Electronic Characteristics of SQUIDs

by

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Submitted to the Department of Physics
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Abstract

Three electronic characteristics of a DC SQUID were studied: magnetic flux quantization, DC Josephson effect, and AC Josephson effect. The magnetic flux quantum was measured to be: $\Phi_0 = 2.06 \cdot 10^{-15} \pm 3\% Wb$, and a measurement of $\frac{e}{h}$ taken from the AC Josephson effect was $2.65 \cdot 10^{14} \pm 6\% \frac{Hz}{V}$.

Thesis Advisor: L. J. Rosenberg, Associate Professor

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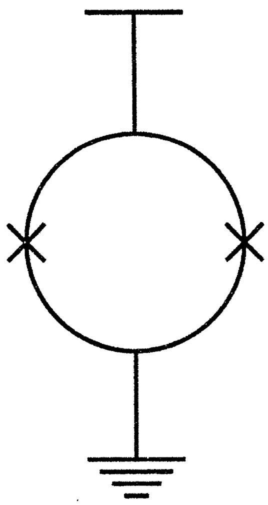

Figure 1: DC SQUID - a two terminal device

1 Introduction

Discussion of SQUIDs (Superconducting **QU**antum **I**nterference **D**evices) begins with Josephson junctions. A Josephson junction consists of two superconducting "pieces" separated by a thin insulator wall.

Normally, if two electrical conductors are separated by an insulating layer, then no current will flow between them in the absence of applied voltage. However, Josephson junctions will have a tunneling current in the absence of voltage. This phenomena was first predicted by Josephson, and is known as the DC Josephson effect.

A SQUID is a superconducting ring broken by one or two Josephson junctions. In this project, a DC SQUID was studied, which consists of two Josephson junctions connected in parallel forming a simple electrical circuit. Magnetic flux can penetrate the ring (see figure 1).

Josephson further predicted that in the presence of a constant dc voltage across a junction, the superconducting current ("supercurrent") will oscillate at high frequencies between the junctions. This is known as the AC Josephson effect. The demonstration of the DC and AC Josephson effects, in addition to the quantization of magnetic flux in superconducting rings, will be the three primary topics of this project.

Thus, a SQUID will be a region of zero resistance in the current-voltage (I-V) curve. Above a certain critical current, however, the superconducting state will be destroyed and the I-V curve will resemble that of a resistor (linear, with the slope being equal to the parallel resistance of the two junctions) (fig. 2).

The AC Josephson effect is a bit more difficult to observe. It is observed

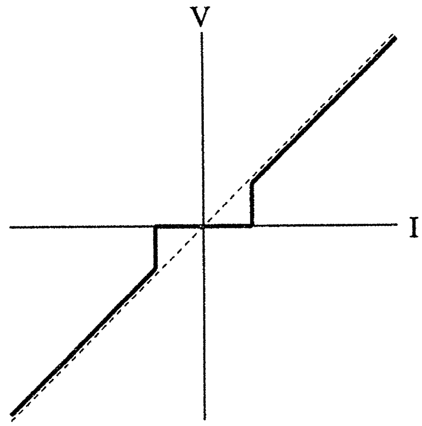

Figure 2: A stylized DC SQUID I-V curve: mostly ohmic except for a region of zero resistance

indirectly by forcing the supercurrents to oscillate at microwave frequencies. A constant voltage then forms across the SQUID, and discontinuous steps in the nonlinear region of the current-voltage (I-V) curve will arise. Inducing these steps (Shapiro steps) to appear, however, is nontrivial, as there are only a few favored microwave frequencies which couple effectively to the SQUID junctions.

The quantization of magnetic flux through the SQUID will be the third topic of this project. Roughly speaking, the flux quantization is similar to a Bohr-Sommerfeld condition, and appears from a non-reducible path integration around the "hole" in the SQUID loop.

This paper will focus on the demonstration of these three effects in a DC SQUID. The background section will treat the basic theory behind the dc SQUID, while the experimental section will detail the procedures and techniques in performing the demonstrations. This will be followed by a discussion of the results, as well as a discussion of errors involved in the measurements. The conclusion will provide a brief look into applications of SQUIDs, particularly as low-noise amplifiers appropriate for ultra-low signal detection.

This project could also serve as a relevant addendum to the superconductivity experiment in the 8.13-8.14 Junior Physics Laboratory sequence [1].

2 Background

In constructing a microscopic understanding of the behavior of superconductors, a good point of departure is Landau's superconducting "wavefunction".

$$\psi(\mathbf{r}) = \sqrt{n_s} \exp i\varphi(\mathbf{r}) \tag{1}$$

The term n_s is the number of superconducting charge carriers in the sample, and φ is the phase of the wavefunction in space. This form for a wavefunction is an extention of single-particle quantum mechanics that was found to explain the empirical features of superconducting systems.

A heuristic justification for this form of the wavefunction follows. By hypothesis, all of the charge carriers in a supercurrent are in the same low-energy state. It then becomes possible to have a macroscopic wavefunction. A simple wavefunction is the one given, where a complex number that varies over space is normalized to the number of charge carriers in the superconducting state. The coherence of the system is contained in the phase φ.

2.1 Flux Quantization

In this section, I'll show how the magnetic flux threading a superconducting loop is quantized. First, let us consider the energy of the system. Following the basics of a well-known argument [2], we will assume the Ginzburg-Landau operator for kinetic energy, given an arbitrary mass m^* and charge e^*:

$$KE = \frac{1}{2m^*}(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A})^2 \tag{2}$$

The vector potential appears in this way so that the Hamiltonian gives Maxwell's equations. From the kinetic energy, we can extract the kinematic velocity:

$$\vec{v} = \frac{1}{m}(\hbar\nabla\varphi - 2e\vec{A}) \tag{3}$$

We have taken the mass m to be the rest mass of an electron, and the charge to be $2e$, since electrons in the superconducting state form Cooper pairs.

