THEORETICAL STUDIES FOR MICROWAVE REMOTE SENSING OF LAYERED RANDOM MEDIA

by

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Abstract

In the microwave remote sensing of earth terrain, the model of a layered random medium can be used to account for volume scattering effects due to random permittivity fluctuations. Applying the first order Born approximation, analytic results for the bistatic scattering coefficients and the backscattering cross-sections have been derived for active remote sensing of a two-layer random medium with arbitrary three-dimensional correlation functions. It is found that as a result of the second boundary, the horizontally polarized return, σ_{hh} can be greater than the vertically polarized return, whereas for a half-space random medium σ_{vv} is always greater than σ_{hh}. The theoretical results are illustrated by matching backscattering data collected from a vegetation field. The bistatic scattering coefficients are used to obtain the emissivity of a two-layer random medium and in the case of thin, low loss layers the emissivity is shown to exhibit strong coherent behavior in the spectral dependence.

As a more realistic simulation of earth terrain for active remote sensing analytic expressions for the backscattering cross-sections are derived for a stratified random medium by applying the first order Born approximation. In the special case of a three-layer random medium two maxima are found in the spectral dependence of the backscattering due to resonance scattering within each random layer. The theoretical results also are found to compare favorably with data obtained from vegetation and snow-ice fields. The Born approximation is carried to second order to obtain backscattering cross-sections that account for depolarization effects in a two-layer random medium. In the half-space limit, additional wave effects are found which are not accounted for by the radiative transfer theory nor by the Bethe-Salpeter equation in the ladder approximation.

The mean dyadic Green's function for a two-layer random medium has been obtained by applying a two-variable expansion technique to solve the non-linear Dyson's equation. The coherent wave is found to propagate in the random medium as in an anisotropic medium with different propagation constants for the characteristic TE and TM polarizations. The effective propagation constants obtained in the zeroth order solution are compared with the scattering coefficients of the radiative transfer theory.

Modified radiative transfer (MRT) equations for the electromagnetic field intensity are derived from the ladder approximated Bethe-Salpeter equation together with the zeroth order solution to Dyson's equation under the non-linear approximation. The MRT equations contain significant wave-like corrections not accounted for by phenomenological radiative transport theories due to the presence of the bottom boundary. The MRT equations are solved in the first order renormalization approximation and comparisons are made with the results obtained in the first order Born approximation. A method for resumming an infinite sequence of terms in the intensity operator is presented. A renormalized Bethe-Salpeter equation is derived which takes the form of a pair of coupled integral equations.

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DEDICATION

TO

JOSEPH R. MERCADO

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LIST OF PRINCIPAL SYMBOLS

In the following list, a superscript single prime is used to denote the real part of a quantity and a superscript double prime to denote the imaginary part of a quantity.

R_{ij}: reflection coefficient of TE wave at $i - j$ interface

S_{ij}: reflection coefficient of TM wave at $i - j$ interface

X_{ij}: transmission coefficient of TE wave at $i - j$ interface

Y_{ij}: transmission coefficient of TM wave at $i - j$ interface

D_2: defined in text

F_2: defined in text

$\bar{\bar{G}}_{01}$, $\bar{\bar{G}}_{11}$: dyadic Green's functions for source in region 1 and observation points in regions 0 and 1, respectively

$\bar{\bar{G}}_{11m}$: mean dyadic Green's function

d_ℓ: depth of ℓ-th layer

$\bar{E}_{1m}$: mean electric field

$\bar{E}_o$, $\bar{E}_1$: electric fields in regions 0 and 1, respectively

$\bar{E}_\ell^{(n)}$: n-th order electric field in region ℓ

$\hat{e}$: unit vector in direction of electric field for TE wave

$\hat{h}$: unit vector in direction of electric field for TM wave

$\bar{k}_\ell$:	wave vector in region ℓ
$\bar{k}_\perp$:	transverse wave vector (k_x, k_y)
$C(\bar{r}_1 - \bar{r}_2)$:	correlation function of random medium
$\Phi(\bar{\beta})$:	spectral density of correlation function
$\gamma_{\mu\nu}$:	bistatic scattering coefficient
$\sigma_{\mu\nu}$:	backscattering cross-section per unit area
ε_ℓ:	permittivity of region ℓ
$<\varepsilon_\ell>$, $\varepsilon_{\ell m}$:	mean permittivity in region ℓ
$\varepsilon_{\ell f}(\bar{r})$:	random permittivity fluctuation
δ:	variance of permittivity fluctuations
ℓ_ρ:	lateral correaltion length
ℓ_z, ℓ:	vertical correlation lengths
e_μ:	emissivity with polarization μ
$\bar{I}$, $\bar{\mathcal{I}}$:	intensity matrix and Stokes vector, respectively
η_v, η_h:	z-component of effective wavevector
$\bar{\bar{P}}_{\mu\nu}$:	scattering phase matrix for incoherent intensity
$\bar{\bar{Q}}_{\mu\nu}$:	scattering phase matrix for mean intensity
ℓ_v, ℓ_h:	long distance scales for TM and TE waves

CHAPTER I

Introduction

In recent years, active and passive microwave remote sensing techniques have proved to be a useful tool in the study of earth terrain, such as snow-ice fields,[1-30] vegetation coverage,[31-45] and meteorological[46-47] as well as oceanographic[48] phenomena. The majority of the work performed in remote sensing has been experimental in nature with theoretical developments lagging far behind. Although past theoretical emphasis has been largely restricted to rough surface scattering,[49-53] recent theoretical models have been proposed to account for volume scattering effects in low loss media such as snow, ice and vegetation. Stogryn[54] considered the scattering of electromagnetic waves by a half-space random medium whose dielectric constant contains a small random part and a non-random part which can vary as a function of depth. Using first order perturbation theory of Karal and Keller,[55] Stogryn derived bistatic scattering coefficients by assuming that the correlation lengths are small compared to the wavelength. The cross-polarized scattering coefficients were shown to vanish in the backscattering direction. This is expected since only first order terms were considered and contributions to the cross-polarized backscattering comes from the higher order terms.

Tsang and Kong[56] also studied the electromagnetic scattering by a half-space random medium with three dimensional correlation functions. The scattered fields in the non-random region are obtained by using the dyadic Green's function for a half-space medium and the Born approximation where the field in the random medium is replaced by the unperturbed field. Following Peake's[57] definition the bistatic scattering coefficients are derived from the scattered fields. The cross-polarized backscattering coefficients also vanish since the Born approximation is a single-scattering approximation which is valid only when the albedo is small.

More recently Tsang and Kong[58] investigated the problem of scattering by a slab of random medium with a laminar structure. A two-variable expansion technique is used to solve for the zeroth order mean Green's function from the scalar Dyson's equation under the non-linear approximation. The mean Green's function is then used to derive modified radiative transfer (MRT) equations from the Bethe-Salpeter equation under the ladder approximation. The MRT equations are solved for a two-layer random medium with laminar structure and the reflectivity at normal incidence is determined. Tsang and Kong[59] extended the renormalization method to the case of a half-space random medium with three dimensional correlations. The zeroth order Green's function is solved from the scalar Dyson's equation under the non-linear approximation and MRT equations are derived from the ladder approximated Bethe-Salpeter equation. In the limit of a laminar structure two effective propagation constants are found to exist.

Tan and Fung[60] also employed the two-variable expansion technique and solved the non-linear Dyson's equation for the zeroth order mean dyadic Green's function in the case of a half-space random medium. Tan and Fung retain terms only to lowest order in correlation lengths, and the resultant vector solution contains only a single propagation constant for all components in the Green's dyadic.

The first order renormalization method also has been employed in the study of electromagnetic scattering by random media. In this method the incoherent scattered intensity is obtained from the ladder approximated Bethe-Salpeter equation by neglecting the scattering of the incoherent field. Fung and Fung[61] applied the first order renormalization method to a half-space characterized by a random permittivity with a cylindrically symmetric correlation function. Fung[62] extended the first order renormalization method to study the scattering of a vegetation layer characterized by a correlation function which is cylindrical laterally and exponential vertically.

Finally, we mention the work of Tsang and Kong,[63] who used radiative transfer theory to calculate the bistatic scattering coefficients and the backscattering cross-sections of a half-space random medium with lateral and vertical fluctuations. They solved the radiative transfer equations iteratively through second order and showed that non-vanishing cross-polarized backscattering coefficients result.

All these past works were carried out either with the model of a half-space random medium or with a two-layer random medium in the case of scalar wave propagation. Therefore the objective of this thesis is to study and develop electromagnetic scattering models which are applicable to the interpretation of active remote sensing data of earth terrain such as vegetation coverage or snow-ice fields. To this end, we consider the model of a two-layer random medium with arbitrary three-dimensional correlation functions. In Chapter 2, we review the dyadic Green's functions appropriate for a two-layer medium where the source and observation points are located in different regions. We then derive the dyadic Green's function where both source and observation points are within the same region.

In Chapter 3, we solve the problem of scattering by a layer of random medium with three dimensional correlation functions with a wave approach by applying Born approximations. Carrying to first order in albedo, bistatic scattering coefficients are derived which reduce to previous results[56] in the limiting case of a half-space.

In Chapter 4, we study the emissivity of a two-layer random medium with three dimensional variations. Using the results of Chapter 3 for the bistatic scattering coefficients, we calculate the emissivity for arbitrary correlation functions. The coherent behavior in the spectral dependence of the two-layer emissivity is illustrated.

In Chapter 5, we extend the first order Born approximation to the case of backscattering by a stratified random medium. Analytical expressions for the backscattering cross-sections are derived for an arbitrary number of random layers. The results are illustrated in the special case of a three-layer random medium, with correlation functions which are gaussian latterally and exponential vertically.

In Chapter 6, we carry the Born approximation to second order to obtain backscattering cross-sections that account for depolarization effects. The results are reduced to the half-space case and wave-like effects not accounted for by radiative transfer theory are discussed.

In order to account for multiple scattering of the electromagnetic field renormalization methods are necessary. The renormalization approach has been widely used to study wave propagation in random unbounded media.[64,55] It gives rise to the Dyson equation for the mean field and the Bethe-Salpeter equation for the covariance of the field. In Chapter 7, we review the derivations of the Dyson and Bethe-Salpeter equations and the various approximations which are applied to the mass and intensity operators.

In Chapter 8, we solve the non-linear Dyson's equation for the zeroth order mean dyadic Green's function for a two-layer random medium. The propagation of the coherent wave in the random medium is similar to that in an anisotropic medium with different propagation constants for the characteristic TE and TM polarizations. The effective propagation constants obtained in the zeroth order solution are compared with the scattering coefficients of radiative transfer theory by taking the limit of a half-space. Comparisons are also made with Tan and Fung's[60] half-space solution for the zeroth order mean dyadic Green's function. The special case of a laminar structure is considered and two effective propagation constants for each polarization state are found to exist.

In Chapter 9, modified radiative transfer (MRT) equations for the electromagnetic field intensity are derived from the ladder approximated Bethe-Salpeter equation together with the zeroth order solution to Dyson's equation under the non-linear approximation. These approximations have been shown[65,66] to be energetically consistent and therefore appropriate in the development of a radiative transport theory. The MRT equations contain significant wave-like corrections not accounted for by phenomenological radiative transport theories due to the presence of the bottom boundary. The significance of these additional contributions is discussed in the context of backscattering by solving the MRT equations in the first order renormalization approximation and comparing with wave solutions obtained in the first order Born approximation.

In Chapter 10, the physical significance of the cross terms in the Neumann series for the field covariance is discussed. A method for resumming an infinite sequence of terms (including cross terms) in the intensity operator is presented. A renormalized Bethe-Salpeter equation is derived which takes the form of a pair of coupled integral equations.

CHAPTER 2

Dyadic Green's Function for Two-Layer Media

The Dyadic Green's function technique of treating electromagnetic boundary-value problems was first formulated by Schwinger in the early 1940's. Since that time the subject matter has been subsequently discussed by Morse and Feshbach,[67] C. T. Tai[68] and Tsang et al.[69] Since a two-layer medium is the basic geometry in this thesis we will review dyadic Green's functions appropriate for a two-layer medium.

By matching boundary conditions we then derive the two-layer dyadic Green's function, $\bar{\bar{G}}_{11}(\bar{r}, \bar{r}')$ where the source and field points are located within the same region.

2.1 Dyadic Green's Function with Source and Observation Points in Different Regions

Consider a point source located within the ℓ-th layer of a stratified medium (Fig. 2.1). Let region 0 be free-space. The field in region 0, is composed of upward going waves only, and has been derived[69] in the form

$$\bar{\bar{G}}_{o\ell}(\bar{r}, \bar{r}') = -\frac{\omega\mu_o}{8\pi^2}\int d^2k_\perp \frac{e^{i\bar{k}_o \cdot \bar{r}}}{k_{oz}} \{\hat{e}(k_{oz})[A_\ell \hat{e}_\ell(-k_{\ell z}) e^{ik_{\ell z}z'}$$

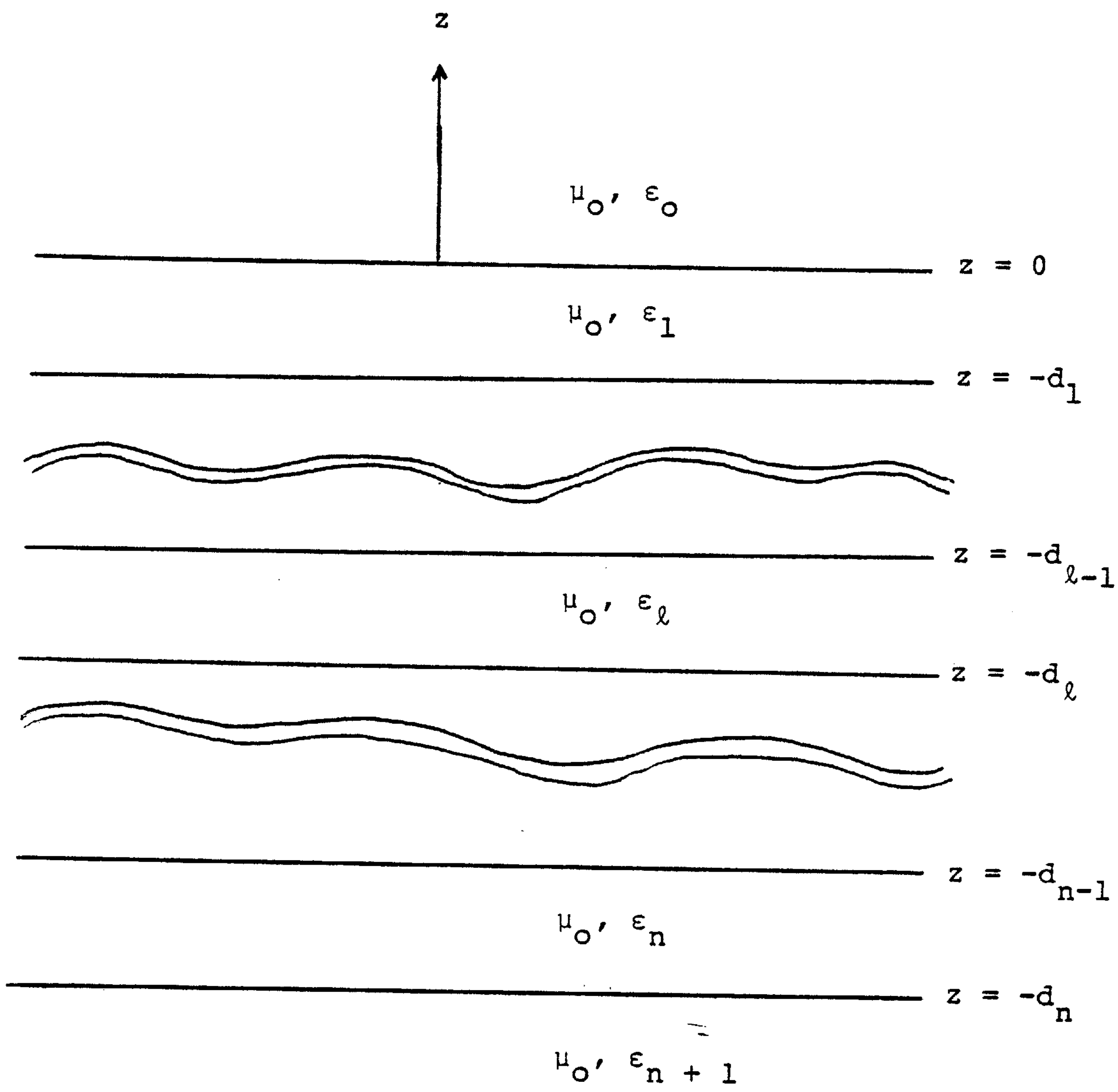
$$+ B_\ell \hat{e}_\ell(k_{\ell z}) e^{-ik_{\ell z}z'}] + \hat{h}(k_{oz})[C_\ell \hat{h}_\ell(-k_{\ell z}) e^{ik_{\ell z}z'}$$

$$+ D_\ell \hat{h}_\ell(k_{\ell z}) e^{-ik_{\ell z}z'}]\} e^{-i\bar{k}_\perp \cdot \bar{r}_\perp'} \tag{2.1}$$

where $\bar{k}_\perp = \hat{x}\, k_x + \hat{y}\, k_y$, $d^2k_\perp = dk_x dk_y$ and

$$\hat{e}_\ell(k_{\ell z}) = \frac{1}{k_\perp}\hat{z} \times \bar{k}_\ell \tag{2.2}$$

$$\hat{h}_\ell(k_{\ell z}) = \frac{1}{k_\ell}\hat{e}_\ell(k_{\ell z}) \times \bar{k}_\ell \tag{2.3}$$



Stratified geometry

Figure 2.1

$$k_{\ell z} = (k_\ell{}^2 - k_\perp{}^2)^{1/2} \tag{2.4}$$

and $k_\ell{}^2 = \omega^2 \mu \varepsilon_\ell$, $\hat{e}_\ell$ is in the direction of the horizontally polarized electric field vector and $\hat{h}_\ell$ is in the direction of the vertically polarized electric field vector. The amplitudes A_ℓ, B_ℓ, C_ℓ and D_ℓ are determined through the propagation matrix formalism.[70] In (2.1) the first subscript of the dyadic Green's function indicates the region of the observation point while the second subscript indicates the region of the source point. Taking $\ell = N = 1$, we obtain from (2.1) the dyadic Green's function for a two-layer medium. The result is

$$\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = \int d^2k_\perp \, \overset{=>}{g}_{01}(\bar{k}_\perp, z, z_1) \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})} \tag{2.5}$$

where

$$\overset{=>}{g}_{01}(\bar{k}_\perp, z, z_1) = - \frac{\omega\mu_o}{8\pi^2} \frac{1}{k_{1z}} \left\{ \frac{X_{10}(k_\perp)}{D_2(k_\perp)} \hat{e}(k_{oz}) [R_{12}(k_\perp) \, e^{i2k_{1z}d_1} \right.$$

$$e^{ik_{1z}z_1}\,\hat{e}_1(-k_{1z}) + e^{-ik_{1z}z_1}\,\hat{e}_1(k_{1z})]$$

$$+ \frac{k_1}{k_o}\,\frac{Y_{10}(k_\perp)}{F_2(k_\perp)}\,\hat{h}(k_{1z})\,[S_{12}(k_\perp)\; e^{i2k_{1z}d_1}\; e^{ik_{1z}z_1}\,\hat{h}(-k_{1z})$$

$$+ e^{-ik_{1z}z_1}\,\hat{h}(k_{1z})]\Big\}\; e^{ik_{1z}z} \qquad (2.6)$$

and,

$$R_{ij}(k_\perp) = \frac{k_{iz} - k_{jz}}{k_{1z} + k_{jz}} \qquad (2.7a)$$

$$S_{ij}(k_\perp) = \frac{\varepsilon_j k_{iz} - \varepsilon_i k_{jz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}} \qquad (2.7b)$$

$$X_{ij}(k_\perp) = 1 + R_{ij}(k_\perp) \qquad (2.8a)$$

$$Y_{ij}(k_\perp) = 1 + S_{ij}(k_\perp) \qquad (2.8b)$$

$$D_2(k_\perp) = 1 + R_{01}(k_\perp)\; R_{12}(k_\perp)\; e^{i2k_{1z}d_1} \qquad (2.9a)$$

$$F_2(k_\perp) = 1 + S_{01}(k_\perp)\; S_{12}(k_\perp)\; e^{i2k_{1z}d_1} \qquad (2.9b)$$

The subscripts i and j in (2.7a)-(2.9b) denote 0, 1 or 2. We note that the portion of $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$ which contains the unit vectors $\hat{e}$ corresponds to TE type waves. In the same way, the portion of $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$ which contains the unit vectors $\hat{h}$ corresponds to TM type waves.

2.2 Dyadic Green's Function with Source and Observation Points in the Same Region

In this section, we derive the dyadic Green's function, $\bar{\bar{G}}_{11}$ where both source and observation points are within region 1 of the two-layer medium. We first write $\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1)$ as the sum of two parts

$$\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1) = \bar{\bar{G}}_{D11}(\bar{r}, \bar{r}_1) + \bar{\bar{G}}_{R11}(\bar{r}, \bar{r}_1) \tag{2.10}$$

where $\bar{\bar{G}}_{D11}$ and $\bar{\bar{G}}_{R11}$ satisfy the vector wave equations

$$\bar{\nabla} \times \bar{\nabla} \times \bar{\bar{G}}_{D11}(\bar{r}, \bar{r}_1) - k_1^2 \, \bar{\bar{G}}_{D11}(\bar{r}, \bar{r}_1) = i\omega\mu\bar{\bar{I}}\delta(\bar{r} - \bar{r}_1) \tag{2.11a}$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{\bar{G}}_{R11}(\bar{r}, \bar{r}_1) - k_1^2 \, \bar{\bar{G}}_{R11}(\bar{r}, \bar{r}_1) = 0. \tag{2.11b}$$

Physically, $\bar{\bar{G}}_{D11}$ represents the direct response to the source at $\bar{r}_1$ and does not contain boundary effects. Alternatively, $\bar{\bar{G}}_{R11}$ represents the response to the image sources produced by the boundaries at $z = 0$ and $z = -d_1$. It is for this reason that $\bar{\bar{G}}_{R11}$ satisfies the homogeneous vector wave equation whereas $\bar{\bar{G}}_{D11}$ satisfies the inhomogeneous vector wave equation. The solution to (2.11a) is just the free space dyadic Green's function, which has been derived by Tsang.[66] The result may be written in the form:

$$\bar{\bar{G}}_{D11}(\bar{r}, \bar{r}_1) = -\frac{i}{\omega\varepsilon_1}\hat{z}\hat{z}\,\delta(\bar{r} - \bar{r}_1) + \bar{\bar{G}}_{F11}(\bar{r}, \bar{r}_1) \tag{2.12}$$

where the free space radiating portion of $\bar{\bar{G}}_{D11}$ is contained in $\bar{\bar{G}}_{F11}(\bar{r}, \bar{r}_1)$. Combining (2.10) and (2.12) we obtain,

$$\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1) = -\frac{i}{\omega\varepsilon_1}\hat{z}\hat{z}\,\delta(\bar{r} - \bar{r}_1) + \bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) \tag{2.13}$$

where

$$\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) = \bar{\bar{G}}_{F11}(\bar{r}, \bar{r}_1) + \bar{\bar{G}}_{R11}(\bar{r}, \bar{r}_1). \tag{2.14}$$

In view of (2.5), (2.6) and in order to match boundary conditions at $z = 0$, the dyadic Green's function $\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1)$ in (2.13) takes the form:

$$\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) = -\frac{\omega\mu_o}{8\pi^2}\int d^2k_\perp \frac{1}{k_{1z}}\{[\alpha\, e^{ik_{1z}z}\,\hat{e}_1(k_{1z})$$

$$+ \beta\, e^{-ik_{1z}z}\,\hat{e}_1(-k_{1z})][\rho\, e^{ik_{1z}z_1}\,\hat{e}_1(-k_{1z}) + \sigma\, e^{-ik_{1z}z_1}\,\hat{e}_1(k_{1z})]$$

$$+$$

$$[\gamma\, e^{ik_{1z}z}\, \hat{h}_1(k_{1z}) + \delta\, e^{-ik_{1z}z_1}\, \hat{h}_1(-k_{1z})]$$

$$[\eta\, e^{ik_{1z}z_1}\, \hat{h}_1(-k_{1z}) + \xi\, e^{-ik_{1z}z_1}\, \hat{h}_1(k_{1z})]\}$$

$$e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_\perp')} \qquad (z > z_1). \tag{2.15}$$

The boundary conditions to be satisfied by $\bar{\bar{G}}_{01}$ and $\bar{\bar{G}}_{11}$ at $z = 0$ are:

$$\hat{z} \times \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = \hat{z} \times \bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1) \tag{2.16a}$$

$$\hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = \hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1). \tag{2.16b}$$

Substituting (2.5), (2.6) and (2.12), (2.13) into boundary conditions (2.16a) and (2.16b), we find the result

$$\alpha = X_{01}^{-1} \tag{2.17a}$$

$$\beta = \frac{R_{10}}{X_{01}} \tag{2.17b}$$

$$\gamma = \frac{k_1}{k_o} Y_{01}^{-1} \tag{2.18a}$$

$$\delta = \frac{k_1}{k_o} \frac{S_{10}}{Y_{01}} \tag{2.18b}$$

$$\rho = \frac{X_{01} R_{12}}{D_2} e^{i2k_{1z}d_1} \tag{2.19a}$$

$$\sigma = \frac{X_{01}}{D_2} \tag{2.19b}$$

$$\eta = \frac{k_o}{k_1} \frac{Y_{01} S_{12}}{F_2} \tag{2.20a}$$

$$\xi = \frac{k_o}{k_1} \frac{Y_{01}}{F_2} . \tag{2.20b}$$

The terms which appear on the right hand sides of (2.17a)-(2.20b) are defined in equations (2.7a)-(2.9b). Combining equations (2.15) and (2.17a)-(2.20b), we find the dyadic Green's function $\bar{\bar{G}}_{S11}$ for $(z > z_1)$ to be:

$$\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) = \int d^2k_\perp \, \bar{\bar{g}}_{11}^{>}(\bar{k}_\perp, z, z_1) \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})}$$

$$(z > z_1) \tag{2.21}$$

where:

$$\bar{\bar{g}}_{11}^{>}(\bar{k}_{\perp}, z, z_1) = -\frac{\omega\mu_o}{8\pi^2}\frac{1}{k_{1z}}\left\{\frac{1}{D_2(k_{\perp})}[e^{ik_{1z}z}\,\hat{e}_1(k_{1z})\right.$$

$$+ R_{10}(k_{\perp})\,e^{-ik_{1z}z}\,\hat{e}_1(-k_{1z})][e^{-ik_{1z}z_1}\,\hat{e}_1(k_{1z})$$

$$\left.+ R_{12}(k_{\perp})\,e^{i2k_{1z}d_1}\,e^{ik_{1z}z_1}\,\hat{e}_1(-k_{1z})]\right\}$$

$$+ \frac{1}{F_2(k_{\perp})}[e^{ik_{1z}z}\,\hat{h}_1(k_{1z}) + S_{10}(k\ )\,e^{-ik_{1z}z}\,\hat{h}_1(-k_{1z})]$$

$$[e^{-ik_{1z}z_1}\,\hat{h}_1(k_{1z}) + S_{12}(k_{\perp})\,e^{i2k_{1z}d_1}\,e^{ik_{1z}z_1}$$

$$\hat{h}_1(-k_{1z})]. \tag{2.22}$$

For $z < z_1$ we use the symmetry condition for dyadic Green's functions

$$\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) = \bar{\bar{G}}_{S11}^{T}(\bar{r}_1, \bar{r}) \tag{2.23}$$

where the superscript T denotes the transpose of the matrix. The dyadic Green's function $\bar{\bar{G}}_{S11}$ for $(z < z_1)$ is found from (2.21)-(2.23) to be

$$\bar{\bar{G}}_{S11}(\bar{r}, \bar{r}_1) = \int d^2k_\perp \, \bar{\bar{g}}_{11}^{<}(\bar{k}_\perp, z, z_1) \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})}$$

$$(z < z_1) \qquad (2.24)$$

where:

$$\bar{\bar{g}}_{11}^{<}(\bar{k}_\perp, z, z_1) = - \frac{\omega\mu_o}{8\pi^2} \frac{1}{k_{1z}} \frac{1}{D_2(k_\perp)} [e^{-ik_{1z}z} \, \hat{e}_1(-k_{1z})$$

$$+ R_{12}(k_\perp) \, e^{i2k_{1z}d_1} \, e^{ik_{1z}z} \, \hat{e}_1(k_{1z})][e^{ik_{1z}z_1} \, \hat{e}_1(-k_{1z})$$

$$+ R_{10}(k_\perp) \, e^{-ik_{1z}z_1} \, \hat{e}_1(k_{1z})] + \frac{1}{F_2(k_\perp)} [e^{-ik_{1z}z} \, \hat{h}_1(-k_{1z})$$

$$+ S_{12}(k_\perp) \, e^{i2k_{1z}d_1} \, e^{ik_{1z}z} \, \hat{h}_1(k_{1z})][e^{ik_{1z}z} \, \hat{h}_1(-k_{1z})$$

$$+ S_{10}(k_\perp) \, e^{-ik_{1z}z_1} \, \hat{h}_1(k_{1z})]. \qquad (2.25)$$

The net dyadic Green's function, $\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1)$ for both source and observation points within Region 1, may be written as

$$\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1) = - \frac{i}{\omega\varepsilon_1} \hat{z}\hat{z} \, \delta(\bar{r} - \bar{r}_1)$$

$$+ \int d^2k_\perp \begin{cases} \bar{\bar{g}}^{>}_{11}(\bar{k}_\perp, z, z_1)\, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})} & (z > z_1) \\ \bar{\bar{g}}^{<}_{11}(\bar{k}_\perp, z, z_1)\, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})} & (z < z_1) \end{cases} \tag{2.26}$$

with $\bar{\bar{g}}^{\gtrless}_{11}$ given by (2.22) and (2.25).

CHAPTER 3

Active Remote Sensing of a Two-Layer Random Medium in the First Order Born Approximation

In this chapter, we make use of dyadic Green's functions to solve the problem of scattering by a layer of random medium with three dimensional correlation functions by applying Born approximations. Integral equations which govern the scattered field are solved by iterating to first order in albedo. Making use of a correlation function that is Gaussian latterally and exponential vertically, we find backscattering cross-sections for the two-layer problem that reduce to previous results[63] in the limiting case of a half-space. A brief discussions of rough surface scattering effects upon the backscattering is presented. The first order Born results are illustrated by matching experimental data.

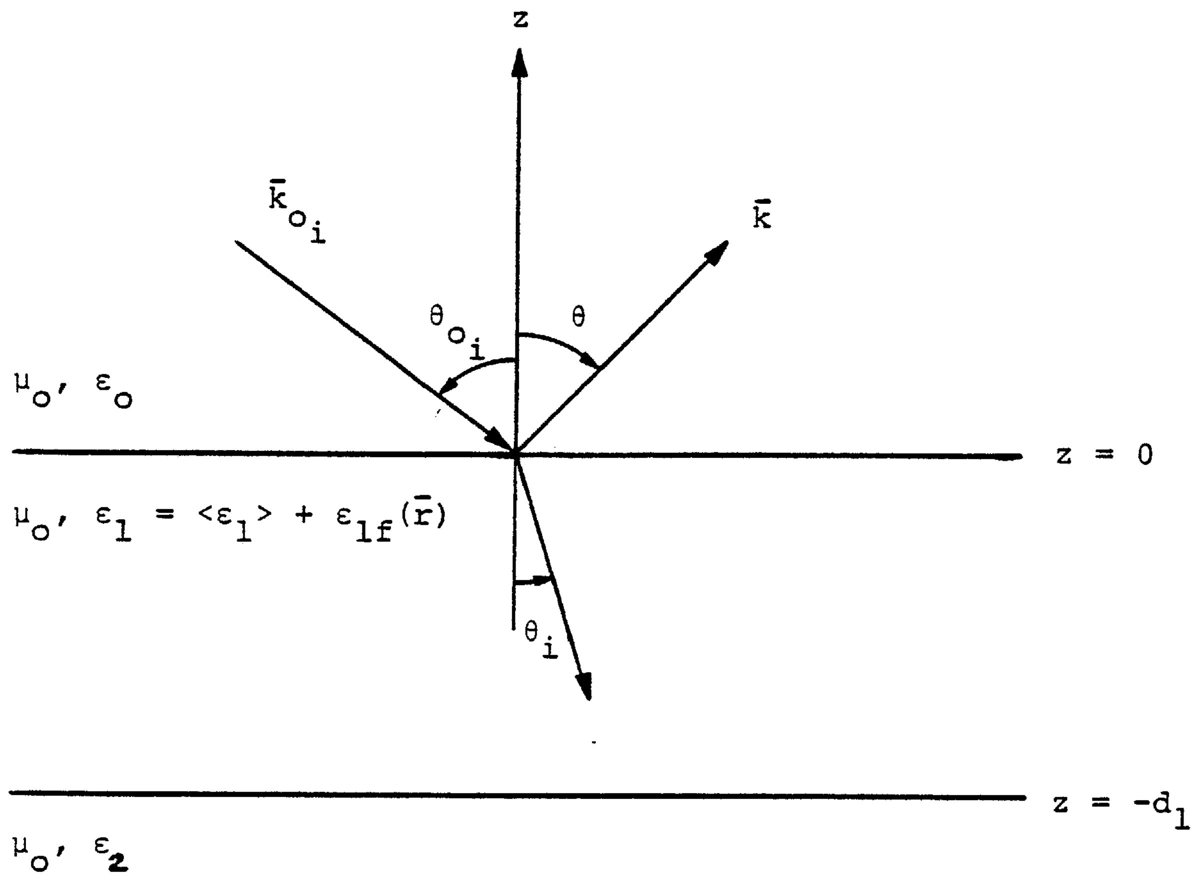
3.1 Far Field Dyadic Green's Function by Saddle Point Method

Consider a layer of random medium with permittivity $\varepsilon_1(\bar{r}) = \langle\varepsilon_1\rangle + \varepsilon_{1f}(\bar{r})$ where $\varepsilon_{1f}(\bar{r})$ is a real function of position characterizing the randomly fluctuating part whose amplitude is very small and whose ensemble average is zero. The layer of random medium has boundaries at $z = 0$ and $z = -d$ (Fig. 3.1). The upper region is free space with permittivity ε_o and the bottom medium is homogeneous with permittivity ε_2. All three regions are assumed to have the same permeability μ_o.

The formal solution to the scattering of electromagnetic waves with time dependent factor $e^{-i\omega t}$ by the two-layer random medium can be cast in terms of dyadic Green's functions. We have

$$\bar{E}_o(\bar{r}) = \bar{E}_o^{(0)}(\bar{r}) + \frac{1}{i\omega\mu_o}\int_{V_1} d^3r_1\ \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)\cdot Q(\bar{r}_1)\ \bar{E}_1(\bar{r}_1) \tag{3.1a}$$

$$\bar{E}_1(\bar{r}) = \bar{E}_1^{(0)}(\bar{r}) + \frac{1}{i\omega\mu_o}\int_{V_1} d^3r_1\ \bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1)\cdot Q(\bar{r}_1)\ \bar{E}_1(\bar{r}_1) \tag{3.1b}$$



Scattering geometry of a two-layer random medium

Figure 3.1

where the integrations extend over region 1 occupied by the random medium. The first and the second subscripts of the dyadic Green's functions $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$ and $\bar{\bar{G}}_{11}(\bar{r}, \bar{r}_1)$ refer to the regions of the field and source points, respectively. The random fluctuating part is accounted for as a source distribution with $Q(\bar{r}) = k_1^2 \varepsilon_{1f}(\bar{r})/\langle\varepsilon_1\rangle$ where $k_1^2 = \omega^2\mu_o\langle\varepsilon_1\rangle$. The superscript zero in $\bar{E}_o^{(0)}(\bar{r})$ and $\bar{E}_1^{(0)}(\bar{r})$ refers to the solutions in the absence of the random fluctuation part which are also the zeroth order terms in an iterative series solution. We shall use parenthesized superscripts 1, 2, 3 etc. to denote higher order terms.

Since we are interested in the scattered far field, we evaluate $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$ which appears in (3.1a) by the saddle point method. From Equations (2.5) and (2.6) we may cast $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$ into the form:

$$\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = \int d^2k_\perp \, e^{i\bar{k}\cdot\bar{r}} \, \bar{\bar{F}}(\bar{k}_\perp, z_1) \, e^{-i\bar{k}_\perp\cdot\bar{r}_{1\perp}} \tag{3.2}$$

where

$$\bar{\bar{F}}(\bar{k}_\perp, z_1) \equiv e^{-ik_{oz}z} \, \bar{\bar{g}}_{01}(\bar{k}_\perp, z, z_1) \tag{3.3}$$

and $\bar{k} = \bar{k}_\perp + \hat{z}\, k_{oz}$. We introduce the transformations

$$k_x = k_\rho \cos \bar{\phi} \tag{3.4a}$$

$$k_y = k_\rho \sin \bar{\phi} \tag{3.4b}$$

$$k_{oz} = \sqrt{k_o{}^2 - k_\rho{}^2} \tag{3.4c}$$

and

$$x = \rho \cos \phi \tag{3.5a}$$

$$y = \rho \sin \phi . \tag{3.5b}$$

Making use of (3.4a)-(3.5b), equation (3.2) becomes

$$\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = \int_0^\infty k_\rho dk_\rho \int_0^{2\pi} d\bar{\phi}\, e^{ik_\rho \rho \cos(\bar{\phi} - \phi) + ik_{oz} z}$$

$$[\bar{\bar{F}}(\bar{k}_\perp, z_1)\, e^{-i\bar{k}_\perp \cdot \bar{r}_{1\perp}}]_{\substack{k_x = k_\rho \cos \bar{\phi} \\ k_y = k_\rho \sin \bar{\phi}}} \tag{3.6}$$

The $\bar{\phi}$ and k_ρ integrations are performed by using the saddle point method. The resultant far field dyadic Green's function is simply:

$$\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) = i\omega\mu_o \frac{e^{ik_o r}}{4\pi r} \{\bar{\bar{H}}\, e^{-i\bar{k}_1 \cdot \bar{r}_1} + \bar{\bar{F}}\, e^{-i\bar{\kappa}_1 \cdot \bar{r}_1}\} \tag{3.7}$$

where

$$\bar{\bar{H}} = \frac{X_{01}}{D_2}\, \hat{e}(k_{oz})\, \hat{e}_1(k_{1z}) + \frac{k_o}{k_1} \frac{Y_{01}}{F_2}\, \hat{h}(k_{oz})\, \hat{h}_1(k_{1z}) \tag{3.8a}$$

$$\bar{\bar{F}} = \left[\frac{X_{01}}{D_2} R_{12}\, \hat{e}(k_{oz})\, \hat{e}_1(-k_{1z}) + \frac{k_o}{k_1} \frac{Y_{01}}{F_2} S_{12}\, \hat{h}(k_{oz})\, \hat{h}_1(-k_{1z})\right]$$

$$e^{i2k_{1z}d_1} \tag{3.8b}$$

$$\bar{k}_1 = \bar{k}_\perp + \hat{z}\, k_{1z} \tag{3.9a}$$

$$\bar{\kappa}_1 = \bar{k}_\perp - \hat{z}\, k_{1z}. \tag{3.9b}$$

3.2 First Order Scattered Intensity

Solutions to (3.1) can be obtained by iteration. Substituting (3.1b) in (3.1a) we find the total solution in region 0 in the form of the Neumann series

$$\bar{E}_o(\bar{r}) = \bar{E}_o^{(0)}(\bar{r}) + \sum_{n=1}^{\infty} \bar{E}_o^{(n)}(\bar{r}) \tag{3.10}$$

where the n-th order field $\bar{E}_o^{(n)}$ is given by:

$$\bar{E}_o^{(n)}(\bar{r}) = \frac{1}{(i\omega\mu_o)^n} \int d^3r_1 \ldots d^3r_n \, \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) \cdot \bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2) \ldots$$

$$\ldots \bar{\bar{G}}_{11}(\bar{r}_{n-1}, \bar{r}_n) \cdot Q(\bar{r}_1) \ldots Q(\bar{r}_n) \cdot \bar{E}_1^{(0)}(\bar{r}_n). \tag{3.11}$$

Physically, the n-th order field represents the n-th scattering of the incident field $\bar{E}_1^{(0)}(\bar{r}_n)$ by the random permittivity fluctuations. In (3.11) it is understood that each of the volume integrations extends over the layer of random medium. Forming the square of the absolute value of $\bar{E}_o(\bar{r})$ and ensemble averaging, we obtain the intensity in region 0

$$<|\bar{E}_o(\bar{r})|^2> = |\bar{E}_o^{(0)}(\bar{r})|^2 + 2\text{Re}\{\bar{E}_o^{(0)*}(\bar{r}) \cdot \sum_{n=1}^{\infty} <\bar{E}_o^{(2n)}(\bar{r})>\}$$

$$+ < \sum_{n=1}^{\infty} \bar{E}_o^{(n)*}(\bar{r}) \cdot \sum_{m=1}^{\infty} \bar{E}_o^{(m)}(\bar{r})> . \tag{3.12}$$

It is to be noted the ensemble average of all odd order moments of $Q(\bar{r})$ vanish in the cross terms.

The first order scattered intensity in (3.12) is given by

$$<|\bar{E}_o(\bar{r})|^2>^{(1)} = <|\bar{E}_o^{(1)}(\bar{r})|^2> + 2\text{Re}\{\bar{E}_o^{(0)*}(\bar{r}) \cdot <\bar{E}_o^{(2)}(\bar{r})>\} .$$

It can be shown that the mean field term $<\bar{E}_o^{(2)}(\bar{r})>$ is specular and is much smaller than $<|\bar{E}_o^{(1)}|^2>$ in the low conductivity regime. We thus have, after making use of (3.11)

$$<|\bar{E}_o(\bar{r})|^2>^{(1)} = \frac{1}{(\omega\mu_o)^2}\int_{V_1} d^3r_1\, d^3r_2\, \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) \cdot \bar{E}_1^{(0)}(\bar{r}_1)$$

$$\cdot \bar{\bar{G}}_{01}^*(\bar{r}, \bar{r}_2) \cdot \bar{E}_1^{(0)*}(\bar{r}_2) <Q(\bar{r}_1)\, Q^*(\bar{r}_2)> . \tag{3.13}$$

The unperturbed field $\bar{E}_1^{(0)}(\bar{r})$ is given by

$$\bar{E}_1^{(0)}(\bar{r}) = E_o[\bar{f} \cdot \bar{\bar{A}}_i \, e^{i\bar{k}_{1i} \cdot \bar{r}} + \bar{f} \cdot \bar{\bar{B}}_i \, e^{i\bar{\kappa}_{1i} \cdot \bar{r}}] \tag{3.14}$$

where

$$\bar{\bar{A}}_i = \frac{X_{01i}R_{12i}}{D_{2i}} e^{i2k_{1zi}d_1} \hat{e}_{1i}\hat{e}_{1i} + \frac{k_o}{k_1} \frac{Y_{01i}S_{12i}}{F_{2i}} e^{i2k_{1zi}d_1}$$

$$\hat{h}_1(k_{1zi}) \; \hat{h}_1(-k_{1zi}) \tag{3.15a}$$

$$\bar{\bar{B}}_i = \frac{X_{01i}}{D_{2i}} \hat{e}_{1i}\hat{e}_{1i} + \frac{k_o}{k_1} \frac{Y_{01i}}{F_{2i}} \hat{h}_1(k_{1zi}) \; \hat{h}_1(-k_{1zi}) \tag{3.15b}$$

$$\bar{f} = f_e \, \hat{e}_{1i} + f_m \, \hat{h}_1(k_{1zi}). \tag{3.16}$$

The subscript i denotes the incident direction, and the vector components denote the fractions of the vertically and horizontally polarized components of the incident wave.

Making use of the unperturbed field $\bar{E}_1^{(0)}(\bar{r})$ and the far field approximated Green's function $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$, we obtain from (3.13)

$$<|\bar{E}_o|^2>^{(1)} = \frac{1}{r^2} \int_{V_1} d^3r_1 \, d^3r_2 \sum_{s,s'} \sum_{p,p'} \bar{\psi}_{s,s'} \cdot \bar{\psi}^*_{p,p'}$$

$$e^{i(sk_{1z} + s'k_{1zi})z_1} e^{-i(pk^*_{1z} + p'k^*_{1zi})z_2}$$

$$e^{i(\bar{k}_{\perp i} - \bar{k}_{\perp}) \cdot \bar{r}_{1\perp}} e^{-i(\bar{k}_{\perp i} - \bar{k}_{\perp}) \cdot \bar{r}_{2\perp}} C(\bar{r}_1 - \bar{r}_2)$$

(3.17)

where s, s', p and p' take values of either -1 or +1, and

$$\bar{\psi}_{-1,1} = \frac{E_o}{4\pi} \bar{\bar{H}} \cdot (\bar{f} \cdot \bar{\bar{A}}_i) \quad (3.18a)$$

$$\bar{\psi}_{-1,-1} = \frac{E_o}{4\pi} \bar{\bar{H}} \cdot (\bar{f} \cdot \bar{\bar{B}}_i) \quad (3.18b)$$

$$\bar{\psi}_{1,1} = \frac{E_o}{4\pi} \bar{\bar{F}} \cdot (\bar{f} \cdot \bar{\bar{A}}_i) \quad (3.18c)$$

$$\bar{\psi}_{1,-1} = \frac{E_o}{4\pi} \bar{\bar{F}} \cdot (\bar{f} \cdot \bar{\bar{B}}_i). \quad (3.18d)$$

The correlation function $C(\bar{r}_1 - \bar{r}_2) = \langle Q(\bar{r}_1)\, Q^*(\bar{r}_2)\rangle$ may be expressed in terms of its Fourier transform

$$C(\bar{r}_1 - \bar{r}_2) = \delta k_1'^4 \int d^3\beta\ \Phi(\bar{\beta})\ e^{-i\bar{\beta} \cdot (\bar{r}_1 - \bar{r}_2)} \quad (3.19)$$

where δ is the variance of the fluctuations and $k_1' = \mathrm{Re}(k_1)$. Substituting (3.19) in (3.17) we may perform the integraions over the spatial variables. One of the transverse spatial integrations yields a delta function and the other transverse spatial integration yields the illuminated area A of the horizontal plane. Furthermore, the delta function enables the $\bar{\beta}_\perp$ integration to be performed. After carrying out the z_1 and z_2 integrations, we obtain

$$<|\bar{E}_o|^2>^{(1)} = \frac{4\pi^2\delta k_1'^4 A}{r^2} \sum_{s,s'} \sum_{p,p'} \bar{\psi}_{s,s'} \cdot \bar{\psi}^*_{p,p'} \int_{-\infty}^{\infty} d\beta_z$$

$$\Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_z) \frac{1}{(sk_{1z} + s'k_{1zi} - \beta_z)(pk^*_{1z} + p'k^*_{1zi} - \beta_z)}$$

$$[1 + e^{-i[(sk_{1z} - pk^*_{1z}) + (s'k_{1zi} - p'k^*_{1zi})]d_1}$$

$$- e^{-i[sk_{1z} + s'k_{1zi} - \beta_z]d_1} - e^{i[pk^*_{1z} + p'k^*_{1zi} - \beta_z]d_1}]. \tag{3.20}$$

The β_z integration is performed by noting that for a low conductivity random layer containing many wavelengths the dominant contribution occurs at $p = s$ and $p' = s'$ for the poles at $\beta_z = pk_{1z} + p'k_{1zi}$. It is to be noted that there also exists an additional significant contribution only in the backscattering direction for the case $p' = s = -p = -s'$. To obtain the residues at these poles we must carefully note the positions of the poles on the complex β_z-plane. For example when $s = +1$ and $s' = -1$, the pole may lie on either the upper or the lower half of the complex β_z-plane according to whether $\mathrm{Im}(k_{1z}) \gtrless \mathrm{Im}(k_{1zi})$. Carrying out the β_z integration in this manner, we find:

$$<|\bar{E}_o|^2>^{(1)} = \frac{\delta k_1'^4 4\pi^3 A}{r^2} \sum_{s,s'} |\bar{\psi}_{ss'}|^2 \, G(sk_{1z} + s'k_{1zi})$$

$$+ 4d_1\Delta \, \Phi(2\bar{k}_{\perp i}, 0) \, \mathrm{Re}[\bar{\psi}_{1,-1} \cdot \bar{\psi}^*_{-1,1}] \qquad (3.21)$$

where:

$$G(sk_{1z} + s'k_{1zi}) = \frac{\Phi(\bar{k}_{\perp i} - \bar{k}_{\perp}, sk_{1z} + s'k_{1zi})}{sk''_{1z} + s'k''_{1zi}}$$

$$[e^{2(sk''_{1z} + s'k''_{1zi})d_1} - 1]. \qquad (3.22)$$

Here, Δ is one in the backscattered direction and is zero for all other scattering directions. Thus given the spectral density Φ of a correlation function, the first order scattered intensity from a two layer random medium is readily determined from (3.21) and (3.22).

3.3 Bistatic Scattering Coefficients and Backscattering Cross Sections

The bistatic scattering coefficients are defined by Peake[57] as:

$$\gamma_{\mu\nu}(\bar{k}_{oi}, \bar{k}) = \lim_{\substack{A\to\infty \\ r\to\infty}} \frac{4\, r^2 (I)_\nu}{\cos\theta_{oi} (I_i)_\mu} \tag{3.23}$$

where $(I_i)_\mu$ is the incident wave intensity with polarization μ, $(I)_\nu$ is the scattered wave intensity with polarization ν, $\bar{k}_{oi}$ is the incident wave vector at the incident angle θ_{oi} and $\phi_{oi} = 0$, $\bar{k}$ is the scattered wave vector at angles θ and ϕ, and r is the distance from the observation point to the surface. Considering spectral densities with an even β_z dependence, we find from (3.21) and (3.23) γ_{hh} and γ_{hv} by letting $f_e = 1$ and $f_m = 0$, and γ_{vh} and γ_{vv} by letting $f_e = 0$ and $f_m = 1$.

$$\gamma_{hh} = \frac{\pi^2\delta\, k_1'^4}{\cos\theta_{oi}} \frac{|k_{oz}|^2}{|k_{1z}|^2} \frac{|X_{10}X_{01i}|^2}{|D_2 D_{2i}|^2} \left\{ \frac{\Phi(\bar{\beta}_+)}{k''_{1zi} + k''_{1z}} [1 - e^{-2(k''_{1zi} + k''_{1z})d_1}] \right.$$

$$[1 + |R_{12}R_{12i}|^2\, e^{-2(k''_{1zi} + k''_{1z})d_1}] + \frac{\Phi(\bar{\beta}_-)}{k''_{1zi} - k''_{1z}}$$

$$[1 - e^{-2(k''_{1zi} - k''_{1z})d_1}][|R_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1}$$

$$+ |R_{12}|^2 e^{-4k''_{1z}d_1}]\Bigg\} \cos^2\phi + \Delta 4\pi^2 \delta k_1'^4 \frac{|X_{01i}|^4}{|D_{2i}|^4} |R_{12i}|^2 d_1$$

$$\Phi(2\bar{k}_{\perp i}, 0)\ e^{-4k''_{1zi}d_1} \qquad (3.24)$$

$$\gamma_{hv} = \frac{\pi^2 \delta k_1'^4}{\cos\theta_{oi}} \frac{|k_{oz}|^2}{k_o^2} \frac{|Y_{10}X_{01i}|^2}{|F_2F_{2i}|^2} \Bigg\{ \frac{\Phi(\bar{\beta}_+)}{k''_{1zi} + k''_{1z}}$$

$$[1 - e^{-2(k''_{1zi} + k''_{1z})d_1}][1 + |S_{12}R_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1}]$$

$$+ \frac{\Phi(\bar{\beta}_-)}{k''_{1zi} - k''_{1z}} [1 - e^{-2(k''_{1zi} - k''_{1z})d_1}]$$

$$[|R_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1} + |S_{12}|^2 e^{-4k''_{1z}d_1}]\Bigg\} \sin^2\phi$$

$$(3.25)$$

$$\gamma_{vh} = \frac{\pi^2 \delta k_1'^4}{\cos\theta_{oi}} \frac{|k_{oz}|^2 |k_{1zi}|^2}{|k_{1z}|^2} \frac{k_o^2}{|k_1|^4} \frac{|X_{10}Y_{01i}|^2}{|D_2F_{2i}|^2} \Bigg\{ \frac{\Phi(\bar{\beta}_+)}{k''_{1zi} + k''_{1z}}$$

$$[1 - e^{-2(k''_{1zi} + k''_{1z})d_1}][1 + |R_{12}S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1}]$$

$$+ \frac{\Phi(\bar{\beta}_-)}{k''_{1zi} - k''_{1z}} [1 - e^{-2(k''_{1zi} - k''_{1z})d_1}]$$

$$[|S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1} + |R_{12}|^2 e^{-4k''_{1z}d_1}] \Bigg\} \sin^2\phi$$

(3.26)

$$\gamma_{vv} = \frac{\pi^2 \delta k_1'^4}{\cos\theta_{oi}} \frac{|k_{oz}|^2}{|k_{1z}|^2} \frac{k_o^4}{|k_1|^4} \frac{|Y_{10}Y_{01i}|^2}{|F_2F_{2i}|^2} \Bigg\{ \frac{\Phi(\bar{\beta}_+)}{k''_{1zi} + k''_{1z}}$$

$$[1 - e^{-2(k''_{1zi} + k''_{z1})d_1}][1 + |S_{12}S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1}]$$

$$\left| \frac{k_{1zi}k_{1z}}{k_o^2} \cos\phi - \sin\theta_{oi} \sin\theta \right|^2 + \frac{\Phi(\bar{\beta}_-)}{k''_{1zi} - k''_{1z}}$$

$$[1 - e^{-2(k''_{1zi} - k''_{1z})d_1}][|S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1z})d_1}$$

$$+ |S_{12}|^2 e^{-4k''_{1z}d_1}] \left| \frac{k_{1zi}k_{1z}}{k_o^2} \cos\phi + \sin\theta_{oi} \sin\theta \right|^2 \Bigg\}$$

$$+ \Delta 4\pi^2 \delta k_1'^4 \frac{k_o^4}{|k_1|^4} \frac{|Y_{01i}|^4}{|F_{2i}|^4} |S_{12i}|^2 d_1 \, \Phi(2\bar{k}_{\perp i}, 0) \, e^{-4k''_{1zi}d_1}$$

$$\left|\frac{k_{1zi}^2}{k_o^2} - \sin^2\theta_{oi}\right|^2 \tag{3.27}$$

where $\bar{\beta}_\pm = (\bar{k}_{\perp i} - \bar{k}_\perp) + \hat{z}(k'_{1zi} \pm k'_{1z})$ and subscripts h and v denote, respectively, horizontal and vertical polarizations.

We consider a correlation function which is Gaussian laterally and exponential vertically:

$$\langle Q(\bar{r}_i)\ Q^*(\bar{r}_j)\rangle = \delta k_1'^4\ e^{-|\bar{r}_{i\perp} - \bar{r}_{ji}|^2/\ell_\rho^2}\ e^{-|z_i - z_j|/\ell}. \tag{3.28}$$

The corresponding spectral density is

$$\Phi(\bar{\beta}) = \frac{\ell\ell_\rho^2}{4\pi^2(1 + \beta_z^2\ell^2)}\ e^{-\beta_\perp^2\ell_\rho^2/4} \ . \tag{3.29}$$

Substituting in (3.24)-(3.27) and letting $\theta = \theta_{oi}$ and $\phi = \pi + \phi_{oi}$, we obtain the backscattering cross section per unit area defined by $\sigma_{pq} = \gamma_{pq} \cos\theta_{oi}$.

$$\sigma_{hh} = \frac{\delta k_1'^4\ell\ell_\rho^2}{4}\ \frac{|X_{10i}|^4}{|D_{2i}|^4}\ \left|\frac{k_{ozi}}{k_{1zi}}\right|^4\ e^{-k_o^2\ell_\rho^2\sin^2\theta_{oi}}$$

$$\left\{ \frac{1 - e^{-4k''_{1zi}d_1}}{2k''_{1zi}(1 + 4k'^2_{1zi}\ell^2)} (1 + |R_{12i}|^4 e^{-4k''_{1zi}d_1}) + 8d_1 |R_{12i}|^2 e^{-4k''_{1zi}d_1} \right\} \tag{3.30a}$$

$$\sigma_{vv} = \frac{\delta k_1'^4 \ell \ell_\rho^2}{4} \frac{|Y_{10i}|^4}{|F_{2i}|^4} \left| \frac{k_{ozi}}{k_{1zi}} \right|^4 e^{-k_o^2 \ell_\rho^2 \sin^2\theta_{oi}}$$

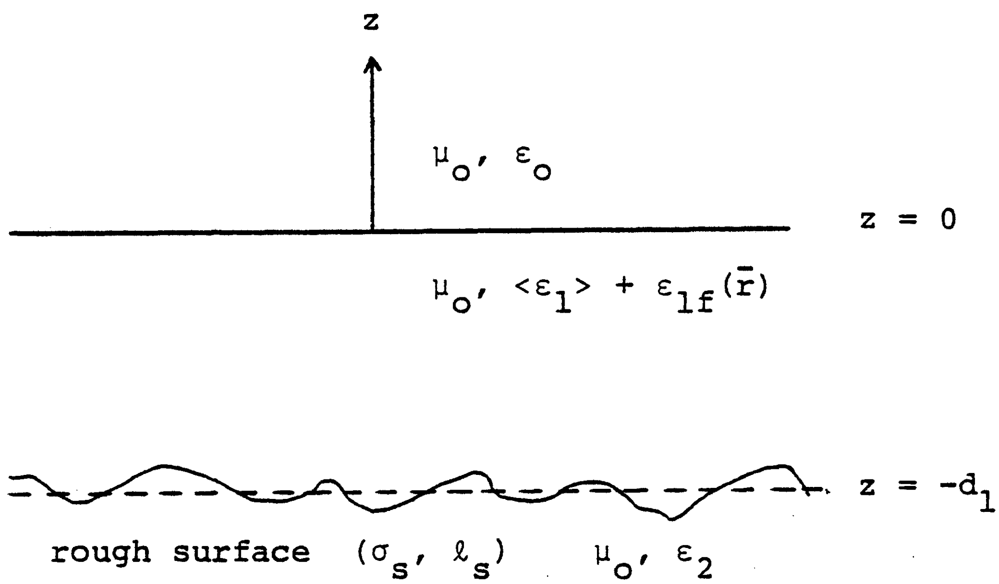
$$\left\{ \frac{1 - e^{-4k''_{1zi}d_1}}{2k''_{1zi}(1 + 4k'_{1zi}\ell^2)} (1 + |S_{12i}|^4 e^{-4k''_{1zi}d_1}) \left| \frac{k^2_{1zi}}{k_o^2} + \sin^2\theta_{oi} \right|^2 + 8d_1 |S_{12i}|^2 e^{-4k''_{1zi}d_1} \left| \frac{k^2_{1zi}}{k_o^2} - \sin^2\theta_{oi} \right|^2 \right\} . \tag{3.30b}$$

It is to be noted that $\sigma_{hv} = \sigma_{vh} = 0$. Thus there is no depolarization effect in the first order scattering theory.

