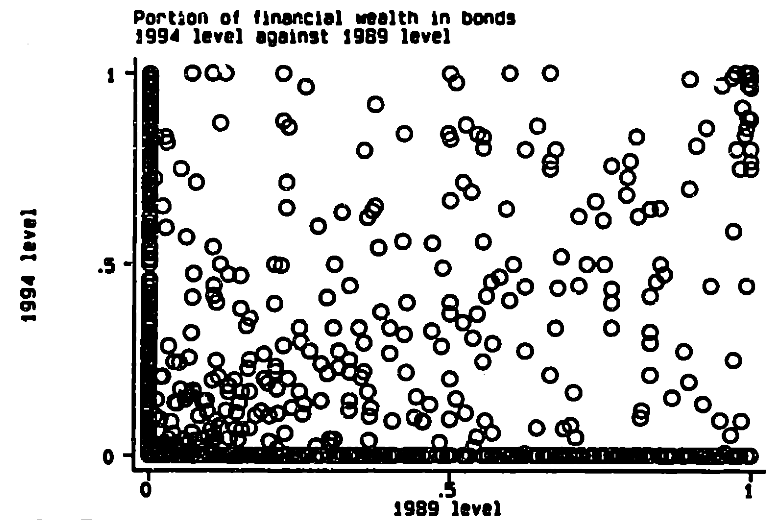


Fig.2a The fixed cost 'bond angle', 1989 and 1994 levels

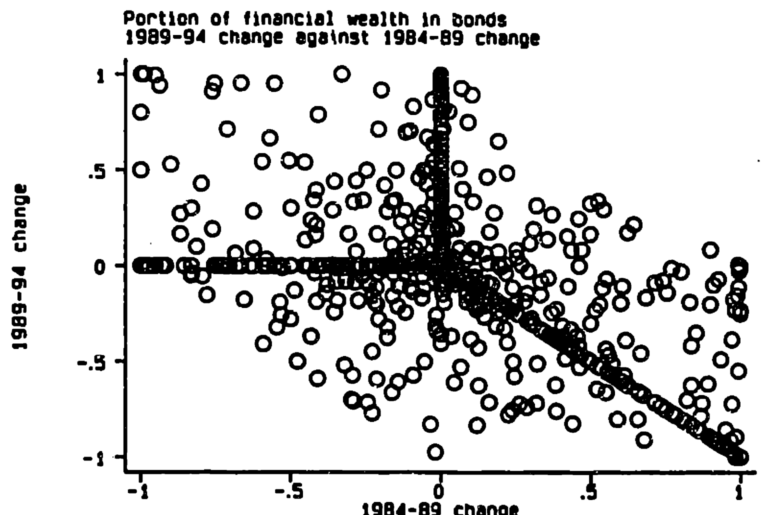


Fig.2c The fixed cost 'bond triplet'

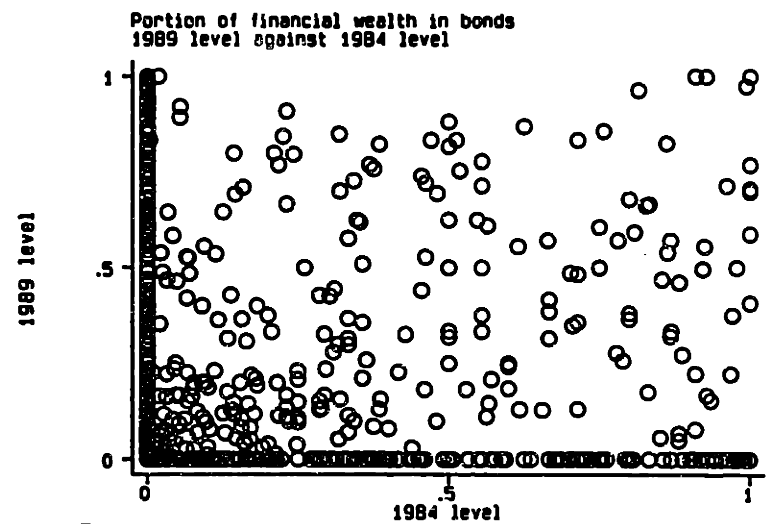


Fig.2b The fixed cost 'bond angle', 1984 and 1989 levels

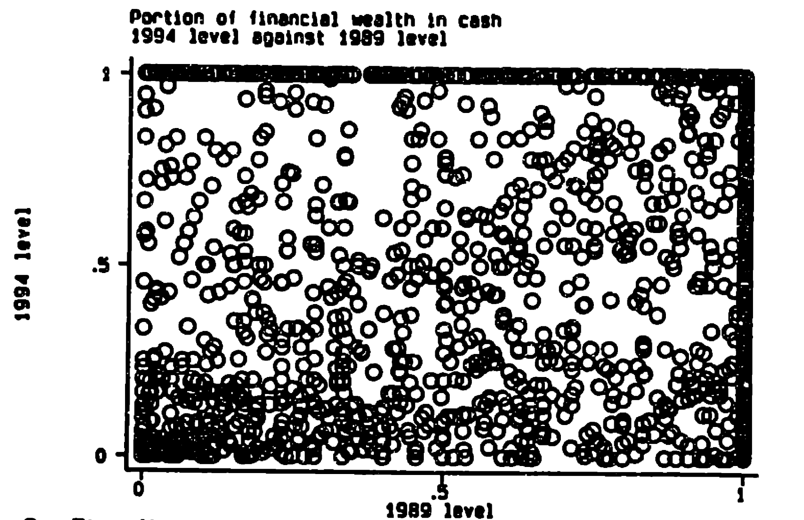


Fig.3a The fixed cost 'cash angle', 1989 and 1994 levels

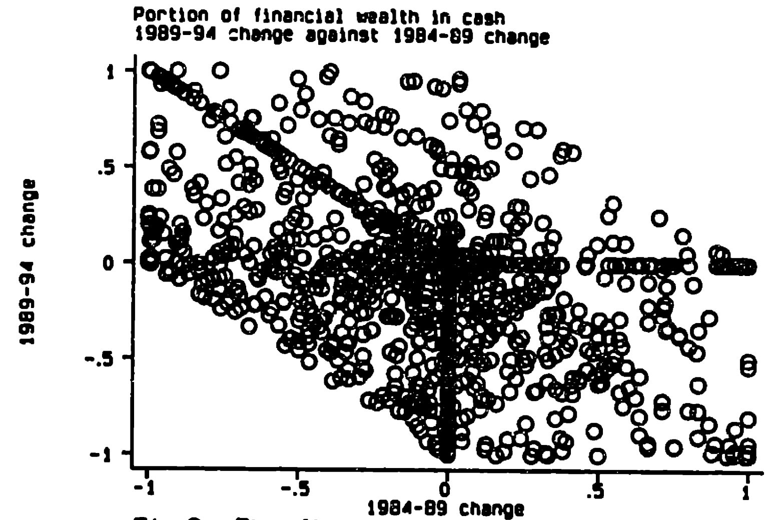


Fig.3c The fixed cost 'cash triplet'

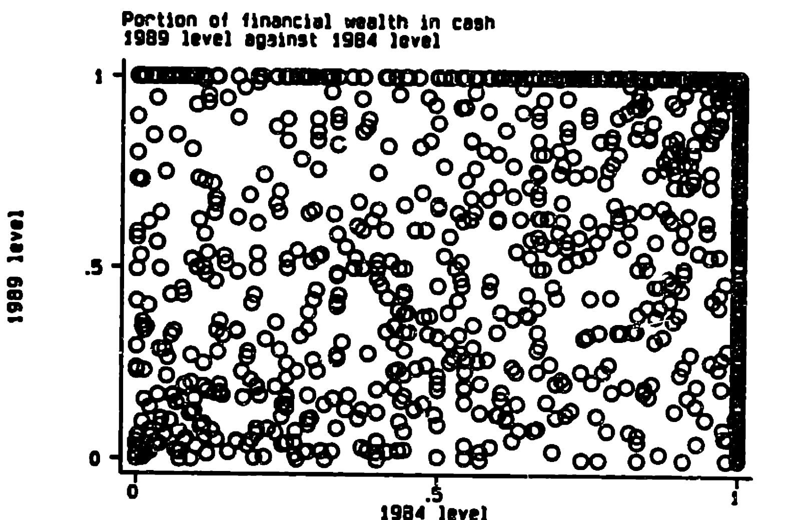


Fig.3b The fixed cost 'cash angle', 1984 and 1989 levels

Chapter 3

General Equilibrium Analysis of Limited Participation and Stock Market Entry

3.1 Motivation

I argued in Chapter 1 that risk aversion estimates based on the CCAPM decrease to much more plausible levels once we focus on stockholders. Therefore limited stock market participation should be considered part of the solution to the equity premium puzzle. These results were all based on Euler equations, but the intuition for why limited stock market participation generate a higher equity premium in general equilibrium is clear. When only a fraction of the population hold all the stock market risk, their consumption endogenously becomes more correlated with dividends than it would have been if all agents shared the risk.

This chapter focuses on the general equilibrium implications on limited stock market participation, with the specific purpose of understanding the effects of the dramatic increase in participation in the US since the 1950s. I first document the increase in participation. An overlapping generations model with a stock and a bond market is then constructed and the effects of random, costless entry are analyzed by comparing steady states with different levels of stock market participation[1]. The main finding is that if risk aversion is sufficiently low (for example under log utility), a steady state with a higher level of participation will have a higher stock price than a steady state with lower participation. This finding suggests that a strong increase in participation is likely to cause a stock market boom, a result which is interesting given the current events in the US stock market. In the OLG model, an increase in the stock market index induced by higher participation drives the price-earnings ratio up and the equity premium down, with little effect on the bond price and the bond return. To understand the nature of this result, I first consider the importance of the use of a production economy and an OLG structure. Basak and Cuoco (1997) construct a finite horizon exchange economy to analyze the effect of limited participation on asset prices and returns. By using an elegant continuous time setup, they are able to derive closed form solutions. For the log utility case they find that the stock price is unaffected by entry. I argue that this result is due to the particular features of an exchange economy and suggest why a (more realistic) production economy can lead to a stock price increase upon entry, consistent with the recent US developments. This is true more so if the production economy takes the form of an OLG model.

