Designing Two-stage Recycling Operations for Increased Usage of Undervalued Raw Materials

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Abstract

Recycling provides a key strategy to move towards a more sustainable society by partially mitigating the impact of fast-growing material consumption. Recent advances in reprocessing technologies enable recyclers to incorporate low-quality secondary materials into higher quality finished products. Despite technological development, the use of these materials in the re-melting stage to produce final alloys is still limited. This thesis addresses this issue by raising the following question: given the complexity of the reprocessing operational environment, what is the most effective way to manage two-stage recycling operations to maximize the usage of low-quality secondary materials? This thesis answers this question for two systems: when outputs from the reprocessing stage can be delivered (1) as sows and (2) as liquid metals to the re-melting stage.

In the first system, the main barrier to use of these materials is the highly variable quality of raw materials. This study suggests the use of data mining as a strategy to manage raw materials with uncertain quality using existing data from the recycling industry. A clustering analysis provides criteria for grouping raw materials by recognizing the pattern of varied compositions. This grouping (binning) strategy using the clustering analysis increases the homogeneity and distinctiveness of uncertain raw materials, allowing recyclers to increase their usage while maintaining minimum information about them.

In the second system, significant energy cost can be saved by immediately incorporating reprocessed secondary raw materials as liquid metal into final alloy production. In this case, the coordination between the reprocessing stage and the re-melting stage is critical. This study suggests integrated production planning for two stages. The mathematical pooling problem is used to model two-stage recycling operations. Integrated planning across the two operations can adjust batch plans and design intermediate products by reflecting demand information of final products. This approach maximizes the use of intermediate products as liquid in the re-melting stage and, therefore, lowers energy cost significantly.

Both strategies are applied to industrial cases of aluminum recycling to explore the benefits and limitations. The results indicate the potential opportunity to significantly reduce material costs and to increase the use of undervalued secondary raw materials.

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1 Introduction

1.1 Motivation for Recycling

Economic development has led to increases in material consumption. As a result, the global material consumption has risen significantly over the past few decades as shown in Figure 1-1. For example, U.S. raw material consumption increased more than nine times in the last century (Matos 2012). Increase in material consumption far exceeds the growth rate of the population, causing various environmental problems such as pollution, waste and depletion of natural resources.

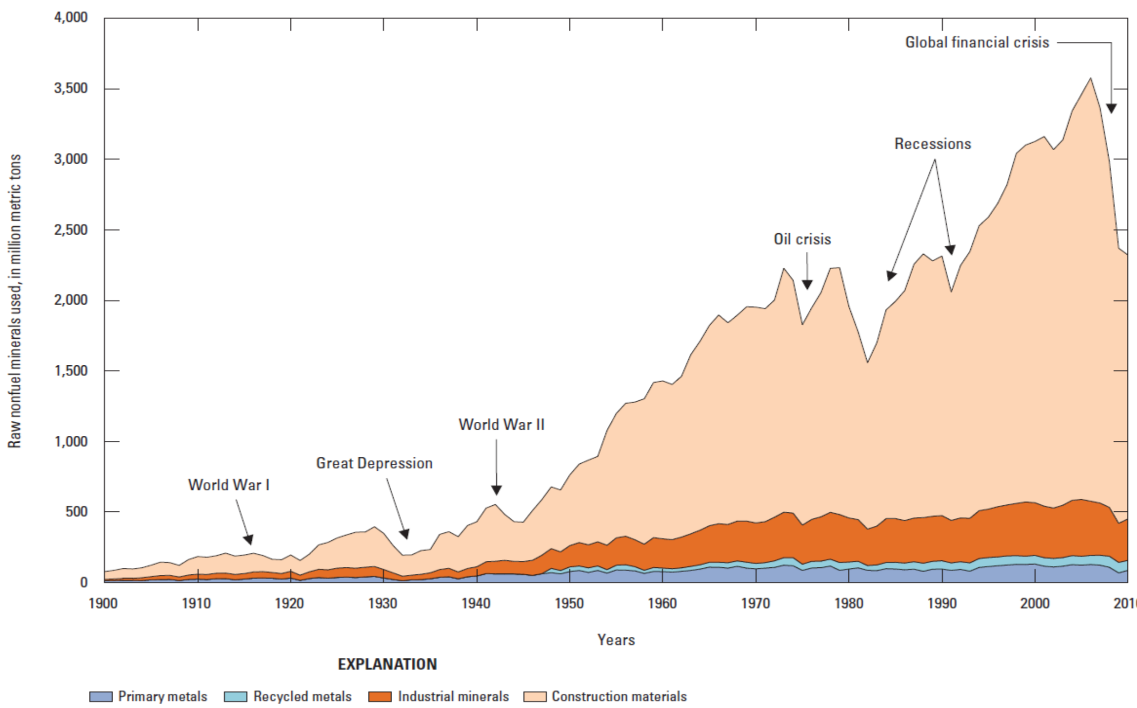


Figure 1-1. U.S raw nonfuel minierals put into use annually from 1900 through 2010. Mineral materials embedded in imported goods are not included. [In million metric tons] Reproduced from (Matos 2012)

Recycling is a promising strategy to partially mitigate problems caused by a material intensive economy. Recycling can conserve natural resources and reduce the amount of waste disposal. In addition, for many materials, the energy requirement to produce secondary materials is lower than the energy requirement to produce primary materials as shown in Figure 1-2 (Grimes 2008).

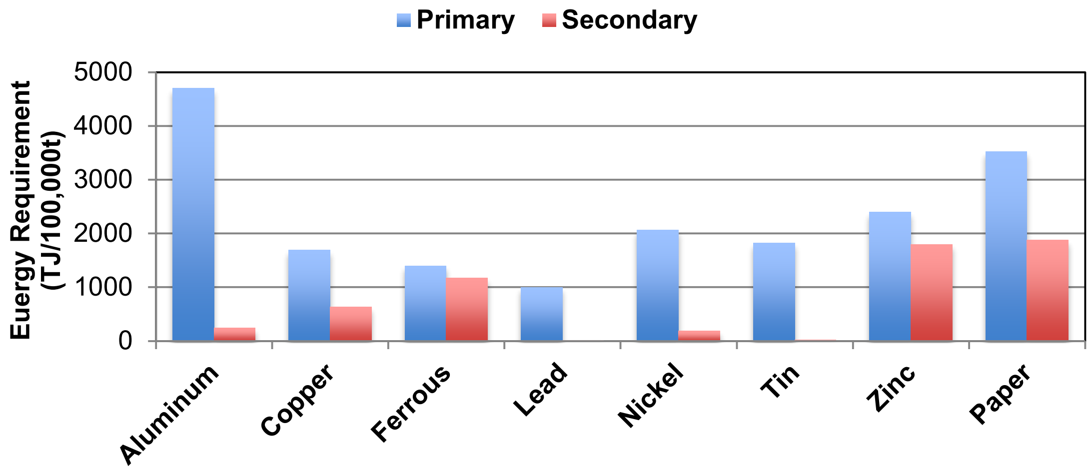


Figure 1-2. The energy requirement to produce primary and secondary materials based on data by (Grimes 2008).

In general, a material is classified as primary if it derives from ore or virgin sources and as secondary if it is produced predominantly from recycled scrap materials. The different energy requirements of primary and secondary production provide additional benefits of recycling. Lower energy requirements for secondary production not only give economic incentive for material producers, but also lower the burden on the environment in terms of resource savings and lower greenhouse gas emissions. Recycling aluminum is particularly attractive because the energy required to produce primary aluminum is 20 times larger than that of secondary aluminum. To produce primary aluminum, 47 MJ/kg of energy is required while the energy required to produce secondary aluminum is only 2.1~2.8MJ/kg (Green 2007; Grimes 2008). The significantly lower energy required to produce secondary aluminum than primary implies lower carbon emission as well. When a unit kg of secondary aluminum is produced, approximately 0.6 kg of CO_2 is released depending on the source of the electricity. This is roughly 95% of emission by producing a unit kg of primary aluminum (Green 2007). Aluminum is currently the most commonly used nonferrous metal and the second-most-commonly used metal behind steel (Goddard 2014). Considering the large consumption volume of aluminum, 45 million tonnes in 2012, and the continued trend of increasing consumption, the impact of recycling aluminum can be significant. In addition, the high corrosion resistance of aluminum and its property of retaining a high level of metal value after use, exposure, or storage make aluminum a good candidate for recycling (Davis, Associates et al. 1993).

1.2 Trends in aluminum recycling market and alternative secondary materials

The benefits offered by aluminum recycling have motivated aluminum producers to incorporate more scrap materials into their alloy production. The recycling rate of aluminum has continuously increased for the last decades. The graph in Figure 1-3 represents the primary and secondary production over the last decades and projection until 2020(IAI 2009; Schlesinger 2013). In 2009, roughly one third of the aluminum demand was satisfied by secondary production (IAI 2009). It is expected that the current trend will continue over the next decade. As aluminum recycling has become important to meet the growing aluminum demand, the scrap material market has become more and more competitive. Aluminum producers have started looking for new sources of secondary raw materials, leading to use of lower quality scrap (in terms of composition and shape) in order to meet increased demand.

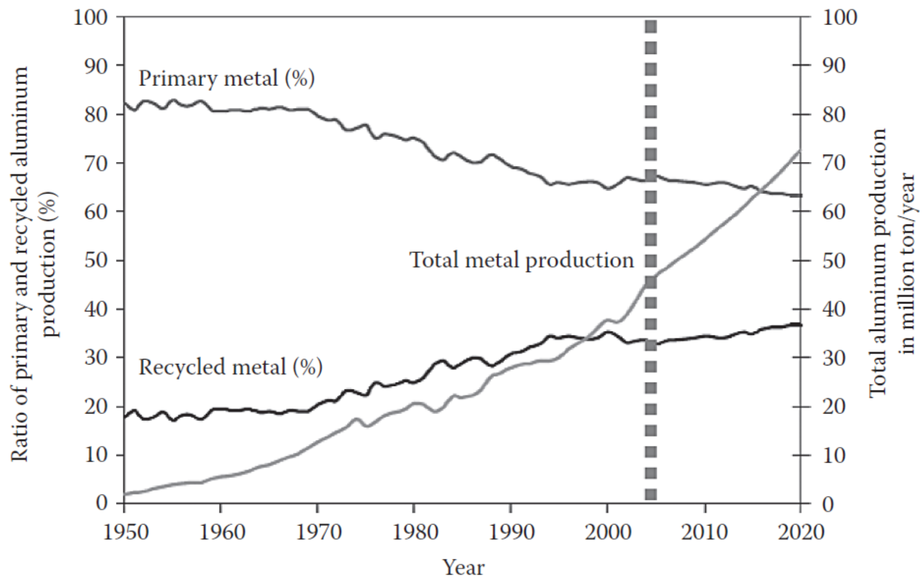


Figure 1-3. Historical and forecasted total aluminum production and percent of aluminum production from primary and recycled sources Reproduced from (IAI 2009; Schlesinger 2013)

It is important to note that there are two types of scrap material: new scrap and old scrap. First, new scrap, also called prompt scrap, is mostly generated during manufacturing processes involving aluminum alloys such as punching and cutting. Old scrap, also called post-consumed scrap, is defined as aluminum discarded after the end of life. Examples of this type of scrap are used beverage cans (UBCs), aluminum foil and automotive wheels. Since new scrap does not go through a use phase, the impurity concentration in prompt scrap is very low. Consequently, it is relatively easy to recycle prompt scrap with a high recovery rate. Conversely, old scrap most likely contains various types of contamination (intended or otherwise) and is more difficult to recycle, compared to new scrap. This type of scrap requires a more intensive reprocessing to get rid of this undesirable contamination. Among various types of old scraps, packaging container is representative of low-quality scrap materials. If other conditions, such as alloy type, are equivalent, new scrap is preferred over old scrap because of its advantages in recycling. However, the increasingly constrained scrap market has prompted aluminum producers to expand their interest to low-quality scrap and alternative raw materials. Although these materials are difficult to recycle, they are more available at a relatively cheaper price in the market. Two sources of secondary materials have recently attracted great attention from aluminum recyclers: packaging containers and aluminum dross.

The packaging container is one of the most popular post-consumed aluminum scrap types. In 2014, the packaging market accounted for 18.4% of the aluminum market. About 75% of packaging aluminum is used for packaging for food, cosmetics and chemical products (Goddard 2014). The most representative type in this stream of scrap is the UBC, which often shoulders the economic burden for recycling of municipal solid waste due to its value compared to other waste streams. The packaging container is an important source of scrap for aluminum recycling. The problem with this scrap stream, however, is heavy contamination by moisture, paint, and other organic components, which are undesirable elements for aluminum recycling.

In addition to old scrap, aluminum dross in Figure 1-4 also has been of great interest to aluminum producers as an alternative secondary material. Aluminum dross, which is a byproduct formed on the top of molten aluminum, has valuable entrapped metals and could therefore be a good candidate for alternative secondary materials. Environmental concern regarding dross disposal gives additional motivation for using it as a resource. The landfill disposal of dross is prohibited in European countries (Council 2000; Council 2000) because dross can react with water and release explosive and noxious gases (Xiao, Reuter et al. 2005; Prillhofer, Prillhofer et al. 2009) causing environmental concerns. Global production of aluminum dross is currently estimated at 760,000 ton/year (Schlesinger 2013) from both primary and secondary smelters. Processing dross can offer economic and environmental benefits for aluminum producers. First, processing dross can have economic value to aluminum recyclers by recovering metal. Second, it also reduces the cost associated with waste treatment processes, particularly in Europe.

Figure 1-4. A pile of aluminum dross.

Table 1-1 shows the various types of scrap sources and their contents (Boin and Bertram 2005). The metallic content of these materials implies the maximum limit of how much aluminum recyclers can recover from them. It is important to note that packaging containers and aluminum dross have lower metal content and higher levels of foreign materials than new scrap. Even though these raw materials are challenging to recycle, these raw materials are cheap and relatively easy to acquire in the secondary materials market, potentially offering aluminum producers economic benefits.

Table 1-1. Selected scrap types listed in European aluminum scrap standard and their average scrap composition. Reproduced from (Boin and Bertram 2005).

Scrap Description	Aluminum Metal (%)	Oxides(%)	Foreign Materials(%)
Wire and cable (new scrap)	98.7	1.3	-
Wire and cable (old scrap)	97.7	1.8	0.5
One single wrought alloy	97.2	1.0	1.8
Two or more wrought alloys of same series	97.2	0.8	2.0
Two or more wrought alloys	94.0	0.8	5.2
Castings	83.4	6.2	10.4
Shredded and density separated scrap	84.5	5.4	10.1
Used beverage cans	94.0	0.8	5.2
Turning, one single alloy	95.3	3.7	1.0
Mixed turnings, two or more alloys	84.0	3.3	12.8
Packaging (coated)	71.5	3.8	24.7
Packaging(de-coated)	86.1	12.9	1.0
Dross	55.7	44.3	

It is estimated that purchasing raw materials absorbs more than 70% of revenue in the aluminum industry. This value is significantly higher than the average of other industries

(Goddard 2014). If aluminum producers can replace primary aluminum with scrap aluminum to produce finished alloy products, this substitution can directly lead to increases in their profit margin. The higher the percentage of low-quality scrap that is used in finished alloy production, the higher the profit margin that can be achieved. Therefore, the size of the profit margin heavily depends on how successfully aluminum producers upgrade and incorporate low-quality materials into alloy products compared to same alloy products produced from primary aluminum. Also the development of various technologies to improve recovery rates of low-valued scrap types contributes to making recycling these materials more feasible.

1.3 Aluminum recycling technology enabling use of low-valued raw materials: Two-stage recycling operation

Recycling low-valued secondary materials is more challenging than recycling clean scrap materials because of concentration of unwanted elements in these raw materials. Another aspect of scrap that determines recycling ease is physical size. It is well known that direct exposure of light-gauge scrap, such as shredded scrap and UBC, to the re-melting furnace atmosphere results in large melt loss. This is because the ratio of surface area to the weight of scrap increases with decreases in the size of scrap. Larger surface area implies greater potential formation of oxide skin layers on the scrap surface, which can cause aluminum metal to be trapped in oxide layers (Peterson 1995; Thornton, Hammond et al. 2007).

Prices of various scrap materials reflect the degree of difficulty of recycling. Schlesinger also pointed out that four characteristics of the scrap determine its price. Those characteristics are alloy purity, contaminants, coatings and attachments, and size (Schlesinger 2013). Contaminants, coating and attachments can be interpreted as factors related to cleanness. The graph in Figure 1-5 represents relative cleanliness and size of different types of scrap materials. The lower left corner is the most difficult to process and the upper right corner is the easiest to recycle (Peterson 1995). Therefore, the dirtier and the smaller the scrap is, the more difficult metal can be recovered from it.

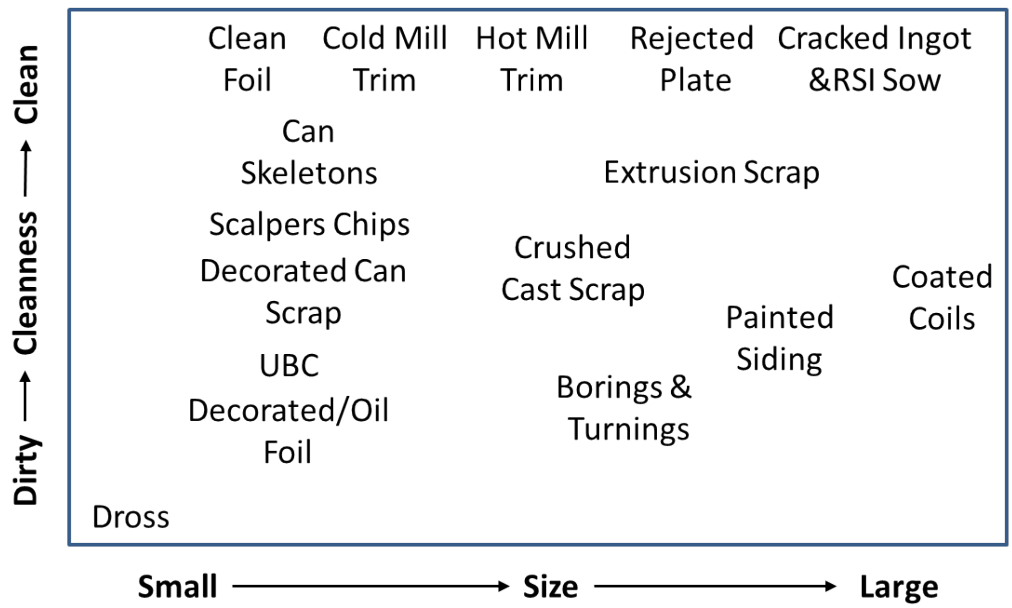


Figure 1-5. Map of sources of aluminum secondary raw materials based on cleanliness and size, Modified from (Peterson 1995).

To use raw materials located in the lower left corner of the graph in Figure 1-5 for finished alloy production, these materials must be reprocessed before blending with clean raw materials in a re-melting furnace. This required pre-processing stage allows aluminum reproducers to protect the expensive aluminum re-melting furnaces from excessive exposure to oxide materials that can accumulate on the furnace wall. Also salt fluxes, which are undesirable inputs in re-melting furnaces, must be used to recover metal from scrap materials with high oxide content. This required pre-processing leads to a two-stage blending operation: blending dross and post-consumed scrap into intermediate products and blending these intermediate products with primary and alloying elements into finished alloy products in a re-melting furnace. Introducing pre-processing technology enables aluminum producers to recover low-valued secondary materials at high recovery rates, which was impossible in one-stage blending operations. Using reprocessed undervalued raw materials effectively in the second blending stage (i.e. re-melting stage) is a key to fully maximize the benefit of reprocessing technologies. To achieve this, a different approach is needed to properly manage two-stage recycling operations. This thesis focuses on two types of popular reprocessing methods to recover valuable metallic content from low-valued scrap and dross: a rotary furnace and a continuous melting system.

1.3.1 Rotary furnace: technology to recover aluminum dross

The rotary furnace is one type of furnace able to reprocess low-quality secondary materials. It has a barrel shape and rotates around its axis as shown in Figure 1-6. The rotary furnace was first introduced in the late 1970s and has undergone several changes to achieve more efficient recovery. One of the major changes in design was switching from a fixed-axis to a tilted-axis rotary furnace to improve the recovery rate and reduce cycle times. Rotary furnaces are able to process a wide range of feed stock from UBC to dross. It is considered the most flexible and universal equipment to process scrap materials (Peterson 1995). Rotary furnaces can process various types of scrap materials including even highly contaminated ones at fairly good recovery rate. Also, it is currently known as the most effective method to recover aluminum dross (Tzonev and Lucheva 2007).

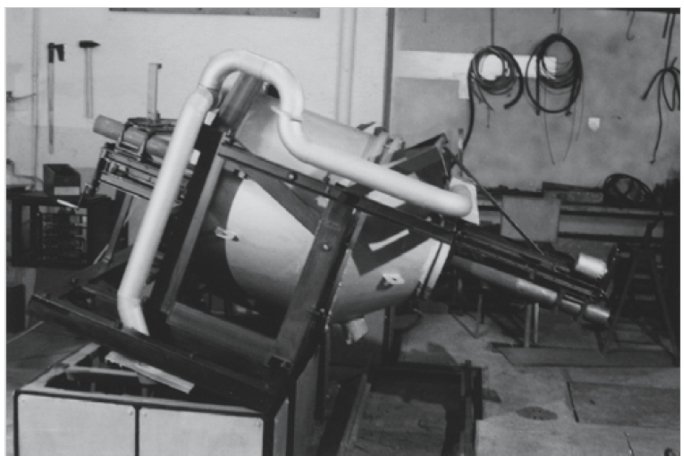


Figure 1-6. Photograph of a direct-current electric-arc rotary furnace (Tzonev and Lucheva 2007).

A mixture of dross, scrap and salts such as NaCl and KCl is charged in a rotary furnace. Using salt flux promotes the coalescence of suspended metal droplets and separation of metal from oxide contamination (Majidi, Shabestari et al. 2007). The rotation action helps to break up oxide particles and recover entrapped metal. In general, rotary furnaces are preferred for melting dross and other oxidized scrap material and for smaller-size of scrap (Schlesinger 2013). These types of materials are ones difficult to process, located at the lower left corner of Figure 1-5. Although the small piece of scrap materials is now preferably processed in continuous melting systems, rotary furnaces have been particularly of interest to aluminum producers due to dross processing, flexibility, and reasonable recovery rate. Outputs of a rotary furnace, recovered metal from dross and scrap materials, can be delivered to downstream in the form of liquid metal or sow, a block of metal solidified in a mold as shown in Figure 1-7.

Figure 1-7. Low –profile secondary aluminum sow (Schlesinger 2006).

1.3.2 Continuous melting system

In the past, UBCs were reprocessed mostly in a rotary furnace. In 1988, 16.5% improvement in recoveries of metal from UBC was reported by switching to continuous melting system (van Linden 1988). Packaging container scrap including UBC is shredded into small pieces and passed through scrap drier/delacquering kiln/decoating kiln processes to remove coatings and various organic contaminants prior to melting. Prepared shredded scrap materials must be submerged quickly to minimize melt loss without direct exposure of re-melting furnaces atmosphere (Green 2007; Thornton, Hammond et al. 2007).