3.4 Rough Surface Effects

The backscattering cross sections derived in the preceding section are based upon the assumption of flat boundaries. This assumption may not be realistic in many situations of interest such as vegetation-ground and snow-air interfaces. A more physical model for such situations is to consider the surface to be irregular and characterized by a surface correlation length, ℓ_s (or ℓ_{sx}, ℓ_{sy} for anisotropic rough surfaces) and R.M.S. height deviation σ_s. In Fig. 3.2 we illustrate a two-layer random medium with a rough surface at the bottom. Such a model would simulate, for example, a vegetation layer with irregular ground. The scattering by a rough surface which separates two homogeneous, dielectric media has been solved only in the cases of $\ell_s \gg \lambda$ or $\ell_s \ll \lambda$. Scattering by a composite rough surface and random medium (e.g. Figure 3.2) is a problem still unsolved at present.

However, in order to account for observed rough surface effects in some fashion we may incoherently superimpose the backscattering of a very rough surface ($\ell_s \gg \lambda$) with the backscattering of the random medium. It is shown in Section 3.5 that such a superposition of intensities matches experimental data remarkably well. The backscattering cross section of a very rough surface is given by Barrick[71] and may be



Two-layer random medium with rough surface

Figure 3.2

written as

$$\sigma_{\substack{hh\\vv}} = \frac{\sec^4\theta_{oi}}{s^2}\,|R_{01}(0)|^2\, e^{-\tan^2\theta_{oi}/s^2} \tag{3.31}$$

where s^2 is the mean square slope of the surface. Comparing (3.31) with (3.30) it is clear that the rough surface backscattering falls off much faster with angle than does the volume backscattering. Physically this is due to less rough surface and more volume being seen at large incident angles.

3.5 Data Matching and Discussions

We have obtained, for a two-layer random medium, the bistatic scattering coefficients in (3.24)-(3.27) and the backscattering cross-sections in (3.30) for a correlation function which is Gaussian laterally and exponential vertically. It is noted that these results can be derived by using a Fourier transform method instead of applying the saddle point method to the evaluation of the dyadic Green's functions. In the case of scattering by a half-space random medium, we let $d_1 \to \infty$ and find

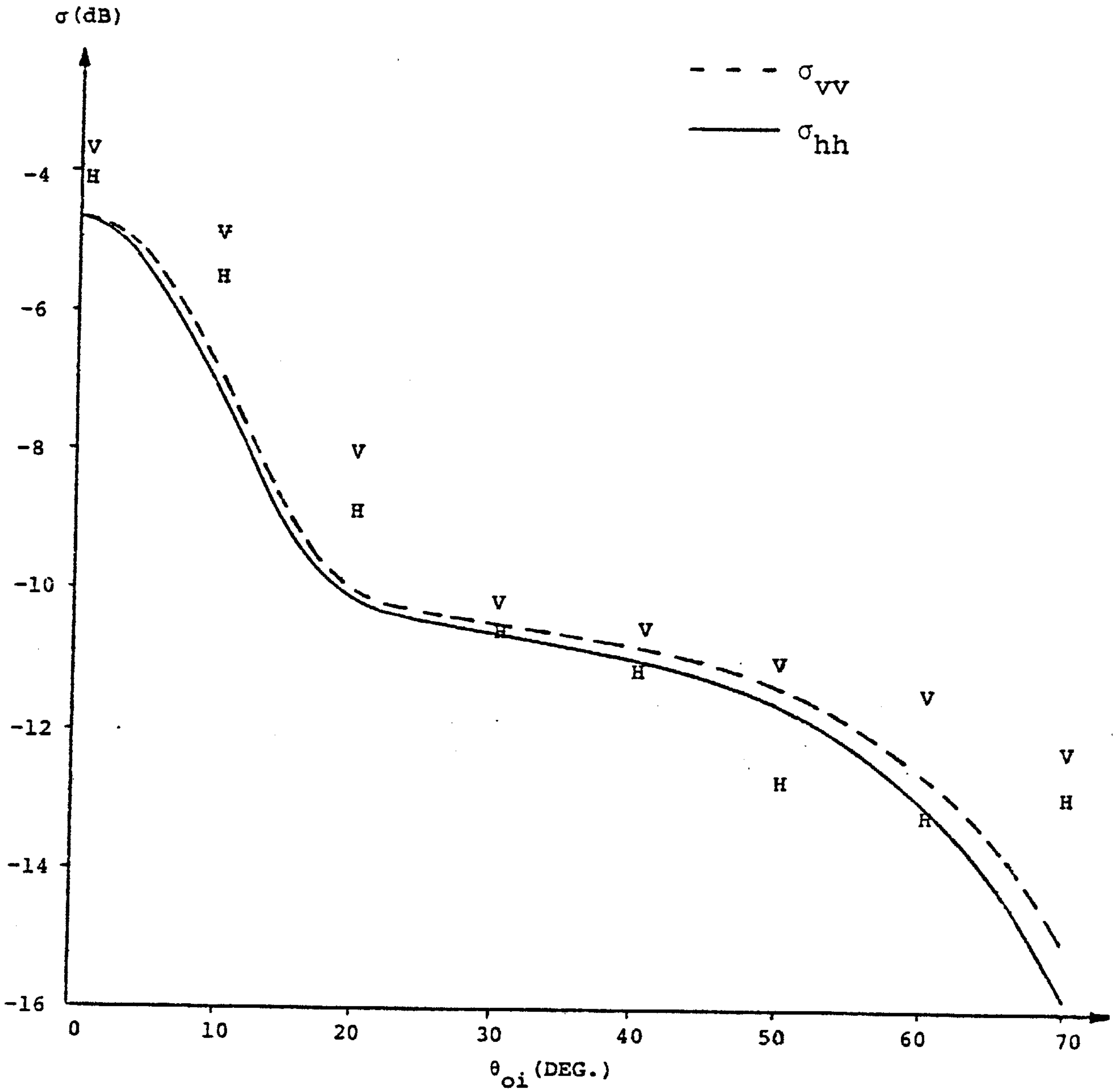
$$\sigma_{hh} = \frac{\delta k_1'^4 \ell \ell_\rho^2}{8k''_{1zi}(1 + 4k_{1zi}^2 \ell^2)} |X_{10i}|^4 \left|\frac{k_{ozi}}{k_{1zi}}\right|^4 e^{-k_o^2 \ell_\rho^2 \sin^2\theta_{oi}}$$

$$\sigma_{vv} = \frac{\delta k_1'^4 \ell \ell_\rho^2}{8k''_{1zi}(1 + 4k_{1zi}^2 \ell^2)} |Y_{10i}|^4 \left|\frac{k_{ozi}}{k_{1zi}}\right|^4 \left|\frac{k_{1zi}^2}{k_o^2} + \sin^2\theta_{oi}\right|^2 e^{-k_o^2 \ell_\rho^2 \sin^2\theta_{oi}}.$$

These are exactly the results obtained with the radiative transfer theory by Tsang and Kong.[63] When plotted as a function of incident angle, σ_{vv} is always larger than σ_{hh}. This is because more vertically polarized wave components are transmitted and backscattered by the random medium.

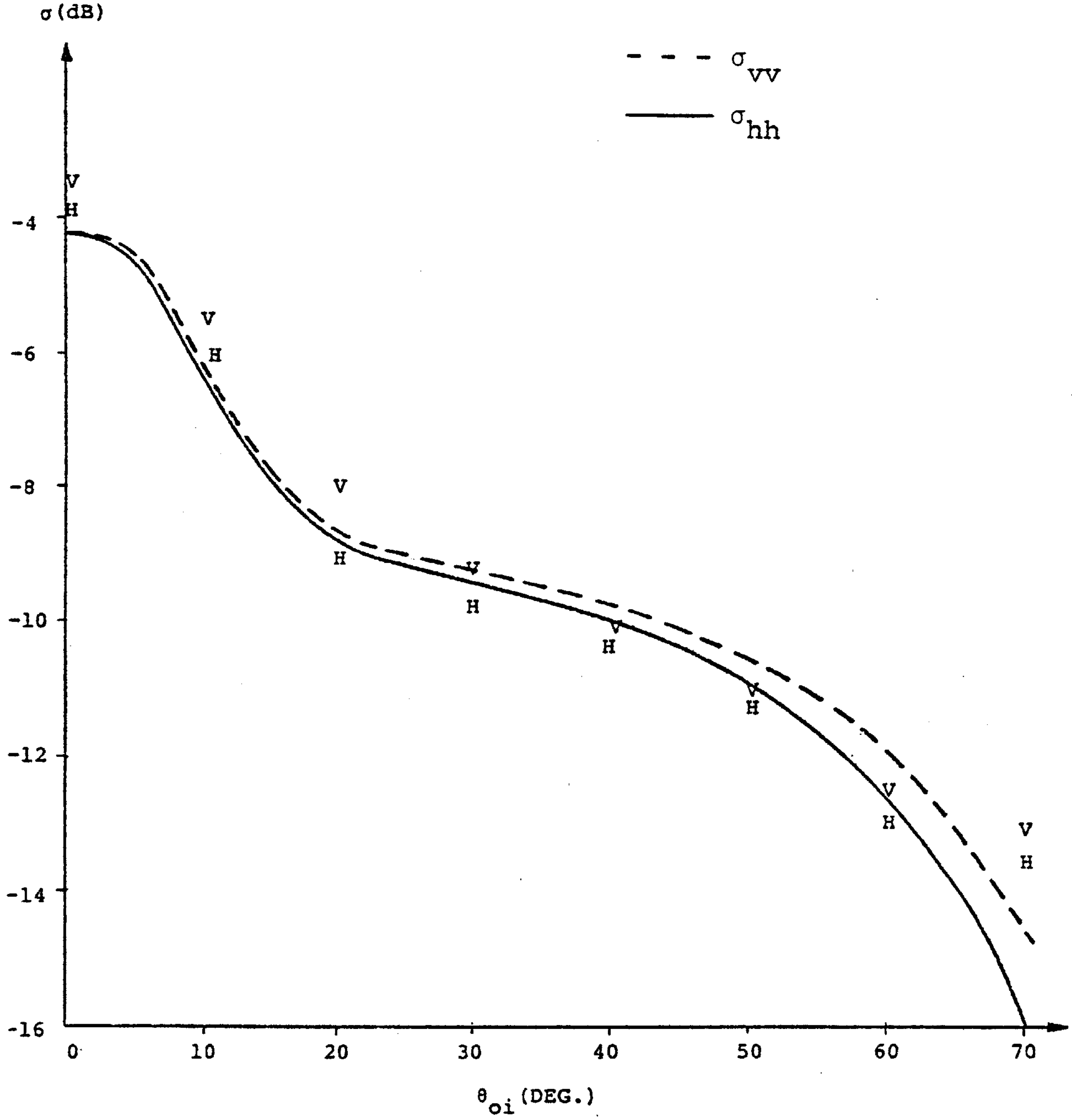
For wave scattering by a two-layer random medium, the results in (3.30) suggest the possibility of σ_{vv} smaller than σ_{hh} when the first terms in the curly brackets become smaller than the second terms. In this case $\sigma_{hh} \sim |X_{10i}|^4 |R_{12i}|^2$, $\sigma_{vv} \sim |Y_{10i}|^4 |S_{12i}|^2$ and $S_{12i} < R_{12i}$ due to the Brewster angle effect at the bottom interface. Thus even though $|Y_{10i}| \geq |X_{10i}|$, it is possible to have $\sigma_{vv} \leq \sigma_{hh}$. Physically this is because more horizontally polarized wave is reflected by the bottom boundary resulting in more horizontally polarized wave in region 1 to be backscattered by the random medium.

The results in (3.30) have been applied to the interpretation of remote sensing data collected in vegetation and snow-ice fields.[42,1] In order to account for rough surface effects, which dominate over volume scattering in the angular region about nadir, we have incorporated the backscattering cross-section of a very rough surface in an incoherent fashion (Section 3.4). In Figs. 3.3-3.5, we have matched the backscattering data of a corn field with a height of 2.1 m at three different frequencies. The letters V and H represent experimental σ_{vv} and σ_{hh} while the continuous curves depict the theoretical results. In order to match these data, we assigned the values $\ell_\rho = 0.32$ cm, $\ell = 1$ cm, $\delta = 0.361$, and $s^2 = 0.03$ where s^2 is the mean square slope of the rough surface. As seen from these figures, the match between the experimental and theoretical results are very good. The volume scattering dominant portion of the curves for $\theta_{oi} \geq 20°$ accounts for frequency change very well which the rough surface dominant portion for $\theta_{oi} \leq 20°$ does not due to the frequency insensitivity of the very rough surface result. It is interesting to note that in order to match these backscattering data, it was necessary to choose $\ell > \ell_\rho$. In Fig. 3.6, we consider the same corn field and fit the frequency dependence of the backscattering cross section at nadir and 30°. The slight frequency variation in the theoretical result at nadir is due to the fact that volume scattering coes contribute at this angle, even though the frequency independent rough surface result dominates.



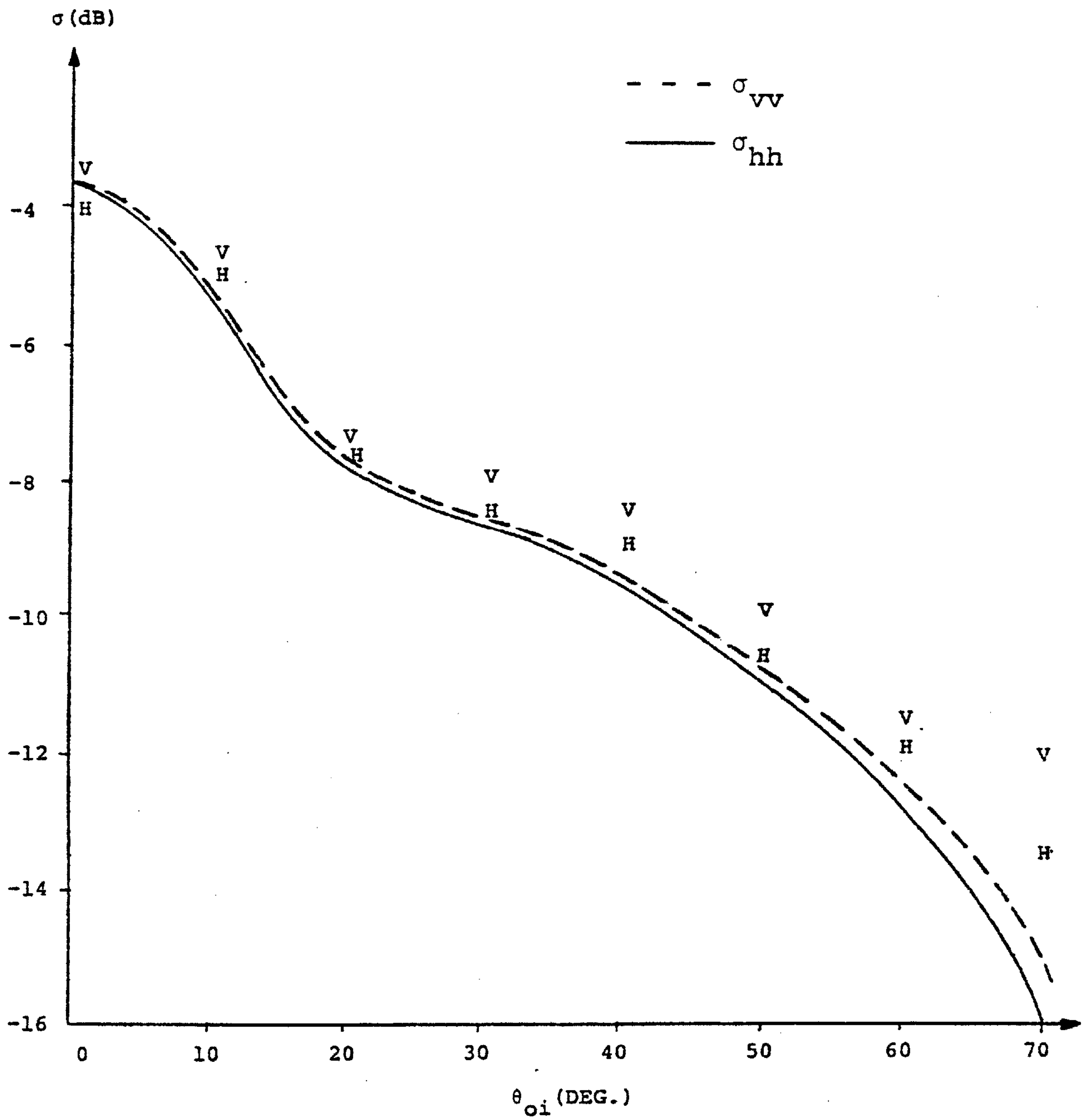
σ_{hh} and σ_{vv} as a function of angle at 9 GHz

Figure 3.3



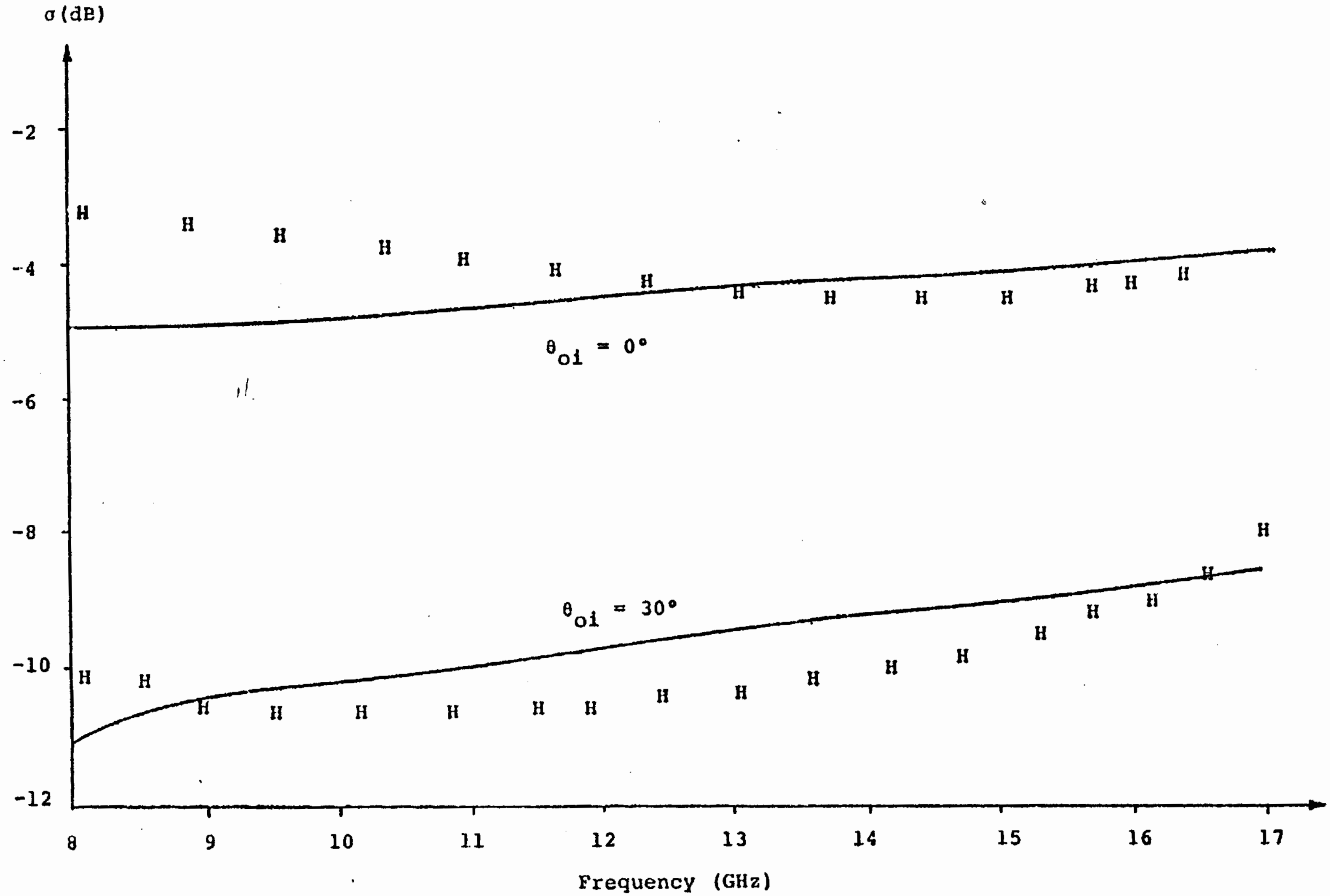
σ_{hh} and σ_{vv} as a function of angle at 13 GHz

Figure 3.4



σ_{hh} and σ_{vv} as a function of angle at 16.6 GHz

Figure 3.5



σ_{hh} as a function of frequency at 0° and 30°

Figure 3.6

CHAPTER 4

Emissivity of a Two-Layer Random Medium

The study of earth terrain can be effected with the method of microwave passive remote sensing. In this method, a radiometer aboard aircraft or satellite records the microwave thermal emissions from naturally occuring media such as snow-ice fields, vegetation coverage and sea-ice. The voluminous data collected and its interpretation has been the subject of considerable theoretical effort in recent years.

Gurvich et al.[72] first derived expressions for the emissivity of a half-space random medium with laminar structure in the single scattering approximation. England[73] examined emission darkening of a half-space containing distributed isotropic point scatterers by employing a radiative transfer approach. Tsang and Kong[69,56,74] have considered thermal microwave emission from a stratified medium with non-uniform temperature distribution, the emissivity of a two-layer laminar random medium as well as thermal microwave emission from half-space random media. In particular, Tsang and Kong[56] employed a wave approach in the first order Born approximation and calculated the emissivities of a half-space random medium. In this chapter, we follow a wave approach similar to Tsang and Kong's[56] and obtain emissivities of a two-layer random medium with arbitrary three dimensional correlation functions.

4.1 Emissivity Using Reciprocity Concept

Peake[57] has defined the emissivity of a natural surface as:

$$e_h(\bar{k}_i) = 1 - r_h - \frac{1}{4\pi}\int d\Omega_s\,[\gamma_{hh}(\bar{k}_i, \bar{k}_s) + \gamma_{hv}(\bar{k}_i, \bar{k}_s)] \qquad (4.1a)$$

$$e_v(\bar{k}_i) = 1 - r_v - \frac{1}{4\pi}\int d\Omega_s\,[\gamma_{vv}(\bar{k}_i, \bar{k}_s) + \gamma_{vh}(\bar{k}_i, \bar{k}_s)] \qquad (4.1b)$$

where r_h and r_v are the Fresnel reflectivities for horizontal and vertical polarizations for a homogeneous two-layer medium. The integrands of (4.1a) and (4.1b) are the bistatic scattering coefficients of a two-layer random medium and the angular integrations extend over the upper hemisphere.

4.2 Emissivity of a Two-Layer Random Medium

The bistatic scattering coefficients of a two-layer random medium have been derived in Chapter 3 and are given in (3.24)-(3.27). In order to develop an expression for the emissivity in as general a form as possible, we expand the spectral density in harmonics of the scattered azimuthal angle, ϕ_s:

$$\Phi(\bar{k}_{\perp i} - \bar{k}_{\perp s}, k_{1zi} \pm k_{1zs}) = \sum_{m=-\infty}^{\infty} e^{im\phi_s} \tilde{\Phi}_m(k_{\rho s}, k_{\rho i}, k_{1zi} \pm k_{1zs}) \tag{4.2}$$

Substituting (4.2) into (4.1a) and (4.1b), the ϕ_s integration may be performed directly. After some algebra, we obtain emissivities in the form:

$$e_h = 1 - r_h - \frac{1}{4}\int_0^{\pi/2} d\theta_s \sin\theta_s \, P_h \Bigg[A_+ \left\{ \frac{|X_{10s}|^2}{|k_{1zi}|^2} \alpha_{is}^{(1)} M_+ + \frac{|Y_{10s}|^2}{k_o^2} \alpha_{is}^{(2)} N_+ \right\} + A_- \left\{ \frac{|X_{10s}|^2}{|k_{1zs}|^2} \beta_{is}^{(1)} M_- + \frac{|Y_{10s}|^2}{k_o^2} \beta_{is}^{(2)} N_- \right\} \Bigg] \tag{4.3a}$$

$$e_v = 1 - r_v - \frac{1}{4}\int_0^{\pi/2} d\theta_s \sin\theta_s \, P_v \left[A_+ \left\{ \frac{|X_{10s}|^2}{2} |k_{1zi}|^2 \, \tilde{\alpha}_{is}^{(1)} N_+ \right.\right.$$

$$+ |Y_{10s}|^2 k_o^2 \, \tilde{\alpha}_{is}^{(2)} \left(\frac{|k_{1zi}|^2 |k_{1zs}|^2}{2k_o^4} M_+ \right.$$

$$+ 2 \sin^2\theta_{oi} \sin^2\theta_s \, \tilde{\Phi}_o(+)$$

$$\left.\left. - 2\mathrm{Re}(k_{1zi}^* k_{1zs}) \frac{\sin\theta_{oi} \sin\theta_s}{k_o^2} [\tilde{\Phi}_1(+) + \tilde{\Phi}_{-1}(+)] \right) \right\}$$

$$+ A_- \left\{ \frac{|X_{10s}|^2}{2} |k_{1zi}|^2 \, \tilde{\beta}_{is}^{(1)} N_- \right.$$

$$+ |Y_{10s}|^2 k_o^2 \, \tilde{\beta}_{is}^{(2)} \left(\frac{|k_{1zi}|^2 |k_{1zs}|}{2k_o^4} M_- + 2 \sin^2\theta_s \, \tilde{\Phi}_o(-) \right.$$

$$\left.\left.\left. + 2\mathrm{Re}(k_{1zi}^* k_{1zs}) \frac{\sin\theta_{oi} \sin\theta_s}{k_o^2} [\tilde{\Phi}_1(-) + \tilde{\Phi}_{-1}(-)] \right) \right\} \right] \qquad (4.3b)$$

where

$$P_h \equiv \frac{\pi^2 \delta \, k_1'^4}{2 \cos\theta_{oi}} \frac{|X_{01i}|^2}{|D_{2i}|^2} k_{ozs}^2 \qquad (4.4a)$$

$$P_v = \frac{\pi^2 \delta k_1'^4}{\cos\theta_{oi}} \frac{|Y_{01i}|^2}{|E_{2i}|^2} \frac{k_{ozs}^2}{|k_{1zs}|^2} \frac{k_o^2}{|k_1|^4} \tag{4.4b}$$

$$A_\pm \equiv \frac{(1 - e^{-2(k''_{1zi} \pm k''_{1zs})d_1})}{(k''_{1zi} \pm k''_{1zs})} \tag{4.5}$$

$$\alpha_{is}^{(1)} = \frac{1 + |R_{12i}R_{12s}|^2 \, e^{-2(k''_{1zi} + k''_{1zs})d_1}}{|D_{2s}|^2} \tag{4.6a}$$

$$\alpha_{is}^{(2)} = \frac{1 + |S_{12i}S_{12s}|^2 \, e^{-2(k''_{1zi} + k''_{1zs})d_1}}{|D_{2s}|^2} \tag{4.6b}$$

$$\beta_{is}^{(1)} = \frac{|R_{12i}|^2 \, e^{-2(k''_{1zi} + k''_{1zs})d_1} + |R_{12s}|^2 \, e^{-4k''_{1zs}d_1}}{|F_{2s}|^2} \tag{4.6c}$$

$$\beta_{is}^{(2)} = \frac{|R_{12i}|^2 \, e^{-2(k''_{1zi} + k''_{1zs})d_1} + |S_{12s}|^2 \, e^{-4k''_{1zs}d_1}}{|F_{2s}|^2} \tag{4.6d}$$

$$\tilde{\alpha}_{is}^{(1)} = \frac{1 + |R_{12s}S_{12i}|^2 \, e^{-2(k''_{1zi} + k''_{1zs})d_1}}{|D_{2s}|^2} \tag{4.7a}$$

$$\tilde{\alpha}_{is}^{(2)} = \alpha_{is}^{(2)} \frac{|D_{2s}|^2}{|F_{2s}|^2} \tag{4.7b}$$

$$\tilde{\beta}_{is}^{(1)} = \frac{|S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1zs})d_1} + |R_{12s}|^2 e^{-4k''_{1zs}d_1}}{|D_{2s}|^2}$$

(4.7c)

$$\tilde{\beta}_{is}^{(2)} = \frac{|S_{12i}|^2 e^{-2(k''_{1zi} + k''_{1zs})d_1} + |S_{12s}|^2 e^{-4k''_{1zs}d_1}}{|F_{2s}|^2}$$

$$M_{\pm} = 2\tilde{\Phi}_o(\pm) + \tilde{\Phi}_2(\pm) + \tilde{\Phi}_{-2}(\pm) \tag{4.8a}$$

$$N_{\pm} = 2\tilde{\Phi}_o(\pm) - \tilde{\Phi}_2(\pm) - \tilde{\Phi}_{-2}(\pm) \tag{4.8b}$$

$$\tilde{\Phi}_n(\pm) = \tilde{\Phi}_n(k_{\rho s}, k_{\rho i}, k_{1zi} \pm k_{1zs}) \qquad n = \text{integer}. \tag{4.9}$$

The subscripts s and i denote that a term is to be evaluated at the scattered and incident wavevector angles, respectively.

4.3 Angular and Spectral Dependence of Two-Layer Emissivity

In (4.3a) and (4.3b) we have obtained the emissivity of a two-layer random medium with arbitrary correlation function in the first order Born approximation. To illustrate the emissivities, we take the spectral density to be

$$\Phi(\bar{\beta}_\perp, \beta_z) = \frac{\ell\ell_\rho{}^2}{4\pi^2(1 + \beta_z{}^2\ell^2)} e^{-\beta_\perp{}^2\ell_\rho{}^2/4} \tag{4.10}$$

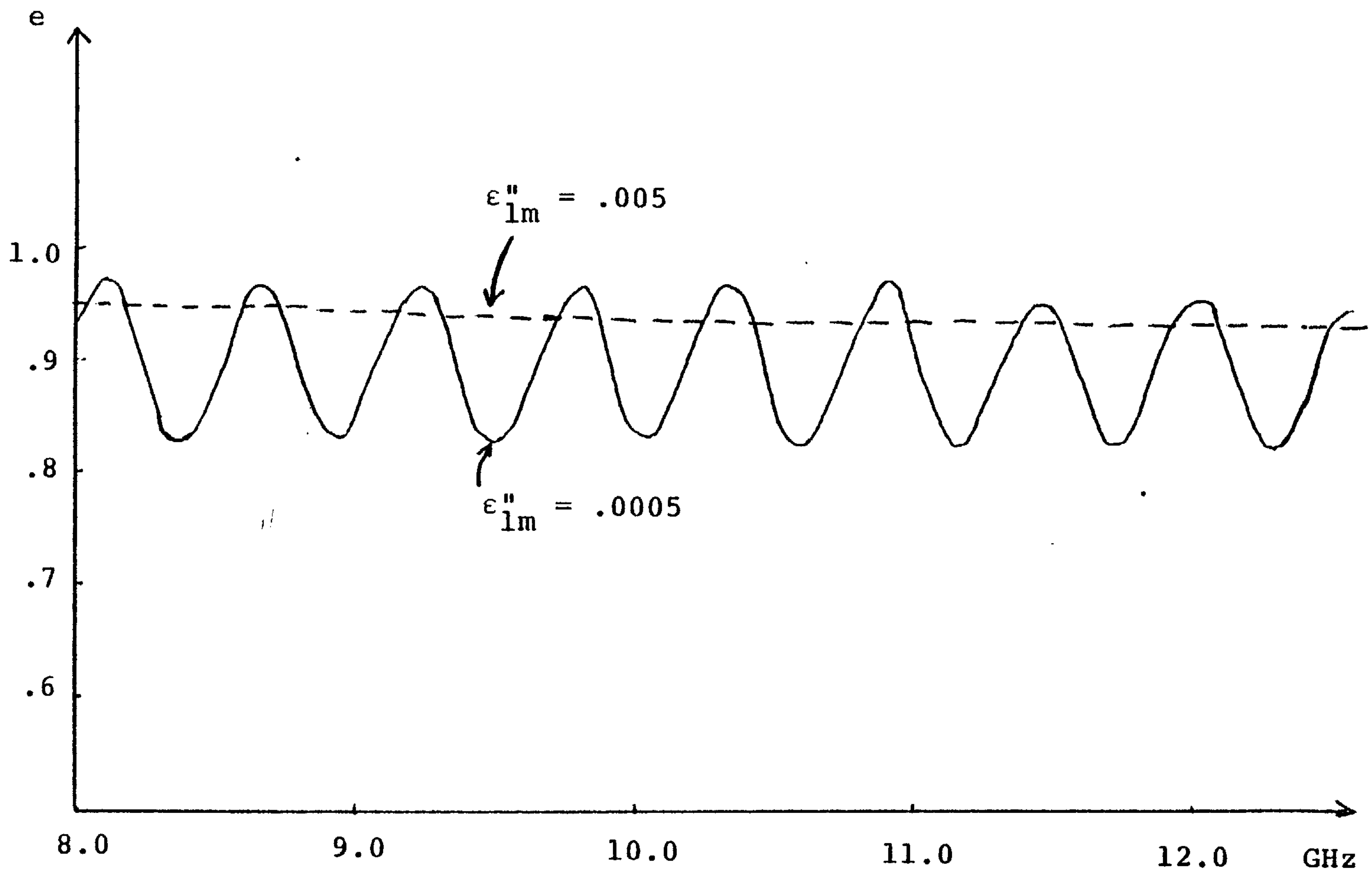
which corresponds to a correlation function that is Gaussian laterally and exponential vertically. Substituting (4.10) in (4.2) we solve for the amplitude $\tilde{\Phi}_m$:

$$\tilde{\Phi}_m(k_{\rho s}, k_{\rho i}, k_{1zi} \pm k_{1zs}) = \frac{\ell\ell_\rho{}^2\, e^{-k_{\rho s}^2\ell_\rho{}^2/4 - k_{\rho i}^2\ell_\rho{}^2/4}}{4\pi^2[1 + (k_{1zi} \pm k_{1zs})^2\ell^2]}$$

$$I_m\left(\frac{k_{\rho s}k_{\rho i}\ell_\rho{}^2}{2}\right) \tag{4.11}$$

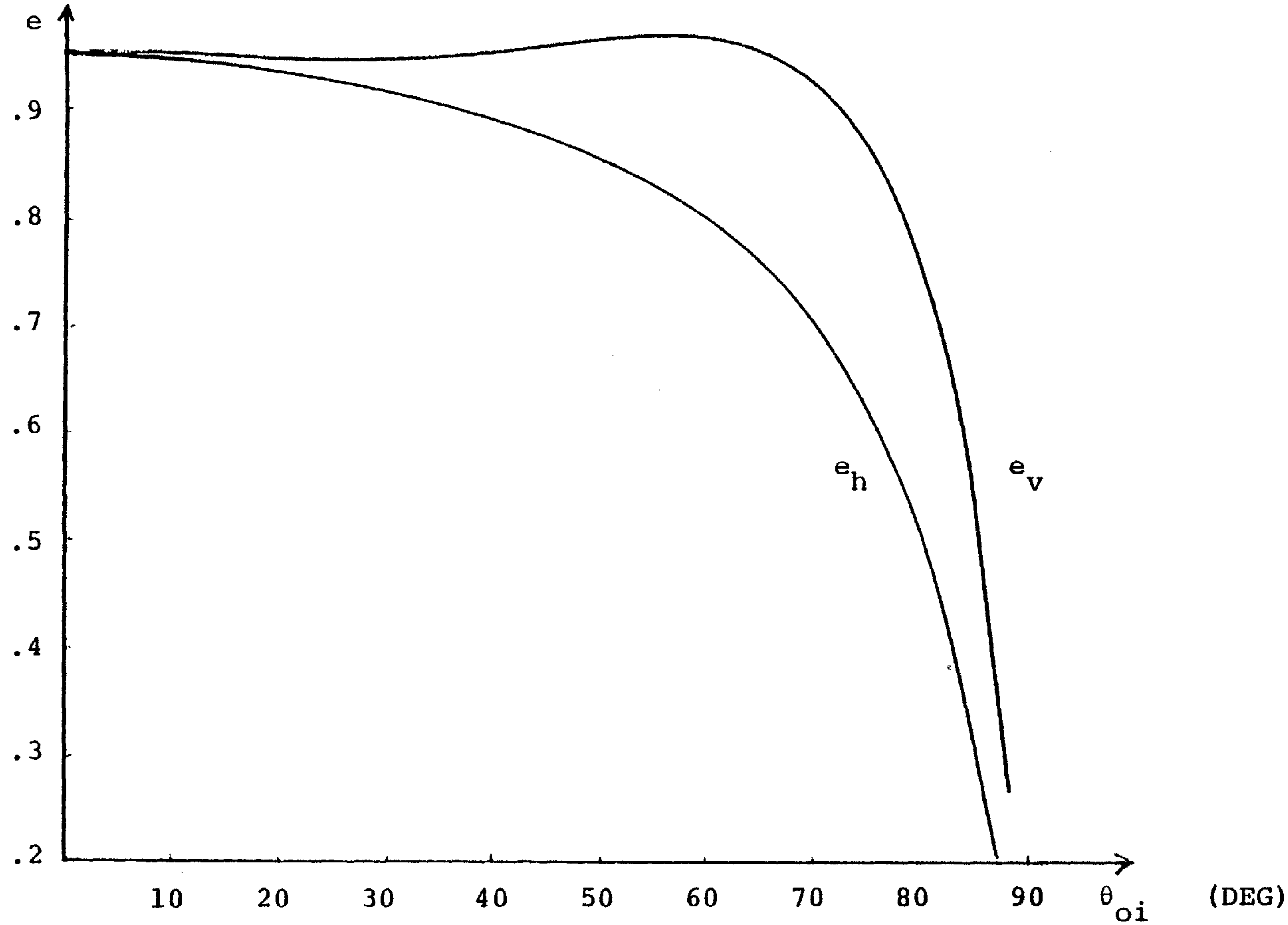
where $I_m(x)$ is the modified Bessel function of order m. In deriving (4.11) we also have taken $\phi_i = 0$, due to the azimuthal symmetry of the assumed correlation function. Inserting

(4.11) into (4.3a) and (4.3b), the θ_s integrations can be completed numerically. We illustrate the results in Fig. 4.1, where we plot the emissivities $e \equiv e_v \equiv e_h$ at nadir as a function of frequency for a 20 cm thick random layer with $\delta = .0005$, $\ell = .002m$, $\ell_\rho = .01m$, $\varepsilon_2 = (6.0 + i0.6)\varepsilon_o$ and $\langle\varepsilon_1\rangle = (1.8 + i\varepsilon''_{1m})\varepsilon_o$. The dashed curve corresponds to $\varepsilon''_{1m} = .005$ whereas the oscillating curve corresponds to $\varepsilon''_{1m} = .0005$. The coherent effects due to the boundary at $z = -d_1 = -20$ cm are apparent in the low loss case where the emissivity oscillates as a function of frequency. As the loss of the medium is increased, the bottom boundary is seen less. This is demonstrated in Fig. 4.1 where for $\varepsilon''_{1m} = .005$ the oscillatory behavior has disappeared, in the spectral dependence of the emissivity. In Fig. 4.2 we plot the emissivites e_v and e_h as a function of angle at 10 GHz, for a 20 cm thick random layer with the same parameter values as Fig. 4.1 and $\varepsilon''_{1m} = .005$. The emissivity of the vertically polarized wave exhibits a slight maximum due to the Brewster angle effect. The horizontally polarized wave has no Brewster angle effect so that e_h decreases monotonically with increasing angle.



Emissivity as a function of frequency at nadir

Figure 4.1



Emissivity as a function of angle at 10 GHz

Figure 4.2

CHAPTER 5

Active Remote Sensing of Stratified Random Media

In active microwave remote sensing of earth terrain the model of a layered random medium has been applied to account for volume scattering effects. Using the model of a half space random medium, backscattering cross sections have been calculated with an iterative approach[56,63] and with the first-order renormalization method.[61] To improve the simulation of earth terrain for vegetation coverage, snow-ice fields or culture targets, two-layer models have been developed, with the random medium bounded by air above and earth below.[62] In these past models, a correlation function with variance and correlation lengths is specified for the entire volume scattering region of interest. However, in remote sensing applications a more realistic model might consist of partitioning the entire scattering region into sub-regions or random media each with a characteristic correlation function. For example, in vegetation cover such as forest terrain the sub-regions would be the leaf and trunk regions of the trees, where the respective lateral and vertical correlation lengths are significantly different. Similarly, in the case of snow-ice fields, one usually finds a complex layered structure in which the lateral and vertical correlation lengths may vary significantly from layer to layer.

In this chapter, we use an iterative wave approach to solve the problem of backscattering by N-layers of random media with three dimensional correlation functions, arbitrarily distributed within (M - N) homogeneous layers. Carrying to first order in albedo and making use of correlation functions which are Gaussian laterally and exponential vertically, we find backscattering cross sections for a three layer problem. The results are illustrated by matching with experimental data collected from vegetation and snow fields.

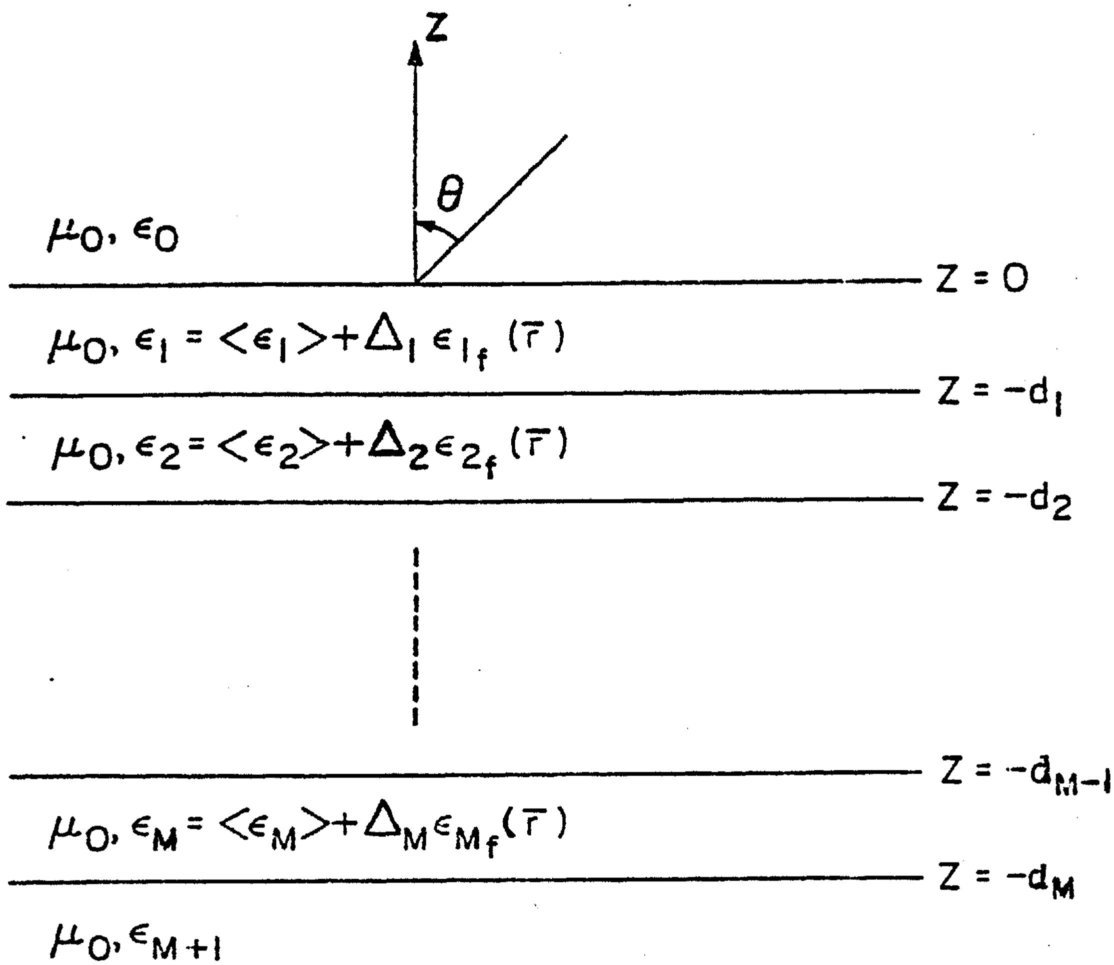
5.1 Backscattered Intensity for a Multilayered Random Medium in the First Order Born Approximation

Consider a vertically stratified medium consisting of N random layers and $M - N$ $(M \geq N)$ homogeneous layers, with boundaries at $z = 0, -d_1, -d_2, \ldots, -d_M$ [Fig. 5.1]. The m-th layer $(m = 1, 2, \ldots, M)$ is characterized by permeability μ_o and permittivity $\varepsilon_m = \langle\varepsilon_m\rangle + \Delta_m \varepsilon_{mf}(\bar{r})$ where $\Delta_m = 0$ or 1 according to whether the m-th layer is homogeneous or random, and $\varepsilon_{mf}(\bar{r})$ is a real random function of position whose magnitude is small and whose ensemble average is zero.

The formal solution to the scattering of time harmonic electromagnetic waves by the stratified medium can be cast in terms of dyadic Green's functions. The first order scattered field is written as

$$\bar{E}_s^{(1)} = \frac{1}{i\omega\mu_o} \sum_{m=1}^{M} \int_{V_m} d^3r_1 \, \bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1) \cdot Q_m(\bar{r}_1) \, \bar{E}_m^{(0)}(\bar{r}_1) \tag{5.1}$$

where $\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1)$ is the dyadic Green's function, $Q_m(\bar{r}_1) = \omega^2\mu_o\Delta_m\varepsilon_{mf}(\bar{r}_1)$, and the superscript zero in $\bar{E}_m^{(0)}(\bar{r}_1)$ refers to the solution in the absence of random fluctuations. The integration in (5.1) extends over the m-th layer occupied by



Scattering geometry of M-layered medium

<u>Figure 5.1</u>

random media and the summation is over the M-layers. The unperturbed field $\bar{E}_m^{(0)}(\bar{r})$ is given by

$$\bar{E}_m^{(0)}(\bar{r}) = E_o[\bar{f} \cdot \bar{\bar{\alpha}}_{mi}\, e^{i\bar{K}_{mi} \cdot \bar{r}} + \bar{f} \cdot \bar{\bar{\beta}}_{mi}\, e^{i\bar{k}_{mi} \cdot \bar{r}}] \tag{5.2}$$

where

$$\bar{k}_{mi} = \bar{k}_{\perp i} + \hat{z}\, k_{mzi} \tag{5.3a}$$

$$\bar{K}_{mi} = \bar{k}_{\perp i} - \hat{z}\, k_{mzi}. \tag{5.3b}$$

The subscript i denotes that a quantity is to be evaluated at the incident wavevector angles. The amplitudes in (5.2) are determined through the propagation matrix formalism[70] (Appendix A) and $\bar{f}$ is a vector which denotes the fraction of TE and TM components of the incident wave. The first and second subscripts of the dyadic Green's function $\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1)$ refer to the regions of the field and source points, respectively. The plane wave representation of $\bar{\bar{G}}_{om}(r, r_1)$ has been derived elsewhere.[69] The result is

$$\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1) = \int d^2k_\perp \, \bar{\bar{g}}_{om}(\bar{k}_\perp, z, z_1) \, e^{i\bar{k}_o \cdot \bar{r} - i\bar{k}_\perp \cdot \bar{r}_1} \tag{5.4}$$

where

$$\bar{\bar{g}}_{om}(\bar{k}_\perp, z, z_1) = -\frac{\omega\mu_o}{8\pi^2}\frac{1}{k_{oz}}\{\hat{e}(k_{oz})[A_m \hat{e}_m(-k_{mz}) e^{ik_{mz}z_1}$$

$$+ B_m \hat{e}_m(k_{mz}) e^{-ik_{mz}z_1}] + \hat{h}(k_{oz})[C_m \hat{h}_m(-k_{mz}) e^{ik_{mz}z_1}$$

$$+ D_m \hat{h}_m(k_{mz}) e^{-ik_{mz}z_1}]\} \; . \tag{5.5}$$

The amplitudes A_m, B_m, C_m and D_m are also determined by using the propagation matrix formulation. Taking the observation point in the far field, $\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1)$ is evaluated with the saddle point method. The result is:

$$\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1) = i\mu_o\omega \{\bar{\bar{H}}_m e^{-ik_{mz}z_1} + \bar{\bar{F}}_m e^{ik_{mz}z_1}\} \frac{e^{ik_or - i\bar{k}_\perp \cdot \bar{r}_1}}{4\pi r} \tag{5.6}$$

where

$$\bar{\bar{H}}_m = B_m \hat{e}(k_{oz}) \hat{e}_m(k_{mz}) + D_m \hat{h}(k_{oz}) \hat{h}_m(k_{mz}) \tag{5.7a}$$

$$\bar{\bar{F}}_m = A_m \hat{e}(k_{oz}) \hat{e}_m(-k_{mz}) + C_m \hat{h}(k_{oz}) \hat{h}_m(-k_{mz}) \tag{5.7b}$$

$$\hat{e}_m(k_{mz}) = \frac{1}{k_\perp} \hat{z} \times \bar{k}_m \tag{5.8a}$$

$$\hat{h}_m(k_{mz}) = \frac{1}{k_m} \hat{e}_m(k_{mz}) \times \bar{k}_m \tag{5.8b}$$

$$k_{mz} = (k_m{}^2 - k_\perp{}^2)^{1/2}, \tag{5.9}$$

$k_m{}^2 = \omega^2\mu\langle\varepsilon_m\rangle$, $\hat{e}_m$ is in the direction of the horizontally polarized electric field vector, and $\hat{h}_m$ is in the direction of the vertically polarized electric field vector.

Forming the absolute square of (5.1) and ensemble averaging, the scattered intensity in region 0 is given by

$$\langle|\bar{E}_s^{(1)}|^2\rangle = \frac{1}{\omega^2\mu_o^2} \sum_{m=1}^{M} \sum_{n=1}^{M} \int_{V_m} \int_{V_n} d^3r_1 d^3r_2\, \bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1)$$

$$\cdot\, \bar{E}_m^{(0)}(\bar{r}_1) \cdot \bar{\bar{G}}^*_{on}(\bar{r}, \bar{r}_2) \cdot \bar{E}_n^{(0)*}(\bar{r}_2) \langle Q_m(\bar{r}_1)\, Q_n{}^*(\bar{r}_2)\rangle \,.$$

(5.10)

Assuming statistical homogeneity throughout the layered medium, the two point correlation function $\langle Q_m(\bar{r}_1)\, Q_n^*(\bar{r}_2)\rangle$ depends only upon the difference of $\bar{r}_1$ and $\bar{r}_2$. Moreover, if we assume that $|d_m - d_{m-1}| >> \ell_{zm}$, $m = 1, 2, \ldots, M$, that is, the thickness of a typical layer is much greater than any of the vertical correlation lengths of interest, then it is clear that random layers separated by one or more homogeneous layers contribute negligibly to the first order scattered intensity (5.10). In the case of adjacent random layers, with $n = m + 1,\ m - 1$, the range of integrations in (5.10) is restricted such that contributions only come from boundary layers on the order of a vertical correlation length thick straddling the $(m, m + 1)$, and $(m, m - 1)$ interfaces. However, since $|d_m - d_{m-1}| >> \ell_{zm}$ $m = 1, 2, \ldots, M$, the volume scattering of the boundary layers is much smaller than the scattering of individual random layers. Therefore, we approximate the two point correlation function as

$$\langle Q_m(\bar{r}_1)\, Q_n^*(\bar{r}_2)\rangle \simeq \delta_{mn}\, C_m(\bar{r}_1 - \bar{r}_2). \tag{5.11}$$

Making use of the unperturbed field $\bar{E}_m^{(0)}(\bar{r})$ and the far field approximated Green's function $\bar{\bar{G}}_{om}(\bar{r}, \bar{r}_1)$, we obtain from (5.10):

$$<|\bar{E}_s^{(1)}|^2> = \frac{1}{r^2} \sum_{m=1}^{M} \sum_{s,s'} \sum_{p,p'} \int_{V_m} d^3r_1 \int_{V_m} d^3r_2 \, \bar{\Psi}_{s,s'}^{(m)}$$

$$\cdot \, \bar{\psi}_{p,p'}^{*(m)} \, e^{i(sk_{mz} + s'k_{mzi})z_1} \, e^{-i(pk_{mz}^* + p'k_{mzi}^*)z_2}$$

$$C_m(\bar{r}_1 - \bar{r}_2) \, e^{i(\bar{k}_{\perp i} - \bar{k}_{\perp}) \cdot \bar{r}_{1\perp}} \, e^{-i(\bar{k}_{\perp i} - \bar{k}_{\perp}) \cdot \bar{r}_{2\perp}} \tag{5.12}$$

where s,s', p and p' take values $+1$ or -1 and,

$$\bar{\psi}_{-1,-1}^{(m)} = \frac{E_o}{4\pi} \bar{\bar{H}}_m \cdot (\bar{f} \cdot \bar{\bar{\beta}}_{mi}) \tag{5.13a}$$

$$\bar{\psi}_{-1,1}^{(m)} = \frac{E_o}{4\pi} \bar{\bar{H}}_m \cdot (\bar{f} \cdot \bar{\bar{\alpha}}_{mi}) \tag{5.13b}$$

$$\bar{\psi}_{1,-1}^{(m)} = \frac{E_o}{4\pi} \bar{\bar{F}}_m \cdot (\bar{f} \cdot \bar{\bar{\beta}}_{mi}) \tag{5.13c}$$

$$\bar{\psi}_{1,1}^{(m)} = \frac{E_o}{4\pi} \bar{\bar{F}}_m \cdot (\bar{f} \cdot \bar{\bar{\alpha}}_{mi}) . \tag{5.13d}$$

The correlation function of random layer m may be expressed in terms of its Fourier transform:

$$C_m(\bar{r}_1 - \bar{r}_2) = \delta_m k_m'^4 \int d^3\beta \, \Phi_m(\bar{\beta}_\perp, \beta_z) \, e^{-i\bar{\beta} \cdot (\bar{r}_1 - \bar{r}_2)} \tag{5.14}$$

where $\delta_m \ll 1$ is the variance of the fluctuations, $k_m' = \mathrm{Re}(k_m)$, and the factor Δ_m has been absorbed into the spectral density $\Phi_m(\bar{\beta}_\perp, \beta_z)$. Substituting (5.14) into (5.12) and proceeding as in Chapter 3, we find

$$<|\bar{E}_s^{(1)}|^2> = \sum_{m=1}^{M} \frac{\delta_m k_m'^4 \, 4\pi^3 A}{r^2} \Big[\sum_{s,s'} |\bar{\psi}_{s,s'}^{(m)}|^2 \, G_m(sk_{mz} + s'k_{mzi})$$

$$+ P \, 4(d_m - d_{m-1}) \, \Phi(2\bar{k}_{\perp i}, 0) \, \mathrm{Re}(\bar{\psi}_{1,-1} \cdot \bar{\psi}^*_{-1,1}) \Big] \tag{5.15}$$

where $P = 1$ for backscattering and $P = 0$ for other scattering directions, A is the illuminated area on the horizontal plane,

$$G_m(sk_{mz} + s'k_{mzi}) = \Phi_m(\bar{k}_{\perp i} - \bar{k}_\perp, sk_{mz} + s'k_{mzi})$$

$$[e^{2(sk''_{mz} + s'k''_{mzi})d_m} - e^{2(sk''_{mz} + s'k''_{mzi})d_{m-1}}],$$

$$\tag{5.16}$$

and $k''_{mz} = \mathrm{Im}(k_{mz})$. Thus, given the spectral densities of the correlation functions the first order scattered intensity of a stratified random medium is readily obtained from (5.15) and (5.16).

5.2 Backscattering Cross Sections for a Three Layer Random Medium

The backscattering cross-sections per unit area are given by

$$\sigma_{\mu\mu} = \cos\theta_{oi}\ \gamma_{\mu\mu}(\bar{k}_{oi}, -\bar{k}_{oi}) \tag{5.17}$$

where $\gamma_{\mu\mu}(\bar{k}_1, \bar{k}_2)$ is the bistatic scattering coefficient as given by Equation (3.23). The subscript μ represents h or v for horizontal or vertical polarization and we have noted in (5.17) that there is no first order depolarized component in the backscattering direction. Combining (5.15), (5.16) and (5.17) we find the backscattering cross sections for the stratified random medium:

$$\sigma_{\mu\mu} = \sum_{m=1}^{M} \delta_m k_m'^4 \left[R_{\mu\mu}^{(m)} (d_m - d_{m-1})\ \Phi_m(2\bar{k}_{\perp i}, 0) + S_{\mu\mu}^{(m)} \frac{\Phi_m(2\bar{k}_{\perp i}, 2k_{mzi})}{k''_{mzi}} \right] \tag{5.18}$$

where

$$R_{hh}^{(m)} = 2\pi^2 [|\beta_{mi}^{TE} A_{mi}|^2 + |\alpha_{mi}^{TE} B_{mi}|^2 + 2\mathrm{Re}(A_{mi} \beta_{mi}^{TE} B_{mi}^{*} \alpha_{mi}^{TE*})] \qquad (5.19a)$$

$$S_{hh}^{(m)} = \frac{\pi^2}{2} [|\alpha_{mi}^{TE} A_{mi}|^2 + |\beta_{mi}^{TE} B_{mi}|^2 e^{-4k''_{mzi}(d_m + d_m - 1)}]$$

$$(e^{4k''_{mzi}d_m} - e^{4k''_{mzi}d_m - 1}) \qquad (5.19b)$$

$$R_{vv}^{(m)} = 2\pi^2 [|\beta_{mi}^{TM} C_{mi}|^2 + |\alpha_{mi}^{TM} D_{mi}|^2 + 2\mathrm{Re}(C_{mi} \beta_{mi}^{TM} D_{mi}^{*} \alpha_{mi}^{*TM}]$$

$$\frac{|k_{\rho i}^2 - k_{mzi}^2|^2}{k_m^4} \qquad (5.20a)$$

$$S_{vv}^{(m)} = \frac{\pi^2}{2} [|\alpha_{mi}^{TM} C_{mi}|^2 + |\beta_{mi}^{TM} D_{mi}|^2 e^{-4k''_{mzi}(d_m + d_m - 1)}]$$

$$(e^{4k''_{mzi}d_m} - e^{4k''_{mzi}d_m - 1}). \qquad (5.20b)$$

The coefficients $\alpha_{mi}^{TE,TM}$, $\beta_{mi}^{TE,TM}$ as well as A_{mi}, B_{mi}, C_{mi} and D_{mi} are given by the propagation matrix formalism in Appendix A.