I next turn to the issue of endogenous entry. Chapter 2 documented the importance of fixed entry costs using data from the PSID, and analyzed which factors makes a household more likely to participate in the stock market. It is clear that the reason for nonparticipation matters for the general equilibrium effects of limited stock market participation. Suppose for example that stockholders are much less risk averse than nonstockholders. Then as more agents enter, the average risk aversion of stockholders increase and it is not a priori clear whether increased participation should still lead to higher stock prices and a lower equity premium. To analyze endogenous entry, I make the fundamental assumptions that the fixed entry cost is a utility cost of stock market entry and that households are distributed along a characteristic which affects the desirability of stockholding. This causes some to stay out of the market and some to enter. The cutoff depends on the size of the utility cost and a decrease in this cost will induce entry. A natural interpretation of the utility cost is that it represents leisure time spent gathering information about the stock market. Decreases in the utility cost then correspond to

[1] By 'random costless' entry, I refer to a scenario with identical agents some of whom are exogenously restricted from participating in the stock market. For agents who are not restricted, participation is assumed to be free (no trading or information costs).

'financial information innovations' with the introduction of mutual funds as the main example. These funds make it much easier to participate in the stock market than it was when direct stockholding was the only option. I deliberately do not introduce monetary costs of entry. This is firstly because I think most of the cost of entering the stock market is time spent understanding it (time which would otherwise have been spent as leisure time, not work time). Secondly, assuming that the entry cost represents a real resource loss would imply that as more households enter, less resources are available for consumption or investment[2]. To the extent that nonparticipation is caused in part by a general lack of knowledge about its potential benefits, my model would overstate the resource costs needed to induce a given fraction of households to stay out of the stock market. This could distort the results, and I therefore choose to assume that the entry cost is purely a utility cost. Given this, I focus on two dimensions of heterogeneity, human capital and risk aversion. In the OLG setup used here, wages are not stochastic and the heterogeneity in human capital is intended to capture differences in mean labor income. The empirical results (probit models) of Chapter 2 pointed towards this as a highly significant determinant of the decision to hold stocks. Future work will consider heterogeneity in human capital uncertainty, consistent with the finding of a significant variance-effect in Chapter 2. Heterogeneity in risk aversion is considered because theory implies that it should be one of the main factors separating households into stockholders and nonstockholders, a prediction which several papers have found born out in the data (see Chapter 2 for references, or refer to Table 9 of that chapter).

The last part of this chapter briefly discusses the issue of expected versus unexpected entry. If stock market entry is expected ahead of time, should asset prices change when the realization of future entry is made or when entry actually takes place? If markets are efficient and thus incorporate the information about future entry, one might think that asset prices should move at the time the realization of future entry is made. On the other hand one could argue that since risk sharing does not increase until entry actually takes place, returns in the periods before entry occurs should be unaffected.

The purpose of the model at this point is mainly qualitative. A precise quantification of

[2]In theory it could be more resources, not less. This would happen if the decrease in entry cost needed to induce entry was so large that the decrease in costs per entrant more than outweighed the effect of more households paying a given cost.

the effects will require a large scale simulation (realistic life times, realistic income processes, careful calibration of actual wealth distributions). The idea is to determine the direction of change in asset returns as participation is increased. In this sense the analysis in this chapter is more speculative than that of the earlier ones.

3.2 Documenting the trend in participation and risk sharing

To document the strong upward trend in stock market participation over the postwar period, data were collected from several sources. Fig. 1 is based on the NYSE shareownership survey. The definition of stockownership used in this survey is broad and includes stocks in equity mutual funds and pension plans[3]. The figure shows an estimate of the total number of stockholders in the US divided by the US population 20 years and older. The increase in participation from about 6 percent in 1952 to 29 percent in 1990 is dramatic. Participation has increased further since then. In the Survey of Consumer Finances, the proportion of families who are stockholders is 31.7 in 1989, 37.3 in 1992 and 41.1 in 1995, cf. Kennickell, Starr-McCluer and Sunden (1997). These numbers are based on a similar broad definition of stockownership and are consistent with the numbers based on the PSID given in Chapter 2.

For comparison Fig. 2a shows the percent of tax returns with positive dividends. This series is based on individual income tax statistics from the IRS. To obtain a series which is comparable to the one in Fig. 1, I furthermore calculated an estimate of the percent of individuals of tax filing age who filed returns with positive dividends. This was done by assuming that all joint filers who report positive dividends hold their stocks jointly. The number of joint returns with positive dividends was then added to the total number of returns with positive dividends. The total was divided by the number of joint returns plus the total number of returns. The upward trend in stock market participation is apparent from both series.

The IRS data are less reliable than the NYSE data for several reasons. However, the sign of the bias is not obvious. The tax data understate the proportion of stockholders by not including dividends on stock held in IRA's, Keogh's or defined contribution pension plans. Furthermore

[3]The methodology varies a bit between years. See the individual surveys for details. The survey in its original form was discontinued in 1990. The 1995 survey relies on the Survey of Consumer Finances, and contains a useful discussion of various sources of information about stockownership.

there is a small dividend exclusion such that filers with dividends below the exclusion are allowed to report zero dividend income. In addition, in any given year there will be some stocks which do not pay dividends. On the other hand, the dividends reported on tax forms include dividends from some non-equity mutual funds[4]. I did not attempt any corrections for these problems. The IRS data are used here despite the problems, because they provide an opportunity to quantify the increase in risk sharing. It is possible that the new entrants only hold small quantities of stock. Thus risk sharing may not have increased as dramatically as Fig. 1 and 2a suggests. By analyzing the concentration of dividend income on tax returns, it can be determined whether this is the case. In the IRS statistics, tabulations sorted by income are provided. Fig. 2b plots on the horizontal axis the cumulative fraction of income earned. The vertical axis shows the cumulative percent of dividends earned (with tax returns still sorted by income). The interpretation of a given point is that the bottom x percent of returns, judged by income, earned y percent of total dividends reported. The diagonal corresponds to full equality. The ratio of the area between a given curve and the diagonal to the total area under the diagonal can be used as a measure of risk sharing[5]. The graph documents a dramatic increase in risk sharing. In 1954 the top 20 percent of income earners earned about three quarters of dividends. In 1994 the number was down to about 40 percent. This finding confirms and extends the results of Poterba and Samwick (1995) based on comparison of the 1983 and the 1992 Survey of Consumer Finances. In 1983 SCF households with incomes of \$100000 or more (for the household head, in 1992 dollars) owned 67.95 percent of the stock owned by all households in this survey. In the 1992 SCF this percentage had decreased to 45.69 despite the fact that a larger fraction of households had incomes of \$100000 or more in 1992 dollars in 1992 than in 1983[6].

[4]I called the Vanguard Group concerning this. They reported that 1099-DIV tax forms (those on which the tax statistics are based) were used for all types of mutual funds except the money market mutual funds.

[5]This measure of dividend inequality is quite similar to the Gini coefficients used as a measure of income inequality. The correspondence is not perfect since the present graph has cumulative percent of income earned on the horisontal axis instead of percent of dividend earners.

[6]See Poterba and Samwick (1995), Table 10.

3.3 An OLG model of limited stock market participation

3.3.1 Setup

The structure of model is a standard OLG setup. Agents live for two periods. Their only endowment is one unit of labor in the first period of life. Labor is supplied inelastically to the market, and wages are partly used for consumption when young, and partly saved to provide for consumption when old. Each generation of measure one. All markets are competitive. There is one consumption good which is numeraire.

Financial structure

Three assets are present in the model, stocks, one-period corporate bonds, and one-period 'household' bonds. Stocks represent ownership of the one existing (aggregate) firm and are in unit net supply. No new stock issues are allowed. Corporate bonds are issued by firms to finance purchase of capital needed for production. In equilibrium corporate bonds will be default free, an issue which I will return to after outlining the model. We can equivalently think of this as the firm renting capital from a bank at the rate $r_t = (1/P^b_{t-1} - 1 + d)$, where d is the rate of depreciation and P^b is the bond price. Household bonds are issued by some young households and purchased by other young households and are in zero net supply. In equilibrium household bonds will be default free. This is ensured by assuming that marginal utility of consumption goes to infinity at zero consumption and that no personal bankruptcy procedures are in force. Therefore the two types of bonds must earn the same rate of interest and it is not necessary to distinguish between them. The net aggregate dollar amount of bonds held will equal the capital stock available for production in the following period.

Technology

A unit of the consumption good can at any time be instantaneously converted to one unit of capital, implying a price of capital of one. The production technology is a described by a Cobb-Douglas production function with labor and capital as inputs, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha-\varepsilon}$. A_t is a productivity shock equal to $\overline{A}_t$ with probability 1/2 and $\underline{A}_t$ with probability 1/2. Y_t denotes physical output. Technology exhibits constant returns to scale if $\varepsilon = 0$, and decreasing returns to scale if $\varepsilon > 0$. I will argue below that constant returns to scale is inconsistent with a positive stock price in general equilibrium (this is well-known for the deterministic case and carries over to the case with productivity shocks). I therefore focus on the case of decreasing returns to scale.

Timing

The timing of the firm's actions is as follows. In period t-1 the firm issues corporate bonds at price P^b_{t-1}. Each of these bonds promise to pay one unit of the consumption good in period t. Firms use the proceeds to buy capital to use for production in period t. At the beginning of period t, firms hire labor L_t at the wage W_t. These inputs are then used to produce output according to the above technology. After output is realized, factors are paid at the agreed upon rates and remaining profits are paid to stockholders as dividends. The setup with factor prices set in advance is similar to Diamond (1967). The existence of a stock market is motivated by residual risk when factor payments are set in advance.