This velocity in:

$$\vec{J} = 2en_s\vec{v_s} \tag{4}$$

gives:

$$\vec{J} = \frac{2e}{m}|\psi|^2(\hbar\nabla\varphi - 2e\vec{A}) \tag{5}$$

Given that the dimensions of practical SQUIDs are much larger than the skin depth, we have the condition:

$$\vec{J} = 0 \tag{6}$$

within most of the sample, which gives us:

$$\hbar\nabla\varphi = 2e\vec{A} \tag{7}$$

We can apply this to the geometry of our DC SQUID. Integrating a path around a loop of the SQUID completely within the superconducting material, we have:

$$\oint \hbar\nabla\varphi \cdot d\vec{l} = \oint 2e\vec{A} \cdot d\vec{l} \tag{8}$$

Since φ is a phase, it is simply expressed in modulo 2π:

$$\oint \hbar\nabla\varphi \cdot d\vec{l} = 2\pi\hbar n \tag{9}$$

leading us to:

$$2\pi\hbar n = \oint 2e\vec{A} \cdot d\vec{l} = 2e\Phi \tag{10}$$

where the magnetic flux Φ comes from electrodynamics. That is, a closed path integral of the vector potential yields the magnetic flux through the path.

We are left with:

$$\Phi = \frac{n2\pi\hbar}{2e} = \frac{nh}{2e} \tag{11}$$

We see that the magnetic flux through a superconducting loop is quantized, similar in fashion to the Bohr-Sommerfeld quantization of angular momentum in a hydrogen atom in quantum mechanics. The expression $\frac{h}{2e}$ is known as the fundamental flux quantum Φ_0, and has a numerical value of $2.0678 \cdot 10^{-15}$ Wb.

2.2 DC Josephson effect

We now discuss the DC Josephson effect in a DC SQUID. We first aim to show that a tunneling current in the absence of voltage occurs, and that the critical current through the junction is dependent on the magnetic flux threading the loop. The basic argument can be found in [3].

Earlier, we found that the superconducting current density was dependent on the phase difference and vector potential:

$$\vec{J} = \frac{2e}{m}|\psi|^2(\hbar\nabla\varphi - 2e\vec{A}) \tag{12}$$

In a weak magnetic field, particularly for a SQUID which is magnetically well shielded, this reduces to:

$$\vec{J} = \frac{2e}{m}|\psi|^2(\hbar\nabla\varphi) \tag{13}$$

The elimination of the last term is justified, in that $\hbar$ is (in MKS) of order 10^{-34}, φ is of order 10^{-2} and the gradient term is the reciprocal of the length scale, about 10^{-9}. Thus, the first term of the right hand side of the equation is of order 10^{-27}. The last term of the right hand side has e of order 10^{-19}, and $\vec{A}$ of order $\frac{\vec{B}}{l}$, from $\oint \vec{A} \cdot d\vec{l} = \Phi$. Then, $|\vec{A}|$ is of order 10^{-11}, as Φ_0, the fundamental fluxoid is of order 10^{-15}, and the length of the SQUID is 90μm, or 10^{-4}. We see that the first term is of order 10^{-27}, while the last term is of order 10^{-30}, such that our approximation rests on a difference of three orders of magnitude.

In a junction, the distance across a junction is very small (about $10\AA$) compared to the correlation length of the phase. We can approximate:

$$\vec{J} = \frac{2e}{md}|\psi|^2(\varphi_2 - \varphi_1) \tag{14}$$

where φ_2 and φ_1 are the phases of the superconducting wavefunction on opposite sides of the junction, and d is the thickness of the junction barrier.

We encapsulate the above relation as such:

$$\vec{J} = \vec{J_0}\gamma \tag{15}$$

where γ is the phase difference and J_0 holds the constants.

A slight alteration of this result is necessary, since the phase exists in modulo 2π. Given that γ is sufficiently small, we still capture our result by writing:

$$\vec{J} = \vec{J_0} \sin\gamma \tag{16}$$

Josephson derived this result in a rigorous manner, and it is generalized to:

$$I = I_0 \sin\gamma \tag{17}$$

We remark that this relation (Josephson's relation) will find frequent use in all that follows.

In deriving the magnetic flux dependence of the critical current, let us begin with a simple depiction of the DC SQUID (fig. 3 , as in [2]). The junctions are labeled A and B, and the superconducting pieces separated by the junctions are labeled 1 and 2, as in [3, p. 157].

From Josephson's relation, the current through the two junctions in parallel is:

$$I = I_0 \sin\gamma_A + I_0 \sin\gamma_B \tag{18}$$

where we have assumed for simplicity that both junctions have the same critical current.

The terms γ_A and γ_B are the customary phase differences across junctions A and B:

$$\gamma_A = \varphi_{2A} - \varphi_{1A} \tag{19}$$

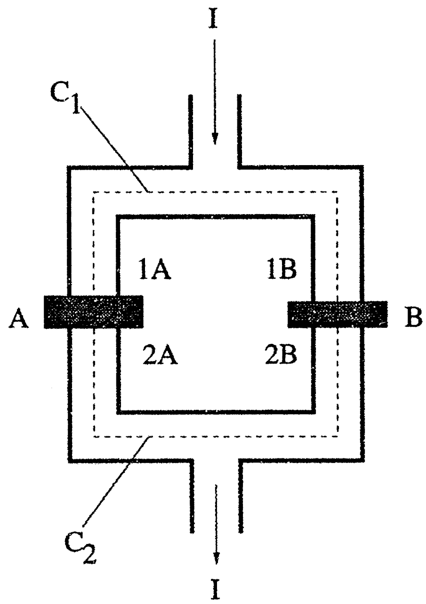

Figure 3: Schematic of DC SQUID. A, B are Josephson junctions [3, p. 158]

$$\gamma_B = \varphi_{2B} - \varphi_{1B} \tag{20}$$

Again assuming that the dimensions of the SQUID are such that the skin depth of magnetic flux penetration is negligible, then the superconducting current on path C_1 or C_2 in fig. 3 is zero. We get:

$$\hbar \nabla \varphi = 2e\vec{A} \tag{21}$$

This naturally leads to:

$$\varphi_{1B} - \varphi_{1A} = \frac{2e}{\hbar} \int_{1A}^{1B} A \cdot dl \tag{22}$$

$$\varphi_{2B} - \varphi_{2A} = \frac{2e}{\hbar} \int_{2A}^{2B} A \cdot dl \tag{23}$$

Summing these two equations yield:

$$\varphi_{1B} + \varphi_{2A} - \varphi_{1A} - \varphi_{2B} = \frac{2e}{\hbar} \oint_{C_1+C_2} Adl = 2\pi \frac{\Phi}{\Phi_0} \tag{24}$$

since $\oint \vec{A} \cdot d\vec{l} = \Phi$ from electrodynamics.