As a special application of (5.18) we consider two adjacent random layers bounded by air above and earth below. In this case $M = 2$ and the amplitudes appearing in Eqs. (5.19) and (5.20) are easily obtained through the method in Appendix A. The results are

$$\sigma_{hh} = \delta_1 k_1'^4 \pi^2 \frac{|X_{01i}|^4}{|D_{2i}|^4} \Bigg[8d_1 \; \Phi_1(2\bar{k}_{\perp i}, 0) |R_{12i} + R_{23i}$$

$$e^{i2k_{2zi}(d_2 - d_1)}|^2 |1 + R_{12i}R_{23i} \; e^{i2k_{2zi}(d_2 - d_1)}|^2$$

$$e^{-4k_{1zi}''d_1} + (|R_{12i} + R_{23i} \; e^{i2k_{2zi}(d_2 - d_1)}|^4 \; e^{-4k_{1zi}''d_1}$$

$$+ |1 + R_{12i}R_{23i} \; e^{i2k_{2zi}(d_2 - d_1)}|^4)(1 - e^{-4k_{1zi}''d_1})$$

$$\frac{\Phi_1(2\bar{k}_{\perp i}, 2k_{1zi})}{2k_{1zi}''}\Bigg] + \delta_2 k_2'^4 \pi^2 \frac{|X_{01i}|^4 |X_{12i}|^4}{|D_{2i}|^4} \Bigg[8(d_2 - d_1)$$

$$\Phi_2(2\bar{k}_{\perp i}, 0)|R_{23i}|^2 \; e^{-4k_{2zi}''(d_2 - d_1)} \; e^{-4k_{1zi}''d_1}$$

$$+ (|R_{23i}|^4 \; e^{-4k_{2zi}''(d_2 - d_1)} + 1)(1 - e^{-4k_{2zi}''(d_2 - d_1)})$$

$$e^{-4k''_{1zi}d_1} \frac{\Phi_2(2\bar{k}_{\perp i}, 2k_{2zi})}{2k''_{2zi}}\Bigg] \tag{5.21a}$$

$$\sigma_{vv} = \delta_1 k_1'^4 \pi^2 \frac{|Y_{01i}|^4}{|E_{2i}|^4} \frac{k_o^4}{|k_1|^4} \Bigg[8d_1\ \Phi_1(2\bar{k}_{\perp i}, 0)|S_{12i} + S_{23i}$$

$$e^{i2k_{2zi}(d_2 - d_1)}|^2|1 + S_{12i}S_{23i}\ e^{i2k_{2zi}(d_2 - d_1)}|^2$$

$$e^{-4k''_{1zi}d_1} \frac{|k^2_{\rho i} - k^2_{1zi}|^2}{|k_1|^4} + (|S_{12i} + S_{23i}$$

$$e^{i2k_{2zi}(d_2 - d_1)}|^4\ e^{-4k''_{1zi}d_1}$$

$$+ |1 + S_{12i}S_{23i}\ e^{i2k_{2zi}(d_2 - d_1)}|^4)(1 - e^{-4k''_{1zi}d_1})$$

$$\frac{\Phi_1(2\bar{k}_{\perp i}, 2k_{1zi})}{2k''_{1zi}}\Bigg] + \delta_2 k_2'^4 \pi^2 \frac{|Y_{01i}|^4|Y_{12i}|^4}{|E_{2i}|^4} \frac{k_o^4}{|k_2|^4}$$

$$\Bigg[8(d_2 - d_1)\ \Phi_2(2\bar{k}_{\perp i}, 0)|S_{23i}|^2\ e^{-4k''_{2zi}(d_2 - d_1)}$$

$$\frac{|k^2_{\rho i} - k^2_{1zi}|^2}{|k_2|^4}\ e^{-4k''_{1zi}d_1} + (|S_{23i}|^4\ e^{-4k''_{2zi}(d_2 - d_1)} + 1)$$

$$(1 - e^{-4k''_{2zi}(d_2 - d_1)}) \; e^{-4k''_{1zi}d_1} \left. \frac{\Phi_2(2\bar{k}_{\perp i},\; 2k_{2zi})}{2k''_{2zi}} \right] \tag{5.21b}$$

where

$$R_{ij} = \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}} \tag{5.22a}$$

$$S_{ij} = \frac{\langle\varepsilon_j\rangle k_{iz} - \langle\varepsilon_i\rangle k_{jz}}{\langle\varepsilon_j\rangle k_{iz} + \langle\varepsilon_i\rangle k_{jz}} \tag{5.22b}$$

$$X_{ij} = 1 + R_{ij} \tag{5.23a}$$

$$Y_{ij} = 1 + S_{ij} \tag{5.23b}$$

$$E_2 = 1 + S_{12}S_{23}\, e^{i2k_{2z}(d_2 - d_1)} + S_{01}[S_{12} + S_{23}\, e^{i2k_{2z}(d_2 - d_1)}]\, e^{i2k_{1z}d_1} \tag{5.24a}$$

$$D_2 = 1 + R_{12}R_{23}\, e^{i2k_{2z}(d_2 - d_1)} + R_{01}[R_{12} + R_{23}\, e^{i2k_{2z}(d_2 - d_1)}]\, e^{i2k_{1z}d_1}. \tag{5.24b}$$

To illustrate the results as given by (5.21a) and (5.21b) we shall consider correlation functions, which are Gaussian laterally and exponential vertically:

$$C_m(\bar{r}_i - \bar{r}_j) = \delta_m k_m'^4 \, e^{-|\bar{r}_{i\perp} - \bar{r}_{j\perp}|^2/\ell_{\rho m}^2 - |z_i - z_j|/\ell_{zm}} \tag{5.25}$$

where $m = 1, 2$ and the corresponding spectral densities are

$$\Phi_m(\bar{\beta}_\perp, \beta_z) = \frac{\Delta_m \ell_{zm} \ell_{\rho m}^2 \, e^{-\beta_\perp^2 \ell_{\rho m}^2/4}}{4\pi^2 (1 + \beta_z^2 \ell_{zm}^2)} \, . \tag{5.26}$$

The choice of correlation functions of the form (5.25) are advantageous in that we may vary the variances as well as the correlation lengths in the lateral and vertical directions independently for each random layer.

5.3 Application to Data Matching and Discussion

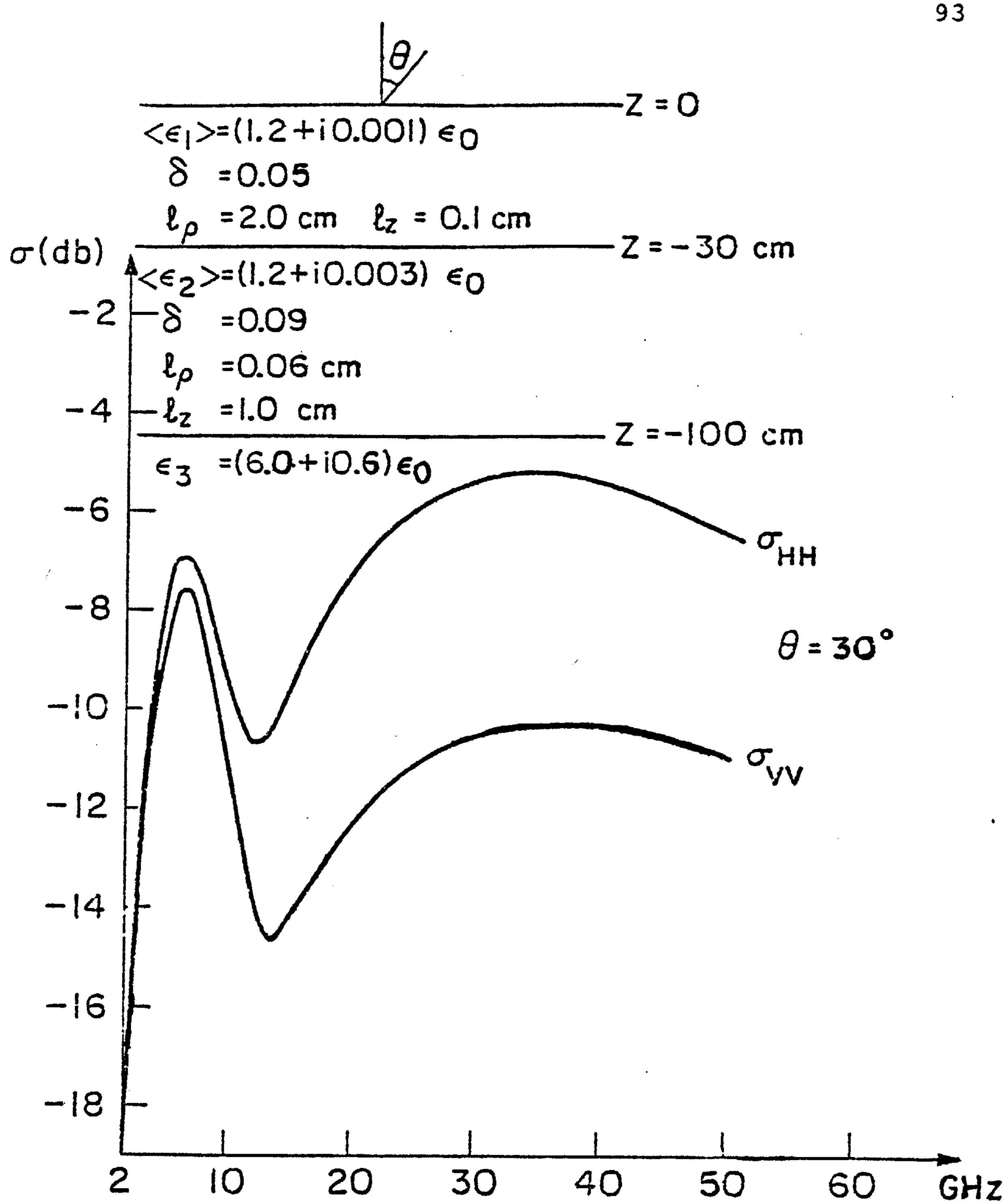
We have obtained, for a stratified random medium, the backscattering cross sections in (5.18) for arbitrary correlation functions. In (5.21a) and (5.21b) we have taken the special case of two adjacent random layers, bounded above and below by homogeneous media. Such a case is of considerable interest, in the active remote sensing of vegetation covers and snow-ice fields where the correlation lengths describing the upper scattering region may differ significantly from the correlation lengths characteristic of the lower scattering regions.

To illustrate the theory, we plot in Fig. 5.2 the TE and TM backscattering cross sections, σ_{hh} and σ_{vv} as given in (5.21a) and (5.21b) as a function of frequency. Note that the spectral variation of the backscattering cross sections exhibits two maxima due to resonant scattering within each random layer. This phenomenon of double resonance (or multiple resonance in the case of many random layers) may explain the spectral behavior observed in some backscattering data. In Fig. 5.3 we have matched TE backscattering cross section as a function of frequency at 30° and 60° for a 50 cm alfalfa field. The letters H (or h) represent experimental σ_{hh} while the continuous curves depict the theoretical results.

As seen from this figure the model of the two-layer random medium can account for the observed minimum in the frequency dependence of the backscattering cross section. In Figs. 5.4, 5.5 and 5.6 we match the observed angular dependence of the TE backscattering cross section at 9.0 GHz, 13.0 GHz and 16.6 GHz for the same alfalfa field. In order to account for rough surface effects at the ground-vegetation interface, we have incoherently superimposed the backscattering cross section of a very-rough surface with mean square slope s^2 and the backscattering cross sections of the random media. As seen from Fig. 5.4 the backscattering exhibits a rough surface effect, which dominates over volume scattering near nadir. In Figs. 5.5 and 5.6 the backscattering data does not manifest ground-vegetation rough surface effects due to the shielding effect of the random layers at 13.0 GHz and 16.6 GHz.

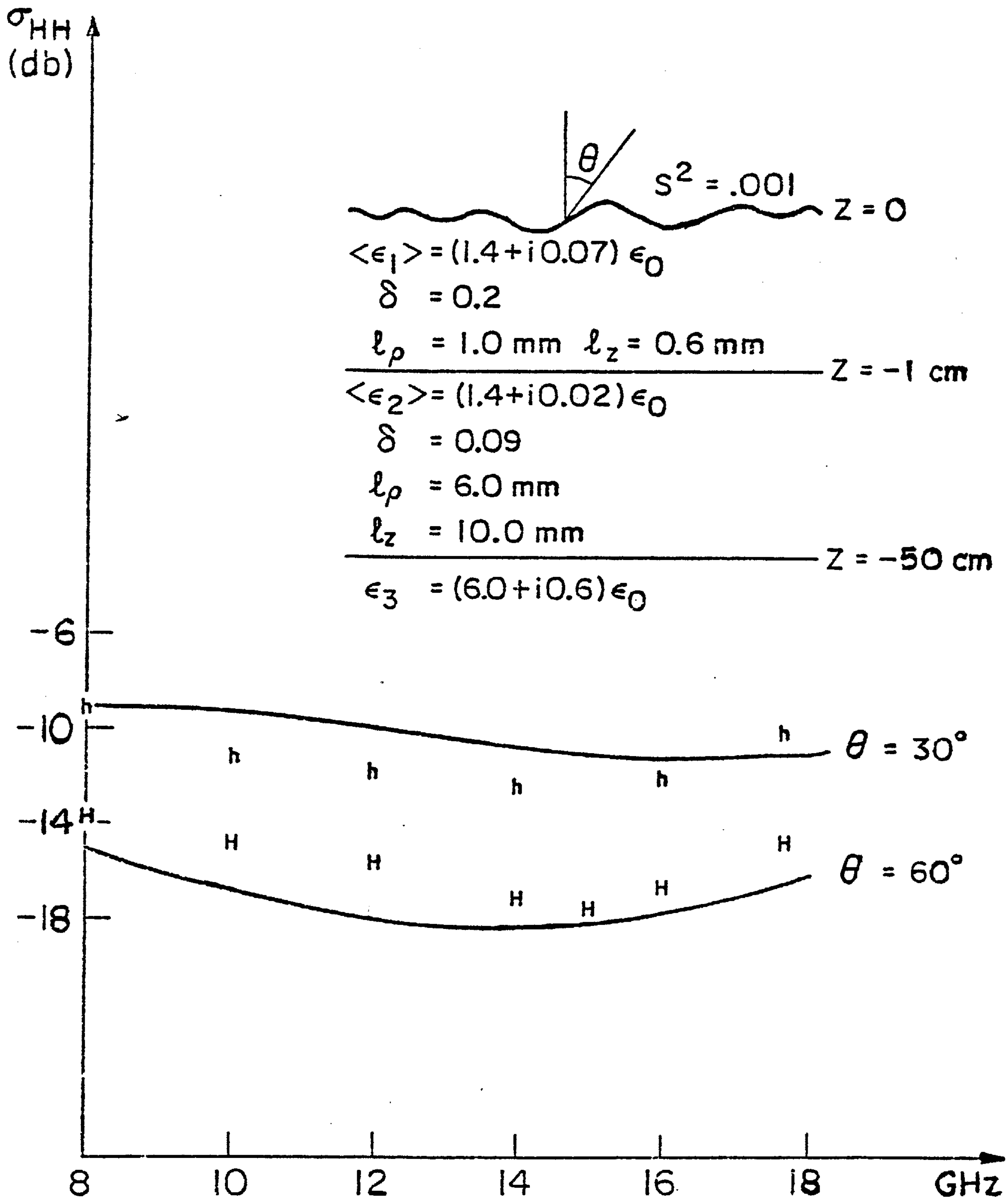
In Figs. 5.7 and 5.8 we match for both morning and afternoon TE backscattering data as a function of frequency at 30° and 50° for a 27 cm snow field. In order to account for the diurnal change in the collected data, we model the snow field as two-random layers: A top layer 4 cm thick and a bottom layer 23 cm thick. The oscillations in the theoretical curves are due to coherent effects of the top

4 cm layer, and are more evident in the morning than in the afternoon. In order to match the afternoon data, only the top layer parameters have been changed, the parameters of the other layers are maintained at the same values used to match the morning data. It was found that the most significant parameter to vary in matching both the morning and afternoon data is the imaginary part of the top layer mean permittivity. The increased value of $\mathrm{Im}\langle\varepsilon_1\rangle$ required to match the afternoon data, is consistent with the expected increase in the free water content of a surface layer due to the solar illumination. The increased free water content causes the surface layer to appear more specular thereby decreasing the amount of backscattering especially at the higher frequencies.



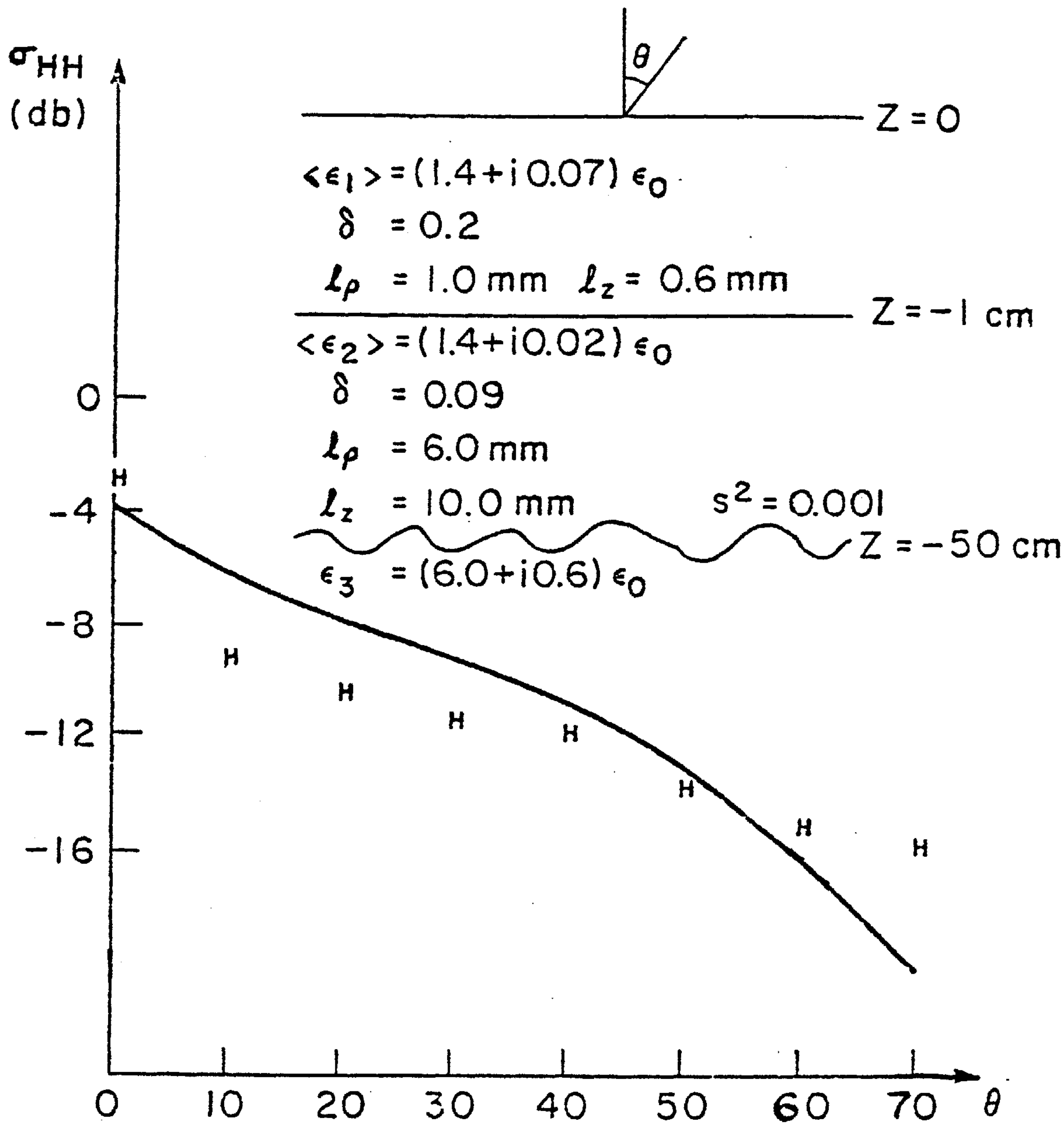
σ_{hh} and σ_{vv} as a function of frequency

Figure 5.2



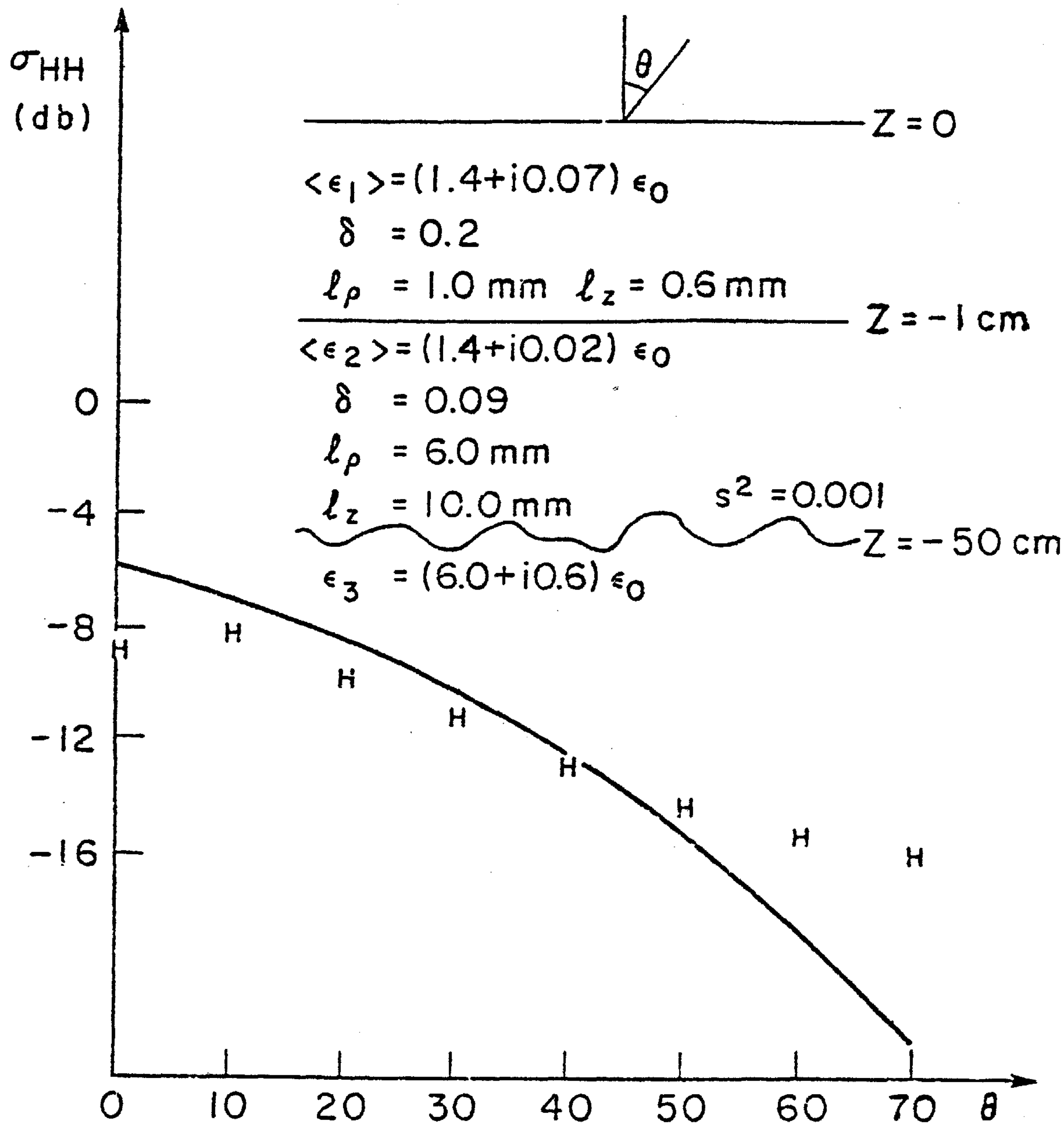
σ_{hh} as a function of frequency at 30° and 60°

<u>Figure 5.3</u>



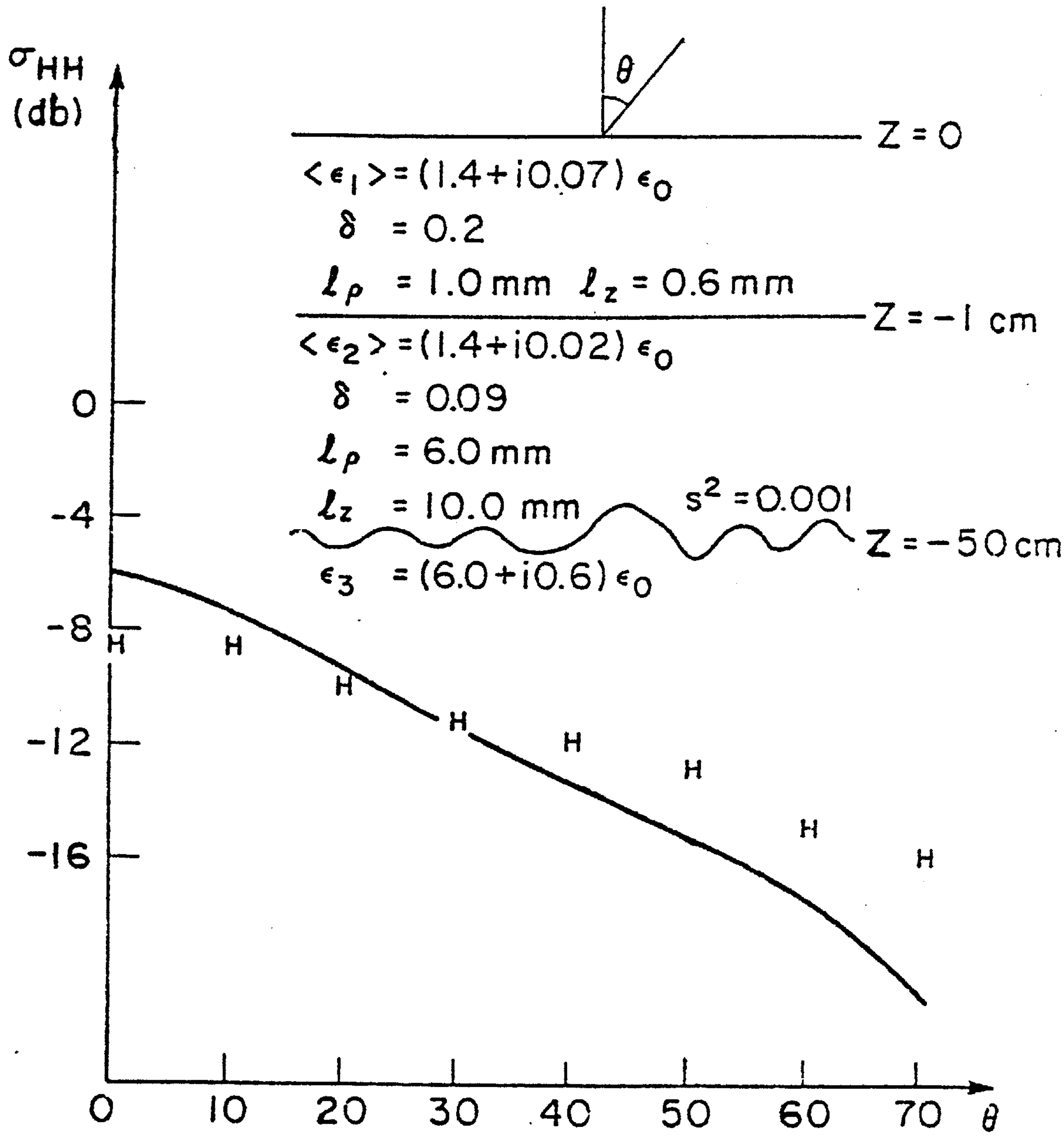
σ_{hh} as a function of angle at 9 GHz

Figure 5.4



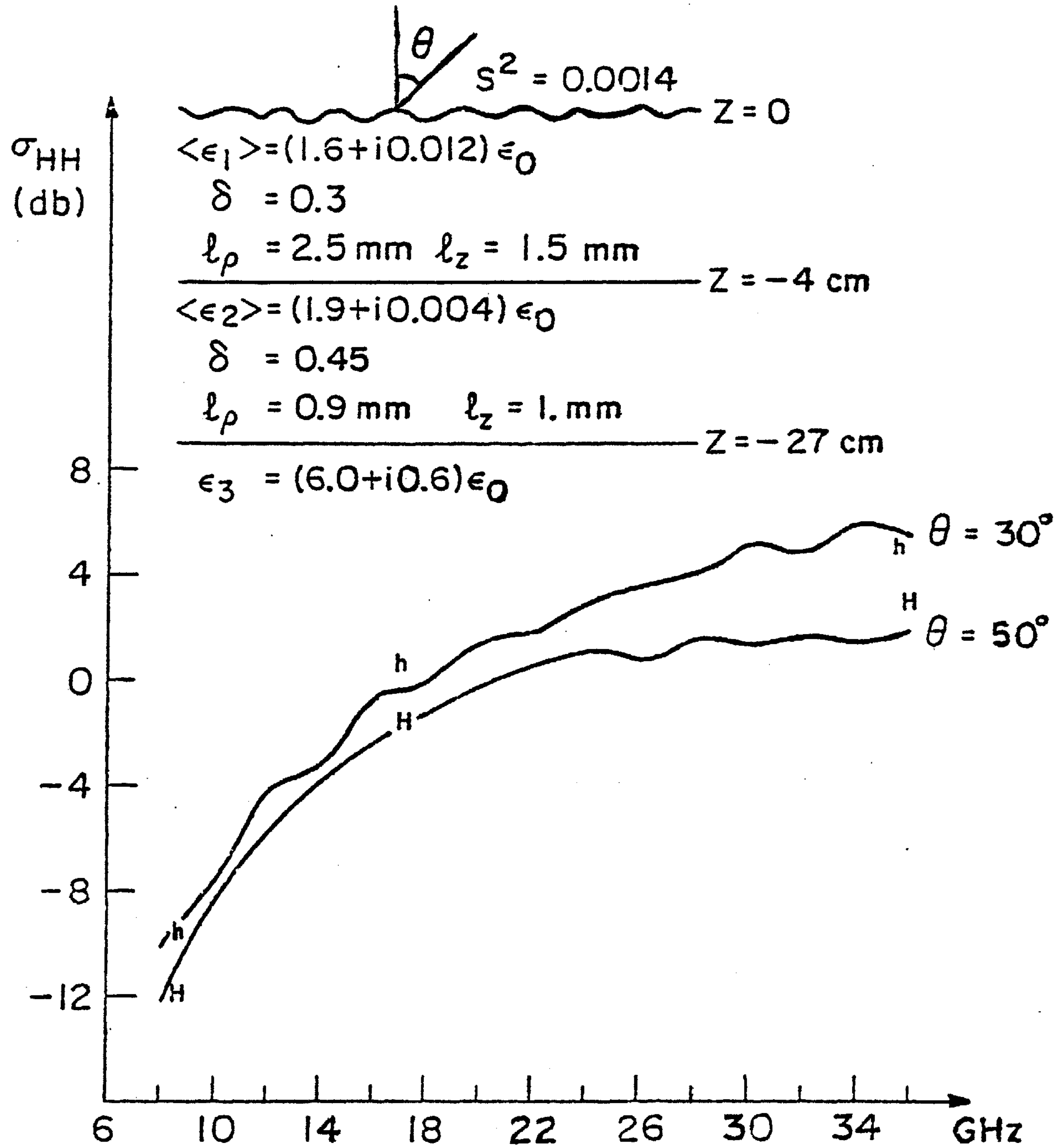
σ_{hh} as a function of angle at 13 GHz

Figure 5.5



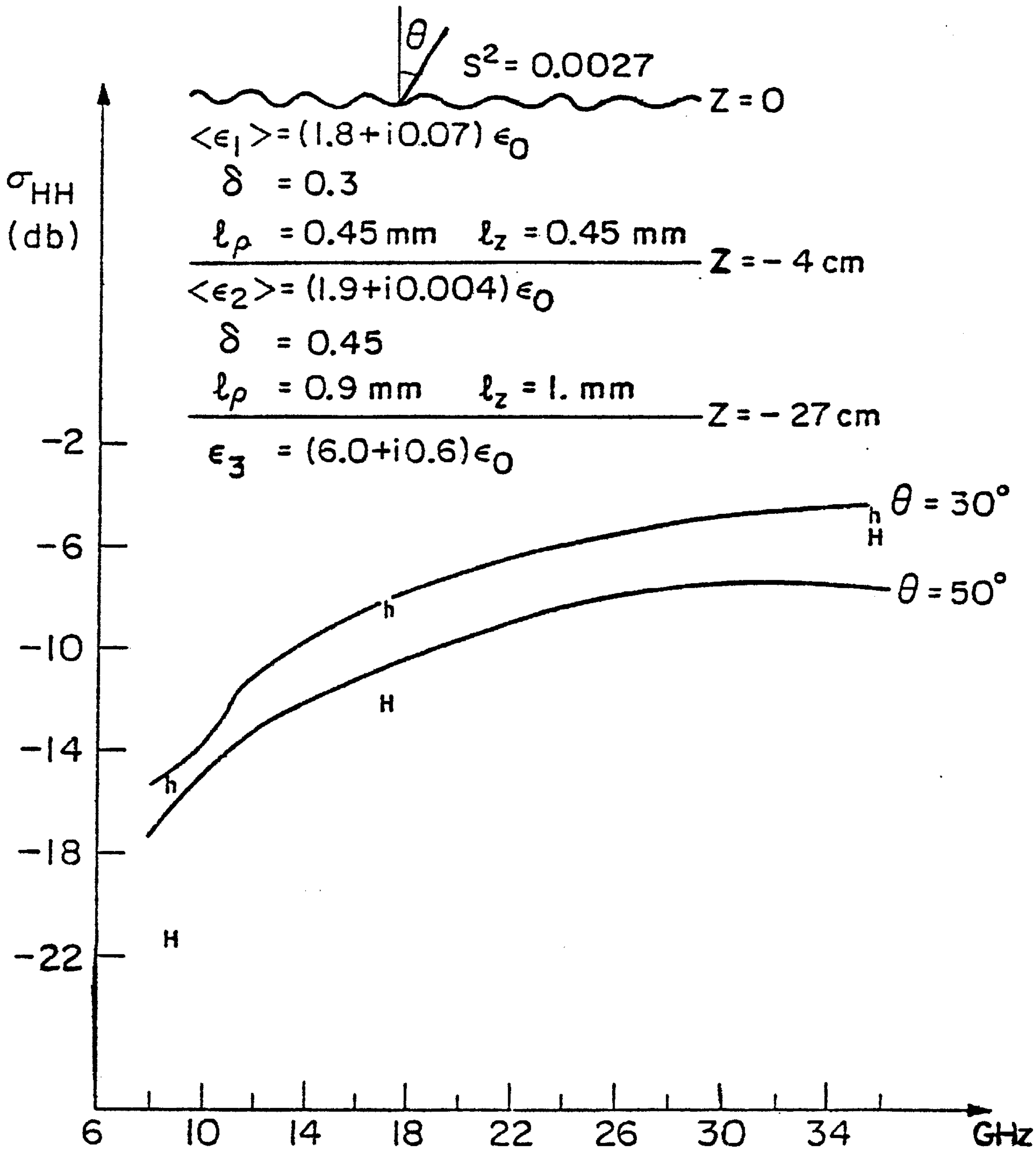
σ_{hh} as a function of angle at 16.6 GHz

Figure 5.6



σ_{hh} as a function of frequency at 30° and 50°

Figure 5.7



σ_{hh} as a function of frequency at 30° and 50°

Figure 5.8

5.4 Appendices

Appendix A

The amplitudes A_m, B_m, α_m^{TE} and β_m^{TE} in each layer are determined through the TE propagation matrix formalism:

$$\begin{bmatrix} \xi_m \, e^{-ik_{mz}d_m} \\ \eta_m \, e^{ik_{mz}d_m} \end{bmatrix} = \bar{\bar{V}}^{TE}_{m,m-1} \begin{bmatrix} \xi_{m-1} \, e^{-ik_{(m-1)z}d_{m-1}} \\ \eta_{m-1} \, e^{ik_{(m-1)z}d_{m-1}} \end{bmatrix}$$

where

$$\bar{\bar{V}}^{TE}_{m,m-1} = \frac{\Gamma}{X_{m,m-1}}$$

$$\begin{bmatrix} e^{-ik_{mz}(d_m - d_{m-1})} & R_{m,m-1} \, e^{-ik_{mz}(d_m - d_{m-1})} \\ R_{m,m-1} \, e^{ik_{mz}(d_m - d_{m-1})} & e^{ik_{mz}(d_m - d_{m-1})} \end{bmatrix}$$

and $X_{m,m-1}$ is given by (5.23a). ξ_m represents A_m or

α_m^{TE}, η_m represents B_m or β_m^{TE}, $\xi_o = R^{TE}$, $\eta_o = 1$ and $\Gamma = 1$. The TE reflection coefficient has the continued fraction representation:

$$R^{TE} = \frac{1}{R_{01}} + \frac{\left[1 - \frac{1}{R_{01}^2}\right] e^{-i2k_{1z}d_1}}{(1/R_{01})\, e^{-i2k_{1z}d_1}} \Bigg/ + \frac{\xi_1}{\eta_1} e^{-i2k_{1z}d_1}.$$

The TM amplitudes C_m, D_m, α_m^{TM} and β_m^{TM}, in each layer, are obtained from the above results by letting $\Gamma = k_{m-1}/k_m$ and by making the replacement $R_{m,m-1} \to S_{m,m-1}$. In this case ξ_m represents C_m or α_m^{TM} and η_m represents either D_m or β_m^{TM}.

CHAPTER 6

Depolarization Effects in the Active Remote Sensing of a Two-Layer Random Medium

For the problem of wave scattering by a half-space random medium, backscattering cross-sections have been calculated with a radiative transfer theory.[63] The radiative transfer theory was derived by a wave approach making use of the nonlinear approximation to the scalar Dyson equation and the ladder approximation to the Bethe-Salpeter equation.[59] Using the technique of Born approximations and evaluating the dyadic Green's function with the saddle point method, in Chapter 3, we calculated to first order in albedo the backscattering cross-sections for a two-layer random medium.

In this paper we carry the Born approximation to second order to obtain backscattering cross sections that account for depolarization effects. Instead of using the saddle point evaluated dyadic Green's functions, we apply Fourier transform methods in the calculation of the bistatic scattering coefficients. The results are reduced to the half-space case and discussed in the limit of radiative transfer theory. The backscattering cross-sections are applied to matching experimental data collected from vegetation and ice-snow fields.

6.1 Depolarization Backscattering Cross Section in the Second Order Born Approximation

The second order scattering intensity takes the form (Chapter 3):

$$<|\bar{E}_o|^2>^{(2)} = <|\bar{E}_o^{(2)}(\bar{r})|^2> + 2\mathrm{Re}\{<\bar{E}_o^{(1)}(\bar{r}) \cdot \bar{E}_o^{(3)*}(\bar{r})>$$

$$+ \bar{E}_o^{(0)}(\bar{r}) \cdot <\bar{E}_o^{(4)*}(\bar{r})>\}. \qquad (6.1)$$

It can be shown that the terms in the curly bracket are negligible [Appendix A]. We can concentrate on the term giving rise to depolarization effects

$$<|\bar{E}_o|^2>_{\mu\nu}^{(2)} = <|\bar{E}_o^{(2)}(\bar{r})|^2>_{\mu\nu} = \frac{1}{\omega^4\mu_o^4}\int_{V_1} d^3r_1 d^3r_2 d^3r_3 d^3r_4$$

$$[\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) \cdot \bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2) \cdot E_1^{(0)}(\bar{r}_2)]_{\mu\nu}$$

$$[\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_3) \cdot \bar{\bar{G}}_{11}(\bar{r}_3, \bar{r}_4) \cdot \bar{E}_1^{(0)}(\bar{r}_4)]^*$$

$$<Q(\bar{r}_1)\, Q(\bar{r}_2)\, Q^*(\bar{r}_3)\, Q^*(\bar{r}_4)> . \qquad (6.2)$$

Here we use the first subscript μ to denote the polarization of the incident wave and the second subscript ν to denote the polarization of the scattered wave. We shall consider the case of $\mu \neq \nu$.

The fourth order moment of $Q(\bar{r})$ in (6.2) may be expanded in clusters:

$$<Q(\bar{r}_1)\ Q(\bar{r}_2)\ Q^*(\bar{r}_3)\ Q^*(\bar{r}_4)> = C(\bar{r}_1 - \bar{r}_2)\ C(\bar{r}_3 - \bar{r}_4)$$

$$+ C(\bar{r}_1 - \bar{r}_3)\ C(\bar{r}_2 - \bar{r}_4) + C(\bar{r}_1 - \bar{r}_4)\ C(\bar{r}_2 - \bar{r}_3) \qquad (6.3)$$

where $C(\bar{r}_i - \bar{r}_j)$ is the two point correlation function for the random medium. Note that the first term of (6.3) when substituted into (6.2) gives the square of the second order mean field which can be neglected in our calculation of depolarization effects. We thus have

$$<|E_o^{(2)}|^2>_{\mu\nu}^{(2)} = \frac{1}{\omega^4 \mu_o^4} \int_{V_1} d^3r_1 d^3r_2 d^3r_3 d^3r_4\ [\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$$

$$\cdot \bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2) \cdot \bar{E}_1^{(0)}(\bar{r}_2)]_{\mu\nu} \cdot [\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_3) \cdot \bar{\bar{G}}_{11}(\bar{r}_3, \bar{r}_4)$$

$$\cdot \bar{E}_1^{(0)}(\bar{r}_4)]^* \quad [C(\bar{r}_1 - \bar{r}_3)\ C(\bar{r}_2 - \bar{r}_4) + C(\bar{r}_1 - \bar{r}_4)$$

$$C(\bar{r}_2 - \bar{r}_3)]. \tag{6.4}$$

The unperturbed incident field may be cast in the following form

$$\bar{E}_1^{(0)}(\bar{r}) = \bar{E}_{1i}^{(0)}(z)\ e^{i\bar{k}_{\perp i} \cdot \bar{r}_\perp} \tag{6.5}$$

where

$$\bar{E}_{1i}^{(0)}(z) = E_o[\bar{f} \cdot \bar{\bar{A}}_i\ e^{ik_{1zi}z} + \bar{f} \cdot \bar{\bar{B}}_i\ e^{-ik_{1zi}z}] \tag{6.6}$$

and $\bar{f}$, $\bar{\bar{A}}_i$ and $\bar{\bar{B}}_i$ have been defined in Chapter 3. Introducing the expressions for the dyadic Green's functions and the correlation functions of Chapters 2 and 3, we first perform the integration over transverse spatial variables which yield delta functions useful in the evaluation of transverse wave vector variables. We obtain

$$<|\bar{E}_o^{(2)}|^2>_{\mu\nu}^{(2)} = \int d^2k_\perp \left\{ \frac{(2\pi)^8(\delta k_1'^4)^2}{\omega^4\mu_o^4} \int d^2\beta_\perp \int_{-\infty}^{\infty} d\beta_z d\alpha_z \right.$$

$$\Phi(\bar{k}_\perp' - \bar{k}_\perp, \beta_z)\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp', \alpha_z) \int_{-d_1}^{0} dz_1 dz_2 dz_3 dz_4$$

$$\cdot\ [\bar{\bar{g}}_{01}^{>}(\bar{k}_\perp, z, z_1) \cdot \bar{\bar{g}}_{11}^{\gtrless}(\bar{k}_\perp', z, z_2) \cdot \bar{E}_{1i}^{(0)}(z_2)]_{\mu\nu}$$

$$[e^{i\beta_z(z_1 - z_3)}\ e^{i\alpha_z(z_2 - z_4)} \cdot \bar{\bar{g}}_{01}(\bar{k}_\perp, z, z_3)$$

$$\cdot\ \bar{\bar{g}}_{11}^{\gtrless}(\bar{k}_\perp', z_3, z_4) \cdot \bar{E}_{1i}^{(0)}(z_4)$$

$$+\ e^{i\beta_z(z_1 - z_4)}\ e^{i\alpha_z(z_2 - z_3)}\ \bar{\bar{g}}_{01}^{>}(\bar{k}_\perp, z, z_3)$$

$$\left.\cdot\ \bar{\bar{g}}_{11}^{\gtrless}(\bar{k}_{\perp i} - \bar{k}_\perp' + \bar{k}_\perp, z_3, z_4) \cdot \bar{E}_{1i}^{(0)}(z_4)]^*_{\mu\nu}\right\} . \qquad (6.7)$$

The integrand of (6.7) is related to the bistatic scattering coefficients (Fung and Fung[61]). In the baskcattered direction, we set $\bar{k} = -\bar{k}_{oi}$ and obtain

$$\gamma_{\mu\nu}^{(2)}(\bar{k}_{oi}, -\bar{k}_{oi}) = \frac{4\pi(2\pi k_1')^8 \delta^2 k_o^2 \cos\theta_{oi}}{\omega^4 \mu_o^4\ |E_o|^2} \int d^2k_\perp \int_{-\infty}^{\infty} d\beta_z d\alpha_z$$

$$\Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_z)\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \alpha_z)$$

$$\cdot \{\bar{\bar{I}}_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z)\cdot[\bar{\bar{I}}^*_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z)$$

$$+ \bar{\bar{I}}^*_{\mu\nu}(-\bar{k}_\perp, -\bar{k}_{\perp i}, \beta_z, \alpha_z)]\} \tag{6.8}$$

where

$$\bar{\bar{I}}_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z) = \int_{-d_1}^{0} dz_1 \left\{ \int_{-d_1}^{z_1} dz_2 [\bar{\bar{g}}_{01}(-\bar{k}_{\perp i}, z, z_1) \right.$$

$$\cdot \bar{\bar{g}}^{>}_{11}(\bar{k}_\perp, z_1, z_2) \cdot \bar{\bar{E}}^{(0)}_{1i}(z_2)]_{\mu\nu}\, e^{-i\alpha_z z_2 - i\beta_z z_1}$$

$$+ \int_{z_1}^{0} dz_2 [\bar{\bar{g}}^{>}_{01}(-\bar{k}_{\perp i}, z, z_1) \cdot \bar{\bar{g}}^{<}_{11}(\bar{k}_\perp, z_1, z_2) \cdot \bar{\bar{E}}^{(0)}_{1i}(z_2)]$$

$$\left. e^{-i\alpha_z z_2 - i\beta_z z_1} \right\} \tag{6.9}$$

Carefully carrying out the integrations over α_z, β_z and the spatial coordinates, we obtain, after considerable manipulations [Appendix B],

$$\gamma^{(2)}_{hv}(\bar{k}_{oi}, -\bar{k}_{oi}) = \gamma^{(2)}_{vh}(\bar{k}_{oi}, -\bar{k}_{oi}) = \frac{8\pi^2\varepsilon_o \cos\theta_{oi}}{\kappa_a{}^2\varepsilon_1 \cos^2\theta_i} \frac{t_h(\theta_i)\; t_v(\theta_i)}{|F_{2i}|^2 |D_{2i}|^2}$$

$$\left\{\int_0^{\pi/2} d\theta \frac{\sin\theta \sec\theta}{(\sec\theta_i + \sec\theta)^2} W_1(\theta, \theta_i) W_3^2(\theta, \theta_i) M_1(\theta, \theta_i)\right.$$

$$+ \int_0^{\pi/2} d\theta \frac{\sin\theta \sec\theta}{(\sec^2\theta_i - \sec^2\theta)} W_1(\theta, \theta_i) W_2(\theta, \theta_i) W_3(\theta, \theta_i)$$

$$M_2(\theta, \theta_i) + \int_0^{\pi/2} d\theta \frac{\sin\theta \sec\theta}{(\sec\theta_i - \sec\theta)^2} W_1(\theta, \theta_i)$$

$$\left. W_2^2(\theta, \theta_i) M_3(\theta, \theta_i)\right\} \tag{6.10}$$

where

$$W_1(\theta, \theta_i) = \left(\frac{\pi\delta k_1'^4}{2}\right)^2 e^{-\frac{k_1^2\ell_\rho^2}{2}(\sin^2\theta + \sin^2\theta_i)} \tag{6.11a}$$

$$W_2(\theta, \theta_i) = \frac{\ell\ell_\rho^2}{4\pi^2[1 + k_1'^2\ell^2(\cos\theta - \cos\theta_i)^2]} \tag{6.11b}$$

$$W_3(\theta, \theta_i) = \frac{\ell\ell_\rho^2}{4\pi^2[1 + k_1'^2\ell^2(\cos\theta + \cos\theta_i)^2]} \tag{6.11c}$$

$$M_1(\theta, \theta_i) = \frac{1}{4}\cos^2\theta_i\left[|R_{12i}|^2|S_{12i}|^2 e^{-4k_{1zi}''d_1}\left|\frac{R_{12}}{D_2} + \frac{S_{12}\cos^2\theta}{F_2}\right|^2\right.$$

$$+ \left|\frac{R_{10}}{D_2} + \frac{S_{10}\cos^2\theta}{F_2}\right|^2\Bigg] (1 - e^{-2(k''_{1zi} + k''_{1z})d_1})^2$$

$$- \frac{1}{2}\left[\frac{1}{4}\cos^2\theta_i \; |R_{12i} - S_{12i}|^2 \left|\frac{R_{10}R_{12}}{D_2} - \frac{S_{10}S_{12}\cos^2\theta}{F_2}\right|^2\right.$$

$$\left. + |R_{12i} + S_{12i}|^2 \frac{|S_{10}S_{12}|^2}{|F_2|^2}\cos^2\theta \sin^2\theta \sin^2\theta_i\right]$$

$$[2d_1(k''_{1zi} + k''_{1z})(e^{-4k''_{1z}d_1} - 1)\; e^{-4k''_{1zi}d_1}$$

$$+ (1 - e^{-2(k''_{1zi} + k''_{1z})d_1})(e^{-2k''_{1zi}d_1} - e^{-2k''_{1z}d_1})\; e^{-2k''_{1zi}d_1}]$$

(6.12a)

$$M_2(\theta, \theta_i) = \frac{(|S_{12i}|^2|R_{12i}|^2\, e^{-4k''_{1zi}d_1} + 1)}{k''_{1zi}} \left[\left[\frac{1}{4}\cos^2\theta_i \left|\frac{1}{D_2}\right.\right.\right.$$

$$\left.\left. - \frac{\cos^2\theta}{F_2}\right|^2 + \frac{\cos^2\theta \sin^2\theta \sin^2\theta_i}{|F_2|^2}\right] [(k''_{1zi} - k''_{1z})$$

$$+ (k''_{1zi} + k''_{1z})\, e^{-4k''_{1zi}d_1} - 2k''_{1zi}\, e^{-2(k''_{1zi} + k''_{1z})d_1}]$$

$$+ \left(\frac{1}{4} \cos^2\theta_i \left|\frac{R_{10}R_{12}}{D_2} - \frac{S_{10}S_{12} \cos^2\theta}{F_2}\right|^2 + \frac{|S_{10}S_{12}|^2}{|F_2|^2}\right.$$

$$\left.\cos^2\theta \sin^2\theta \sin^2\theta_i\right) [(k''_{1zi} - k''_{1z}) e^{-4(k''_{1zi} + k''_{1z})d_1}$$

$$\left. + (k''_{1zi} + k''_{1z}) e^{-4k''_{1z}d_1} - 2k''_{1zi} e^{-2(k''_{1zi} + k''_{1z})d_1}]\right]$$

$$- \frac{1}{2} |R_{12i} - S_{12i}|^2 \left[\frac{1}{4} \cos^2\theta_i \left(\left|\frac{R_{12}}{D_2} + \frac{S_{12} \cos^2\theta}{F_2}\right|^2\right.\right.$$

$$\left. + \left|\frac{R_{10}}{D_2} + \frac{S_{10} \cos^2\theta}{F_2}\right|^2\right)$$

$$\left. + \frac{|S_{10}|^2 + |S_{12}|^2}{|F_2|^2} \cos^2\theta \sin^2\theta \sin^2\theta_i\right] [1 - e^{-2(k''_{1zi} + k''_{1z})d_1}]$$

$$[e^{-2k''_{1zi}d_1} - e^{-2k''_{1z}d_1}] e^{-2k''_{1zi}d_1} \tag{6.12b}$$

$$M_3(\theta, \theta_i) = \frac{1}{4} \cos^2\theta_i \left(|S_{12i}|^2 |R_{12i}|^2 e^{-4k''_{1zi}d_1} \left|\frac{R_{10}}{D_2} + \frac{S_{10} \cos^2\theta}{F_2}\right|^2\right.$$

$$+ \left| \frac{R_{12}}{D_2} + \frac{S_{12} \cos^2\theta}{F_2} \right|^2 \Bigg) (e^{-2k''_{1zi} d_1} - e^{-2k''_{1z} d_1})$$

$$- \frac{1}{2} \left(\frac{1}{4} \cos^2\theta_i \; |R_{12i} - S_{12i}|^2 \left| \frac{1}{D_2} - \frac{\cos^2\theta}{F_2} \right|^2 \right.$$

$$\left. + |R_{12i} + S_{12i}|^2 \frac{\cos^2\theta \sin^2\theta \sin^2\theta_i}{F_2} \right) e^{-2k''_{1zi} d_1}$$

$$[2(k''_{1zi} - k''_{1z}) d_1 \; e^{-2k''_{1zi} d_1} + e^{-2k''_{1zi} d_1} - e^{-2k''_{1z} d_1}]$$

$$+ \frac{1}{2} \left(\frac{1}{4} \cos^2\theta_i \; |R_{12i} - S_{12i}|^2 \left| \frac{R_{10} R_{12}}{D_2} - \frac{S_{10} S_{12} \cos^2\theta}{F_2} \right|^2 \right.$$

$$\left. + |R_{12i} + S_{12i}|^2 \frac{\cos^2\theta \sin^2\theta \sin^2\theta_i}{|F_2|^2} |S_{10} S_{12}|^2 \right)$$

$$e^{-4k''_{1zi} d_1} \; e^{-2k''_{1z} d_1} \; [2(k''_{1zi} - k''_{1z}) d_1 \; e^{-2k''_{1z} d_1}$$

$$- e^{-2k''_{1z} d_1} + e^{-2k''_{1zi} d_1}] \qquad (6.12c)$$

$$t_h(\theta_i)\ t_v(\theta_i) = \frac{k_o^{\ 2}}{k_1^{\ 2}} |X_{10i}|^2 |Y_{01i}|^2 = \frac{k_1^{\ 2}}{k_o^{\ 2}} |X_{01i}|^2 |Y_{10i}|^2 \quad (6.13)$$

where $\kappa_a = 2k_1''$. The backscattering cross-section per unit area, $\sigma_{\mu\nu}^{(2)} = \gamma_{\mu\nu}^{(2)} \cos\theta_{oi}$ readily follows from (6.10).

6.2 Half-Space Limit and Comparison with Backscattering Cross Sections of Radiative Transfer Theory

The backscattering coefficient (6.10) reduces to that for a half-space when we let $d_1 \to \infty$. We find

$$\gamma_{hv}^{(2)}(\bar{k}_{oi}, -\bar{k}_{oi}) = \gamma_{vh}^{(2)}(\bar{k}_{oi}, -\bar{k}_{oi}) = \gamma_{+}^{(2)} + \gamma_{-}^{(2)} \tag{6.14}$$

where

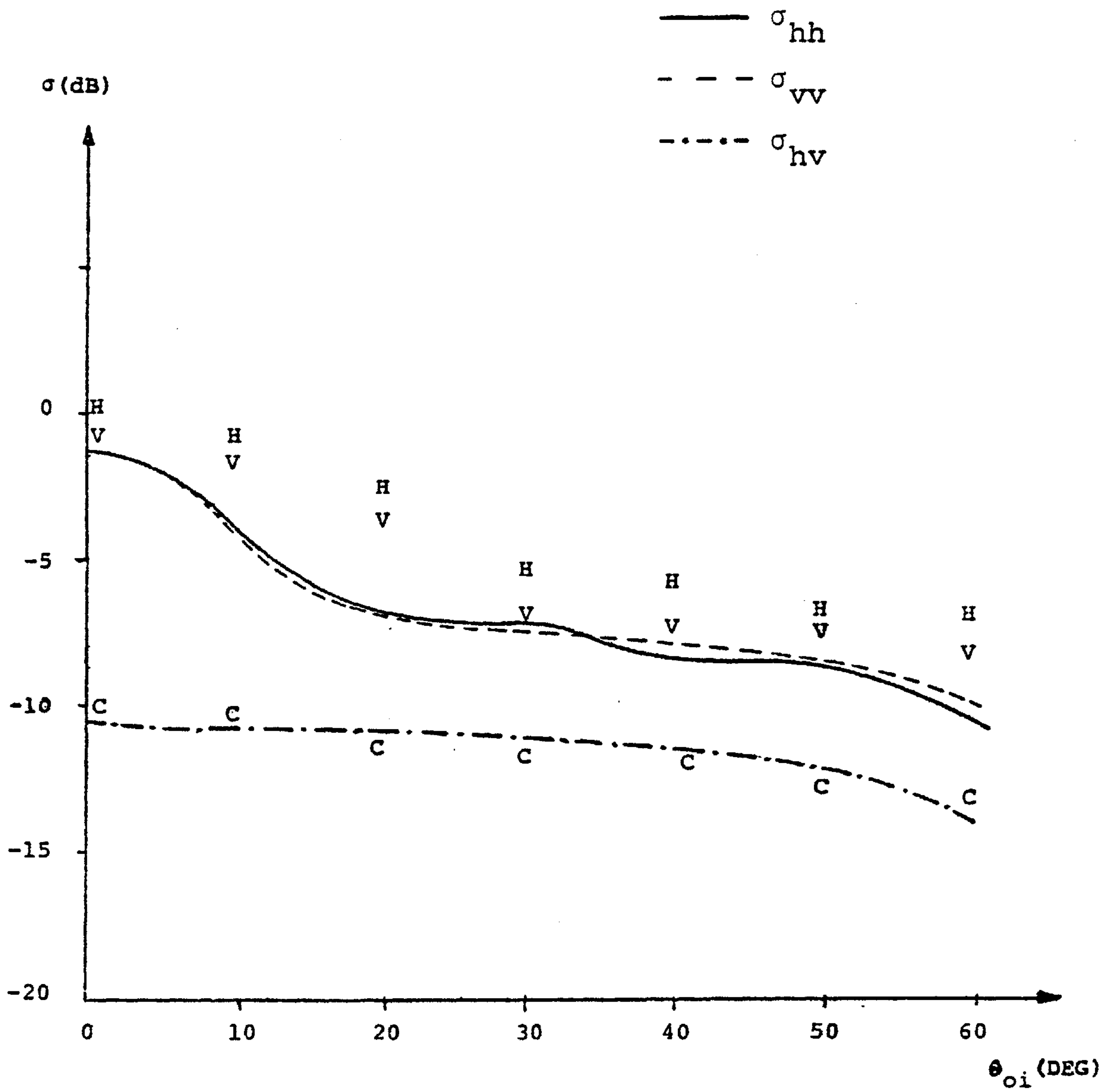
$$\gamma_{\pm}^{(2)} = \frac{4\pi^2\varepsilon_o \cos\theta_{oi}}{\kappa_a{}^2\varepsilon_1 \cos\theta_i} t_v(\theta_i)\, t_h(\theta_i) \int_0^{\pi/2} d\theta \frac{\sin\theta \sec\theta}{\sec\theta_i + \sec\theta}$$

$$W_1(\theta, \theta_i)\, W_3(\theta, \theta_i)\{W_2(\theta, \theta_i) \sin^2\theta \left(\frac{1}{4}\cos^2\theta_i \sin^2\theta\right.$$

$$\left. + \cos^2\theta \sin^2\theta_i\right) + W_3(\theta, \theta_i) \frac{\sec\theta_i}{4(\sec\theta_i + \sec\theta)} [|R_{10}$$

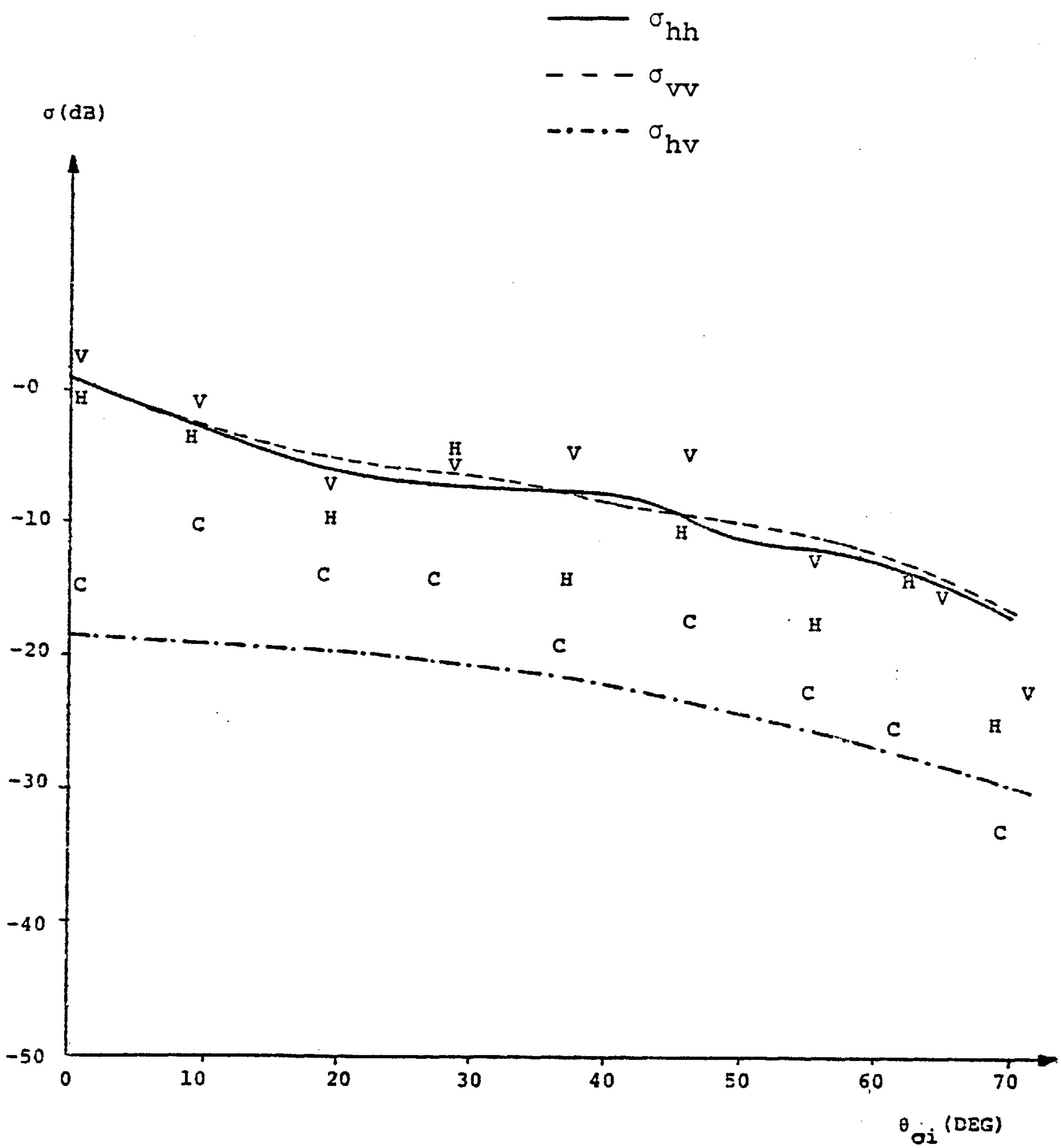
$$+ S_{10}\cos^2\theta|^2\cos^2\theta_i \pm |S_{10}|^2 \cos^2\theta \sin^2\theta \sin^2\theta_i]\}. \tag{6.15}$$

The first term $\gamma_{+}^{(2)}$ is also obtained from the radiative transfer theory derived with the nonlinear approximation to the Dyson equation and with the ladder approximation to the Bethe-Salpeter equation.[59] The second term $\gamma_{-}^{(2)}$ is an additional contribution not accounted for by the ladder approximation nor by the radiative transfer theory. The physical significance of the additional contribution especially in the context of renormalization methods will be discussed in Chapter 10.