Participation

Of the generation born at t, a fraction λ_t are allowed to hold stocks and both types of bonds, the remaining fraction 1-λ_t are restricted from participating in the stock market but may participate in the bond markets. For now λ_t is exogenous, but will be endogenized in the section on the effects of different reasons for entry.

3.3.2 Optimization problems

Stockholders born at t maximize the expected net present value of utility. Preferences are assumed time separable and of the constant relative risk aversion form.

$$\begin{aligned} &\underset{a^s_t,\omega_t}{Max}\ \tfrac{1}{1-\gamma}\left(C^s_{1t}\right)^{1-\gamma} + \delta E_t\left[\tfrac{1}{1-\gamma}\left(C^s_{2t+1}\right)^{1-\gamma}\right] \\ st.\ & \begin{array}{l} C^s_{1t} = a^s_t W_t \\ C^s_{2t+1} = \left(1/P^b_t + \omega_t\left(\Pi_{t+1} + P^s_{t+1} - 1/P^b_t\right)\right)\left(1 - a^s_t\right) W_t \end{array} \end{aligned} \tag{3.1}$$

a^s_t denotes the average propensity to consume out of wages, W_t. ω_t refers to the share of savings invested in stocks. Given the simple structure for dividends, closed form solutions for a_t^s and ω_t can be derived. Nonstockholders born at t maximize the same objective function, but are exogenously restricted to not hold stocks.

$$\begin{aligned} &\underset{a_t^{ns}}{Max}\ \tfrac{1}{1-\gamma}\left(C_{1t}^{ns}\right)^{1-\gamma} + \delta E_t\left[\tfrac{1}{1-\gamma}\left(C_{2t+1}^{ns}\right)^{1-\gamma}\right] \\ &st. \quad \begin{array}{l} C_{1t}^{ns} = a_t^{ns} W_t \\ C_{2t+1}^{ns} = \left(1/P_t^b\right)\left(1-a_t^{ns}\right) W_t \end{array} \end{aligned} \tag{3.2}$$

The firm maximizes the stock market value to current stockholders. This objective function is well defined since stockholders face dynamically complete markets and thus unique Arrow-Debreu prices. Without adjustment costs, current choices of K_t, L_t do not affect P_{t+1}^s. The firm's maximization problem then reduces to period by period maximization of expected one period profits evaluated at the Arrow-Debreu prices.

$$\begin{aligned} &\underset{K_t,L_t}{Max}\left(\overline{AD}_t\overline{\Pi}_t + \underline{AD}_t\underline{\Pi}_t\right) \\ &st. \quad \begin{array}{l} \overline{\Pi}_t = \overline{A}_t K_t^{\alpha} L_t^{1-\alpha-\varepsilon} - w_t L_t - (1/P_{t-1}^b - 1 + d)K_t \\ \underline{\Pi}_t = \underline{A}_t K_t^{\alpha} L_t^{1-\alpha-\varepsilon} - w_t L_t - (1/P_{t-1}^b - 1 + d)K_t \end{array} \end{aligned} \tag{3.3}$$

The Arrow-Debreu prices implicit in P_{t-1}^s, P_{t-1}^b are given by:

$$\begin{bmatrix} \overline{AD}_t \\ \underline{AD}_t \end{bmatrix} = \begin{bmatrix} \overline{\Pi}_t + \overline{P}_t^s & \underline{\Pi}_t + \underline{P}_t^s \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} P_{t-1}^s \\ P_{t-1}^b \end{bmatrix} \tag{3.4}$$

In other words,

$$\begin{aligned} P_t^s &= E_t\left[MRS_t^s\left(\Pi_t + P_{t+1}^s\right)\right] = \tfrac{1}{2}\left[\overline{MRS^s}_t\left(\overline{\Pi}_t + \overline{P}_{t+1}^s\right) + \underline{MRS^s}_t\left(\underline{\Pi}_t + \underline{P}_{t+1}^s\right)\right] \\ &= \tfrac{1}{2}\left(\overline{AD}_t\left(\overline{\Pi}_t + \overline{P}_{t+1}^s\right) + \underline{AD}_t\left(\underline{\Pi}_t + \underline{P}_{t+1}^s\right)\right) \end{aligned} \tag{3.5}$$

where MRS_t^s denotes the marginal rate of substitution between consumption at t and t+1 for stockholders. At the optimal levels of K_t and L_t the firms objective function and thus P_t^s equals zero in each period and state if the production function exhibits constant returns to scale to (K, L). Since stock prices are positive in reality, I restrict the analysis to the case of $\varepsilon > 0$.

Two comments about the way I have modelled firms are in order. Firstly, by using decreasing returns to scale to ensure a positive stock price in equilibrium, I am implicitly assuming restricted entry of firms. If not, new smaller firms could enter and be more profitable (the optimal firm size goes to zero under decreasing returns to scale). Alternatively, assume that there is a fixed, pecuniary, entry cost for firms of just the right size to make one firm optimal. Or think of the firm as an aggregate of many smaller but heterogeneous firms facing a fixed entry cost. The last entrant makes zero profits evaluated at the Arrow-Debreu prices. The inframarginal firms make profits and have a positive stock price. The latter interpretation is most plausible and is consistent with the assumption of competitive behavior.

Secondly, I have assumed pure debt financing of the firm's need to raise capital for production factors. Since stockholders face dynamically complete markets, the Modigliani-Miller theorem applies and stockholders are indifferent between debt and equity financing. Whether it issues corporate bonds or new shares, the firm must give away claims with time t value $\left(1-(1-d)\left(\overline{AD}_t+\underline{AD}_t\right)\right)K_t$ to raise enough money to buy K_t units of capital for production. Of course, the firm cannot issue unlimited amounts of riskless debt. In all results shown below, parameter values are chosen to ensure that the firm has non-zero profits in both the high and the low productivity state (ignoring this issue would imply that limited liability for firms was not assumed).

3.3.3 Equilibrium relations

A competitive rational expectations equilibrium consists of a process for prices $\{P_t^b, P_t^s, W_t\}$, and quantities $\{C_{1t}^s, C_{2t+1}^s, C_{1t}^{ns}, C_{2t+1}^{ns}, K_t, L_t\}$ such that households maximize utility given current and expected future prices, the firm maximizes its value to stockholders given current and expected future prices, and markets for goods, assets and production factors clear at each date. With consumption as numeraire, this results in four equilibrium relations for each period.

$$\begin{aligned}
&\text{Stocks: } \lambda_t\omega_t\left(1-a_t^s\right)W_t = P_t^s \\
&\text{Bonds: } \lambda_t\left(1-\omega_t\right)\left(1-a_t^s\right)W_t + \left(1-\lambda_t\right)\left(1-a_t^{ns}\right)W_t = K_{t+1} \\
&\text{Labor: } \left(1-\alpha-\varepsilon\right)\left(\tfrac{K_t}{L_t}\right)^{\alpha} L_t^{-\varepsilon}\left(\overline{AD}_t\overline{A}_t + \underline{AD}_t\underline{A}_t\right) = W_t\left(\overline{AD}_t+\underline{AD}_t\right) \\
&\text{Capital: } \alpha\left(\tfrac{K_t}{L_t}\right)^{\alpha-1} L_t^{-\varepsilon}\left(\overline{AD}_t\overline{A}_t + \underline{AD}_t\underline{A}_t\right) = \left(1/P_{t-1}^b - 1 + d\right)\left(\overline{AD}_t+\underline{AD}_t\right)
\end{aligned} \tag{3.6}$$

where the Arrow-Debreu prices can be stated explicitly in terms of profits, the stock price and the bond price using 3.4 above. $L_t = 1$ in equilibrium given the unit endowment of labor for each young agent. The only state variable at t is K_t. λ_t enters the problem as a parameter. Past values of λ are not relevant over and above their importance for determining, K_t. The equilibrium relations for period t and t+1 are linked through the definition of the Arrow-Debreu prices. This makes it impossible to solve the problem without further assumptions. I assume that productivity is i.i.d over time i.e. that

$$A_t = \left\{ \begin{array}{l} \overline{A}, \text{ probability } 1/2 \\ \underline{A}, \text{ probability } 1/2 \end{array} \right\} \text{for all t.} \tag{3.7}$$

Given this, and given the assumption that the wage is set before the realization of the productivity shock, there exists a steady state in which all prices and the levels of capital (and labor) are constant over time. This follows from the two period OLG structure in which the realization of A_t only affects the old, whereas K_{t+1} is determined by the savings of those who are young at t. Dropping the time subscripts from the above system of equations then results in four equations in four unknowns, P^b, P^s, W and K which can be solved numerically.

The parameter values used for the results presented below are:

$\overline{A} = 1.05^{20}$, $\underline{A} = 1^{20}$: Productivity growth is either 5 percent per year or 0 percent per year (with agents being young for 20 years and old for 20 years).

$d = 0.5$: Half of capital depreciates during one 20-year period.

$\varepsilon = 0.3$: Decreasing returns to scale with coefficients in the Cobb-Douglas production function summing to 0.7.

$\alpha = 0.2$: Coefficient on capital in the Cobb-Douglas production function.

$\delta = (1/1.02)^{20}$: Agents discount the future at 2 percent per year.

The above parameters are fixed for all cases. As emphasized in the introduction, the model is too simple to generate precise quantitative predictions. The qualitative results are robust to changes in the above parameter values.

Risk aversion is varied and is heterogeneous in case 2a. Labor endowments are heterogeneous in case 2b but otherwise equal to one unit per household when young.

3.3.4 Case 1. Random costless entry, $\lambda \in]0;1]$

Log utility

The solution for the log utility case is shown in Fig.3. Comparing steady states, the effect of higher stock market participation is to push up the stock price with no effect on the bond price. The expected stock return and the equity premium decreases. Capital and wages increase. The intuition for these results is as follows.