However, the path $C_1 + C_2$ is essentially a loop integral, as the junction barrier lengths are negligible in relation to the SQUID dimensions. We are then led to:

$$\gamma_A - \gamma_B = 2\pi \frac{\Phi}{\Phi_0} \tag{25}$$

with $\Phi_0 = \frac{2\pi\hbar}{2e}$.
We can rearrange our phase differences (γs), writing:

$$\gamma_A = \gamma_0 + \pi \frac{\Phi}{\Phi_0} \tag{26}$$

$$\gamma_B = \gamma_0 - \pi \frac{\Phi}{\Phi_0} \tag{27}$$

Inserting these two phases into (18) and use of a simple trig identity, we have:

$$I = 2I_0 \sin\gamma_0 \cos\pi \frac{\Phi}{\Phi_0} = I_{max} \sin\gamma_0 \tag{28}$$

with:

$$I_{max} = 2I_0 \cos\pi \frac{\Phi}{\Phi_0} \tag{29}$$

We finally see that the term I_{max}, which we recognize to be the critical current of the system, is sinusoidally dependent on the magnetic flux threading the SQUID loop.

2.3 AC Josephson effect

As mentioned earlier, the AC Josephson effect is the oscillation of superconducting current between the junctions when a constant DC voltage is applied across the junctions. Note that this cannot take place in a one-junction (RF) SQUID.

The next four equations are an attempt to "reverse engineer" a relation in which an oscillating current, and therfore, an oscillating phase, will entail a nonzero voltage across the junctions.

Turning to our now-familiar expression for the phase difference across a given junction, we obtain:

$$\gamma = \varphi_2 - \varphi_1 - \frac{2\pi}{\Phi_0}\int_1^2 A_z dz \tag{30}$$

Differentiating with respect to time, we arrive at:

$$\frac{\partial\gamma}{\partial t} = \frac{2\pi}{\Phi_0}\int_1^2 \frac{\partial A_z}{\partial t} dz \tag{31}$$

With the aid of Faraday's law:

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) \tag{32}$$

we have:

$$E_z = -\frac{\partial A_z}{\partial t} \tag{33}$$

and thus:

$$\frac{\partial \gamma}{\partial t} = \frac{2\pi}{\Phi_0}\int_1^2 E_z dz = \frac{2\pi}{\Phi_0}V \tag{34}$$

This is our fundamental result.

Notice that if V across the device is constant, then γ is linear in time. Then, our DC Josephson current equation:

$$J = J_0 \sin \gamma_0 \tag{35}$$

is transformed into an oscillating, time-dependent, current:

$$J = J_0 \sin \omega t \tag{36}$$

where:

$$\omega = \frac{2e}{\hbar}V_0 \tag{37}$$

In my opinion, we get a nontrivial result for the minimal investment of time differentiation.

Since viewing alternating currents (at microwave frequencies) within the junctions themselves is difficult, alternative methods of observing the AC Josephson effect include applying a constant current through the device or radiating the SQUID with microwaves to induce oscillations. Our experiment took the latter road. Radiating the SQUID with microwaves will deform the I-V relation of the device. In the resistanceless superconducting region, discontinuous current steps will form, the so-called Shapiro steps (fig. 4).

The final connection to make is to show that an oscillating superconducting current through a junction implies discontinuous current (Shapiro) steps in the I-V relation.

We can slightly modify the above discussion by making V an external sinusoidal source. The added term gives:

$$\frac{\partial \gamma}{\partial t} = \frac{2eV_0}{\hbar} + \frac{2ev}{\hbar}\cos(\omega t + \theta) \tag{38}$$

$$\gamma = \frac{2eV_0}{\hbar}t + \frac{2ev}{\hbar\omega}\sin(\omega t + \theta) \tag{39}$$

where v and ω are the amplitude and frequency of the injected microwave signal.

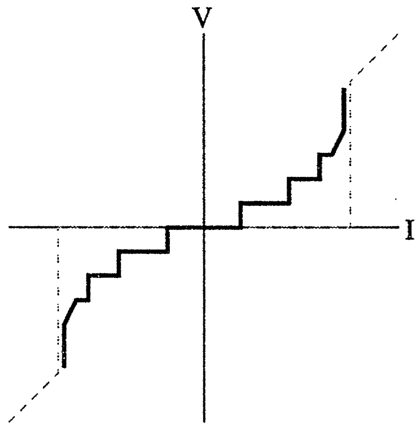

Figure 4: A depiction of Shapiro steps in near zero resistance in the I-V curve

We introduce a necessary mathematical identity:

$$\exp(\imath a \sin x) = \sum_{n=-\infty}^{\infty} J_n(a) \exp \imath n x \tag{40}$$

where $J_n(a)$ are Bessel functions of order n.

By applying the above phase difference and this identity to Josephson's relation, I can be written as:

$$I = I_0 \sum_{-\infty}^{\infty} (-1)^n J_n\left(\frac{2ev}{\hbar\omega}\right) \sin\left(\left(\frac{2eV_0}{\hbar} - n\omega\right)t - n\theta + \gamma_0\right) \tag{41}$$

We can make the following observation: when $2eV_0 = n\hbar\omega$, the current in the expression for I becomes independent of time. In addition, the DC Josephson current can vary from $-I_0 J_n(\frac{2ev}{\hbar\omega})$ to $I_0 J_n(\frac{2ev}{\hbar}\omega)$.