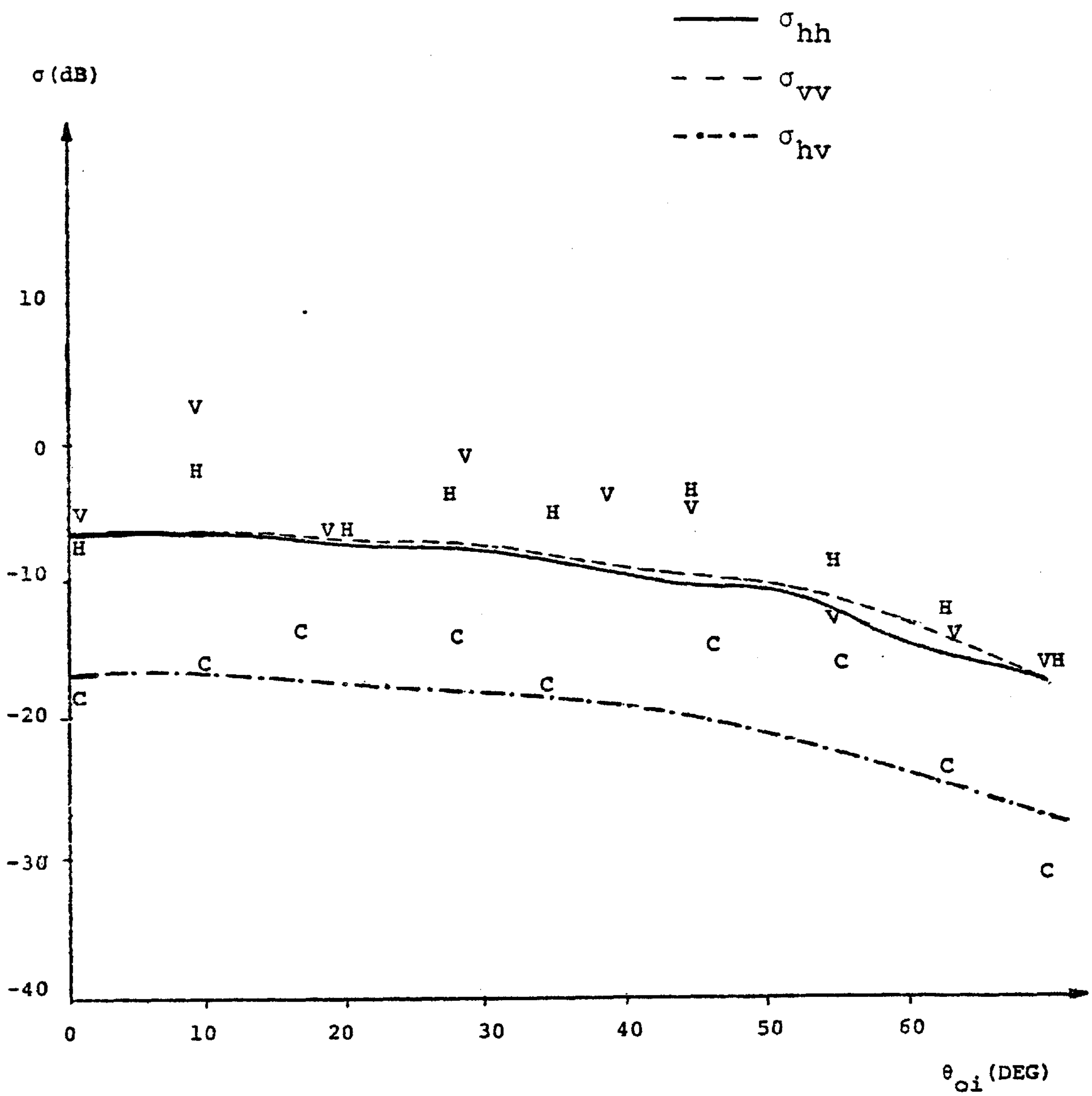
σ_{hh}, σ_{vv} and σ_{hv} as a function of angle at 5.9 GHz

Figure 6.1



σ_{hh}, σ_{vv} and σ_{hv} as a function of angle at 35 GHz

Figure 6.2



σ_{hh}, σ_{vv} and σ_{hv} as a function of angle at 35 GHz

Figure 6.3

6.4 Appendices

Appendix A

The Cross Terms of Equation (6.1)

According to Equation (6.1) the cross terms are

$$2\mathrm{Re}\{<\bar{E}_o^{(1)}(\bar{r}) \cdot \bar{E}_o^{(3)*}(\bar{r})> + \bar{E}_o^{(0)}(\bar{r}) \cdot <\bar{E}_o^{(4)*}(\bar{r})>\}. \qquad (A.1)$$

The first cross term, $<\bar{E}_o^{(1)}(\bar{r}) \cdot \bar{E}_o^{(3)*}(\bar{r})>$ does not contribute to the depolarized backscattering according to the following argument. It is well known that the first order field $\bar{E}_o^{(1)}$ has no depolarized component in the backscattering direction. Therefore, even though $\bar{E}_o^{(3)}(\bar{r})$ may have depolarized components the product $<\bar{E}_o^{(1)}(\bar{r}) \cdot \bar{E}_o^{(3)*}(\bar{r})>$ is easily seen to produce no depolarization effect in the backscattering direction.

The fourth order scattered mean field is given by equation (3.11):

$$<\bar{E}_o^{(4)}(\bar{r})> = \frac{1}{\omega^4\mu_o^4}\int d^3r_1 d^3r_2 d^3r_3 d^3r_4\ \bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) \cdot \bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2)$$

$$\cdot \bar{\bar{G}}_{11}(\bar{r}_2, \bar{r}_3) \cdot \bar{\bar{G}}_{11}(\bar{r}_3, \bar{r}_4) \cdot \bar{E}_1^{(0)}(\bar{r}_4)$$

$$\langle Q(\bar{r}_1)\ Q(\bar{r}_2)\ Q(\bar{r}_3)\ Q(\bar{r}_4)\rangle. \tag{A.2}$$

Substituting Equation (6.3) as well as Equations (2.5), (2.26) and (3.14) of the preceding chapters into (A.2), we obtain

$$\langle \bar{E}_o^{(4)}(\bar{r})\rangle = \frac{1}{\omega^4 \mu_o{}^4} \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 \int d^2k_\perp d^2k_\perp' d^2k_\perp'' d^2k_\perp'''$$

$$\cdot \bar{\bar{g}}_{01}^{>}(\bar{k}_\perp, z, z_1) \cdot \bar{\bar{g}}_{11}(\bar{k}_\perp', z_1, z_2) \cdot \bar{\bar{g}}_{11}(\bar{k}_\perp'', z_2, z_3)$$

$$\cdot \bar{\bar{g}}_{11}(\bar{k}_\perp''', z_3, z_4) \int d^3\alpha d^3\beta\ \Phi(\bar{\beta})\ \Phi(\bar{\alpha})$$

$$[e^{-i\bar{\beta} \cdot (\bar{r}_1 - \bar{r}_2)} e^{-i\bar{\alpha} \cdot (\bar{r}_3 - \bar{r}_4)} + e^{-i\bar{\beta} \cdot (\bar{r}_1 - \bar{r}_3)}$$

$$e^{-i\bar{\alpha} \cdot (\bar{r}_2 - \bar{r}_4)} + e^{-i\bar{\beta} \cdot (\bar{r}_1 - \bar{r}_4)} e^{-i\bar{\alpha} \cdot (\bar{r}_2 - \bar{r}_3)}]$$

$$\cdot \bar{E}_{1i}^{(0)}(z_4)\ e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{1\perp})} e^{i\bar{k}_\perp' \cdot (\bar{r}_{1\perp} - \bar{r}_{2\perp})}$$

$$e^{i\bar{k}_\perp'' \cdot (\bar{r}_{2\perp} - \bar{r}_{3\perp})} e^{i\bar{k}_\perp''' \cdot (\bar{r}_{3\perp} - \bar{r}_{4\perp})} e^{i\bar{k}_{\perp i} \cdot \bar{r}_4}.$$

$$\tag{A.3}$$

It is understood here that $\bar{\bar{g}}_{11}(\bar{k}_\perp, z_i, z_j)$ actually consists of two parts corresponding to $z_i \lessgtr z_j$. Performing the transverse spatial integrations first we then may carry out the integrations over $\bar{\alpha}_\perp$, $\bar{\beta}_\perp$, $\bar{k}_\perp''$ and $\bar{k}_\perp'''$. The result takes the form:

$$<\bar{E}_o^{(4)}(\bar{r})> = \bar{E}_o^{(4)}(\bar{k}_{\perp i})\ e^{i\bar{k}_{\perp i} \cdot \bar{r}_\perp + i\bar{k}_{ozi} z} \tag{A.4}$$

where

$$\bar{E}_o^{(4)}(\bar{k}_{\perp i}) \equiv \frac{(\delta k_1'^4)^2 (2\pi)^3}{\omega^4 \mu_o^4} \int d^2k_\perp d^2k_\perp' \int d\alpha_z d\beta_z \int dz_1 dz_2 dz_3 dz_4$$

$$\Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_z)\ e^{-ik_{ozi} z}\ \bar{\bar{g}}_{01}^{>}(\bar{k}_{\perp i}, z, z_1)$$

$$\cdot\ \bar{\bar{g}}_{11}(\bar{k}_\perp', z_1, z_2) [\Phi(\bar{k}_\perp - \bar{k}_{\perp i}, \alpha_z)\ \bar{\bar{g}}_{11}(\bar{k}_{\perp i}, z_2, z_3)$$

$$\cdot\ \bar{\bar{g}}_{11}(\bar{k}_\perp, z_3, z_4)\ e^{-i\beta_z(z_1 - z_2)}\ e^{-i\alpha_z(z_3 - z_4)}$$

$$+\ \Phi(\bar{k}_\perp - \bar{k}_\perp', \alpha_z) \cdot \bar{\bar{g}}_{11}(\bar{k}_\perp, z_2, z_3)$$

$$\cdot\ \bar{\bar{g}}_{11}(\bar{k}_\perp - \bar{k}_\perp' - \bar{k}_{\perp i}, z_3, z_4)\ e^{-i\beta_z(z_1 - z_3)}$$

$$\cdot\ e^{-i\alpha_z(z_2 - z_4)} + \Phi(\bar{k}_\perp - \bar{k}_\perp', \alpha_z)\ \bar{\bar{g}}_{11}(\bar{k}_\perp, z_2, z_3)$$

$$\bar{\bar{g}}_{11}(\bar{k}_\perp', z_3, z_4)\ e^{-i\beta_z(z_1 - z_4)}\ e^{-i\alpha_z(z_2 - z_3)}]$$

$$\cdot\ \bar{E}_{1i}^{(0)}(z_4). \qquad (A.5)$$

It is to be noted that the term $e^{-ik_{ozi}z}\ \bar{\bar{g}}_{01}^{>}(\bar{k}_{\perp i}, z, z_1)$ in (A.5) is independent of z. It is clear from (A.4) that the fourth order scattered mean field is specular and can only contribute to backscattering at normal incidence. However, even for normal incidence it is important to recognize that the zeroth order reflected field $\bar{E}_o^{(0)}$ has no depolarized component. Therefore the cross term $2\mathrm{Re}[\bar{E}_o^{(0)}(\bar{r}) \cdot \langle\bar{E}_o^{(4)*}(\bar{r})\rangle]$ does not contribute to depolarization in the backscattering direction.

Appendix B

Derivation of Equation (6.10)

The integrand of Equations (6.9) may be cast into the form:

$$[\bar{\bar{g}}^{>}_{01}(-\bar{k}_{\perp i}, z, z_1) \cdot \bar{\bar{g}}_{11}(\bar{k}_\perp, z_1, z_3) \cdot \bar{E}^{(0)}_1(z_3)]_{\mu\nu}$$

$$= \sum_{s,s'} \sum_{p,p'} (\bar{A}^{\gtrless}_{\mu\nu}(\phi))_{ss'pp'} \, e^{i(pk_{1zi} + p'k_{1z})z_1}$$

$$e^{i(sk_{1zi} + s'k_{1z})z_3} \, e^{ik_{ozi}z} \tag{B.1}$$

where we have used Equations (2.6), (2.22), and (2.25) of Chapter 2. The indices s, s', p and p' take the values of $+1$ and -1 and the amplitudes $(\bar{A}^{\gtrless}_{\mu\nu}(\phi))_{ss'pp'}$ are listed in Appendix C. Substituting (B.1) into (6.9) we form the following expression:

$$\bar{I}_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z) \cdot \bar{I}^*_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z)$$

$$= \sum_{s,s'} \sum_{p,p'} \sum_{\bar{s},\bar{s}'} \sum_{\bar{p},\bar{p}'} \{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 \, F^{(p,\, p',\, s,\, s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \alpha_z,\, \beta_z)}$$

$$+ \ (\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{z_3}^{0} dz_4 \, F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \alpha_z,\, \beta_z)}$$

$$+ \ (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{>}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{z_1}^{0} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 \, F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \alpha_z,\, \beta_z)}$$

$$+ \ (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{z_1}^{0} dz_2 \int_{-d_1}^{0} dz_3 \int_{z_3}^{0} dz_4 \, F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}', \bar{s},\bar{s}')}_{(z_3, z_4, \alpha_z, \beta_z)} \tag{B.2}$$

where:

$$F^{(p,p', s,s')}_{(z_1, z_2, \alpha_z, \beta_z)} \equiv e^{i(pk_{1zi} + p'k_{1z} - \beta_z)z_1}$$

$$e^{i(sk_{1zi} + s'k_{1z} - \alpha_z)z_2}. \tag{B.3}$$

Similarly, we obtain,

$$\bar{I}_{\mu\nu}(\bar{k}_\perp, -\bar{k}_{\perp i}, \alpha_z, \beta_z) \cdot \bar{I}^*_{\mu\nu}(-\bar{k}_\perp, -\bar{k}_{\perp i}, \beta_z, \alpha_z)$$

$$= \sum_{s,s'} \sum_{p,p'} \sum_{\bar{s},\bar{s}'} \sum_{\bar{p},\bar{p}'} \{(\bar{A}^>_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^>_{\mu\nu}(\phi + \pi))^*_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 \; F^{(p,p', s,s')}_{(z_1, z_2, \alpha_z, \beta_z)}$$

$$F^{*(p,p', s,s')}_{(z_3, z_4, \beta_z, \alpha_z)}$$

$$+ (\bar{A}^>_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^<_{\mu\nu}(\phi + \pi))^*_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{z_3}^{0} F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \beta_z,\, \alpha_z)}$$

$$+ \, (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{>}_{\mu\nu}(\phi + \pi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{z_1}^{0} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 \; F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \beta_z,\, \alpha_z)}$$

$$+ \, (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi + \pi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}$$

$$\int_{-d_1}^{0} dz_1 \int_{z_1}^{0} dz_2 \int_{-d_1}^{0} dz_3 \int_{z_3}^{0} dz_4 \; F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \beta_z,\, \alpha_z)} \} \; . \qquad \text{(B.4)}$$

Substituting Equations (B.2) and (B.4) into (6.8), we then perform the α_z, β_z integrations before carrying out the z integrations. The only contributions must come from the poles of the spectral densities. However, in order to determine the direction in which to close the contours in the complex α_z and β_z planes, it is necessary to subdivide the domain of z integrations further. In Equation (B.2) we must break the integrations into regions in which $z_2 \gtrless z_4$ and $z_1 \gtrless z_3$. In Equation (B.4) we break the integrations into regions in which $z_1 \gtrless z_4$ and $z_2 \gtrless z_3$. For example, consider the first term of Equation (B.2) when substituted into (6.8)

$$\gamma^{(2)}_{\mu\nu}(\bar{k}_{oi}, -\bar{k}_{oi})\Big|_{\substack{\text{first term}\\ \text{of (B.2)}}} \propto \sum_{s,s'} \sum_{p,p'} \sum_{\bar{s},\bar{s}'} \sum_{\bar{p},\bar{p}'} \int d^2k_\perp$$

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 \, d\alpha_z \, d\beta_z$$

$$\Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_z) \, \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \alpha_z) \, F^{(p,p', s,s')}_{(z_1, z_2, \alpha_z, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}', \bar{s},\bar{s}')}_{(z_3, z_4, \alpha_z, \beta_z)} \, (\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{ss'pp'} . \tag{B.5}$$

We then break up the integrations over z, as

$$\int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{0} dz_3 \int_{-d_1}^{z_3} dz_4 = \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2$$

$$\int_{-d_1}^{z_2} dz_3 \int_{-d_1}^{z_3} dz_4 + \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_2}^{z_1} dz_3 \int_{-d_1}^{z_2} dz_4$$

$$+ \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_1}^{0} dz_3 \int_{-d_1}^{z_2} dz_4$$

$$+ \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_2}^{z_1} dz_3 \int_{z_2}^{z_3} dz_4$$

$$+ \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_1}^{0} dz_3 \int_{z_2}^{z_3} dz_4 \qquad \text{(B.6)}$$

when (B.6) is substituted into (B.5) the direction in which to close the contours in the complex α_z, β_z planes is well defined. We denote the poles of Φ as $\beta_m^{\pm}$ $m = 1, 2, \ldots$ where the superscripts + and - signify the location of the pole in the upper or lower half complex plane, accordingly. Therefore, upon performing the α_z and β_z integrations, Equation (B.5) becomes

$$\gamma^{(2)}_{\mu\nu}(\bar{k}_{oi}, -\bar{k}_{oi})\Big|_{\substack{\text{first term} \\ \text{of (B.2)}}} \propto \sum_{s,s'} \sum_{p,p'} \sum_{\bar{s},\bar{s}'} \sum_{\bar{p},\bar{p}'} (2\pi i)^2$$

$$\sum_{m,n} [\text{Res } \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^-) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^-)$$

$$W_{1,\mu\nu}^{(p,p', s,s', \bar{p},\bar{p}', \bar{s},\bar{s}')}(\beta_m^-, \beta_n^-)$$

$$+ \text{ Res } \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^-) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^-)$$

$$W_{2,\mu\nu}^{(p,p', ss', \bar{p}\bar{p}', \bar{s}\bar{s}')}(\beta_m^-, \beta_n^-)$$

$$- \text{ Res } \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^+) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^-)$$

$$W_{3,\mu\nu}^{(p,p', s,s', \bar{p},\bar{p}', \bar{s},\bar{s}')}(\beta_m^+, \beta_n^-)$$

$$- \text{ Res } \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^-) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^+)$$

$$W_{4,\mu\nu}^{(p,p', s,s', \bar{p},\bar{p}', \bar{s},\bar{s}')}(\beta_m^-, \beta_n^+)$$

$$+ \text{Res}\ \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^{\,+})\ \text{Res}\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^{\,+})$$

$$W_{5,\mu\nu}^{(p,p',\ s,s',\ \bar{p},\bar{p}',\ \bar{s},\bar{s}')}(\beta_m^{\,+}, \beta_n^{\,+})\ \Big] \tag{B.7}$$

where the summations over m and n extend over the poles of Φ and

$$W_{1,\mu\nu}^{(p,p',\ s,s',\ p,p')}(\alpha_z, \beta_z) \equiv \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{-d_1}^{z_2} dz_3 \int_{-d_1}^{z_3} dz_4$$

$$\{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}',\bar{p}\bar{p}'}\ F^{(p,p',\ s,s')}_{(z_1,\ z_2,\ \alpha_z,\ \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{(z_3,\ z_4,\ \alpha_z,\ \beta_z)}\} \tag{B.8a}$$

$$W_{2,\mu\nu}^{(p,p',\ s,s',\ \bar{p},\bar{p}')}(\alpha_z, \beta_z) \equiv \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_2}^{z_1} dz_3 \int_{-d_1}^{z_2} dz_4$$

$$\{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}\, F^{(p,p',\ s,s')}_{(z_1,\ z_2,\ \alpha_z,\ \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{(z_3,\ z_4,\ \alpha_z,\ \beta_z)}\} \tag{B.8b}$$

$$W^{(p,p',\ s,s',\ \bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{3,\mu\nu(\alpha_z,\ \beta_z)} \equiv \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_1}^{0} dz_3 \int_{-d_1}^{z_2} dz_4$$

$$\{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{ss'pp'}\, F^{(p,p',\ s,s')}_{(z_1,\ z_2,\ \alpha_z,\ \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{(z_3,\ z_4,\ \alpha_z,\ \beta_z)}\} \tag{B.8c}$$

$$W^{(p,p',\ s,s',\ \bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{4,\mu\nu(\alpha_z,\ \beta_z)} \equiv \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_2}^{z_1} dz_3 \int_{z_2}^{z_3} dz_4$$

$$\{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}\, F^{(p,p',\ s,s')}_{(z_1,\ z_2,\ \alpha_z,\ \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\ \bar{s},\bar{s}')}_{(z_3,\ z_4,\ \alpha_z,\ \beta_z)}\} \tag{B.8d}$$

$$W_{5,\mu\nu}^{(p,p',\, s,s',\, \bar{p},\bar{p}',\, \bar{s},\bar{s}')}(\alpha_z,\, \beta_z) \equiv \int_{-d_1}^{0} dz_1 \int_{-d_1}^{z_1} dz_2 \int_{z_1}^{0} dz_3 \int_{z_2}^{z_3} dz_4$$

$$\{(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'} \cdot (\bar{A}^{<}_{\mu\nu}(\phi))^{*}_{\bar{s}\bar{s}'\bar{p}\bar{p}'}\, F^{(p,p',\, s,s')}_{(z_1,\, z_2,\, \alpha_z,\, \beta_z)}$$

$$F^{*(\bar{p},\bar{p}',\, \bar{s},\bar{s}')}_{(z_3,\, z_4,\, \alpha_z,\, \beta_z)}\} \,. \tag{B.8e}$$

The next step is to perform the z integrations retaining only those terms which are dominant in the low conductivity regime. The result is:

$$\gamma^{(2)}_{\mu\nu}(\bar{k}_{oi},\, -\bar{k}_{oi})\Big|_{\substack{\text{first term}\\ \text{of (B.2)}}} \propto - \sum_{s,s'} \sum_{p,p'} \sum_{\bar{s},\bar{s}'} \sum_{\bar{p},\bar{p}'}$$

$$\frac{\delta_{s\bar{s}}\, \delta_{s'\bar{s}'}\, \delta_{p\bar{p}}\, \delta_{p'\bar{p}'} \,|(\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'}|^2}{4\kappa''_{ss'}}$$

$$\Bigg[\text{Res}\ \Phi(\bar{k}_\perp + \bar{k}_{\perp i},\, \beta_m^-) \times \frac{\text{Res}\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp,\, \beta_n^-)}{(\kappa^*_{ss'} - \beta_n^-)(\kappa^*_{pp'} - \beta_m^-)}$$

$$+ \frac{\text{Res } \Phi(\bar{k}_{\perp i} + \bar{k}_{\perp}, \beta_m^{+}) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_{\perp}, \beta_n^{-})}{(\kappa^*_{ss'} - \beta_n^{-})(\kappa^*_{pp'} - \beta_m^{+})}$$

$$+ \frac{\text{Res } \Phi(\bar{k}_{\perp i} + \bar{k}_{\perp}, \beta_m^{-}) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_{\perp}, \beta_n^{+})}{(\kappa^*_{ss'} - \beta_n^{+})(\kappa^*_{pp'} - \beta_m^{-})}$$

$$+ \left. \frac{\text{Res } \Phi(\bar{k}_{\perp i} + \bar{k}_{\perp}, \beta_m^{+}) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_{\perp}, \beta_n^{+})}{(\kappa^*_{ss'} - \beta_n^{+})(\kappa^*_{pp'} - \beta_m^{+})} \right] M^{(s,s',\, p,p')}_{(k_\rho)} \tag{B.9}$$

where $M^{(s,s',\, p,p')}_{(k_\rho)}$ is defined below in (B.11a). Following steps analogous to those which led to (B.9), we include the remainder of the terms in (B.2) as well as all of (B.4) when evaluating (6.8). The result is:

$$\gamma^{(2)}_{\mu\nu}(\bar{k}_{oi}, -\bar{k}_{oi}) = \frac{4\pi(2\pi)^8(\delta k_1'^4)^2 k_o^2 \cos\theta_{oi}}{\omega^4 \mu_o^4 |E_o|^2} \int d^2k_{\perp} (2\pi i)^2$$

$$\sum_{m,n} \sum_{s,s'} \sum_{p,p'} \{ (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'} \cdot [(\bar{A}^{>}_{\mu\nu}(\phi + \pi))_{pp'ss'}$$

$$+ (\bar{A}^{<}_{\mu\nu}(\phi))_{ss'pp'}]^* N^{(s,s',\, p,p')}_{(k_\rho)} + (\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'}$$

$$\cdot\ [(\bar{A}^{<}_{\mu\nu}(\phi + \pi))_{pp'ss'} + (\bar{A}^{>}_{\mu\nu}(\phi))_{ss'pp'}]^*$$

$$\times\ M^{(s,s'\ p,p')}_{(k_\rho)}\} \ \frac{J_{mn}}{4\kappa''_{ss'}} \qquad \text{(B.10)}$$

where

$$M^{(s,s',\ p,p')}_{(k_\rho)} \quad \frac{1 - e^{2(\kappa''_{pp'} + \kappa''_{ss'})d_1}}{(\kappa''_{pp'} + \kappa''_{ss'})} - \frac{e^{2\kappa''_{ss'}d_1}(1 - e^{2\kappa''_{pp'}d_1})}{\kappa''_{pp'}}$$

(B.11a)

$$N^{(ss'pp')}_{(k_\rho)} \equiv \frac{1 - e^{2(\kappa''_{pp'} + \kappa''_{ss'})d_1}}{(\kappa''_{ss'} + \kappa''_{pp'})} - \frac{(1 - e^{2\kappa''_{pp'}d_1})}{\kappa''_{pp'}} \qquad \text{(B.11b)}$$

$$J_{mn} \equiv \left[\frac{\text{Res}\ \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^-)\ \text{Res}\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^-)}{(\kappa^*_{ss'} - \beta_n^-)(\kappa^*_{pp'} - \beta_m^-)}\right.$$

$$+ \frac{\text{Res}\ \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^-)\ \text{Res}\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^+)}{(\kappa^*_{ss'} - \beta_n^+)(\kappa^*_{pp'} - \beta_m^-)}$$

$$+ \frac{\text{Res}\ \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \beta_m^+)\ \text{Res}\ \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \beta_n^-)}{(\kappa^*_{ss'} - \beta_n^-)(\kappa^*_{pp'} - \beta_m^+)}$$

$$+ \frac{\text{Res } \Phi(\bar{k}_{\perp} + \bar{k}_{\perp i}, \beta_m^{+}) \text{ Res } \Phi(\bar{k}_{\perp i} - \bar{k}_{\perp}, \beta_n^{+})}{(\kappa_{ss'}^{*} - \beta_n^{+})(\kappa_{pp'}^{*} - \beta_m^{+})} \Bigg] . \tag{B.11c}$$

The term $\sum_{m,n} J_{mn}$ may be simplified by utilizing the following argument. Let

$$I \equiv \int_{-\infty}^{\infty} d\alpha_z \frac{\Phi(\bar{k}_{\perp}, \alpha_z)}{(z_p - \alpha_z)} \tag{B.12}$$

where $\Phi(\bar{k}_{\perp}, \alpha_z)$ vanishes everywhere on the circle at infinity and has an equal distribution of poles $\{\alpha_m^{+}\}$ and $\{\alpha_n^{-}\}$ in the upper and lower half complex planes. z_p is a complex number which lies anywhere in the complex plane. To evaluate (B.12) we may close the contour either up or down. Closing up and assuming $\text{Im } z_p > 0$ we find:

$$I = (2\pi i)\left[-\Phi(\bar{k}_{\perp}, z_p) + \sum_m \frac{\text{Res } \Phi(\bar{k}_{\perp}, \alpha_m^{+})}{(z_p - \alpha_m^{+})}\right] \tag{B.13}$$

and closing down, we find

$$I = -2\pi i \sum_n \frac{\text{Res } \Phi(\bar{k}_{\perp}, \alpha_n^{-})}{(z_p - \alpha_n^{-})} . \tag{B.14}$$

Therefore, it follows that

$$\Phi(\bar{k}_\perp, z_p) = \sum_m \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_m^+)}{(z_p - \alpha_m^+)} + \sum_n \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_n^-)}{(z_p - \alpha_n^-)} . \tag{B.15}$$

Multiplying (B.15) by $\Phi(\bar{k}_\perp', z_s)$ yields

$$\begin{aligned}\Phi(\bar{k}_\perp, z_p)\,\Phi(\bar{k}_\perp', z_s) = \sum_{m,n} \Bigg[& \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_m^+)\ \text{Res } \Phi(\bar{k}_\perp', \alpha_n^+)}{(z_p - \alpha_m^+)(z_s - \alpha_n^+)} \\ & + \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_m^+)\ \text{Res } \Phi(\bar{k}_\perp', \alpha_n^-)}{(z_p - \alpha_m^+)(z_s - \alpha_n^-)} \\ & + \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_m^-)\ \text{Res } \Phi(\bar{k}_\perp', \alpha_n^+)}{(z_p - \alpha_m^-)(z_s - \alpha_n^+)} \\ & + \frac{\text{Res } \Phi(\bar{k}_\perp, \alpha_m^-)\ \text{Res } \Phi(\bar{k}_\perp', \alpha_n^-)}{(z_p - \alpha_m^-)(z_s - \alpha_n^-)} \Bigg] . \end{aligned} \tag{B.16}$$

This result would not change if we had assumed $\text{Im}(z_p) < 0$ rather than $\text{Im}(z_p) > 0$ as we did. Therefore, utilizing the result (B.16) we may sum equation (B.11c) as:

$$\sum_{m,n} J_{mn} = \Phi(\bar{k}_\perp + \bar{k}_{\perp i}, \kappa^*_{pp'}) \; \Phi(\bar{k}_{\perp i} - \bar{k}_\perp, \kappa^*_{ss'}) . \qquad \text{(B.17)}$$

Substituting (B.17), Equation (3.29) of Chapter 3, and the values of $(A^{\gtrless}_{\mu\nu})_{ss'pp'}$ from Appendix C, into (B.10) we obtain after some algebra; Equation (6.10)

$$\gamma^{(2)}_{hv}(\bar{k}_{oi}, -\bar{k}_{oi}) = \gamma^{(2)}_{vh}(\bar{k}_{oi}, -\bar{k}_{oi}) = \frac{8\pi^2\varepsilon_o \cos\theta_{oi}}{\varepsilon_1 \cos\theta_i}$$

$$\frac{t_h(\theta_i)\; t_v(\theta_i)}{|D_{2i}|^2 |F_{2i}|^2} \frac{\sec\theta_i}{\kappa_a{}^2} \left\{ \int_0^{\pi/2} d\theta \, \frac{\sin\theta \sec\theta}{(\sec\theta_i + \sec\theta)^2} \right.$$

$$W_1(\theta, \theta_i)\; W_3{}^2(\theta, \theta_i)\; M_1(\theta, \theta_i)$$

$$+ \int_0^{\pi/2} d\theta \, \frac{\sin\theta \sec\theta}{(\sec\theta_i - \sec\theta)} W_1(\theta, \theta_i)\; W_2(\theta, \theta_i)$$

$$W_3(\theta, \theta_i)\; M_2(\theta, \theta_i) + \int_0^{\pi/2} d\theta \, \frac{\sin\theta \sec\theta}{(\sec\theta_i - \sec\theta)^2}$$

$$\left. W_1(\theta, \theta_i)\; W_2{}^2(\theta, \theta_i)\; M_3(\theta, \theta_i) \right\} . \qquad \text{(6.10)}$$

APPENDIX C

The Coefficients $(\bar{A}^{\lessgtr}_{\mu\nu}(\phi))_{s,s',p,p'}$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{1,1,1,1} = \bar{\Gamma}_v\, e^{i2k_{1z}d_1}\,(R_{12i}\, S_{12i}\, e^{i4k_{1zi}d_1})$$

$$\left\{ - \frac{k_{1zi}\; R_{12}}{D_2} \cos\phi \sin\phi - \frac{k_{1z}\; S_{12}}{F_2\; k_1{}^2}\, \alpha_+(\phi) \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{1,1,-1,1} = \bar{\Gamma}_v\, e^{i2k_{1z}d_1}\,(R_{12i}\, e^{i2k_{1zi}d_1})$$

$$\left\{ \frac{k_{1zi}\; R_{12}}{D_2} \cos\phi \sin\phi + \frac{k_{1z}\; S_{12}}{D_2 k_1{}^2}\, \alpha_-(\phi) \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{1,1,1,-1} = \bar{\Gamma}_v (R_{12i}\; S_{12i}\; e^{i4k_{1zi}d_1})$$

$$\left\{ \begin{array}{l} e^{i2k_{1z}d_1}\left[- \dfrac{k_{1zi}\; R_{10}R_{12}}{D_2} \cos\phi \sin\phi + \dfrac{k_{1z}\; S_{10}S_{12}}{F_2 k_1{}^2}\, \alpha_-(\phi) \right] \\ - \dfrac{k_{1zi}}{D_2} \cos\phi \sin\phi + \dfrac{k_{1z}}{F_2 k_1{}^2}\, \alpha_-(\phi) \end{array} \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{1,-1,1,1} = \bar{\Gamma}_v \; (R_{12i} \; S_{12i} \; e^{i4k_{1zi}d_1})$$

$$\begin{Bmatrix} -\dfrac{k_{1zi}}{D_1} \cos\phi \sin\phi + \dfrac{k_{1z}}{F_2 k_1{}^2} \alpha_+(\phi) \\ e^{i2k_{1z}d_1} \left[-\dfrac{k_{1zi} \; R_{10}R_{12}}{D_2} \cos\phi \sin\phi + \dfrac{k_{1z}S_{10}S_{12}}{F_2 k_1{}^2} \alpha_+(\phi) \right] \end{Bmatrix}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,1,1,1} = \bar{\Gamma}_v \; (S_{12i} \; e^{i2k_{1zi}d_1}) \; e^{i2k_{1z}d_1}$$

$$\left\{ -\frac{k_{1zi} \; R_{12}}{D_2} \cos\phi \sin\phi - \frac{k_{1z} \; S_{12}}{F_2 k_1{}^2} \alpha_+(\phi) \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{1,1,-1,-1} = \bar{\Gamma}_v \; (R_{12i} \; e^{i2k_{1zi}d_1})$$

$$\begin{Bmatrix} e^{i2k_{1z}d_1} \left[\dfrac{k_{1zi} \; R_{10}R_{12}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z} \; S_{10}S_{12}}{F_2 k_1} \alpha_+(\phi) \right] \\ \dfrac{k_{1zi}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}}{F_2 k_1{}^2} \alpha_+(\phi) \end{Bmatrix}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,-1,1,1} = \bar{\Gamma}_v \; (S_{12i} \; e^{i2k_{1zi}d_1})$$

$$\left\{ \begin{aligned} & - \frac{k_{1zi}}{D_2} \cos\phi \sin\phi + \frac{k_{1z}}{F_2 k_1^{\,2}} \alpha_+(\phi) \\ & e^{i2k_{1z}d_1} \left[- \frac{k_{1z} \; R_{10} R_{12}}{D_2} \cos\phi \sin\phi + \frac{k_{1z} \; S_{10} S_{12}}{F_2 k_1^{\,2}} \alpha_+(\phi) \right] \end{aligned} \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,1,-1,1} = \bar{\Gamma}_v \; e^{i2k_{1z}d_1} \left\{ \frac{k_{1zi} \; R_{12}}{D_2} \cos\phi \sin\phi \right.$$

$$\left. + \frac{k_{1z} \; S_{12}}{F_2 k_1^{\,2}} \alpha_-(\phi) \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,1,1,-1} = \bar{\Gamma}_v \; (S_{12i} \; e^{i2k_{1zi}d_1})$$

$$\left\{ \begin{aligned} & e^{i2k_{1z}d_1} \left[- \frac{k_{1zi} \; R_{10} R_{12}}{D_2} \cos\phi \sin\phi + \frac{k_{1z} \; S_{10} S_{12}}{F_2 k_1^{\,2}} \alpha_-(\phi) \right] \\ & - \frac{k_{1zi}}{D_2} \cos\phi \sin\phi + \frac{k_{1z}}{F_2 k_1^{\,2}} \alpha_-(\phi) \end{aligned} \right\}$$

$$\bar{A}_{hv}^{\gtrless}(\phi))_{1,-1,-1,1} = \bar{\Gamma}_v \; (R_{12i} \; e^{i2k_{1zi}d_1})$$

$$\left\{ \begin{array}{l} \dfrac{k_{1zi}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}}{F_2 k_1{}^2} \alpha_-(\phi) \\ e^{i2k_{1z}d_1} \left[\dfrac{k_{1zi} \; R_{10} R_{12}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z} \; S_{10} S_{12}}{F_2 k_1{}^2} \alpha_-(\phi) \right] \end{array} \right\}$$

$$(\bar{A}_{hv}^{\gtrless}(\phi))_{1,-1,1,-1} = \bar{\Gamma}_v \; (R_{12i} \; S_{12i} \; e^{i4k_{1zi}d_1})$$

$$\left\{ - \frac{k_{1zi} \; R_{10}}{D_2} \cos\phi \sin\phi - \frac{k_{1z} \; S_{10}}{F_2 k_1} \alpha_-(\phi) \right\}$$

$$(\bar{A}_{hv}^{\gtrless}(\phi))_{-1,-1,-1,-1} = \bar{\Gamma}_v \left\{ \frac{k_{1zi} \; R_{10}}{D_2} \cos\phi \sin\phi + \frac{k_{1z} \; S_{10}}{F_2 k_1{}^2} \alpha_+(\phi) \right\}$$

$$(\bar{A}_{hv}^{\gtrless}(\phi))_{1,-1,-1,-1} = \bar{\Gamma}_v \; (R_{12i} \; e^{i2k_{1zi}d_1}) \left\{ \frac{k_{1zi} \; R_{10}}{D_2} \cos\phi \sin\phi \right.$$

$$\left. + \frac{k_{1z} \; S_{10}}{F_2 k_1{}^2} \alpha_+(\phi) \right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,-1,-1,1} = \bar{\Gamma}_v$$

$$\left\{\begin{array}{l} \dfrac{k_{1zi}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}}{F_2 k_1^2} \alpha_-(\phi) \\ e^{i2k_{1z}d_1}\left[\dfrac{k_{1zi}\ R_{10}R_{12}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}\ S_{10}S_{12}}{F_2 k_1^2} \alpha_-(\phi)\right] \end{array}\right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,1,-1,-1} = \bar{\Gamma}_v$$

$$\left\{\begin{array}{l} e^{i2k_{1z}d_1}\left[\dfrac{k_{1zi}\ R_{10}R_{12}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}\ S_{10}S_{12}}{F_2 k_1^2} \alpha_+(\phi)\right] \\ \dfrac{k_{1zi}}{D_2} \cos\phi \sin\phi - \dfrac{k_{1z}}{F_2 k_1^2} \alpha_+(\phi) \end{array}\right\}$$

$$(\bar{A}^{\gtrless}_{hv}(\phi))_{-1,-1,1,-1} = \bar{\Gamma}_v\ (S_{12i}\ e^{i2k_{1zi}d_1}) \left\{- \dfrac{k_{1zi}\ R_{10}}{D_2} \cos\phi \sin\phi \right.$$

$$\left. - \dfrac{k_{1z}\ S_{10}}{F_2 k_1^2} \alpha_-(\phi)\right\}$$

where

$$\alpha_{\pm}(\phi) \equiv (k_{1z}k_{1zi} \cos \phi \pm k_{\rho}k_{\rho i}) \sin \phi$$

$$\bar{\Gamma}_{\mu} \equiv \hat{\mu}\; E_o \left[\left(\frac{\omega\mu_o}{8\pi^2} \right)^2 \frac{Y_{10i}X_{01i}}{F_{2i}D_{2i}\; k_o k_{1z} k_{1zi}} \right]$$

$$\hat{\mu} = \begin{cases} \hat{h}(k_{ozi}) & \text{for scattered TM waves} \\ \hat{e}(k_{ozi}) & \text{for scattered TE waves} \end{cases}$$

The coefficients $(\bar{A}^{\gtrless}_{vh}(\phi))_{ss'pp'}$ are obtained from the relation

$$(\bar{A}^{\gtrless}_{vh}(\phi))_{ss'pp'} = (\bar{A}^{\lessgtr}_{hv}(\phi + \pi))_{pp'ss'} \;.$$

CHAPTER 7

Renormalization Methods in the Active Remote Sensing of Random Media

In the preceeding chapters the problem of electromagnetic scattering by a two-layer random medium is solved with an iterative approach. However, if multiple scattering effects are to be included renormalization methods are necessary in which the Neumann series for the mean and covariance of the electromagnetic field are resummed. Tatarskii[64] employed a Feynman diagrammatic technique to develop the Dyson equation for the mean field and the Bethe-Salpeter equation for the covariance of the field. He also considered the bilocal and ladder approximations to the Dyson and Bethe-Salpeter equations, respectively. Rosenbaum[65] also investigated the coherent wave motion by applying a non-linear approximation to the Dyson equation.

In this chapter we review the development of the Dyson and Bethe-Salpeter equations and discuss the various approximations made in the solution of these equations.

7.1 The Dyson and Bethe-Salpeter Equations

In Chapter 3, we obtained the integral equation which governs the scattering of electromagnetic waves by a random medium. In the case of a point source, we have:

$$\bar{\bar{G}}(\bar{r}, \bar{r}_o) = \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o) + \int_V d^3r' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}') \, Q(\bar{r}') \cdot \bar{\bar{G}}(\bar{r}', \bar{r}_o) \tag{7.1}$$

where $Q(\bar{r}) = \omega^2 \mu_o \varepsilon_{1f}(\bar{r})$. Iterating (7.1) leads to the infinite Neumann series given by:

$$\bar{\bar{G}}(\bar{r}, \bar{r}_o) = \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o) + \int_V d^3r' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}') \, Q(\bar{r}') \cdot \bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}_o)$$

$$+ \int_V d^3r' \int_V d^3r'' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}') \cdot \bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}'')$$

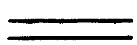
$$\cdot \, Q(\bar{r}') \, Q(\bar{r}'') \cdot \bar{\bar{G}}^{(0)}(\bar{r}'', \bar{r}_o) + \ldots \tag{7.2}$$

Ensemble averaging (7.2) yields

$$\langle \bar{\bar{G}}(\bar{r}, \bar{r}_o) \rangle = \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o) + \int_V d^3r' \int_V d^3r'' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}')$$

$$\bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}'') \cdot \bar{\bar{G}}^{(0)}(\bar{r}'', \bar{r}_o)\ C(\bar{r}' - \bar{r}'') + \ldots \qquad (7.3)$$

where it should be noted that all odd order moments of $Q(\bar{r})$ vanish and all even moments of $Q(\bar{r})$ are cluster expanded in terms of the two-point correlation function, $C(\bar{r} - \bar{r}')$. A convenient method of handling cumbersome equations of the type (7.3) is through the use of Feynman diagrams. The following symbols are used in constructing the Feynman diagrams

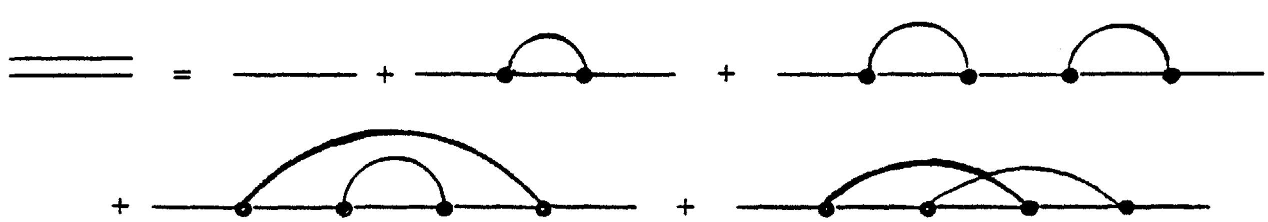
 $\equiv \langle\bar{\bar{G}}(\bar{r}, \bar{r}_o)\rangle$

 $\equiv \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o)$

 $\equiv C(\bar{r} - \bar{r}')$

• ≡ vertex over which integration is implied.

Thus (7.3) may be expressed as

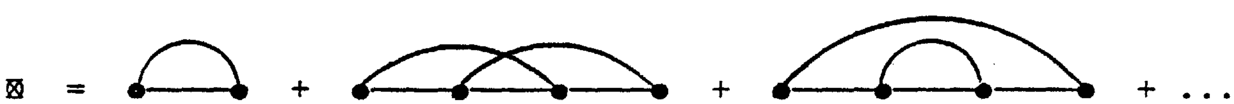


$+ \ldots$ (7.4)

where, for example:

$$= \int_V d^3r' d^3r'' \, \overline{\overline{G}}^{(0)}(\bar{r}, \bar{r}') \cdot \overline{\overline{G}}^{(0)}(\bar{r}', \bar{r}'') \cdot \overline{\overline{G}}^{(0)}(\bar{r}'', \bar{r}_o) \, C(\bar{r}' - \bar{r}''). \tag{7.5}$$

We now define diagrams (minus end connectors) as strongly connected if it is impossible to bisect the diagram without breaking the correlation connections. All other diagrams are called weakly connected. The sum of all strongly connected diagrams defines the mass operator, given by



(7.6)

We note that with (7.6), Equation (7.4) may be written in the form

(7.7)

(7.8)

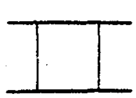
(7.9)

where (7.9) is the diagrammatical representation of the Dyson equation for the mean Dyadic Green's function. In analytic form (7.9) reads:

$$\langle \bar{\bar{G}}(\bar{r}, \bar{r}_o) \rangle = \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o) + \int_V d^3r' d^3r'' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}') \cdot \bar{\bar{Q}}(\bar{r}', \bar{r}'') \cdot \langle \bar{\bar{G}}(\bar{r}'', \bar{r}_o) \rangle \tag{7.10}$$

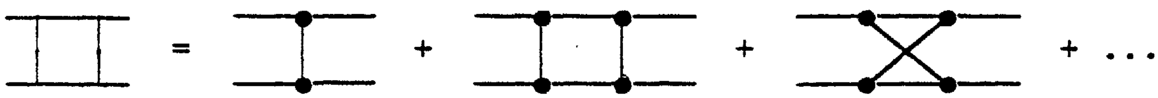
Here $\bar{\bar{Q}}$ is used to denote the mass operator.

In a similar fashion we introduce the symbol for the field covariance:



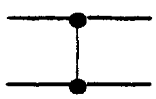
$$\equiv \langle \bar{\bar{G}}(\bar{r}, \bar{r}_o) \, \bar{\bar{G}}^*(\bar{r}', \bar{r}_o') \rangle - \langle \bar{\bar{G}}(\bar{r}, \bar{r}_o) \rangle \langle \bar{\bar{G}}^*(\bar{r}', \bar{r}_o') \rangle \tag{7.11}$$

The Neumann series for the field covariance has the following diagrammatical representation:



(7.12)

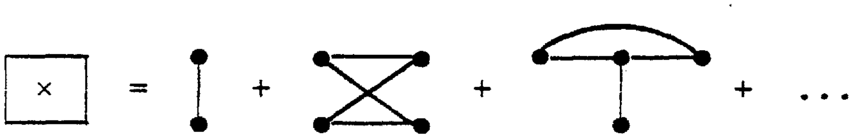
where for example



$$= \int_V d^3r_1 d^3r_1' [\bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_1) \cdot \bar{\bar{G}}^{(0)}(\bar{r}_1, \bar{r}_o)]$$

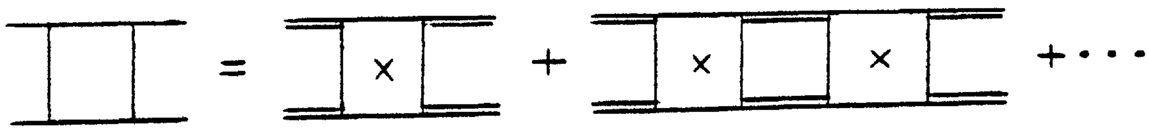
$$[\bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}_1') \cdot \bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}_1')]^* C(\bar{r}_1 - \bar{r}_1'). \qquad (7.13)$$

The sum of all strongly connected diagrams is defined as the intensity operator:

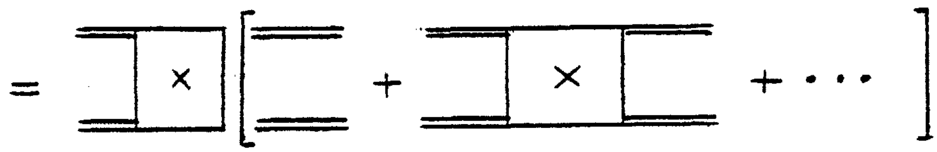


(7.14)

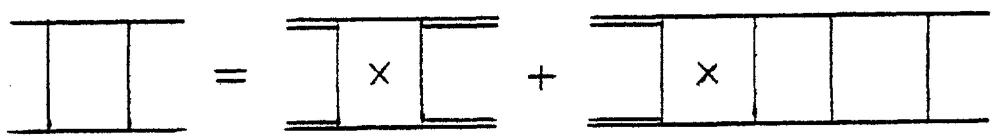
This allows the infinite Neumann series (7.12) to be written as:



(7.15)



(7.16)



(7.17)

Equation (7.17) is the diagrammatic representation of the Bethe-Salpeter equation, which will be given in the next section in analytical form.

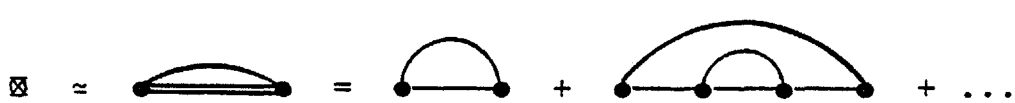
7.2 Approximations to the Mass and Intensity Operators

In (7.10), $\bar{\bar{Q}}(\bar{r}', \bar{r}'')$ denotes the mass operator, which must be approximated if Dyson's equation is to be solved analytically. The most popular approximation to the mass operator is the bi-local approximation, which consists of retaining only the first term in the series for $\bar{\bar{Q}}$, that is,

$$\bar{\bar{Q}}(\bar{r}', \bar{r}'') \approx \bar{\bar{G}}^{(0)}(\bar{r}', \bar{r}'')\, C(\bar{r}' - \bar{r}''). \tag{7.18}$$

Physically the bilocal approximation corresponds to a single scattering of the mean field as can be seen by substituting (7.18) into (7.10). The validity of the bilocal approximation has been discussed by Tatarskii,[64] and Rosenbaum[65] has shown that the bilocally approximated Dyson equation together with the ladder approximated Bethe-Salpeter equation do not lead to an energy conserving formalism.

Another approximation to the mass operator which circumvents these difficulties is the non-linear approximation, in which an infinite sequence of terms in (7.6) is summed.

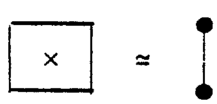

(7.19)

Substituting (7.19) into (7.10), we obtain a non-linear integral equation for the mean dyadic Green's function

$$<\bar{\bar{G}}(\bar{r}, \bar{r}_o)> = \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}_o) + \int_V d^3r' \, d^3r'' \, \bar{\bar{G}}^{(0)}(\bar{r}, \bar{r}') \cdot <\bar{\bar{G}}(\bar{r}', \bar{r}'')> \cdot <\bar{\bar{G}}(\bar{r}'', \bar{r}_o)> C(\bar{r}' - \bar{r}'') \qquad (7.20)$$

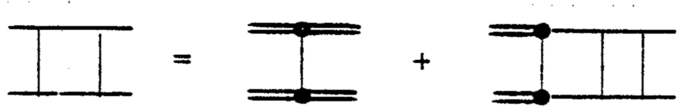
Physically, the non-linear approximation accounts for multiple scattering of the mean field and in this regard is superior to the bi-local approximation.

The usual approximation made for the intensity operator is the so called ladder approximation in which only the first term of the series (7.14) is retained. That is,



(7.21)

In which case the Bethe-Salpeter equation reduces to:



(7.22)

or:

$$\langle \underset{\sim}{\bar{\bar{G}}}(\bar{r}, \bar{r}_o)\, \underset{\sim}{\bar{\bar{G}}}{}^*(\bar{r}', \bar{r}_o')\rangle = \int_V d^3r_1 d^3r_1'\, C(\bar{r}_1 - \bar{r}_1')[\langle\bar{\bar{G}}(\bar{r}, \bar{r}_1)\rangle$$

$$\cdot \langle\bar{\bar{G}}(\bar{r}_1, \bar{r}_o)\rangle\langle\bar{\bar{G}}{}^*(\bar{r}', \bar{r}_1')\rangle \cdot \langle\bar{\bar{G}}{}^*(\bar{r}_1', \bar{r}_o')\rangle$$

$$+\langle\langle\bar{\bar{G}}(\bar{r}, \bar{r}_1)\rangle \cdot \underset{\sim}{\bar{\bar{G}}}(\bar{r}_1, \bar{r}_o)\langle\bar{\bar{G}}{}^*(\bar{r}', \bar{r}_1')\rangle \cdot \underset{\sim}{\bar{\bar{G}}}{}^*(\bar{r}_1', \bar{r}_o')\rangle] \tag{7.23}$$

where $\underset{\sim}{\bar{\bar{G}}}(\bar{r}, \bar{r}_o) \equiv \bar{\bar{G}}(\bar{r}, \bar{r}_o) - \langle\bar{\bar{G}}(\bar{r}, \bar{r}_o)\rangle$, is the incoherent mean dyadic Green's function.

CHAPTER 8

Mean Dyadic Green's Function of a Two Layer Random Medium

The study of the mean dyadic Green's function for a two layer random medium is of special importance in the fields of scattering, radiation, and diffraction of electromagnetic waves as applied to optical communications in the atmosphere, radar backscattering from earth terrain, and active remote sensing of the terrestrial environment. It is well known that the coherent wave motion in a random medium can be described by Dyson's equation, which is an exact equation for the mean field. Dyson's equation expresses the coherent field in terms of a mass operator Q which is in the form of an infinite series and must be approximated. The most commonly used approximation to Q is the so called bilocal approximation which follows by retaining only the first term in the infinite series representation for Q. Solutions to the bilocally approximated Dyson's equation have been the subject of extensive investigation in the literature.[64,65] However, as pointed out by Rosenbaum,[65] the bilocal approximation not only leads to solutions with potentially severe range restrictions; due to the omission of higher order terms in the mass operator, it also leads to solutions which are energetically inconsistent with the Bethe-Salpeter equation under the ladder approximation. An approximation to the mass operator, which circumvents these difficulties is the non-linear approximation[65] in which an infinite sequence of higher order terms in the series for Q is summed. This approximation to the mass operator results in an intractable non-linear integral-differential equation for the coherent field. Rosenbaum[65,75] found approximate solutions to the non-linearly approximated scalar Dyson's equation, for unbounded random media in the limit of large and small scale fluctuations. Tsang and Kong[58,59] using a two variable expansion technique have solved the scalar Dyson's equation in the non-linear approximation for the cases of a one-dimensional two-layer laminar structure and a three dimensional half-space random medium. In the limit of a laminar structure they found the coherent wave motion to possess two effective propagation constants. More recently, Tan and Fung[60] also employed the two-variable expansion technique and solved the non-linear Dyson's equation for the zeroth order mean dyadic Green's function in the case of a half space random medium. Their vector solution contains only a single propagation constant for all components in the Green's dyadic.

In this paper we employ the two variable expansion technique to obtain the complete zeroth order solutions for the mean dyadic Green's functions of a two layer random medium with arbitrary three dimensional correlation functions. It is found that the coherent vector field in general propagates in the random layer as if in an anisotropic medium with different propagation constants for the characteristic TE and TM polarizations. Moreover, in the limit of a laminar structure two propagation constants for each polarization state are found to exist.