Therefore, continuous melting systems are designed to satisfy this melting condition. In continuous melting systems, molten metal is pumped at high velocity and forms a vortex. Small pieces of shredded scrap materials are fed into the circulation (or vortex) of hot liquid metal as shown in Figure 1-8. This process allows scrap to melt quickly before it develops an additional oxide layer on the surface. There are some variations in the design in terms of scrap size tolerance or production capacity (van Linden, Herrick et al. 1976; van Linden and Gross 1981; Claxton 1982; Cooper 1986). All these types of furnace generally generate minimal skim layers and are able to reprocess shredded scrap at high recovery rates. This kind of furnace is currently the most popular technology to recycle UBC and other shredded scrap materials.

Another advantage of using a continuous system is that it does not require any salt flux unlike a rotary furnace. Therefore, continuous furnace operation does not generate any salt cake, which is a byproduct of reprocessing scrap and dross with salt fluxes. The landfill of salt cake poses environmental concerns as described above for aluminum dross and, therefore, requires proper treatment. Because of these advantages, as their design has been optimized continuous melting systems have become popular to recover shredded scrap materials. Recovered metal can be delivered to downstream as liquid metal or as sows as in the case of using rotary furnaces. However, in most practices, a continuous melting system is attached directly to the re-melting furnace. In this setup, molten scrap can be directly transferred as liquid metal to a re-melting furnace without opening the hearth.

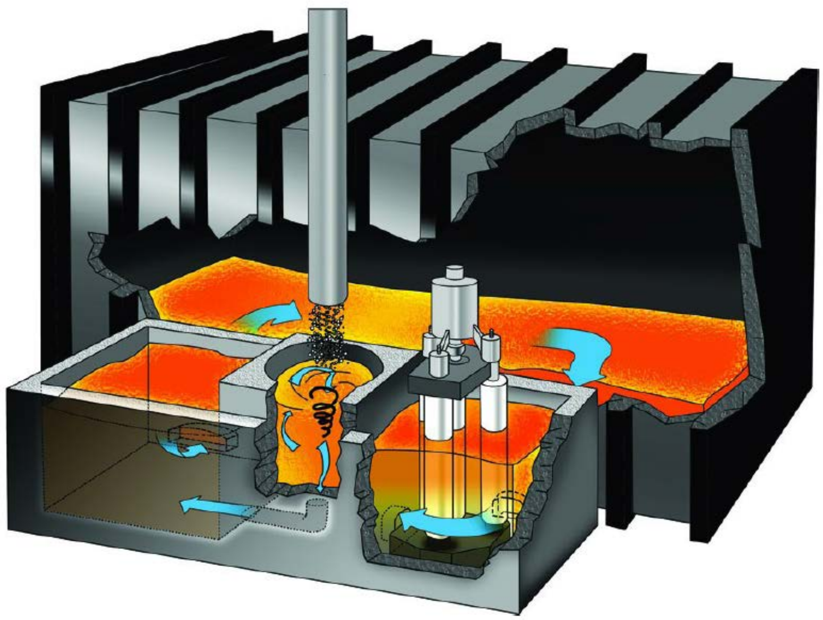


Figure 1-8. LOTUSS (Low Turbulence Scrap Submergence) System(Pyrotek).

1.3.3 Challenges in two-stage recycling operations

Once dross and scrap materials are reprocessed into intermediate products, they can be used as feed materials for a re-melting furnace to produce finished alloy products. To maximize the benefit of recycling, aluminum recyclers must be able to incorporate intermediate products into final products as effectively as possible. Failure to use intermediate products effectively causes dramatic increases in raw material costs by using more expensive primary aluminum (or more expensive scrap materials) to meet demand. Depending on design layout and equipment available within a recycling facility, the recovered metal from reprocessing furnaces can be cast as sows or can be delivered as liquid to downstream re-melters. The way intermediate products are delivered to the downstream furnace within a facility is an important factor in determining optimal design of a two-stage recycling operation. Different operational environments cause different challenges to maximize usage of low-quality raw materials.

For the case where operators are delivering intermediate products to downstream re-melters as liquid metal, the biggest challenge is to maximize the amount of intermediate products that remain molten. Since aluminum liquid metal is highly perishable (i.e. it will solidify), liquid metal must be cast as sows and be stored if it is not immediately used in finished alloy production (some facilities have equipment to keep the metal molten, but this becomes expensive). However, casting produced liquid metal as sows requires additional energy costs to subsequently remelt them. For this reason, the benefit of delivering liquid metal can be largest if all intermediate products can be incorporated as liquid metal in finished alloy productions. To achieve this goal, the reprocessing operation and re-melting operation must be closely coordinated. An operator of a rotary furnace or continuous furnace needs to produce intermediate products that can be immediately used in alloy production. To do that, an operator of the reprocessing stage must be aware of demand information for finished alloy products in downstream re-melters.

When intermediate products cannot be delivered as liquid metals, the compositional uncertainty of intermediate products is the major challenge. Secondary material compositional uncertainty has been identified as one of major problems to achieve successful recycling operation (Peterson 1999). When intermediate products are delivered as sows, there is no incentive for downstream re-melter operators to immediately use sows freshly produced on the day of production versus ones produced on earlier days. Consequently, production in the reprocessing stage and re-melting stage need not be coordinated. Intermediate products are not particularly designed to fit the specifications of alloy products produced in a downstream re-melter. This is often due to the physical location of the reprocessing stage and the re-melting stage. For example, an off-site dross re-processor who engages in tolling operations is not necessarily aware of demand information of their customer. This leads to the situation where there is no planning for the reprocessing stage. The goal of a reprocessing stage operator is to maximize recovery of dross and scrap materials rather than to produce designed intermediate products. Hence, the resulting compositions of sows are highly variable. An operator of downstream re-melting furnaces receives partial information on the composition of sows such as the average and variation values. It is not easy to obtain accurate compositional information on individual sows unless they are melted.

As a result, in actual practice, it is common to use sows in production of low-quality alloy products (i.e., ones with wider compositional specifications) or use only limited amounts of them. This practice occurs in order to avoid violating specification of final alloy products caused by sows having compositions largely deviated from the average or estimation. All intermediate products must be cast as sows in the following cases: off-site dross processing in which an outside contractor processes aluminum dross and returns it to the re-melter for a fee, or on-site processing facilities without equipment for delivering liquid metal from a rotary furnace to a downstream re-melter.

1.4 Description of Thesis

This thesis will explore the following question,

- Given the complexity of the reprocessing operational environment, what is the most effective way to operate a two-stage recycling operation to maximize the usage of low-quality secondary materials?

Figure 1-9 schematically describes the two-stage recycling operation including reprocessing and downstream re-melting stages, and material inputs and outputs of each stage. As explained earlier, issues between the reprocessing stage and the re-melting stage vary depending on the plant setup. Therefore, different approaches are required to design an efficient batch operation in an aluminum recycling facility. This thesis answers the research question above in two situations: when delivery of intermediate products from a rotary furnace or a continuous melting furnace to downstream re-melters is (1) in a cast form (a sow) or (2) as liquid metal.

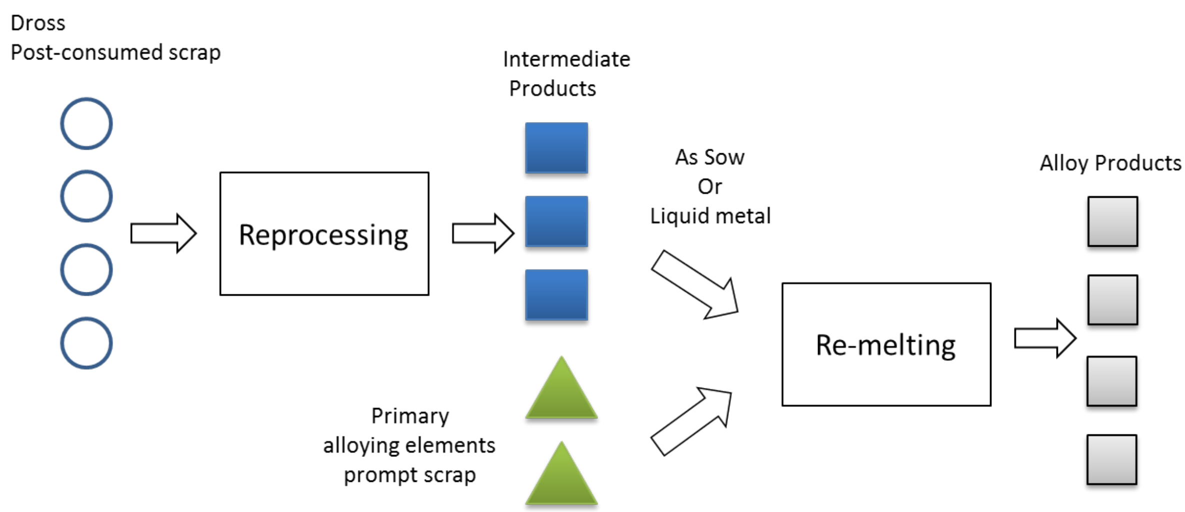


Figure 1-9 Schematic description of a two-stage recycling operation

First, for the case where intermediate products are delivered as sows, managing compositional variation is the major challenge for increasing the usage of low-quality raw materials. This thesis suggests grouping (binning) sows to resolve this issue. A clustering analysis is used to recognize the pattern of compositional variation of sows in order to provide criteria for grouping sows. Assigning sows into different categories by cluster analysis enables each bin to have a more homogeneous composition. The idea of using cluster analysis is motivated by actual practice in which the compositions of recovered dross and scrap are measured after reprocessing stage but not used except when calculating the average compositions of them. This study will explore the opportunity for employing existing data in recycling firms and evaluate the impact of grouping sows by cluster analysis on the usage of low quality raw materials in batch planning for re-melting furnaces.

Second, for the case where intermediate products are delivered as liquid metal, the major issue is coordinating production plan between the reprocessing stage and the re-melting stage. This study proposes integrating production planning for two different stages simultaneously to achieve this coordination. The mathematical pooling problem is used to model this integrated production planning. This thesis work attempts to understand benefits and limitations of integrated production planning and compare with independent production planning in which operations of reprocessing and re-melting furnaces are independently planned without any coordination. The performance of two different approaches will be evaluated in terms of total material production cost as well as the amount of intermediate products incorporated as liquid in the re-melting stage. In addition, this thesis studies the interaction between blending behaviors of intermediate pools and various operational parameters.

This thesis is structured in the following way. Chapter 2 examines previous research on technology to improve recovery of low-quality raw materials, managing uncertainty of raw materials, and mathematical batch planning tools. Chapter 3 describes methodologies used in this thesis. Chapter 4 presents operational strategies when intermediate products cannot be delivered as liquid metals. Chapter 5 presents integrated production planning approach when intermediate products can be delivered as liquid metal. Chapter 5 discusses the fundamental understanding of the pooling problem with a small case study. This case study is designed to understand benefits and limitations of the pooling problem as a mathematical tool for integrated production planning and blending behaviors of intermediate products in the context of metallurgical batch planning. Chapter 6 extends the discussions of Chapter 5 and demonstrates the impact of integrated production planning in an actual recycling operation. Chapter 7 describes the limitations of the presented work and presents a proposal for further study.

2 Literature Review

2.1 Researches on reprocessing technology to improve recovery of undervalued raw materials

A variety of reprocessing methods have been studied and developed to improve recovery of undervalued raw materials. These studies often aim to improve recovery of particular types of secondary raw materials. Approaches and scopes can vary widely depending on the target materials to be reprocessed. This section is divided into two parts: studies of reprocessing technologies and practices (1) for aluminum dross and (2) for shredded scrap of UBC and other packaging scrap materials.

2.1.1 Previous work studying reprocessing dross

Currently, dross management has involved metal recovery, either mechanically or chemically (David and Kopac 2013), as well as repurposing as refractory materials, along with composites, and slag, among others (Hermsmeyer, Diekmann et al. 2002; Shinzato and Hypolito 2005; Bajare, Korjakins et al. 2012; Dai and Apelian 2012). These latter methods have shown promise and lead to lower waste by volume due to salt management (David and Kopac 2012), but in some cases, metal recovery may be most beneficial either economically or environmentally (Nakajima, Osuga et al. 2007).

For metal recovery, the compositional characteristic of recovered dross is the key information for aluminum manufacturers who use it as a feed material because the composition of raw materials is directly related to profitability in alloy production. Previous research characterized the chemical and physical properties of aluminum dross and provided general estimates for those compositions (Manfredi, Wuth et al. 1997; Kevorkijan 2002). As several authors have pointed out, many factors influence the composition of dross. These factors include the skimming method, composition of the molten alloy, added salt flux composition, and dross-cooling process (Hiraki and Nagasaka 2014). For example, Manfredi and co-authors found that metal content in the industrial aluminum dross can range from 47% ~ 93%. These authors also found that the total alloying element content ranges from 1.03 – 6.80% depending on types and sources of dross (Manfredi, Wuth et al. 1997). Although limiting the number of dross suppliers may reduce the range of chemical compositions, the results from many studies suggest that the uncertainty from dross composition is unavoidable even if dross is separated by the original melt (Manfredi, Wuth et al. 1997; Xiao, Reuter et al. 2005).

Previous researchers have tried to find the optimal set of operational conditions for rotary furnaces in reprocessing aluminum dross (Zhou, Yang et al. 2006; Tzonev and Lucheva 2007). Tzonev and his colleague studied the influences of different factors on dross recovery. The authors examined how aluminum recovery rate from different types of dross (compact versus granular) changes with the rotation speed of the rotary furnace, retaining time before tapping, and tapping temperature. This study identified the optimal technological parameters for both compact and granular dross and also found that crushing bodies during dross processing can increase aluminum recovery by 10% when other operational parameters are optimized (Tzonev and Lucheva 2007). Zhou et al. developed a computational model to understand the complex metallurgical reactions of dross and scrap reprocessing in a rotary furnace (Zhou, Yang et al. 2006).

Some researchers explored the possibility of increasing the efficiency of aluminum dross reprocessing by manipulating salt fluxes added in dross reprocessing. Utigard et al. found that adding a salt flux can significantly reduce aluminum oxidation and liquid metal losses during recycling operations (Utigard 1998). They analyzed characteristics of various types of salt fluxes and identified some chemistries of fluxes are more proper for dross processing. For example, fluoride salt fluxes can reduce the entrapment of aluminum metal inside oxide by increasing the interfacial tension between oxide and metallic aluminum. This research informed aluminum recyclers of how to choose fluxes based on the operational conditions such as alloy chemistry or operating temperature. Some researchers have tried to find a salt-free dross reprocessing technology. As stated above, salt cake is environmentally undesirable and requires expensive treatment methods to be discarded. Different salt-free dross reprocessing technologies are well reviewed in the paper by Unlu and Drouet (Unlu and Drouet 2002). Although these technologies are more environmental friendly, they require either high capital investment or high electricity cost and are therefore not used commercially (Schlesinger 2013).

2.1.2 Previous work researching reprocessing shredded scrap

The benefits of shredding and de-lacquering scrap materials before melting them were recognized early in the growth of the aluminum recycling industry (Li and Qiu 2013). The benefits include exposing the lacquered inside surface of the UBCs to make them easier to decoat and allowing trapped moisture, liquid, or contaminants in the cans to be released (Schlesinger 2013). Due to these benefits, shredding UBCs or other packaging scrap became a necessary part of scrap preparation in aluminum recycling. This process is certainly beneficial in terms of removing unwanted elements and making scrap chemically cleaner. However, it adds a difficulty by reducing the physical size of scrap materials. As shown in Figure 1-5, the physical size of scrap is also an important aspect of determining the level of difficulty of melting. As the size of scrap materials decreases, the relative surface area per unit weight of scrap greatly increases. It is easy to develop oxide layers on the surface of scrap in a high temperature environment, which causes metal loss (Xiao and Reuter 2002). Continuous melting systems have been used to resolve this issue. It has been reported that the recovery rate of metal from UBCs has been increased from 80% to 90% when switching to a continuous melting system (Thornton, Hammond et al. 2007). The key characteristic of this type of approach is to quickly submerge the pieces of scrap into the molten metal and melt them while minimizing oxidation of scrap pieces (Green 2007; Schlesinger 2013). As a result, dynamics of melting small pieces of scrap in the circulation of the molten metal have been studied by many researchers (Farner 2000; Thornton, Hammond et al. 2007).

The study by Thornton et al. calculated the effect of shredded thickness and oxide strength (Thornton, Hammond et al. 2007). This study informs aluminum recyclers about the optimal size of shredded scrap that can minimize melt loss to get the best metal yield in melting scrap in the continuous furnace. The authors of this paper investigated the optimal flow rate for circulation of molten aluminum in a continuous furnace. The key operational parameters of continuous furnaces are temperature and flow rates that provide an enough shear force to break up the existing oxide layer on the surface of shredded scrap. Farner studied re-melting aluminum scrap by continuous submersion. Farner examined the effect of lacquer and temperature on the melting rate of scrap and compared with the mathematical model to understand heat transfer mechanism of one-dimensional steady state models (Farner 2000). The study by Farner suggested that preheating and increasing the melt temperature increased the melting rate, whereas lacquer and oil on the surface of scrap reduced the melting rate. Farner also pointed out the dimensions of the scrap were an important factor to determine the efficiency of melting scrap because the heat transfer coefficient varies with the dimension.

The studies described in last two sections greatly contribute to finding optimal conditions for reprocessing technologies that maximize the recovery rate of dross and shredded scrap. The results presented in these studies help identify a theoretical recovery. However, many of these studies did not address how to effectively incorporate processed scrap and dross into finished alloy production. In other words, studies reviewed in this section mostly focus on how to improve the recycling efficiency only in the first stage of aluminum recycling operations rather than how to achieve successful overall recycling operations which consist of both reprocessing and re-melting stage. This question is as important as improving the recovery rate of the reprocessing stage because the relative amounts of low-quality raw materials used in final alloy products determines the profit margin for aluminum producers. This thesis attempts to answer the question: how can we achieve higher overall recycling efficiency by increasing the use of recovered dross and scrap materials in final alloy products with given reprocessing technologies.

2.2 Research on managing uncertainty of raw materials

The compositional uncertainty of scrap materials has been identified as one of the most critical issues to improve recycling as mentioned in the previous section. Variation in raw material compositions results in difficulty in satisfying the target specifications of finished alloy products. Therefore, the use of raw materials with compositional uncertainty is often very limited despite its economic incentives for aluminum producers.

Uncertain raw material quality has been one of the major barriers for recycling as well as remanufacturing. Despite the fact that remanufacturing focuses on component recovery while recycling recovers materials, these two activities are similar in that the goal is to redirect or repurpose waste as input resources. Although recycling and remanufacturing have different operational constraints originating from the nature of the different processes, blending and assembly, respectively, they share common difficulties in practice. Both recycling and remanufacturing processes are inherently subject to uncertainty in input quality because secondary materials and components come from varied sources under varied conditions. Guide addressed the need for research in production planning and control of remanufacturing, given the uncertainties in recovered materials (Guide Jr 2000). Thierry et al. pointed out that the benefits of remanufacturing and recycling processes can be maximized when companies are able to manage the quality of returned materials (Thierry, Salomon et al. 1995). Many studies take the uncertainty of raw material quality into consideration in recycling and remanufacturing firms' decision-making processes. Previous work focused on improving the quality of secondary streams by increasing local homogeneity. Some studies explored different management strategies regarding material acquisition to manage the quality of returned products in remanufacturing (Guide and Van Wassenhove 2001). Thierry et al. reported the case of a copier manufacturer with the strategy of reducing the types of materials in products in order to achieve a simplified and cost-effective recycling operation (Thierry, Salomon et al. 1995).

One of the most popular approaches to increase homogeneity of raw materials is sorting. Lund et al. performed an analysis of a centralized material-recovery facility to sort municipal solid waste using linear programming (Lund, Tchobanoglous et al. 1994). Research by Stuart and Lu demonstrated a model for reprocessing options for electronic scrap (Stuart and Lu 2000). These papers explored the interaction between operational decision making and sorting in various contexts.

Sorting requires knowledge of the stream quality (identification) as well as a strategy to separate (group). In some cases, however, the composition of materials (i.e., the target objects of sorting) is unknown or not a constant. Galbreth and Blackburn considered the variability of used product conditions in remanufacturing and perform an analysis of optimal acquisition and sorting policies. The authors also pointed out the common assumption (often unsubstantiated) regarding homogeneous quality of returned products made in many other papers (Galbreth and Blackburn 2006). In most of these papers, there has been only one decision around sorting: sort or not? There is no further discussion on the necessary degree of homogeneity of materials streams or required levels of sorting to achieve profitability. A recent study by Li et al. investigated the economic feasibility of separating various scraps into two categories, cast and wrought, and identifies the context that maximizes the benefit of sorting (Li, Dahmus et al. 2011). This study evaluated the impact of different recovery rates for cast and wrought scrap. However, the discussion around how to determine criteria to categorize raw materials and the effectiveness of these grouping methods has not been sufficiently addressed.

Chapter 4 of this thesis suggests a way to improve the homogeneity of raw materials, using existing data from a recycler before investments are made into sorting technology. In this thesis, a clustering analysis is used as a strategy to segment or categorize raw materials, specifically dross from aluminum re-melting, across a broad compositional space into a more homogeneous stream. This approach is shown for a case where the identification has been made but the method to group the raw materials is not clear.

2.3 Research on blending problems

2.3.1 One-stage blending models

Many researchers have developed mathematical tools to describe batch planning for aluminum alloy production as well as for other industries. Batch planning tools help firms to make decisions about allocating raw materials efficiently throughout their production with consideration of various operational constraints. This category of optimization problem is called a blending problem. The blending problem determines optimal blends of various raw materials that maximize profit or minimize raw material cost, while satisfying a set of constraints, including product quality specification, product demand and raw material availability. The applications of the blending problem vary from gasoline (Symonds 1955) and steel production (Fabian 1958) to bio diesel (Gulsen, Olivetti et al. 2014) and chemical fertilizer (Ashayeri, van Eijs et al. 1994) due to its relative simplicity of mathematical formulation and the opportunity to significantly increase profitability.

A linear one-stage blending model can provide not only optimal production plans but also valuable information about sensitivity analysis. Sensitivity analysis, obtained by running a linear optimization model, reveals information such as shadow prices and reduced costs. These values are critical information for industry because they represent how much production cost can be saved by adjusting operational constraints. Kirchain and Cosquer studied how to design better aluminum recycling using a sensitivity analysis obtained from a linear blending model (Kirchain and Cosquer 2007). In this paper, the authors described how aluminum recyclers can use sensitivity analysis information to improve recycling practice.

However, linear blending models are often not able to capture all the complexities associated with recycling low-valued materials. One of these complexities is compositional uncertainty in raw materials, which has been a major barrier to prevent further improvement in aluminum recycling. The chance-constrained (CC) optimization method was developed to explicitly incorporate uncertainty into a mathematical model. The CC method, introduced by Charnes and Cooper (Charnes and Cooper 1959), allows users to explicitly indicate the confidence level for each batch meeting the specifications of final products (Shih and Frey 1995; Gaustad, Li et al. 2007; Olivetti, Gaustad et al. 2011). This method provides an optimal batch plan based on the statistical parameters of the input materials. Due to its capability to control the batch error rate, it has been recently applied to the recycling area to model the blending operation for scrap with uncertain quality (Gaustad, Li et al. 2007; Olivetti, Gaustad et al. 2011).