Thus, at integral values of $\frac{2eV_0}{\hbar\omega}$, current steps of amplitude $2I_0 J_n(\frac{2ev}{\hbar\omega})$ will arise. An item to note is that Shapiro steps are not always induced. However, you will still see a perturbed $I - V$ with a reduced critical current I_{red}:

$$I_{red} = I_0 J_0\left(\frac{2ev}{\hbar\omega}\right) \tag{42}$$

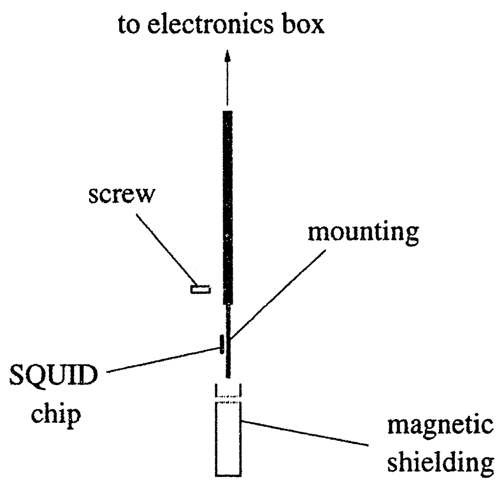

Figure 5: DC SQUID probe

3 Experimental

The basic experiment proceeded as follows: a DC SQUID made from $YB_2C_3O_7$ housed in a probe was immersed in liquid nitrogen. While either the current through or magnetic flux threading the SQUID were varied, the voltage across the SQUID was monitored. In analyzing these data, the flux quantum was determined, and the DC Josephson effect was observed. For the AC Josephson effect, microwave power was radiated on the SQUID via a dipole antenna. Details in the experiment components as well as the procedure follow.

3.1 Components

The DC SQUID was part of a commercially produced educational kit. Manufactured by Conductus, Inc. [4], it was housed on a probe with its own magnetic shielding (fig. 5). Accompanying the SQUID was the electronics box which biased the SQUID current, applied flux bias, and amplified and isolated the SQUID output.

The DC SQUID was a square loop of superconducting material, 90 μm by 90 μm, with a 24 μm by 24 μm hole in the middle (fig. 6). The Josephson junctions were situated near one another, but this geometry made no difference in SQUID behavior; this was adopted for fabrication purposes. Embedded on the superconducting material were two coils: an "internal" coil which was used to modulate magnetic flux through the hole (fig. 7), and an "external" coil, which could also modulate magnetic flux from external current sources. The "external" coil was not used (fig. 8).

One can note from the figure that the SQUID resembles the schematic dia-

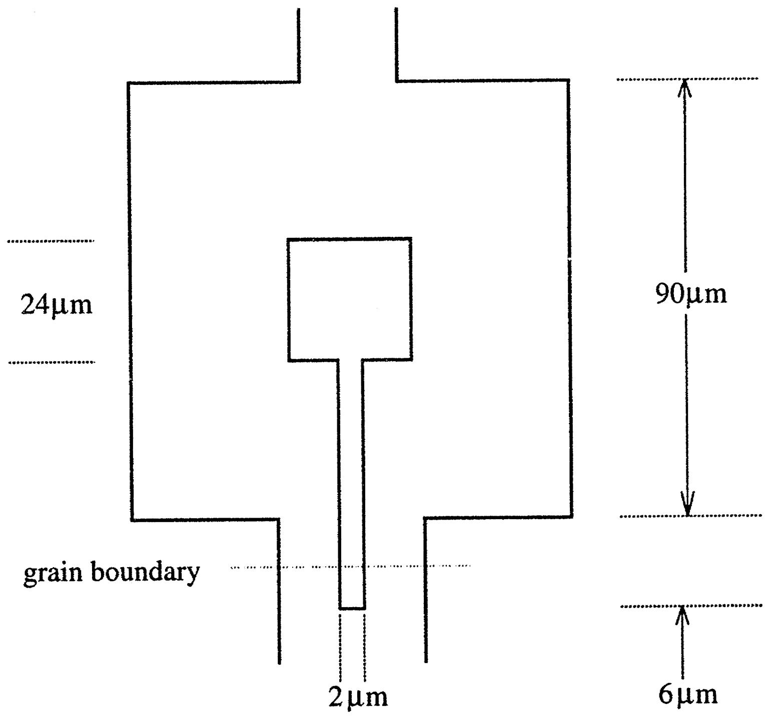

Figure 6: Dimensions of DC SQUID in experiment

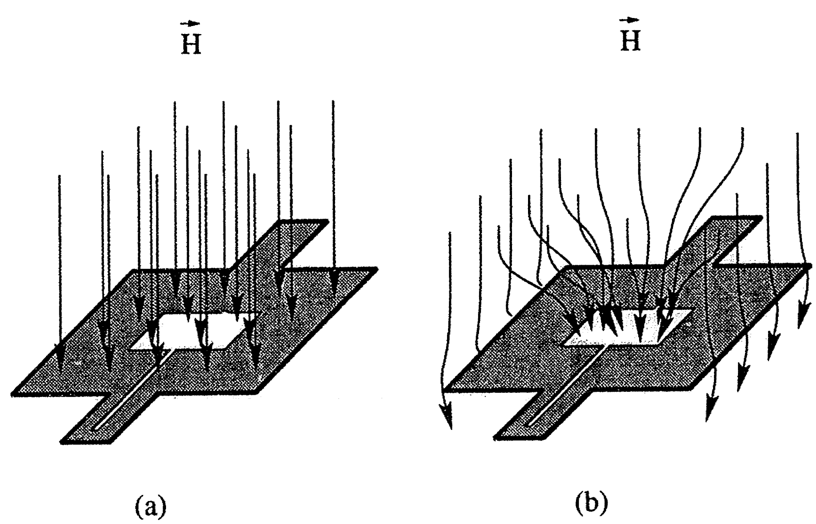

Figure 7: Flux exclusion in the SQUID: (a) Above transition temperature, (b) Below transition temperature