8.1 Zeroth Order Mean Dyadic Green's Function Using the Two Variable Expansion Technique

Consider a two-layer random medium with boundaries at $z = 0$ and $z = -d_1$ [Fig. 3.1]. The random medium has a permittivity consisting of the sum of a mean part $\varepsilon_{1m} \equiv \langle\varepsilon_1(\bar{r})\rangle$ and a random part $\varepsilon_{1f}(\bar{r})$ whose ensemble average vanishes. The media in the regions $z > 0$ and $z < -d_1$ are nonrandom having permittivities ε_o and ε_2, respectively. All regions are characterized by permeability μ_o. The random fluctuations $\varepsilon_{1f}(\bar{r})$ will be assumed to be statistically homogeneous so that the two-point correlation function of the fluctuations is a function only of the difference in the two points. The coherent dyadic Green's function of a point source imbedded in the random medium satisfies Dyson's equation which under the nonlinear approximation takes the form

$$\bar{\nabla} \times \bar{\nabla} \times \bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_o) - k_{1m}^2 \bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_o) = \bar{\bar{I}}\, \delta(\bar{r} - \bar{r}_o)$$

$$+ \int_V d^3r_2\, \bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_2) \cdot \bar{\bar{G}}_{11m}(\bar{r}_2, \bar{r}_o)\, C(\bar{r} - \bar{r}_2) \qquad (8.1)$$

where $k_{1m}^2 = \omega^2\mu_o\varepsilon_{1m}$ and the spatial integration extends over the layer of the random medium. The first subscript of the dyadic Green's function indicates the region containing the observation point, the second subscript indicates the region containing the source point, and the third subscript indicates that the dyadic Green's function is the mean dyadic Green's function. We introduce the Fourier transforms of the mean dyadic Green's function and of the correlation function:

$$\bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_o) = \frac{1}{(2\pi)^2} \int d^2k_\perp \, \bar{\bar{g}}_{11m}(\bar{k}_\perp, z, z_o) \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \tag{8.2}$$

$$C(\bar{r} - \bar{r}_2) = \int d^3\alpha \, \Phi(\bar{\alpha}) \, e^{-i\bar{\alpha} \cdot (\bar{r} - \bar{r}_2)} \tag{8.3}$$

where $\bar{k}_\perp \equiv \hat{x} \, k_x + \hat{y} \, k_y$ and $d^2k_\perp \equiv dk_x dk_y$. Substituting (8.2) and (8.3) into (8.1) and performing the transverse spatial integrations, we obtain

$$\left(\frac{\partial^2}{\partial z^2} + k_{1mz}^2\right) \bar{\bar{g}}_{11m}(\bar{k}_\perp, z, z_o) - \left(i\bar{k}_\perp + \hat{z} \frac{\partial}{\partial z}\right)\left(i\bar{k}_\perp + \hat{z} \frac{\partial}{\partial z}\right)$$

$$\cdot \, \bar{\bar{g}}_{11m}(\bar{k}_\perp, z, z_o) = -\bar{\bar{I}} \, \delta(z - z_o) - \frac{1}{(2\pi)^2}$$

$$\int_{-\infty}^{\infty} d^2k_\perp' \int_{-\infty}^{\infty} d\alpha_z \int_{-d_1}^{0} dz_2 \, \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_z) \, e^{-i\alpha_z(z - z_2)}$$

$$\bar{\bar{g}}_{11m}(\bar{k}_\perp', z, z_2) \cdot \bar{\bar{g}}_{11m}(\bar{k}_\perp, z_2, z_o) \tag{8.4}$$

where

$$k_{1mz} = [k_{1m}^2 - k_\perp^2]^{1/2}. \tag{8.5}$$

To solve equation (8.4) we make use of the two variable expansion technique, which has been used to solve for the long distance behavior of the wave propagation in a random medium with laminar structure.[58] Following the procedure as in Tsang and Kong[58,59] and defining a bookeeping parameter, s, we find that to zeroth order of s^0

$$\left(\frac{\partial^2}{\partial z^2} + k_{1mz}^2\right) \bar{\bar{g}}_{11mo}(\bar{k}_\perp; z, \xi, z_o, \xi_o) - \left(i\bar{k}_\perp + \hat{z}\frac{\partial}{\partial z}\right)\left(i\bar{k}_\perp + \hat{z}\frac{\partial}{\partial z}\right)$$

$$\cdot\ \bar{\bar{g}}_{11mo}(\bar{k}_\perp, z, \xi; z_o, \xi_o) = -\bar{\bar{I}}\,\delta(z - z_o) \tag{8.6}$$

and to order, s for $z \gtrless z_o$,

$$\left(\frac{\partial^2}{\partial z^2} + k_{1mz}^2\right) \bar{\bar{g}}_{11m1}(\bar{k}_\perp; z, \xi; z_o, \xi_o) = -2\,\frac{\partial^2}{\partial z\,\partial \xi}$$

$$\bar{\bar{g}}_{11mo}(\bar{k}_\perp, z, \xi; z_o, \xi_o)$$

$$- \int_{-d_1}^{0} dz_2 \int_{-\infty}^{\infty} d\alpha_z \int_{-\infty}^{\infty} d^2k_\perp{}' \, \Phi(\bar{k}_\perp{}' - \bar{k}_\perp, \alpha_z) \, e^{-i\alpha_z(z - z_2)}$$

$$\cdot \, \bar{\bar{g}}_{11mo}(\bar{k}_\perp{}', z, \xi, z_2, \xi_2) \cdot \bar{\bar{g}}_{11mo}(\bar{k}_\perp, z_2, \xi_2; z_o, \xi_o)$$

(8.7)

where $\xi = sz$, $\xi_o = sz_o$, $\xi_2 = sz_2$ are long distance scales and the subscripts o and 1 which follow the subscript m on the mean dyadic Green's function denote, respectively, the zeroth and the first order solution. In deriving (8.7) we have used the divergence relation for the mean dyadic Green's function in Fourier transform space, which for $z > z_o$ and $z < z_o$ is given by

$$\left[i\bar{k}_\perp + \hat{z}\left(\frac{\partial}{\partial z} + s\,\frac{\partial}{\partial \xi}\right)\right] \cdot \left[\bar{\bar{g}}_{11mo}(\bar{k}_\perp; z, \xi; z_o, \xi_o)\right.$$

$$\left. + \, s\bar{\bar{g}}_{11m1}(\bar{k}_\perp, z, \xi; z_o, \xi_o) + \ldots\right] = 0. \qquad (8.8)$$

From which it follows that:

<u>Zeroth Order</u>: $$\left(i\bar{k}_\perp + \hat{z}\frac{\partial}{\partial z}\right) \cdot \bar{\bar{g}}_{11mo}(\bar{k}_\perp, z, \xi; z_o, \xi_o) = 0 \tag{8.9a}$$

<u>First Order</u>: $$\left(i\bar{k}_\perp + \hat{z}\frac{\partial}{\partial z}\right) \cdot \bar{\bar{g}}_{11m1}(\bar{k}_\perp, z, \xi; z_o, \xi_o)$$

$$= - \hat{z} \cdot \frac{\partial \bar{\bar{g}}_{11mo}(\bar{k}_\perp, z, \xi; z_o, \xi_o)}{\partial \xi} . \tag{8.9b}$$

The zeroth order Fourier transform mean dyadic Green's function satisfies an equation identical to that satisfied by the Fourier transformed dyadic Green's function of the non-random problem. This is not surprising since in the limit of vanishing random fluctuations the zeroth order solution obtained from (8.6) must reduce to the dyadic Green's function of the corresponding homogeneous problem. Moreover, the two variable expansion technique carried to zeroth order, accounts for the random fluctuations essentially by introducing corrections to the phase of the unperturbed dyadic Green's function. Therefore the zero order solution takes the form

$$\bar{\bar{g}}_{11mo}(\bar{k}_\perp, z, \xi; z_o, \xi_o) = - \hat{z}\hat{z}\,\frac{\delta(z - z_o)}{k_{1m}^2}$$

$$+\begin{cases}\bar{\bar{g}}^{>}_{11mo}(\bar{k}_\perp,\ z,\ \xi;\ z_o,\ \xi_o) & (z > z_o)\\ \bar{\bar{g}}^{<}_{11mo}(\bar{k}_\perp,\ z,\ \xi;\ z_o,\ \xi_o) & (z < z_o)\end{cases} \tag{8.10a}$$

where

$$\bar{\bar{g}}^{>}_{11mo}(\bar{k}_\perp,\ z,\ \xi;\ z_o,\ \xi_o) =$$

$$\begin{cases}[A_1(k_\perp,\ \xi)\ \hat{e}(k_{1mz})\ e^{ik_{1mz}z} + A_2(k_\perp,\ \xi)\ \hat{e}(-k_{1mz})\ e^{-ik_{1mz}z}]\\ [B_1(k_\perp,\ \xi_o)\ \hat{e}(-k_{1mz})\ e^{ik_{1mz}z_o} + B_2(k_\perp,\ \xi_o)\ \hat{e}(k_{1mz})\ e^{-ik_{1mz}z_o}]\\ \qquad\qquad + \\ [C_1(k_\perp,\ \xi)\ \hat{h}(k_{1mz})\ e^{ik_{1mz}z} + C_2(k_\perp,\ \xi)\ \hat{h}(-k_{1mz})\ e^{-ik_{1mz}z}]\\ [D_1(k_\perp,\ \xi_o)\ \hat{h}(-k_{1mz})\ e^{ik_{1mz}z_o} + D_2(k_\perp,\ \xi_o)\ \hat{h}(k_{1mz})\ e^{-ik_{1mz}z_o}]\end{cases}$$

$$(z > z_o) \tag{8.10b}$$

$$\bar{\bar{g}}^{<}_{11mo}(\bar{k}_\perp,\ z,\ \xi;\ z_o,\ \xi_o) =$$

$$\left\{\begin{array}{l} [B_1(k_\perp, \xi)\, \hat{e}(k_{1mz})\, e^{ik_{1mz}z} + B_2(k_\perp, \xi)\, \hat{e}(-k_{1mz})\, e^{-ik_{1mz}z}] \\ [A_1(k_\perp, \xi_o)\, \hat{e}(-k_{1mz})\, e^{ik_{1mz}z_o} + A_2(k_\perp, \xi_o)\, \hat{e}(k_{1mz})\, e^{-ik_{1mz}z_o}] \\ \\ \qquad\qquad + \\ \\ [D_1(k_\perp, \xi)\, \hat{h}(k_{1mz})\, e^{ik_{1mz}z} + D_2(k_\perp, \xi)\, \hat{h}(-k_{1mz})\, e^{-ik_{1mz}z}] \\ [C_1(k_\perp, \xi_o)\, \hat{h}(-k_{1mz})\, e^{ik_{1mz}z_o} + C_2(k_\perp, \xi_o)\, \hat{h}(k_{1mz})\, e^{-ik_{1mz}z_o}] \end{array}\right.$$

$$(z < z_o) \qquad\qquad (8.10c)$$

The unit vectors $\hat{e}(k_{1mz})$ and $\hat{h}(k_{1mz})$ point in the directions of the electric field for the TE and TM polarized waves, respectively, we next substitute (8.10a)-(8.10c) into (8.7) and eliminate secular terms independently for each polarization. This yields four differential equations per polarization in the variable, ξ, for A_1, A_2, B_1, B_2 and for C_1, C_2, D_1, D_2. After careful manipulations, the differential equations may be cast into the following compact form:

$$0 = -2ipk_{1mz} \frac{\partial}{\partial\xi} \bar{\alpha}_p^{\,T}(\bar{k}_\perp, \xi) + (2\pi i) \sum_m \sum_s \sum_{T'} \left\{ \int d^2k_\perp{}' \right.$$

$$\frac{\text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp, \alpha_n^-)}{i[-sk_{1mz}' + pk_{1mz} + \alpha_n^-]}\, \bar{\alpha}_s^{\,T'}(\bar{k}_\perp{}', \xi)\, [\bar{\beta}_{-s}^{\,T'}(\bar{k}_\perp{}', \xi)$$

$$\cdot\ \bar{\alpha}_p{}^T(\bar{k}_\perp,\ \xi)] + \int d^2k_\perp{}'\ \frac{\text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp,\ \alpha_n{}^+)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n{}^+]}$$

$$\left.\bar{\beta}_s^{T'}(-\bar{k}_\perp{}',\ \xi)[\bar{\alpha}_{-s}^{T'}(-\bar{k}_\perp{}',\ \xi)\ \cdot\ \bar{\alpha}_p{}^T(\bar{k}_\perp,\ \xi)]\right\}$$

$$+ \frac{i2\pi}{k_{1m}^2}\sum_n \int d^2k_\perp{}'\ \text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp,\ \alpha_n{}^+)\ \hat{z}\hat{z}\ \cdot\ \bar{\alpha}_p{}^T(\bar{k}_\perp,\ \xi)$$

(8.11a)

$$0 = -2ipk_{1mz}\frac{\partial}{\partial\xi}\bar{\beta}_p{}^T(-\bar{k}_\perp,\ \xi) + (2\pi i)\sum_n\sum_s\sum_{T'}\int d^2k_\perp{}'$$

$$\frac{\text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp,\ \alpha_n{}^-)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n{}^-]}\ \bar{\alpha}_s^{T'}(\bar{k}_\perp{}',\ \xi)[\bar{\beta}_{-s}^{T'}(\bar{k}_\perp{}',\ \xi)$$

$$\cdot\ \bar{\beta}_p{}^T(-\bar{k}_\perp,\ \xi)] + \int d^2k_\perp{}'\ \frac{\text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp,\ \alpha_n{}^+)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n{}^+]}$$

$$\left.\bar{\beta}_s^{T'}(-\bar{k}_\perp{}',\ \xi)[\bar{\alpha}_{-s}^{T'}(-\bar{k}_\perp{}',\ \xi)\ \cdot\ \bar{\beta}_p{}^T(-\bar{k}_\perp,\ \xi)]\right\}$$

$$+ \frac{i2\pi}{k_{1m}^2}\sum_n \int d^2k_\perp{}'\ \text{Res}\ \Phi(\bar{k}_\perp{}' - \bar{k}_\perp,\ \alpha_n{}^+)\ \hat{z}\hat{z}\ \cdot\ \bar{\beta}_p{}^T(\bar{k}_\perp,\ \xi)$$

(8.11b)

where

$$\bar{\alpha}_1^{T}(\bar{k}_\perp, \xi) = \begin{cases} A_1(k_\perp, \xi)\ \hat{e}(k_{1mz}) & T = TE \qquad (8.12a) \\ C_1(k_\perp, \xi)\ \hat{h}(k_{1mz}) & T = TM \qquad (8.12b) \end{cases}$$

$$\bar{\alpha}_{-1}^{T}(\bar{k}_\perp, \xi) = \begin{cases} A_2(k_\perp, \xi)\ \hat{e}(-k_{1mz}) & T = TE \qquad (8.13a) \\ C_2(k_\perp, \xi)\ \hat{h}(-k_{1mz}) & T = TM \qquad (8.13b) \end{cases}$$

$$\bar{\beta}_1^{T}(\bar{k}_\perp, \xi) = \begin{cases} B_1(k_\perp, \xi)\ \hat{e}(-k_{1mz}) & T = TE \qquad (8.14a) \\ D_1(k_\perp, \xi)\ \hat{h}(-k_{1mz}) & T = TM \qquad (8.14b) \end{cases}$$

$$\bar{\beta}_{-1}^{T}(\bar{k}_\perp, \xi) = \begin{cases} B_2(k_\perp, \xi)\ \hat{e}(k_{1mz}) & T = TE \qquad (8.15a) \\ D_2(k_\perp, \xi)\ \hat{h}(k_{1mz}) & T = TM \qquad (8.15b) \end{cases}$$

In (8.11a) and (8.11b) the indices p and s take the values of $+1$ or -1, whereas the superscripts T and T' stand for either TE or TM. Moreover we have performed the α_z integration using residue calculus, with α_n^+ and α_n^- denoting, respectively, the n-th poles of $\Phi(\bar{\alpha}_\perp, \alpha_z)$ in the upper and lower halves of the complex α_z plane. In the case of laminar structures, for which $k_{1m}\ell_\rho \to \infty$, equations (8.11a) and (8.11b) are different. Treatment of this case is deferred to Section 8.5.

Solutions to (8.11a) and (8.11b) have the form:

$$\bar{\alpha}_p^T(\bar{k}_\perp, \xi) = \bar{a}_p^T(\bar{k}_\perp)\, e^{ip\lambda^T(\bar{k}_\perp)\xi} \tag{8.16a}$$

$$\bar{\beta}_p^T(\bar{k}_\perp, \xi) = \bar{b}_p^T(\bar{k}_\perp)\, e^{ip\lambda^T(\bar{k}_\perp)\xi} \tag{8.16b}$$

where

$$\bar{a}_1^T(\bar{k}_\perp) \equiv \begin{cases} \hat{e}(k_{1mz}) & T = TE \quad (8.17a) \\ \hat{h}(k_{1mz}) & T = TM \quad (8.17b) \end{cases}$$

$$\bar{a}_{-1}^T(\bar{k}_\perp) \equiv \begin{cases} A_2(k_\perp)\ \hat{e}(-k_{1mz}) & T = TE \quad (8.18a) \\ C_2(k_\perp)\ \hat{h}(-k_{1mz}) & T = TM \quad (8.18b) \end{cases}$$

$$\bar{b}_1^T(\bar{k}_\perp) \equiv \begin{cases} B_1(k_\perp)\ \hat{e}(-k_{1mz}) & T = TE \qquad (8.19a) \\ D_1(k_\perp)\ \hat{h}(-k_{1mz}) & T = TM \qquad (8.19b) \end{cases}$$

$$\bar{b}_1^T(\bar{k}_\perp) \equiv \begin{cases} B_2(k_\perp)\ \hat{e}(k_{1mz}) & T = TE \qquad (8.20a) \\ D_2(k_\perp)\ \hat{h}(k_{1mz}) & T = TM \qquad (8.20b) \end{cases}$$

In (8.17a) through (8.20b), we have redefined the coefficients in the right hand side of (8.12a)-(8.15b). First note that the ξ-dependence of A_1, A_2, B_1, B_2, C_1, C_2, D_1 and D_2 is exponential in behavior and thus may be combined with their multiplying exponentials, $e^{\pm ik_{1mz}z}$ which appear in the mean dyadic Green's function (8.10a). We then factor out the amplitudes of A_1 and C_1 which depend only on $k_\perp$, and combine these with the other amplitudes in (8.10a). This defines a new set of coefficients which are given in (8.17a)-(8.20b) and are determined through the boundary conditions on the zeroth order mean dyadic Green's function. Substituting (8.16a) and (8.16b) into (8.11a) and (8.11b), we obtain

$$0 = 2\lambda^T(\bar{k}_\perp)\ k_{1mz}\ \bar{a}_p^T(k_\perp) + (2\pi i) \sum_n \sum_s \sum_{T'} \left\{ \int d^2k_\perp' \right.$$

$$\frac{\text{Res }\Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^-)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n^-]} \bar{a}_s^{T'}(\bar{k}_\perp')[\bar{b}_{-s}^{T'}(\bar{k}_\perp')$$

$$\cdot \bar{a}_p^{\,T}(\bar{k}_\perp)] + \int d^2k_\perp' \frac{\text{Res }\Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n^+]}$$

$$\left.\bar{b}_s^{T'}(-\bar{k}_\perp')\ [\bar{a}_{-s}^{T'}(-\bar{k}_\perp') \cdot \bar{a}_p^{\,T}(\bar{k}_\perp)]\right\}$$

$$+ \frac{i2\pi}{k_{1m}^2} \sum_n \int d^2k_\perp' \text{ Res }\Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)\ \hat{z}\hat{z}$$

$$\cdot \bar{a}_p^{\,T}(\bar{k}_\perp, \xi) \qquad (8.21a)$$

$$0 = 2\lambda^T(-\bar{k}_\perp)\ k_{1mz}\ \bar{b}_p^{\,T}(-\bar{k}_\perp) + (2\pi i) \sum_n \sum_s \sum_{T'} \left\{\int d^2k_\perp'\right.$$

$$\frac{\text{Res }\Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^-)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n^-]} \bar{a}_s^{T'}(\bar{k}_\perp')[\bar{b}_{-s}^{T'}(\bar{k}_\perp')$$

$$\cdot \bar{b}_p^{\,T}(-k_\perp)] + \int d^2k_\perp' \frac{\text{Res }\Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)}{i[-sk'_{1mz} + pk_{1mz} + \alpha_n^+]}$$

$$\bar{b}_s^{T'}(-\bar{k}_\perp')[\bar{a}_{-s}^{T'}(-\bar{k}_\perp') \cdot \bar{b}_p{}^T(-\bar{k}_\perp)]$$

$$+ \frac{i2\pi}{k_{1m}^2} \sum_n \int d^2k_\perp' \text{ Res } \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n{}^+) \hat{z}\hat{z} \cdot \bar{b}_p{}^T(-\bar{k}_\perp)$$

(8.21b)

It is to be noted that equations (8.21a) and (8.21b) are consistent, yielding the same result for $\lambda^T(\bar{k}_\perp)$. This may be seen as follows. In (8.21a) let $p \to -p$ and $s \to -s$ then dot with the vector $\bar{b}_p{}^T(\bar{k}_\perp)$. In (8.21b), let $\bar{k}_\perp \to -\bar{k}_\perp$, then change integration variables as $\bar{k}_\perp' \to -\bar{k}_\perp'$, and finally, dot with the vector $\bar{a}^T_{-p}(\bar{k}_\perp)$. We also make use of the following properties of the spectral density, which are valid for a large class of physically interesting correlation functions:

$$\Phi(\bar{\alpha}_\perp, \alpha_z) = \Phi(-\bar{\alpha}_\perp, \alpha_z) \tag{8.22a}$$

$$\text{Res } \Phi(\bar{\alpha}_\perp, \alpha_n{}^-) = - \text{ Res } \Phi(\bar{\alpha}_\perp, \alpha_n{}^+) \tag{8.22b}$$

$$\alpha_n{}^- = -\alpha_n{}^+. \tag{8.22c}$$

Upon performing the described operations on (8.21a) and (8.21b), we find the consistent result:

$$\lambda^{T}(\bar{k}_{\perp}) = -\frac{\pi}{k_{1mz}[\bar{a}_{-p}^{T}(\bar{k}_{\perp}) \cdot \bar{b}_{p}^{\;T}(\bar{k}_{\perp})]} \sum_{n} \sum_{s} \sum_{T'} \int d^{2}k_{\perp}'$$

$$\text{Res } \Phi(\bar{k}_{\perp}' - k_{\perp}, \alpha_{n}^{\;+})$$

$$\frac{[\bar{a}_{-p}^{T}(\bar{k}_{\perp}) \cdot \bar{a}_{s}^{T'}(\bar{k}_{\perp}')][\bar{b}_{-s}^{T'}(-\bar{k}_{\perp}') \cdot \bar{b}_{p}^{\;T}(\bar{k}_{\perp})]}{(sk_{1mz}' - pk_{1mz} + \alpha_{n}^{\;+})}$$

$$\left. - \frac{[\bar{a}_{-p}^{T}(\bar{k}_{\perp}) \cdot \bar{b}_{s}^{T'}(\bar{k}_{\perp}')][\bar{a}_{-s}^{T'}(\bar{k}_{\perp}') \cdot \bar{b}_{p}^{\;T}(\bar{k}_{\perp})]}{(sk_{1mz}' - pk_{1mz} - \alpha_{n}^{\;+})} \right\}$$

$$- \frac{i\pi}{k_{1mz}k_{1m}^{2}} \sum_{n} \int d^{2}k_{\perp}' \text{ Res } \Phi(\bar{k}_{\perp}' - \bar{k}_{\perp}, \alpha_{n}^{\;+})$$

$$\frac{[\hat{z} \cdot \bar{a}_{-p}^{T}(\bar{k}_{\perp})][\hat{z} \cdot \bar{b}_{p}^{\;T}(\bar{k}_{\perp})]}{(sk_{1mz}' - pk_{1mz} - \alpha_{n}^{\;+})} . \qquad (8.23)$$

Utilizing (8.17a)-(8.20b), we evaluate (8.23) for the cases $T = TE$ and $T = TM$. The results are:

$$\lambda^{TE}(\bar{k}_\perp) = -\frac{2\pi}{k_{1mz}} \sum_n \int d^2k_\perp' \ \text{Res}\ \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)$$

$$\left\{ \frac{B_2(k_\perp') \cos^2(\phi - \phi')(k'_{1mz} + \alpha_n^+)}{(k'_{1mz} + \alpha_n^+)^2 - k^2_{1mz}} \right.$$

$$- \frac{A_2(k_\perp')\ B_1(k_\perp') \cos^2(\phi - \phi')(k'_{1mz} - \alpha_n^+)}{(k'_{1mz} - \alpha_n^+)^2 - k^2_{1mz}}$$

$$+ \frac{D_2(k_\perp') \frac{k^2_{1mz}}{k^2_{1m}} \sin^2(\phi - \phi')(k'_{1mz} + \alpha_n^+)}{(k'_{1mz} + \alpha_n^+)^2 - k^2_{1mz}}$$

$$- \frac{C_2(k_\perp')\ D_1(k_\perp') \frac{k'^2_{1mz}}{k^2_{1m}} \sin^2(\phi - \phi')(k'_{1mz} - \alpha_n^+)}{(k'_{1mz} - \alpha_n^+)^2 - k^2_{1mz}} \qquad (8.24a)$$

$$\lambda^{TM}(\bar{k}_\perp) = -\frac{2\pi}{k_{1mz}} \sum_n \int d^2k_\perp' \ \text{Res}\ \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)$$

$$\left\{ \frac{B_2(k_\perp') \frac{k^2_{1mz}}{k^4_{1m}} (k'_{1mz} + \alpha_n^+) \sin^2(\phi - \phi')}{(k'_{1mz} + \alpha_n^+)^2 - k^2_{1mz}} \right.$$

$$- \frac{A_2(k_\perp')\ B_1(k_\perp') \frac{k^2_{1mz}}{k^2_{1m}} (k'_{1mz} - \alpha_n^+) \sin^2(\phi - \phi')}{(k'_{1mz} - \alpha_n^+)^2 - k^2_{1mz}}$$

$$+ \frac{\dfrac{D_2(k_\perp')}{k_{1m}^4}\,(k_{1mz}'^2 k_{1mz}^2 \cos^2(\phi - \phi') + k_\perp^2 k_\perp'^2)}{(k_{1mz}' + \alpha_n^+)^2 - k_{1mz}^2}$$

$$\cdot\ (k_{1mz}' + \alpha_n^+) + \frac{\dfrac{2D_2(k_\perp')}{k_{1m}^4}\, k_{1mz}^2 k_{1mz}' k_\perp k_\perp' \cos(\phi - \phi')}{(k_{1mz}' + \alpha_n^+)^2 - k_{1mz}^2}$$

$$- \frac{\dfrac{C_2(k_\perp')D_1(k_\perp')}{k_{1m}^4}\,(k_{1mz}' - \alpha_n^+)(k_{1mz}'^2 k_{1mz}^2 \cos^2(\phi - \phi') + k_\perp^2 k_\perp'^2)}{(k_{1mz}' - \alpha_n^+)^2 - k_{1mz}^2}$$

$$\left. + \frac{2C_2(k_\perp')\ D_1(k_\perp')\ \dfrac{k_{1mz}^2}{k_{1m}^4}\, k_{1mz}' k_\perp k_\perp' \cos(\phi - \phi')}{(k_{1mz}' - \alpha_n^+)^2 - k_{1mz}^2} \right\}$$

$$- \frac{i\pi}{k_{1mz}} \frac{k_\perp^{\ 2}}{k_{1m}^4} \sum_n \int d^2k_\perp' \ \mathrm{Res}\ \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+). \qquad (8.24b)$$

Here, ϕ and ϕ' are the azimuthal angles subtended by $\bar{k}_\perp$ and $\bar{k}_\perp'$, respectively. The coefficients, A_2, B_1, B_2, C_2, D_1 and D_2 must now be determined by imposing the boundary conditions on the zeroth order mean dyadic Green's function.

The Zeroth Order Mean Dyadic Green's Function

Combining Equations (8.16a)-(8.20b), together with (8.10b) and (8.10c), the zeroth order mean dyadic Green's function may be written in the form:

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = \frac{1}{(2\pi)^2} \int d^2k_\perp \; \{[\hat{e}(k_{1mz})\, e^{i(k_{1mz} + \lambda^{TE})z}$$

$$+ A_2(k_\perp)\, \hat{e}(-k_{1mz})\, e^{-i(k_{1mz} + \lambda^{TE})z}]$$

$$[B_1(k_\perp)\, \hat{e}(-k_{1mz})\, e^{i(k_{1mz} + \lambda^{TE})z_o}$$

$$+ B_2(k_\perp)\, \hat{e}(k_{1mz})\, e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+$$

$$[\hat{h}(k_{1mz})\, e^{i(k_{1mz} + \lambda^{TM})z}$$

$$+ C_2(k_\perp)\, \hat{h}(-k_{1mz})\, e^{-i(k_{1mz} + \lambda^{TM})z}]$$

$$[D_1(k_\perp)\, \hat{h}(-k_{1mz})\, e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$+ D_2(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z_o}]\}\ e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})}$$

$$(z > z_o) \qquad (8.25a)$$

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = \frac{1}{(2\pi)^2} \int d^2k_\perp\ \{[B_1(k\ )\ \hat{e}(k_{1mz})\ e^{i(k_{1mz} + \lambda^{TE})z}$$

$$+ B_2(k_\perp)\ \hat{e}(-k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TE})z}]$$

$$[\hat{e}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TE})z_o}$$

$$+ A_2(k_\perp)\ \hat{e}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+$$

$$[D_1(k_\perp)\ \hat{h}(k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM})z}$$

$$+ D_2(k_\perp)\ \hat{h}(-k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z}]$$

$$[\hat{h}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$+ C_2(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z_o]\}}$$

$$e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \qquad (z < z_o). \tag{8.25b}$$

The boundary conditions which the zeroth order mean dyadic Green's function must satisfy are:

At z = 0:

$$\hat{z} \times \bar{\bar{G}}_{01mo}(\bar{r}, \bar{r}_o) = \hat{z} \times \bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) \tag{8.26a}$$

$$\hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{01mo}(\bar{r}, \bar{r}_o) = \hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o). \tag{8.26b}$$

At $z = -d_1$:

$$\hat{z} \times \bar{\bar{G}}_{21mo}(\bar{r}, \bar{r}_o) = \hat{z} \times \bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) \tag{8.27a}$$

$$\hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{21mo}(\bar{r}, \bar{r}_o) = \hat{z} \times \bar{\nabla} \times \bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o). \tag{8.27b}$$

From (8.6) it follows that

$$\hat{k}_\perp \cdot \left[\lim_{z \to z_o^+} \frac{\partial \bar{\bar{g}}_{11mo}}{\partial z} - \lim_{z \to z_o^-} \frac{\partial \bar{\bar{g}}_{11mo}}{\partial z} \right] \cdot \hat{k}_\perp - ik_\perp \hat{z}$$

$$\cdot \left[\lim_{z \to z_o^+} \bar{\bar{g}}_{11mo} - \lim_{z \to z_o^-} \bar{\bar{g}}_{11mo} \right] \cdot \hat{k}_\perp = -1 \qquad (8.28a)$$

$$\hat{e}(k_{1mz}) \cdot \left[\lim_{z \to z_o^+} \frac{\partial \bar{\bar{g}}_{11mo}}{\partial z} - \lim_{z \to z_o^-} \frac{\partial \bar{\bar{g}}_{11mo}}{\partial z} \right]$$

$$\cdot \hat{e}(k_{1mz}) = -1 \qquad (8.28b)$$

where $\hat{k} \equiv \bar{k}/k$. In region 0, the Fourier transformed mean dyadic Green's function assumes the following form in order to match the boundary conditions at $z = 0$:

$$\bar{\bar{g}}_{01mo}(\bar{k}_\perp, z, z_o) = \Gamma_1 \hat{e}(k_{oz}) e^{ik_{oz}z} [B_1(k_\perp) \hat{e}(-k_{1mz})$$

$$e^{i(k_{1mz} + \lambda^{TE})z_o} + B_2(k_\perp) \hat{e}(k_{1mz}) e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+$$

$$\Gamma_2 \hat{h}(k_{oz}) e^{ik_{oz}z} [D_1(k_\perp) \hat{h}(-k_{1mz}) e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$+ D_2(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM}) z_o}] . \tag{8.29}$$

Similarly, in Region 2, the mean dyadic Green's function is taken to be:

$$\bar{\bar{g}}_{21mo}(\bar{k}_\perp, z, z_o) = \Gamma_3\ \hat{e}(-k_{2z})\ e^{-ik_{2z}z}\ [\hat{e}(-k_{1mz})$$

$$e^{i(k_{1mz} + \lambda^{TE}) z_o} + A_2(k_\perp)\ \hat{e}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TE}) z_o}]$$

$$+$$

$$\Gamma_4\ \hat{h}(-k_{2z})\ e^{-ik_{2z}z}\ [\hat{h}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM}) z_o}$$

$$+ C_2(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM}) z_o}] . \tag{8.30}$$

Applying boundary conditions (8.26a)-(8.28b) to (8.25a), (8.25b), (8.29) and (8.30), results in the following equations for the unknown coefficients contained in (8.25a), (8.25b), (8.29) and (8.30):

$$\Gamma_1 = 1 + A_2 \tag{8.31}$$

$$-\Gamma_2 = \frac{k_o}{k_{1m}} \frac{k_{1mz}}{k_{oz}} (-1 + C_2) \tag{8.32}$$

$$\Gamma_1 = \frac{k_{1mz}}{k_{oz}} (1 - A_2) \tag{8.33}$$

$$k_o\Gamma_2 = k_{1m}(1 + C_2) \tag{8.34}$$

$$\Gamma_3 e^{ik_{2z}d_1} = B_1 e^{-iK_{1m}^{TE}d_1} + B_2 e^{iK_{1m}^{TE}d_1} \tag{8.35}$$

$$-\Gamma_4 e^{ik_{2z}d_1} = \frac{k_{1mz}}{k_{2z}} \frac{k_2}{k_{1m}} [D_1 e^{-iK_{1m}^{TM}d_1} - D_2 e^{iK_{1m}^{TM}d_1}] \tag{8.36}$$

$$-\Gamma_3 e^{ik_{2z}d_1} = \frac{k_{1mz}}{k_{2z}} [B_1 e^{-iK_{1m}^{TE}d_1} - B_2 e^{iK_{1m}^{TE}d_1}] \tag{8.37}$$

$$k_2\Gamma_4 e^{ik_{2z}d_1} = k_{1m}[D_1 e^{-iK_{1m}^{TM}d_1} + D_2 e^{iK_{1m}^{TM}d_1}] \tag{8.38}$$

$$B_2 - A_2B_1 = -\frac{1}{2ik_{1mz}} \tag{8.39}$$

$$D_2 - C_2D_1 = -\frac{1}{2ik_{1mz}} \tag{8.40}$$

where $K^T_{1m} \equiv k_{1mz} + \lambda^T$. Equations (8.31)-(8.40) are easily solved for the unknown coefficients. The resulting zeroth order mean dyadic Green's functions are:

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = - \frac{1}{(2\pi)^2} \int d^2k_\perp \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \frac{1}{2ik_{1mz}}$$

$$\left\{ \frac{1}{D_2(k_\perp)} [\hat{e}(k_{1mz}) \, e^{i(k_{1mz} + \lambda^{TE})z}\right.$$

$$+ R_{10}(k_\perp) \, \hat{e}(-k_{1mz}) \, e^{-i(k_{1mz} + \lambda^{TE})z}]$$

$$[R_{12}(k_\perp) \, e^{i2K^{TE}_{1m}d_1} \, \hat{e}(-k_{1mz}) \, e^{i(k_{1mz} + \lambda^{TE})z_o}$$

$$+ \hat{e}(k_{1mz}) \, e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+$$

$$\frac{1}{F_2(k_\perp)} [\hat{h}(k_{1mz}) \, e^{i(k_{1mz} + \lambda^{TM})z}$$

$$+ S_{10}(k_\perp) \, \hat{h}(-k_{1mz}) \, e^{-i(k_{1mz} + \lambda^{TM})z}]$$

$$[S_{12}(k_\perp)\ e^{i2K_{1m}^{TM}d_1}\ \hat{h}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$\left. + \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z_o}] \right\} \qquad (z > z_o) \qquad (8.41)$$

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2} \int d^2k_\perp\ e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})}\ \frac{1}{2ik_{1mz}}$$

$$\left\{ \frac{1}{D_2(k_\perp)}\ [R_{12}(k_\perp)\ e^{i2K_{1m}^{TE}d_1}\ \hat{e}(k_{1mz})\ e^{i(k_{1mz} + \lambda^{TE})z} \right.$$

$$+ \hat{e}(-k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TE})z}][\hat{e}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TE})z_o}$$

$$+ R_{10}(k_\perp)\ \hat{e}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+$$

$$\frac{1}{F_2(k_\perp)}\ [S_{12}(k_\perp)\ e^{i2K_{1m}^{TM}d_1}\ \hat{h}(k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM})z}$$

$$+ \hat{h}(-k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z}][\hat{h}(-k_{1mz})\ e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$\left. + S_{10}(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda^{TM})z_o}] \right\} \qquad (z < z_o) \qquad (8.42)$$

$$\bar{\bar{G}}_{01mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2} \int d^2k_\perp \; e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \frac{1}{2ik_{1mz}} e^{ik_{oz}z}$$

$$\left\{ \frac{X_{10}(k_\perp)}{D_2(k_\perp)} \hat{e}(k_{oz}) [R_{12}(k\;) \; e^{i2K_{1m}^{TE}d_1} \hat{e}(-k_{1mz}) \; e^{i(k_{1mz} + \lambda^{TE})z_o} \right.$$

$$+ \hat{e}(k_{1mz}) \; e^{-i(k_{1mz} + \lambda^{TE})z_o}] + \frac{k_{1m}}{k_o} \frac{Y_{10}(k_\perp)}{F_2(k_\perp)} \hat{h}(k_{oz})$$

$$[S_{12}(k_\perp) \; e^{i2K_{1m}^{TM}d_1} \hat{h}(-k_{1mz}) \; e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$\left. + \hat{h}(k_{1mz}) \; e^{-i(k_{1mz} + \lambda^{TM})z_o}] \right\} \tag{8.43}$$

$$\bar{\bar{G}}_{21mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2} \int d^2k_\perp \; e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \frac{e^{-ik_{2z}z}}{2ik_{1mz}}$$

$$\left\{ \frac{X_{12}(k_\perp)}{D_2(k_\perp)} \hat{e}(-k_{2z}) [\hat{e}(-k_{1mz}) \; e^{i(k_{1mz} + \lambda^{TE})z_o} \right.$$

$$+ R_{10}(k_\perp) \; \hat{e}(k_{1mz}) \; e^{-i(k_{1mz} + \lambda^{TE})z_o}]$$

$$+ \frac{k_{1m}}{k_2} \frac{Y_{12}(k_\perp)}{F_2(k_\perp)} \hat{h}(-k_{2z}) [\hat{h}(-k_{1mz}) \; e^{i(k_{1mz} + \lambda^{TM})z_o}$$

$$\left. + S_{10}(k_\perp) \; \hat{h}(k_{1mz}) \; e^{-i(k_{1mz} + \lambda^{TM})z_o}] \right\} \tag{8.44}$$

where:

$$\lambda^{TE}(\bar{k}_\perp) = \frac{\pi}{ik_{1mz}} \sum_n \int d^2k_\perp' \, \text{Res} \, \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)$$

$$\left\{ \frac{\cos^2(\phi - \phi')}{k_{1mz}' \, D_2(k_\perp')} \left[\frac{(k_{1mz}' + \alpha_n^+)}{(k_{1mz}' + \alpha_n^+)^2 - k_{1mz}^2} \right. \right.$$

$$\left. - \frac{R_{10}(k_\perp') \, R_{12}(k_\perp') \, e^{i2K_{1m}'^{TE} d_1} (k_{1mz}' - \alpha_n^+)}{(k_{1mz}' - \alpha_n^+)^2 - k_{1mz}^2} \right]$$

$$+ \frac{k_{1mz}' \sin^2(\phi - \phi')}{k_{1m} \, F_2(k_\perp')} \left[\frac{(k_{1mz}' + \alpha_n^+)}{(k_{1mz}' + \alpha_n^+)^2 - k_{1mz}^2} \right.$$

$$\left. \left. - \frac{S_{10}(k_\perp') \, S_{12}(k_\perp') \, e^{i2K_{1m}'^{TM} d_1} (k_{1mz}' - \alpha_n^+)}{(k_{1mz}' - \alpha_n^+)^2 - k_{1mz}^2} \right] \right\} \qquad (8.45)$$

$$\lambda^{TM}(\bar{k}_\perp) = \frac{\pi}{ik_{1mz}} \sum_n \int d^2k_\perp' \, \text{Res} \, \Phi(\bar{k}_\perp' - \bar{k}_\perp, \alpha_n^+)$$

$$\left\{ \frac{k_{1mz}^2 \sin^2(\phi - \phi')}{k_{1m}^2 \, k_{1mz}' \, D_2(k_\perp')} \left[\frac{(k_{1mz}' + \alpha_n^+)}{(k_{1mz}' + \alpha_n^+)^2 - k_{1mz}^2} \right. \right.$$

$$\left. - \frac{R_{10}(k_\perp') \, R_{12}(k_\perp') \, e^{i2K_{1m}'^{TE} d_1} (k_{1mz}' - \alpha_n^+)}{(k_{1mz}' - \alpha_n^+)^2 - k_{1mz}^2} \right]$$

$$+ \frac{1}{k'_{1mz}\ k^4_{1m}\ F_2(k_\perp')}$$

$$\left[\frac{(k'^2_{1mz}k^2_{1mz}\cos^2(\phi - \phi') + k^2_\perp k'^2_\perp)(k'_{1mz} + \alpha_n^+) + 2k^2_{1mz}k'_{1mz}k_\perp k'_\perp\cos(\phi - \phi')}{(k'_{1mz} + \alpha_n^+)^2 - k^2_{1mz}}\right]$$

$$+ \frac{S_{10}(k_\perp')\ S_{12}(k_\perp')\ e^{i2K'^{TM}_{1m}d_1}}{k'_{1mz}\ k^4_{1m}\ F_2(k_\perp')}$$

$$\left.\left[\frac{2k^2_{1mz}k'_{1mz}k_\perp k'_\perp\cos(\phi - \phi') - (k'_{1mz} - {}_n^+)(k'^2_{1mz}k^2_{1mz}\cos^2(\phi - \phi') + k^2_\perp k'^2_\perp)}{(k'_{1mz} - \alpha_n^+)^2 - k^2_{1mz}}\right]\right\}$$

$$- \frac{i\pi}{k_{1mz}}\frac{k_\perp^{\ 2}}{k^4_{1m}}\sum_n \int d^2k_\perp'\ \text{Res}\ \Phi(\bar{k}_\perp' - \bar{k}_\perp,\ \alpha_n^{\ +}) \tag{8.46}$$

and

$$R_{ij}(k_\perp) \equiv \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}} \tag{8.47a}$$

$$S_{ij}(k_\perp) \equiv \frac{\varepsilon_j k_{iz} - \varepsilon_i k_{jz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}} \tag{8.47b}$$

$$X_{ij}(k_\perp) = 1 + R_{ij}(k_\perp) \tag{8.48a}$$

$$Y_{ij}(k_\perp) = 1 + S_{ij}(k_\perp) \tag{8.48b}$$

$$D_2(k_\perp) = 1 + R_{01}R_{12}\, e^{i2K_{1m}^{TE}d_1} \tag{8.49a}$$

$$F_2(k_\perp) \equiv 1 + S_{01}S_{12}\, e^{i2K_{1m}^{TM}d_1} \tag{8.49b}$$

$$K_{1m}'^{T} \equiv k_{1mz}' + \lambda^T(\bar{k}_\perp') \qquad T = TE,\ TM. \tag{8.50}$$

In (8.47a)-(8.49b) i and j take the values 0, 1 and 2, where k_{1z} and ε_1 are to be interpreted as k_{1mz} and ε_{1m}, respectively. Equations (8.41)-(8.49b) represent the complete zeroth order solution for the mean dyadic Green's function of a two layer random medium in the non-linear approximation to Dyson's equation.

8.2 Effective Propagation Constants for TE and TM Modes

The two variable expansion technique has been applied to Dyson's equation in the non-linear approximation to obtain the zeroth order mean dyadic Green's function for a two layer random medium. The results as given by (8.41)-(8.49b) indicate the anisotropic effect of the random medium to vector fields resulting in effective propagation constants which in general are different for the characteristic TE and TM modes. Physically, this is not surprising if we consider wave propagation within a random medium with distinctive vertical and lateral correlation lengths. Waves with vertical polarization will experience an effective decay differing from that of waves with horizontal polarization. However, for correlation functions with azimuthal symmetry, and for wave propagation along the z-direction, we expect idential propagation characteristics for both the TE and TM modes. This observation is substantiated by the results given in (8.45) and (8.46) by letting $\bar{k}_\perp \to 0$ in which case $\lambda^{TM} = \lambda^{TE}$.

To illustrate the results given by (8.45) and (8.46), we take the spectral density to be

$$\Phi(\bar{\beta}_\perp, \beta_z) = \frac{\delta k_{1m}^4 \ell_\rho^2 \ell}{4\pi^2(1 + \beta_z^2 \ell^2)} e^{-\beta_\perp^2 \ell_\rho^2/4} \tag{8.51a}$$

which corresponds to a correlation function of the form:

$$C(\bar{r}_1 - \bar{r}_2) = \delta k_{1m}^4 \, e^{-|\bar{r}_{1\perp} - \bar{r}_{2\perp}|^2/\ell_\rho{}^2 - |z_1 - z_2|/\ell}$$

where ℓ_ρ and ℓ denote the lateral and vertical correlation lengths, respectively and δ is the variance of the fluctuations. Taking the residue of (8.51a) at the pole $\alpha_n{}^+ = \delta_{n,1}$ (i/ℓ), where $\delta_{n,1}$ is the Kronecker delta symbol, we find

$$\text{Res } \Phi(\bar{\beta}_\perp, i/\ell) = \frac{\delta k_{1m}^4 \ell_\rho{}^2}{8\pi^2 i} \, e^{-\beta_\perp{}^2 \ell_\rho{}^2/4} . \tag{8.52}$$

Substituting (8.52) into (8.45) and (8.46) yields

$$\lambda^{TE}(\bar{k}_\perp) = - \frac{\ell_\rho{}^2 \delta k_{1m}^4}{8\pi k_{1mz}} \int d^2k_\perp{}' \, e^{-k_\perp{}^2 \ell_\rho{}^2/4 - k_\perp'^2 \ell_\rho{}^2/4}$$

$$e^{\frac{k_\perp k_\perp{}' \ell_\rho{}^2}{2} \cos(\phi - \phi')} \left\{ \frac{\cos^2(\phi - \phi')}{k_{1mz}' D_2(k_\perp{}')} M_1(\bar{k}_\perp{}') \right.$$

$$\left. + \frac{k_{1mz}'}{k_{1m}^2} \frac{\sin^2(\phi - \phi')}{F_2(k_\perp{}')} M_2(\bar{k}_\perp{}') \right\} \tag{8.53a}$$

and,

$$\lambda^{TM}(\bar{k}_\perp) = -\frac{\ell_\rho^{\,2}\delta k_{1m}^4}{8\pi k_{1mz}} \int d^2k_\perp' \; e^{-k_\perp'^{\,2}\ell_\rho^{\,2}/4 \;-\; k_\perp^{\,2}\ell_\rho^{\,2}/4} \cdot$$

$$e^{\frac{k_\perp k_\perp' \ell_\rho^{\,2}}{2}\cos(\phi - \phi')} \left\{ \frac{k_{1mz}\sin^2(\phi - \phi')}{k_{1m}^2 k_{1mz}' \, D_2(\bar{k}_\perp')} M_1(\bar{k}_\perp') \right.$$

$$\left. + \frac{1}{k_{1mz}' k_{1m}^4 \, F_2(\bar{k}_\perp')} \left(M_3(\bar{k}_\perp') + M_4(\bar{k}_\perp')\right) \right\} \qquad (8.53b)$$

where

$$M_1(\bar{k}_\perp') \equiv \left[\frac{\left(k'_{1mz} + \frac{i}{\ell}\right)}{\left(k'_{1mz} + \frac{i}{\ell}\right)^2 - k_{1mz}^2} - R_{10}(k_\perp')\, R_{12}(k_\perp') \right.$$

$$\left. \frac{e^{i2k'_{1mz}d_1}\left(k'_{1mz} - \frac{i}{\ell}\right)}{\left(k'_{1mz} - \frac{i}{\ell}\right)^2 - k_{1mz}^2} \right] \qquad (8.54a)$$

$$M_2(\bar{k}_\perp') \equiv \left[\frac{\left(k'_{1mz} + \frac{i}{\ell}\right)}{\left(k'_{1mz} + \frac{i}{\ell}\right)^2 - k_{1mz}^2} - S_{10}(k_\perp')\, S_{12}(k_\perp') \right.$$

$$e^{i2k'_{1mz}d_1} \frac{\left(k'_{1mz} - \frac{i}{\ell}\right)}{\left(k'_{1mz} - \frac{i}{\ell}\right)^2 - k^2_{1mz}}\Bigg] \tag{8.54b}$$

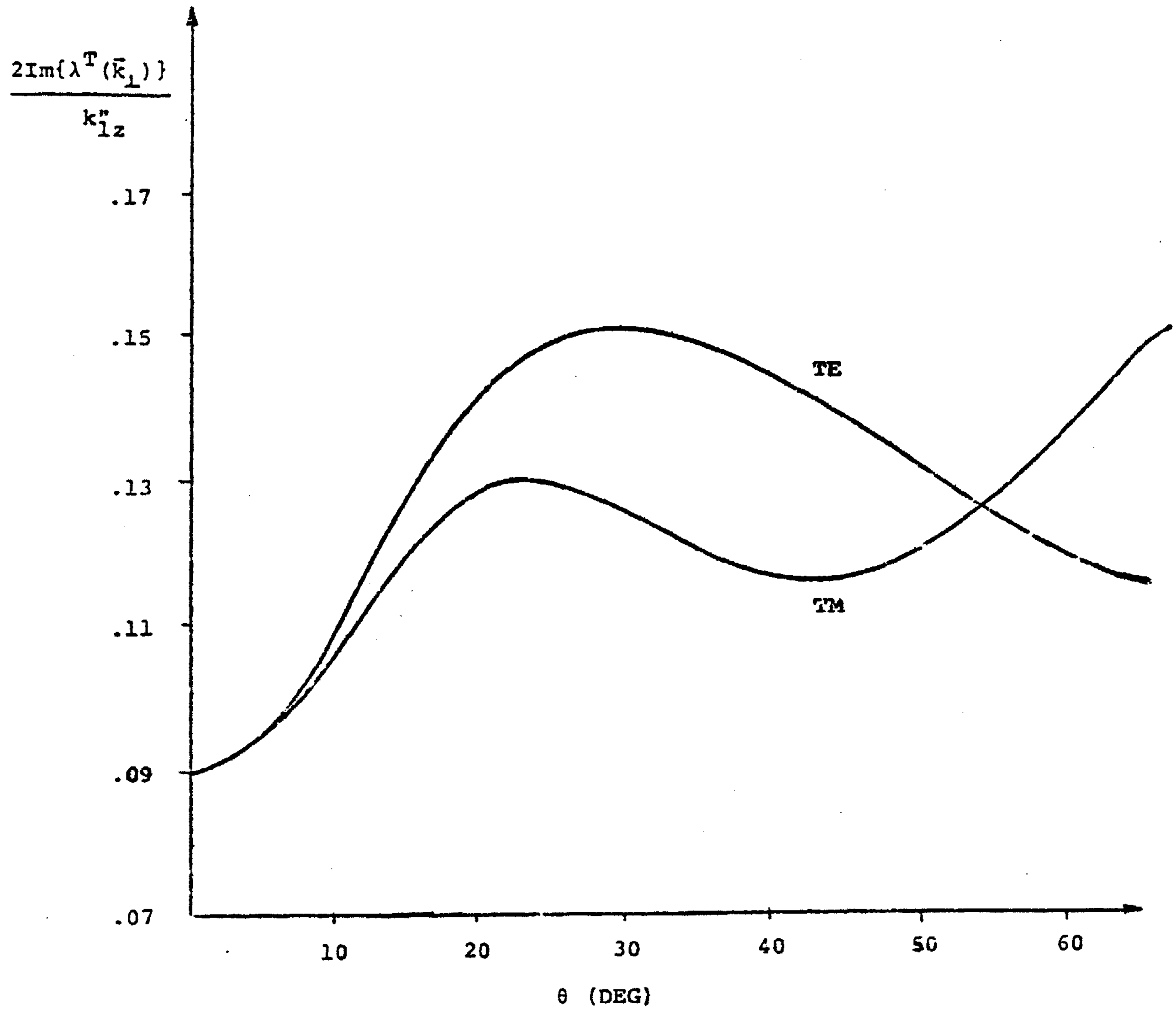
$$M_{\substack{3\\4}}(\bar{k}_\perp') = S_{10}(k_\perp')\; S_{12}(k_\perp')\; e^{i2k'_{1mz}d_1}$$

$$\left[\frac{2k^2_{1mz}k'_{1mz}k_\perp k_\perp'\cos(\phi - \phi') \pm (k'^2_{1mz}k^2_{1mz}\cos^2(\phi - \phi') + k_\perp^2 k_\perp'^2)\left(k'_{1mz} \pm \frac{i}{\ell}\right)}{\left(k'_{1mz} \pm \frac{i}{\ell}\right)^2 - k^2_{1mz}}\right] \tag{8.54c}$$

In (8.53a) and (8.53b), $\phi - \phi'$ is the angle between $\bar{k}_\perp$ and $\bar{k}_\perp'$. If we make the transformation $k_x' = k_{1m}\sin\theta'\cos\phi'$, $k_y' = k_{1m}\sin\theta'\sin\phi'$ and neglect the evanescent portion of $\bar{k}$-space, we obtain $d^2k_\perp' = k^2_{1m}\cos\theta'\sin\theta'\,d\theta' d\phi'$, where the angular integrations extend over the hemisphere defined by $0 \leq \phi' \leq 2\pi$ and $0 \leq \theta' \leq \pi/2$. The ϕ' integration may be performed directly. However, the θ' integration can only be carried out numerically. The numerical results are illustrated in Figs. 8.1, 8.2 and 8.3 where we have plotted $\Gamma^T \equiv 2\mathrm{Im}[\lambda^T(k_\perp)]/k''_{1z}$ versus angle and frequency for $T = TE$

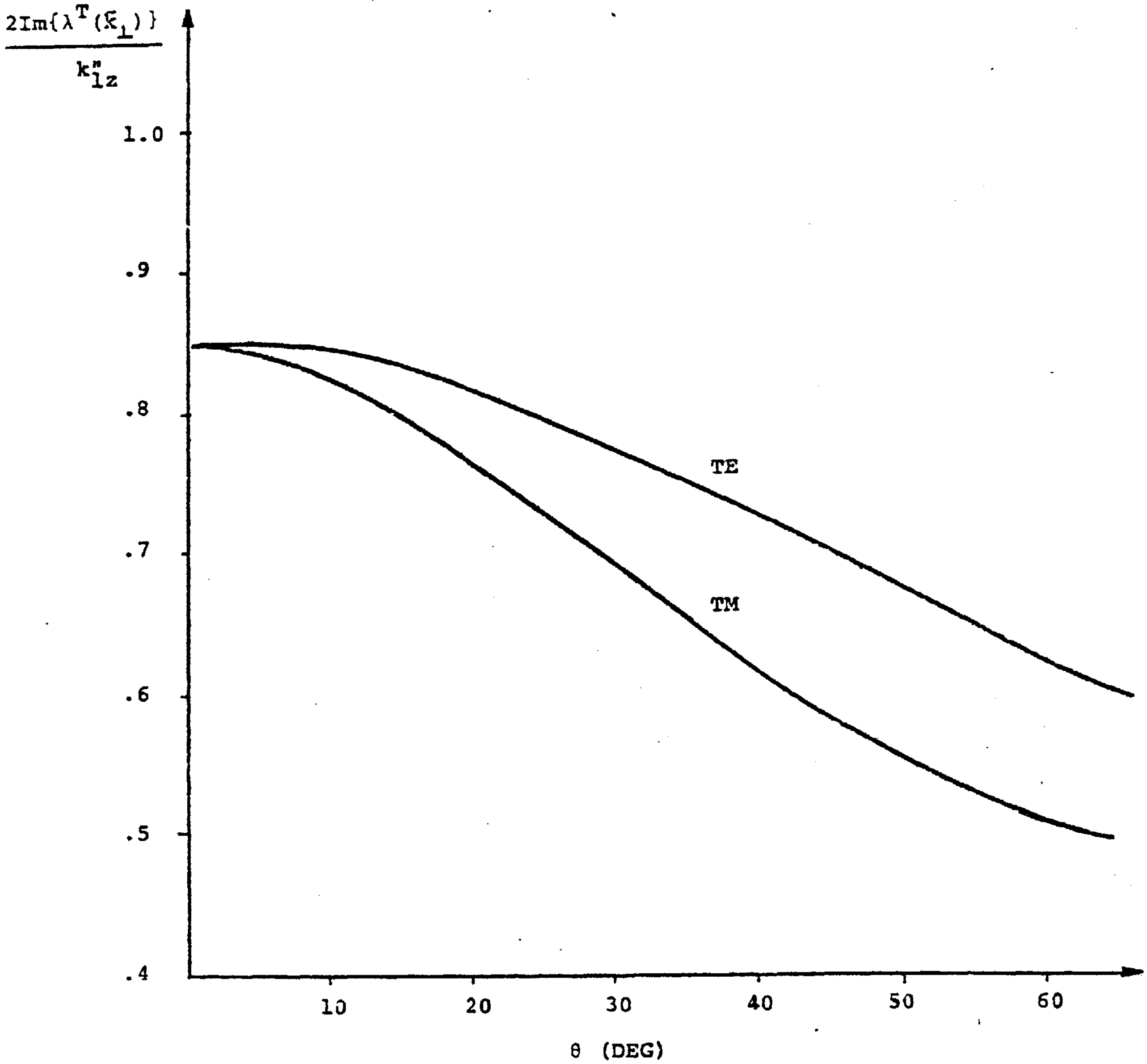
and TM waves.

In Fig. 8.1 we have plotted Γ^{T} at 10 GHz as a function of propagation angle, θ, with $\ell_{\rho} = .01$ m, $\ell = .1$ m and $\delta = .1$ for a 40 cm thick random layer. Note that at nadir $\Gamma^{TE} = \Gamma^{TM}$. This follows from the azimuthal symmetry of the correlation function used. At large angles, Γ^{TM} becomes greater than Γ^{TE}, indicating a grater scattering loss for the TM wave than for the TE wave. This is because $\ell > \ell_{\rho}$, which presents a greater scattering cross section for TM waves than for TE waves. To illustrate the case of $\ell_{\rho} > \ell$, we have plotted in Fig. 8.2 Γ^{T} at 10 GHz as a function of angle, with $\ell_{\rho} = .009$ m, $\ell = .0004$ m and $\delta = .1$ for a 40 cm thick random layer. It is interesting to note that in this case, both Γ^{TE} and Γ^{TM} decrease with angle with $\Gamma^{TE} > \Gamma^{TM}$, indicating a greater scattering loss for the TE wave than for the TM wave. In Fig. 8.3 we have plotted Γ^{T} at 20° as a function of frequency for a 40 cm thick random layer. We have taken $\ell_{\rho} = .95$ m, $\ell = .002$ m and $\delta = .0005$. Both Γ^{TE} and Γ^{TM} are seen to exhibit a broad maximum around 27 GHz, and a slow decay at higher frequencies. This behavior is due to the effect of resonant scattering, upon the propagation constant corrections, $\lambda^{TE}(\bar{k}_{\perp})$, and $\lambda^{TM}(\bar{k}_{\perp})$.