2.3.2 Two-stage blending models (the pooling problem)

The batch planning tools described in the previous section model a one-stage blending process. However, in two-stage recycling operations consisting of both a reprocessing stage and re-melting stage, require a different tool able to model a two-stage blending process. The type of optimization problem involving more than one blending process is called the pooling problem. The pooling problem was introduced by Haverly in the 1970s (Haverly 1978) and has been investigated by many researchers in recent decades (Foulds, Haugland et al. 1992; Adhya, Tawarmalani et al. 1999; Audet, Brimberg et al. 2004; Misener and Floudas 2009). In the petroleum industry, it is common to have intermediate blends of crude oils due to the limited number of reservoir tanks for storing different types of crude oils. Consequently, the pooling problem has been intensively studied in the petroleum industry.

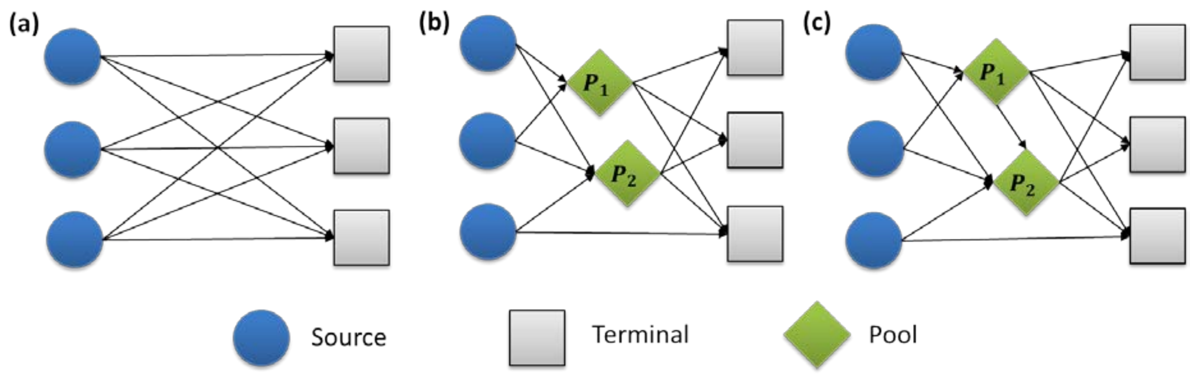


Figure 2-1. Schematic of (a) a linear blending problem (b) the standard pooling problem, and (c) the generalized pooling problem.

Figure 2-1 shows the difference between a general linear blending and the pooling problem. In a linear blending problem, all materials flows are expressed as direct arcs from source nodes to terminal nodes. In the pooling problem, some source nodes are not directly connected to terminal nodes but linked to pools. The structure of the pooling problem is clearly analogous to two-stage aluminum recycling operations which consist of raw materials as source nodes, intermediate products as pools, and final alloy products as terminal nodes.

The pooling problem can be further categorized into two: the standard pooling problem and the generalized pooling problem. In the standard pooling problem, it is assumed that the allowed flows are from sources to pools, from sources to terminals, and from pools to terminals as shown in Figure 2-1(b). In the generalized the pooling problem of Figure 2-1(c), introduced by Audet et al., the additional connections between pools are allowed in addition to the ones in the standard pooling problem (Audet, Brimberg et al. 2004). In the literature, the generalized pooling problem is often distinguished from the standard pooling problem. Because of its structural difference, the generalized pooling problem is more complex than the standard the pooling problem.

Solving either the standard or the generalized pooling problem is computationally more intensive than solving linear blending problem. In addition to the constraints in a traditional one-stage blending problem, the pooling problem has the constraints of mass conservation and quality conservation in the pools. These constraints introduce nonlinearity and nonconvexity into optimization models due to bilinear terms (multiplication of two decision variables), consequently causing computational difficulties. Many studies on the pooling problem have tried to resolve this issue using various methods. These methods have included developing new algorithms to find the global optimal solution of the pooling problem, and reformulating mathematical expressions to reduce the number of nonlinear constraints or to make the problem more suitable for particular algorithms (Ben-Tal, Eiger et al. 1994; Audet, Brimberg et al. 2004).

In general, the formulations to model the pooling problem can be divided into two categories. One is to express the mass balance with total flows and the fractions of quality components in each flow (quality formulation). The other is to express the mass balance with the individual component flows. Also this formulation expresses either flow from source to pools or from pools to terminals as the proportion of flows to the size of the pools (proportional formulation). The former one is the most straightforward formulation proposed by Haverly, often referred to as the P-formulation. Two formulations are mathematically equivalent. But it has been found that the first formulation has more bilinear terms when the number of entering arcs into pools is higher than the number of leaving arcs from pools. A more detailed description of the difference between the two formulations will be described in Chapter 3.

The practical application of the pooling problem has been demonstrated by several research groups (Baker and Lasdon 1985; Dewitt, Lasdon et al. 1989). These studies demonstrated the benefit of using the pooling problem to formulate the multi-stage blending practice in the refinery industry even though the solutions were local optimal. Meyer et al. and Misener et al. extended the pooling problem to determine the optimal design of wastewater treatment network (Meyer and Floudas 2006; Misener and Floudas 2010). In this application, there is a fixed initial cost for opening a new arc and node. Binary variables are used to describe decisions associated with a fixed initial cost. Therefore, the resulting model becomes a mixed integer nonlinear program (MINLP).

Decision makers in many industries often need to account for the uncertainty existing in various operational parameters. In recent decades, algorithms to solve the pooling problem have been developed, and its computational time has been greatly reduced. Commercial global optimal solvers such as BARON now enable solving larger problems with more quality attributes, products, and sources. However, considering the uncertainty into the pooling problem is still computationally challenging. As a result, there has been little research on stochastic pooling problem. A recent study by Li et al. attempts to solve this issue for the application of designing networks of natural gas (Li, Armagan et al. 2011). Li and co-authors proposed two-stage recourse approach for the stochastic pooling problem. In their model, the first-stage decisions associated with sources, pools, product terminals or pipeline investment decision are expressed as binary variables. The second-stage decision variables are associated with the actual plan of operating the natural gas system with the actual realization of uncertainty of source quality and product demand. As the authors pointed out, the stochastic pooling problem is a potentially large-scale nonconvex MINLP problem depending on the number of scenarios considered in the model. The authors of this study proposed a new decomposition strategy (Li, Armagan et al. 2011; Li, Tomasgard et al. 2012). The authors demonstrated that the solving time of the proposed decomposition method increases moderately with the number of scenarios, where the solving time of BARON exponentially increases.

All these studies of the pooling problem focus on finding the fastest algorithms to obtain global optimal solutions rather than understanding the characteristics of blending behaviors in the pools and the interaction between such characteristics and other operational parameters.

It is also important to note the difference of using the pooling problem for aluminum recycling operations and for other popular research fields such as natural gas, or wastewater management networks. The studies in the latter cases model continuous blending operations whereas aluminum recycling operations can be predominantly batch operations. In the continuous blending operations, there are continuous streams of feed materials and output products without any starting or ending point. A production facility is generally designed to produce only a limited number of products. The models for the continuous blending operations assume the steady state. Parameters and variables of the continuous blending operations are expressed as flow rates rather than absolute amounts. Once flow rates are optimized in a given system, those values are constant throughout multiple periods. For batch production, on the other hand, equipment is typically shared and used to produce multiple products (Barrera, Evans et al. 1989). Consequently, there is a clearly defined start and end by which job is assigned to the equipment. In batch blending models, decision variables are expressed as the absolute amount of raw materials used between specific time intervals. Each batch requires a different recipe. For example, when two consecutive batches produce the same product, blending ratios of raw materials in two batches are not necessarily equivalent depending on the inventory of raw materials in contrast to continuous blending operations. The varying availability of raw materials can lead to different optimal blending recipes. Therefore, batch operations have flexibility to adjust the optimal blending based on operational conditions (Barrera, Evans et al. 1989; Goršek and Glavič 1997). Also, the numbers of final products and raw materials in the facility of the batch operation are often higher than the number of those in the facility of the continuous operation (Barrera, Evans et al. 1989).

The differences between continuous and batch blending operation create different issues in addressing uncertainty in the pooling problem. For example, the study by Li et al. suggests the decomposition strategy for the stochastic pooling problem. In their study, the first stage decision variables are associated with designing the network, the second stage decisions represents the operational decisions which are typical variables in decision variables. Each scenario represents one pooling problem. Since this model is designed for the continuous blending operations (i.e. natural gas), there is no consideration of time constraints in this model. In other words, all operational decision variables, the quality of pools and the flow between nodes, are determined by assuming a stead state. However, aluminum recycling operations are predominantly batch operations. Since they therefore cannot be modeled as a stead state, there is a gap between operations of the first blending process and the second blending process. When the design of reprocessing facility is given, the first-stage decisions are decisions for the reprocessing stage and the second-stage decision are decisions for the re-melting stage. The first decisions are continuous variables, unlike existing models for the stochastic pooling problem where the first-stage decision variables are discrete (usually binary). Therefore, current existing models cannot fully capture the characteristics of two-stage aluminum recycling operations.

Recent work by Brommer first applied the pooling problem to aluminum recycling operations (Brommer 2013). This study was the first attempt to use the pooling problem to model metallurgical blending operations. Due to the limited capability of applying the pooling problem to large-scale problems, this study aggregated batch information for a one-month period. In other words, the approach in the study by Brommer uses the pooling problem for the overall long-term planning to determine only the specifications of intermediate products. This approach does not capture operational constraints at the batch level. The optimal specifications of intermediate products determined from solving the pooling problem with the sum of all demands for finished alloys during a one-month period are used as the predetermined compositions of intermediate products in two-stage batch planning. However, at the operational level, the demand for all products does not occur simultaneously. The predetermined compositions of intermediate products obtained from the long-term planning are not necessarily optimal in any conditions of batch operations. Also, the actual batch planning tool used in the study by Brommer is mathematically no longer a pooling problem. Once the compositions of intermediate products are fixed, the bilinear terms disappear. Thus, the resulting formulation of two-stage blending models with the predetermined composition of the pool becomes linear. In addition, this study does not include any discussion of blending behaviors of intermediate pools and does not investigate the benefits and limitations of using the pooling problem as a method to model two-stage recycling operations.

To summarize this section, there are two academic gaps identified. First, the existing models of the pooling problems are not able to describe two-stage aluminum recycling batch operations. Second, fundamental understanding of the pooling problem as a batch planning tool, including benefits and limitations as well as key drivers of those benefits and limitations, particularly in metallurgical recycling operations, has not been discussed. This thesis explores these unanswered topics in literature.

2.4 Thesis contribution

This thesis will address the academic gaps identified in previous sections and attempt to fill these gaps. For the first case, which will be introduced in Chapter 4, where there is no direct liquid metal delivery, the following questions will be addressed.

- Is clustering analysis an effective method for binning raw materials with uncertain composition?

- What is the impact of grouping raw materials by their compositions on the usage of raw materials?

For the second case, which will be covered in Chapter 5 and 6, where intermediate products are delivered as liquid metal, the following questions will be answered

- What is the value of integrating production planning for the reprocessing and the re-melting stage, compared to independent planning?
- What are the key drivers that determine the benefit of integrated planning?
- How does considering downstream uncertainty explicitly impact the performance of recycling?

3 Methodology

This chapter discusses the methods used in this thesis. Section 3.1 describes the clustering analysis to identify the patterns of compositional characteristics of sows and batch planning to evaluate the impact of grouped sows by the clustering analysis on their usage in final alloy productions, which will be discussed in Chapter 4. Section 3.2 introduces the mathematical models of integrated production planning and two different models of independent production planning, which is introduced to quantify the value of integrated production planning in Chapter 5 and Chapter 6. The differences between three models are discussed. Furthermore, a simulated screening analysis will be described to understand the optimal blending behaviors and explore conditions that maximize the benefit of integrated production planning and in more complex systems.

3.1 Two-stage recycling operation when intermediate products are delivered as sows

3.1.1 Cluster analysis on compositions of sows

The cluster analysis method forms groups or clusters of similar records based on several measurements made on these records. Among clustering methods, hierarchical algorithms are characterized as sequential clustering procedures, meaning each subsequent cluster cascades from the previous grouping. They can be categorized into two types of methods: agglomerative methods and divisive methods. Agglomerative methods start with a single point in each cluster and choose the pair of clusters to merge at each step, based on the optimal value of an objective function, until only one cluster is left. Divisive clustering methods are the reverse of the agglomerative methods. Divisive clustering methods begin with all data in one cluster and split a cluster at each stage until each cluster has only a single entity (Milligan and Cooper 1987; Xu and Wunsch 2005). Compared to partitioning algorithms that require the number of clusters a priori (Milligan and Cooper 1987), hierarchical algorithms do not require any knowledge of the number of clusters. As a result, this category of algorithms produces a map of hierarchy that represents the procedure by which clusters are merged or separated at every step, often described as a dendrogram or binary tree. The researcher can either use the entire hierarchy or select a level representing the specific number of clusters as needed (Xu and Wunsch 2005).

The proposed binning strategy is analogous to the process of divisive hierarchical clustering methods. However, divisive hierarchical algorithms are not commonly used due to their computational complexity (Milligan and Cooper 1987; Xu and Wunsch 2005). We choose Ward's minimum variance method in this study. Starting with many different clusters having only one object, this method finds the pair of clusters that leads to minimum increases in the total within-cluster variance at each step (Ward 1963). Since the goal of clustering analysis in this study is to reduce the uncertainty of raw materials, this method meets this goal. The distance between the two clusters A and B in Ward's method is calculated as shown in equation (1)

$$D_{AB} = \frac{\left\| \overline{x_A} - \overline{x_B} \right\|^2}{\left(\frac{1}{n_A} + \frac{1}{n_B} \right)} \tag{1}$$

where $\overline{x_j}$ is the center of cluster j and n_j is the number of points in it.

The historical composition data of outputs from the rotary furnace in a recycling facility that produces multiple alloy products for a six-month period are used as clustering objects in this study. The commercial statistical software JMP is used to perform clustering analysis. Six elements of composition are chosen to calculate the distances because these elements are key components of alloy products in this facility. The six key elements are Si, Fe, Cu, Mn, Mg, and Zn. Also, other compositional elements vary relatively less. Although including other elements to calculate the distance between objects is possible, it decreases the contribution of these six elements to the overall distance. Therefore, using major alloy elements to calculate the distance leads to clearer distinctions between clusters for these elements.

3.1.2 Chance-constrained batch planning

In order to answer the first part of research questions in this thesis, it is essential to evaluate the impact of binned sows on their usage in a batch plan for finished alloy production. The goal of batch planning is to combine a variety of feeds such that the composition of their blend falls below maximum and above minimum targets. Therefore, this allowable range of final blends is often interpreted as a compositional window. In Chapter 4, the chance-constrained (CC) method is used to model batches for finished alloy production. As explained in Chapter 2, the CC method is capable of explicitly modeling uncertainty of compositions of raw materials. The mathematical model of the blending problem with chance constraints can be written as follows. Table 3-1 describes the nomenclature used throughout this thesis.

Table 3-1. Nomenclature used in chance-constrained batch planning model.

Type	Symbol	Description
Indices	$i, l \in I$	The set for scraps and group of sows
	$j \in J$	The set of primary and alloying materials
	$t \in T$	The set of batches for finished alloy products
	$k \in K$	The set of compositional elements
Parameters	A_i	Max available amount of dross and scrap material i
	A_j	Max available amount of primary and alloying material j
	u_i	Material gross yield of scrap and group of sows i
	u_j	Material gross yield of primary or alloying material j
	c_i	Unit cost of scrap and group of sows i
	c_j	Unit cost of primary and alloying material j
	$\overline{e_{\iota,k}}$	The mean of an compositional element k of scrap or group of sows i
	$\sigma_{(e)ik}$	Standard deviation of the composition of an element k of scrap or group of sows i
	$\rho_{(e)ilk}$	Correlation of composition of element k between scrap or group of sows i and l
	$e_{j,k}$	Weight fraction of an compositional element k in primary or alloying material j
	D_t	Demand of final product batch t
	$\varepsilon_{t,k}^{max}$	Upper limit of weight fraction of an element k in final product of batch t
	$\varepsilon_{t,k}^{min}$	Lower limit of weight fraction of an element k in final product of batch t
	α	Confidence level to satisfy the maximum specification of finished alloy products
	β	Confidence level to satisfy the minimum specification of finished alloy products
	$X(\cdot)$	The inverse normalized cumulative Gaussian distribution function
Decision Variables	$x_{i,t}$	The amount of scrap or group of sows i used in final product batch t
	$y_{j,t}$	The amount of primary or alloying material j used in final product batch t

Objective Function

$$\min \sum_i \sum_t c_i x_{i,t} + \sum_l \sum_t c_j y_{j,t} \tag{2}$$

Subject to

$$\sum_t x_{i,t} \leq A_i \qquad \forall i \tag{3}$$

$$\sum_t y_{j,t} \leq A_j \qquad \forall j \tag{4}$$

$$\sum_i u_i x_{i,t} + \sum_j u_j y_{j,t?} \geq D_t \qquad \forall t \tag{5}$$

$$\Pr\left\{ \sum_i e_{i,k} x_{i,t} + \sum_j e_{j,k} y_{j,t} \leq \varepsilon_{t,k}^{max} D_t \right\} \geq \alpha \qquad \forall t, k \tag{6}$$

$$\Pr\left\{ \sum_i e_{i,k} x_{i,t} + \sum_j e_{j,k} y_{j,t} \geq \varepsilon_{t,k}^{min} D_t \right\} \geq \beta \qquad \forall\, t, k \tag{7}$$

$$x_{i,t} \geq 0 \qquad \forall i, t \tag{8}$$

$$y_{j,t} \geq 0 \qquad \forall j, t \tag{9}$$

The objective function (2) is to minimize the sum of all raw material costs used in alloy production. Constraint (3) ensures that each raw material with quality uncertainty, such as each scrap or sow group, is used in alloy products less than its availability, A_i. Similarly, constraint (4) limits the total amount of each primary material or alloying element used in alloy production to not more than its availability, A_j. Constraint (5) ensures that production volume of each alloy product satisfies demand. Constraints (6) and (7) enforce the maximum and minimum quality requirement for each final alloy product. Instead of two linear inequality constraints for quality requirement, the CC method requires those two inequality constraints to be satisfied with a given probability level, α and β, where $0 \leq \alpha, \beta \leq 1$. Therefore, parameters α and β represent the likelihood that the actual composition of blends will fall within the upper and lower limits of an alloy specification, respectively. Constraints (8) and (9) represent the non-negativity of decision variables. Assuming that the compositions of raw materials with compositional uncertainty, which are indexed by i, follow a normal distribution, the two probabilistic constraints (6) and (7) can be transformed into their deterministic equivalents:

$$\sum_i \overline{e_{i,k}}\, x_{i,t} + \sum_j e_{j,k} y_{j,t} + X(\alpha)\left(\sum_i \sum_l \rho_{(\varepsilon)ilk}\sigma_{(\varepsilon)ik}\sigma_{(\varepsilon)lk}x_{i,t}x_{l,t}\right)^{\frac{1}{2}} \le \varepsilon_{t,k}^{max} D_t \quad \forall t,k \tag{10}$$

$$\sum_i \overline{e_{i,k}}\, x_{i,t} + \sum_j e_{j,k} y_{j,t} + X(1-\beta)\left(\sum_i \sum_l \rho_{(\varepsilon)ilk}\sigma_{(\varepsilon)ik}\sigma_{(\varepsilon)lk}x_{i,t}x_{l,t}\right)^{\frac{1}{2}} \ge \varepsilon_{t,k}^{min} D_t \quad \forall t,k \tag{11}$$

In this study, it is assumed that the compositions of raw materials follow a normal distribution. The compositional distribution of each bin of sows obtained from the cluster analysis varies with the element and the total number of bins, so this assumption may be limiting in some cases. The same six elements of composition used in the clustering analysis are tracked, and 99% is used as a confidence level for the compositional constraint for each element.

3.2 Two-stage recycling operation when intermediate products are delivered as liquid metals

Integrated production planning is proposed to model two-stage aluminum recycling operations. In order to quantify the value of the integrated production planning for two-stage recycling operations, the independent production planning approach is used as a benchmark. Figure 3-1 describes the scope of decision making in two different production planning approaches. The red dashed line represents independent production planning where each operation units are independently planned and operated. However, integrated production planning determines batch decisions by considering two stages simultaneously as presented in blue lines of Figure 3-1. This chapter discusses the mathematical formulation of two different planning approaches.

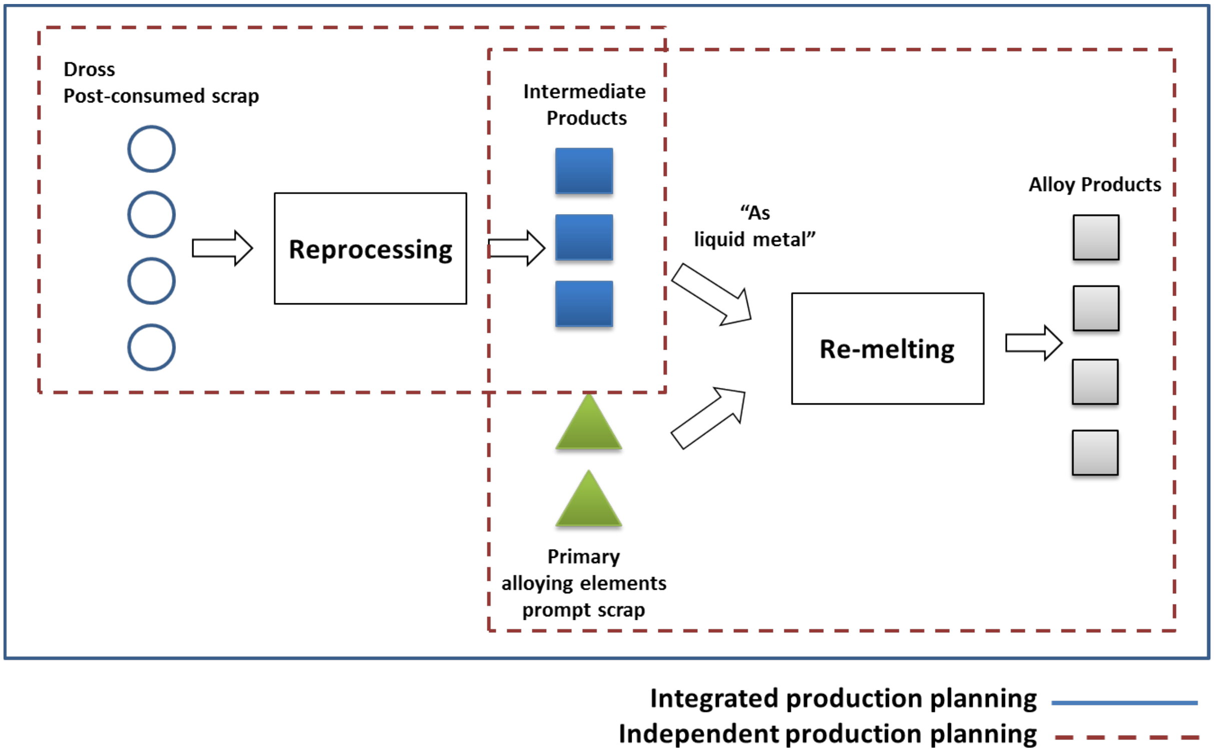


Figure 3-1. Schematic diagram of the scope of decision making in two different production planning approaches.