Under log utility the savings choice of a household is unaffected by whether the household is allowed to hold stocks or not. The only effect of lifting the nonparticipation restriction for a given household is a change in its portfolio choice. As more households are allowed to buy stocks, the stock demand increase and the bond demand decrease implying a tendency for the stock price to increase and the bond prices to decrease. At the initial choice of labor and capital this has three effects on firms' factor demands. Firstly, the value of each unit of capital left over from production $(1-d)\left(\overline{AD} + \underline{AD}\right) = (1-d)P^b$ falls. This increases firm's cost of capital and decreases the demand for capital and labor (larger effect on capital). Secondly, wage costs $W\left(\overline{AD} + \underline{AD}\right) = WP^b$ fall, which increases the firm's demand for capital and labor (larger effect on labor). Thirdly, the value of the firm's revenues evaluated at the Arrow-Debreu prices $K^{\alpha}L^{1-\alpha-\varepsilon}\left(\overline{AD}\,\overline{A} + \underline{AD}\,\underline{A}\right)$ changes which in turn changes the optimal level of inputs (the size of this effect is proportionally the same for capital and labor). Since the equilibrium illustrated in Fig. 3 shows that the wage is increasing in participation, the net effect on labor demand must be positive. Higher wages in turn increase savings of the young and thus the demand for bonds and stocks. This tends to push up the bond price and explains why the bond price can be unaffected by the level of participation.

It is important to emphasize that the stock price is increasing in participation despite the fact that capital increases and technology exhibits decreasing returns to scale, and despite the increase in wage costs in equilibrium. Profits in both states of the world are decreasing in participation, and the price-earnings ratio (calculated as the ratio of the price to one 20th of the average profits in the two states) is higher in steady states with higher participation. Furthermore, the equity premium is decreasing in participation, despite the fact that capital, and thus the aggregate amount of production risk to be borne, is higher. These two observations show the strength of increased participation as a vehicle to increase risk sharing and push up the value of the stock market. For the parameter values chosen, the stock price increases by about 40 percent when participation is changed (exogenously) from 5 percent to 100 percent of the population. An increase in participation from 5 to 40 percent of the population increases the stock price by about 16 percent. The effects on the equity premium are equally strong since the bond rate is unaffected by the level of stock market participation. However, given the highly stylized model, these numbers are only suggestive and the main point is to illustrate the qualitative effects and the underlying mechanisms.

Understanding the nature of the stock market boom

The lower equity premium in steady states with higher levels of participation is consistent with the decreasing price of risk found by Basak and Cuoco (1997) in their exchange economy. But whereas the stock price was unaffected by participation in their model (for log utility), my model generates a large stock market boom upon entry[7]. The difference lies partly in the use of a (more realistic) production economy and partly in the OLG structure. To show this I constructed a very simple two period model of an exchange economy (a tree-model).

In this model households do not earn wages but are endowed with $e_0 = 1$ units of the consumption good (fruit) at time 0 and ownership of the one existing tree. There is a measure one of households of which a fraction λ is not allowed to participate in the stock market. Stocks represent ownership of the tree. Fruit at t=1 is stochastic and equals a high value D_g with probability $\frac{1}{2}$ and a low value D_b with probability $\frac{1}{2}$. Bonds are in zero net supply. The optimization problem of stockholders and nonstockholders are as for the OLG model but with the value of their endowment $(e_0 + P_0^s)$ replacing the wage and the fruit of the tree replacing profits from production. Choosing the consumption good as numeraire, the equilibrium relations are simply:

$$\begin{aligned} &\text{Bonds: } \lambda(1-\omega_0)(1-a_0^s)(P_0^s+e_0)+(1-\lambda)(1-a_0^{ns})(P_0^s+e_0)=0 \\ &\text{Stocks: } \lambda\omega_0(1-a_0^s)(P_0^s+e_0)=P_0^s \end{aligned} \tag{3.8}$$

[7] Again, this choice of words is not quite accurate since I am comparing prices across steady states of different economies.

The solution for the stock and bond price and the date 0 consumption levels in the log utility case is[8]:

$$
\begin{aligned}
&P_0^s = \delta e_0 .\ P_0^b = \frac{-\left(\frac{2}{\lambda}-1\right)\delta e_0(D_g+D_b)-\sqrt{\left(\left(\frac{2}{\lambda}-1\right)\delta e_0(D_g+D_b)\right)^2+\frac{8}{\lambda}D_g D_b(\delta e_0)^2\left(2-\frac{2}{\lambda}\right)}}{-\frac{4}{\lambda}D_g D_b} \\
&C_0^s = C_0^{ns} = \frac{1}{1+\delta}\left(P_0^s + e_0\right)
\end{aligned}
\tag{3.9}
$$

The stock price and thus the expected stock return is unaffected by the level of stock market participation, just like Basak and Cuoco (1997) found in their more complicated exchange economy. The bond price is decreasing in participation. Thus as more agents are allowed to hold stocks the bond return increases pushing the equity premium down as illustrated in Fig. 4.

This result is driven by the combination of a fixed aggregate level of fruit at date 0 and by an endowment effect caused by households being endowed with the tree at date 0. As more agents are given the opportunity to hold stocks, an upward pressure on the stock price and a downward pressure on the bond price forms as the 'new' stockholders switch some of their savings from bonds to stocks. But with a higher stock price, the value of each agent's endowment increases, and all agents attempt to consume more at date 0. In an exchange economy this is impossible in equilibrium. Furthermore, in the log utility case the average propensity to consume is unaffected by asset returns and the only way to prevent excess demand for goods at date 0 is for the stock price to stay constant. For constant relative risk aversion above or below one it is possible for the stock price to change in equilibrium, but only for risk aversion less than one does the model generate a stock market boom upon entry.

It follows from this example, that moving to a production economy will enable movements in the stock price even for the log utility case, since the production economy allows resources to be transferred between periods (a storage technology would have a similar effect)[9]. If the production economy in addition take the form of an OLG model, even larger stock price changes are possible because no one is endowed with the stock market at birth, thus removing the

[8] I have picked the one of two solutions for P_0^b which makes economic sense.

[9] I constructed a two period production economy to confirm this intuition. As long as capital depreciates less than fully in production, the equilibrium stock price in increasing in the level of participation, whereas the bond price is unaffected as was the case for the OLG model.

endowment effect. In reality some agents do inherit stocks. Allowing bequests will reintroduce an endowment effect in the OLG model. This would tend to diminish the size of the stock market boom which is consistent with the model.

Relative risk aversion different from one

The above has focused on the log utility case, partly to simplify the analysis and more clearly be able to discuss the differences between an exchange and a production economy. Although Chapter 1 argued that risk aversion estimates are much lower once we focus on stockholders, risk aversion estimates were most often above one even for the set of stockholders. I therefore consider the effect of higher risk aversion (but with risk aversion still homogenous across agents). Fig. 5 shows the results for the OLG model with $\gamma = 2.75$. As risk aversion is increased above one, the qualitative features of the solution change. For a risk aversion level of about 2.75 and higher, the steady state stock price and price earnings ratio is monotonically decreasing in the level of stock market participation. The equity premium is increasing in participation at low participation levels but decreasing for participation levels of about 50 percent and upwards.

The fact that the equity premium may increase as risk sharing increases seems counterintuitive, but consider the consumption CAPM expression for the equity premium between date t and date t+1, $E_t R_{s,t+1} - R_{f,t+1} = -R_{f,t+1} cov\left(\left(\frac{C^s_{t+1}}{C^s_t}\right)^{-\gamma}, R_{s,t+1}\right)$. Increased risk sharing decreases the absolute value of the covariance factor but, for risk aversion above one, forces up the bond rate. The net effect on the equity premium is positive for λ low and γ sufficiently above one.

The reason that the bond rate is increasing in stock market participation when agents are more risk averse than log utility, is related to the propensity to consume out of wealth being dependent of asset returns in this case. Allowing an agent to hold stocks has several effects on her date 0 consumption. The mean return on savings increases, causing a negative substitution effect and a positive income effect on present consumption. However, stocks are risky so by holding a positive fraction of savings in stocks, the agent endogenously causes her future consumption to be risky. This again has two effects on present consumption. Risky consumption tomorrow is less attractive than certain consumption today leading the agent to save less (a risk aversion or 'bird in the hand' effect). But higher uncertainty of future returns makes it necessary to save more to avoid very low levels of consumption when stock returns are low. With a slight abuse of terms we can refer to this as a precautionary savings effect (this term is commonly used with reference to exogenous income risk, but the present effect of asset return risk is similar). For log utility all four effects cancel. For more risk averse agents, $\gamma > 1$, the income effect dominates the substitution effect causing an increase in date 0 consumption, but the precautionary savings effect dominates the risk aversion effect causing a decrease in date 0 consumption. See Sandmo (1969), (1970) and Merton (1969) for a discussion of these effects. For the stock returns generated by the present OLG model, the net effect of being allowed to participate in the stock market is an increase in date 0 consumption. The implication for the present model is that as more households are allowed to hold stocks, a larger fraction of wages are consumed instead of saved. This reduces the demand for both stocks and bonds, leading both stock and bond prices to decrease in participation for risk aversion sufficiently above one. As a consequence of the lower supply of resources to be used as capital in production, the equilibrium level of capital is decreasing in participation for high levels of risk aversion.

If a value of risk aversion below one is used, the converse results obtain and the model generates an even larger stock market boom than for log utility.

I conclude that within an OLG model, the effect of random costless entry is most likely to be a stock market boom and a decrease in the equity premium. The model generates this prediction for the log utility case or for risk aversion levels above but close to one (and certainly for risk aversion lower than one, but this is less realistic)[10]. This conclusion is based on comparisons of steady states in economies with different levels of stock market participation. I have not solved for the transitional dynamics within a given economy, since this is complicated in the OLG setup. Since the steady state stock price is higher in steady states with higher participation, capital gains will tend to be positive in during a transition period with increasing participation. Thus realized stock returns (and the realized premium of stocks over bonds) can be high during the transition period despite the lower equity premium in the new steady state. Thus the joint observation of massive entry and high stock returns for the US during recent years is not

[10] In a related literature, several authors have argued for a positive relation between the real stock price and the fraction of the population in age groups with traditionally large proportions of stockholders/large stockholdings per stockholder. Bakshi and Chen (1994) and Bergantino (1997) present evidence in favor of such an effect. Poterba (1997) finds less conclusive results.

inconsistent with the model presented here.