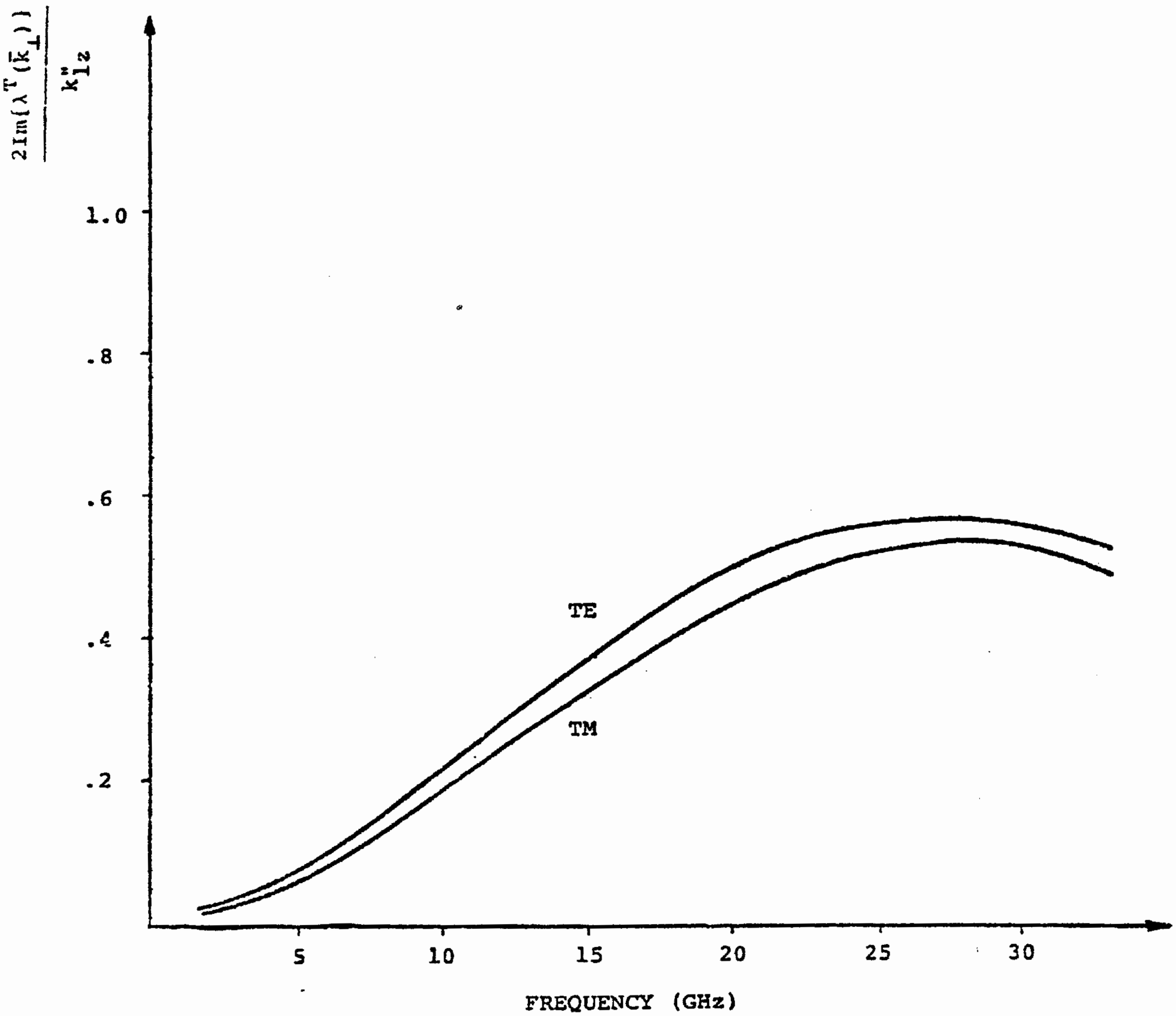
Γ^{TE} and Γ^{TM} as a function of angle at 10 GHz

Figure 8.1



Γ^{TE} and Γ^{TM} as a function of angle at 10 GHz

Figure 8.2



Γ^{TE} and Γ^{TM} as a function of frequency at 20°

Figure 8.3

8.3 Comparison with Scattering Coefficients of Radiative Transfer Theory

It is of interest to compare in the half space limit the imaginary parts of the effective propagation constants λ^{TM} and λ^{TE} as obtained from the zeroth order solution to Dyson's equation with the scattering coefficients $K_h(\theta)$ and $K_v(\theta)$ as given by the radiative transfer theory.[63] Taking the half space limit of (8.53a) and (8.53b), we obtain

$$\mathrm{Im}[\lambda^{TE}] = \frac{\delta k_{1m}^4 \ell_\rho^2 \ell}{8\pi \cos\theta} \int d\Omega'$$

$$e^{-\frac{k_{1m}^2 \ell_\rho^2}{2}(\sin^2\theta + \sin^2\theta' - 2\sin\theta\sin\theta'\cos(\phi-\phi'))}$$

$$\frac{[1 + k_{1m}^2 \ell^2(\cos^2\theta + \cos^2\theta')]}{[1 + k_{1m}^2\ell^2(\cos\theta + \cos\theta')^2][1 + k_{1m}^2\ell^2(\cos\theta - \cos\theta')^2]}$$

$$\{\cos^2(\phi-\phi') + \cos^2\theta' \sin^2(\phi-\phi')\} \tag{8.55a}$$

$$\mathrm{Im}[\lambda^{TM}] = \frac{\delta k_{1m}^4 \ell_\rho^2 \ell}{8\pi \cos\theta} \int d\Omega'$$

$$\frac{e^{-\frac{k_{1m}^2 \ell_\rho^2}{4}(\sin^2\theta + \sin^2\theta' - 2\sin\theta\sin\theta'\cos(\phi - \phi'))}}{[1 + k_{1m}^2\ell^2(\cos\theta + \cos\theta')^2][1 + k_{1m}^2\ell^2(\cos\theta - \cos\theta')^2]}$$

$$\{[1 + k_{1m}^2\ell^2(\cos^2\theta + \cos^2\theta')][\sin^2\theta \sin^2\theta'$$

$$+ \cos^2\theta \cos^2\theta' \cos^2(\phi - \phi') + \cos^2\theta \sin^2(\phi - \phi')]$$

$$+ 4k_{1m}^2\ell^2 \cos^2\theta \cos^2\theta' \sin\theta \sin\theta' \cos(\phi - \phi')\}. \qquad (8.55b)$$

The scattering coefficients as defined in radiative transfer theory are given by Tsang and Kong:[63]

$$K_h(\theta) = \frac{\delta k_{1m}^4 \pi}{2} \int d\Omega' \, [\Phi(\theta', \phi'; \theta, \phi) + \Phi(\pi - \theta', \phi'; \theta, \phi)]$$

$$[(\hat{v}' \cdot \hat{h})^2 + (\hat{h}' \cdot \hat{h})^2] \qquad (8.56a)$$

$$K_v(\theta) = \frac{\delta k_{1m}^4 \pi}{2} \int d\Omega' \, \{\Phi(\theta', \phi'; \theta, \phi)[(\hat{h}' \cdot \hat{v})^2 + (\hat{v}' \cdot \hat{v})^2]$$

$$+ \Phi(\pi - \theta', \phi'; \theta, \phi)[(\hat{h}' \cdot \hat{v})^2 + (\hat{v}' \cdot \hat{v})^2]_{\pi - \theta'}\}$$

$$(8.56b)$$

where

$$\Phi(\theta', \phi'; \theta, \phi)$$

$$= \frac{\ell_\rho^{\,2}\ell\, e^{-\frac{k_1^{\,2}\ell_\rho^{\,2}}{4}(\sin^2\theta' + \sin^2\theta - 2\sin\theta'\sin\theta\cos(\phi - \phi'))}}{4\pi^2[1 + k_{1m}^2\ell^2(\cos\theta - \cos\theta')^2]} \tag{8.57}$$

$$(\hat{v}' \cdot \hat{v})^2 = \sin^2\theta \sin^2\theta + \cos^2\theta \cos^2\theta' \cos^2(\phi - \phi')$$

$$+ 2 \sin\theta \sin\theta' \cos\theta \cos\theta' \cos(\phi - \phi') \tag{8.58a}$$

$$(\hat{h}' \cdot \hat{v})^2 = \cos^2\theta \sin^2(\phi - \phi') \tag{8.58b}$$

$$(\hat{v}' \cdot \hat{h})^2 = \cos^2\theta' \sin^2(\phi - \phi') \tag{8.58c}$$

$$(\hat{h}' \cdot \hat{h})^2 = \cos^2(\phi - \phi'). \tag{8.58d}$$

Combining (8.56a)-(8.58d), we obtain

$$K_h(\theta) = \frac{\delta k_{1m}^4 \ell_\rho{}^2 \ell}{4\pi} \int d\Omega'$$

$$e^{-\frac{k_1{}^2 \ell_\rho{}^2}{4}(\sin^2\theta + \sin^2\theta' - 2\sin\theta'\sin\theta\cos(\phi - \phi'))}$$

$$\frac{[1 + k_{1m}^2 \ell^2(\cos^2\theta + \cos^2\theta')]}{[1 + k_{1m}^2\ell^2(\cos\theta - \cos\theta')^2][1 + k_{1m}^2\ell^2(\cos\theta + \cos\theta')^2]}$$

$$[\cos^2\theta' \sin^2(\phi - \phi') + \cos^2(\phi - \phi')] \tag{8.59a}$$

$$K_v(\theta) = \frac{\delta k_{1m}^4 \ell_\rho{}^2 \ell}{4\pi} \int d\Omega'$$

$$\frac{e^{-\frac{k_1{}^2 \ell_\rho{}^2}{4}(\sin^2\theta + \sin^2\theta' - 2\sin\theta'\sin\theta\cos(\phi - \phi'))}}{[1 + k_{1m}^2\ell^2(\cos\theta - \cos\theta')^2][1 + k_{1m}^2\ell^2(\cos\theta + \cos\theta')^2]}$$

$$\{[1 + k_{1m}^2\ell^2(\cos^2\theta + \cos^2\theta')][\sin^2\theta \sin^2\theta'$$

$$+ \cos^2\theta \cos^2\theta' \cos^2(\phi - \phi') + \cos^2\theta \sin^2(\phi - \phi')]$$

$$+ 4k_{1m}^2\ell^2 \cos^2\theta \cos^2\theta' \sin\theta \sin\theta' \cos(\phi - \phi')\}. \tag{8.59b}$$

Comparing (8.59a), (8.59b) with (8.55a), (8.55b), we find that

$$2\ \mathrm{Im}[\lambda^{TE}]\cos\theta = K_h(\theta) \tag{8.60a}$$

$$2\ \mathrm{Im}[\lambda^{TM}]\cos\theta = K_v(\theta). \tag{8.60b}$$

It is clear from (8.60a) and (8.60b) that the scattering coefficients as defined in the radiative transfer theory are related to the imaginary parts of the effective TE and TM propagation constants obtained in the zeroth order solution to the non-linear Dyson's equation for a half space (or unbounded) random medium. In the case of a two (or more) layer random medium the terms in (8.53a) and (8.53b) which contain Fresnel reflection coefficients, are present in the effective propagation constant and represent coherent effects due to the presence of the bottom boundary.

8.4 Half-Space Limit-Comparison with the Result of Tan and Fung

As another interesting case, we consider the limit of small scale fluctuations for a half space random medium, characterized by a correlation function of the form:

$$C(\bar{r}_1 - \bar{r}_2) = \delta k_{1m}'^4 \, e^{-|\bar{r}_{1\perp} - \bar{r}_{2\perp}|/\ell_\rho - |z_1 - z_2|/\ell} . \qquad (8.61)$$

where $k_{1m}' \equiv \mathrm{Re}(k_{1m})$. The spectral density of this correlation function is:

$$\Phi(\bar{\beta}_\perp, \beta_z) = \frac{\delta k_{1m}'^4 \ell_\rho{}^2 \ell}{2\pi^2 (1 + \beta_\perp{}^2 \ell_\rho{}^2)^{3/2} \, (1 + \beta_z{}^2 \ell^2)} \qquad (8.62)$$

Substituting (8.62) into (8.45) and (8.46), and retaining terms through first order in the vertical correlation length we obtain, the leading corrections to the propagation constant, k_{1mz}.

$$\lambda^{TE} \simeq i \, \frac{\delta k_{1m}^5 \ell_\rho{}^2 \ell}{3k_{1mz}} \qquad (8.63a)$$

$$\lambda^{TM} \simeq - \frac{1}{4k_{1mz}} k_{\perp}^{2} \frac{k_{1m}^{\prime 4}}{k_{1m}^{4}} \delta \ell_{\rho}^{2} k_{1m}^{\prime 2} + i \frac{\delta k_{1m}^{3} \ell_{\rho}^{2} \ell}{6k_{1mz}} (k_{1m}^{2} + k_{1mz}^{2}).$$

(8.63b)

We again observe that in the limit of small scale fluctuations the propagation constant corrections as given by (8.63a) and (8.63b) are distinct, but reduce to an identical result for propagation in the z-direction. The results (8.63a) and (8.63b) should be compared to those of Tan and Fung,[60] who for the same correlation function, (8.61) obtained a single propagation constant correction for all components in the mean dyadic Green's function. They found the zeroth order mean dyadic Green's function to be:

$$g_{ij}^{o}(u, v, z, z') = [C_{ij}(u, v)\, e^{i\gamma'|z - z'|}$$

$$+ \Gamma_{ij}(u, v)\, e^{-i\gamma'[z + z']}] \qquad (8.64a)$$

with,

$$\gamma'(u, v) = \gamma\left[1 - \frac{1}{4\gamma^2}(u^2 + v^2)\frac{k_o^{\,4}}{k_a^{\,4}}\sigma^2 (k_a'\ell_\rho)^2\right] \qquad (8.64b)$$

$$\gamma = (k_a^{\,2} - u^2 - v^2)^{1/2} \qquad (8.64c)$$

where k_a is the mean wavenumber, σ^2 is the variance of fluctuations and the transverse Fourier transform variables are denoted by u and v. Comparing the second term in the square bracket of (8.64b) with the first term of (8.63b), it is clear that Tan and Fung's result represents the lowest order correction to the real part of the TM propagation constant. The terms of order $k_{1m}^3 \ell_\rho{}^2 \ell$ in (8.63a) and (8.63b) correct the imaginary parts of the propagation constants and account for the decay of the coherent field as it propagates in the random medium.

In the case of laminar structures, with $k_{1m}\ell_\rho \to \infty$, it is found (Section 8.5) that additional secular terms arise in the first order equation and that elimination of the secularities results in two propagation constants per mode of polarization. The existence of two propagation constants also has been found in the case of scalar wave propagation in a half-space random medium[59] when the limit of a laminar structure is taken. However, in the limit of an unbounded, laminar random medium, we recover one propagation constant per polarization in the mean dyadic Green's function.

8.5 Special Case of a Laminar Structure

In the limit of a laminar structure, we have

$$\Phi(\bar{\alpha}_\perp, \alpha_z) = (2\pi)^2 \, \delta(\bar{\alpha}_\perp) \, \Phi(\alpha_z). \tag{8.65}$$

Substituting (8.65) into (8.7) and eliminating secular terms results in four coupled differential equations per polarization to be solved for the ξ dependence of the coefficients:

$$0 = -\, 2ip \, k_{1mz} \frac{\partial}{\partial \xi} \bar{\alpha}_p{}^T(\bar{k}_\perp, \xi) - (2\pi)^2 \, (2\pi i) \sum_{n,s} \text{Res} \, \Phi(\alpha_n{}^+)$$

$$\left\{ \frac{\bar{\alpha}_s{}^T(\bar{k}_\perp, \xi)[\bar{\beta}_{-s}{}^T(\bar{k}_\perp, \xi) \cdot \bar{\alpha}_p{}^T(\bar{k}_\perp, \xi)]}{i[(p - s)k_{1mz} + \alpha_n{}^-]} \right.$$

$$\left. - \frac{\bar{\beta}_s{}^T(-\bar{k}_\perp, \xi)[\bar{\alpha}_{-s}{}^T(\bar{k}_\perp, \xi) \cdot \bar{\alpha}_p{}^T(\bar{k}_\perp, \xi)]}{i[(p - s)k_{1mz} + \alpha_n{}^+]} \right\}$$

$$+ \, (2\pi)^2 \, (2\pi i) \sum_n \text{Res} \, \Phi(\alpha_n{}^+)$$

$$\left\{ \frac{\bar{\beta}_p^{\,T}(-\bar{k}_\perp,\ \xi)\,[\bar{\alpha}_p^{\,T}(-\bar{k}_\perp,\ \xi)\ \cdot\ \bar{\alpha}_{-p}^{\,T}(\bar{k}_\perp,\ \xi)]}{i\alpha_n^{\,+}} \right.$$

$$- \frac{\bar{\alpha}_p^{\,T}(\bar{k}_\perp,\ \xi)\,[\bar{\beta}_p^{\,T}(\bar{k}_\perp,\ \xi)\ \cdot\ \bar{\beta}_{-p}^{\,T}(\bar{k}_\perp,\ \xi)]}{i\alpha_n^{\,-}}$$

$$+ \frac{i(2\pi)}{k_{1m}^2} \sum_n \text{Res}\ \Phi(\alpha_n^{\,+})\ \hat{z}\hat{z}\ \cdot\ \bar{\alpha}_p^{\,T}(\bar{k}_\perp,\ \xi). \qquad (8.66a)$$

$$0 = -\ 2ip\ k_{1mz} \frac{\partial}{\partial \xi}\ \bar{\beta}_p^{\,T}(-\bar{k}_\perp,\ \xi)\ -\ (2\pi)^2\ (2\pi i) \sum_{n,s} \text{Res}\ \Phi(\alpha_n^{\,+})$$

$$\left\{ \frac{\bar{\alpha}_s^{\,T}(\bar{k}_\perp,\ \xi)\,[\bar{\beta}_{-s}^{\,T}(\bar{k}_\perp,\ \xi)\ \cdot\ \bar{\beta}_p^{\,T}(-\bar{k}_\perp,\ \xi)]}{i[(p\ -\ s)k_{1mz}\ +\ \alpha_n^{\,-}]} \right.$$

$$\left. - \frac{\bar{\beta}_s^{\,T}(-\bar{k}_\perp,\ \xi)\,[\bar{\beta}_{-s}^{\,T}(-\bar{k}_\perp,\ \xi)\ \cdot\ \bar{\beta}_p^{\,T}(-\bar{k}_\perp,\ \xi)]}{i[(p\ -\ s)k_{1mz}\ +\ \alpha_n^{\,+}]} \right\}$$

$$+\ (2\pi)^2\ (2\pi i) \sum_n \text{Res}\ \Phi(\alpha_n^{\,+})$$

$$\left\{ \frac{\bar{\beta}_p^{\,T}(-\bar{k}_\perp,\ \xi)\,[\bar{\alpha}_p^{\,T}(-\bar{k}_\perp,\ \xi)\ \cdot\ \bar{\beta}_{-p}^{\,T}(-\bar{k}_\perp,\ \xi)]}{i\alpha_n^{\,+}} \right.$$

$$- \left. \frac{\bar{\alpha}_p^{\,T}(\bar{k}_\perp, \xi)[\bar{\beta}_p^{\,T}(\bar{k}_\perp, \xi) \cdot \bar{\beta}_{-p}^{\,T}(-\bar{k}_\perp, \xi)]}{i\alpha_n^-} \right\}$$

$$+ \frac{i(2\pi)}{k_{1m}^2} \sum_n \text{Res}\, \Phi(\alpha_n^+)\; \bar{\beta}_p^{\,T}(-\bar{k}_\perp, \xi) \cdot \hat{z}\hat{z} \tag{8.66b}$$

where $\bar{\alpha}_p^{\,T}$ and $\bar{\beta}_p^{\,T}$ are given by (8.12a)-(8.15b). For example with $T = TM$, and after careful manipulations, the coupled differential equations to be solved take the form:

$$0 = -\frac{k_{1mz}}{4\pi^3}\frac{\partial C_1}{\partial \xi} + C_1 \sum_n \text{Res}\,\Phi(\alpha_n^-)\left[D_2C_1\left\{\frac{1}{i\alpha_n^-} - \frac{1}{i(2k_{1mz} + \alpha_n^+)}\right\}\right.$$

$$\left. + C_2D_1\left\{\frac{-3}{i\alpha_n^+} + \frac{1}{i(2k_{1mz} + \alpha_n^-)}\right\} + \frac{k_\perp^2}{(2\pi)^2 k_{1m}^2 (k_\perp^2 - k_{1mz}^2)}\right] \tag{8.67a}$$

$$0 = \frac{k_{1mz}}{4\pi^3}\frac{\partial D_2}{\partial \xi} + D_2 \sum_n \text{Res}\,\Phi(\alpha_n^-)\left[D_2C_1\left\{\frac{1}{i\alpha_n^-} - \frac{1}{i(2k_{1mz} + \alpha_n^+)}\right\}\right.$$

$$\left. + C_2D_1\left\{\frac{-3}{i\alpha_n^+} + \frac{1}{i(2k_{1mz} + \alpha_n^-)}\right\} + \frac{k_\perp^2}{(2\pi)^2 k_{1m}^2 (k_\perp^2 - k_{1mz}^2)}\right] \tag{8.67b}$$

$$0 = \frac{k_{1mz}}{4\pi^3}\frac{\partial C_2}{\partial \xi} + C_2 \sum_n \text{Res}\ \Phi(\alpha_n^-)\left[D_2C_1\left\{\frac{-3}{i\alpha_n^+} - \frac{1}{i(2k_{1mz}+\alpha_n^+)}\right\}\right.$$

$$\left. + C_2D_1\left\{\frac{1}{i\alpha_n^-} + \frac{1}{i(2k_{1mz}+\alpha_n^-)}\right\} + \frac{k_\perp^2}{(2\pi)^2k_{1m}^2(k_\perp^2 - k_{1mz}^2)}\right]$$

(8.67c)

$$0 = -\frac{k_{1mz}}{4\pi^3}\frac{\partial D_1}{\partial \xi} + D_1 \sum_n \text{Res}\ \Phi(\alpha_n^-)\left[D_2C_1\left\{\frac{-3}{i\alpha_n^+} - \frac{1}{i(2k_{1mz}+\alpha_n^+)}\right\}\right.$$

$$\left. + C_2D_1\left\{\frac{1}{i\alpha_n^-} + \frac{1}{i(2k_{1mz}+\alpha_n^-)}\right\} + \frac{k_\perp^2}{(2\pi)^2k_{1m}^2(k_\perp^2 - k_{1mz}^2)}\right]$$

(8.67d)

Solutions to (8.67a)-(8.67c) have the form

$$C_1 = C_1^{TM}\, e^{\sum_n g_n (M_{1n}^{TM} + M_{2n}^{TM})\xi} \qquad (8.68a)$$

$$C_2 = C_2^{TM}\, e^{-\sum_n g_n (N_{1n}^{TM} + N_{2n}^{TM})\xi} \qquad (8.68b)$$

$$D_1 = \frac{L_2^{TM}}{C_2^{TM}} e^{\sum_n g_m (N_{1n}^{TM} + N_{2n}^{TM}) \xi} \tag{8.69a}$$

$$D_2 = \frac{L_1^{TM}}{C_1^{TM}} e^{-\sum_n g_n (M_{1n}^{TM} + M_{2n}^{TM}) \xi} \tag{8.69b}$$

where

$$M_{1n}^{TM} = L_1^{TM}\left[\frac{1}{i\alpha_n^-} - \frac{1}{i(2k_{1mz} + \alpha_n^+)}\right] + \frac{k_\perp^{\,2}}{(2\pi)^2 k_{1m}^2 (k_\perp^{\,2} - k_{1mz}^2)} \tag{8.70a}$$

$$M_{2n}^{TM} = L_2^{TM}\left[\frac{-3}{i\alpha_n^+} + \frac{1}{i(2k_{1mz} + \alpha_n^-)}\right] \tag{8.70b}$$

$$N_{1n}^{TM} = L_1^{TM}\left[\frac{-3}{i\alpha_n^+} - \frac{1}{i(2k_{1mz} + \alpha_n^+)}\right] + \frac{k_\perp^{\,2}}{(2\pi)^2 k_{1m}^2 (k_\perp^{\,2} - k_{1mz}^2)} \tag{8.71a}$$

$$N_{2n}^{TM} = L_2^{TM}\left[\frac{1}{i\alpha_n^-} + \frac{1}{i(2k_{1mz} + \alpha_n^-)}\right] \tag{8.72b}$$

$$g_n = \frac{4\pi^3}{k_{1mz}} \operatorname{Res} \Phi(\alpha_n^-). \tag{8.73}$$

Here, $L_1^{TM} = C_1 D_2$ $L_2^{TM} = C_2 D_1$. A similar set of equations and corresponding solutions follow from (8.66a) and (8.66b) for $T = TE$. Substituting these results as well as (8.68a)-(8.73) into (8.10a) and (8.10b) and applying boundary conditions (8.26a)-(8.28b), yields the zeroth order mean dyadic Green's functions for a laminar structure:

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2} \int d^2k_\perp \, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})} \frac{1}{2ik_{1mz}}$$

$$\left\{ \frac{1}{D_2(k_\perp)} [\hat{e}(k_{1mz}) \, e^{i(k_{1mz} + \lambda_u^{TE})z} + R_{10}(k_\perp) \, \hat{e}(-k_{1mz}) \right.$$

$$e^{-i(k_{1mz} + \lambda_D^{TE})z}][R_{12}(k_\perp) \, \hat{e}(-k_{1mz}) \, e^{i2k_{1m}^{TE}d_1} \, e^{i(k_{1mz} + \lambda_D^{TE})z_o}$$

$$+ \hat{e}(k_{1mz}) \, e^{-i(k_{1mz} + \lambda_u^{TE})z_o}]$$

$$+ \frac{1}{F_2(k_\perp)} [\hat{h}(k_{1mz}) \, e^{i(k_{1mz} + \lambda_u^{TM})z}$$

$$+ S_{10}(k_\perp) \, \hat{h}(-k_{1mz}) \, e^{-i(k_{1mz} + \lambda_D^{TM})z}][S_{12}(k_\perp) \, \hat{h}(-k_{1mz})$$

$$e^{i2K_{1m}^{TM}d_1}\, e^{i(k_{1mz} + \lambda_D^{TM}) z_o} + \hat{h}(k_{1mz})\, e^{-i(k_{1mz} + \lambda_u^{TM}) z_o}]$$

$$(z > z_o) \qquad (8.74)$$

$$\bar{\bar{G}}_{11mo}(\bar{r}, \bar{r}_o) = - \frac{1}{(2\pi)^2} \int d^2k_\perp\, e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - r_{o\perp})} \frac{1}{2ik_{1mz}}$$

$$\Bigg\{ \frac{1}{D_2(k_\perp)} [R_{12}(k_\perp)\, \hat{e}(k_{1mz})\, e^{i2K_{1m}^{TE}d_1}\, e^{i(k_{1mz} + \lambda_D^{TE}) z}$$

$$+ \hat{e}(-k_{1mz})\, e^{-i(k_{1mz} + \lambda_u^{TE}) z}] [\hat{e}(-k_{1mz})\, e^{i(k_{1mz} + \lambda_u^{TE}) z_o}$$

$$+ R_{10}(k_\perp)\, \hat{e}(k_{1mz})\, e^{-i(k_{1mz} + \lambda_D^{TE}) z_o}]$$

$$+ \frac{1}{F_2(k_\perp)} [S_{12}(k_\perp)\, \hat{h}(k_{1mz})\, e^{i2K_{1m}^{TM}d_1}\, e^{i(k_{1mz} + \lambda_D^{TM}) z}$$

$$+ \hat{h}(-k_{1mz})\, e^{-i(k_{1mz} + \lambda_u^{TM}) z}] [\hat{h}(-k_{1mz})\, e^{i(k_{1mz} + \lambda_u^{TM}) z_o}$$

$$+ S_{10}(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda_D^{TM})z_o}]$$

$$(z < z_o) \tag{8.75}$$

$$\bar{\bar{G}}_{01mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2}\int d^2k_\perp\ e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})}\ \frac{e^{ik_{oz}z}}{2ik_{1mz}}$$

$$\left\{\frac{X_{10}(k_\perp)}{D_2(k_\perp)}\ \hat{e}(k_{oz})[R_{12}(k_\perp)\ \hat{e}(-k_{1mz})\ e^{i2K_{1m}^{TE}d_1}\ e^{i(k_{1mz} + \lambda_D^{TE})z_o}\right.$$

$$+ \hat{e}(k_{1mz})\ e^{-i(k_{1mz} + \lambda_u^{TE})z_o}] + \frac{k_{1m}}{k_o}\frac{Y_{10}(k_\perp)}{F_2(k_\perp)}\ \hat{h}(k_{oz})$$

$$[S_{12}(k_\perp)\ \hat{h}(-k_{1mz})\ e^{i2K_{1m}^{TM}d_1}\ e^{i(k_{1mz} + \lambda_D^{TM})z_o}$$

$$\left. + \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda_u^{TM})z_o}]\right\} \tag{8.76}$$

$$\bar{\bar{G}}_{21mo}(\bar{r}, \bar{r}_o) = -\frac{1}{(2\pi)^2}\int d^2k_\perp\ e^{i\bar{k}_\perp \cdot (\bar{r}_\perp - \bar{r}_{o\perp})}\ \frac{e^{-ik_{2z}z}}{2ik_{1mz}}$$

$$\left\{\frac{X_{12}(k_\perp)}{D_2(k_\perp)}\ \hat{e}(-k_{2z})[\hat{e}(-k_{1mz})\ e^{i(k_{1mz} + \lambda_u^{TE})z_o}\right.$$

$$+ R_{10}(k_\perp)\ \hat{e}(k_{1mz})\ e^{-i(k_{1mz} + \lambda_D^{TE})z_o}]$$

$$+ \frac{k_{1m}}{k_o}\frac{Y_{12}(k_\perp)}{F_2(k_\perp)}\ \hat{h}(-k_{2z})[\hat{h}(-k_{1mz})\ e^{i(k_{1mz} + \lambda_u^{TM})z_o}$$

$$+ S_{10}(k_\perp)\ \hat{h}(k_{1mz})\ e^{-i(k_{1mz} + \lambda_D^{TM})z_o}]\Bigg\} \tag{8.77}$$

where

$$\lambda_U{}^T = \sum_n g_n(M_{1n}^T + M_{2n}^T) \tag{8.78a}$$

$$\lambda_D{}^T = \sum_n g_n(N_{1n}^T + N_{2n}^T) \tag{8.78b}$$

$$L_1^{TM} = -\frac{1}{2ik_{1mz}\ F_2(k_\perp)} \tag{8.79}$$

$$L_2^{TM} = -\frac{S_{10}S_{12}\ e^{i2K_{1m}^{TM}d_1}}{2ik_{1mz}\ F_2(k_\perp)} \tag{8.80}$$

$$L_1^{TE} = -\frac{1}{2ik_{1mz}\ D_2(k_\perp)} \tag{8.81}$$

$$L_2^{TE} = - \frac{R_{10}R_{12}\, e^{i2K_{1m}^{TE}d_1}}{2ik_{1mz}\, D_2(k_\perp)} . \qquad (8.82)$$

The two effective propagation constants per polarization of a laminar structure follow from (8.78a) and (8.78b).

$$\eta_1{}^T(\bar{k}_\perp) = k_{1mz} + \lambda_U{}^T \qquad (8.83a)$$

$$\eta_2{}^T(\bar{k}_\perp) = k_{1mz} + \lambda_D{}^T . \qquad (8.83b)$$

CHAPTER 9

Modified Radiative Transfer Theory for a Two-Layer Random Medium

Modified radiative transfer (MRT) equations appropriate for electromagnetic wave propagation in bounded random media are derived from the Bethe-Salpeter equation in the ladder approximation together with the solution to the non-linearly approximated Dyson equation. The MRT equations are solved in the first order renormalization approximation to obtain analytical results for the backscattering cross-sections, of a two-layer random medium with arbitrary three-dimensional correlation functions. The coherent effects of MRT theory are illustrated and comparisons are made with backscattering cross sections obtained in the first order Born approximation to the wave equation.

9.1 Derivation of MRT Equations from Bethe-Salpeter Equation

The Bethe-Salpeter equation for the dyadic field covariance, under the ladder approximation may be written as

$$<\bar{\varepsilon}_1(\bar{r})\ \bar{\varepsilon}_1{}^*(\bar{r}')> = \int_V d^3r_1 \int_V d^3r_2\ C(\bar{r}_1 - \bar{r}_2)\{\bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_1)$$

$$\cdot\ \bar{E}_{1m}(\bar{r}_1)\ \bar{\bar{G}}^*_{11m}(\bar{r}', \bar{r}_2)\ \cdot\ \bar{E}^*_{1m}(\bar{r}_2)$$

$$+ <\bar{\bar{G}}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{\varepsilon}_1(\bar{r}_1)\ \bar{\bar{G}}^*_{11m}(\bar{r}', \bar{r}_2)\ \cdot\ \bar{\varepsilon}_1{}^*(\bar{r}_2)>\} \tag{9.1}$$

where $\bar{\varepsilon}_1(\bar{r}) = \bar{E}_1(\bar{r}) - \bar{E}_{1m}(\bar{r})$ and $\bar{E}_{1m} = <\bar{E}_1>$ is the mean electric field in Region 1 (Fig. 3.1) and is a solution to Dyson's equation. The zeroth order mean field propagating within the random layer, due to an incident plane wave $\bar{E}_{oi}$, takes the general form:

$$\bar{E}_{1m}(\bar{r}) = [E_{hui}\ e^{i\eta_{hi}z}\ \hat{e}(k_{1mzi}) + E_{hdi}\ e^{-i\eta_{hi}z}\ \hat{e}(-k_{1mzi})]\ e^{i\bar{k}_{\perp i}\cdot\bar{r}_\perp}$$

$$+$$

$$[E_{vui}\ e^{i\eta_{vi}z}\ \hat{h}(k_{1mzi}) + E_{vdi}\ e^{-i\eta_{vi}z}\ \hat{h}(-k_{1mzi})]\ e^{i\bar{k}_{\perp i}\cdot\bar{r}_\perp} \tag{9.2}$$

where the subscript i denotes that a quantity is to be evaluated at the incident wave vector angles. The unknown amplitudes which appear in (9.2) are determined by matching boundary conditions on the mean fields to zeroth order, and are listed in Appendix B. The effective propagation constants η_v and η_h are defined in (8.50).

We first decompose the incoherent field into a spectrum of upward and downward propagating plane waves:

$$\bar{\varepsilon}_1(\bar{r}) = \int d^2\beta_\perp \; e^{i\bar{\beta}_\perp \cdot \bar{r}_\perp} \, [\bar{\varepsilon}_u(z, \bar{\beta}_\perp) \, e^{i\beta'_{1mz} z} + \bar{\varepsilon}_d(z, \bar{\beta}_\perp) \, e^{-i\beta'_{1mz} z}] \tag{9.3}$$

where

$$\bar{\varepsilon}_{\substack{u\\d}}(z, \bar{\beta}_\perp) = \varepsilon_{\substack{hu\\d}}(z, \bar{\beta}_\perp) \, \hat{e}(\pm\beta'_{1mz}) + \varepsilon_{\substack{vu\\d}}(z, \bar{\beta}_\perp) \, \hat{h}(\pm\beta'_{1mz}). \tag{9.4}$$

In (9.3) and (9.4), $\beta'_{1mz} = \mathrm{Re}(\beta_{1mz})$. The z-dependence retained in $\bar{\varepsilon}_u(z, \bar{\beta}_\perp)$ and $\bar{\varepsilon}_d(z, \bar{\beta}_\perp)$ is a long distance scale characterized by the quantities $\ell_v = \{\mathrm{Im}(\eta_v)\}^{-1}$ and $\ell_h = \{\mathrm{Im}(\eta_h)\}^{-1}$. Therefore, two points can be close together on the ℓ_v, ℓ_h scale and yet far apart on a scale characterized by λ_{1m} or ℓ_z (ℓ_z being the vertical correlation length). We will assume that for points z and z' close together on the ℓ_v, ℓ_h scale:

$$\langle \varepsilon_{ju}(z, \bar{\alpha}_\perp)\, \varepsilon^*_{ku}(z', \bar{\beta}_\perp)\rangle = \delta(\bar{\alpha}_\perp - \bar{\beta}_\perp)\, J_{jku}(z, z', \bar{\alpha}_\perp) \tag{9.5a}$$

$$\langle \varepsilon_{jd}(z, \bar{\alpha}_\perp)\, \varepsilon^*_{kd}(z', \bar{\beta}_\perp)\rangle = \delta(\bar{\alpha}_\perp - \bar{\beta}_\perp)\, J_{jkd}(z, z', \bar{\alpha}_\perp) \tag{9.5b}$$

$$\langle \varepsilon_{ju}(z, \bar{\alpha}_\perp)\, \varepsilon^*_{kd}(z', \bar{\beta}_\perp)\rangle = \delta(\bar{\alpha}_\perp - \bar{\beta}_\perp)\, J_{jkc1}(z, z', \bar{\alpha}_\perp) \tag{9.5c}$$

$$\langle \varepsilon_{jd}(z, \bar{\alpha}_\perp)\, \varepsilon^*_{ku}(z'_\perp, \bar{\beta}_\perp)\rangle = \delta(\bar{\alpha}_\perp - \bar{\beta}_\perp)\, J_{jkc2}(z, z', \bar{\alpha}_\perp) \tag{9.5d}$$

where $j, k = h$ or v for horizontal or vertical polarizations. Thus incoherent fields with different transverse directions of propagation are uncorrelated. However, for a given transverse direction there exist correlations between upward and downward propagating incoherent fields due to the presence of reflecting dielectric interfaces at $z = 0$ and $z = -d_1$. These correlations are represented by J_{jkc1} and J_{jkc2} in (9.5c) and (9.5d).

Combining Equations (9.3) and (9.5a)-(9.5d), the left hand side of the Bethe-Salpeter equation (9.1) may be written as

$$<\bar{\varepsilon}_1(\bar{r})\ \bar{\varepsilon}_1{}^*(\bar{r}')> = \int d^2\beta_\perp\ e^{i\bar{\beta}_\perp \cdot (\bar{r}_\perp - \bar{r}_\perp{}')} \{\bar{\bar{\Gamma}}_u(z, z', \bar{\beta}_\perp)$$

$$e^{i\beta'_{1mz}(z - z')} + \bar{\bar{\Gamma}}_d(z, z', \bar{\beta}_\perp)\ e^{-i\beta'_{1mz}(z - z')}$$

$$+ \bar{\bar{\Gamma}}_{c1}(z, z', \bar{\beta}_\perp)\ e^{i\beta'_{1mz}(z + z')} + \bar{\bar{\Gamma}}_{c2}(z, z', \bar{\beta}_\perp)$$

$$e^{-i\beta'_{1mz}(z + z')}\} \tag{9.6}$$

where

$$\bar{\bar{\Gamma}}_u(z, z', \bar{\beta}_\perp) = J_{hhu}(z, z', \bar{\beta}_\perp)\ \hat{e}(\beta_{1mz})\ \hat{e}(\beta_{1mz})$$

$$+ J_{vvu}(z, z', \bar{\beta}_\perp)\ \hat{h}(\beta_{1mz})\ \hat{h}(\beta_{1mz}) + J_{hvu}(z, z', \bar{\beta}_\perp)$$

$$\hat{e}(\beta_{1mz})\ \hat{h}(\beta_{1mz}) + J_{vhu}(z, z', \bar{\beta}_\perp)\ \hat{h}(\beta_{1mz})\ \hat{e}(\beta_{1mz})$$

$$\tag{9.7a}$$

$$\bar{\bar{\Gamma}}_d(z, z', \bar{\beta}_\perp) = J_{hhd}(z, z', \bar{\beta}_\perp)\ \hat{e}(-\beta_{1mz})\ \hat{e}(-\beta_{1mz})$$

$$+ J_{vvd}(z, z', \bar{\beta}_\perp)\ \hat{h}(-\beta_{1mz})\ \hat{h}(-\beta_{1mz})$$

$$+ J_{hvd}(z, z', \bar{\beta}_\perp) \hat{e}(-\beta_{1mz}) \hat{h}(-\beta_{1mz}) + J_{vhd}(z, z', \bar{\beta}_\perp)$$

$$\hat{h}(-\beta_{1mz}) \hat{e}(-\beta_{1mz}) \tag{9.7b}$$

$$\bar{\bar{\Gamma}}_{c1}(z, z', \bar{\beta}_\perp) = J_{hhc1}(z, z', \bar{\beta}_\perp) \hat{e}(\beta_{1mz}) \hat{e}(-\beta_{1mz})$$

$$+ J_{vvc1}(z, z', \bar{\beta}_\perp) \hat{h}(\beta_{1mz}) \hat{h}(-\beta_{1mz}) + J_{hvc1}(z, z', \bar{\beta}_\perp)$$

$$\hat{e}(\beta_{1mz}) \hat{h}(-\beta_{1mz}) + J_{vhc1}(z, z', \bar{\beta}_\perp) \hat{h}(\beta_{1mz}) \hat{e}(-\beta_{1mz}) \tag{9.7c}$$

$$\bar{\bar{\Gamma}}_{c2}(z, z', \bar{\beta}_\perp) = J_{hhc2}(z, z', \bar{\beta}_\perp) \hat{e}(-\beta_{1mz}) \hat{e}(\beta_{1mz})$$

$$+ J_{vvc2}(z, z', \bar{\beta}_\perp) \hat{h}(-\beta_{1mz}) \hat{h}(\beta_{1mz}) + J_{hvc2}(z, z', \bar{\beta}_\perp)$$

$$\hat{e}(-\beta_{1mz}) \hat{h}(\beta_{1mz}) + J_{vhc2}(z, z', \bar{\beta}\ ) \hat{h}(-\beta_{1mz})$$

$$\hat{e}(\beta_{1mz}). \tag{9.7d}$$

The right hand side (RHS) of the Bethe-Salpeter equation (9.1) may be expanded as:

$$\text{R.H.S.} = \int_V d^2r_{1\perp} \int_V d^2r_{2\perp} \left\{ \int_{-d_1}^{z} dz_1 \int_{-d_1}^{z'} [\bar{\bar{G}}^{>}_{11m}(\bar{r}, \bar{r}_1)\right.$$

$$\cdot\ \bar{E}_{1m}(\bar{r})\ \bar{\bar{G}}^{>*}_{11m}(\bar{r}', \bar{r}_2)\ \cdot\ \bar{E}^{*}_{1m}(\bar{r}_2)$$

$$+ <\bar{\bar{G}}^{>}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{\varepsilon}_1(\bar{r}_1)\ \bar{\bar{G}}^{>*}_{11m}(\bar{r}', \bar{r}_2)\ \cdot\ \bar{\varepsilon}_1{}^*(\bar{r}_2)>]C(\bar{r}_1 - \bar{r}_2)$$

$$+$$

$$\int_{-d_1}^{z} dz_1 \int_{z'}^{0} dz_2 [\bar{\bar{G}}^{>}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{E}_{1m}(\bar{r}_1)\ \bar{\bar{G}}^{<*}_{11m}(\bar{r}', \bar{r}_2)$$

$$\cdot\ \bar{E}_{1m}{}^*(\bar{r}_2)\ +\ <\bar{\bar{G}}^{>}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{\varepsilon}_1(\bar{r}_1)\ \bar{\bar{G}}^{<*}(\bar{r}', \bar{r}_2)$$

$$\cdot\ \bar{\varepsilon}_1{}^*(\bar{r}_2)>]\ C(\bar{r}_1 - \bar{r}_2)$$

$$+$$

$$\int_{z}^{0} dz_1 \int_{-d_1}^{z'} dz_2 [\bar{\bar{G}}^{<}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{E}_{1m}(\bar{r}_1)\ \bar{\bar{G}}^{>*}_{11m}(\bar{r}', \bar{r}_2)$$

$$\cdot\ \bar{E}^{*}_{1m}(\bar{r}_2)\ +\ <\bar{\bar{G}}^{<}_{11m}(\bar{r}, \bar{r}_1)\ \cdot\ \bar{\varepsilon}_1(\bar{r}_1)\ G^{>*}_{11m}(\bar{r}', \bar{r}_2)$$

$$\cdot \bar{\varepsilon}_1{}^*(\bar{r}_2)>] \ C(\bar{r}_1 - \bar{r}_2)$$

$$+$$

$$\int_z^0 dz_1 \int_{z'}^0 dz_2 [\bar{\bar{G}}^{<}_{11m}(\bar{r}, \bar{r}_1) \cdot \bar{E}_{1m}(\bar{r}_1) \ \bar{\bar{G}}^{<*}_{11m}(\bar{r}', \bar{r}_2) \cdot \bar{E}^*_{1m}(\bar{r}_2)$$

$$+ <\bar{\bar{G}}^{<}_{11m}(\bar{r}, \bar{r}_1) \cdot \bar{\varepsilon}_1(\bar{r}_1) \ \bar{\bar{G}}^{<*}_{11m}(\bar{r}', \bar{r}_2)$$

$$\left. \cdot \bar{\varepsilon}_1{}^*(\bar{r}_2)>] \ C(\bar{r}_1 - \bar{r}_2) \right\} \qquad (9.8)$$

where $\bar{\bar{G}}^{>}_{11m}$ and $\bar{\bar{G}}^{<}_{11m}$ are defined by (8.25a) and (8.25b) respectively. In the limit $z \to z'$ it can be shown by substituting (8.25a), (8.25b) and (9.2) together with (9.3) and a typical correlation function into (9.8) that the second and third terms in (9.8) do not contribute as significantly as the first and fourth terms. The same conclusion may be reached by the following argument. The correlation functions which appear in (9.8) restrict the integration variables z_1 and z_2 to be separated by not more than the order of a vertical correlation length, ℓ_z. Therefore, in the limit $z \to z'$, it is clear that in (9.8) the first and fourth terms contribute over the entire range of the z_1, z_2 integration whereas the second and third terms can contribute only over a narrow range on the order of a correlation length which straddle the point $z = z'$. Thus, we approximate (9.8) by retaining only the first and fourth terms.

Introducing the Fourier transform of the correlation function

$$C(\bar{r}_1 - \bar{r}_2) = \int d^3\alpha \; \Phi(\bar{\alpha}) \; e^{-i\bar{\alpha} \cdot (\bar{r}_1 - \bar{r}_2)} . \tag{9.9}$$

We substitute (8.25a), (8.25b), (9.2), (9.3) and (9.9) into the approximated form of (9.8) and make use of (9.7a)-(9.7d). Equating this result to (9.6), we next balance terms of similar phase and polarization. This yields:

<u>Phase Factor $e^{i\beta'_{1mz}(z - z')}$</u>:

$$J_{hhu}(z, z', \bar{\beta}_\perp) \; e^{i\beta'_{1mz}(z - z')} = e^{i\eta_h z - i\eta_h^* z'} \; [|A_1(\beta_\perp)|^2$$

$$I_1^>(z, z', \bar{\beta}_\perp) + |B_1(\beta_\perp)|^2 \; I_1^<(z, z', \bar{\beta}_\perp)$$

$$+ I_5^>(z, z', \bar{\beta}_\perp) + I_5^<(z, z', \bar{\beta}_\perp)] \tag{9.10a}$$

$$J_{vvu}(z, z', \bar{\beta}_\perp)\, e^{i\beta'_{lmz}(z - z')} = e^{i\eta_v z - i\eta_v^* z'} [|C_l(\beta_\perp)|^2$$

$$I_2^{>}(z, z', \bar{\beta}_\perp) + |D_l(\beta_\perp)|^2\, I_2^{<}(z, z', \bar{\beta}_\perp)$$

$$+ I_6^{>}(z, z', \bar{\beta}_\perp) + I_6^{<}(z, z', \bar{\beta}_\perp)] \tag{9.10b}$$

$$J_{hvu}(z, z', \bar{\beta}_\perp)\, e^{i\beta'_{lmz}(z - z')} = e^{i\eta_h z - i\eta_v^* z'} [A_l(\beta_\perp)\, C_l^*(\beta_\perp)$$

$$I_3^{>}(z, z', \bar{\beta}_\perp) + B_l(\beta_\perp)\, D_l^*(\beta_\perp)\, I_3^{<}(z, z', \bar{\beta}_\perp)$$

$$+ I_7^{>}(z, z', \bar{\beta}_\perp) + I_7^{<}(z, z', \bar{\beta}_\perp)] \tag{9.10c}$$

$$J_{vhu}(z, z', \bar{\beta}_\perp)\, e^{i\beta'_{lmz}(z - z')} = e^{i\eta_v z - i\eta_h^* z'} [C_l(\beta_\perp)\, A_l^*(\beta_\perp)$$

$$I_4^{>}(z, z', \bar{\beta}_\perp) + D_l(\beta_\perp)\, B_l^*(\beta_\perp)\, I_4^{<}(z, z', \bar{\beta}_\perp)$$

$$+ I_8^{>}(z, z', \bar{\beta}_\perp) + I_8^{<}(z, z', \bar{\beta}_\perp)]. \tag{9.10d}$$

Phase Factor $e^{-i\beta'_{1mz}(z - z')}$:

$$J_{hhd}(z, z', \bar{\beta}_\perp)\, e^{-i\beta'_{1mz}(z - z')} = e^{-i\eta_h z + i\eta_h{}^* z'}$$

$$[|A_2(\beta_\perp)|^2\, I_1{}^>(z, z', \bar{\beta}_\perp) + I_1{}^<(z, z', \bar{\beta}_\perp)$$

$$+ I_9{}^>(z, z', \bar{\beta}_\perp) + I_9{}^<(z, z', \bar{\beta}_\perp)] \tag{9.11a}$$

$$J_{vvd}(z, z', \bar{\beta}_\perp)\, e^{-i\beta'_{1mz}(z - z')} = e^{-i\eta_v z + i\eta_v{}^* z'}$$

$$[|C_2(\beta_\perp)|^2\, I_2{}^>(z, z', \bar{\beta}_\perp) + I_2{}^<(z, z', \bar{\beta}_\perp)$$

$$+ I_{10}^>(z, z', \bar{\beta}_\perp) + I_{10}^<(z, z', \bar{\beta}_\perp)] \tag{9.11b}$$

$$J_{hvd}(z, z', \bar{\beta}_\perp)\, e^{-i\beta'_{1mz}(z - z')} = e^{-i\eta_h z - i\eta_v{}^* z'}$$

$$[A_2(\beta_\perp)\, C_2{}^*(\beta_\perp)\, I_3{}^>(z, z', \bar{\beta}_\perp) + I_3{}^<(z, z', \bar{\beta}_\perp)$$

$$+ I_{11}^>(z, z', \bar{\beta}_\perp) + I_{11}^<(z, z', \bar{\beta}_\perp)] \tag{9.11c}$$

$$J_{vhd}(z, z', \bar{\beta}_\perp)\, e^{-i\beta'_{1mz}(z - z')} = e^{-i\eta_v z + i\eta_h^* z'}$$

$$[C_2(\beta_\perp)\, A_2^*(\beta_\perp)\, I_4^>(z, z', \bar{\beta}_\perp) + I_4^<(z, z', \bar{\beta}_\perp)$$

$$+ I_{12}^>(z, z', \bar{\beta}_\perp) + I_{12}^<(z, z', \bar{\beta}_\perp)] \qquad (9.11d)$$

where the coefficients A_1, B_1, C_1, D_1, A_2, C_2 and the terms $I_j^{\gtrless}$ (j = 1, 2, ..., 12) are listed in Appendices A and C respectively. A similar set of equations may be obtained for the cross correlation terms J_{hhc1}, J_{hhc2} ... by balancing phase factors of the type

$$e^{i\beta'_{1mz}(z + z')} \qquad \text{and} \qquad e^{-i\beta'_{1mz}(z + z')}.$$

The resultant set of equations, however, is not required in the development of MRT equations for three dimensional random media. In the case of a one dimensional laminar structure in which $k_{1m}\ell_\rho \to \infty$ the set of equations for the cross correlation terms is required in the derivation of MRT equations. In order to see this point more clearly and moreover to illustrate the evaluation and reduction of the $I_j^{\lessgtr}$ terms in Appendix C, we will consider $I_5^>(z, z', \bar{\beta}_\perp)$ in detail.

The Term $I_5^>(z, z', \bar{\beta}_\perp)$

The term $I_5^>$ takes the form:

$$I_5^>(z, z', \bar{\beta}_\perp) = (2\pi)^4 \int_{-\infty}^{\infty} d\alpha_z \int d^2k_\perp \; \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_z)$$

$$\int_{-d_1}^{z} dz_1 \int_{-d_1}^{z'} dz_2 \; e^{-i\alpha_z(z_1 - z_2)} \; \bar{F}(\bar{z}_1, \bar{\beta}_\perp)$$

$$\cdot \{\bar{\bar{\Gamma}}_u(z_1, z_2, \bar{k}_\perp) \; e^{ik'_{1mz}(z_1 - z_2)}$$

$$+ \bar{\bar{\Gamma}}_d(z_1, z_2, \bar{k}_\perp) \; e^{-ik'_{1mz}(z_1 - z_2)}$$

$$+ \bar{\bar{\Gamma}}_{c1}(z_1, z_2, \bar{k}_\perp) \; e^{ik'_{1mz}(z_1 + z_2)}$$

$$+ \bar{\bar{\Gamma}}_{c2}(z_1, z_2, \bar{k}_\perp) \; e^{-ik'_{1mz}(z_1 + z_2)}\} \cdot \bar{F}^*(z_2, \bar{\beta}_\perp) \qquad (9.12)$$

where $k'_{1mz} = \mathrm{Re}(k_{1mz})$ and

$$\bar{F}(z_1, \bar{\beta}_\perp) = A_1(\beta_\perp)[B_1(\beta_\perp) \; e^{i\eta_h z_1} \; \hat{e}(-\beta_{1mz}) + e^{-i\eta_h z_1} \; \hat{e}(\beta_{1mz})].$$

$$(9.13)$$

We note from Equations (9.7a)-(9.7d) that $\overline{\overline{\Gamma}}_u$, $\overline{\overline{\Gamma}}_d$, $\overline{\overline{\Gamma}}_{c1}$ and $\overline{\overline{\Gamma}}_{c2}$ vary on the long distance z_1 and z_2 scales. Therefore, in (9.12) due to the extremely sharp peak in the correlation function relative to the long distance scale, we may replace z_2 by z_1 in the arguments of $\overline{\overline{\Gamma}}_u$, $\overline{\overline{\Gamma}}_d$, $\overline{\overline{\Gamma}}_{c1}$ and $\overline{\overline{\Gamma}}_{c2}$. Next we take the limit $z \rightarrow z'$ and break up the z_2 integration into the intervals $-d_1 < z_2 < z_1$, $z_1 < z_2 < z$. The α_z integration then is performed using residue calculus followed by the z_2-integration. In performing the z_2 integration we retain only constructive interference terms of the form $e^{\pm 2\eta_h'' z_1}$ or $e^{\pm 2\eta_v'' z_1}$, where $\eta_{\substack{h\\v}}'' = \mathrm{Im}(\eta_{\substack{h\\v}})$. Destructive interference terms of the form

$$e^{i(2\eta' + \alpha_n^{\pm})z_1}$$

or

$$e^{i2\eta' z_1}$$

(where $\eta' \equiv \mathrm{Re}(\eta_h)$ or $\mathrm{Re}(\eta_v)$ and $\alpha_n^{\pm}$ is explained below) will contribute a small fraction $\eta''/(2\eta' + \alpha_n^{\pm})$ or η''/η' of the constructive interference terms when integrated over z_1. Upon performing the operations discussed above, the expression for $I_5^>$ becomes:

$$I_5^>(z, z, \bar{\beta}_\perp) = i(2\pi)^5 \sum_n \int d^2k_\perp \int_{-d_1}^{z} dz_1 \text{ Res } \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_n^+)$$

$$i(\alpha_n^- - \alpha_n^+) \; J_{hhu}(z_1, z_1, \bar{k}_\perp)[M_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{-2\eta_h'' z_1}$$

$$+ M_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{vvu}(z_1, z_1, \bar{k}_\perp)[M_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{-2\eta_h'' z_1} + M_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{hvu}(z_1, z_1, \bar{k}_\perp)[M_{5n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{-2\eta_h'' z_1} + M_{6n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{vhu}(z_1, z_1, \bar{k}_\perp)[M_{7n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{-2\eta_h'' z} + M_{8n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{hhd}(z_1, z_1, \bar{k}_\perp)[N_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{-2\eta_h'' z_1} + N_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) \; e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{vvd}(z_1, z_1, \bar{k}_\perp)[N_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{-2\eta_h'' z_1} + N_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{hvd}(z_1, z_1, \bar{k}_\perp)[N_{5n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{-2\eta_h'' z_1} + N_{6n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{vhd}(z_1, z_1, \bar{k}_\perp)[N_{7n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{-2\eta_h'' z_1} + N_{8n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{2\eta_h'' z_1}]$$

$$+$$

$$J_{hhc1}(z_1, z_1, \bar{k}_\perp)[W_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{i(\eta_h + \eta_h^* + 2k'_{1mz})z_1}$$

$$+ W_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{-i(\eta_h + \eta_h^* - 2k'_{1mz})z_1}]$$

$$+$$

$$J_{hhc2}(z_1, z_1, \bar{k}_\perp)[W_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{i(\eta_h + \eta_h^* - 2k'_{1mz})z_1}$$

$$+ W_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) e^{-i(\eta_h + \eta_h^* + 2k'_{1mz})z_1}] \qquad (9.14)$$

where α_n^+ and α_n^- represent the n-th poles of $\Phi(\bar{\alpha}_\perp, \alpha_z)$ in the upper and lower half complex-α_z plane. The functions M_{in}, N_{in} and W_{jn} $(i = 1, 2, \ldots, 8 \quad j = 1, 2, 3, 4)$ are listed in Appendix D, for reference. In deriving (9.14), we have made use of the following properties of the spectral density which are valid for a large class of physically interesting correlation functions:

$$\alpha_n^+ = - \alpha_n^- \tag{9.15a}$$

$$\text{Res } \Phi(\bar{\alpha}_\perp, \alpha_n^+) = - \text{Res } \Phi(\bar{\alpha}_\perp, \alpha_n^-). \tag{9.15b}$$

It is important to note in (9.14) that the terms η_h and η_h'' which appear in the phases are function of $\bar{\beta}_\perp$. However, for laminar structures the spectral density attains a delta function dependence $\delta(\bar{k}_\perp - \bar{\beta}_\perp)$ so that in (9.14) $k'_{1mz} \to \beta'_{1mz}$. In this case a typical cross correlation term in (9.14) takes the form:

$$J_{hhc1}(z_1, z_1, \bar{\beta}_\perp)\, W_{2n}(\bar{\beta}_\perp, \bar{\beta}_\perp)\, e^{-i[\eta_h(\bar{\beta}_\perp) + \eta_h^*(\bar{\beta}_\perp) - 2\beta'_{1mz}]z_1}. \tag{9.16}$$

A typical equation obtained by balancing phases and polarization terms (as was done in (9.10a)-(9.11d)) and then letting $z \to z'$ is:

$$J_{hhc1}(z, z, \bar{\beta}_\perp)\, e^{-i(\eta_h(\bar{\beta}_\perp) + \eta_h^*(\bar{\beta}_\perp) - 2\beta'_{1mz})z}$$

$$= A_1(\beta_\perp)\, A_2^*(\beta_\perp) I_1^>(z, z, \bar{\beta}_\perp) + B_1(\bar{\beta}_\perp)\, I_1^<(z, z, \bar{\beta}_\perp)$$

$$+ I_{13}^>(z, z, \bar{\beta}_\perp) + I_{13}^<(z, z, \bar{\beta}_\perp). \qquad (9.17)$$

It is clear that equation (9.17) may be used to express (9.16) in terms of $I_1^>$, $I_1^<$, $I_{13}^>$ and $I_{13}^<$ each of which contains constructive interference terms of the form $e^{2\eta_v'' z_1}$ or $e^{2\eta_h'' z_1}$. However, for general non-laminar structures, the cross correlation terms of (9.14) do not have the form (9.16) and therefore cannot be expressed in terms of contructive interference terms by means of equations like (9.17). Therefore, as discussed above, the equations obtained by balancing phases and polarizations for cross-correlations are not required in the development of MRT equations for three dimensional random media. Finally, the residue of the spectral density which appears in (9.14), may be reduced by the method described in Appendix E, and the final form of $I_5^>$ is:

$$I_5^{>}(z, z, \bar{\beta}_\perp) = (2\pi)^5 \int d^2k_\perp \int_{-d_1}^{z} dz_1 \{[|A_1(\beta_\perp)\, B_1(\beta_\perp)|^2$$

$$\bar{P}^{+}_{hu}(\bar{\beta}_\perp, \bar{k}_\perp)\, e^{-2\eta_h'' z_1} + |A_1(\beta_\perp)|^2\, \bar{P}^{-}_{hu}(\bar{\beta}_\perp, \bar{k}_\perp)\, e^{2\eta_h'' z_1}]$$

$$\cdot\, \bar{I}_u(z_1, \bar{k}_\perp) + [|A_1(\beta_\perp)\, B_1(\beta_\perp)|^2\, \bar{P}^{-}_{hd}(\bar{\beta}_\perp, \bar{k}_\perp)\, e^{-2\eta_h'' z_1}$$

$$+ |A_1(\beta_\perp)|^2\, \bar{P}^{+}_{hd}(\bar{\beta}_\perp, \bar{k}_\perp)\, e^{2\eta_h'' z_1}] \cdot \bar{I}_d(z, \bar{k}_\perp)\} \qquad (9.18)$$

where, $\bar{P}^{\pm}_{hu}$ and $\bar{P}^{\pm}_{hd}$ are row matrices, defined as:

$$\bar{P}^{\pm}_{hu}(\bar{\beta}_\perp, \bar{k}_\perp) = \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \beta_\pm)[\{\hat{e}(\mp\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}^2,$$

$$\{\hat{e}(\mp\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2,$$

$$\{\hat{e}(\mp\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\mp\beta_{1mz}) \cdot \hat{h}(k_{1mz})\},\ 0] \qquad (9.19a)$$

$$\bar{P}^{\pm}_{hd}(\bar{\beta}_\perp, \bar{k}_\perp) = \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \beta_\pm)[\{\hat{e}(\pm\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}^2,$$

$$\{\hat{e}(\pm\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}^2,$$

$$\{\hat{e}(\pm\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}\{\hat{e}(\pm\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}, \quad 0] \tag{9.19b}$$

where $\beta_\pm \equiv k'_{1mz} \pm \beta_{1mz}$. $\bar{I}_u$ and $\bar{I}_d$ are column matrices given by:

$$I_{\substack{u\\d}}(z, \bar{k}_\perp) = \begin{bmatrix} J_{hh\substack{u\\d}}(z, z, \bar{k}_\perp) \\ J_{vv\substack{u\\d}}(z, z, \bar{k}_\perp) \\ 2\mathrm{Re}\{J_{vh\substack{u\\d}}(z, z, \bar{k}_\perp)\} \\ 2\mathrm{Im}\{J_{vh\substack{u\\d}}(z, z, \bar{k}_\perp)\} \end{bmatrix} \tag{9.20}$$

The other terms $I_j^{\gtrless}$ listed in Appendix C, may be reduced and cast into a form similar to that of the $I_5^>$ term (9.18).