3.2.1 Mathematical formulation for integrated production planning

The formulation in this section describes the model for integrated production planning. Table 3-2 includes the descriptions for parameters and decision variables used in the integrated production planning model.

Table 3-2. Nomenclature used in integrated production planning.

Type	Symbol	Description
Indices	$i \in I$	The set of dross scrap materials (raw materials for the first stage of blending)
	$j \in J$	The set of primary and alloying materials (raw materials for the second stage of blending)
	$l \in L$	The set of batches for intermediate products
	$t \in T$	The set of batches for finished alloys
	$k \in K$	The set of compositional elements
Parameters	A_i	Max available amount of dross and scrap material i
	A_j	Max available amount of primary and alloying material j

	u_i	Material gross yield of dross and scrap material i
	u_j	Material gross yield of primary or alloying material j
	c_i	Unit cost of dross and dross and scrap material i
	c_j	Unit cost of primary and alloying material j
	$e_{i,k}$	Weight fraction of an element k in dross and scrap material i
	$e_{j,k}$	Weight fraction of an element k in primary or alloying material j
	D_t	Demand of final product batch t
	$e_{t,k}^U$	Upper limit of weight fraction of an element k in final product of batch t
	$e_{t,k}^L$	Lower limit of weight fraction of an element k in final product of batch t
	V_{max}	Upper limit of reprocessing furnace capacity
	V_{min}	Lower limit of reprocessing furnace capacity
Decision Variables (quality formulation)	$f_{i,l}$	Weight of dross and scrap material i used in batch for intermediate product l
	$f_{j,t}$	Weight of primary or alloying material j used in batch for product t
	$f_{l,t}$	Weight of intermediate product l used in batch for product t
	$\varepsilon_{l,k}$	Weight fraction of an element k in intermediate product batch l
	R_l	Weight of intermediate product produced in batch l but not used in final alloy production
Decision Variables (proportional formulation)	$f_{i,l}$	Weight of dross and scrap material i used in batch for intermediate product l
	$f_{j,t}$	Weight of primary or alloying material j used in batch for product t
	$E_{l,k}$	Weight of an element k in intermediate product batch l
	$q_{l,t}$	The proportion of flow of an intermediate product batch l used in final product t
	r_l	The proportion of an intermediate product produced in batch l but not used in final alloy production

Objective Function

$$\min \sum_i c_i f_{i,l} + \sum_j c_j f_{j,t} \tag{12}$$

Subject to

$$\sum_l f_{i,l} \le A_i \qquad \forall i \tag{13}$$

$$\sum_{j} f_{j,t} \le A_j \qquad \forall j \tag{14}$$

$$\sum_{i} f_{i,l} \le V_{max} \qquad \forall l \tag{15}$$

$$\sum_{i} f_{i,l} \ge V_{min} \qquad \forall l \tag{16}$$

$$\sum_{i} u_i f_{i,l} = \sum_{t} f_{l,t} + R_l \qquad \forall l \tag{17}$$

$$\sum_{i} e_{i,k}\, u_i f_{i,l} = \varepsilon_{l,k} \, (\sum_{i} u_i f_{i,l}) \qquad \forall l.k \tag{18}$$

$$\sum_{l} f_{l,t} + \sum_{j} u_j f_{j,t} = D_t \qquad \forall t \tag{19}$$

$$\sum_{l} \varepsilon_{l,k} \; f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \le e^{U}_{t,k} \; D_t \qquad \forall \; t,k \tag{20}$$

$$\sum_{l} \varepsilon_{l,k} \; f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \ge e^{L}_{t,k} \; D_t \qquad \forall \; t,k \tag{21}$$

$$f_{i,l} \ge 0 \qquad \forall i,l \tag{22}$$

$$f_{j,t} \ge 0 \qquad \forall j,t \tag{23}$$

$$f_{l,t} \ge 0 \qquad \forall l,t \tag{24}$$

Objective function (12) minimizes the sum of cost of all raw materials used in a two-stage recycling operation. Constraints (13) and (14) ensure that the total amount of each raw material used in production is within the upper limits of its availability. Constraint (15) and constraint (16) represent the minimum and maximum limit of each batch of intermediate products. Constraint (17) represents the weight balance at each batch of intermediate products in reprocessing stage. In other words, the sum of flows from dross and scrap materials to intermediate products must be equal to the sum of intermediate products used in finished alloy and unused intermediate products. Constraint (18) describes the mass balance for each compositional element, meaning that the sum of each compositional element of dross and other scrap materials used in an intermediate product is equal to total weight of that element in an intermediate product. Constraint (19) expresses the demand requirement for each batch of final products. Constraints (20) and (21) ensure that the final blends satisfy the maximum and minimum quality specification of final products. Constraints (22)-(24) represents the non-negativity of decision variables. Three constraints (18), (20), and (21) are nonlinear in this formulation. The bilinear terms in these three constraints exist because of the multiplication of the quality variable of intermediate products, $\varepsilon_{l,k}$, and the flow variables, $f_{i,l}$ and $f_{l,t}$.

This formulation is very straightforward. However, the size of the problem quickly increases with the number of quality attributes and the number of products. In this study, the quality formulation is only used in the small case study developed in Chapter 5 with one quality attribute, four sources, one pool and two terminals. In the case study approaching an industrial scale of Chapter 6, the following mathematical formulation, the proportional formulation is used.

Objective function

$$\min \sum_i c_i f_{i,l} + \sum_j c_j f_{j,t} \tag{25}$$

Subject to

$$\sum_l f_{i,l} \leq A_i \qquad \forall i \tag{26}$$

$$\sum_j f_{j,t} \leq A_j \qquad \forall j \tag{27}$$

$$\sum_i f_{i,l} \leq V_{max} \qquad \forall l \tag{28}$$

$$\sum_{i} f_{i,l} \geq V_{min} \qquad \forall l \tag{29}$$

$$\sum_{t} q_{l,t} + r_l = 1 \qquad \forall l \tag{30}$$

$$\sum_{i} e_{i,k}\, u_i f_{i,l} = E_{l,k} \qquad \forall l,k \tag{31}$$

$$\sum_{l} q_{l,t}\left(\sum_{i} u_i f_{i,l}\right) + \sum_{j} u_j f_{j,t} = D_t \qquad \forall t \tag{32}$$

$$\sum_{l} E_{l,k}\, q_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \leq e_{t,k}^{U}\, D_t \qquad \forall\, t,k \tag{33}$$

$$\sum_{l} E_{l,k}\, q_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \geq e_{t,k}^{L}\, D_t \qquad \forall\, t,k \tag{34}$$

$$f_{i,l} \geq 0 \qquad \forall i,l \tag{35}$$

$$f_{j,t} \geq 0 \qquad \forall j,t \tag{36}$$

$$q_{l,t} \geq 0 \qquad \forall l,t \tag{37}$$

$$q_{l,t} \leq 1 \qquad \forall l,t \tag{38}$$

$$r_l \geq 0 \qquad \forall l \tag{39}$$

$$r_l \leq 1 \qquad \forall l \tag{40}$$

In the proportional formulation, we define the new variable $q_{l,t} = \frac{f_{l,t}}{\sum_i f_{i,l}}$ as the proportion of flow of an intermediate product, $l \in L$, destined to a finished product $t \in T$. Similarly, the proportional of intermediate products that is not used in batches for final products can be defined as $r_l = \frac{R_l}{\sum_i f_{i,l}}$. In the proportional formulation, the equation (17) can be rewritten as the equation (30) which describes the sum of proportional flows from an unused intermediate product must be 1. The constraint (31) expresses the mass balance of compositional elements in each intermediate product using the mass of composition instead of fractional quality expression $\varepsilon_{l,k}$ in the constraint (18). Consequently, the constraints for the specifications of final products in (20) and (21) can be also replaced with (33) and (34)

The compositions of intermediate products can be calculated using given variables. The compositional variable in the quality formulation, $\varepsilon_{l,k} = \frac{\sum_i e_{i,k} f_{i,l}}{\sum_i f_{i,l}} = \frac{E_{l,k}}{\sum_i f_{i,l}}$ is no longer necessary in the proportional formulation. Replacing this variable reduces the number of nonlinear constraints by removing nonlinear constraint (18).

3.2.2 Mathematical formulation for independent production planning

To evaluate the value of the integrated production planning in two-stage recycling operations, independent production planning is used as a benchmark in this thesis. Independent production planning is designed to mimic actual practice where reprocessing and the re-melting operations are planned separately without any coordination. Therefore, in independent production planning, the batch plans for the reprocessing furnace is determined without any consideration of demand for final products in the re-melting stage.

Two models of independent production planning are compared to integrated product planning model in this thesis: fixed recipe model and fixed composition model. In the fixed recipe model, the composition of the intermediate products and the batch plans for these intermediate products are both predetermined. In the fixed composition model, the only predetermined parameter in addition to parameters in independent production planning model is the composition of intermediate products. The actual batch plan to achieve that pre-determined composition of intermediate products is decided by the model. Therefore, fixed composition model have more flexibility than the fixed recipe model. Table 3-3 summarizes the differences of three different models used in this thesis.

The pre-determined composition of the intermediate products can be anything. In practice in which there is not any coordination between the reprocessing stage and the re-melting stage, the compositions of the intermediate products are often random. In this thesis, it is assumed that the average compositions of dross and scrap are the predetermined composition of the intermediate products in two models of independent planning. In other words, intermediate products are produced by blending all available dross and scrap in equal proportion.

Table 3-3. Comparison of integrated production planning model and two independent production planning models (fixed recipe model and fixed composition model).

	Integrated production planning	Independent production planning (fixed recipe model)	Independent production planning (fixed composition model)
Model Type	Nonconvex nonlinear programming	Linear programming	Mixed-integer linear programming
Pre-determined decisions		-Compositions of intermediate products -Batch plans for the reprocessing stage	- Compositions of intermediate products
Decisions	- Batch plans for the reprocessing stage - Batch plans for the re-melting stage	- Batch plans for the re-melting stage	- Batch plans for the reprocessing stage - Decisions of production (on/off) for each batch of reprocessing stage - Batch plans for the re-melting stage

3.2.2.1 Independent production planning – fixed recipe model

In the fixed recipe model of independent production planning, all decision variables for reprocessing furnaces are predetermined unlike the integrated production planning model. Thus, the resulting model is simple linear one-stage blending problem. Table 3-4 describes the nomenclature used in the fixed recipe model of independent production model.

Table 3-4 Nomenclature for independent production planning fixed recipe model

Type	Symbol	Description
Indices	$i \in I$	The set of dross and scrap materials (raw materials for the first stage of blending)

	$j \in J$	The set of primary and alloying materials (raw materials for the second stage of blending)
	$l \in L$	The set of batches for intermediate products
	$t \in T$	The set of batches for finished alloys
	$k \in K$	The set of compositional elements
Parameters	A_i	Max available amount of dross and scrap material i
	A_j	Max available amount of primary and alloying material j
	u_i	Material gross yield of dross and scrap material i
	u_j	Material gross yield of primary or alloying material j
	c_i	Unit cost of dross and dross and scrap material i
	c_j	Unit cost of primary and alloying material j
	c_l	Unit production cost of intermediate product l
	$e_{i,k}$	Weight fraction of an element k in dross and scrap material i
	$e_{j,k}$	Weight fraction of an element k in primary or alloying material j
	D_t	Demand of final product batch t
	$e_{t,k}^U$	Upper limit of weight fraction of an element k in final product of batch t
	$e_{t,k}^L$	Lower limit of weight fraction of an element k in in finished alloy of batch t
	V_{max}	Upper limit of reprocessing furnace capacity
	V_{min}	Lower limit of reprocessing furnace capacity
	$e_{l,k}$	Weight fraction of an element k in intermediate product l
	$g_{i,l}$	Weight of dross and scrap material i used in intermediate product l
	P_l	Penalty cost of the intermediate product produced in batch l but not used in final alloy production
Decision Variables	$f_{j,t}$	Weight of primary or alloying material j used in batch for final product t
	$f_{l,t}$	Weight of intermediate product l used in batch for final product t
	R_l	Weight of intermediate product produced in batch l but not used in final alloy production

Objective function

$$\min \sum_j c_j f_{j,t} + \sum_l c_l f_{l,t} + \sum_l P_l R_l \tag{41}$$

Subject to

$$\sum_{j} f_{j,t} \leq A_j \qquad \forall j \tag{42}$$

$$\sum_{t} f_{l,t} + R_l = \sum_{i} u_i g_{i,l} \qquad \forall l \tag{43}$$

$$\sum_{l} f_{l,t} + \sum_{j} u_j f_{j,t} = D_t \qquad \forall t \tag{44}$$

$$\sum_{l} e_{l,k}\, f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \leq e_{t,k}^{U}\, D_t \qquad \forall\, t, k \tag{45}$$

$$\sum_{l} e_{l,k}\, f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \geq e_{t,k}^{L}\, D_t \qquad \forall\, t, k \tag{46}$$

$$f_{j,t} \geq 0 \qquad \forall j, t \tag{47}$$

$$f_{l,t} \geq 0 \qquad \forall l, t \tag{48}$$

Eq(41) is the objective function that minimize the cost of raw materials used in the re-melting stage. In addition, the penalty cost for intermediate products that is not incorporated in finished alloy production is explicitly included in the objective function. In the integrated production planning model or the fixed composition model of the independent production planning which is introduced in the next section, there is no direct penalty cost for unused intermediate products. In these two models, the objective functions include the production cost of reprocessing furnace. If intermediate products are produced but not used in the re-melting stage, it will overly cost raw materials used in the re-processing stage and raw materials used in the re-melting stage. Therefore, unused intermediate products are somewhat indirectly penalized in the independent production planning model and the fixed composition model of the independent production planning.

Eq(42) describes the availability limit of primary and alloying element. Eq(43) describes that the intermediate products produced are either used in final products or cast as sow. Eq(44) expresses the demand requirement for final alloy products. Eq(45) and (46) ensures the all final blends in the re-melting furnace must satisfy the upper and lower specification of final products.

There is no longer a quadratic term involving the multiplication of decision variables since the recipe (dross and scrap used in production the intermediate products) is predetermined. The compositions of the intermediate products can be easily calculated as

$$e_{l,k} = \frac{\sum_i e_{i,k} u_i g_{i,l}}{\sum_i u_i g_{i,l}} \tag{49}$$

3.2.2.2 Independent production planning – fixed composition model

As explained earlier, the fixed composition model of the independent production planning has more flexibility than the fixed recipe model. This model uses the same compositions of the intermediate products as the fixed recipe model. However, depending on price, the results of this formulation may use different combination of raw materials to satisfy these compositions of the intermediate products.

Table 3-5 summarizes descriptions of parameters and decision variables used in the fixed composition model of independent production planning. The specifications of intermediate products are no longer decision variables in contrast to the integrated production planning. Although the overall formulation looks very similar to the quality formulation of integrated production planning, it is not the pooling problem but a linear model for two-stage blending.

Also, the binary variable is introduced in the fixed composition model. Unlike how the amount of intermediate product produced is predetermined in the fixed recipe model, the fixed composition model has flexibility in terms of the intermediate product produced. However, in actual practice, the minimum capacity of the reprocessing furnace must be filled in each batch for energy efficiency reasons. If the compositions of the intermediate product are not well matched to the specification of final products, the reprocessing furnace operator may choose to skip a batch instead of casting most of the produced intermediate products as sows. The binary variables are introduced to model this decision. Therefore, the resulting model becomes mixed integer linear programming.

Table 3-5. Nomenclature for independent production planning fixed composition model.

Type	Symbol	Description
Indices	$i \in I$	The set of dross scrap materials (raw materials for the first stage of blending)
	$j \in J$	The set of primary and alloying materials (raw materials for the second stage of blending)
	$l \in L$	The set of batches for intermediate products
	$t \in T$	The set of batches for finished alloys
	$k \in K$	The set of compositional elements
Parameters	A_i	Max available amount of dross and scrap material i
	A_j	Max available amount of primary and alloying material j
	u_i	Material gross yield of dross and scrap material i
	u_j	Material gross yield of primary or alloying material j
	c_i	Unit cost of dross and dross and scrap material i
	c_j	Unit cost of primary and alloying material j
	$e_{i,k}$	Weight fraction of an element k in dross and scrap material i
	$e_{j,k}$	Weight fraction of an element k in primary or alloying material j
	D_t	Demand of final product batch t
	$e^U_{t,k}$	Upper limit of weight fraction of an element k in finished alloy in batch t
	$e^L_{t,k}$	Lower limit of weight fraction of an element k in in finished alloy in batch t
	$e_{l,k}$	Weight fraction of an element k in intermediate product l
	V_{max}	Upper limit of reprocessing furnace capacity
	V_{min}	Lower limit of reprocessing furnace capacity
Decision Variables	$f_{i,l}$	Weight of dross and scrap material i used in batch for intermediate product l
	$f_{j,t}$	Weight of primary or alloying material j used in batch for final product t
	$f_{l,t}$	Weight of intermediate product l used in batch for final product t
	R_l	Weight of intermediate product produced in batch l but not used in final alloy production
	y_l	Binary variables that determines production of a batch of the intermediate product l

Objective function

$$\min \sum_i c_i f_{i,l} + \sum_j c_j f_{j,t} \tag{50}$$

Subject to

$$\sum_{l} f_{i,l} \leq A_i \qquad \forall i \tag{51}$$

$$\sum_{j} f_{j,t} \leq A_j \qquad \forall j \tag{52}$$

$$\sum_{i} f_{i,l} \leq y_l V_{max} \qquad \forall l \tag{53}$$

$$\sum_{i} f_{i,l} \geq y_l V_{min} \qquad \forall l \tag{54}$$

$$\sum_{i} u_i f_{i,l} = \sum_{t} f_{l,t} + R_l \qquad \forall l \tag{55}$$

$$\sum_{i} e_{i,k}\, u_i f_{i,l} = e_{l,k}\, (\sum_{i} u_i f_{i,l}) \qquad \forall l, k \tag{56}$$

$$\sum_{l} f_{l,t} + \sum_{j} u_j f_{j,t} = D_t \qquad \forall t \tag{57}$$

$$\sum_{l} \varepsilon_{l,k}\, f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \leq e^{U}_{t,k}\, D_t \qquad \forall\, t, k \tag{58}$$

$$\sum_{l} \varepsilon_{l,k}\, f_{l,t} + \sum_{j} e_{j,k}\, f_{j,t} \geq e^{L}_{t,k}\, D_t \qquad \forall\, t, k \tag{59}$$

$$f_{i,l} \geq 0 \qquad \forall i, l \tag{60}$$

$$f_{j,t} \geq 0 \qquad \forall j, t \tag{61}$$

$$f_{l,t} \geq 0 \qquad \forall l, t \tag{62}$$

3.2.3 Mathematical formulation for stochastic two-stage blending operations

In this thesis, we use the recourse approach to formulate the demand uncertainty for finished alloy products. Considering the demand uncertainty introduces the additional set of scenarios, M. Quantifying the value of the integrated production plans with explicitly considering the uncertainty of demands for final products requires the comparison with the production plans that do not consider the demand uncertainty.

The two-period recourse approach is used in this study. There are two-types of decisions depending on the timing of decision made. The first-period decisions are made before the resolution of uncertainty (product demand uncertainty in this case). The second-period decision is made after the resolution of uncertainty. The first decision, therefore, should not depend on the future observation since at the time of the first decision made the possible outcome has not been realized.

The recourse formulation explicitly considers all potential scenarios of demand for final products and determines the optimal first-decision based on the expected value of outcomes. In a deterministic approach, instead of considering all scenarios, it considers one scenario that is obtained by taking the average of parameters in all scenarios.

In the aluminum recycling context, the first stage decision is determining the composition of the intermediate product. The second-stage decision is the actual batch planning for both stages, the reprocessing stage and the re-melting stage. Table 3-6 describes the nomenclature used in stochastic two-stage blending operations.

Table 3-6 Nomenclature for stochastic integrated production planning model.

Type	Symbol	Description
Indices	$i \in I$	The set of dross scrap materials (raw materials for the first stage of blending)
	$j \in J$	The set of primary and alloying materials (raw materials for the second stage of blending)
	$l \in L$	The set of batches for intermediate products
	$t \in T$	The set of batches for finished alloy
	$k \in K$	The set of compositional elements
	$m \in M$	The set of demand scenarios of finished alloy
Parameters	A_i	Max available amount of dross and scrap material i
	A_j	Max available amount of primary and alloying material j
	u_i	Material gross yield of dross and scrap material i
	u_j	Material gross yield of primary or alloying material j
	c_i	Unit cost of dross and dross and scrap material i
	c_j	Unit cost of primary and alloying material j
	$e_{i,k}$	Weight fraction of an element k in dross and scrap material i
	$e_{j,k}$	Weight fraction of an element k in primary or alloying material

		j
	$D_{t,m}$	Demand of final product batch t in scenario m
	$e_{t,k,m}^{U}$	Upper limit of weight fraction of an element k in final product batch t in scenario m
	$e_{t,k,m}^{L}$	Lower limit of weight fraction of an element k in in final product batch t in scenario m
	V_{max}	Upper limit of reprocessing furnace capacity
	V_{min}	Lower limit of reprocessing furnace capacity
	α_m	The probability of scenario m
First period decision variables	$\varepsilon_{l,k}$	Weight fraction of an element k in intermediate product l
Second period decision variables	$f_{i,l,m}$	Weight of dross and scrap material i used in batch for intermediate product l in scenario m
	$f_{j,t,m}$	Weight of primary or alloying material j used in final product batch t in scenario m
	$f_{l,t,m}$	Weight of intermediate product l used in final product batch t in scenario m
	$R_{l,m}$	Weight of intermediate product produced in batch l but not used in final alloy production in scenario m

Objective function

$$\text{Minimize} \quad \sum_{m} \alpha_m \left(\sum_{i} \sum_{l} c_i f_{i,l,m} + \sum_{j} \sum_{t} c_j f_{j,t,m} \right) \tag{63}$$

Subject to

$$\sum_{l} f_{i,l,m} \leq A_i \qquad \forall i, m \tag{64}$$

$$\sum_{t} f_{j,t,m} \leq A_j \qquad \forall j, m \tag{65}$$

$$\sum_{i} f_{i,l,m} \leq V_{max} \qquad \forall l, m \tag{66}$$

$$\sum_{i} f_{i,l,m} \geq V_{min} \qquad \forall l, m \tag{67}$$

$$\sum_{i} u_i f_{i,l,m} = \sum_{t} f_{l,t,m} + R_{l.m} \qquad \forall l, m \tag{68}$$

$$\sum_{i} e_{i,k}\, u_i f_{i,l,m} = \varepsilon_{l,k} \sum_{i} u_i f_{i,l,m} \qquad \forall l, k, m \tag{69}$$

$$\sum_{l} f_{l,t,m} + \sum_{j} u_j f_{j,t,m} = D_{t,m} \qquad \forall t, m \tag{70}$$

$$\sum_{l} \varepsilon_{l,k}\ f_{l,t,m} + \sum_{j} e_{j,k}\, f_{j,t,m} \leq e^{U}_{t,k,m}\ D_{t,m} \qquad \forall\ t, k, m \tag{71}$$

$$\sum_{l} \varepsilon_{l,k}\ f_{l,t,m} + \sum_{j} e_{j,k}\, f_{j,t,m} \leq e^{L}_{t,k,m}\ D_{t,m} \qquad \forall\ t, k, m \tag{72}$$

$$f_{i,l,m} \geq 0 \qquad \forall i, l, m \tag{73}$$

$$f_{j,t,m} \geq 0 \qquad \forall j, t, m \tag{74}$$

$$R_{l,m} \geq 0 \qquad \forall l, m \tag{75}$$

The overall formulation is similar to the deterministic formulation in 3.2.1. The main difference is that the objective function is expressed as the expected value weighted by the probability of each scenario. Also, all the constraints must be satisfied in each scenario, $m \in M$. Unlike the many recourse formulations in literature, the objective function does not contain the first-stage decision variables. However, all the second stage variables in the objective function are affected by the first-stage decision variables, which is the composition of the intermediate products.