Case 3 below is a simple attempt to address a part of the transitional dynamics and the timing of changes in asset prices and returns. In particular, I consider a setup where an expectation exists of a future step change in participation.

3.3.5 Case 2. Fixed costs and endogenous entry

As discussed in the introduction I assume a fixed utility cost of entry. This cost is assumed to be the same for all agents[11]. The assumption of a homogeneous utility cost of stock market entry simplifies the solution of the model with endogenous entry. With agents distributed along a given characteristic, it is immediately apparent in which order they will self-select into the stockholder category as the utility cost is lowered. Thus no formal modelling of the decrease in the utility cost is needed. When agents are heterogeneous in terms of (mean) human capital endowments, those with higher endowments will gain more from investing in the stock market. With heterogeneous risk aversion, the least risk averse agents will be the ones who chose to enter the stock market at a given entry cost.

With heterogeneity, stock and bond demands must be integrated over households. For heterogeneous human capital this is straightforward, since consumption is linear in wealth and portfolio weights are independent of wealth under CRRA utility. In other words, there exists an aggregate stockholder and an aggregate nonstockholder. This is an implication of the results of Rubinstein (1974) and, as pointed out there, only holds when all individuals have identical discount rates (and beliefs). For heterogeneous coefficients of relative risk aversion, I numerically integrate over households.

a. Heterogeneous human capital endowments

In the standard OLG setup used above, each agent is endowed with one unit of labor when young. Empirically, households differ dramatically in the level of their labor income. Assume therefore as an alternative that agents have heterogeneous human capital endowments. I chose

[11] The fixed entry cost assumption has been used in the asset pricing literature by e.g. Merton (1987) (a fixed information cost per stock purchased leading to limited diversification) and Allen and Gale (1994) (a limited participation model of high volatility in the presence of liquidity shocks). In Merton's model agents are endowed with a given information set. In the model of Allen and Gale, the fixed cost is modelled as a dollar cost.

the following functional form:

$L(h) = \underline{L} + \beta h^2$, $\beta = 3\left(1 - \underline{L}\right)$, $h \in [0;1]$. The least productive agent has human capital endowment $\underline{L}$, which is set equal to 0.2. For comparison, β is chosen to ensure that the total human capital endowment equals one as for Case 1. A higher L for some households than others is intended to model that some are more productive for the same input of hours. Thus, we can interpret L is as the product of a unit endowment of time and a heterogeneous productivity parameter.

For simplicity I assume log utility. The solution is shown in Fig. 6. With agents sorted in increasing order of human capital endowments, households with higher index h will enter the stock market first. The main difference from Case 1 is that the stock price is higher and the expected stock return and the equity premium lower at each level of participation less than one. Bond rates are as for Case 1. These results are intuitive. At low levels of participation, the effect of increased participation is stronger in this setup, because each of the new participants is rich compared to the homogeneous agents of Case 1. Thus a large switch of resources from the bond market to the stock market is induced when a new agent enters. As the utility cost decreases enough to make many agents participate in the stock market, the effect of each additional participant is lower, since the last entrants are the poorest agents who have little savings to reallocate between financial markets. At full participation all variables are the same in the two models since aggregate resources are identical.

In sum, the effects of limited stock market participation on stocks and bond returns are qualitatively similar for the case with heterogeneity in human capital and the case of random costless entry. The quantitative effects of increased participation are larger at low initial participation levels in the case of human wealth heterogeneity than in the case of homogeneous agents.

b. Heterogeneous risk aversion

In order to get closed form solutions, the model of Basak and Cuoco (1997) assume log utility for restricted agents and $\gamma \geq 1$ for unrestricted agents. Thus as more agents are allowed to participate in the stock market, the average risk aversion of the stockholding population decreases unless all agents have log utility. This makes it harder to isolate the effect of entry, since it is unclear whether the price of risk decreases because more agents participate or because the average participant is less risk averse. With a fixed cost of entry the stock market entrants will tend to be more, not less, risk averse than the current participant. Is it then still the case that entry leads to a lower equity premium?

To analyze this issue I assume agents have heterogenous relative risk aversion and that the heterogeneity takes the following form:

$\gamma(h) = \overline{\gamma} - h(\overline{\gamma} - \underline{\gamma})$, $\overline{\gamma} > \underline{\gamma}$, $h \in [0;1]$.

The solution for $\underline{\gamma} = 0.5$ and $\overline{\gamma} = 2$ is shown in Fig. 7. As for Case 1 with $\gamma = 1$ the stock price and the price-earnings ratio is still higher and the equity premium still lower in steady states with higher levels of participation. If the whole range of risk aversion is shifted upwards by choosing $\underline{\gamma} = 1$ and $\overline{\gamma} = 6$ (not shown), the equity premium is still lower in steady states with higher participation levels, but the stock price is now lower with higher participation.

3.3.6 Case 3. Expected entry

To circumvent the problems of analyzing transitional dynamics of stock market entry in an OLG model, I use the simple exchange economy discussed earlier, but add one more period. This is obviously unsatisfactory in the sense that the use of a production economy and an OLG structure was shown to be important for the nature of the results. The reason for proceeding despite this is to show that entry does have an effect on returns even if it is expected ahead of time and that the expectation of future entry affects behavior ahead of time if risk aversion differs from one. Future work will extend the analysis of expected entry to a three period production economy.

With these caveats, suppose it is known at date 0 that all present nonstockholders will be allowed to hold stocks at date 1 with probability one. I am interested in determining firstly, whether this expectation has any effect on returns to holding stocks or bonds between date 0 and date 1, and secondly, whether the effect of entry, once it occurs, is different if it was anticipated. I analyze these issues in the context of a setup with random costless entry.

The basic setup is as outlined for the two period exchange economy. A continuum of households of measure one is present. Households are endowed with $e_0 = 1$ units of the consumption good (fruit) at time 0 and one unit of the tree. States are defined by realizations of dividends.

With an extra period added to the model, there are two possible states at date 1, $\Omega_1 = \{g, b\}$, and four possible states at date 2, $\Omega_2 = \{gg, gb, bg, bb\}$. g denotes a high (good) dividend realization and b a low (bad) dividend realization. At date 1 dividends equal D_g units of the consumption good in the good state of nature and $D_b < D_g$ in the bad state. Each of these two states occur with probability $\frac{1}{2}$. The realization of states is independent over time. Thus gross dividend growth between time 1 and 2 is K_g with probability $\frac{1}{2}$ and $K_b < K_g$ with probability $\frac{1}{2}$, independent of which state was realized at time 1. This implies that the four states at t=2 are equally probable with dividends $D_{gg} = D_g K_g$, $D_{gb} = D_g K_b$, $D_{bg} = D_b K_g$, and $D_{bb} = D_b K_b$. Dividends are perishable.

A spot market for the consumption good is open at each state at each date. In each state I choose the consumption good as numeraire and denote prices of stocks and bonds in units of the consumption good. Stocks and bonds can be traded at t=0 and in each state at t=1. As before, agents who are allowed to hold stocks face dynamically complete markets. Nonstockholders face incomplete markets. At date 0, a fraction $\lambda_0 \in]0;1]$ of households are allowed to hold both stocks and bonds, the remaining $1 - \lambda_0$ can only hold bonds. At date 1 all households are allowed to hold stocks, i.e. $\lambda_1 = 1$ with probability 1, and this is known to all agents at date 0. With only two possible realizations for dividends at each date, closed form solutions to the optimization problems for stockholders and nonstockholders can be derived.

Using ω to denote portfolio shares, a to denote propensities to consume, and W to denote wealth, the equilibrium relations are as follows:

Bond, t=0: $\lambda_0 (1 - \omega_0)(1 - a_0^s) W_0 + (1 - \lambda_0)(1 - a_0^{ns}) W_0 = 0$

Stock, t=0: $\lambda_0 \omega_0 (1 - a_0^s) W_0 = P_{s0}$

Bond, t=1, $\omega = g$: $\lambda_0 (1 - \omega_{1,g}) \left(1 - a_{1,g}^s\right) W_{1,g}^s + (1 - \lambda_0)(1 - \omega_{1,g}) \left(1 - a_{1,g}^{ns}\right) W_1^{ns} = 0$

Bond, t=1, $\omega = b$: $\lambda_0 (1 - \omega_{1,b}) \left(1 - a_{1,b}^s\right) W_{1,b}^s + (1 - \lambda_0)(1 - \omega_{1,b}) \left(1 - a_{1,b}^{ns}\right) W_1^{ns} = 0$

Stock, t=1, $\omega = g$: $\lambda_0 \omega_{1,g} \left(1 - a_{1,g}^s\right) W_{1,g}^s + (1 - \lambda_0) \omega_{1,g} \left(1 - a_{1,g}^{ns}\right) W_1^{ns} = P_{s1,g}$

Stock, t=1, $\omega = b$: $\lambda_0 \omega_{1,b} \left(1 - a_{1,b}^s\right) W_{1,b}^s + (1 - \lambda_0) \omega_{1,b} \left(1 - a_{1,b}^{ns}\right) W_1^{ns} = P_{s1,b}$

These six equations can be solved for the equilibrium prices of stocks and bonds, P_{b0}, P_{s0}, $P_{b1,g}$, $P_{b1,b}$, $P_{s1,g}$, and $P_{s1,b}$. Notice that all agents are allowed to hold stocks at date 1, but the wealth of a household at date 1 depends on whether is was a stockholder or a nonstockholder at date 0. The solution can then be compared with the solution to the same model with λ_1

restricted to equal λ_0, i.e. with no entry at date 1.