We illustrate the development of MRT equations by considering equations (9.10a)-(9.10d) in the limit $z' \to z$. Equation (9.10a) may be written as

$$J_{hhu}(z, z, \bar{\beta}_\perp)\, e^{2\eta_h'' z} = |A_1(\beta_\perp)|^2\, I_1^>(z, z, \bar{\beta}_\perp)$$

$$+ |B_1(\beta_\perp)|^2\, I_1^<(z, z, \bar{\beta}_\perp) + [I_5^>(z, z, \bar{\beta}_\perp)$$

$$+ I_5^{<}(z, z, \beta_\perp)]. \tag{9.21}$$

Operating on both sides of (9.21) with d/dz, we obtain

$$\frac{dJ_{hhu}(z, z, \bar{\beta}_\perp)}{dz} = -2\eta_h'' J_{hhu}(z, z, \bar{\beta}_\perp)$$

$$+ e^{-2\eta_h'' z}[|A_1(\beta_\perp)|^2 \frac{dI_1^{>}(z, z, \bar{\beta}_\perp)}{dz}$$

$$+ |B_1(\beta_\perp)|^2 \frac{d}{dz} I_1^{<}(z, z, \bar{\beta}_\perp) + \frac{d}{dz}(I_5^{>}(z, z, \bar{\beta}_\perp)$$

$$+ I_5^{<}(z, z, \bar{\beta}_\perp))]. \tag{9.22}$$

The transport equation for $J_{vvu}(z, z, \bar{\beta}_\perp)$ is similar to that for $J_{hhu}(z, z, \bar{\beta}_\perp)$ and can be obtained from (9.22) by making the replacements $h \to v$, $I_1^{\gtrless} \to I_6^{\gtrless}$, $A_1 \to C_1$ and $B_1 \to D_1$. Equations (9.10c) and (9.10d) may be cast into the following forms:

$$\frac{d}{dz} 2\mathrm{Re}\{J_{vhu}(z, z, \bar{\beta}_\perp)\} = -2\mathrm{Im}[\kappa_{vh} J_{vhu}(z, z, \bar{\beta}_\perp)]$$

$$+ \left[e^{-i\kappa^*_{vh}z}\left(A_1C_1^* \frac{dI_3^{>}(z, z, \bar{\beta}_\perp)}{dz} + B_1D_1^* \frac{dI_3^{<}(z, z, \bar{\beta}_\perp)}{dz}\right)\right.$$

$$\left. + e^{i\kappa_{vh}z}\left(C_1A_1^* \frac{dI_4^{>}(z, z, \beta_\perp)}{dz} + D_1B_1^* \frac{dI_4^{<}(z, z, \bar{\beta}_\perp)}{dz}\right)\right]$$

$$+ \left[e^{-i\kappa^*_{vh}z} \frac{d}{dz} (I_7^{>}(z, z, \bar{\beta}_\perp) + I_7^{<}(z, z, \bar{\beta}_\perp)\right.$$

$$\left. + e^{i\kappa_{vh}z} \frac{d}{dz} (I_8^{>}(z, z, \bar{\beta}_\perp) + I_8^{<}(z, z, \bar{\beta}_\perp))\right] \quad (9.23)$$

$$\frac{d}{dz} 2\text{Im}\{J_{vhu}(z, z, \beta_\perp)\} = 2\text{Re}[\kappa_{vh} J_{vhu}(z, z, \bar{\beta}_\perp)]$$

$$+ i\left[e^{-i\kappa^*_{vh}z}\left(A_1C_1^* \frac{d}{dz} I_3^{>}(z, z, \bar{\beta}_\perp) + B_1D_1^* \frac{d}{dz} I_3^{<}(z, z, \bar{\beta}_\perp)\right)\right.$$

$$\left. - e^{i\kappa_{vh}z}\left(C_1A_1^* \frac{d}{dz} I_4^{>}(z, z, \bar{\beta}_\perp) + D_1B_1^* \frac{d}{dz} I_4^{<}(z, z, \bar{\beta}_\perp)\right)\right]$$

$$+ i\left[e^{i\kappa^*_{vh}z} \frac{d}{dz} (I_7^{>}(z, z, \bar{\beta}_\perp) + I_7^{<}(z, z, \bar{\beta}_\perp))\right.$$

$$\left. - e^{i\kappa_{vh}z} \frac{d}{dz} (I_8^{>}(z, z, \bar{\beta}_\perp) + I_8^{<}(z, z, \bar{\beta}_\perp))\right] \quad (9.24)$$

where $\kappa_{vh} \equiv \eta_v - \eta_h^*$. Reducing the terms $I_j^{\gtrless}$ listed in

Appendix C, according to the method outlined above for the $I_5^>$ term and then substituting into (9.21)-(9.24), we may cast the MRT equations governing the upward propagating intensity into matrix form:

$$|\beta_{1mz}|^2 \frac{d}{dz} \bar{I}_u(z, \bar{\beta}_\perp) = - |\beta_{1mz}|^2 \bar{\bar{\eta}}(\bar{\beta}_\perp) \cdot \bar{I}_u(z, \bar{\beta}_\perp)$$

$$+ \bar{\bar{Q}}_{uu}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mu}(z, \bar{k}_{\perp i}) + \bar{\bar{Q}}_{ud}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{md}(z, \bar{k}_{\perp i})$$

$$+ \Delta_1 \bar{\bar{Q}}_{c1}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mc1}(z, \bar{k}_{\perp i}) + \int d^2k_\perp [\bar{\bar{P}}_{uu}(\bar{\beta}_\perp, \bar{k}_\perp)$$

$$\cdot \bar{I}_u(z, \bar{k}_\perp) + \bar{\bar{P}}_{ud}(\bar{\beta}_\perp, \bar{k}_\perp) \cdot \bar{I}_d(z, \bar{k}_\perp)]. \qquad (9.25a)$$

Similarly, equations (9.11a)-(9.11d) may be reduced to matrix form governing the downward propagating intensity:

$$-|\beta_{1mz}|^2 \frac{d}{dz} \bar{I}_d(z, \bar{\beta}_\perp) = - |\beta_{1mz}|^2 \bar{\bar{\eta}}(\bar{\beta}_\perp) \cdot \bar{I}_d(z, \bar{\beta}_\perp)$$

$$+ \bar{\bar{Q}}_{dd}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{md}(z, \bar{k}_{\perp i}) + \bar{\bar{Q}}_{du}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mu}(z, \bar{k}_{\perp i})$$

$$- \Delta_1 \,\bar{\bar{Q}}_{c1}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mc2}(z, \bar{k}_{\perp i}) + \int d^2k_\perp [\bar{\bar{P}}_{dd}(\bar{\beta}_\perp, \bar{k}_\perp)$$

$$\cdot \bar{I}_d(z, \bar{k}_\perp) + \bar{\bar{P}}_{du}(\bar{\beta}_\perp, \bar{k}_\perp) \cdot \bar{I}_u(z, \bar{k}_\perp)] \qquad (9.25b)$$

where in (9.25a) and (9.25b), $\Delta_1 = 1$ if $\bar{\beta}_\perp = \pm\bar{k}_{\perp i}$ and is zero otherwise. Mathematically this arises in the reduction of $I_j^{\gtrless}$ $(j = 1, 2, 3, 4)$, where terms of the form

$$e^{i[\eta_h(\bar{\beta}_\perp) - \eta_h^*(\bar{k}_{\perp i})]z_1}$$

occur. It is clear that these terms are of the constructive interference type only for $\bar{\beta}_\perp = \pm\bar{k}_{\perp i}$ and otherwise are of the destructive interference type. Discussion of the physical significance of this result is deferred to Section 9.5.

The $\bar{\bar{Q}}$ and $\bar{\bar{P}}$ matrices in (9.25a) and (9.25b) are scattering matrices for the mean and incoherent field intensities respectively and are defined in Appendix F, together with the other matrices in (9.25a) and (9.25b). In order to complete the derivation of MRT equations we must obtain boundary conditions satisfied by the intensity matrices, $\bar{I}_u$ and $\bar{I}_d$.

9.2 Boundary Conditions for MRT Equations

The boundary conditions appropriate for the modified radiative transfer equations are obtained from Equations (9.10a) -(9.11d), by first taking the limit $z' \to z$ and then setting $z = 0$ and $-d_1$. To illustrate the technique, consider (9.10a) and (9.11a) at $z = 0$. We have

$$J_{hhu}(0, 0, \bar{\beta}_\perp) = |A_1(\beta_\perp)|^2 I_1^>(0, 0, \bar{\beta}_\perp) + I_5^>(0, 0, \bar{\beta}_\perp) \tag{9.26a}$$

$$J_{hhd}(0, 0, \bar{\beta}_\perp) = |A_2(\beta_\perp)|^2 I_1^>(0, 0, \bar{\beta}_\perp) + I_9^>(0, 0, \bar{\beta}_\perp). \tag{9.26b}$$

Upon evaluating $I_5^>$ and $I_9^>$ according to the method described above for $I_5^>$, it is easily shown that

$$I_5^>(0, 0, \bar{\beta}_\perp) = \frac{|A_1(\beta_\perp)|^2}{|A_2(\beta_\perp)|^2} I_9^>(0, 0, \bar{\beta}_\perp). \tag{9.27}$$

From Appendix A, we obtain $|A_2|^2 = |R_{10}(\beta_\perp)|^2 |A_1|^2$, which upon combining with (9.27), (9.26a) and (9.26b) yields

$$J_{hhd}(0,\ 0,\ \bar{\beta}_{\perp}) = R_{10}(\beta_{\perp}) \cdot J_{hhu}(0,\ 0,\ \bar{\beta}_{\perp}). \tag{9.28}$$

A similar procedure may be applied to the other equations in (9.10a)-(9.11d), so that the complete set of boundary conditions satisfied by $\bar{I}_u$ and $\bar{I}_d$ takes the form:

$$\bar{I}_d(0,\ \bar{\beta}_{\perp}) = \bar{\bar{R}}_{10}(\beta_{\perp}) \cdot \bar{I}_u(0,\ \bar{\beta}_{\perp}) \tag{9.29a}$$

$$\bar{I}_u(-d_1,\ \bar{\beta}_{\perp}) = \bar{\bar{R}}_{12}(\beta_{\perp}) \cdot \bar{I}_d(-d_1,\ \bar{\beta}_{\perp}) \tag{9.29b}$$

where:

$$\bar{\bar{R}}_{ij}(\beta_{\perp}) = \begin{bmatrix} |R_{ij}|^2 & 0 & 0 & 0 \\ 0 & |S_{ij}|^2 & 0 & 0 \\ 0 & 0 & \mathrm{Re}(R_{ij}^* S_{ij}) & -\mathrm{Im}(R_{ij}^* S_{ij}) \\ 0 & 0 & \mathrm{Im}(R_{ij}^* S_{ij}) & \mathrm{Re}(R_{ij}^* S_{ij}) \end{bmatrix} \tag{9.30}$$

where $i = 1$ and $j = 0$ or 2.

The incoherent intensity transmitted from region 1 into region 0, is given by

$$\bar{I}_{ou}(0,\ \bar{\beta}_\perp) = \bar{\bar{T}}_{10}(\beta_\perp) \cdot \bar{I}_u(0,\ \bar{\beta}_\perp) \tag{9.31}$$

where:

$$\bar{\bar{T}}_{10}(\beta_\perp) = \begin{bmatrix} |X_{10}|^2 & 0 & 0 & 0 \\ 0 & \left|\frac{\eta_o}{\eta_1} Y_{10}\right|^2 & 0 & 0 \\ 0 & 0 & \mathrm{Re}\left[\frac{\eta_o}{\eta_1} Y_{10}X_{10}^*\right] & -\mathrm{Im}\left[\frac{\eta_o}{\eta_1} Y_{10}X_{10}^*\right] \\ 0 & 0 & \mathrm{Im}\left[\frac{\eta_o}{\eta_1} Y_{10}X_{10}^*\right] & \mathrm{Re}\left[\frac{\eta_o}{\eta_1} Y_{10}X_{10}^*\right] \end{bmatrix} \tag{9.32}$$

and,

$$\bar{I}_{ou}(0,\ \bar{\beta}_\perp) = \begin{bmatrix} \langle \varepsilon_{ohu}(0,\ \bar{\beta}_\perp)\ \varepsilon^*_{ohu}(0,\ \bar{\beta}_\perp)\rangle \\ \langle \varepsilon_{ovu}(0,\ \bar{\beta}_\perp)\ \varepsilon^*_{ovu}(0,\ \bar{\beta}_\perp)\rangle \\ 2\mathrm{Re}\langle \varepsilon_{ovu}(0,\ \bar{\beta}_\perp)\ \varepsilon^*_{ohu}(0,\ \bar{\beta}_\perp)\rangle \\ 2\mathrm{Im}\langle \varepsilon_{ovu}(0,\ \bar{\beta}_\perp)\ \varepsilon^*_{ohu}(0,\ \bar{\beta}_\perp)\rangle \end{bmatrix} \tag{9.33}$$

Boundary condition (9.31) is obtained by noting that the plane wave components of the incoherent fields in regions 1 and 0 satisfy the boundary conditions

$$\varepsilon_{ovu}(0,\ \bar{\beta}_{\perp}) = \frac{\eta_o}{\eta_1} Y_{10}(\beta_{\perp})\ \varepsilon_{1vu}(0,\ \bar{\beta}_{\perp}) \tag{9.34a}$$

$$\varepsilon_{ohu}(0,\ \bar{\beta}_{\perp}) = X_{10}(\beta_{\perp})\ \varepsilon_{1hu}(0,\ \bar{\beta}_{\perp}). \tag{9.34b}$$

Upon forming the appropriate ensemble averages, boundary condition (9.31) results.

9.3 Comparison of MRT and RT Equations and Solutions in the First Order Renormalization Approximation

It is of interest to compare the MRT equations (9.25a) and (9.25b) with the conventional RT equations in current use. For example the RT scattering phase matrix coupling the four Stokes parameters of the scattered wave in direction Ω to the four Stokes parameters of the incident wave in direction Ω' is given in Ref. 63 and may be cast into the notation of this thesis as:

$$\bar{\bar{P}}_{uu}(\bar{\beta}_{\perp}, \bar{k}_{\perp}) = \frac{\pi}{2}\,\Phi(\bar{k}_{\perp} - \bar{\beta}_{\perp}, \beta_{1mz} - k_{1mz})$$

$$\begin{bmatrix} (\hat{e}\cdot\hat{e}')^2 & (\hat{e}\cdot\hat{h}')^2 & (\hat{e}\cdot\hat{e}')(\hat{e}\cdot\hat{h}') & 0 \\ (\hat{h}\cdot\hat{e}')^2 & (\hat{h}\cdot\hat{h}')^2 & (\hat{h}\cdot\hat{e}')(\hat{h}\cdot\hat{h}') & 0 \\ 2(\hat{h}\cdot\hat{e}')(\hat{e}\cdot\hat{e}') & 2(\hat{h}\cdot\hat{h}')(\hat{e}\cdot\hat{h}') & (\hat{h}\cdot\hat{h}')(\hat{e}\cdot\hat{e}') + (\hat{e}\cdot\hat{h}')(\hat{h}\cdot\hat{e}') & 0 \\ 0 & 0 & 0 & (\hat{e}\cdot\hat{e}')(\hat{h}\cdot\hat{h}') - (\hat{e}\cdot\hat{h}')(\hat{h}\cdot\hat{e}') \end{bmatrix} \tag{9.35}$$

where $\hat{e} \equiv \hat{e}(\beta_{1mz})$, $\hat{h} \equiv \hat{h}(\beta_{1mz})$, $\hat{e}' \equiv \hat{e}(k_{1mz})$ and $\hat{h}' \equiv \hat{h}(k_{1mz})$.

Comparing (9.35) with (F.16)-(F.20), we note that the MRT scattering phase matrix has additional terms not present in the RT scattering phase matrix. These additional terms represent wave like corrections due to the bottom interface at $z = -d_1$. This can be seen by removing the effects of the bottom boundary (i.e. by letting $R_{12}, S_{12} \to 0$) in which case $\alpha_h, \alpha_v, \alpha_{c1} \to 1$, $\alpha_{c2} \to 0$ and the MRT scattering phase matrix (F.16) reduces identically to the RT result (9.35).

We now consider the MRT scattering coefficient matrix as given by (F.1) and (G.13). The RT scattering coefficient matrix is given in Reference 63, and in the notation of this thesis is given by

$$\bar{\bar{K}}(\theta) = \begin{bmatrix} K_h(\theta) & 0 & 0 & 0 \\ 0 & K_v(\theta) & 0 & 0 \\ 0 & 0 & \dfrac{(K_v(\theta) + K_h(\theta))}{2} & 0 \\ 0 & 0 & 0 & \dfrac{(K_v(\theta) + K_h(\theta)}{2} \end{bmatrix} \tag{9.36}$$

In Chapter 8 it was shown that,

$$2\mathrm{Im}[\lambda^{TE}]\cos\theta = K_h(\theta) \tag{9.37a}$$

$$2\mathrm{Im}[\lambda^{TM}]\cos\theta = K_v(\theta) \tag{9.37b}$$

where

$$\lambda^{TE}(\bar{k}_\perp) = \eta_h(\bar{k}_\perp) - k_{1mz} \tag{9.38a}$$

$$\lambda^{TM}(\bar{k}_\perp) = \eta_v(\bar{k}_\perp) - k_{1mz}. \tag{9.38b}$$

Therefore, it is clear that (G.13) reduces to the form

$$\bar{\bar{K}}(\Omega) = \begin{bmatrix} K_h(\theta) & 0 & 0 & 0 \\ & K_v(\theta) & 0 & 0 \\ 0 & 0 & \dfrac{(K_v(\theta) + K_h(\theta))}{2} & (\eta_v' - \eta_h') \\ 0 & 0 & -(\eta_v' - \eta_h') & \dfrac{(K_v(\theta) + K_h(\theta))}{2} \end{bmatrix} \tag{9.39}$$

Comparing (9.39) and (9.36) it is apparent that the MRT result (9.39) exhibits additional wave-like correction terms of the form $\pm(\eta_v' - \eta_h')$. Physically these terms arise from the fact that TE and TM polarized waves in general propagates in the random medium with distinct phase velocities, thereby affecting their cross correlation. However, should the random medium be such that TE and TM polarized waves have identical phase velocities, then the MRT result (9.39) reduces to the standard RT result (9.36).

As an application of the MRT equations (9.25a), (9.25b) and to illustrate the significance of the $\bar{\bar{Q}}_{\mu\nu}$ matrix terms (particularly the term with factor Δ_1) we compute the back-scattering cross section for a two-layer random medium by applying the first order renormalization approximation.[61] In the first order renormalization method, the scattering of the incoherent intensity is neglected in the Bethe-Salpeter equation. In this approximation the MRT equations reduce to

$$\frac{d}{dz} \bar{I}_u(z, \bar{\beta}_\perp) = - \bar{\bar{\eta}}(\bar{\beta}_\perp) \cdot \bar{I}_u(z, \bar{\beta}_\perp) + \frac{1}{|\beta_{1mz}|^2} \{\bar{\bar{Q}}_{uu}(\bar{\beta}_\perp, \bar{k}_{\perp i})$$

$$\cdot \bar{I}_{mu}(z, \bar{k}_{\perp i}) + \bar{\bar{Q}}_{ud}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{md}(z, \bar{k}_{\perp i})$$

$$+ \Delta_1 \bar{\bar{Q}}_{c1}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mc1}(z, \bar{k}_{\perp i})\} \tag{9.40a}$$

$$\frac{d}{dz}\bar{I}_d(z, \bar{\beta}_\perp) = \bar{\bar{\eta}}(\bar{\beta}_\perp) \cdot \bar{I}_d(z, \bar{\beta}_\perp) - \frac{1}{|\beta_{1mz}|^2}\{\bar{\bar{Q}}_{dd}(\bar{\beta}_\perp, \bar{k}_{\perp i})$$

$$\cdot \bar{I}_{md}(z, \bar{k}_{\perp i}) + \bar{\bar{Q}}_{du}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mu}(z, \bar{k}_{\perp i})$$

$$- \Delta_1 \bar{\bar{Q}}_{c1}(\bar{\beta}_\perp, \bar{k}_{\perp i}) \cdot \bar{I}_{mc2}(z, \bar{k}_{\perp i})\} \ . \tag{9.40b}$$

Since the z-dependence of the terms within the curly brackets of (9.40a) and (9.40b) is known, we can readily solve for $\bar{I}_u$ and $\bar{I}_d$ in conjunction with boundary conditions (9.29a), (9.29b). Moreover we consider the backscattering direction, $\bar{\beta}_\perp = -\bar{k}_{\perp i}$ in which case $\Delta_1 = 1$. The results are

$$|k_{1mzi}|^2 \; I_{uh}(0, -\bar{k}_{\perp i}) = \frac{\pi}{2} \frac{|X_{01i}|^2}{|D_{2i}|^4} f_e^{\,2} \Bigg[\Phi(2\bar{k}_{\perp i}, 2k_{1mzi})$$

$$\frac{(1 - e^{-4\eta''_{hi} d_1})}{4\eta''_{hi}} (1 + |R_{12i}|^4 \; e^{-4\eta''_{hi} d_1})$$

$$+ 4d_1 \; \Phi(2\bar{k}_{\perp i}, 0)|R_{12i}|^2 \; e^{-4\eta''_{hi} d_1} \Bigg] \tag{9.41a}$$

$$|k_{1mzi}|^2 \; I_{uv}(0, -\bar{k}_{\perp i}) = \frac{\pi}{2} \frac{k_o^{\,2}}{k_{1m}} \frac{|Y_{01i}|^2}{|F_{2i}|^4} f_m^{\,2} \Bigg[\Phi(2\bar{k}_{\perp i}, 2k_{1mzi})$$

$$\frac{(1 - e^{-4\eta''_{vi} d_1})}{4\eta''_{vi}} (1 + |S_{12i}|^4 e^{-4\eta''_{vi} d_1}) + 4d_1 \, \Phi(2\bar{k}_{\perp i}, 0)$$

$$|S_{12i}|^2 \frac{(k_{\perp i}^2 - k_{1mzi}^2)}{k_{1m}^2} e^{-4\eta''_{vi} d_1} \Big] \tag{9.41b}$$

where I_{uh} and I_{uv} signify respectively the first and second elements of $\bar{I}_u$. The intensity transmitted into region 0, is found by applying boundary condition (9.31) to (9.41a) and (9.41b).

$$I_{ouh} = |X_{10i}|^2 \, I_{uh}(0, -\bar{k}_{\perp i}) \tag{9.42a}$$

$$I_{ouv} = \left|\frac{\eta_o}{\eta_1} Y_{10i}\right|^2 I_{uv}(0, -\bar{k}_{\perp i}). \tag{9.42b}$$

The backscattering cross sections per unit area follow from the relation

$$\begin{Bmatrix} \sigma_{hh} \\ \sigma_{vv} \end{Bmatrix} = 4\pi \, k_o^2 \cos^2\theta_{oi} \begin{Bmatrix} I_{ouh} \\ I_{ouv} \end{Bmatrix} \tag{9.43}$$

with $f_e = 1$ in I_{ouh} and $f_m = 1$ in I_{ouv}.

We now consider a spectral density of the form

$$\Phi(\bar{\beta}_\perp, \beta_z) = \frac{\delta k_{1m}'^4 \ell_\rho^2 \ell_z}{4\pi^2(1 + \beta_z^2 \ell_z)} e^{-\beta_\perp^2 \ell_\rho^2/4} \tag{9.44}$$

which corresponds to a correlation function which is Gaussian laterally and exponential vertically. Combining equations (9.41a)-(9.43), we obtain the backscattering cross sections per unit area for a two-layer random medium in the first order renormalization:

$$\sigma_{hh} = \frac{\delta k_{1m}'^4 \ell_\rho^2 \ell_z}{4} \frac{|X_{10i}|^4}{|D_{2i}|^4} \frac{|k_{ozi}|^4}{|k_{1zi}|^4} e^{-k_o^2 \ell_\rho^2 \sin^2\theta_{oi}}$$

$$\left\{ \frac{(1 - e^{-4\eta_{hi}'' d_1})}{2\eta_{hi}''(1 + 4k_{1mzi}'^2 \ell_z^2)} (1 + |R_{12i}|^4 e^{-4\eta_{hi}'' d_1}) + 8d_1 |R_{12i}|^2 e^{-4\eta_{hi}'' d_1} \right\} \tag{9.45a}$$

$$\sigma_{vv} = \frac{\delta k_{1m}'^4 \ell_\rho^2 \ell_z}{4} \frac{|Y_{10i}|^4}{|F_{2i}|^4} \frac{|k_{ozi}|^4}{|k_{1zi}|^4} e^{-k_o^2 \ell_\rho^2 \sin^2\theta_{oi}}$$

$$\left\{ \frac{(1 - e^{-4\eta_{vi}'' d_1})}{2\eta_{vi}''(1 + 4k_{1mzi}'^2 \ell_z^2)} (1 + |S_{12i}|^4 e^{-4\eta_{vi}'' d_1}) \left| \frac{k_{1mzi}^2}{k_o^2} + \sin^2\theta_{oi} \right|^2 + 8d_1 |S_{12i}|^2 e^{-4\eta_{vi}'' d_1} \left| \frac{k_{1mzi}^2}{k_o^2} - \sin^2\theta_{oi} \right|^2 \right\} \tag{9.45b}$$

9.4 Illustration of Backscattering Cross-Sections

In order to illustrate the backscattering cross sections (9.45a) and (9.45b) obtained from MRT theory, we plot in Fig. 9.1 $\sigma_{hh} = \sigma_{vv} = \sigma$ at nadir, as a function of frequency for a 20 cm thick random layer. The coherent effects of MRT theory are apparent in the oscillatory behavior exhibited by the spectral dependence of σ. In Fig. 9.2 we plot σ as a function of frequency for the same parameters as Fig. 9.1 but with increased δ. We see that increased scattering dampens the interference pattern by shielding the bottom boundary to a greater extent.

A particularly significant coherent effect arises from the Δ_1 terms in (9.25a) and (9.25b). As discussed in Section 9.2 these terms constructively interfere only for $\bar{\beta}_\perp = \pm\bar{k}_{\perp i}$ (i.e. forward or backward scattering). Physically, this is illustrated in Fig. 9.3 where we have sketched the path lengths traversed by the single scattered mean field and its conjugate in the two cases of forward and backscattering. Constructive interference terms of this type are not included in phenomenological radiative transport theories. In order to gauge the error produced by omitting these constructive interference terms, we set $\Delta_1 = 0$ in (9.25a) and (9.25b) and rederive backscattering cross sections, σ_{hh} and

σ_{vv}. The results are similar to (9.45a) and (9.45b) except that the second terms within the curly brackets are reduced by a factor of 2. In the case of thin layers with low extinction the contribution of the Δ_1 terms can be significant. However, for thick lossy layer the error produced by omitting the Δ_1 terms is minimal.

Figure 9.1

$\sigma_{hh} = \sigma_{vv}$ as a function of frequency at nadir

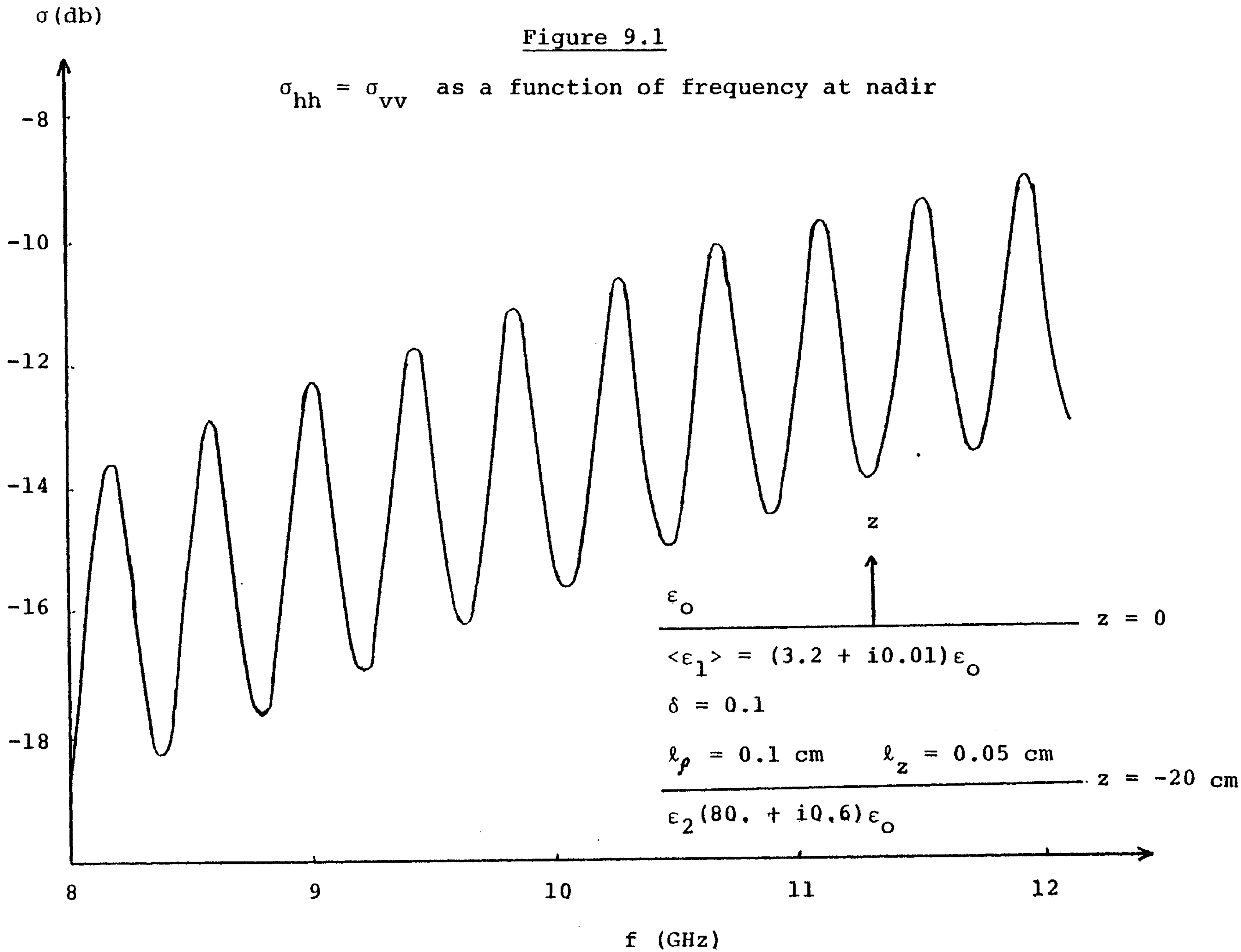
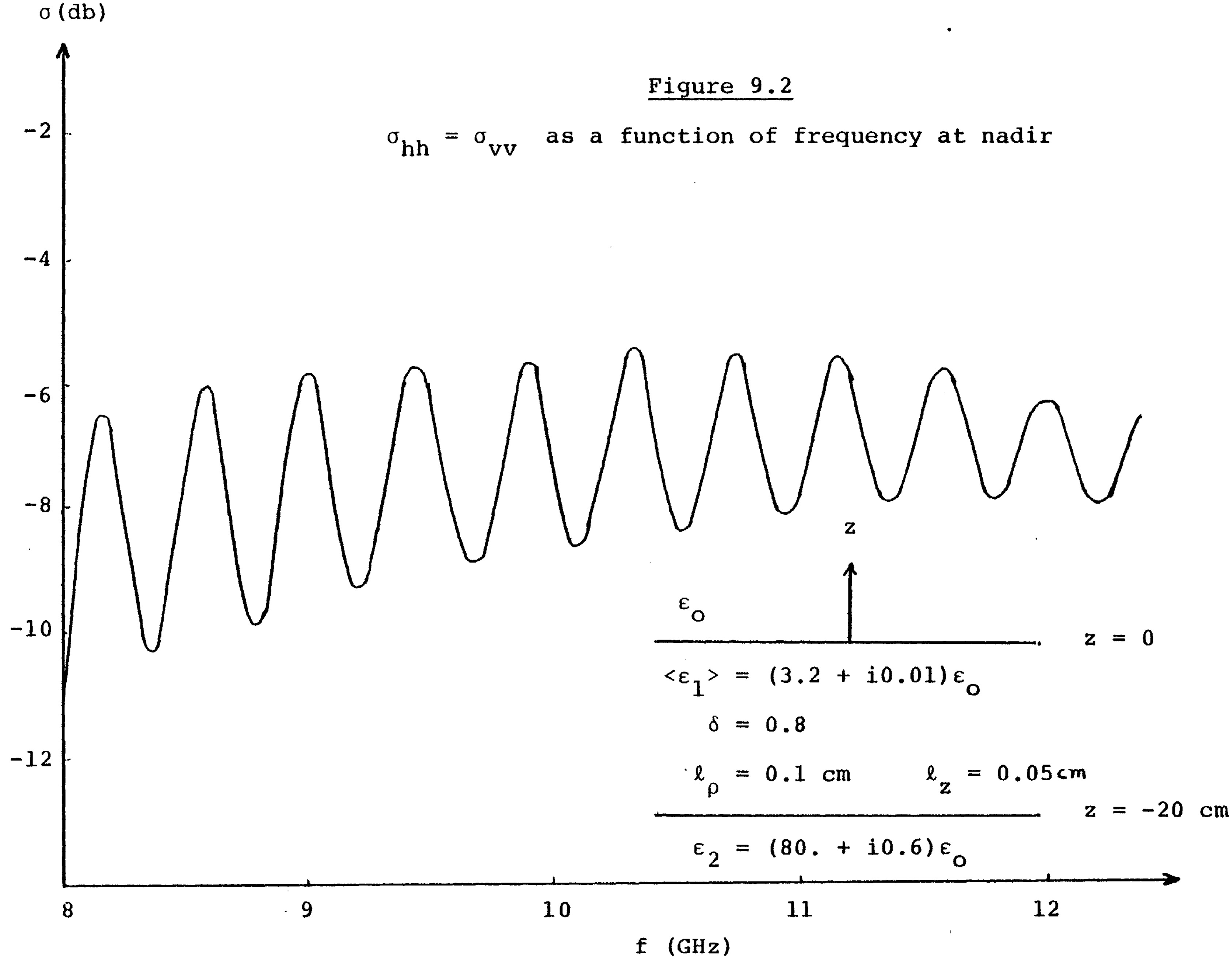
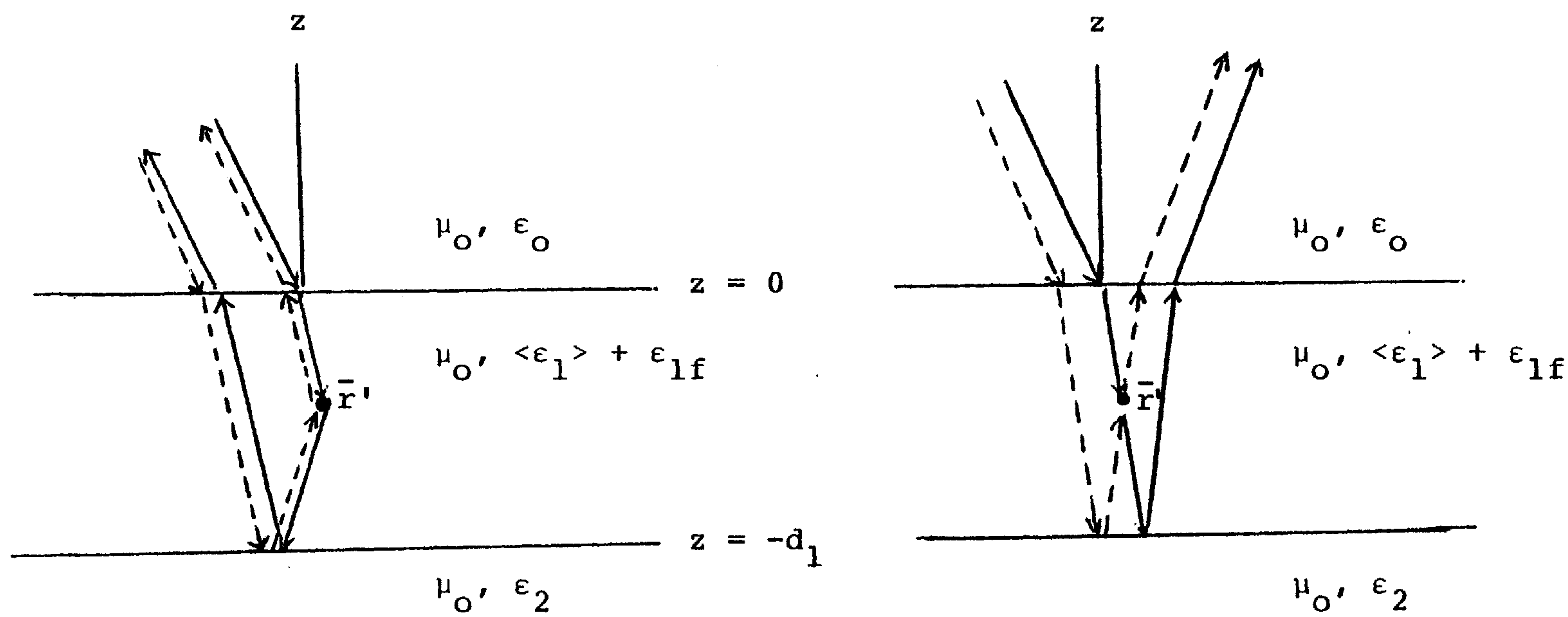


Figure 9.2

$\sigma_{hh} = \sigma_{vv}$ as a function of frequency at nadir





(a) Backscattering (b) Forward scattering

Constructive interference path lengths

Figure 9.3

9.5 Comparison with First Order Born Approximation Cross Sections

As another case of interest, we compare the backscattering cross sections of MRT theory, (9.45a) and (9.45b) with those obtained in the first Born approximation to the wave equation. In Chapter 3, the Born approximated cross sections for a two-layer random medium with spectral density (9.44) are given in (3.30a) and (3.30b). Comparing (3.30) with (9.45), the MRT results are seen to have the same form as the Born-wave results but with renormalized decay constants η''_{hi} and η''_{vi} in place of the decay constant k''_{1mzi}. In the limit of small permittivity fluctuations (small scattering) $\delta \to 0$ and η''_{hi}, $\eta''_{vi} \to k''_{1mzi}$ and the MRT backscattering cross sections reduce to the first order Born results. This is illustrated in Fig. 9.4 where for fixed incident angle θ_{oi}, frequency, f, and decay constant k''_{1m}, we plot backscattering cross sections, σ_{vv} (in db) as a function of the variance, δ, in both the wave-Born and MRT-first order renormalization approximations.

For large permittivity fluctuations the MRT result is bounded by the asymptote σ_{vv} = constant. Physically, this is due to a balance between the increased backscattering and increased shielding of the medium produced by large permittivity fluctuations. Alternatively, the Born result is unbounded as

δ increases. This follows from the absence of multiple scattering and the associated shielding effect in the first Born approximation.

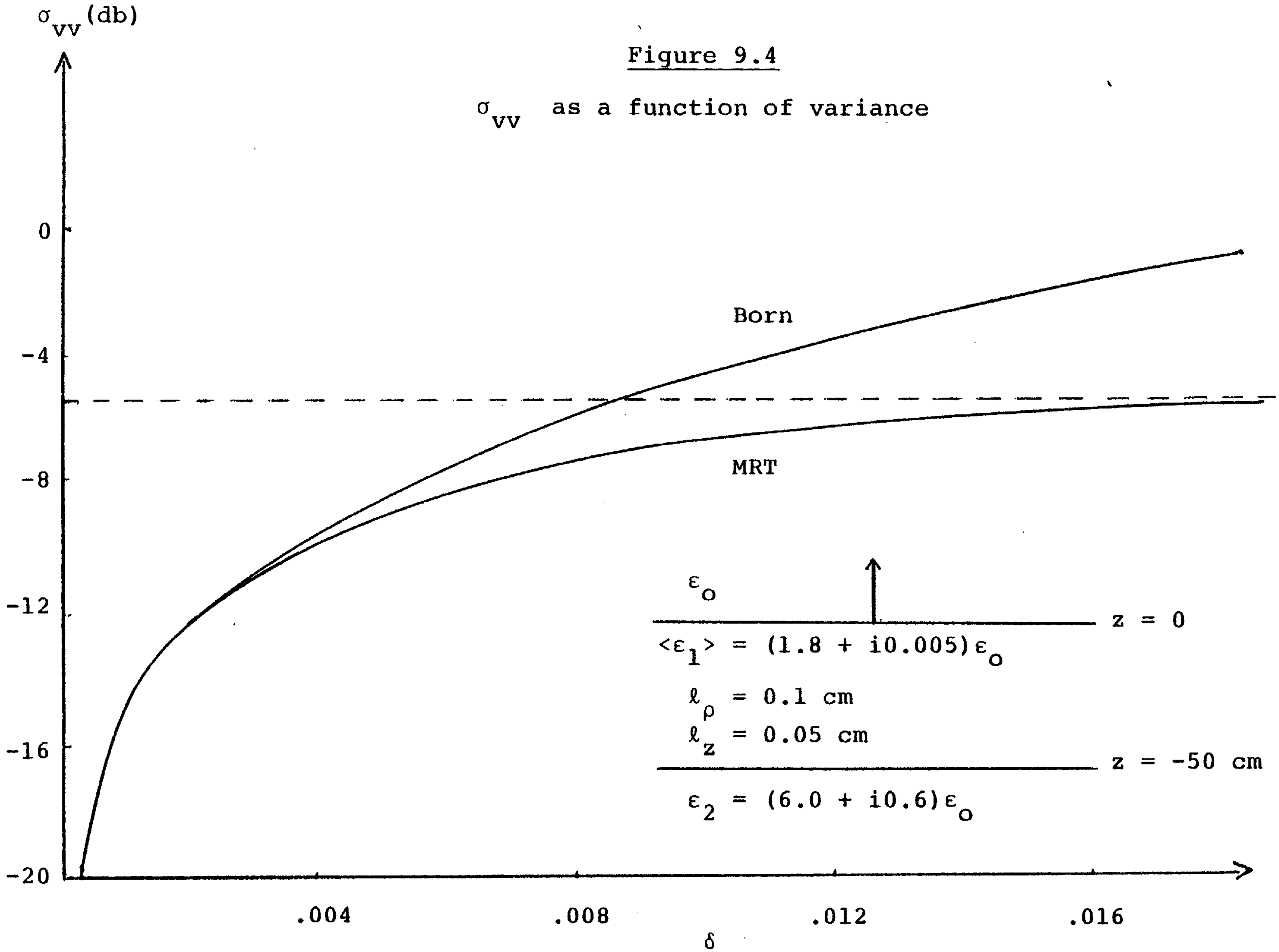


Figure 9.4

σ_{vv} as a function of variance

9.6 Appendices

APPENDIX A

$$A_1(k_\perp) = \frac{\sigma}{D_2(k_\perp)} \tag{A.1}$$

$$A_2(k_\perp) = \sigma \frac{R_{10}(k_\perp)}{D_2(k_\perp)} \tag{A.2}$$

$$B_1(k_\perp) = R_{12}(k_\perp)\ e^{i2\eta_h d_1} \tag{A.3}$$

$$C_1(k_\perp) = \frac{\sigma}{F_2(k_\perp)} \tag{A.4}$$

$$C_2(k_\perp) = \sigma \frac{S_{10}(k_\perp)}{F_2(k_\perp)} \tag{A.5}$$

$$D_1(k_\perp) = S_{12}(k_\perp)\ e^{i2\eta_v d_1} \tag{A.7}$$

where:

$$\sigma = -\frac{1}{(2\pi)^2\ 2ik_{1mz}} \tag{A.8}$$

$$D_2(k_\perp) = 1 + R_{01}(k_\perp)\ R_{12}(k_\perp)\ e^{i2\eta_h d_1} \tag{A.9}$$

$$F_2(k_\perp) = 1 + S_{01}(k_\perp)\ S_{12}(k_\perp)\ E^{i2\eta_v d_1} \tag{A.10}$$

$$R_{ij} = \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}} \tag{A.11}$$

$$S_{ij} = \frac{\varepsilon_j k_{iz} - \varepsilon_i k_{jz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}} \tag{A.12}$$

where $i, j = 0, 1, 2$ and k_{1z} as well as ε_1 are to be interpreted as k_{1mz} and ε_{1m}, respectively.

APPENDIX B

Here we list the amplitudes in Equation (9.2) for the zeroth order mean field in the two-layer random medium. Assuming a unit amplitude incident plane wave, we have:

$$E_{hui} = f_e \frac{X_{01i}}{D_{2i}} R_{12i} \, e^{i2\eta_{hi} d_1} \tag{B.1}$$

$$E_{hdi} = f_e \frac{X_{01i}}{D_{2i}} \tag{B.2}$$

$$E_{vui} = f_m \frac{k_o}{k_{1m}} \frac{Y_{01i}}{F_{2i}} S_{12i} \, e^{i2\eta_{vi} d_1} \tag{B.3}$$

$$E_{vdi} = f_m \frac{k_o}{k_{1m}} \frac{Y_{01i}}{F_{2i}} \tag{B.4}$$

$$X_{01i} = 1 + R_{01i} \tag{B.5}$$

$$Y_{01i} = 1 + S_{01i} \tag{B.6}$$

where f_e and f_m denote the fraction of TE and TM components in the incident wave.

APPENDIX C

The terms $I_j^{\gtrless}(z, z', \bar{\beta}_\perp)$ $(j = 1, 2, \ldots, 12)$ which appear in Equations (13a)-(14d) are listed. For $j = 1, 2, 3, 4$, $I_j^{\gtrless}$ has the generic form:

$$I_j^{>}(z, z', \bar{\beta}_\perp) = (2\pi)^4 \int_{-\infty}^{\infty} d\alpha_z\, \Phi(\bar{k}_{\perp i} - \bar{\beta}_\perp, \alpha_z) \int_{-d_1}^{z} dz_1$$

$$\int_{-d_1}^{z'} dz_2\, e^{-i\alpha_z(z_1 - z_2)} \{\bar{P}_j^{>}(z_1, \bar{\beta}_\perp) \cdot \bar{E}_{1m}(z_1, \bar{k}_{\perp i})\}$$

$$\{\bar{Q}_j^{>*}(z_2, \bar{\beta}_\perp) \cdot \bar{E}_{1m}^{*}(z_2, \bar{k}_{\perp i})\} \qquad \text{(C.1)}$$

where

$$\bar{E}_{1m}(z, \bar{k}_{\perp i}) = E_{hui}\, e^{i\eta_{hi}z}\, \hat{e}(k_{1mzi}) + E_{hdi}\, e^{-i\eta_{hi}z}\, \hat{e}(-k_{1mzi})$$

$$+ E_{vui}\, e^{i\eta_{vi}z}\, \hat{h}(k_{1mzi}) + E_{vdi}\, e^{-i\eta_{vi}z}\, \hat{h}(-k_{1mzi}). \qquad \text{(C.2)}$$

The coefficients in (C.2) are given in Appendix B. $I_j^{<}(z, z', \bar{\beta}_\perp)$ has the same form as (C.1) except the integration ranges become $z < z_1 < 0$, $z' < z_2 < 0$ and $\bar{P}_j^{>}$, $\bar{Q}_j^{>}$ are replaced by $\bar{P}_j^<$, $\bar{Q}_j^<$.

$$\bar{P}_1^>(z, \bar{\beta}_\perp) = B_1(\beta_\perp)\,\hat{e}(-\beta_{1mz})\,e^{i\eta_h(\bar{\beta}_\perp)z} + \hat{e}(\beta_{1mz})\,e^{-i\eta_h(\bar{\beta}_\perp)z} \tag{C.3}$$

$$\bar{Q}_1^>(z, \bar{\beta}_\perp) = \bar{P}_1^>(z, \bar{\beta}_\perp) \tag{C.4}$$

$$\bar{P}_1^<(z, \bar{\beta}_\perp) = A_1(\beta_\perp)\,\hat{e}(-\beta_{1mz})\,e^{i\eta_h(\bar{\beta}_\perp)z} + A_2(\beta_\perp)\,\hat{e}(\beta_{1mz})\,e^{-i\eta_h(\bar{\beta}_\perp)z} \tag{C.5}$$

$$\bar{Q}_1^<(z, \bar{\beta}_\perp) = \bar{P}_1^<(z, \bar{\beta}_\perp) \tag{C.6}$$

$$\bar{P}_2^>(z, \bar{\beta}_\perp) = D_1(\beta_\perp)\,\hat{h}(-\beta_{1mz})\,e^{i\eta_v(\bar{\beta}_\perp)z} + \hat{h}(\beta_{1mz})\,e^{-i\eta_v(\bar{\beta}_\perp)z} \tag{C.7}$$

$$\bar{Q}_2^>(z, \bar{\beta}_\perp) = \bar{P}_2^>(z, \bar{\beta}_\perp) \tag{C.8}$$

$$\bar{P}_2^<(z, \bar{\beta}_\perp) = C_1(\beta_\perp)\,\hat{h}(-\beta_{1mz})\,e^{i\eta_v(\bar{\beta}_\perp)z} + C_2(\beta_\perp)\,\hat{h}(\beta_{1mz})$$

$$e^{-i\eta_v(\beta_\perp)z} \tag{C.9}$$

$$\bar{Q}_2^{<}(z,\ \bar{\beta}_\perp) = \bar{P}_2^{<}(z,\ \bar{\beta}_\perp) \tag{C.10}$$

$$\bar{P}_3^{>}(z,\ \bar{\beta}_\perp) = \bar{P}_1^{>}(z,\ \bar{\beta}_\perp) \tag{C.11}$$

$$\bar{Q}_3^{>}(z,\ \bar{\beta}_\perp) = \bar{P}_2^{>}(z,\ \bar{\beta}_\perp) \tag{C.12}$$

$$\bar{P}_3^{<}(z,\ \bar{\beta}_\perp) = \bar{P}_1^{<}(z,\ \bar{\beta}_\perp) \tag{C.13}$$

$$\bar{Q}_3^{>}(z,\ \bar{\beta}_\perp) = \bar{P}_2^{<}(z,\ \bar{\beta}_\perp) \tag{C.14}$$

$$\bar{P}_4^{>}(z,\ \bar{\beta}_\perp) = \bar{P}_1^{>}(z,\ \bar{\beta}_\perp) \tag{C.15}$$

$$\bar{Q}_4^{>}(z,\ \beta_\perp) = \bar{P}_2^{>}(z,\ \bar{\beta}_\perp) \tag{C.16}$$

$$\bar{P}_4^{<}(z,\ \bar{\beta}_\perp) = \bar{P}_1^{<}(z,\ \bar{\beta}_\perp) \tag{C.17}$$

$$\bar{Q}_4^{<}(z,\ \bar{\beta}_\perp) = \bar{P}_2^{<}(z,\ \bar{\beta}_\perp). \tag{C.18}$$

For $j = 5, 6, \ldots, 12$ the $I_j^<$ terms which appear in (9.10a)-(9.11d) have the generic form:

$$I_j^>(z, z', \bar{\beta}_\perp) = (2\pi)^4 \int_{-\infty}^{\infty} d\alpha_z \int_{-\infty}^{\infty} d^2k_\perp \; \phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_z)$$

$$\int_{-d_1}^{z} dz_1 \int_{-d_1}^{z'} dz_2 \; e^{-i\alpha_z(z_1 - z_2)} \; \bar{F}_j^>(z_1, \bar{\beta}_\perp)$$

$$\cdot \{\bar{\bar{\Gamma}}_u(z_1, z_2, \bar{k}_\perp) \; e^{ik'_{1mz}(z_1 - z_2)}$$

$$+ \bar{\bar{\Gamma}}_d(z_1, z_2, \bar{k}_\perp) \; e^{-ik'_{1mz}(z_1 - z_2)}$$

$$+ \bar{\bar{\Gamma}}_{c1}(z_1, z_2, \bar{k}_\perp) \; e^{ik'_{1mz}(z_1 + z_2)}$$

$$+ \bar{\bar{\Gamma}}_{c2}(z_1, z_2, \bar{k}_\perp) \; e^{-ik'_{1mz}(z_1 + z_2)}\} \cdot \bar{H}_j^{>*}(z_2, \bar{\beta}_\perp) \qquad \text{(C.19)}$$

where the dyads $\bar{\bar{\Gamma}}$ are defined in (9.7a)-(9.7d). The expression for $I_j^<$ $(j = 1, 2, \ldots, 12)$ is identical to (C.19) except the ranges of the z_1, z_2 integrations become $z < z_1 < 0$, $z' < z_2 < 0$, and $\bar{F}_j^>$ and $\bar{H}_j^>$ are replaced by $\bar{F}_j^<$ and $\bar{H}_j^>$.

$$\bar{F}_5^{>}(z,\ \bar{\beta}_\perp) = \bar{H}^{>}(z,\ \bar{\beta}_\perp) = A_1(\beta_\perp)\ B_1(\beta_\perp)\ e^{i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(-\beta_{1mz})$$

$$+ A_1(\beta_\perp)\ e^{-i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(\beta_{1mz}) \tag{C.20}$$

$$\bar{F}_5^{<}(z,\ \bar{\beta}_\perp) = \bar{H}_5^{<}(z,\ \bar{\beta}_\perp) = A_1(\beta_\perp)\ B_1(\beta_\perp)\ e^{i\eta_h(\beta_\perp)z}\ \hat{e}(-\beta_{1mz})$$

$$+ B_1(\beta_\perp)\ A_2(\beta_\perp)\ e^{-i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(\beta_{1mz}) \tag{C.21}$$

$$\bar{F}_6^{>}(z,\ \bar{\beta}_\perp) = \bar{H}_6^{>}(z,\ \bar{\beta}_\perp) = C_1(\beta_\perp)\ D_1(\beta_\perp)\ e^{i\eta_v(\bar{\beta}_\perp)z}\ \hat{h}(-\beta_{1mz})$$

$$+ C_1(\beta_\perp)\ e^{-i\eta_v(\bar{\beta}_\perp)z}\ \hat{h}(\beta_{1mz}) \tag{C.22}$$

$$\bar{F}_6^{<}(z,\ \bar{\beta}_\perp) = \bar{H}_6^{<}(z,\ \bar{\beta}_\perp) = C_1(\beta_\perp)\ D_1(\beta_\perp)\ e^{i\eta_v(\bar{\beta}_\perp)z}\ \hat{h}(-\beta_{1mz})$$

$$+ D_1(\beta_\perp)\ C_2(\beta_\perp)\ e^{-i\eta_v(\bar{\beta}_\perp)z}\ \hat{h}(\beta_{1mz}) \tag{C.23}$$

$$\bar{F}_7^{>}(z,\ \bar{\beta}_\perp) = \bar{F}_5^{>}(z,\ \bar{\beta}_\perp) \tag{C.24}$$

$$\bar{H}_7^{>}(z,\ \bar{\beta}_\perp) = \bar{F}_6^{>}(z,\ \bar{\beta}_\perp) \tag{C.25}$$

$$\bar{F}_7^{<}(z, \bar{\beta}_\perp) = \bar{F}_5^{<}(z, \bar{\beta}_\perp) \tag{C.26}$$

$$\bar{H}_7^{<}(z, \bar{\beta}_\perp) = \bar{F}_6^{<}(z, \bar{\beta}_\perp) \tag{C.27}$$

$$\bar{F}_8^{>}(z, \bar{\beta}_\perp) = \bar{F}_6^{>}(z, \bar{\beta}_\perp) \tag{C.28}$$

$$\bar{H}_8^{>}(z, \bar{\beta}_\perp) = \bar{F}_5^{>}(z, \bar{\beta}_\perp) \tag{C.29}$$

$$\bar{F}_8^{<}(z, \bar{\beta}_\perp) = \bar{F}_6^{<}(z, \bar{\beta}_\perp) \tag{C.30}$$

$$\bar{H}_8^{<}(z, \bar{\beta}_\perp) = \bar{F}_5^{<}(z, \bar{\beta}_\perp) \tag{C.31}$$

$$\bar{F}_9^{>}(z, \bar{\beta}_\perp) = \bar{H}_9^{>}(z, \bar{\beta}_\perp) = A_2(\beta_\perp)\ e^{-i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(\beta_{1mz}) + B_1(\beta_\perp)\ A_2(\beta_\perp)\ e^{i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(-\beta_{1mz}) \tag{C.32}$$

$$\bar{F}_9^{<}(z, \bar{\beta}_\perp) = \bar{H}_9^{<}(z, \bar{\beta}_\perp) = A_2(\beta_\perp)\ e^{-i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(\beta_{1mz}) + B_2(\beta_\perp)\ A_1(\beta_\perp)\ e^{i\eta_h(\bar{\beta}_\perp)z}\ \hat{e}(-\beta_{1mz}) \tag{C.33}$$

$$\bar{F}^{>}_{10}(z, \bar{\beta}_{\perp}) = \bar{H}^{>}_{10}(z, \bar{\beta}_{\perp}) = C_2(\beta_{\perp})\, e^{-i\eta_v(\bar{\beta}_{\perp})z}\, \hat{h}(\beta_{1mz})$$

$$+ C_2(\beta_{\perp})\, D_1(\beta_{\perp})\, e^{i\eta_v(\bar{\beta}_{\perp})z}\, \hat{h}(-\beta_{1mz}) \quad (C.34)$$

$$F^{<}_{10}(z, \bar{\beta}_{\perp}) = \bar{H}^{<}_{10}(z, \beta_{\perp}) = C_2(\beta_{\perp})\, e^{-i\eta_v(\bar{\beta}_{\perp})z}\, \hat{h}(\beta_{1mz})$$

$$+ C_1(\beta_{\perp})\, e^{i\eta_v(\bar{\beta}_{\perp})z}\, \hat{h}(-\beta_{1mz}) \quad (C.35)$$

$$\bar{F}^{>}_{11}(z, \bar{\beta}_{\perp}) = \bar{F}_9^{>}(z, \bar{\beta}_{\perp}) \quad (C.36)$$

$$\bar{H}^{>}_{11}(z, \bar{\beta}_{\perp}) = \bar{F}^{>}_{10}(z, \bar{\beta}_{\perp}) \quad (C.37)$$

$$\bar{F}^{<}_{11}(z, \bar{\beta}_{\perp}) = \bar{F}_9^{<}(z, \beta_{\perp}) \quad (C.38)$$

$$\bar{H}^{<}_{11}(z, \bar{\beta}_{\perp}) = \bar{F}^{<}_{10}(z, \bar{\beta}_{\perp}) \quad (C.39)$$

$$\bar{F}^{>}_{12}(z, \bar{\beta}_{\perp}) = \bar{F}^{>}_{10}(z, \bar{\beta}_{\perp}) \quad (C.40)$$

$$\bar{H}^{>}_{12}(z, \bar{\beta}_{\perp}) = \bar{F}_9^{>}(z, \bar{\beta}_{\perp}) \quad (C.41)$$

$$\bar{F}_{12}^{<}(z,\ \bar{\beta}_{\perp}) = \bar{F}_{10}^{<}(z,\ \bar{\beta}_{\perp}) \tag{C.42}$$

$$\bar{H}_{12}^{<}(z,\ \bar{\beta}_{\perp}) = \bar{F}_{9}^{<}(z,\ \bar{\beta}_{\perp}). \tag{C.43}$$

APPENDIX D

$$M_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}^2 \tag{D.1}$$

$$M_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}^2 \tag{D.2}$$

$$M_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2 \tag{D.3}$$

$$M_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2 \tag{D.4}$$

$$M_{5n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \tag{D.5}$$

$$M_{6n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \tag{D.6}$$

$$M_{7n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} \tag{D.7}$$

$$M_{8n}(\bar{\beta}_\perp, \bar{k}_\perp) = f^+ \cdot \{e(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}$$

(D.8)

$$N_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}^2 \qquad (D.9)$$

$$N_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}^2 \qquad (D.10)$$

$$N_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}^2 \qquad (D.11)$$

$$N_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}^2 \qquad (D.12)$$

$$N_{5n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}$$

(D.13)

$$N_{6n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}$$

(D.14)

$$N_{7n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^- \cdot \{\hat{e}(-\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(-\beta_{1mz})\}$$

(D.15)

$$N_{8n}(\bar{\beta}_\perp, \bar{k}_\perp) = g^+ \cdot \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(-k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}$$

(D.16)

where:

$$f^\pm \equiv \frac{C_\pm}{(\alpha_n^- - k'_{1mz} \pm \beta^*_{1mz})(\alpha_n^+ - k'_{1mz} \pm \beta^*_{1mz})} \tag{D.17}$$

$$g^\pm \equiv \frac{C_\pm}{(\alpha_n^- + k'_{1mz} \pm \beta^*_{1mz})(\alpha_n^+ + k'_{1mz} \pm \beta^*_{1mz})} \tag{D.18}$$

$$C_+ \equiv |A_1(\beta_\perp)\ B_1(\beta_\perp)|^2 \tag{D.19}$$

$$C_- \equiv |A_1(\beta_\perp)|^2 \tag{D.20}$$

Also,

$$W_{1n}(\bar{\beta}_\perp, \bar{k}_\perp) = b\left(\frac{g^+}{C_+}\right)\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}$$

(D.21)

$$W_{2n}(\bar{\beta}_\perp, \bar{k}_\perp) = b^*\left(\frac{g^-}{C_-}\right)\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}$$

(D.22)

$$W_{3n}(\bar{\beta}_\perp, \bar{k}_\perp) = b\left(\frac{f^+}{C_+}\right)\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} \tag{D.23}$$

$$W_{4n}(\bar{\beta}_\perp, \bar{k}_\perp) = b^*\left(\frac{f^-}{C_-}\right)\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(-k_{1mz})\}\{\hat{e}(-\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} \tag{D.24}$$

where:

$$b = |A_1(\beta_\perp)|^2 \, B_1(\beta_\perp).$$

APPENDIX E

In this appendix, we illustrate the technique used to sum the residue terms which appear in (9.14). Referring to equation (9.14) and Appendix D, a typical sum over residues has the form:

$$\sum_n \frac{\text{Res } \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_n^+)}{(\alpha_n^- - z_p)(\alpha_n^+ - z_p)} \tag{E.1}$$

where the complex variable z_p represents $(k'_{lmz} + \beta^*_{lmz})$ or $(k'_{lmz} - \beta^*_{lmz})$ or $-(k'_{lmz} + \beta^*_{lmz})$ or $-(k'_{lmz} - \beta^*_{lmz})$, according to the specific term in (9.14) we are trying to sum. Let $\text{Im}(z_p) > 0$ and consider the following integral, along the real α_z axis:

$$I \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\alpha_z \frac{\Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_z)}{(\alpha_z - z_p)} . \tag{E.2}$$

The spectral density Φ has an equal distribution of poles $\{\alpha_n^+\}$ and $\{\alpha_n^-\}$ in the upper and lower half complex-α_z plane and vanishes everywhere on the circle at infinity. In this case we evaluate (E.2) by closing the contour both up and down in the complex α_z plane:

$$I_{up} = \Phi(\bar{k}_\perp - \bar{\beta}_\perp, z_p) + \sum_n \frac{\text{Res } \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_n^+)}{\alpha_n^+ - z_p} \tag{E.3}$$

$$I_{down} = - \sum_n \frac{\text{Res } \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_n^-)}{\alpha_n^- - z_p} . \tag{E.4}$$

Equating I_{up} and I_{down}, we obtain

$$\Phi(\bar{k}_\perp - \bar{\beta}_\perp, z_p) = \sum_n \text{Res } \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \alpha_n^+) \frac{(\alpha_n^- - \alpha_n^+)}{(\alpha_n^- - z_p)(\alpha_n^+ - z_p)} \tag{E.5}$$

where we have made use of Equations (9.15). Result (E.5) is unchanged if the initial assumption $\text{Im}(z_p) > 0$ is replaced by $\text{Im}(z_p) < 0$. Therefore, (E.5) may be used to sum the residue terms of the form (E.1) which appear in (9.14).