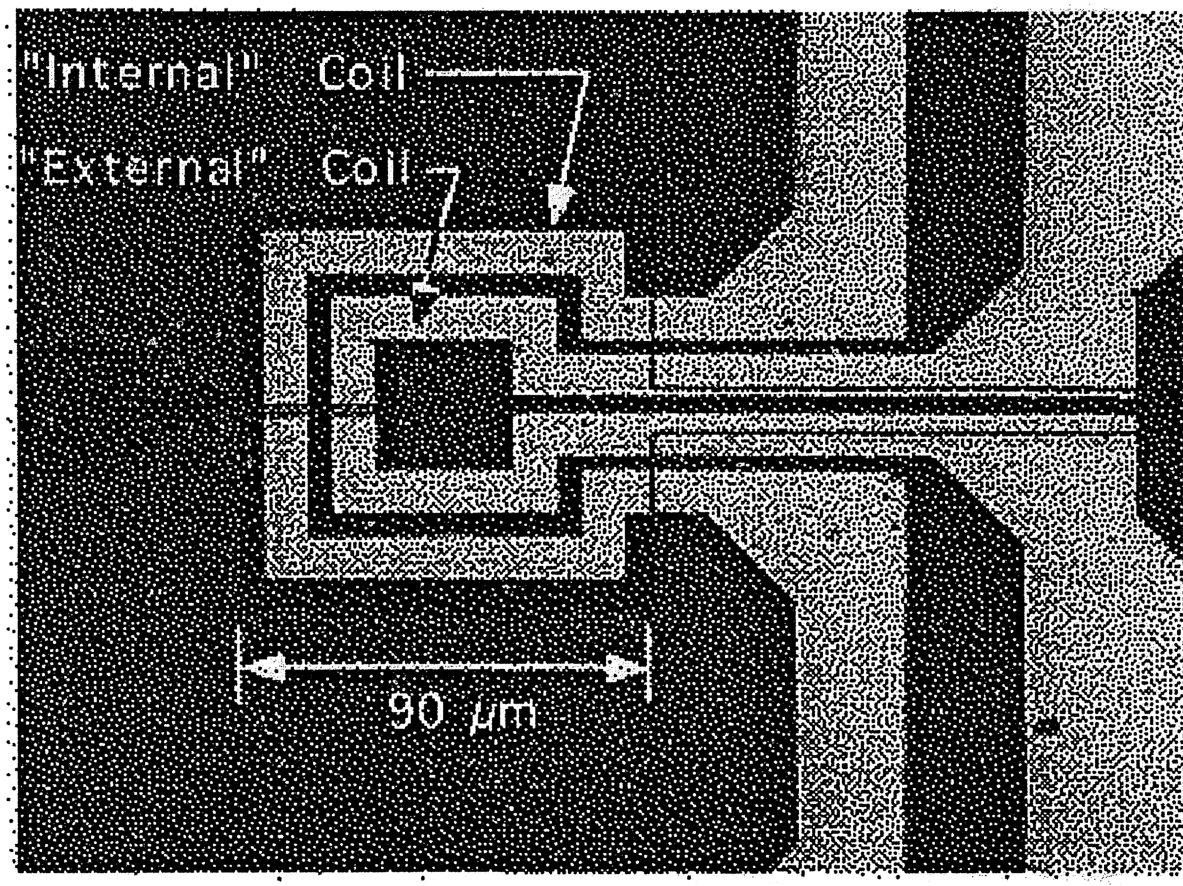

Figure 8: Micrograph of DC SQUID showing internal and external flux modulation coils on SQUID chip

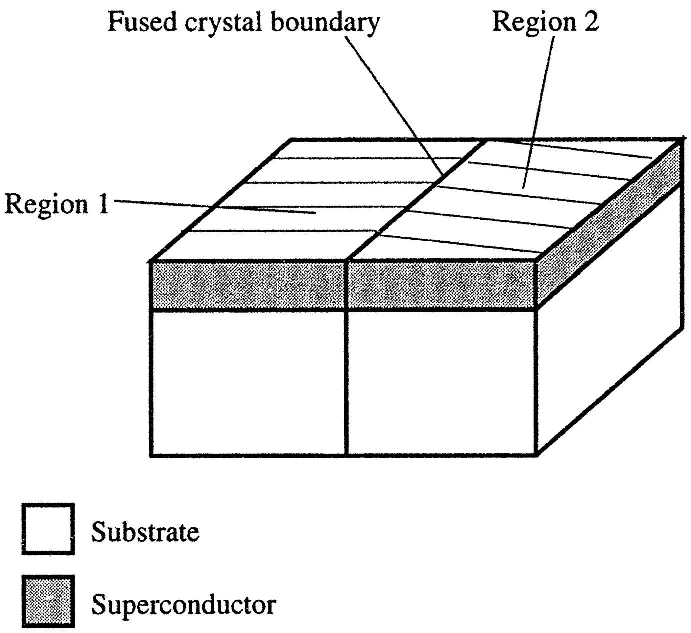

Figure 9: Schematic of bicrystal grain boundary Josephson junction

gram (fig. 1), in that it is a two terminal device, with the terminals on opposite sides of the square loop.

The superconducting material was $YB_2C_3O_7$, a $high-T_c$ ceramic. Actually, the stochiometric ratio of the material is $YB_2C_3O_x$, where x ranges from 6.3 to 6.9. For this SQUID, x was about 6.9, which yields the highest transition temperature. A typical Josephson junction, as outlined in the introduction, is a sandwich: superconducting pieces separated by a non-superconducting layer, typically an insulator. However, a non-superconducting layer isn't necessary to construct a SQUID. All that need exist is a weak link in the flow of current.

For instance, Josephson junctions have been fabricated wherein a superconducting piece tapers to a point and then contacts to another superconducting piece. Flow of current is restricted at the point, which is sufficient for Josephson effects.

Recently, grain-boundary Josephson junctions have been developed. Since $high-T_c$ materials are highly anisotropic, the direction of supercurrents will be dependent on crystal direction. Grain-boundary junctions are those in which one superconducting piece is fused to another superconducting piece such that their crystal orientations are different (fig. 9). A forced change of direction will provide the weak link necessary, and makes up the Josephson junctions in the SQUID. The "cut", or grain boundary is outlined in figure 9.