3.2.4 Simulated screening

In Chapter 5, the analytical solution of integrated production planning is derived for a simplified case setup with some assumptions made. This approach helps us to understand the interaction between parameters and the optimal solution and identify what drives the benefit of integrated production planning. However, a simplified case setup cannot fully capture operational constraints in real practice that may lead to different characteristics of optimal solutions as well as key drivers that maximize the benefit of integrated production planning. However, it is too complicated to derive the relationship between parameters and optimal solution with analytical approach for a complex system with these operational constraints. The simulated screening analysis is a numerical approach to overcome this limitation of analytical approach. The study investigates key drivers for optimal composition of the intermediate products as well as the value of integrated production planning versus independent production planning with consideration of operational constraints. The purpose of this study is to provide an insight that which circumstance that integrated production planning of two-stage recycling operations can be particularly beneficial to aluminum recyclers.

3.2.4.1 Optimal composition of the intermediate product

Figure 3-2 is the flow chart of the screening process to identify the most influential parameters on the optimal composition of the intermediate product determined by the integrated production planning model. Intervals for values of price of scrap, composition of scrap, availability of scrap material, price of primary aluminum and alloying element, and specification of products, are from data of an aluminum recycling production facility, located in Norway. It is assumed that each parameter has uniform distribution from its lower and upper bound. For each iteration, input parameters required for integrated production planning are sampled from its distribution. Then, sampled parameters each parameter is fed into the optimization model and solved to find the optimal solution. Commercial optimization modeling software, LINGO, is used in this study to solve this optimization model. The optimal composition of the intermediate product and sampled input parameters are recorded in every iteration. After repeating this iteration 1000 times, generated data that include sampled input parameters and the optimal solution based on those parameters are used to perform regression analysis. Least square regression analysis is used in this screening process. All sampled input parameters are normalized. Only first order parameters are included in the regression analysis. Since the optimal composition of the intermediate product obtained by solving the integrated production planning model is of interest, it is chosen as the dependent variable.

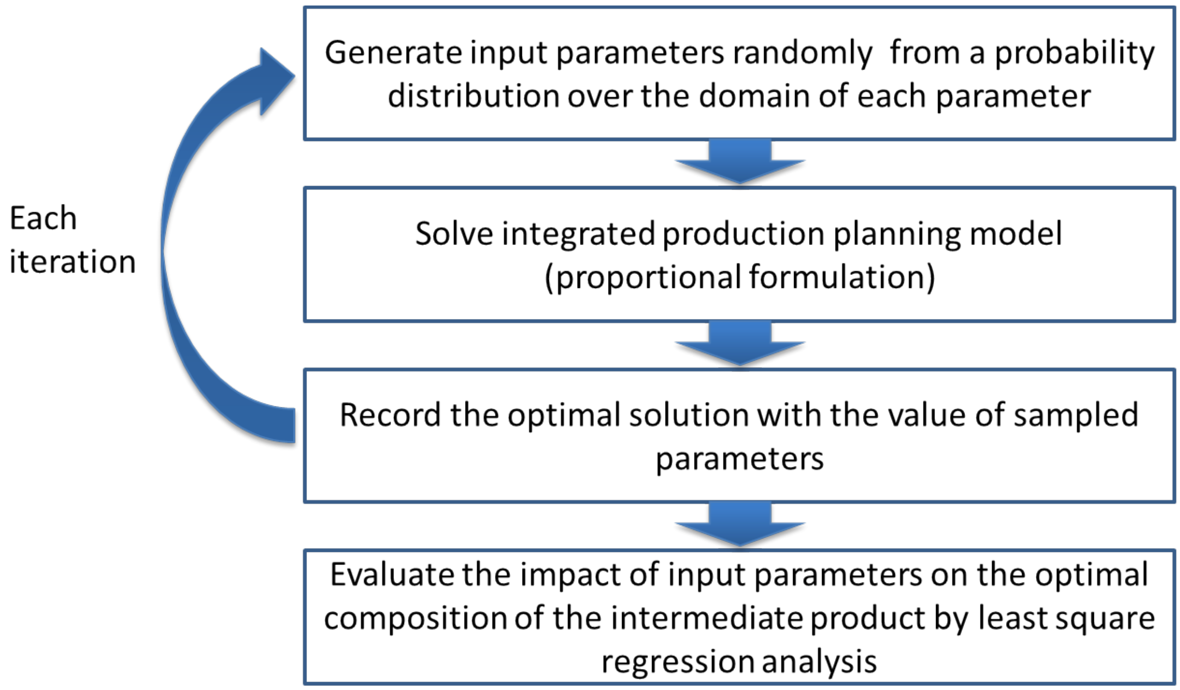


Figure 3-2. Flow chart of simulated screening process to identify the interaction between operational parameters and the optimal composition of the intermediate product.

3.2.4.2 The values of the integrated production planning

A similar simulated screening study is performed to identify the characteristics of operational conditions that allow integrated production planning to bring more benefits compared to the independent production planning. Figure 3-3 describes the screening process for this study. All processes are similar as the study for the optimal composition. All procedures to determine the probability distribution of input parameters and sampling process follow the same process described in the previous section. In addition, for this study, both the integrated production planning model and the independent production planning models are solved with each sampled parameters instead of solving only the integrated production planning model. However, there is an additional input for the independent production planning. Unlike the composition of the intermediate product is determined by solving the integrated production plan, the composition of the intermediate product is input parameters for the independent production planning model. In this study, it is assumed that the predetermined composition of the intermediate product in the independent production planning uses all scrap materials in the equal proportion. Therefore, in this particular study, the average composition of Scrap 1 and

Scrap 2 is used as the input parameter for the independent production planning model. In each iteration, the optimal solutions as well as the optimal objective values from both integrated production planning model and the independent production planning model are recorded. Our interest is to find when the benefit of the integrated production planning can be largest, compared to the independent production planning. Therefore, in this study, the dependent variable is the difference between the optimal objective value of independent production planning and that of the integrated production planning. The resulting data of this variable obtained from simulations have highly skewed distribution and values always higher than zero. The dependent variable is log transformed in the regression analysis.

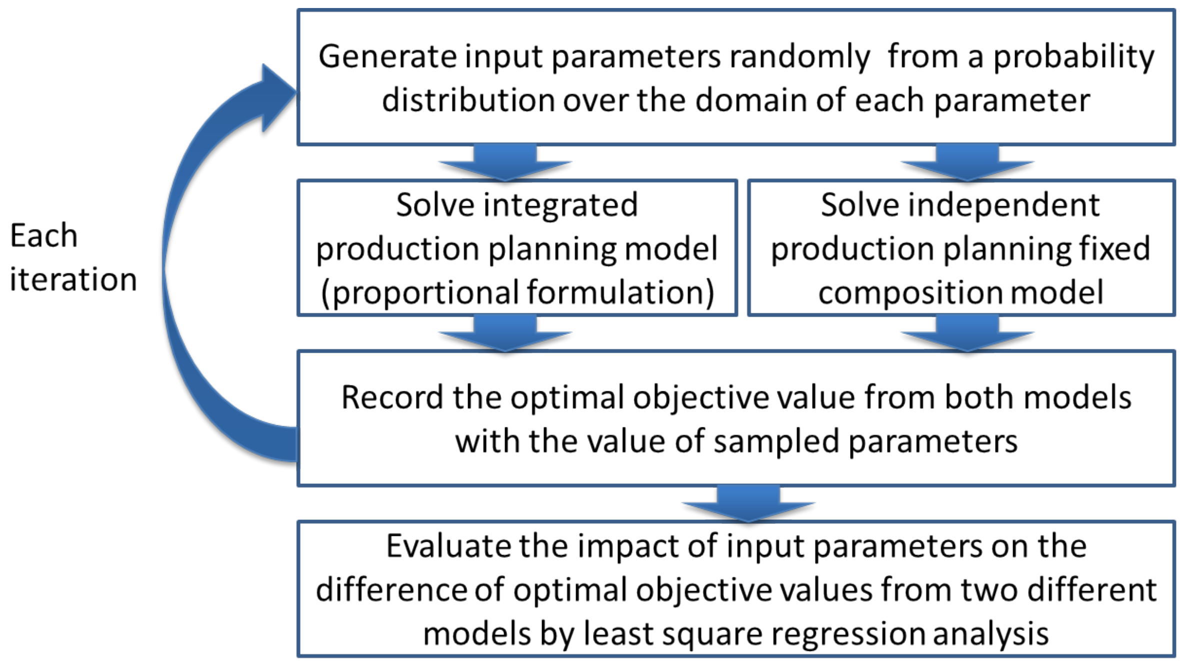


Figure 3-3. Flow chart of simulated screening process to identify the characteristics of operational parameters that maximize the benefit of the integrated production planning model, compared to the independent production planning model.

4 Two-stage operation with uncertainty of incoming materials

4.1 Problem description

Recovering metal from in-house dross (in other words, within the same facility where it was generated) may be especially beneficial since the metal will have a composition similar to alloy products being made in that facility. This benefit can be maximized when the metal from dross for a given alloy is used again to produce the same alloy. Achieving this goal, however, is challenging in practice. In a cast house that produces multiple finished alloys, it is difficult to track the product from which each dross originates. Collecting dross separately by alloy product may provide a solution, but this requires as many dedicated lots for each type of dross as the number of products. Separate storage for dross is even more constrained for aluminum producers in Europe and Japan where storing dross outdoors is restricted due to the potential reaction with water. Therefore, this strategy is not practical in many cases and dross materials are combined before preprocessing. The significant loss in its economic value due to commingled dross is an issue not only for in-house processing, but also for off-site processing.

In addition, rotary furnace operators may add different scraps or dross from other sources to in-house dross in order to leverage the energy efficiency gains of operating a furnace at full capacity. Moreover, it is difficult to estimate the composition of dross from external sources until it is processed in the rotary furnace. This practice results in a situation in which the composition of output from the rotary furnace is different in every batch and potentially quite variable. If the measured composition of output after operating the rotary furnace happens to be similar enough to a product for the next batch in a melting furnace, it can be immediately used. Otherwise, it must be cast as an output (or sow) from the rotary furnace. Typically, the sows are aggregated and stored together within a facility. Because of the myriad challenges described above resulting in increased compositional uncertainty, the use of the rotary furnace output may be limited to low-quality alloys (in other words, those with wider compositional specification). Considering the fact that some outputs can be used for a higher quality alloy if the composition fits well, aggregating is not the most efficient recycling strategy. However, for many plants, it is practically impossible to separate and store each output from every batch of the rotary furnace in an individual bin. A trade-off exists between having one aggregated bin or having several individual bins for each output from one batch of rotary furnace, as described in Figure 4-1(a) and (b), respectively.

The former provides logistical simplicity but loses the collected compositional information of each sow by aggregating them and increases the uncertainty of raw materials for alloy production. The latter provides perfect information about raw materials for batch planning; however, separating is expensive and requires many lots or bins to store each material stream. It raises a question of how much information is enough or how much binning is enough for effective usage of cast sows.

Figure 4-1 describes the general idea of our approach. In the suggested recycling operation, each output from the rotary furnace is assigned to a different bin based on its measured composition. Binning outputs from the rotary furnace as shown in Figure 4-1(c) allows each bin to have relatively more similar raw materials compared to common recycling operations where all outputs from the rotary furnaces are mixed regardless of their composition as shown in Figure 4-1(a). Binning enables melting furnace operators to distinguish the specification of raw materials in different bins and use this information to model batches for the melting furnace. The clustering analysis in this study provides a way to define these bins.

Clustering analysis is one of several data mining methods able to find patterns without any prior knowledge of what pattern exists. This method segments larger data sets into subsets, each of which are more homogeneous clusters of observations than the aggregate set as a whole. Therefore, clustering analysis can be used to recognize the patterns of raw materials with varied composition and group them into several categories.

Recent improvements in information-gathering techniques in manufacturing allow firms to collect and store many types of data. Consequently, data mining has attracted attention as a tool for extracting information from these accumulated data pools (Wang 2007). However, applying data mining methods is less frequently done in manufacturing environments than in other areas such as finance or business. Several authors also point out that the use of accumulated data in manufacturing firms has been very limited, although the collected data embody valuable insights and knowledge (Wang 2007; Choudhary, Harding et al. 2009) Many studies in recycling and remanufacturing that have employed historical data mostly focus on forecasting the expected outcome using statistical analyses(Goh and Varaprasad 1986; Clottey, Benton et al. 2012)

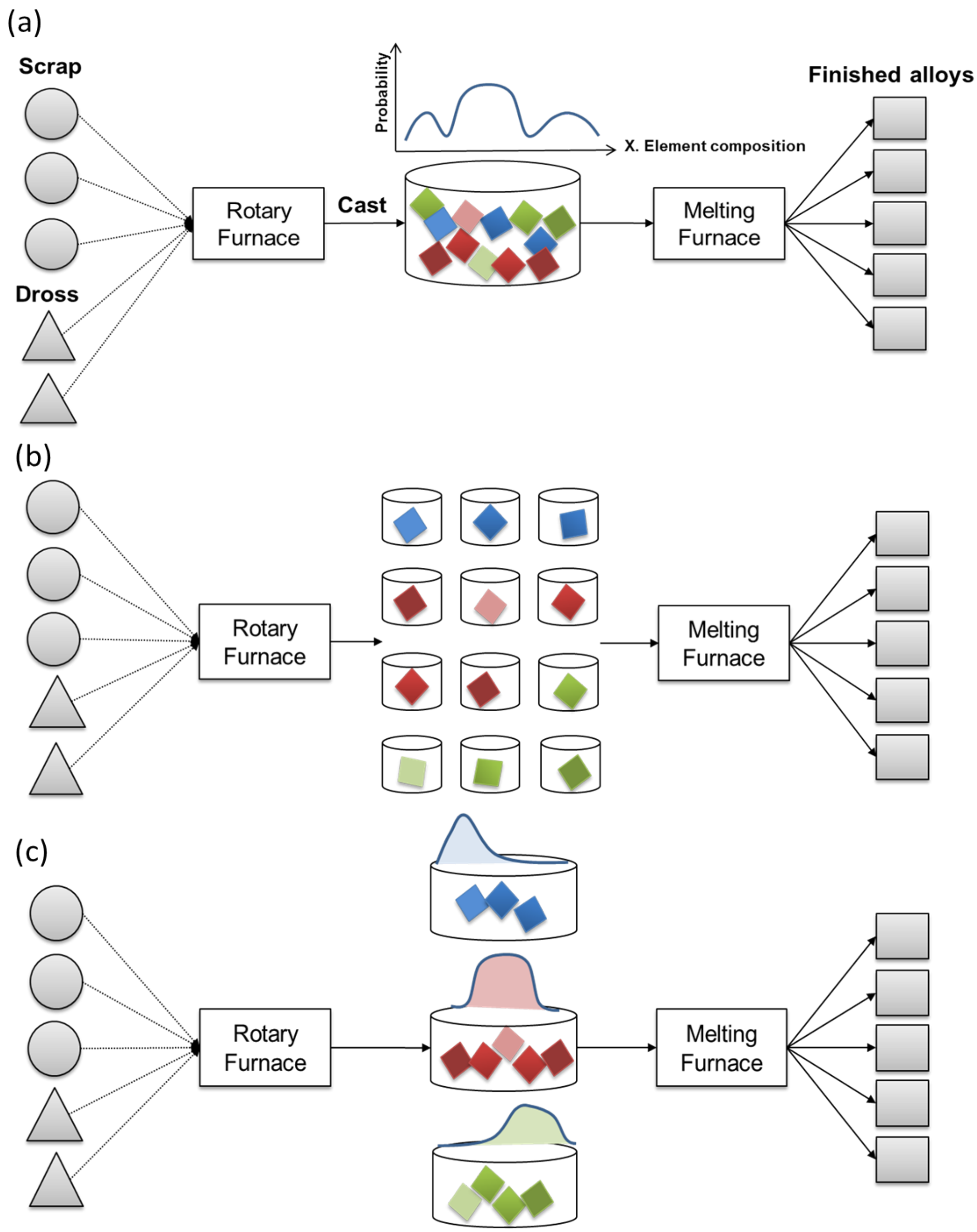


Figure 4-1. The diagram of aluminum dross recycling operation (a) current operation setup in which all cast sows are aggregated in the single bin (b) ideal operation setup where each output from the rotary furnace is individually binned (c) proposed operation setup where sows with relatively similar composition are binned together.

The approach of this study is motivated by actual practices in the recycling and remanufacturing industries. Most of these firms are likely able to acquire data about outputs from the first process, such as preprocessing at the rotary furnace or disassembly stage. Given the current common practice of measuring the composition of the outputs from the rotary furnace, data mining methods can provide valuable insights to improve current recycling operations. In this chapter, the opportunity of using data mining method to improve the two-stage recycling operation when the intermediate products are delivered as sows from the reprocessing stage and the re-melting stage will be explored.

4.2 Result of the clustering analysis for cast sows

The clustering results can be obtained by cutting the dendrogram at different levels which represent the number of clusters as shown in Figure 4-2. The result from each selected level of the dendrogram contains information about which sow belongs to which group. Each group can be interpreted as one separated bin for sows in the context of a production environment. Various levels are selected since there is no prior knowledge of which level will be most effective to indicate sows for use in alloy production. The compositional specification of each bin can be described by the statistical parameters, including the mean and standard deviation, of sows assigned in that bin.

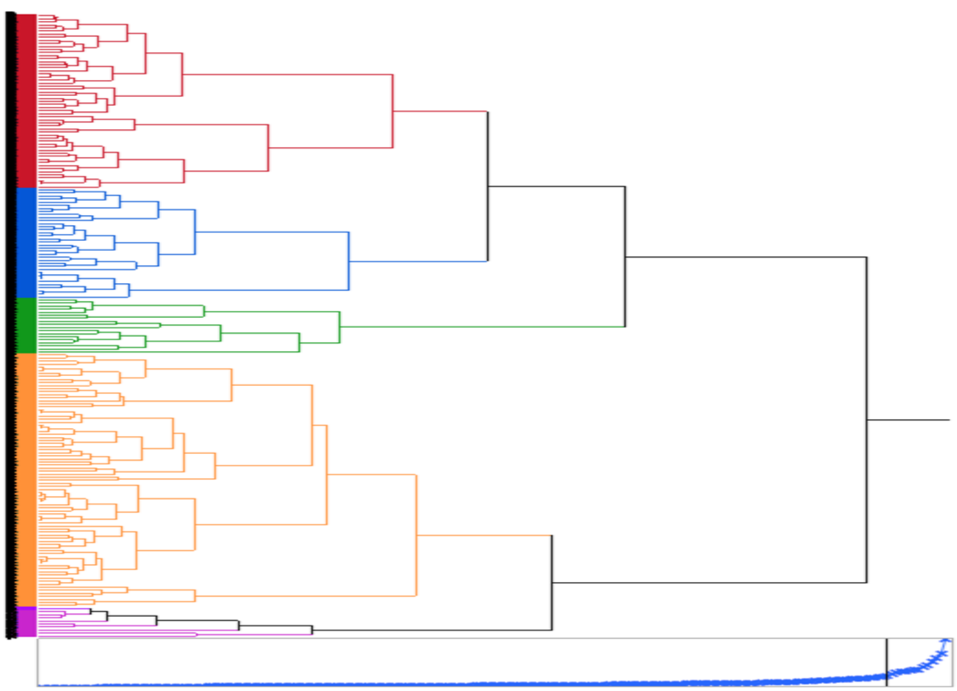


Figure 4-2. Dendrogram image obtained from hierarchical clustering analysis. The box in the bottom represents distance measurement at each stage of clustering.

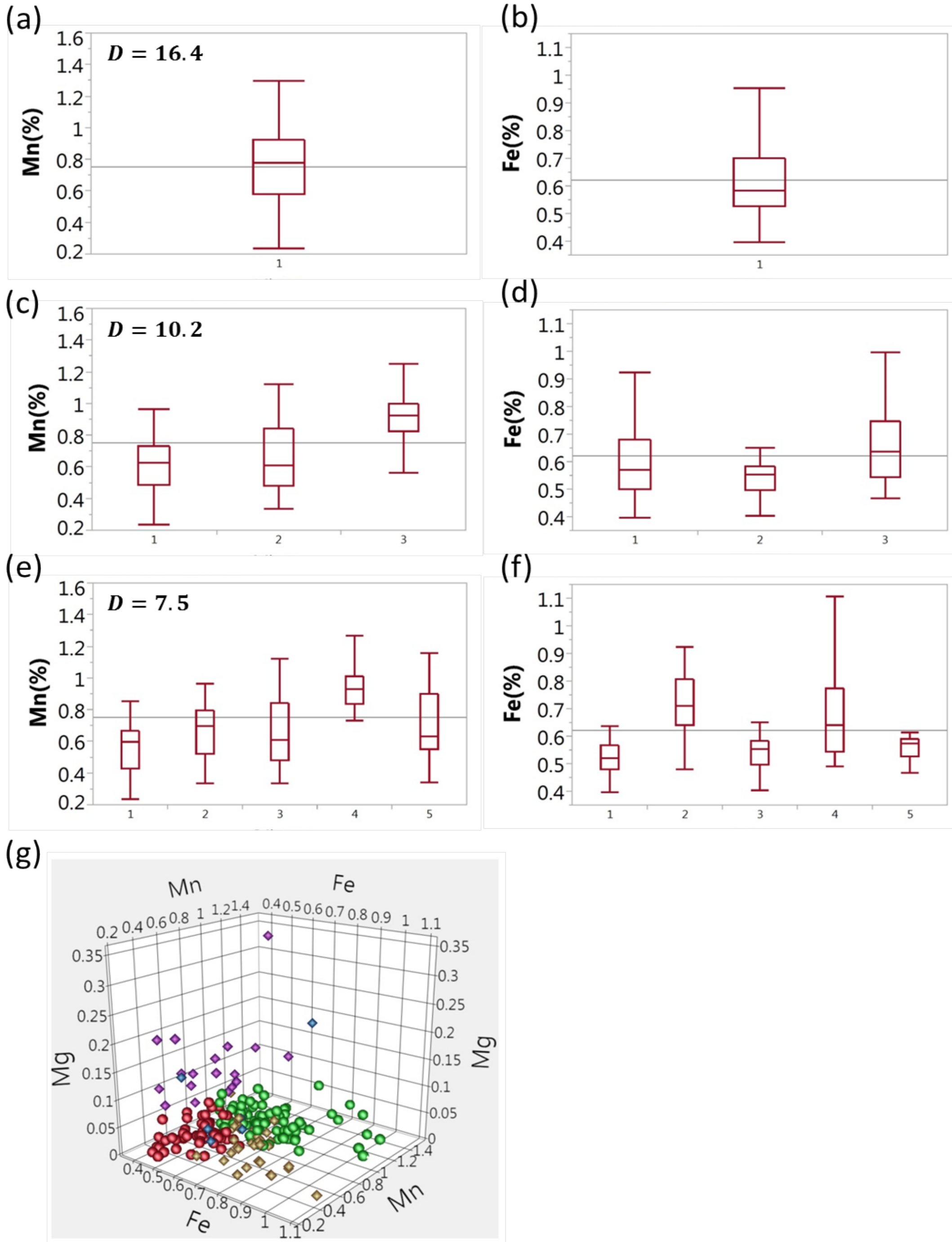


Figure 4-3. The statistical characteristics of each bin of sows with the selected number of bins, the case of one bin for (a) Mn and (b) Fe three bins for (c) Mn and (d) Fe, and five bins for (e) Mn and (f) Fe. The number in the left top corner in (a), (c), and (e) represents the distance of bins, defined as total within-variance, when the number of bins is one, three and five, respectively. This metric considers all six elements. (g) The scatter plot of

sow compositions with limited elements, Fe, Mn, and Mg in the five-bin cases. Each color represents one bin and each dot represents one output from the rotary furnace batch.