APPENDIX F

Here we list the matrices of the MRT equations (9.25a) and (9.25b).

$$\bar{\bar{\eta}}(\bar{\beta}_\perp) = \begin{bmatrix} 2\eta_h''(\bar{\beta}_\perp) & 0 & 0 & 0 \\ 0 & 2\eta_v''(\bar{\beta}_\perp) & 0 & 0 \\ 0 & 0 & (\eta_v'' + \eta_h'') & (\eta_v' - \eta_h') \\ 0 & 0 & -(\eta_v' - \eta_h') & (\eta_v'' + \eta_h'') \end{bmatrix}$$

(F.1)

where $\eta_{\substack{h\\v}}'' \equiv \mathrm{Im}(\eta_{\substack{h\\v}})$ and $\eta_{\substack{h\\v}}' \equiv \mathrm{Re}(\eta_{\substack{h\\v}})$.

$$\bar{\bar{Q}}_{uu}(\bar{\beta}_\perp, \bar{k}_{\perp i}) = \frac{\pi}{2}\,\Phi(\bar{k}_{\perp i} - \bar{\beta}_\perp, \beta_{1mz} - k_{1mzi})$$

$$\begin{bmatrix} \alpha_h\{\hat{e}(\beta_{1mz})\cdot\hat{e}(k_{1mzi})\}^2 & \alpha_h\{\hat{e}(\beta_{1mz})\cdot\hat{h}(k_{1mzi})\}^2 & 0 & 0 \\ \alpha_v\{\hat{h}(\beta_{1mz})\cdot\hat{e}(k_{1mzi})\}^2 & \alpha_v\{\hat{h}(\beta_{1mz})\cdot\hat{h}(k_{1mzi})\}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(F.2)

$$\bar{\bar{Q}}_{ud}(\bar{\beta}_\perp, \bar{k}_{\perp i}) = \text{same as } \bar{\bar{Q}}_{uu} \text{ except let } k_{lmzi} \to -k_{lmzi} \quad (F.3)$$

$$\bar{\bar{Q}}_{dd}(\bar{\beta}_\perp, \bar{k}_{\perp i}) = \text{same as } \bar{\bar{Q}}_{uu} \text{ except let } \beta_{lmz} \to -\beta_{lmz}$$

$$\text{and } k_{lmzi} \to -k_{lmzi}. \quad (F.4)$$

$$\bar{\bar{Q}}_{du}(\bar{\beta}_\perp, \bar{k}_{\perp i}) = \text{same as } \bar{\bar{Q}}_{uu} \text{ except let } \beta_{lmz} \to -\beta_{lmz} \quad (F.5)$$

$$\bar{\bar{Q}}_{cl}(\bar{\beta}_\perp, \bar{k}_{\perp i}) = \frac{\pi}{2}\,\Phi(\bar{k}_{\perp i} - \bar{\beta}_\perp, 0)$$

$$\cdot \begin{bmatrix} \begin{matrix}\{\hat{e}(\beta_{lmz}) \cdot \hat{e}(k_{lmzi})\} \\ \{\hat{e}(-\beta_{lmz}) \cdot \hat{e}(-k_{lmzi})\}\end{matrix} & 0 & 0 & 0 \\ 0 & \begin{matrix}\{\hat{h}(\beta_{lmz}) \cdot \hat{h}(k_{lmzi})\} \\ \{\hat{h}(-\beta_{lmz}) \cdot \hat{h}(-k_{lmzi})\}\end{matrix} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(F.6)$$

$$\bar{I}_{\substack{mu\\d}}(z, \bar{\beta}_\perp) = \begin{bmatrix} |E_{\substack{hui\\d}}|^2 \, e^{\mp 2\eta''_{hi}(\bar{\beta}_\perp)z} \\ |E_{\substack{vui\\d}}|^2 \, e^{\mp 2\eta''_{vi}(\bar{\beta}_\perp)z} \\ 0 \\ 0 \end{bmatrix} \tag{F.7}$$

$$\bar{I}_{mc1}(z, \bar{k}_{\perp i}) = \begin{bmatrix} 2\mathrm{Re}\{E_{hui} \, E^*_{hdi} \, \gamma_{hi}\} \, e^{-2\eta''_{hi}z} \\ 2\mathrm{Re}\{E_{vui} \, E^*_{vdi} \, \gamma_{vi}\} \, e^{-2\eta''_{vi}z} \\ 0 \\ 0 \end{bmatrix} \tag{F.8}$$

$$\bar{I}_{mc2}(z, \bar{k}_{\perp i}) = \begin{bmatrix} 2\mathrm{Re}\{E_{hui} \, E^*_{hdi} \, \tilde{\gamma}_{hi}\} \, e^{2\eta''_{hi}z} \\ 2\mathrm{Re}\{E_{vui} \, E^*_{vdi} \, \tilde{\gamma}_{vi}\} \, e^{2\eta_{vi}z} \\ 0 \\ 0 \end{bmatrix} \tag{F.9}$$

where:

$$\alpha_h = \frac{1 - |R_{10}(\beta_\perp)|^2 |R_{12}(\beta_\perp)|^2 \, e^{-4\eta_h''(\bar{\beta}_\perp)d_1}}{|D_2(\beta_\perp)|^2} \tag{F.10}$$

$$\alpha_v = \frac{1 - |S_{10}(\beta_\perp)|^2 |S_{12}(\beta_\perp)|^2 \, e^{-4\eta_v''(\beta_\perp)d_1}}{|F_2(\beta_\perp)|^2} \tag{F.11}$$

$$\gamma_{hi} = R_{12i}^* \, e^{-i2\eta_h^* d_1} \, \frac{(1 + R_{01i}R_{12i} \, e^{i2\eta_{hi}d_1})}{|D_{2i}|^2} \tag{F.12}$$

$$\gamma_{vi} = S_{12i}^* \, e^{-i2\eta_{vi}^* d_1} \, \frac{(1 + S_{01i}S_{12i} \, e^{i2\eta_{vi}d_1})}{|F_{2i}|^2} \tag{F.13}$$

$$\tilde{\gamma}_{hi} = - R_{10i} \frac{D_{2i}^*}{|D_{2i}|^2} \tag{F.14}$$

$$\tilde{\gamma}_{vi} = - S_{10i} \frac{F_{2i}^*}{|F_{2i}|^2} \; . \tag{F.15}$$

We also have:

$$\bar{\bar{P}}_{uu}(\bar{\beta}_\perp, \bar{k}_\perp) = \frac{\pi}{2} \, \Phi(\bar{k}_\perp - \bar{\beta}_\perp, \beta_{1mz} - k_{1mz}) \left[\begin{array}{c|c} \bar{\bar{A}} & \bar{\bar{B}} \\ \hline \bar{\bar{C}} & \bar{\bar{D}} \end{array}\right] \tag{F.16}$$

where:

$$\bar{\bar{A}} = \begin{bmatrix} \alpha_h\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}^2 & \alpha_h\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(\beta_{1mz})\}^2 \\ \alpha_v\{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}^2 & \alpha_v\{h(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2 \end{bmatrix} \quad \text{(F.17)}$$

$$\bar{\bar{B}} = \begin{bmatrix} \alpha_h\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2 & 0 \\ \alpha_v\{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}^2 & 0 \end{bmatrix} \quad \text{(F.18)}$$

$$\bar{\bar{C}} = \begin{bmatrix} 2\alpha_{c1}\{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} & 2\alpha_{c1}\{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \\ -2\alpha_{c2}\{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} & -2\alpha_{c2}\{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\}\{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \end{bmatrix}$$

(F.19)

$$\bar{\bar{D}} = \begin{bmatrix} \alpha_{c1}[\{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} & \alpha_{c2}[\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} \\ \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} & \{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \\ + \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} & - \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \\ \{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}] & \{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}] \\ & \\ -\alpha_{c2}[\{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} & \alpha_{c1}[\{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} \\ \{\hat{e}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\} & \{\hat{h}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \\ + \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} & - \{\hat{e}(\beta_{1mz}) \cdot \hat{h}(k_{1mz})\} \\ \{\hat{h}(k_{1mz}) \cdot \hat{e}(k_{1mz})\}] & \{\hat{h}(\beta_{1mz}) \cdot \hat{e}(k_{1mz})\}] \end{bmatrix}$$

(F.20)

$\bar{\bar{P}}_{ud}(\bar{\beta}_\perp, \bar{k}_\perp)$ = same as $\bar{\bar{P}}_{uu}(\bar{\beta}_\perp, \bar{k}_\perp)$ except let $k_{1mz} \to -k_{1mz}$

(F.21)

$\bar{\bar{P}}_{du}(\bar{\beta}_\perp, \bar{k}_\perp)$ = same as $\bar{\bar{P}}_{uu}(\bar{\beta}_\perp, \bar{k}_\perp)$ except let $\beta_{1mz} \to -\beta_{1mz}$

(F.22)

$\bar{\bar{P}}_{dd}(\bar{\beta}_\perp, \bar{k}_\perp)$ = same as $\bar{\bar{P}}_{uu}(\bar{\beta}_\perp, \bar{k}_\perp)$ except let $\beta_{1mz} \to -\beta_{1mz}$

and $k_{1mz} \to -k_{1mz}$ (F.23)

$$\alpha_{c1} = \mathrm{Re}\left[\frac{1 - R_{01}R_{12}\ S_{01}^* S_{12}^*\ e^{i2(\eta_h - \eta_v^*)d_1}}{D_2 F_2^*}\right] \quad (F.24)$$

$$\alpha_{c2} = \mathrm{Im}\left[\frac{1 - R_{01}R_{12}\ S_{01}^* S_{12}^*\ e^{i2(\eta_h - \eta_v^*)d_1}}{D_2 F_2^*}\right] \quad (F.25)$$

where Appendices A, B and C should be referenced for the definition of the variables in (F.1)-(F.25).

APPENDIX G

In this appendix we convert the MRT equations (9.25a) and (9.25b) to standard form by using solid angles and specific intensities. To illustrate we consider the angular decomposition of the upward Poynting flux associated with the vertical polarization:

$$S_{vu} = \sqrt{\varepsilon_{1m}/\mu} \int d^2\beta_\perp \langle \varepsilon_{vu}(z, \bar{\beta}_\perp)\, \varepsilon^*_{vu}(z, \bar{\beta}_\perp) \rangle$$

$$= \sqrt{\varepsilon_{1m}/\mu} \int d^2\beta_\perp\, J_{vvu}(z, z, \bar{\beta}_\perp). \qquad (G.1)$$

Let

$$\beta_x = \beta_{1m} \sin\theta \cos\phi \qquad (G.2)$$

$$\beta_y = \beta_{1m} \sin\theta \sin\phi \qquad (G.3)$$

then the integral operator $\int d^2\beta_\perp$ becomes

$$\int d^2\beta_\perp = \int_0^{\pi/2} \sin\theta d\theta \int_0^{2\pi} d\phi \beta^2_{1m} \cos\theta \equiv \int d\Omega \beta^2_{1m} \cos\theta \qquad (G.4)$$

where we have neglected the evanescent portion of $\bar{\beta}_\perp$ space. Therefore (G.1) becomes:

$$S_{vu} = \int d\Omega \; [\sqrt{\varepsilon_{1m}/\mu} \; \beta_{1m}^2 \cos\theta \; J_{vvu}(z, z, \bar{\beta}_\perp)] \; . \qquad (G.5)$$

We define the quantity within the square brackets of (G.5) as the upward specific intensity, $\mathcal{I}_{vu}(z, \Omega)$ of the vertical polarization. Therefore we define upward and downward specific intensity matrices $\bar{\mathcal{I}}_{\substack{u\\d}}(z, \Omega)$ as:

$$\bar{\mathcal{I}}_{\substack{u\\d}}(z, \Omega) = \frac{\beta_{1m}^2}{\eta_1} \cos\theta \; \bar{I}_{\substack{u\\d}}(z, \bar{\beta}_\perp) \qquad (G.6)$$

where $\eta_1 = \sqrt{\mu/\varepsilon_{1m}}$ and $\bar{I}_{\substack{u\\d}}(z, \bar{\beta}_\perp)$ is given by (9.20). Using (G.2), (G.3), and (G.4) together with (G.6) the MRT equations (9.25a) and (9.25b) may be cast into the form:

$$\cos \; \frac{d}{dz}\bar{\mathcal{I}}_u(z, \Omega) = - K_a \bar{\mathcal{I}}_u(z, \Omega) - \bar{\bar{K}}(\Omega) \cdot \bar{\mathcal{I}}_u(z, \Omega)$$

$$+ \bar{\bar{Q}}_{ud}(\Omega, \Omega_i) \cdot \bar{\mathcal{I}}_{md}(z, \Omega_i) + \Delta_1 \bar{\bar{Q}}_{c1}(\Omega, \Omega_i) \cdot \bar{\mathcal{I}}_{mc1}(z, \Omega_i)$$

$$+ \int d\Omega' [\bar{\bar{P}}_{uu}(\Omega, \Omega') \cdot \bar{\mathcal{I}}_u(z, \Omega') + \bar{\bar{P}}_{ud}(\Omega, \Omega') \cdot \bar{\mathcal{I}}_d(z, \Omega')]$$

(G.7)

$$-\cos\theta \frac{d}{dz}\bar{\mathcal{I}}_d(z, \Omega) = -K_a \bar{\mathcal{I}}_d(z, \Omega) - \bar{\bar{K}}(\Omega) \cdot \bar{\mathcal{I}}_d(z, \Omega)$$

$$+ \bar{\bar{Q}}_{du}(\Omega, \Omega_i) \cdot \bar{\mathcal{I}}_{mu}(z, \Omega_i) + \bar{\bar{Q}}_{dd}(\Omega, \Omega_i) \cdot \bar{\mathcal{I}}_{md}(z, \Omega_i)$$

$$- \Delta_1 \bar{\bar{Q}}_{c1}(\Omega, \Omega_i) \cdot \bar{\mathcal{I}}_{mc3}(z, \Omega_i) + \int d\Omega' [\bar{\bar{P}}_{du}(\Omega, \Omega')$$

$$\cdot \bar{\mathcal{I}}_u(z, \Omega') + \bar{\bar{P}}_{dd}(\Omega, \Omega') \cdot \bar{\mathcal{I}}_d(z, \Omega')] \quad \text{(G.8)}$$

$$\bar{\bar{P}}_{\mu\nu}(\Omega, \Omega') = \bar{\bar{P}}_{\mu\nu}(\bar{\beta}_\perp, \bar{k}_\perp') \quad \text{(G.9)}$$

$$\bar{\bar{Q}}_{\mu\nu}(\Omega, \Omega_i) = \frac{\bar{\bar{Q}}_{\mu\nu}(\bar{\beta}_\perp, \bar{k}_{\perp i})}{\beta_{1m}^2 \cos\theta_i} \quad \text{(G.10)}$$

$$\bar{\bar{Q}}_{c1}(\Omega, \Omega_i) = \frac{\bar{\bar{Q}}_{c1}(\bar{\beta}_\perp, \bar{k}_{\perp i})}{\beta_{1m}^2 \cos\theta_i} \quad \text{(G.11)}$$

$$K_a = 2\beta''_{1m} \equiv 2\mathrm{Im}(\beta_{1m}) \tag{G.12}$$

$$\bar{\bar{K}}(\Omega) = \cos\theta\ \bar{\bar{\eta}}(\bar{\beta}_{\perp}) - K_a\bar{\bar{I}} \tag{G.13}$$

where $\bar{\bar{I}}$ is the identity matrix and it is understood that $\bar{\beta}_{\perp}$, is to be expressed in terms of θ and ϕ by means of (G.2) and (G.3). The subscripts μ and ν signify, respectively u or d. Combining (G.6) with (9.29a) and (9.29b) the boundary conditions relating $\bar{\mathscr{L}}_u$ and $\bar{\mathscr{L}}_d$ are easily derived.

$$\bar{\mathscr{L}}_d(0,\ \Omega_1) = \bar{\bar{R}}_{10}(\Omega_1)\ \cdot\bar{\mathscr{L}}_u(0,\ \Omega_1) \tag{G.14}$$

$$\bar{\mathscr{L}}_u(-d_1,\ \Omega_1) = \bar{\bar{R}}_{12}(\Omega_1)\ \cdot\bar{\mathscr{L}}_d(-d_1,\ \Omega_1). \tag{G.15}$$

Conversion of boundary condition (9.31) to standard form may be accomplished by first multiplying both sides of (9.31) by $d^2\beta_{\perp}$ and then making use of the transformation implied by (G.4):

$$d^2\beta_{\perp}\ \bar{I}_{ou}(0,\ \bar{\beta}_{\perp}) = \bar{\bar{T}}_{10}(\beta_{\perp})\ \cdot\ \bar{I}_u(0,\ \bar{\beta}_{\perp})\ d^2\beta \tag{G.16}$$

$$d\Omega_o \beta_o^2 \cos\theta_o \bar{I}_{ou}(0, \bar{\beta}_\perp) = \bar{\bar{T}}_{10}(\beta_\perp) \cdot \bar{I}_u(0, \bar{\beta}_\perp) \beta_{1m}^2 \cos\theta_1 \, d\Omega_1 \tag{G.17}$$

$$\bar{\mathcal{I}}_{ou}(0, \Omega_o) = \frac{\varepsilon_o}{\varepsilon_{1m}} \left[\frac{\cos\theta_o}{\cos\theta_1} \frac{\eta_1}{\eta_o} \bar{\bar{T}}_{10}(\Omega_1)\right] \cdot \bar{\mathcal{I}}_u(0, \Omega_1). \tag{G.18}$$

In going from (G.17) to (G.18) we have used (G.6) and the result:

$$\frac{d\Omega_o}{d\Omega_1} \sqrt{\varepsilon_{1m}/\varepsilon_o} = \frac{\cos\theta_1}{\cos\theta_o} \frac{\varepsilon_{1m}}{\varepsilon_o} \frac{\eta_o}{\eta_1} . \tag{G.19}$$

CHAPTER 10

Renormalization of the Bethe-Salpeter Equation

In Chapter 6, it was found that the cross-polarized backscattering cross-section computed in the second order Born approximation does not agree with the cross-polarized backscattering cross section obtained from radiative transfer theory. In this chapter we examine the reason for the discrepancy between the wave and radiative transfer results. It is found that an infinite sequence of terms in the intensity operator may be summed resulting in a renormalized Bethe-Salpeter equation which contain coherent effects not accounted for by the ladder approximated Bethe-Salpeter equation nor by the radiative transfer theory.

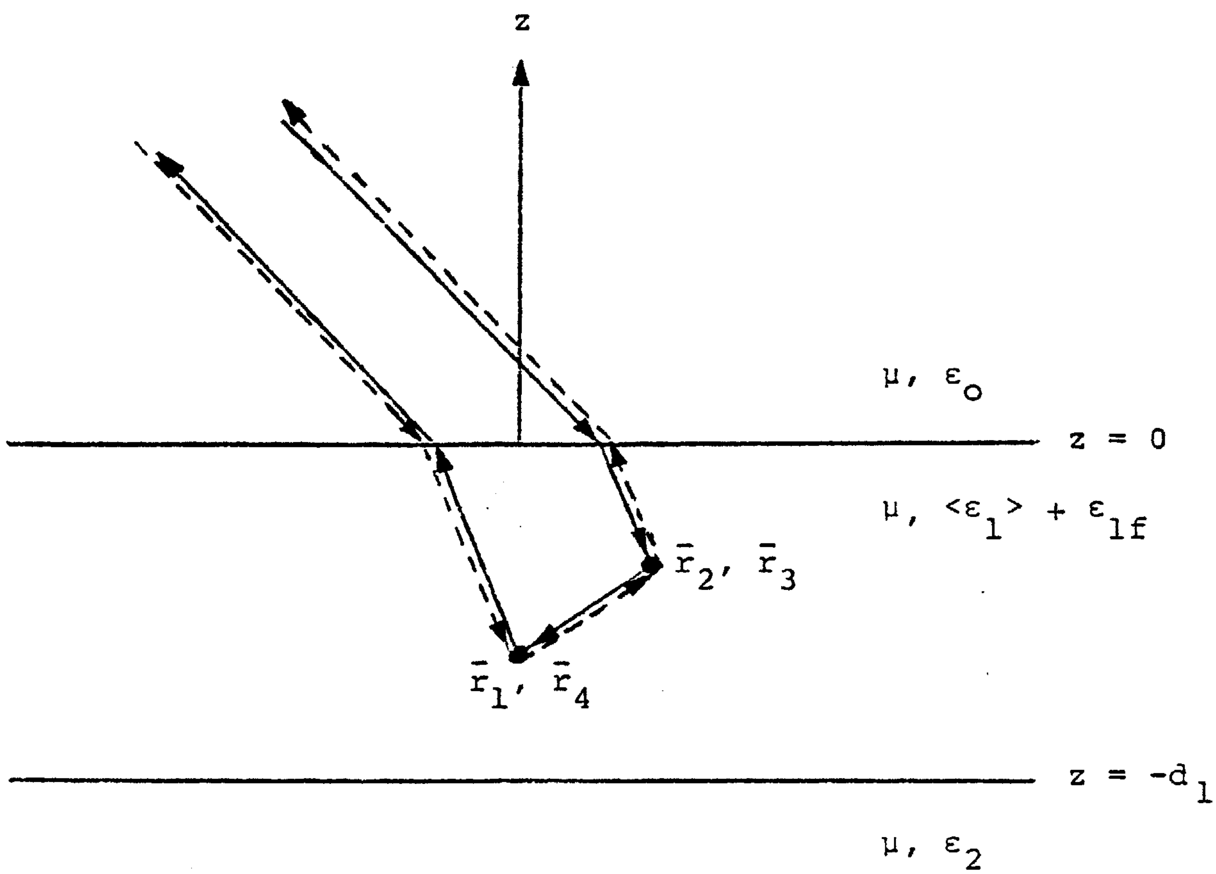
10.1 Physical Significance of Cross Terms

The additional contribution to the depolarized backscattering as shown in (6.14) originates from the last term of the cluster expansion (6.3) which correlates $\bar{r}_1$ with $\bar{r}_4$ and $\bar{r}_2$ with $\bar{r}_3$. To see this more clearly, consider the last term of (6.4) in the limit of very strong correlations. It becomes

$$\int d^3r_1 d^3r_2 [\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1) \cdot \bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2) \cdot \bar{E}_1^{(0)}(\bar{r}_2)]_{\mu\nu}$$

$$\cdot [\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_2) \cdot \bar{\bar{G}}_{11}(\bar{r}_2, \bar{r}_1) \cdot \bar{E}_1^{(0)}(\bar{r}_1)]^*_{\mu\nu}.$$

The backscattering path of the field represented by the first of the square brackets is depicted in Fig. 10.1 by a solid line. The backscattering path of the field contained in the second set of square brackets is depicted in Fig. 10.1 by a dashed line. We may consider each represents the wave vector of a plane wave. Thus in the backscattering direction the two path lengths are equal and the two waves interfere constructively. In the radiative transfer theory only the ladder terms are included and the constructive interference is con-



Constructive interference path lengths for second order backscattering

Figure 10.1

sidered due to coincidental ray paths rather than phase fronts. Thus only the first term of (6.14) is accounted for in the radiative transfer theory.

Mathematically we see that the phases of $\bar{E}_1^{(0)}(\bar{r}_2)$ and $\bar{\bar{G}}^*_{01}(\bar{r}, \bar{r}_2)$, $\bar{E}_1^{(0)*}(\bar{r}_1)$ and $\bar{\bar{G}}_{01}(\bar{r}, \bar{r}_1)$, as well as $\bar{\bar{G}}_{11}(\bar{r}_1, \bar{r}_2)$ and $\bar{\bar{G}}^*_{11}(\bar{r}_2, \bar{r}_1)$ combine in such a way that when integrated over the respective z-coordinates, a contribution of second order in albedo is produced. This argument may be applied to higher order intensities with the result that additional significant contributions besides the ladder terms in the radiative transfer theory may be identified. Therefore in the limit of low loss and high scattering these additional contributions may be included in a renormalization of the Neumann series for the covariance of the field. We then obtain a Bethe-Salpeter equation with a renormalized intensity operator in which the infinite sequence of cross type terms are summed.

10.2 Resummation of Cross Terms in the Intensity Operator

The dyadic Green's function for a random medium satisfies an integral equation similar to that of the field $\bar{E}_1(\bar{r})$.

$$G_{ij}(\bar{r}, \bar{r}_o) = G_{ij}^{(0)}(\bar{r}, \bar{r}_o)$$

$$+ \int_{V_1} d^3r_1\, G_{ik}^{(0)}(\bar{r}, \bar{r}_1)\, Q(\bar{r}_1)\, G_{kj}^{(0)}(\bar{r}_1, \bar{r}_o)$$

$$+ \int_{V_1} d^3r_1 d^3r_2\, G_{ik}^{(0)}(\bar{r}, \bar{r}_1)\, G_{k\ell}^{(0)}(\bar{r}_1, \bar{r}_2)$$

$$Q(\bar{r}_1)\, Q(\bar{r}_2)\, G_{\ell j}^{(0)}(\bar{r}_2, \bar{r}_o) + \ldots \qquad (10.1)$$

To simplify the analysis we associate Feynman diagrams with the various terms in (10.1) according to the following rules

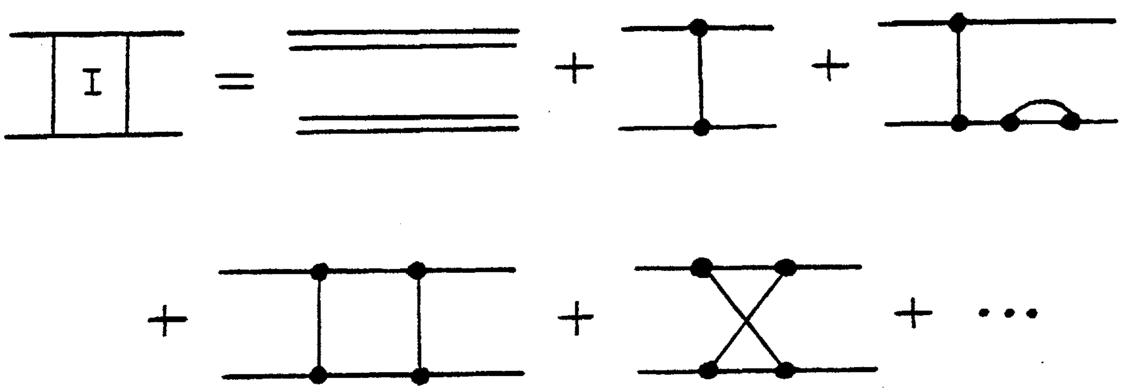
$\langle G_{ij}(\bar{r}, \bar{r}_o)\rangle =$ (10.2a)

$G_{ij}^{(0)}(\bar{r}, \bar{r}_2) =$ (10.2b)

$C(\bar{r}_1 - \bar{r}_2) =$  or  (10.2c)

$<G_{ij}(\bar{r}, \bar{r}_o)\ G^*_{\ell k}(\bar{r}', \bar{r}_o')> =$  (10.2d)

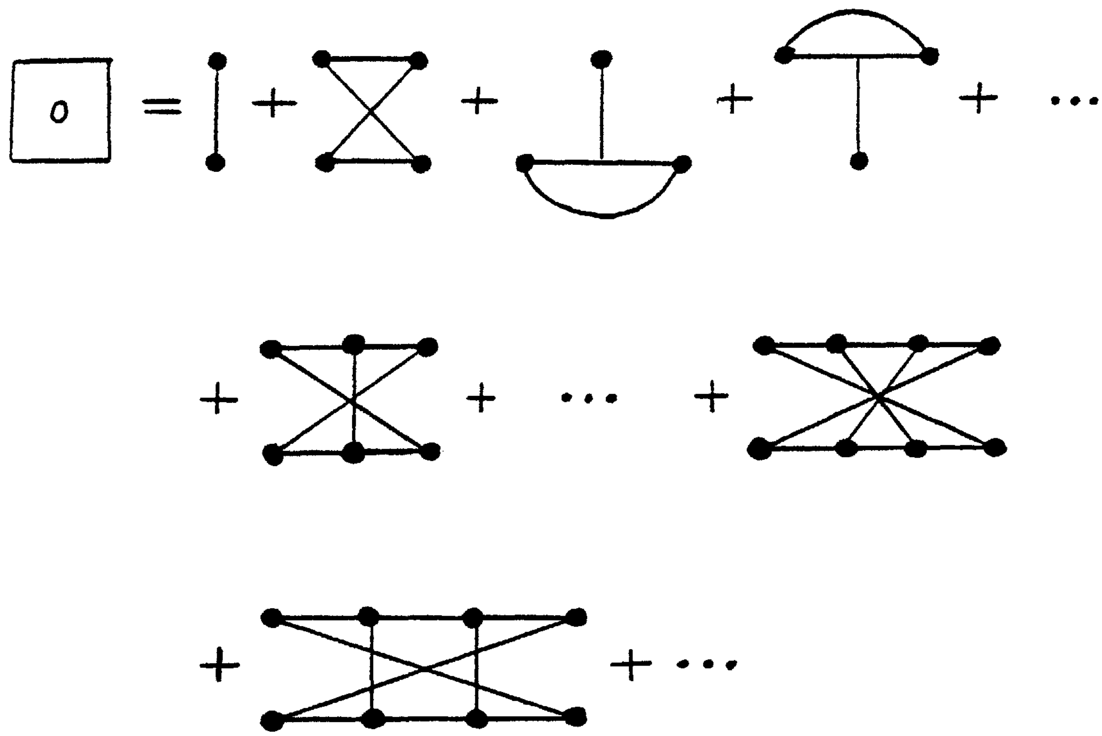
The correlation of the dyadic field in (10.1) is given by



(10.3)

Integration is performed over the coordinates of all internal vertices and convolution is carried out over all indices of internal vertices. The second order ladder and cross terms which lead to an equally significant contribution are readily recognized in (10.3).

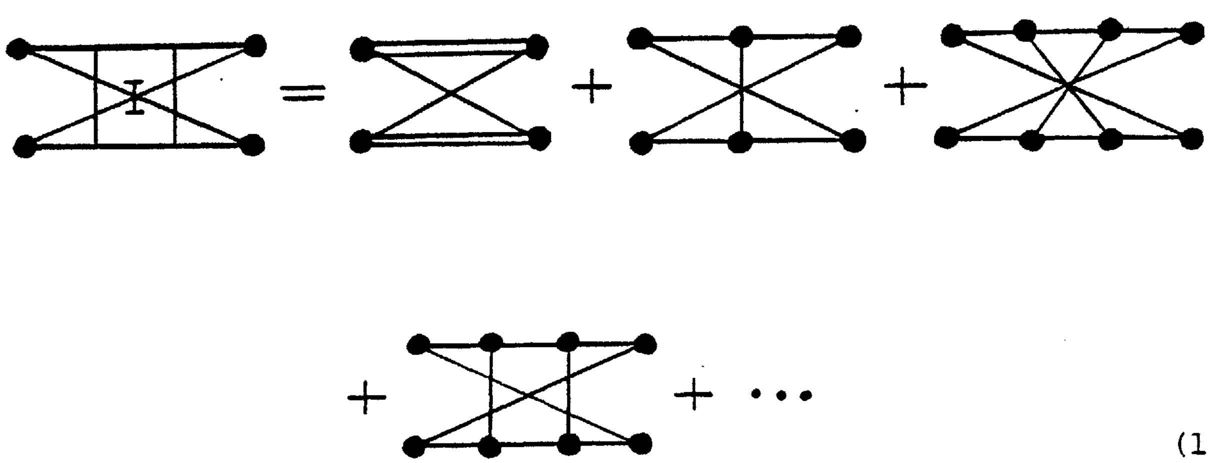
We consider the sum of the strongly connected diagrams and identify the "kernel" of the this expression as the intensity operator:



(10.4)

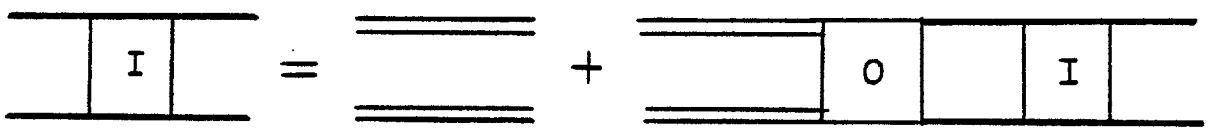
The approximation usually made is to retain only the first term of (10.4) resulting in the so-called ladder approximation. The ladder approximation reproduces, for example, the second and fourth terms on the right hand side of (10.3) as well as higher order ladder terms.

In order to reproduce the cross terms of (10.4) which contribute significantly to the backscattering cross sections, we renormalize (10.4) by considering the following expression



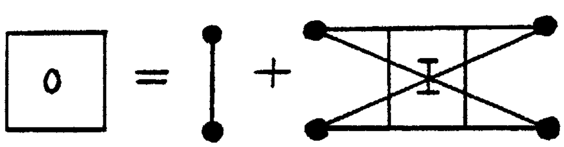
(10.5)

Equation (10.5) resums an infinite sequence of cross terms in the intensity operator. We may proceed with the usual development of the Bethe-Salpeter equation to obtain



(10.6)

and approximate the intensity operator by



(10.7)

Equations (10.6) and (10.7) lead to a nonlinear integral equation from the correlation of the dyadic Green's function. In analytic form, (10.6) and (10.7) may be written as

$$<G_{ij}(\bar{r}, \bar{r}_o)\ G^*_{\ell k}(\bar{r}', \bar{r}_o')> = <G_{ij}(\bar{r}, \bar{r}_o)><G^*_{\ell k}(\bar{r}', \bar{r}_o')>$$

$$+ \int d^3r_1 d^3r_2 d^3r' d^3r_2' <G_{ip}(\bar{r}, \bar{r}_1)><G^*_{\ell q}(\bar{r}', \bar{r}_1)>$$

$$[C(\bar{r}_1 - \bar{r}_1')\ \delta(\bar{r}_1 - \bar{r}_2)\ \delta(\bar{r}_1' - \bar{r}_2')\ \delta_{ps}\ \delta_{qr}$$

$$+ C(\bar{r}_1 - \bar{r}_2')\ C(\bar{r}_1' - \bar{r}_2) <G_{ps}(\bar{r}_1, \bar{r}_2)\ G^*_{qr}(\bar{r}_1', \bar{r}_2')>]$$

$$<G_{sj}(\bar{r}_2, \bar{r}_o)\ G^*_{rk}(\bar{r}_2', \bar{r}_o')>. \qquad (10.8)$$

Consequently the covariance of the scattering field takes the form

$$<\varepsilon_i(\bar{r})\ \varepsilon_j^*(\bar{r}')> = \int d^3r_1 d^3r_2 d^3r_1' d^3r_2' <G_{ip}(\bar{r}, \bar{r}_1)><G^*_{jq}(\bar{r}', \bar{r}_1')>$$

$$[C(\bar{r}_1 - \bar{r}_1')\ \delta(\bar{r}_1 - \bar{r}_2)\ \delta(\bar{r}_1' - \bar{r}_2')\ \delta_{ps}\ \delta_{rq}$$

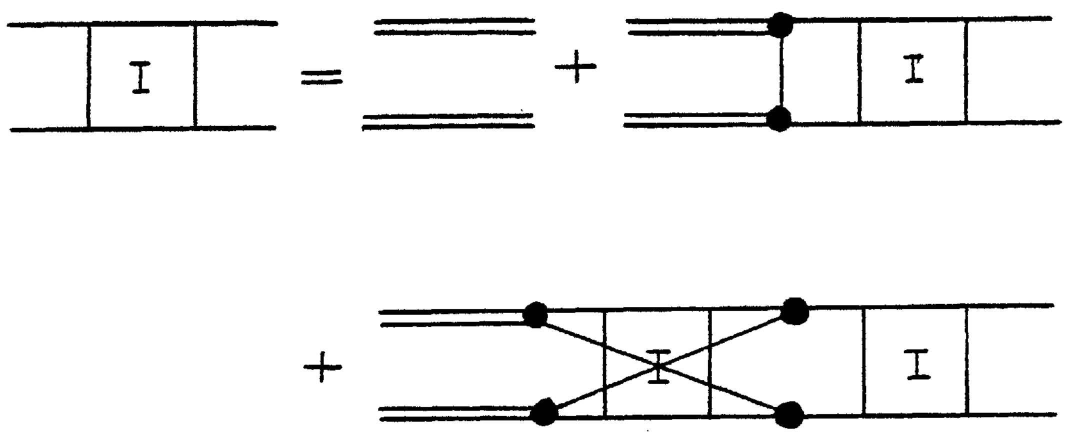
$$+ C(\bar{r}_1 - \bar{r}_2')\ C(\bar{r}_1' - \bar{r}_2)\ <G_{ps}(\bar{r}_1, \bar{r}_2)\ G^*_{qr}(\bar{r}_1', \bar{r}_2')>]$$

$$[<E_s(\bar{r}_2)><E_r(\bar{r}_2')> + <\varepsilon_s(\bar{r}_2)\ \varepsilon_r^*(\bar{r}_2')>] \qquad (10.9)$$

where $\varepsilon_i = E_i - \langle E_i \rangle$.

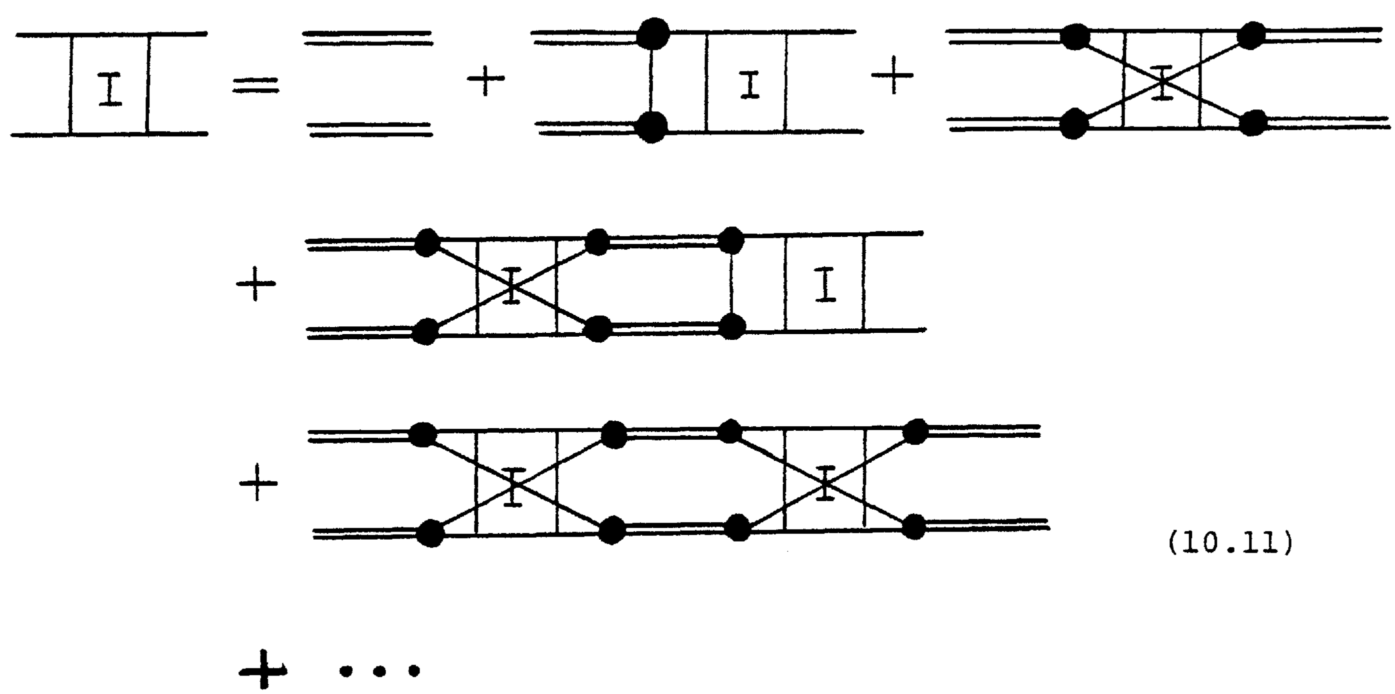
10.3 Renormalized Bethe-Salpeter Equation and Discussion

Continuing with the renormalization we write (10.8) in diagrammatic form:



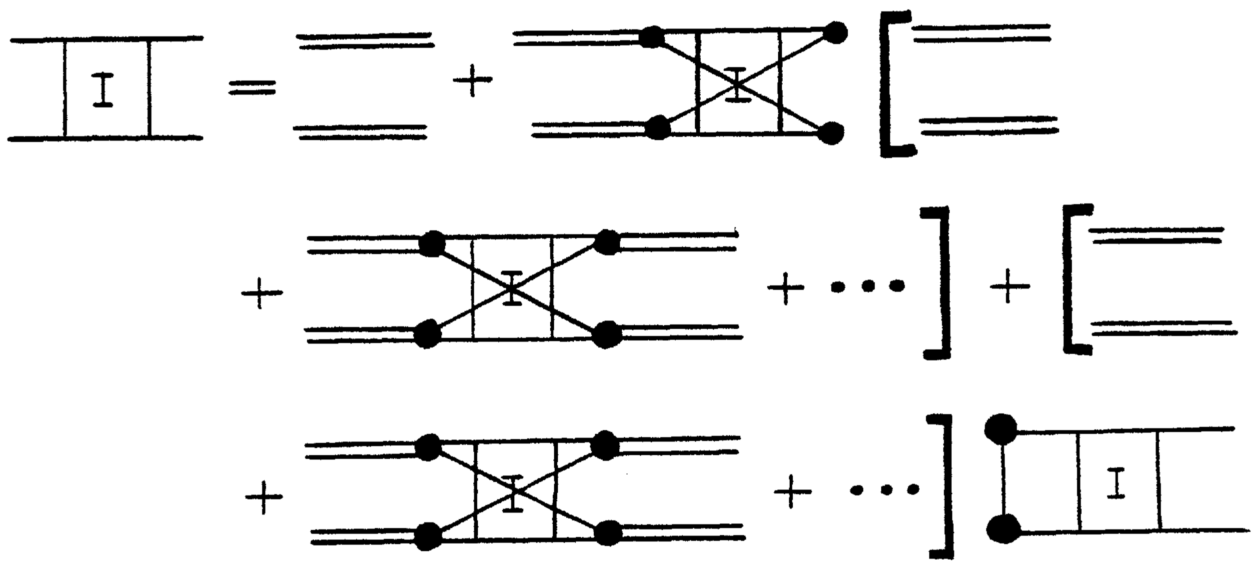
(10.10)

Using the first two-terms on the right hand side of (10.10) as the zeroth order iteration, and iterating with respect to the correlation which appears on the right side of the third term in (10.10), we obtain:



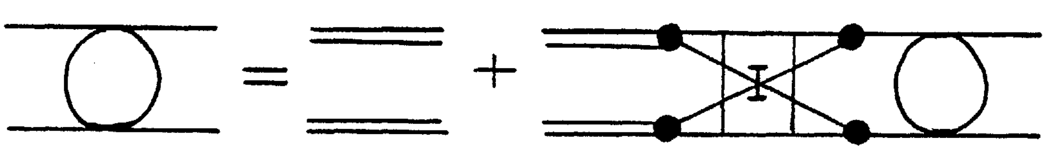
(10.11)

We regroup the terms of (10.11) as follows:



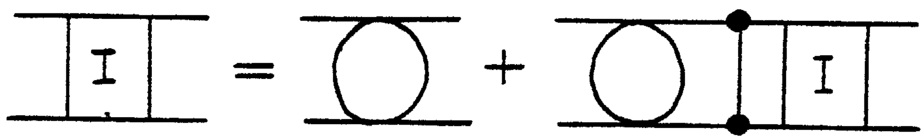
(10.12)

In (10.12) we define the sum of all the terms which appear in the square brackets by the symbol  . It is easily proven that  satisfies the following integral equation:



(10.13)

in which case (10.12) takes the form:



(10.14)

Equations (10.13) and (10.14) are a pair of coupled integral equations for the correlation of the dyadic Green's function which analytically takes the form:

$$U_{ij,\ell k}(\bar{r}, \bar{r}_o|\bar{r}', \bar{r}_o') = \langle G_{ij}(\bar{r}, \bar{r}_o)\rangle\langle G^*_{\ell k}(\bar{r}', \bar{r}_o')\rangle$$

$$+ \int d^3r_1 d^3r_1' d^3r_2 d^3r_2' \; C(\bar{r}_1 - \bar{r}_2') \; C(\bar{r}_1' - \bar{r}_2)$$

$$\langle G_{ip}(\bar{r}, \bar{r}_1)\rangle\langle G^*_{\ell q}(\bar{r}', \bar{r}_1')\rangle\langle G_{ps}(\bar{r}_1, \bar{r}_2) \; G^*_{qr}(\bar{r}_1', \bar{r}_2')\rangle$$

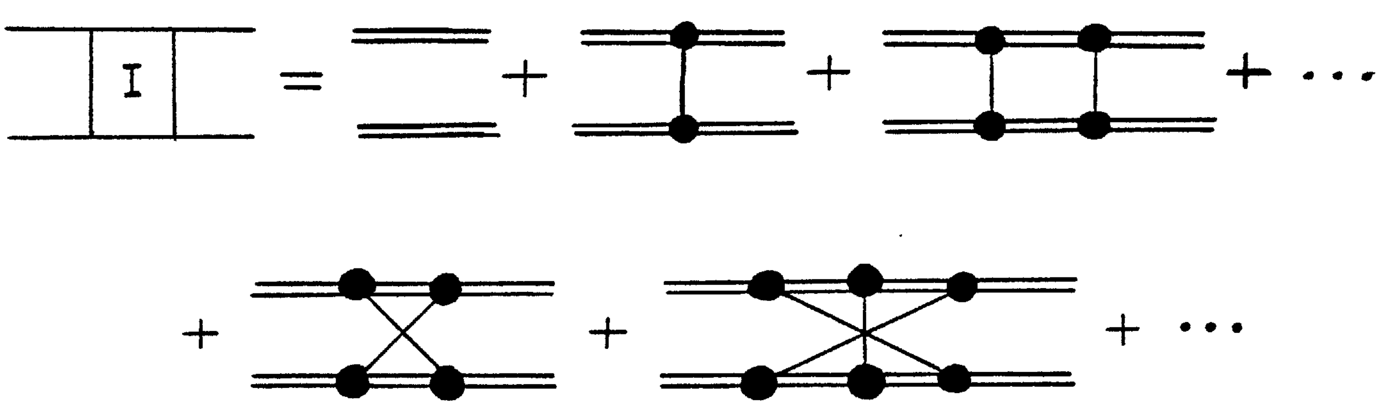
$$U_{sj,rk}(\bar{r}_2, \bar{r}_o|\bar{r}_2', \bar{r}_o') \qquad (10.15)$$

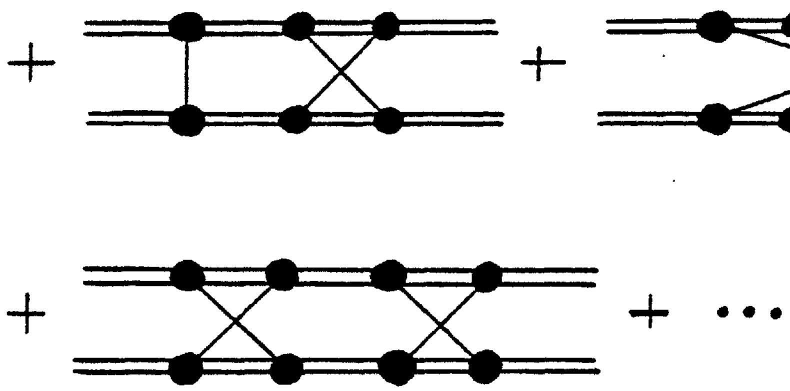
$$\langle G_{ij}(\bar{r}, \bar{r}_o) \; G^*_{\ell k}(\bar{r}', \bar{r}_o')\rangle = U_{ij,\ell k}(\bar{r}, \bar{r}_o|\bar{r}', \bar{r}_o')$$

$$+ \int d^3r_1 d^3r_1' \, U_{ip,js}(\bar{r}, \bar{r}_1 | \bar{r}', \bar{r}_1') \, C(\bar{r}_1 - \bar{r}_1')$$

$$\langle G_{pj}(\bar{r}_1, \bar{r}_o) \, G^*_{sk}(\bar{r}_1', \bar{r}_o') \rangle \qquad (10.16)$$

where we have defined $U_{ij,\ell k}$ as  . It is interesting to note that the renormalized equation (10.16) has a structure similar to the ladder approximated Bethe-Salpeter equation. In fact, if $U_{ij,\ell k}$ is approximated by the first term on the right hand side of (10.15), then (10.16) reduces identically to the Bethe-Salpeter equation in the ladder approximation. In order to determine which terms have been picked up in the resummation of (10.1) (or equivalently, (10.6), (10.7)), we may solve equations (10.15) and (10.16) by successive iterations. The result is:





(10.17)

It is clear from (10.17) that the renormalization includes ladder terms, strongly and weakly connected cross terms as well as weakly connected ladder-cross terms.

CHAPTER 11

Conclusions and Suggestions for Further Study

In this thesis, theoretical models have been developed for electromagnetic wave scattering from layered random media with applications to microwave remote sensing. Applying Born approximations which are valid for small albedo, backscattering cross-sections have been derived with a wave approach for a two-layer random medium with arbitrary three-dimensional correlation functions. Carrying to first order the backscattering cross sections illustrate the possibility of $\sigma_{hh} > \sigma_{vv}$ due to the Brewster angle effect at the bottom boundary. Previous models of a half-space random medium do not reproduce the effect of $\sigma_{hh} > \sigma_{vv}$ which is observed in certain backscattering data.

The first order Born approximation also has been applied to the case of backscattering by a stratified random medium with arbitrary three-dimensional correlation functions. It has been found that multiple resonances occur which may explain the spectral dependence observed in active remote sensing data. The multiple resonances are due to resonant scattering in each random layer and to illustrate this important effect the special case of a three-layer random medium has been used.

In addition the three-layer model also has been found useful in interpreting the diurnal change of snow-ice field due to solar illumination.

Cross polarized backscattering cross sections for a two-layer random medium have been derived by applying the second order Born approximation to the integral equations of scattering theory. In the half-space limit the cross-polarized backscattering cross sections do not reduce to previous results[63] obtained using radiative transfer theory. The discrepancy is due to cross terms not accounted for by radiative transfer theory nor by the Bethe-Salpeter equation under the ladder approximation. In order to account for these additional cross terms the Bethe-Salpeter equation has been renormalized by summing an infinite sequence of terms in the intensity operator. The result takes the form of coupled integral equations for the covariance of the electromagnetic field. It is found that the renormalization accounts not only for cross terms but many other terms in the infinite series representation for the intensity operator.

In this thesis we have applied renormalization methods to study the multiple scattering of electromagnetic waves by a two-layer random medium. Due to the presence of a bottom boundary there exist significant coherent effects in a two-layer random medium.

We have solved Dyson's equation in the non-linear approximation for the zeroth order mean dyadic Green's function. The coherent field is found to propagate within the random medium as if in an anistropic medium with characteristic TE and TM polarizations. The zeroth order solution to Dyson's equation in conjunction with the ladder approximated Bethe-Salpeter equation have been used to derive modified radiative transfer (MRT) equations appropriate for electromagnetic scattering within a two-layer random medium. The MRT equations have been solved in the first order renormalization approximation and significant coherent effects not accounted for by phenomenological radiative transport theories are found.

The task of developing theoretical models is by no means complete. We have considered primarily the case of a two-layer random medium with three dimensional correlation functions. Although the first order Born approximation has been applied to a stratified random medium, the second order Born approximation as well as the renormalization approach should be extended to multi-layer random media. The coupled-integral equations of the renormalized Bethe-Salpeter equation need to be developed into a wave radiative transfer (WRT) theory which accounts for the cross terms discussed in Chapter 10.

Finally, scattering by a composite medium including rough surface and random permittivity fluctuations is an important and challenging problem to be solved especially when we consider the inability of volume scattering to account for backscattering data near normal incidence.

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