When λ_1 is restricted to equal λ_0 the results are as would be expected based on the two period exchange economy. The stock price at date 0 and in each state at date 1 is unaffected by the level of participation under log utility whereas the bond price is decreasing in participation. Thus the bond return is higher and the equity premium lower when more agents participate in the stock market.

Comparing the solution for $\lambda_0 = \lambda_1$ with the solution for $\lambda_1 = 1$ shows that for the log utility case, the anticipation of entry has no effect on equilibrium returns on asset held between date 0 and date 1. Furthermore, returns between date 1 and 2 are unaffected by whether some households were out of the stock market in the first period. These results are driven by two factors. Under log utility, entry at date 1 does not affect the stock price at date 1 (the intuition is as described earlier), and thus expected returns between date 0 and 1 are unchanged. Furthermore, for log utility, consumption and portfolio choices are myopic in the sense of being unaffected by expectations about future returns. Thus the expectation of changed returns between date 1 and 2 does not affect behavior in the first period.

Once entry occurs, the returns between date 1 and 2 are the same whether some households were out of the stock market in the previous period. This is partly because portfolio choice and the propensity to consume out of wealth is unaffected by whether an agent was a stockholder in the previous period or not, and partly because aggregate wealth at date 1 does not depend on past participation levels when the capital stock is exogenous.

When agents are more risk averse than log utility, consumption and portfolio behavior and asset returns are changed by the expectation of future entry. The expectation of future entry improves the lifetime investment opportunities for present nonstockholders. For $\gamma > 1$ this leads them to increase date 0 consumption[12]. For a given level of participation at date 0, the bond return and the expected stock return for assets held between date 0 and 1 is lower than for Case 1 where the present level of stockholding correctly was expected to persist, and the equity premium is higher. To ensure goods market equilibrium asset returns must change so

[12] With risk aversion different from one, the possibility of a hedging component in portfolio weights arises. However, with all agents participating at date 1, expected returns between date 1 and 2 are the same in the good and the bad state at date 1. Therefore portfolio choice at date 0 will not contain a hedging component even for risk aversion different from one.

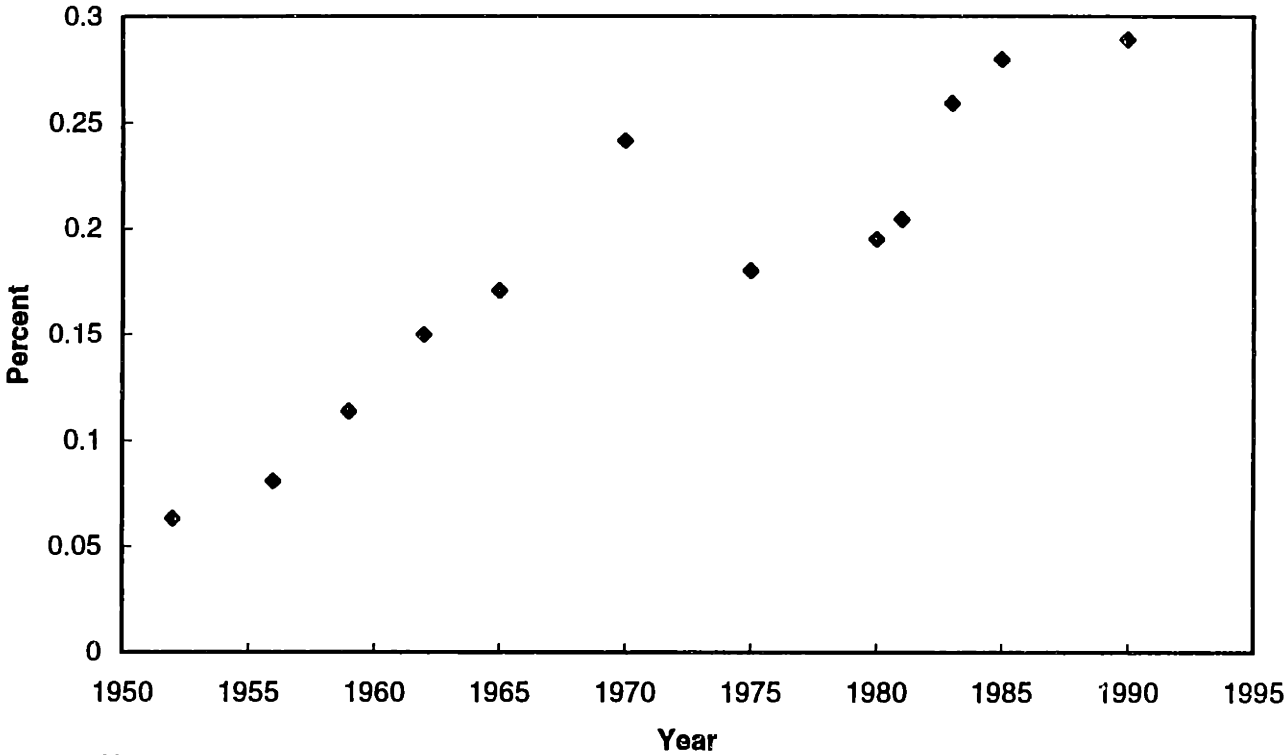
as to induce higher savings, which requires lower asset returns between date 0 and 1. The bond return is depressed more than the expected stock return and thus the equity premium is increased.

3.4 Conclusion

This chapter has provided a general equilibrium analysis of limited stock market participation with the purpose of understanding the effects of the strong upward trend in stock market participation in the US over the postwar period.

Based on an OLG model, I concluded that the effect of random costless entry is likely to be a stock market boom and a decrease in the equity premium if risk aversion is sufficiently small. I emphasized the importance of a production economy and an OLG structure for higher participation to induce higher stock prices. These findings were shown to be robust to endogenizing the entry decision by assuming a fixed utility cost of entry and a distribution of agents along human capital or risk aversion. A preliminary analysis of the issue of expected entry within an exchange economy showed that entry still has affects even if it is expected ahead of time. For the log utility case the equity premium did not decline until entry occurred and risk sharing increased.

Fig. 1. NYSE data
Number of stockholders/US population 20 years or older



Note:

The NYSE shareownership survey in its original form ends in 1990. The 1995 NYSE shareownership survey is based on the SCF. In the 1989, 1992 and 1995 SCF the proportions of stockholders are 31.7, 37.2 and 41.1 percent.

Fig. 2a. IRS data
Percent of returns and percent of individuals with positive dividends