As the direct connection to the SQUID, the electronics box served three

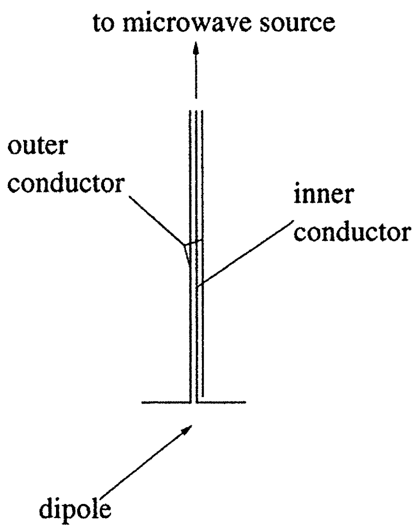

Figure 10: Dipole antenna for AC Josephson effect. Approximate dimensions: 40cm long (probe length), 4cm wide (dipole length).

functions: a) it acted as an amplifier such that the minute voltage signals from the SQUID could be displayed on an oscilloscope; b) it provided a μA current source directly to the SQUID to act as a current bias, as well as a current sweep such that a current-voltage curve could be easily obtained; and c) it provided a stable μA current source to the "internal" coil of the SQUID, acting as a magnetic flux bias.

The box had two modes: current-voltage (V-I) mode and voltage-flux (V-Φ) mode. In the current-voltage mode, the box would sweep the current over a certain range, fixed by current bias and current amplitude controls. The amplified voltage and the driving current signal could both be sent to an oscilloscope on XY mode, thus graphing the current-voltage relation. In voltage-flux mode, essentially the same thing was occuring, except that the voltage was responding to changes of magnetic flux threading the loop via the "internal" coil on the SQUID chip.

The Dewar held $1l$ of liquid. The SQUID's junctions were fabricated from $YB_2C_3O_7$, with a superconducting transition temperature of about 92K. Liquid nitrogen, with a boiling point of 77K, was of adequately low temperature. Some caution was needed when filling the Dewar, to avoid splashing of liquid nitrogen.

For microwave signal injection into the Dewar and observation of the AC Josephson effect, a crude dipole antenna made from rigid coaxial cable (type RG-401) was inserted into the dewar. Antenna shape and dimensions are shown in figure 10.

The signal power was produced by an Advantest vector network analyzer with a 50Ω S-parameter test kit (Advantest). Due to the sensitivity of the SQUID, low power levels (-5 dBm to 0 dBm at the signal generator) were optimal for inducing Shapiro steps.

3.2 Procedure

Operation of the SQUID was straightforward due to the well-designed commercial packaging of the unit. In doing a typical run, I would cool the SQUID to liquid nitrogen temperature by filling a Dewar and then immersing the probe in cryogen. I would then turn on the electronics box, setting the display mode to either the V-I [swept current, voltage output] mode or the V-Φ [swept magnetic flux, voltage output] mode via a switch on the electronics box. The choice depended upon the effect I was demonstrating. I would then observe the output on an oscilloscope. A small amount of flux bias adjustment was necessary to optimize the critical current of the SQUID, giving the sharpest possible I-V curve.

This simple procedure was used to demonstrate the DC Josephson effect and magnetic flux quantization. The electronics box would automatically sweep the current, with the voltage response and swept current acting as inputs to an XY mode on the oscilloscope. The electronics box would perform a similar task for the V-Φ relation.

The biggest problem encountered was the rf interference that could come from all sorts of sources (i.e., harmonics from 60 Hz ac power, WMBR MIT student radio, etc.). Even the tiny amount of radiation from the dipole antenna (and most of the power from the source was not radiated, from impedance mismatching) was enough to observe the ac Josephson effect, suggesting the nominal amounts necessary to distort an I-V curve. The original experiment location, room 4-332 (in the Junior Physics Laboratory), had heavy rf interference in certain areas.

Changing experiment location or changing the orientation of the probe was often enough to get a good display. Magnetic shielding in the form of a mu-metal shield attached to the probe was adequate.

The demonstration of the ac Josephson effect utilized the same procedure and electronics box settings as that of the DC Josephson effect (V-I mode), with one extra complication. The extra task was finding a frequency and power level of microwave radiation such that Shapiro steps could be seen.

It was found that only two small frequency ranges in the microwave bandwidth available induced Shapiro steps. As predicted in the introduction and background sections, all frequencies of microwave power will decrease the critical current of the SQUID, and shows up as a deformation of the I-V relation. However, at constant microwave frequency, the general deformations are Shapiro steps if and only if the shape of the curve remains constant when power levels are altered, since the discontinuous voltage steps created are only dependent on the frequency of the microwave radiation. A typical set of Shapiro steps are shown in the Results section.

Once Shapiro steps were acquired, their voltage step heights were measured off of the oscilloscope display, with a measurement error of about the width of the displayed line.

The initial stage of the experiment was centered on getting an I-V curve with a superconducting region. When cooling the SQUID, the resistance of the device would fall by about two orders of magnitude. This displayed the unusual nature of this superconductor, $YB_2C_3O_7$, as an insulator at room temperature. Initially, it was difficult to obtain an I-V curve due to RF interference, but changing experiment location removed most of the trouble.

After a clear I-V curve was obtained, the V-Φ curve was obtained, with a measurement of the fundamental flux quantum Φ_0, outlined in the Results section.

At this stage, a crude measurement of the transition temperature was made. A zener diode has a temperature sensitive reverse bias voltage, and was the thermometer for this measurement. Calibration was a crude two point linear calibration: bias voltage values were taken at room temperature and liquid nitrogen temperature, with the voltage being linearly dependent on temperature.

The diode was attached (taped) onto the back of the probe and inserted into the Dewar. This time, the probe was not completely immersed in liquid, but allowed to be cooled via nitrogen vapor, such that the temperature of the probe changed gradually. When the slope of the SQUID I-V curve changed dramatically (change in resistance), the bias voltage of the diode, and thus the temperature, was noted. The transition temperature of the SQUID material was found to be about 90K $\pm$5K.