Figure 4-3 shows the statistical characteristics of two of the compositional elements, Mn and Fe, with the selected numbers of bins as examples. Figure 4-3 (a) and (b) represent the case when all raw materials are aggregated into one bin, which corresponds to the current operation at the case facility. The beginning of the clustering process, starting with only one object in each cluster (not included in Figure 4-3), represents the opposite situation, where all outputs from batches of the rotary furnace are completely separated. In that case, the number of bins is equal to the number of batches in the rotary furnace. The compositional range of a bin is more distinctive as the number of clusters increases.

For example, the one bin having compositional characteristics as shown in Figure 4-3(a) and (b) is separated into three bins which are relatively characterized as low Mn and medium Fe, medium Mn and low Fe, high Mn and medium Fe, as shown in Figure 4-3(c) and (d). The ranges of different bins are not completely distinguishable because of the multi-dimensionality of composition, which consists of six elements. Figure 4-3(g) represents how sows are assigned to different bins based on their composition in the case of five bins for three elements, Fe, Mn and Mg. Each dot represents one batch output from the rotary furnace. Each color represents one bin. Dots with the same colors clearly congregate. This representation indicates that sows with relatively similar compositions are binned together. Although the graph is plotted with three elements, the distance between clusters is calculated based on all six elements.

4.3 Batch planning with binned raw materials

We evaluate the effects of varying the number of bins in daily batch planning of finished alloy production. Two performance metrics are used in this study: the percentage of the amount of sows used in production to the total available amount, and the ratio of material production cost to that of the current recycling operation where there is only one aggregated bin.

Figure 4-4 represents the result of a selected day's batch planning as an example. The recycling facility in this industrial case study produces total eighteen different alloy products. Since this re-melting facility produce different final products every day, the impact of binning sows on daily batch planning varies depending on which finished alloy product is produced on a given day. The result in Figure 4-4 is the one that has the average performance compared to other days. As the number of bins increases, more sows are incorporated to produce final alloys, replacing expensive primary metals and alloying elements. In the case of one bin, only 22% of total available cast sows are used in alloy production, while all available cast sows are completely used in the case of ten bins. The production cost ratio of the 10-bin case to the single-bin case is 0.93.

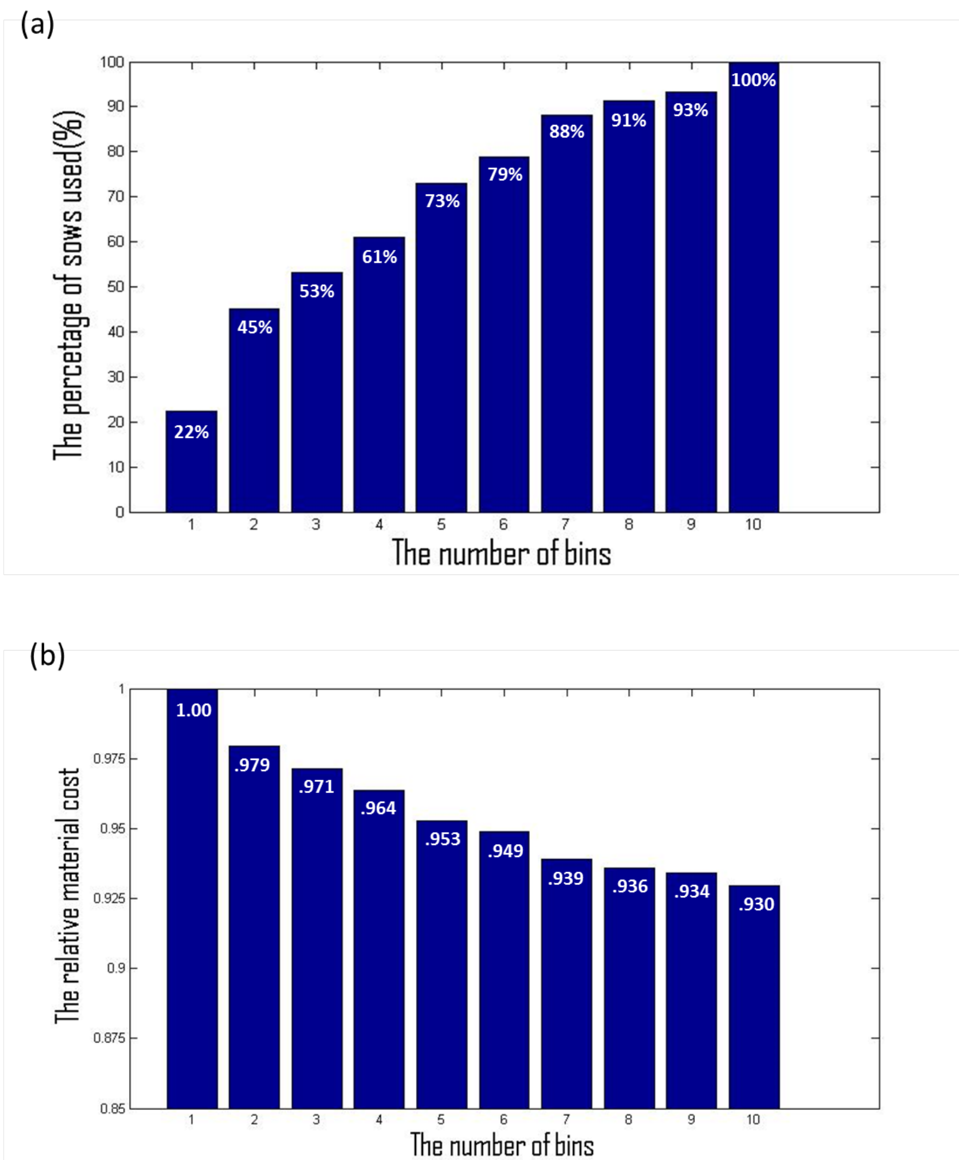


Figure 4-4. (a) The percentage of the amount of sows used in alloy production to total available amount (b) the ratio of material production cost to that of the single bin case with the different number of bins.

As a benchmark, we also ran the batch optimization model for the 204-bin case where individual sows are separately binned as Figure 4-1(b). This case, therefore, represents the situation in which there is no uncertainty associated with compositions of sows. In this case, 100% of available cast sows are used in alloy production and its production cost ratio is 0.924. Compared to the 204-bin case, binning sows into ten bins by their compositions allows using the same amount of cast sows at a significantly lower number of bins and similar material cost. This result suggests that the clustering by the compositions of sows is an effective binning strategy to increase usage of low-quality raw materials such as scrap and dross while reducing that of primary and alloying elements. This benefit is an incentive for material recyclers to maintain some compositional information from cast sows by grouping them into several categories rather than aggregating all of them.

Two different mechanisms explain the increase in performance with a higher number of carefully designed bins. The first mechanism is the reduced uncertainty of raw materials in each bin produced by binning sows. As discussed around Figure 4-3, the composition of a bin in the case of higher number of bins has a narrower distribution than in the case of a single bin. This reduced uncertainty of the sows allows use of more secondary raw materials, instead of using expensive primary metal or alloying elements. However, more use of cast sows, rather than other scrap, is attributed to their lower price. Second, a re-melting furnace operator can take advantage of the more distinctive composition with the higher number of bins. In other words, the compositional distribution of each bin covers a relatively more distinct range and becomes more directly customized with particular products as the number of bins increases. For example, when the alloy specification is characterized as having high manganese content, one can reduce use of sows from the bin 1 and increase use of those from the bin 4 in the five-bin case if only considering the element manganese.

Understanding these mechanisms is easier if we look at constraints for the maximum and minimum specification requirements in the CC batch optimization model. Mathematically, the first benefit from the reduced uncertainty is related to the second term in Equation (10) and (11). It should be noticed that $\mathrm{X}(\alpha)$ is a positive number, whereas $\mathrm{X}(1-\beta)$ is negative. These second terms play a role in narrowing the window of alloy specification depending on the compositional uncertainty of raw materials. The second term in Equation (10), $X(\alpha)(\sum_i \sum_l \rho_{(e)ilk}\sigma_{(e)ik}\sigma_{(e)lk}x_{it}x_{lt})^{\frac{1}{2}}$, lowers the maximum limit of specification according to statistical parameters of uncertain raw materials and their usage. The terms in Equation (11), $X(1-\beta)(\sum_i \sum_j \rho_{(e)ilk}\sigma_{(e)ik}\sigma_{(e)lk}x_{it}x_{lt})^{\frac{1}{2}}$, elevates the minimum of specification. When the aggregated bin splits into two separate bins, the standard deviations of these bins become smaller. The smaller standard deviation of each newly formed bin, $\sigma_{(e)ik}$ or $\sigma_{(e)lk}$, results in broadening the width of the given windows when the same amounts of raw materials are used. This broadening allows incorporating more raw materials with uncertainty into a batch plan if other conditions are unchanged.

The second mechanism of improved performance is relevant to the first term in Equations (10) and (11). As a consequence of clustering, the values of $\overline{e_{ik}}$ of bins are adjusted depending on the compositions of assigned raw materials. A re-melting furnace operator can accordingly differentiate the composition of sows in different bins so that sows in each bin can be more customized to alloy products with similar compositions.

As the number of bins increases, the marginal increase in sow usage generally decreases, but not necessarily monotonically. The fact that the benefits of binning sows originate from two different mechanisms explains this behavior. The benefit from the first mechanism, the reduced compositional uncertainty, becomes less significant as the number of bins increases. This can be explained relative to the clustering procedure. In an agglomerative hierarchical clustering process, the earlier two clusters merge into one, therefore the total-within cluster variation increases. In general, merging the final two clusters into one leads to the greatest increase in total-within cluster variation because these last two clusters are most unlike. Therefore we see a large benefit going from one bin to two bins and so on. In other words, the first split results in the greatest decreases in compositional variation within each bin and further binning has a diminishing decrease in compositional uncertainty.

Once the compositional distribution of sows in each bin becomes smaller than the final alloy specification window, the sows in that bin can be fully utilized. Eventually, at a certain stage in the binning process, all available sows can be completely used in alloy production. However, the benefit from the second mechanisms, a more distinctive composition, is not necessarily related to the number of bins. For example, because the final alloy specification is not used in the clustering, there is no guarantee that the mean composition of sows in a bin for a two-bin case is more customized to the specification of alloy products than that of a bin in ten-bin case. Therefore, whether the benefit from the second mechanism is significant or not depends on the final alloy specification. Overall, the optimal number of bins that allows complete usage is determined by the relationship between the statistical characteristics of bins at each level of clustering and the final alloy specification.

Although we observe improved performance from binning sows by employing the CC method to determine batch recipes, the mechanisms described above imply that similar results could be observed in other batch modeling approaches that consider uncertainty.

4.4 Economic analysis of binning strategies

Binning sows by clustering based on their compositions increases the homogeneity of raw materials available for production. Since sorting materials can be defined as an activity to separate the mixture of different materials into more homogeneous sets, this strategy can be considered as a different way to sort materials.

In that context, binning is an effective method to boost usage of cast sows because it improves the uniformity of raw materials. Since purchasing raw materials is one of the major cost factors for material manufacturers, the substitution of this new secondary material for expensive primary material can bring significant economic benefits. However, obtaining more compositional information on sows requires firms to purchase additional property (e.g., land) to accommodate sorting and storage in order to separate existing raw materials. Therefore, it is certainly meaningful for recycling firms to weigh the capital cost of bin setup versus the benefit of sorting. We perform a simple analysis to evaluate the expected economic benefits of the binning strategy based on our results of the daily batch planning above. Several assumptions are made for the purpose of this analysis. It is assumed that the rate of material substitution of cast sows for primary materials will be similar to our results above throughout the payback period. We also assume that there is no additional cost, such as a maintenance cost, other than the fixed cost to purchase a lot for storing raw materials. Other parameters used in the analysis are summarized in Table 4-1.

Table 4-1. Parameters used to calculate the expected cost saving of binning strategy.

Operation days per year	240 days/ year
Discount Rate	10%
Payback period	3 years

Table 4-2 represents the present value of total material cost savings over three years if bins are added to separate raw materials, compared to the current operation, which is equivalent to the single-bin case. For example, with an addition of two bins in which cast sows are separated into three groups, the expected cost saving from material substitution is US$7.6 million or US$3.8 million per bin. This value suggests an upper limit at which firms can invest to set up additional bins. Therefore, the benefit of adding a bin becomes less significant as the number of bins grows, because the average cost saving per bin decreases with the increase in the number of added bins as shown in the third column of Table 4-2.

In reality, expanding storage places for raw materials is often complicated and contextual. The cost of expansion varies from firm to firm and determining the size of lots for raw materials depends on many different factors. The optimal number of bins to bring firms the largest economic benefits must be chosen after careful consideration of the expected cost saving from material substitution as well as the capital costs needed to expand inventory spaces. However, the simple economic analysis in this study suggests that binning raw materials by their composition allows firms not only to increase the usage of low-quality raw materials in their production but also to realize an economic benefit.

Table 4-2. Total expected material cost saving and average expected material cost saving per bin with the different number of bins.

(Units : million dollar)

The number of added bins (Total number of bins)	Total material costs savings	Average cost saving per bin
+1 (2)	(5.5)	(5.5)
+2 (3)	(7.6)	(3.8)
+3 (4)	(9.6)	(3.2)
+4 (5)	(12.5)	(3.1)
+5 (6)	(13.5)	(2.7)
+6 (7)	(16.2)	(2.7)
+7 (8)	(17.0)	(2.4)
+8 (9)	(17.5)	(2.2)
+9 (10)	(18.7)	(2.1)

The economic analysis included in this thesis is for the case where the aluminum producer processed dross and other low-quality scrap materials in-house. However, it is also easy to consider the case of off-site processing. In general, off-site dross processor recovers dross for a fee, which is called tolling. The results obtained in this study quantify when aluminum producers have motivation to pay outsider contractors instead of grouping processed dross into multiple bins. For example, aluminum producers can save 2.1% of total material cost in alloy production by separating sows into two bins. If the additional fee to separate is less than cost saving in the re-melting furnace batches, aluminum producers are willing to pay additional cost to an off-site dross processor.

5 Integrated production planning for two-stage recycling operations

5.1 Problem Description

When intermediate products produced in the reprocessing stage are delivered as liquid metal to a downstream re-melter, a tension arises between the reprocessing stage and the re-melting stage. As mentioned earlier, molten aluminum is highly perishable. Because of this perishability, metal must be immediately used for alloy production in re-melting furnaces or must be cast as sow. The latter requires additional energy costs for the subsequent re-melting of sows. When a two-stage recycling operation is operated with a vortex system, it does not require casting intermediate products as sows since this continuous furnace always stores some amount of liquid metals. However, the composition of intermediate products is a key factor that determines how much liquid metal can be transferred to a re-melting furnace. If the composition of the liquid metal in a continuous furnace does not match with the specification of alloy products to be made in a re-melting furnace, only limited amounts of liquid metal can be transferred into the re-melting furnace. Therefore, in any case, designing intermediate products is a critical decision to achieve successful two-stage recycling operations.

In this chapter, two case studies are introduced to understand how the operational parameters influence the design of intermediate products when the demand information in the reprocessing furnace is considered. The first case study is designed to understand the benefits and limitations of the pooling problem for integrated production planning and the differences between the integrated production planning and a typical one–stage blending operation. The second case study is extended to consider the uncertainty in demand for final products. The analytical approach is used to understand the interactions between operational parameters and the optimal solution in both case studies.

5.2 Understanding blending behavior of two-stage recycling operation

One goal of this study is to understand blending behavior in the pooling problem particularly in the material recycling context. The small case study introduced in this chapter provides a simplified version of the pooling problem. This case study determines the opportunities and limitations of integrated production planning. Figure 5-1 provides a schematic diagram of the simplest system of two-stage recycling operations. This system consists of a total four raw materials (two low-quality scrap materials, $1,2 \in I$, and aluminum and one alloying element, $3,4 \in J$) and two final products ($A, B \in T$). Two different kinds of scrap materials, 1 and 2, require pre-processing before blending with other pure raw materials, the primary (3) and the alloying element (4) in the re-melting stage. Throughout this chapter, it is assumed that Scrap 1 is more compositionally pure than Scrap 2 ($e_1 < e_2$) and Product A is more compositionally pure than Product B ($e_A < e_B$). The pool is the resulting product of blending two scrap materials, called the intermediate product, $p \in L$ in the material recycling process. The goal of this problem is to find blends that minimize the cost to produce alloy products A and B while satisfying demands for them and their specifications.

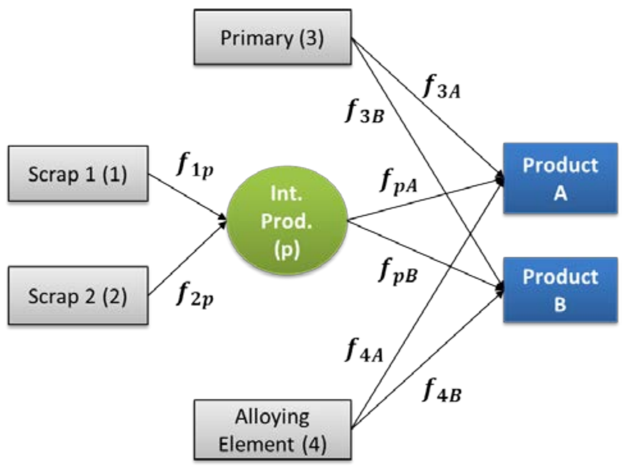


Figure 5-1. Schematic of material flow of the case study where two scrap materials are blended as an intermediate product and this intermediate product is blended with the primary and an alloying element to produce two products, A and B.

5.2.1 Problem without availability constraints (P1)

To simplify the problem, we assume that there is no availability constraint on the scrap material in this section. The x-axis of the graph in Figure 5-2 represents the composition of alloying element within Al in percent. The y-axis of the graph represents the price of raw materials. Therefore, the black dots in the graph show the prices and compositions of raw materials within the system. Primary aluminum is located at the left side of the graph, and the alloying element is located at the right side of the graph due to their composition. Scrap materials are located between two pure materials, primary and the alloying element, on the x-axis and mostly below of them on the y-axis. This is because most scrap materials are relatively cheaper than primary aluminum and alloying elements. The collection of line segments connecting adjacent black dots are convex in most cases of aluminum recycling. Let's assume all raw materials can be blended in one stage, without the reprocessing stage. Then the minimum price of a final blend will be determined by convex hull of these points as lines in Figure 5-2. For example, the minimum cost to make an unit weight of the blend with the composition, e_f, is c_f. The raw materials to make a blend at the minimum cost will be determined by two points connecting the line segment of the convex hull at c_f and their blending ratio is determined by the lever rule. In the example in Figure 5-2, the cheapest way to make the blend e_f is to blend scrap 1 and scrap 2. The amount of scrap 1 and scrap 2 used in the blend f with unit weight, x_{1f} and x_{2f}, becomes

$$x_{1f} = \frac{e_2 - e_f}{e_2 - e_1}, x_{2f} = \frac{e_f - e_1}{e_2 - e_1} \tag{76}$$

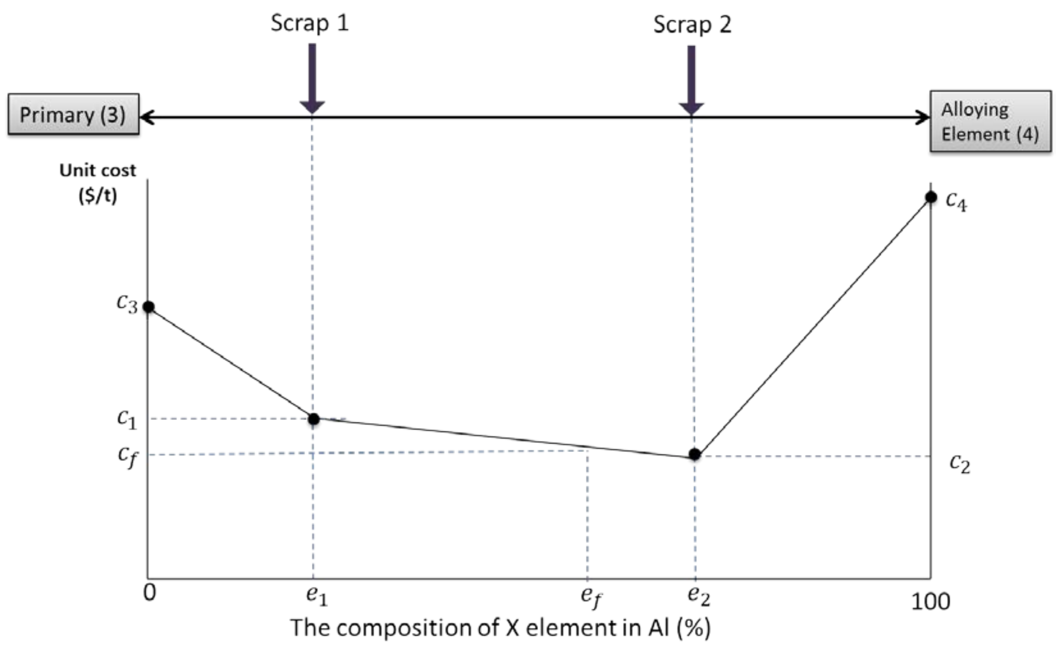


Figure 5-2. Composition-price graph of raw materials with one alloying element. This graph also shows the price of blends with various composition when each blend is produced independently.