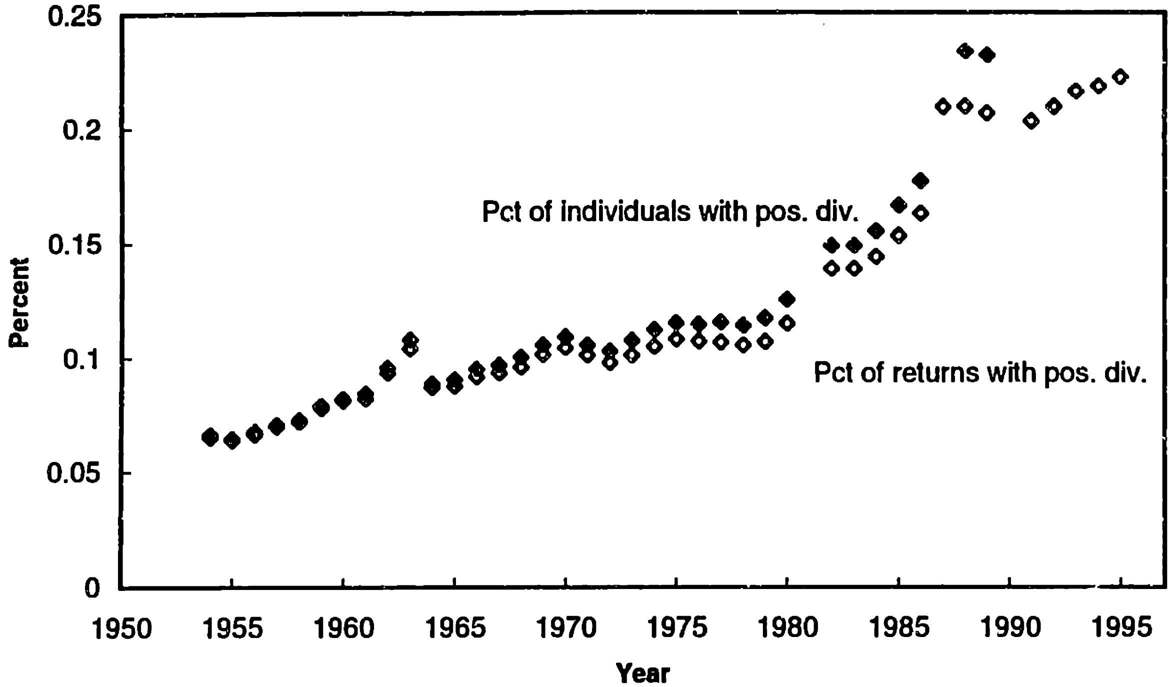


Fig. 2b. IRS data
Concentration of dividend income, selected years 1954-1994

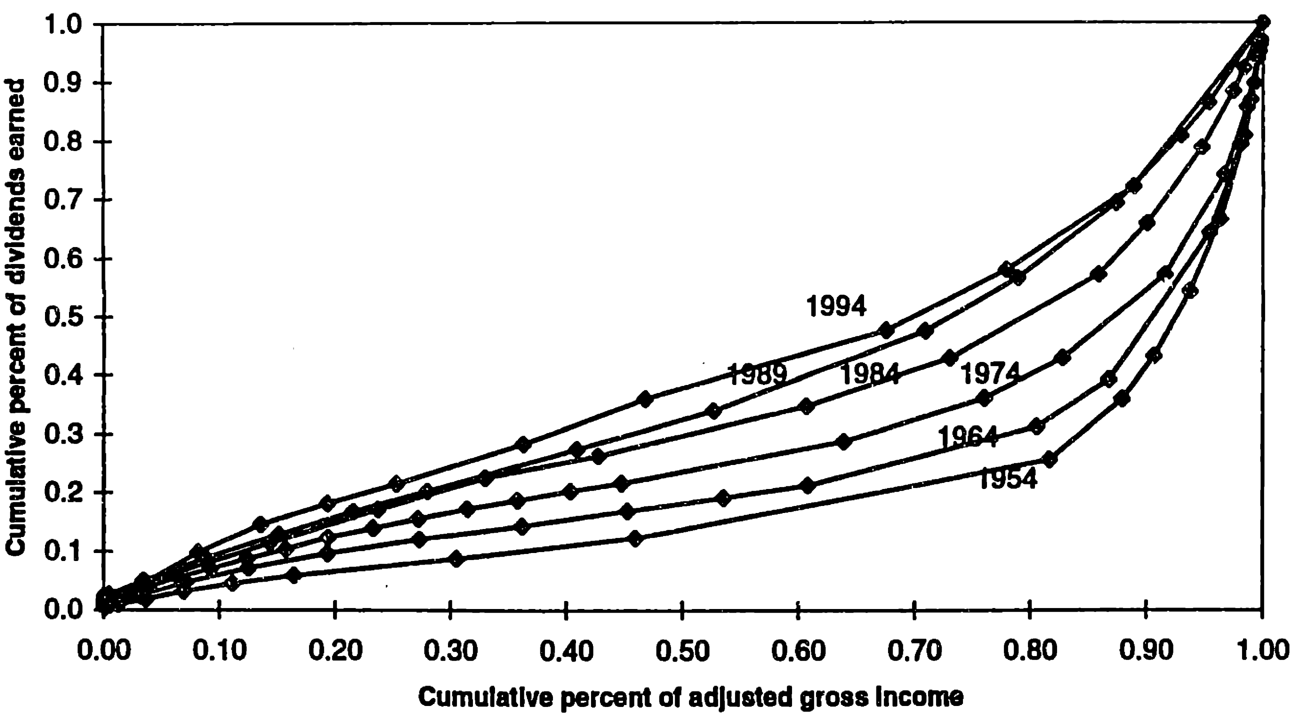


Fig. 3. OLG model.
Case 1. Random costless entry, γ=1

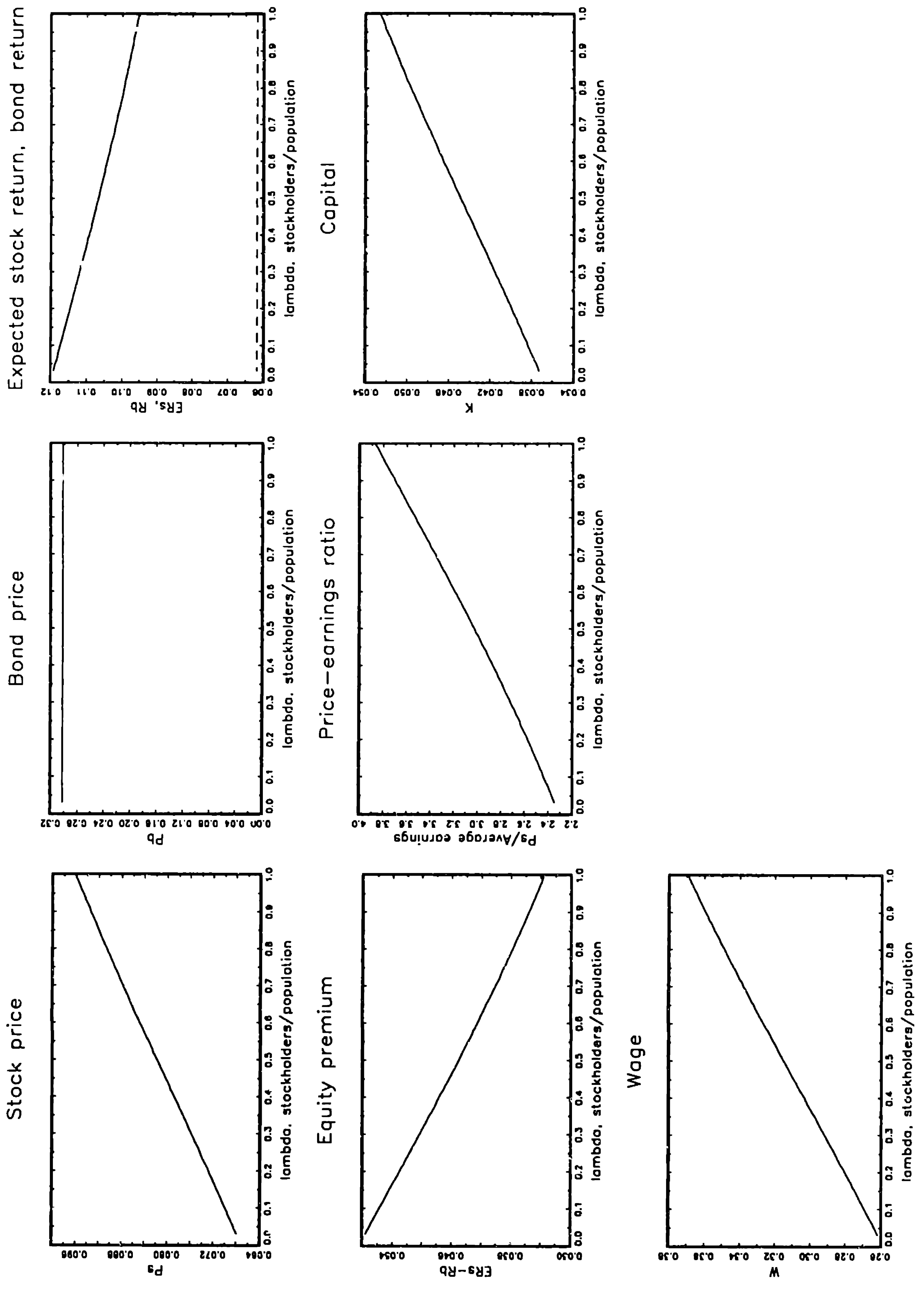


Fig. 4. Exchange economy.
Case 1. Random costless entry, γ=1

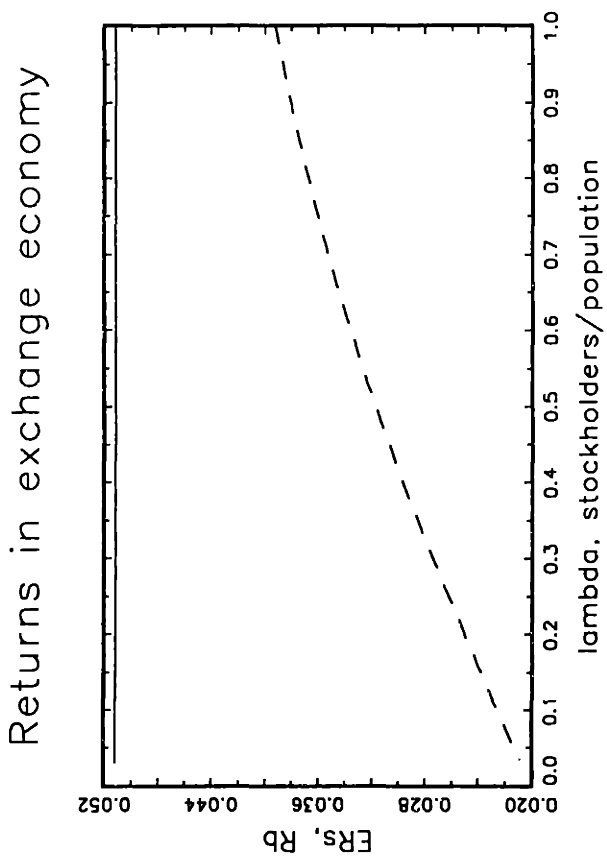


Fig. 5. OLG model.