Since $YB_2C_3O_7$ has a variable oxygen doping concentration (that is, $YB_2C_3O_7$ is really $YB_2C_3O_x$ where x ranges from 6.3 to 6.9) which effects the transition temperature by an order of 50K, the transition temperature itself is not well-defined. Empirically, it is simply measured when each batch is made. Upon conferring with Conductus, Inc., the transition temperature of this particular SQUID was 92K.

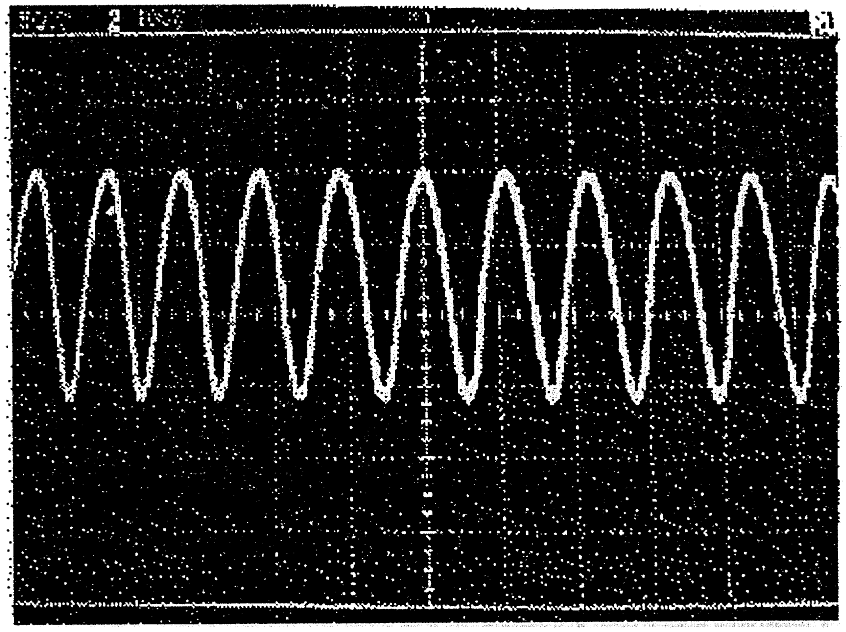

Figure 11: Voltage-flux curve. Scale: 500mV per div. for voltage (Y), $1.85 \cdot 10^{-15}$Wb per div. for flux (X)

4 Results

There are three main results from the experiment. The first is a measurement of the fundamental flux quantum, approximately $\frac{h}{2e} = 2.07 \cdot 10^{-15} Wb$. This comes from the periodicity of the voltage-flux relation. The second is the qualitative observation of the voltage-current relation in the dc Josephson effect, with a measurement of the critical current of the SQUID. This critical current is a parameter specific to each device. The last result is a measurement of $\frac{e}{h}$ that is gleaned from the ac Josephson effect.

To measure the flux quantum, all that we must find is the periodicity in the V-Φ curve. Observing figure 11, it was noted from the oscilloscope display that the periodicity of the oscillating curve was about $55.6\mu A \pm 0.5\mu A$ (90%), obtained by observing 10 oscillations, reading the distance between the 10 oscillations, and then dividing by 10. The error came from the width of the line in the oscilloscope display.

This value was the current injected into the flux bias, or "internal coil" on the SQUID. If the mutual inductance of the SQUID is known, then converting current through the coil to magnetic flux threading the coil is readily done:

$$\Phi = M\Delta I \tag{43}$$

where M is the mutual inductance, Φ is the flux through the SQUID and "internal" coil, and ΔI is the periodicity of the current through the coil.

With the parameters given from the user's manual, the mutual inductance of the device is 37pH, which yields:

$$\Phi_0 = (37pH) \cdot 55.6\mu A = 2.06 \cdot 10^{-15} Wb \tag{44}$$

The errors in this measurement arise from error in the mutual inductance of the device as well as error from the current measurement off of the screen. The readout error was roughly the width of the line on the oscilloscope display.

On-line data acquisition would have resulted in lower readout error. However, given the initial qualitative nature of the experiment, this was not implemented.

The error in the mutual inductance is assumed to be small, since the coil geometry is well-defined. However, it turned out to be on the order of 1%, due to fabrication errors of approximately 1μm on a 90μm long SQUID. Added in quadrature, the error was about 3%. This appears to corroborate accepted values, which is $2.0678 \cdot 10^{-15}$ Wb.

In the second measurement, a critical current at zero applied voltage was observed (fig. 12). This current was measured to be 1.6μA $\pm$0.3μA. This value was obtained directly off of the display by taking the value at which the curve leaves V=0 and the x-intercept of the intermediate straight line. If we assume that the probability density of the actual value is uniform within these limits (a probabilistically conservative assumption), then they can be taken as confidence limits.

This value is temperature sensitive, as well as dependent on the fabrication quality of the SQUID. An improvement on the critical current can be made by lowering the temperature of the cryogen bath. This can be accomplished by pumping on the liquid, lowering the partial pressure of the nitrogen. In obtaining Shapiro steps, we probably pumped on the liquid to about $\frac{1}{10}$ atm, lowering the nitrogen temperature to about 60K, increasing the critical current by a factor of about 1.5.

The third result was an $\frac{e}{h}$ measurement from the observation of the AC Josephson effect. From the output of the voltage-current curve (fig. 13), we see approximate voltage steps. From (0.37):

$$\frac{e}{h} = \frac{\nu}{2V_0} \tag{45}$$

we find that $\frac{e}{h} = 2.65 \cdot 10^{14} \frac{Hz}{V}$.

The error was about $\pm$6% , which arose from error in the thickness of the line, as well as position of the almost-discontinuous steps. This was orders of magnitude greater than the error in microwave signal frequency. With the literature value for $\frac{e}{h} = 2.4179671 \cdot 10^{14} \frac{Hz}{V}$, our obtained value was off by about 10%, or within errors.