Let's consider the case when the connected lines of adjacent dots representing individual raw materials are not convex. There are two possibilities: (1) when the point representing

Scrap 1 is above the line connecting primary aluminum and Scrap 2 (Figure 5-3(a)) (2) when the point representing Scrap 2 is above the line connecting Scrap 1 and alloying elements (Figure 5-3(b)). In the first case, the price of Scrap 1(c_1) is more expensive than the cost of the blend of primary aluminum and Scrap 2(c_{b1}) that has the equivalent composition with Scrap1. In this case, using Scrap 1 to produce final alloy products is not an efficient production plan because it can be replaced by the blend of primary and Scrap 1 at lower cost. Similarly, in the second case, using Scrap 2 is not cost-effective. Even if the specification of the alloy product is exactly the same as the composition of Scrap 2, blending Scrap 1 and the alloying element can make the same alloy product at a cheaper price, which is presented as c_{b2} in the Figure 5-3(b). However, the second case is unrealistic. The price of scrap is predominantly determined by the purity of alloy. Scrap price generally decreases or at least does not increase with an increase in its composition of alloying elements. Consequently, the price of Scrap 1 is generally higher than that of Scrap2 in reality.

Regardless of the details of a case, it is true that the cost of final blends will be determined by the collection of lines that defines the convex hull of a point set whose each point represents the price and composition of each raw material in the composition-price plot.

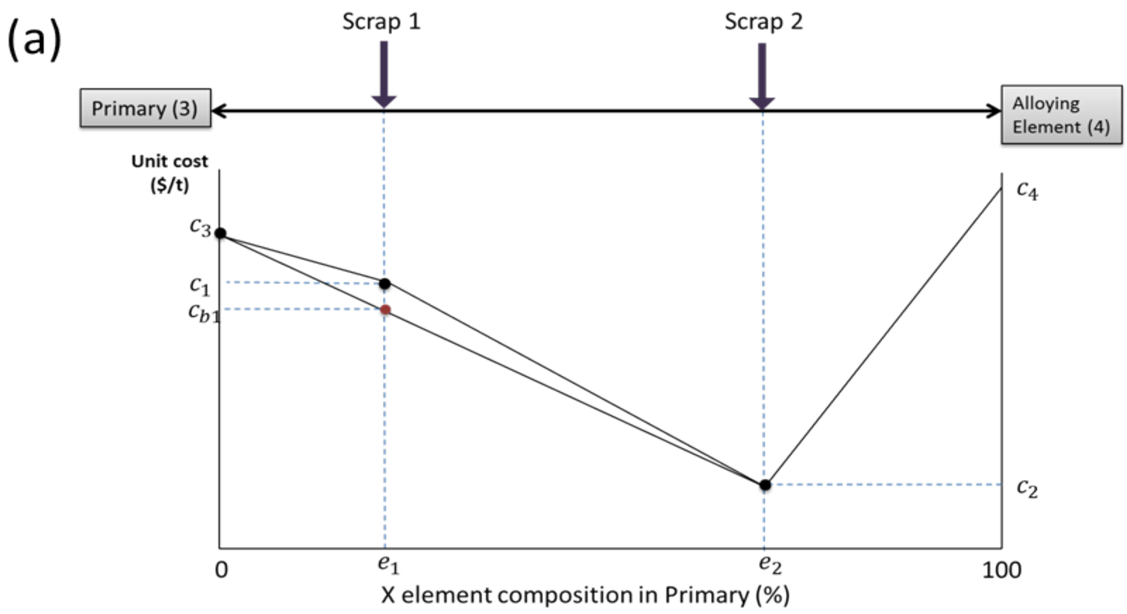


Figure 5-3. Composition – price graphs of raw materials when the lines connecting adjacent points of raw materials are not convex (a) when the price of scrap 1 is higher than the cost of blend of primary and Scrap 2 (b) when the price of Scrap 2 is higher than the cost of blend of Scrap 1 and the alloying element.

One of the main goals of this chapter is to understand how a two-stage blending operation differs from a one-stage blending operation. It is important to address the constraints posed by the reprocessing stage in aluminum recycling, or having pools (e.g. intermediate products). The key difference between one-stage and two-stage recycling operations is that scrap materials must be blended prior to blending with primary and alloying elements. Because of this operational constraint, the blending ratio of the first-stage raw materials must be the same in all final products. In this particular case study, the blending ratio of Scrap 1 and Scrap 2 must be equal in product A and product B. This is because individual Scrap 1 and Scrap 2 are not available in the second stage of the blending process. The only available raw materials in the second stage are primary, the alloying element and the blend of Scrap 1 and Scrap 2. This constraint posed by a two-stage recycling operation changes the shape of lines in a composition-price graph from Figure 5-2 to Figure 5-4. Instead of two black dots representing Scrap 1 and Scrap 2, there will be only one green dot since the blend will be the only available raw material in the second stage. Therefore, the line segment that represents the final blend cost will be newly defined and lifted as shown in Figure 5-4.

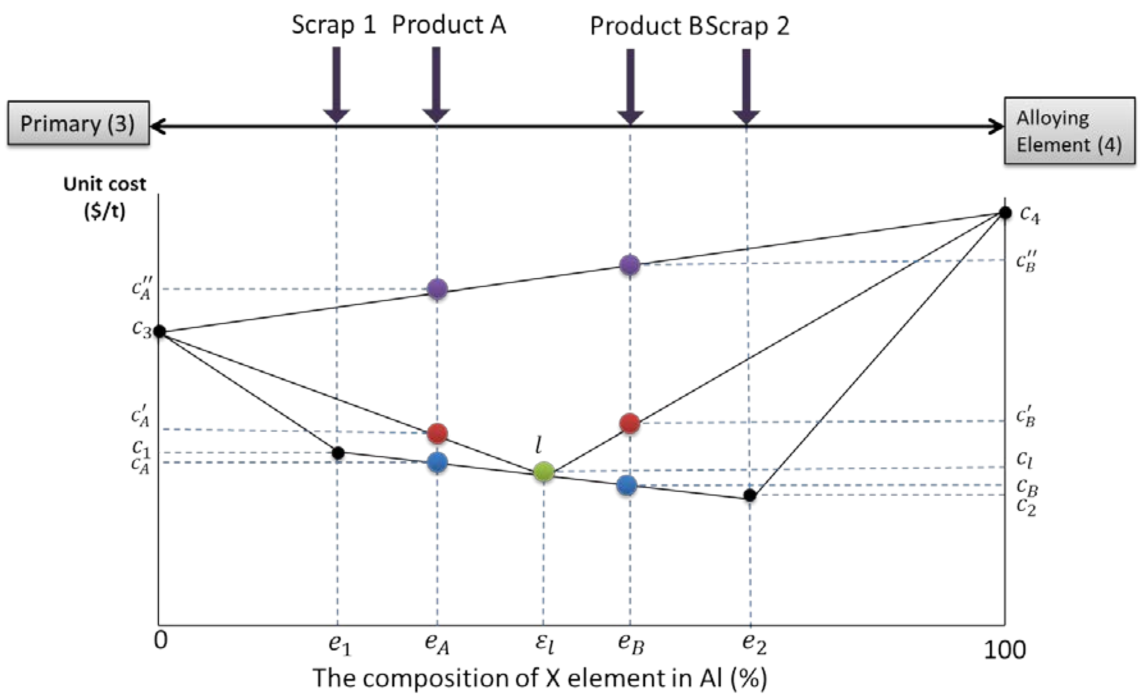


Figure 5-4. Unit production cost of individual product as a function of the composition of the final blend.

For an arbitrary composition ε_l of the intermediate product, the cost of producing unit volume of Product A and Product B consequently increase from c_A and c_B to c'_A and c'_B, respectively. The total production cost is calculated as

$$Total\ Production\ Cost(TPC) = c'_A D_A + c'_B D_B \tag{77}$$

Therefore, the problem can be restated as finding the composition of the intermediate product, or the optimal location of the green dot that makes the lowest total production cost.

In this case, if the number of intermediate products is two, then there is no penalty to reprocess low-quality scrap prior to the re-melting stage. For example, one easy solution can be making two intermediate products, one made from only Scrap A and one made from only Scrap B. The another solution can be making two intermediate products, one with the same specifications as Product A and the other with the same specification as Product B. This finding implies the number of intermediate products compared to the number of products is the key information. This value determines the upper limit of the performance of integrated production planning. The number of intermediate products in batch operations is determined by the available equipment in a recycling facility. For example, a facility may have one rotary furnace and outputs of this rotary furnace can be used in more than one re-melting furnace. If the number of rotary furnace is equal to the number of re-melting furnaces, each rotary furnace can produce intermediate products customized to final products made in each re-melting furnace. Such a facility can maximize the benefit of reprocessing technology without any penalty, even compared to the hypothetical case where all low-quality scrap materials can be blended in the re-melting furnace directly.

Although there is a penalty for having one joint intermediate product rather than two intermediate products customized for each product, the production of final products with the reprocessing stage is still cheaper than production without the reprocessing stage. If a recycling operation involves only high quality raw materials that do not require any pre-processing, the cost of producing unit weight of Product A and Product B is c''_A and c''_B, respectively. These two values are certainly higher than c'_A and c'_B. Thus, the reprocessing stage that enables using low-quality raw materials is economically beneficial even though when there is a penalty for having fewer intermediate products than the number of products.

Since the minimum cost curve to produce a blend independently is a piecewise linear function, mathematical expression for cost function produce both Product A and B will have different forms depending on their relative specifications to the compositions of Scrap 1 and 2. The mathematical formulation of this particular case study of Figure 5-1 with unit demand for each product can be written as

$$\min c_1 f_{1p} + c_2 f_{2p} + c_3 \left(f_{3A} + f_{3B} \right) + c_4 \left(f_{4A} + f_{4B} \right) \quad (78)$$

Subject to

$$f_{1p} + f_{2p} = f_{pA} + f_{pB} \quad (79)$$

$$e_1 f_{1p} + e_2 f_{2p} = \varepsilon_p \left(f_{1p} + f_{2p} \right) \quad (80)$$

$$f_{pA} + f_{3A} + f_{4A} = 1 \quad (81)$$

$$f_{pB} + f_{3B} + f_{4B} = 1 \quad (82)$$

$$\varepsilon_p f_{pA} + e_3 f_{3A} + e_4 f_{4A} \leq e_A^U \quad (83)$$

$$\varepsilon_p f_{pA} + e_3 f_{3A} + e_4 f_{4A} \geq e_A^L \quad (84)$$

$$\varepsilon_p f_{pB} + e_3 f_{3B} + e_4 f_{4B} \leq e_B^U \quad (85)$$

$$\varepsilon_p f_{pB} + e_3 f_{3B} + e_4 f_{4B} \geq e_B^L \quad (86)$$

The cost of an intermediate product with an arbitrary composition, e_p, can be determined by

$$c_p = \frac{c_2 - c_1}{e_2 - e_1} \varepsilon_p + \frac{c_1 e_2 - c_2 \varepsilon_p}{e_2 - e_1} \quad (87)$$

This equation can be rearranged as

$$c_p = c_1 \frac{e_2 - \varepsilon_p}{e_2 - e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2 - e_1} \quad (88)$$

Since the ratio of scrap 1 and scrap 2 in the intermediate product with the composition, ε_p, is by the level rule

$$f_{1p} : f_{2p} = \frac{e_2 - \varepsilon_p}{e_2 - e_1} : \frac{\varepsilon_p - e_1}{e_2 - e_1} \quad (89)$$

Therefore, the cost of a unit weight of the intermediate product with the composition, ε_p, also can be expressed as

$$c_p = \frac{c_1 f_{1p} + c_2 f_{2p}}{f_{1p} + f_{2p}} \quad (90)$$

Then, the cost function of a unit weight of final blend with the composition, e_f, is defined by the location of the intermediate product with an arbitrary composition, ε_p, and the cost of primary and alloying elements. The cost function of the final blend will be

$$c_f = \begin{cases} \dfrac{c_1 - c_3}{\varepsilon_p} e_f + c_3, & where \;\; 0 < e_f < \varepsilon_p \\ \dfrac{c_4 - c_1}{1 - \varepsilon_p} e_f + \dfrac{c_1 - c_4 \varepsilon_p}{1 - \varepsilon_p} & where \;\; \varepsilon_p < e_f < 1 \end{cases} \tag{91}$$

For the purpose of developing intuition about a simplified problem, we assume that the specifications of two final products are expressed as the point rather than the window. In other words, it is assumed that $e_A^U = e_A^L = e_A$ and $e_B^U = e_B^L = e_B$. Then the compositions of the two final products must be e_A and e_B. The relative compositions of two final blends to the composition of the intermediate product will determine the production cost of each product. The possible intervals are as summarized in Table 5-1.

Table 5-1 Possible interval ranges of the composition of the intermediate products relative to the specification of final alloys.

Case I)	$\mathrm{e_A} < \mathrm{e_B} < \varepsilon_p$
Case II)	$\mathrm{e_A} < \varepsilon_p < \mathrm{e_B}$
Case III)	$\varepsilon_\mathrm{p} < e_A < e_B$

Therefore, the total production cost becomes

Case I) $e_A < e_B < e_p$,

$$TPC = \left(\frac{c_p - c_3}{\varepsilon_p} e_A + c_3 \right) + \left(\frac{c_p - c_3}{\varepsilon_p} e_B + c_3 \right) \tag{92}$$

Case II) $e_A < e_l < e_B$,

$$TPC = \left(\frac{c_p - c_3}{\varepsilon_p} e_A + c_3 \right) + \left(\frac{c_4 - c_p}{1 - \varepsilon_p} e_B + \frac{c_1 - c_4 \varepsilon_p}{1 - \varepsilon_p} \right) \tag{93}$$

Case III) $e_l < e_A < e_B$,

$$TPC = \left(\frac{c_4 - c_p}{1 - \varepsilon_p} e_A + \frac{c_1 - c_4 \varepsilon_p}{1 - \varepsilon_p} \right) + \left(\frac{c_4 - c_p}{1 - \varepsilon_p} e_B + \frac{c_1 - c_4 \varepsilon_p}{1 - \varepsilon_p} \right) \tag{94}$$

In case I), since the demand of each product is assumed to be 1, the amounts of each raw material usage are the same as their ratios which can be obtained by the level rule.

$$f_{pA} = \frac{e_A}{\varepsilon_p}, f_{3A} = \frac{\varepsilon_p - e_A}{\varepsilon_p}, f_{4A} = 0\ , f_{pB} = \frac{e_B}{\varepsilon_p}, f_{3B} = \frac{\varepsilon_p - e_B}{\varepsilon_p},\ \ f_{4B} = 0 \qquad (95)$$

If demand for each product is not 1, each flow variables will be weighted by the size of demand D_A or D_B. Rearranging parameters in the function in Eq (92) leads to

$$TPC = c_p\left(\frac{e_A}{\varepsilon_p}\right) + c_3\left(\frac{\varepsilon_p - e_A}{\varepsilon_p}\right) + c_p\left(\frac{e_B}{\varepsilon_p}\right) + c_3\left(\frac{\varepsilon_p - e_B}{\varepsilon_p}\right)$$

$$= c_p f_{pA} + c_3 f_{3A} + c_4 f_{4A} + c_p f_{pB} + c_3 f_{3B} + c_4 f_{4B} \qquad (96)$$

where $c_4 f_{4A}\ and\ c_4 f_{4B}$ terms will be vanished because $f_{4A} = 0, and\ f_{4B} = 0$.

Similarly, the amount of each raw material used in final products and the total cost function in case II) are

$$f_{pA} = \frac{e_A}{\varepsilon_p}, f_{3A} = \frac{\varepsilon_p - e_A}{\varepsilon_p}, f_{4A} = 0\ ,\ f_{pB} = \frac{1 - e_B}{1 - \varepsilon_p}, f_{3B} = 0,\ \ f_{4B} = \frac{e_B - \varepsilon_p}{1 - \varepsilon_p} \qquad (97)$$

$$TPC = c_p\left(\frac{e_A}{\varepsilon_p}\right) + c_3\left(\frac{\varepsilon_p - e_A}{\varepsilon_p}\right) + c_p\left(\frac{1 - e_B}{1 - \varepsilon_p}\right) + c_4\left(\frac{e_B - \varepsilon_p}{1 - \varepsilon_p}\right)$$

$$= c_p f_{pA} + c_3 f_{3A} + c_4 f_{4A} + c_p f_{pB} + c_3 f_{3B} + c_4 f_{4B} \qquad (98)$$

And in case III)

$$f_{pA} = \frac{1 - e_A}{1 - \varepsilon_p}, f_{3A} = 0, f_{4A} = \frac{e_A - \varepsilon_p}{1 - \varepsilon_p}\ ,\ f_{pB} = \frac{1 - e_B}{1 - \varepsilon_p}, f_{3B} = 0,\ \ f_{4B} = \frac{e_B - \varepsilon_p}{1 - \varepsilon_p} \qquad (99)$$

$$TPC = c_p\left(\frac{1 - e_A}{1 - \varepsilon_p}\right) + c_4\left(\frac{e_A - \varepsilon_p}{1 - \varepsilon_p}\right) + c_p\left(\frac{1 - e_B}{1 - \varepsilon_p}\right) + c_4\left(\frac{e_B - \varepsilon_p}{1 - \varepsilon_p}\right)$$

$$= c_p f_{pA} + c_3 f_{3A} + c_4 f_{4A} + c_p f_{pB} + c_3 f_{3B} + c_4 f_{4B} \qquad (100)$$

The rearranged total production cost function (96), (98) and (100) are also consistent with the original objective function. If you arrange the original objective function,

$$c_1 f_{1p} + c_2 f_{2p} + c_3(f_{3A} + f_{3B}) + c_4(f_{4A} + f_{4B})$$

$$= c_p\left(f_{1p} + f_{2p}\right) + c_3(f_{3A} + f_{3B}) + c_4(f_{4A} + f_{4B})$$

$$= c_p\left(f_{pA} + f_{pB}\right) + c_3(f_{3A} + f_{3B}) + c_4(f_{4A} + f_{4B})$$

$$= \left(c_p f_{pA} + c_3 f_{3A} + c_4 f_{4A}\right) + \left(c_p f_{pB} + c_3 f_{3B} + c_4 f_{4B}\right) \tag{101}$$

In Eq (101), the term: $c_p f_{pA} + c_3 f_{3A} + c_4 f_{4A}$, is associated with the production cost of Product A and the term, $c_p f_{pB} + c_3 f_{3B} + c_4 f_{4B}$, represents the production cost of Product B. Since $f_{pA}, f_{3A}, f_{4A}, f_{pB}, f_{3B}, f_{4B}$ and c_p are all a function of one variable, $\varepsilon_p \in [e_1, e_2]$, we can express the total objective function as the function of e_p. However, $f_{pA}, f_{3A}, f_{4A}, f_{pB}, f_{3B}$ *and* f_{4B} have different function of form depending on the range of ε_p. Therefore, the objective function becomes:

$$TPC =$$
$$\begin{cases} (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{e_A}{\varepsilon_p}\right) + c_3 \left(\frac{\varepsilon_p - e_A}{\varepsilon_p}\right) + (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{e_B}{\varepsilon_p}\right) + c_3 \left(\frac{\varepsilon_p - e_B}{\varepsilon_p}\right) & where\ \mathrm{e_A} < \mathrm{e_B} < \varepsilon_p \\ (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{e_A}{\varepsilon_p}\right) + c_3 \left(\frac{\varepsilon_p - e_A}{\varepsilon_p}\right) + (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{1-e_B}{1-\varepsilon_p}\right) + c_4 \left(\frac{e_B - \varepsilon_p}{1-\varepsilon_p}\right) & where\ \mathrm{e_A} < \varepsilon_p < \mathrm{e_B} \\ (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{e_A}{\varepsilon_p}\right) + c_3 \left(\frac{\varepsilon_p - e_A}{\varepsilon_p}\right) + (c_1 \frac{e_2-\varepsilon_p}{e_2-e_1} + c_2 \frac{\varepsilon_p - e_1}{e_2-e_1}) \left(\frac{1-e_B}{1-\varepsilon_p}\right) + c_4 \left(\frac{e_B - \varepsilon_p}{1-\varepsilon_p}\right) & where\ \mathrm{\varepsilon_p} < e_A < e_B \end{cases}$$

(102)

5.2.2 Problem with availability constraints in practice

It is useful to identify the shape of total production cost function in the aluminum recycling production context. This section demonstrates the case study with actual values derived from an aluminum recycling context. Table 5-2 summarizes two examples of the composition of scrap materials and the price of raw materials.

Table 5-2. Examples of the composition and price of raw materials.

	Example 1	Example 2
Scrap 1 composition e_1	0.2%	0.19%
Scrap 2 composition e_2	1.39%	1.84%
Scrap 1 price c_1	\$1700/t	\$1789/t
Scrap 2 price c_2	\$1700/t	\$1700/t
Primary price c_3	\$2137/t	\$2137/t
Alloying element price c_4	\$2689/t	\$5000/t

The graphs in Figure 5-5 are the composition-cost curves for two examples, Example 1 and Example 2. These graphs define the minimum unit cost as a function of the composition of a final blend when each blend is independently produced. There is an interesting aspect to these graphs. First, the intervals representing by two scrap materials are very narrow, compared to overall compositional range, and much closer to pure aluminum side than alloying element side in Figure 5-5(a) and (c). This is because scrap materials are aluminum alloy-derived. The majority composition within scrap materials consists of aluminum rather than alloying elements such as Zn or Mn. For the same reason, the slope of the line connecting pure aluminum (primary) and Scrap1 is much steeper than the slope of the line connecting Scrap2 and an alloying element. The difference of slopes implies that the cost structure is asymmetric. When the specification of final blend is out of the range of scrap compositions, the price to make a final alloy having the low alloying element content is much more expensive than making a final alloy with a higher alloying element content. In other words, if the specification of an alloy product is not achievable using only scrap materials, the adjustment of using primary aluminum or an alloying element is necessary to satisfy the specification ($e_f \in [0, e_1]$ or $e_f \in [e_2, 1]$). However, the price adjusting composition using primary aluminum (to reduce alloying content) is much higher than the price adjusting composition using alloying element (to increase alloying content) when the equal size of adjustment is required.