Case 1. Random costless entry, γ=2.75

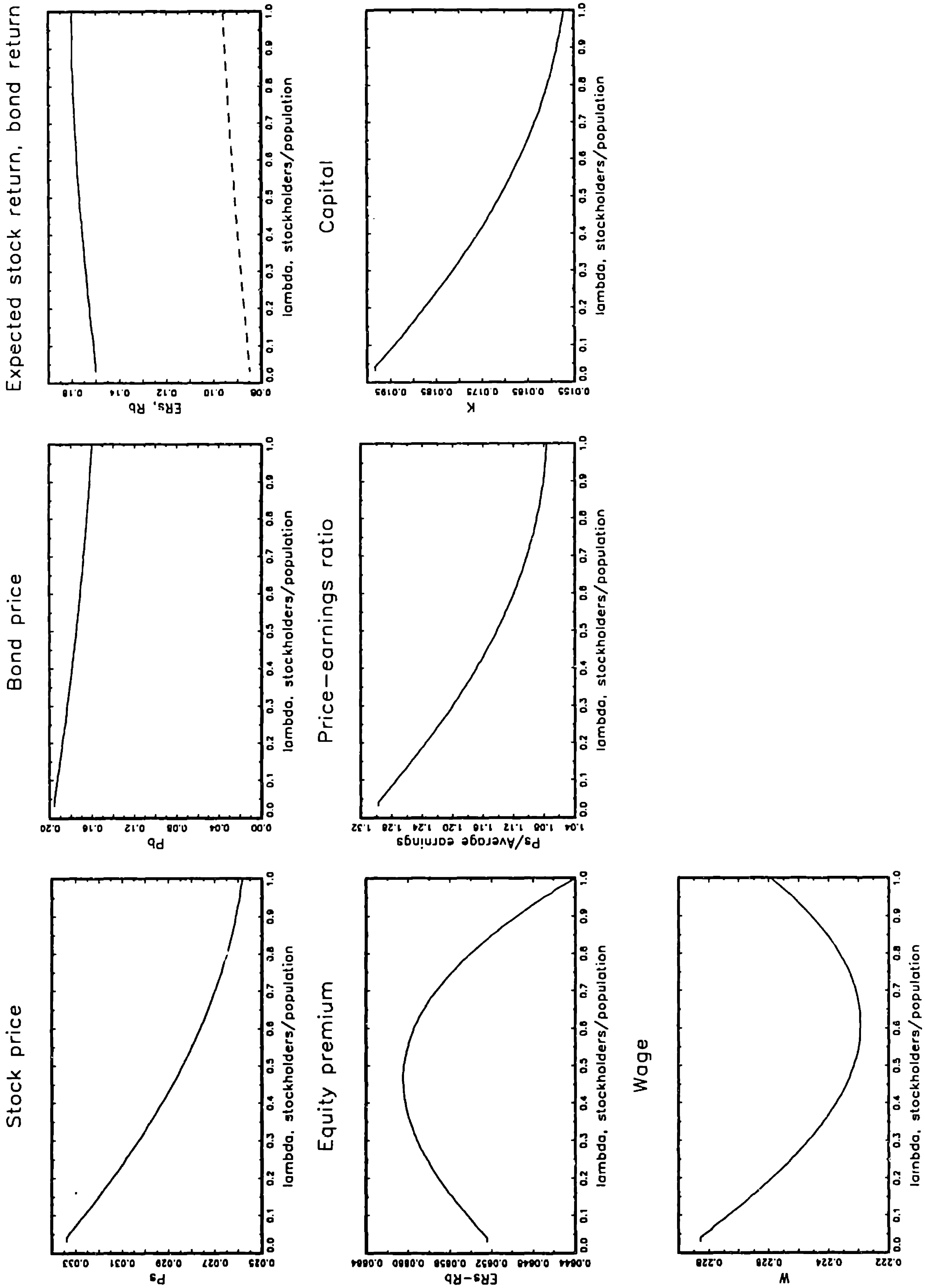


Fig. 6. OLG model.
Case 2a. Endogenous entry, heterogeneous human capital, γ=1

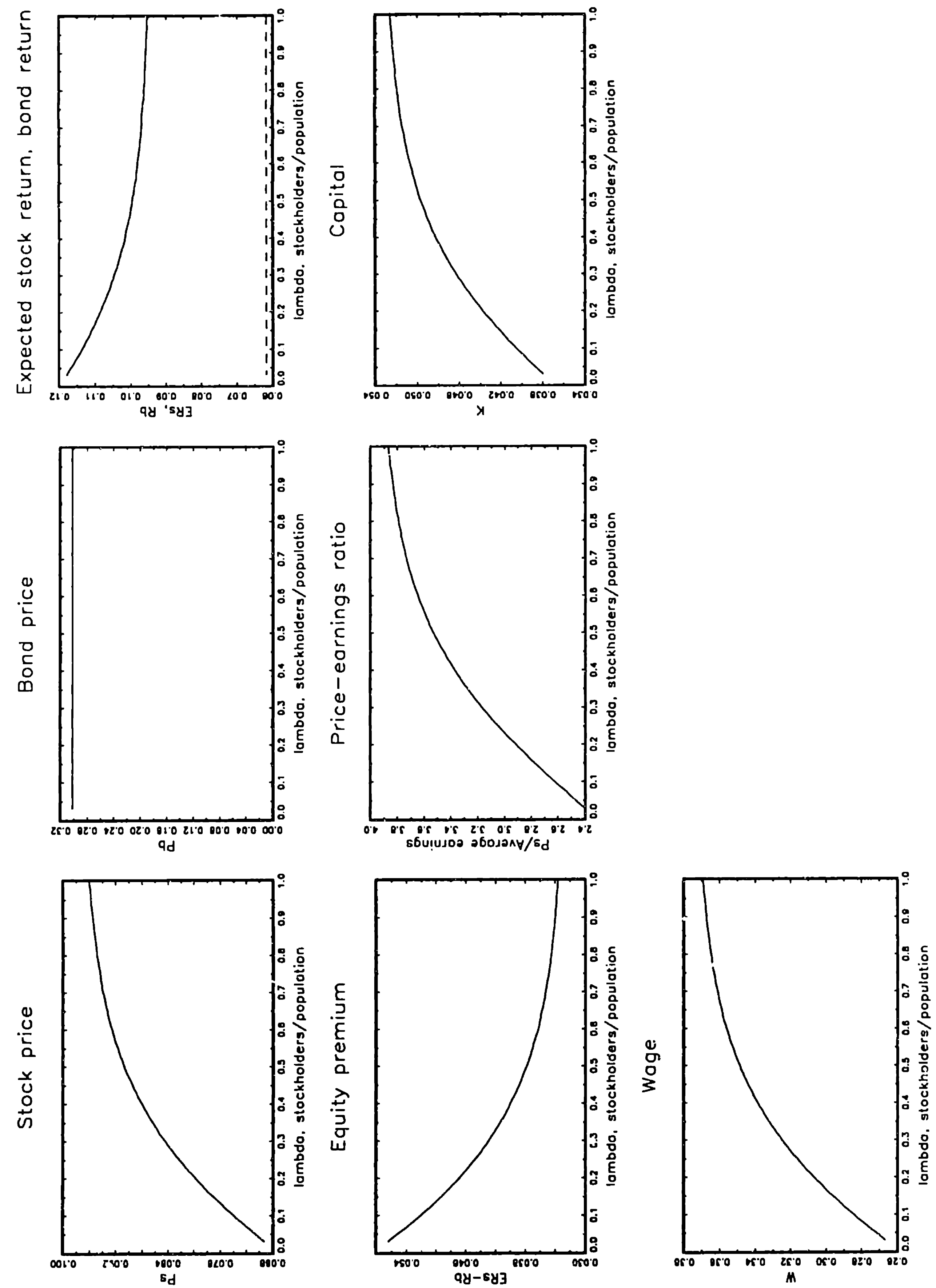
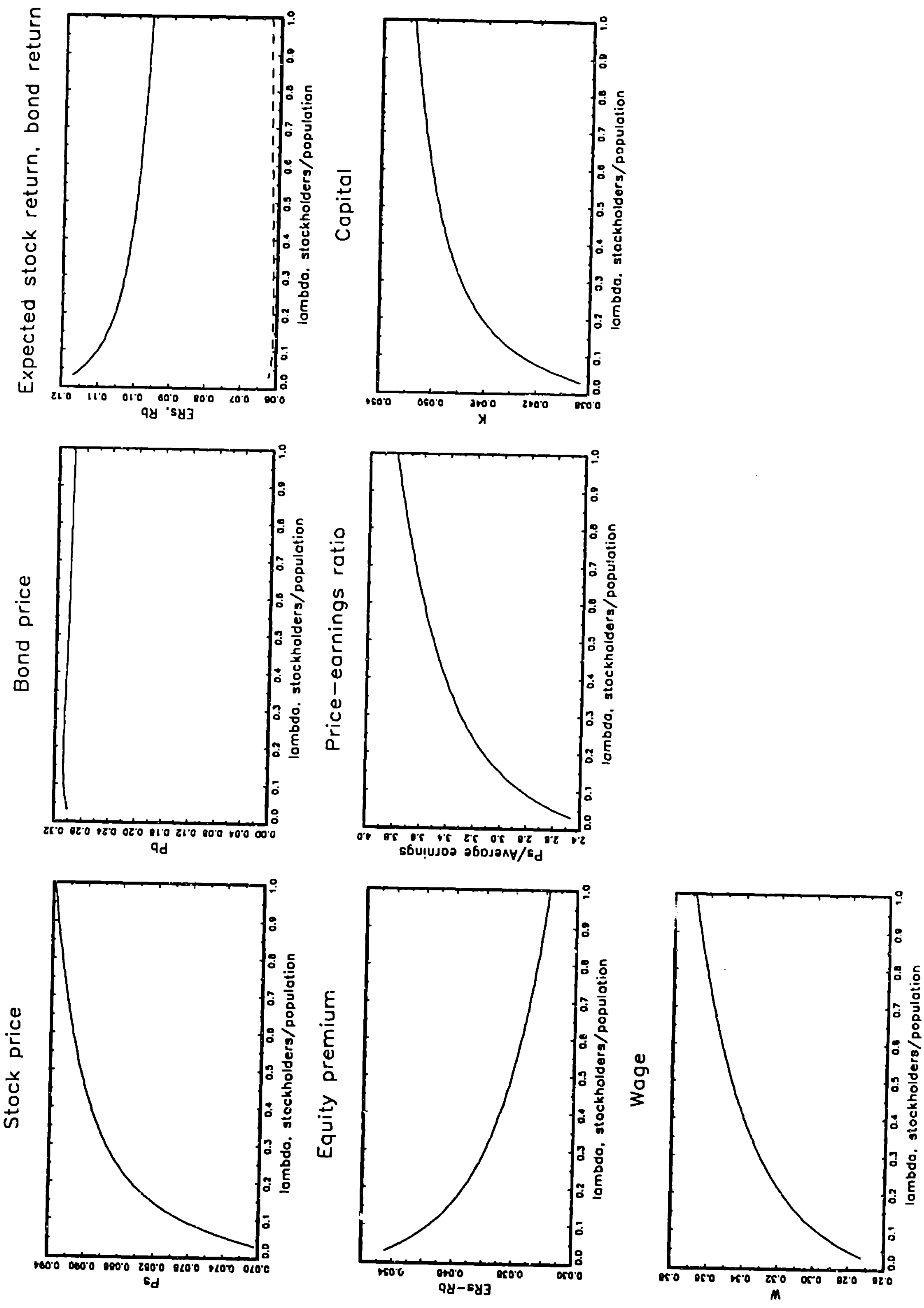


Fig. 7. OLG model.
Case 2b. Endogenous entry, heterogeneous risk aversion



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