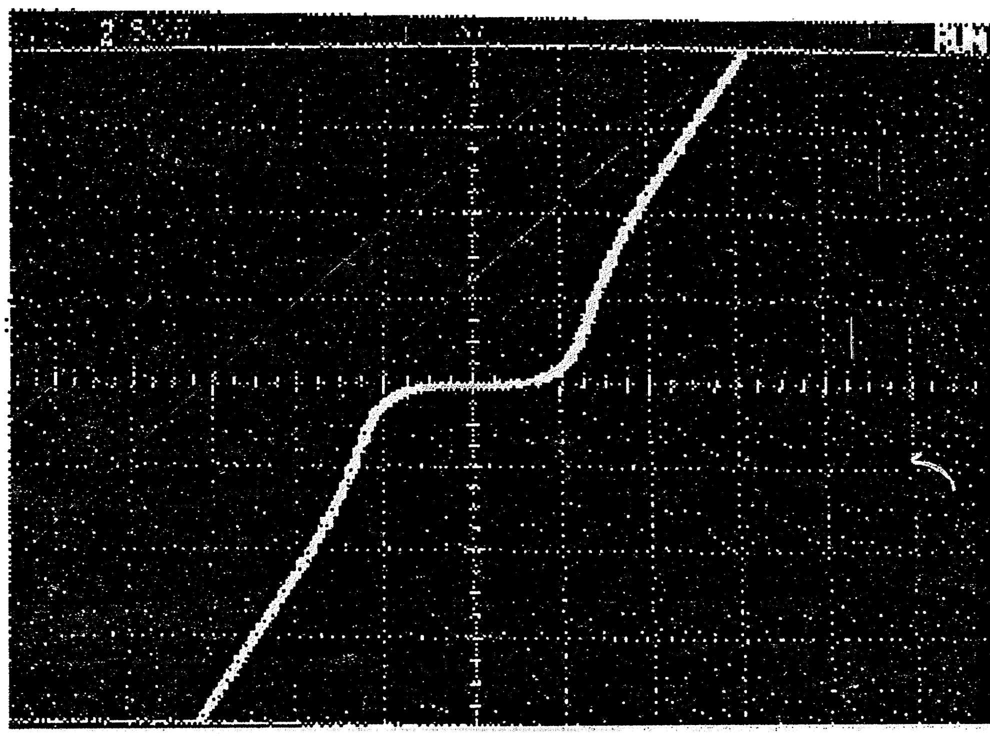

Figure 12: DC SQUID I-V curve. Note superconducting region of zero resistance. Scale: 500mV/div (Y), 1μA/div (X)

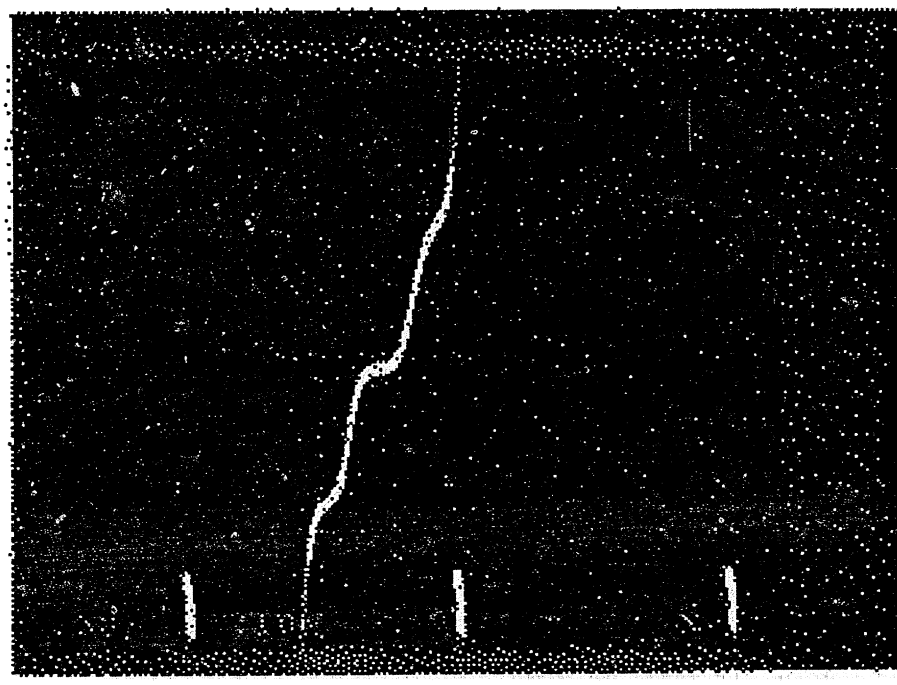

Figure 13: Shapiro steps in AC Josephson effect. Scale: 200mV/div (Y) 0.5μA/div (X)

5 Conclusion

Three effects in DC SQUIDs were observed, and were understood quantitatively. Since this experiment is a good candidate to supplement an existing Junior Physics Laboratory experiment, a well-written write-up to accompany the experiment will allow for a suitable addition to the existing superconductivity experiment [1].

I will make some recommendations if this is to be implemented. A difficulty of this experiment that could be alleviated involves the selecting of a frequency to induce Shapiro steps (AC Josephson effect). Given the geometry and dimensions of the SQUID, it may be possible to actually calculate a set of acceptable frequencies for which radiation would induce Shapiro steps. This calculation could serve as an addendum to the work done here.

Another recommendation I will make is to acquire an analog microwave source. While the digital Advantest unit worked beautifully, it was cumbersome to use, in that changing the frequency could only be done by going through several menus. Changing the frequency from 2.00 GHz to 2.01 GHz required pushing 10 buttons in sequence, where only a twist of a knob should have been necessary. Thus, searching for a frequency at which Shapiro steps appear was time-consuming: a source with a knob controlling frequency would simplify this task.

A final recommendation is that an rf shielded apparatus would be nice to have. A copper box about three feet cubed is an affordable luxury that would eliminate worries of rf interference.

In closing, a logical direction of work in this area is in exploring and exploiting the application of SQUID detectors in ultra-low signal measurements. SQUIDs are already being implemented as the next generation of amplifiers for the MIT Axion Search, and amplifier properties such as the noise temperature need to be measured.

6 References

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