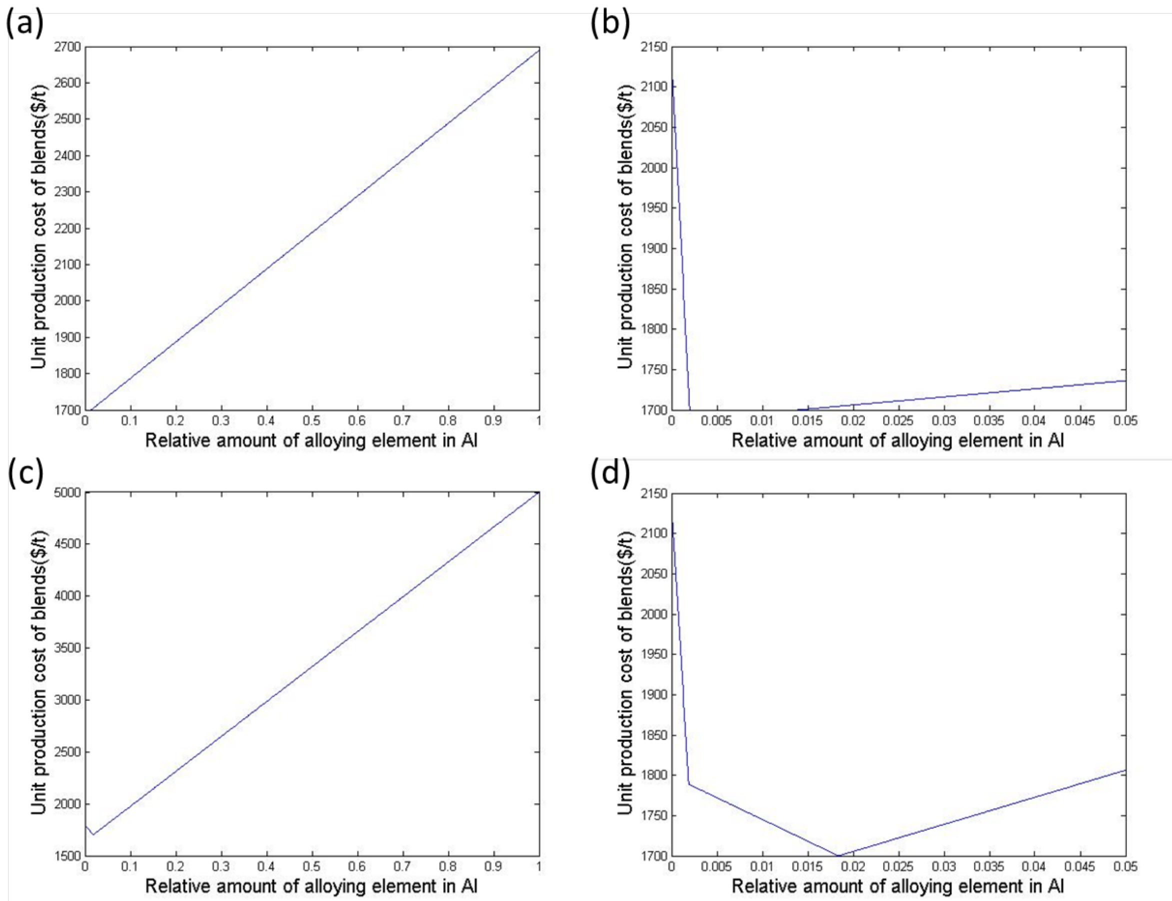


Figure 5-5. (a) Cost-composition curve of example 1, (b) the enlarged image of (a), (c) cost-composition curve of Example 2, (d) the enlarged image of Example 2. These graphs represent the cost of blends when each final blend is independently produced.

When the two final products are jointly produced together as a schematic of Figure 5-1, the graph of the total production cost cannot be directly read from Figure 5-5 because two ratios of two scrap materials must be equal in the both products. Since the total production cost function can be expressed as a function of one variable, which is the composition of the intermediate product as found in the previous section, the total production cost curve can be plotted. Figure 5-6 is the graph of the total production cost and production cost of each product of Example 3. The values of parameters used in Example 3 are summarized in Table 5-3.

Table 5-3. Data of product specification and demand, scrap composition, and price of raw materials used in Example 3.

	Values in Example 3
Scrap 1 composition(e_1)	0.2%
Scrap 2 composition (e_2)	1.39%
Scrap 1 price (c_1)	\$1700/t
Scrap 2 price (c_2)	\$1700/t
Primary price (c_3)	\$2137/t
Alloying element price (c_4)	\$2689/t
Product A min specification (e_A^L)	0.4%
Product A max specification (e_A^U)	0.6%
Product B min specification (e_B^L)	0.9%
Product B max specification (e_B^U)	1.1%
Product A Demand (D_A)	1
Product B Demand (D_B)	1

Figure 5-6 (a) represents the cost of producing product A and product B as a function of the composition of the intermediate product. Figure 5-6(b) shows the total production cost as a function of the composition of the intermediate product. Therefore, Figure 5-6(b) is the sum of two curves in Figure 5-6(a) when both products are produced with a unit weight. Since the possible composition of the intermediate product is bound by the range of scrap composition, the x-axis of two graphs in Figure 5-6ranges from the composition of Scrap 1 to Scrap 2. In this particular example, Scrap 1 and Scrap 2 have the same price as \$1700/t. Therefore, the unit price of any intermediate blends becomes \$1700/t as well.

When the composition of the intermediate product is 0.6%, the cost of producing a unit weight of Product A is the cheapest at \$1700. Since the composition of the intermediate product is within the specification window of the final product specification, the adjustment using primary aluminum or alloying element is not required in the second-stage of blending. In other words, Product A can be made of using only the intermediate product, the blend of two scrap materials. Meanwhile, Product B requires the addition of alloying elements to satisfy at least its minimum specification, 0.9%. This addition results in the production cost of product B more expensive than \$1700/t as shown in Figure 5-6(a). If the composition of the intermediate product is 0.9%, the cost of producing a unit weight of Product B is \$1700. In this case, Product B does not require any compositional adjustment in the second-stage of blending. It can be made of only using the intermediate product. However, making Product A requires dilution with primary aluminum down the intermediate product composition 0.9% to 0.6% in the re-melting stage. Therefore, the resulting production cost of Product A is more expensive than \$1700.

It is important to note that the cost of adjustment of Product B in the first case is much cheaper than the cost of adjustment of Product A in the second case. This is because of asymmetric slopes in Figure 5-5. Although both cases require adjustment of one of the product as much as 0.3%, the adjustment of the composition to reduce 0.3% is much more expensive than adjustment to increase 0.3%. Therefore, the optimal composition that minimizes the total production cost is 0.6%, which is the max specification of product A as shown in Figure 5-6(b).

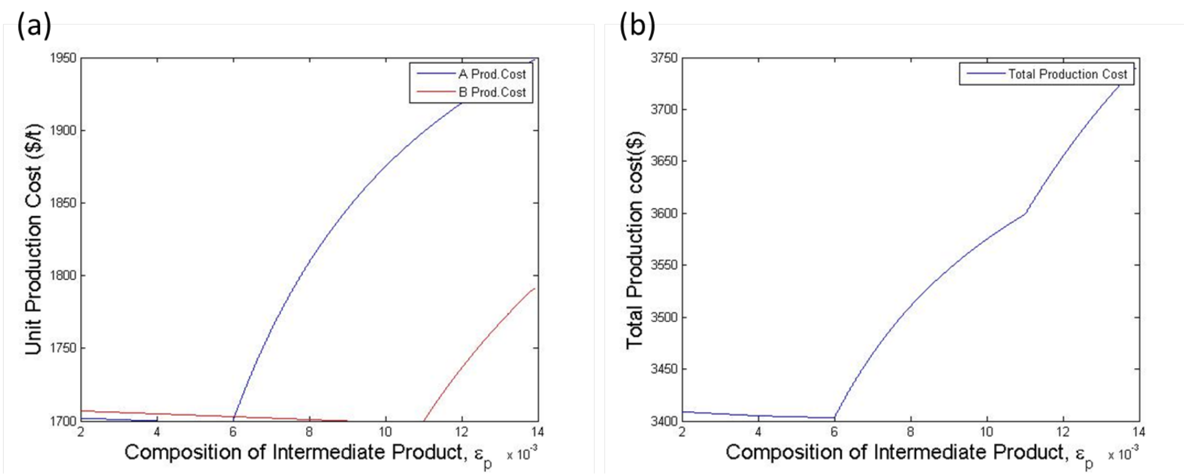


Figure 5-6. Unit production cost of Product A and Product B, (b) total production cost as the function of the composition of intermediate product of Example 3.

If the range of scrap composition is wide enough to cover the range of specifications of both products as Example 3, the optimal composition of the intermediate product is generally the minimum of max specifications of both product ($\min\left(e_A^U, e_B^U\right)$) as we have seen from the example in Figure 5-6 .

Since using primary aluminum and alloying elements is more expensive than using scrap material, the solution tends to minimize the usage of those two raw materials. In this system, there is the only one intermediate product. Thus, its composition can be same with the specification of only one of two products. Given this limitation, making the intermediate product with the minimum of max specifications is the cheapest solution. When the composition of the intermediate product is the minimum of max specifications, it can produce one product with higher purity at the lowest possible cost and produce the other one with the addition of alloying element. Let's assume that the composition of the intermediate product is the maximum of max specification, then one product can be produced using only intermediate product. Producing the other product requires the addition of primary aluminum. As we have seen earlier, the dilution with primary aluminum is much more expensive than concentrating with alloying element in the composition range of aluminum alloy. As a result, having the intermediate product with the maximum of max specifications has higher overall production cost than having the intermediate product with the minimum of max specifications.

5.2.2.1 Compositional range of scrap relative to Product Specification

So far, we assumed that the range of compositions of scrap materials cover the specification of products. ($e_A^U, e_A^L, e_B^U, e_B^L \in [e_1, e_2]$). For aluminum producers, the desirable scrap is the one with similar composition to the product portfolio or a high purity alloy. However, sourcing scrap materials relies on the condition of scrap market. Depending on the market situation, desirable scrap materials may be unavailable. This section studies how the compositions of scrap materials relative to products composition affect the solution space and consequently, the optimal solution.

Table 5-4. Scrap composition data used in Example 4-7.

	Example 4	Example 5	Example 6	Example 7
Scrap 1 composition(e_1)	1.2%	1.0%	0.1%	0.1%
Scrap 2 composition (e_2)	1.5%	1.4%	0.3%	0.5%

Let's consider the following examples in Table 5-4. All other parameters other than the compositions of scrap remain same as Example 3 in Table 5-3. In Example 4, both scrap materials have higher composition than the specification of two products. As shown in Figure 5-7(a), then the optimal composition of the intermediate product will be the composition of

Scrap 1. Since the composition of Scrap 1 is relatively close to the product specifications, increasing the amount of Scrap 2 used in the intermediate product only leads to using more primary aluminum in the second stage blending process. Therefore, total production cost function monotonically increases within the given range of the potential composition of intermediate product. The compositional range of blend of scrap ($[e_1, e_2]$) share the part of the specification of Product B. However, the minimum possible composition of the intermediate product is 1.0% in terms of alloying element content which is the composition of Scrap 1. This value is still higher than the minimum of max specification (0.6%). Figure 5-7(b) presents the total production cost graph as the function of the composition of the intermediate product. It should be noted that a knee is formed when the composition of the intermediate product is 1.1%. When $\varepsilon_p \in [0.01, 0.011]$, Product B can be fully made of using only the intermediate product without addition of alloying element or primary aluminum. Within this range, the increase in the total production cost is solely attributed to the addition of primary aluminum usage in Product A. When the composition of the intermediate product is higher than 1.1%, primary aluminum must be used to produce Product B as well. Total production cost increases much quickly with increase of composition of the intermediate product within this range than when $\varepsilon_p \in [0.01, 0.011]$.

On the contrary, Example 6 consists of two scrap materials which are cleaner than product specifications. In this example, the optimal composition of intermediate product will be the composition of Scrap2. Any blends of two scrap materials require addition of alloying element to satisfy the specifications of final products. The intermediate product made of only Scrap 2 requires less addition of alloying element in the re-melting stage. In Example 6, the compositional range of scrap materials contains the part of specification of Product A. When $\varepsilon_p \in [0.001, 0.004]$, alloying element must be used in both Product A and B. Within this compositional range, using more Scrap 2 in the intermediate product reduces the amount of alloying element usage in Product A and B. However, when the composition of the intermediate product is higher than 0.4%, Product A does not require any adjustment using alloying element. Thus, using more Scrap 2 in the intermediate product is only beneficial to Product B.

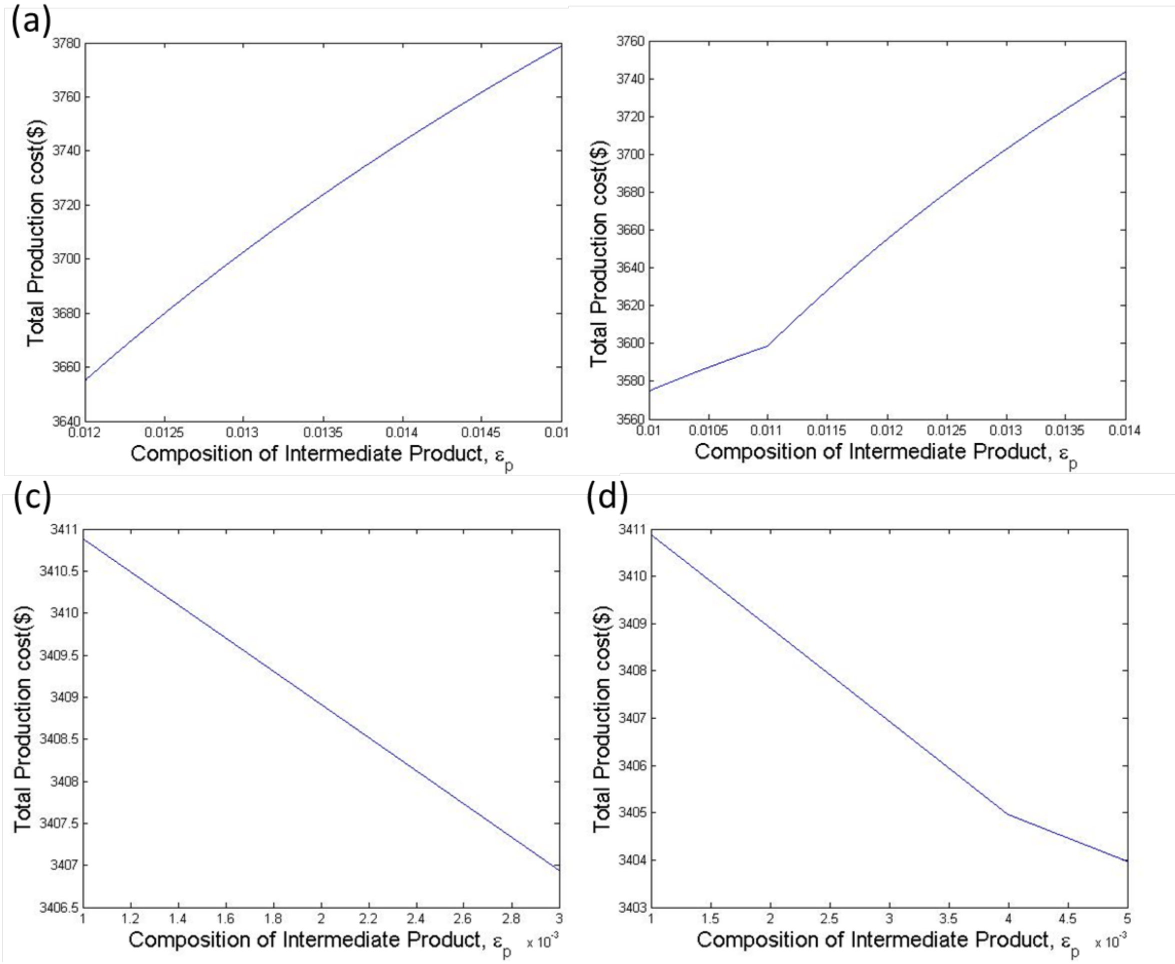


Figure 5-7. Total production cost curve of (a) Example 4 (b) Example 5 (c) Example 6 and (d) Example 7.

In the example of the previous section, the total production cost function is expressed as the piecewise nonlinear function. The four examples demonstrated in this section are different from the examples in the previous section. The feasible space in terms of the composition of the intermediate product is more limited than the example in the previous section. Therefore, the total production cost function in the examples of this section contains only the some intervals of the piecewise nonlinear function in the previous section. Also, in the previous section, we have identified that the minimum of the total cost production cost exist where the composition of the intermediate product is equal to the minimum of max specification. When the feasible range does not contain this point, the total production cost function either monotonically increases or decreases depending on the relative location of the compositional range of scrap to the minimum of max specification. In such cases, the optimal solution is to have intermediate products made of only one of scrap materials rather than blends of them.

Example 6 and 7 are certainly more desirable situation than Example 4 and 5 for aluminum producers because adding alloying elements is cheaper than diluting with primary aluminum as we have seen earlier. Also the total production cost in Example 6 and 7 are relatively lower than Example 4 and 5 as shown in Figure 5-7. However, the situation like Example 6 or 7 may not be common. One possible case could be using wrought alloy scrap materials into production of cast alloy product. Since wrought alloy has much low alloying contents than the cast alloys, it is easy to incorporate wrought scrap in the production of cast product.

5.2.2.2 Relative price of scrap materials

As mentioned earlier, the price of scrap can be determined by its purity. In the previous section, all examples are when two scrap materials have the same price. However, often scrap of relatively more pure alloy (more aluminum than alloying element) is more expensive than scrap of less pure alloy. Let's consider the following example in Table 5-5.

Table 5-5. Data of product specification and demand, scrap composition, and price of raw materials used in Example 8.

	Example 8
Scrap 1 composition(e_1)	0.223%
Scrap 2 composition (e_2)	1.618%
Scrap 1 price (c_1)	\$1937/t
Scrap 2 price (c_2)	\$1740/t
Primary price (c_3)	\$2137/t
Alloying element price (c_4)	\$3769/t
Product A min specification (e_A^L)	0.209%
Product A max specification (e_A^U)	0.329%
Product B min specification (e_B^L)	0.693%
Product B max specification (e_B^U)	1.163%
Product A Demand (D_A)	1
Product B Demand (D_B)	1

In Example 8, Scrap 1 has much higher price than Scrap 2. In this case, the price of intermediate product is different depending on the blending ratios. The cost of unit weight of the intermediate product is defined as Eq (88). Since Scrap 2 is much cheaper than Scrap1, the more Scrap 2 is used, the higher the composition of intermediate product, the cheaper the intermediate product that can be produced.

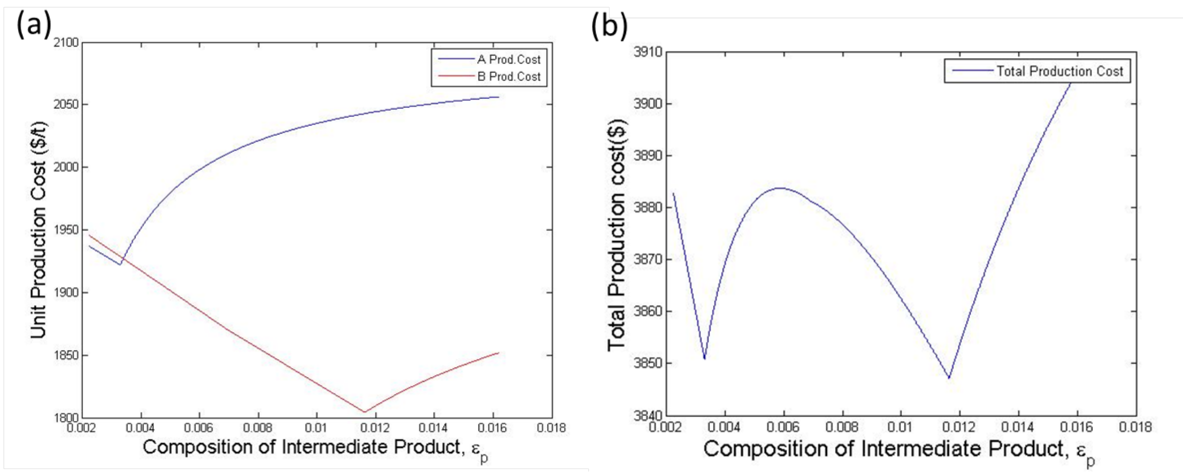


Figure 5-8. (a) Production cost of each product and (b) Total production cost as the function of the composition of the intermediate product of Example 9.

Figure 5-8 represents the cost function of two products and the total production cost as the function of the composition of the intermediate product. As shown in Figure 5-8(b), the total production cost is minimum at $\varepsilon_p = 1.163\%$. Thus, the optimal composition of the intermediate product is not the minimum of max specifications but the maximum of max specifications. This is different from the case where the prices of two scrap materials are the same or similar enough. When the composition of the intermediate product is higher the minimum of max specifications, which is Product A max specification in this example, the cost of Product A becomes significantly increased. Meanwhile, within the range, $\varepsilon_p \in [e_A^U, e_B^U]$, the unit production cost of Product B continuously decreases as the composition of the intermediate product increases. In other words, within this range, the slope of production cost of Product A is positive whereas the slope of production cost of Product B is negative as presented in Figure 5-8(a). Toward $\varepsilon_p = e_B^U$, the slope of production cost of Product A gradually saturated and the slope of the production cost of Product B also decreases at $\varepsilon_p = e_B^L$ but almost remains same.

This difference in the slope of production cost of two products results in the total production cost function has bell shape curve within the range of $\varepsilon_p \in [e_A^U, e_B^U]$. As a result, to make the intermediate product with $\varepsilon_p = e_B^U$ by using more Scrap 2, is cheaper even though this requires using more primary aluminum in Product A. Scrap 2 is cheap enough to compensate that penalty to Product A.

5.2.3 Problem with availability constraints (P2)

In the previous section, it is assumed that there is no availability constraint of raw materials. This allows us to simplify the problem by expressing objective function as a function of one variable, the composition of the intermediate product (ε_p) . The objective function can be easily visualized as a function of the composition of the intermediate product. However, it is common that the amount of available scrap materials is limited. This situation is particularly true for relatively high quality scrap materials. This section explores the situation with the limited amount of the relatively pure scrap (Scrap 1) to understand how the availability constraints of raw materials impact on the blending behaviors in intermediate pools. In this section, we use slightly different formulation by introducing a new variable, $x_{i,t}$, the actual amount of raw material i in the final product t instead of flow variables between nodes.

(P2)

Objective Function

$$\min c_1(x_{1A} + x_{1B}) + c_2(x_{2A} + x_{2B}) + c_3(x_{3A} + x_{3B}) + c_4(x_{4A} + x_{4B}) \qquad (103)$$

Subject to

$$x_{1A} + x_{1B} \leq A_1 \qquad (104)$$

$$x_{2A} + x_{2B} \leq A_2 \qquad (105)$$

$$x_{3A} + x_{3B} \leq A_3 \qquad (106)$$

$$x_{4A} + x_{4B} \leq A_4 \qquad (107)$$

$$\frac{x_{2A}}{x_{1A}} = \frac{x_{2B}}{x_{1B}} \tag{108}$$

$$x_{1A} + x_{2A} + x_{3A} + x_{4A} = D_A \tag{109}$$

$$x_{1B} + x_{2B} + x_{3B} + x_{4B} = D_B \tag{110}$$

$$e_1x_{1A} + e_2x_{2A} + e_3x_{3A} + e_4x_{4A} \; = e_AD_A \tag{111}$$

$$e_1x_{1B} + e_2x_{2B} + e_3x_{3B} + e_4x_{4B} \; = e_BD_B \tag{112}$$

$$x_{1A},\ x_{1B}, x_{2A},\ x_{2B}, x_{3A},\ x_{3B},\ x_{4A},\ x_{4B} \ \geq 0 \tag{113}$$

In this formulation, there is no variable that explicitly represents flows into the pool and flow out of the pool such as Eq (80) in the section 5.2.1. However, the expression to describe the two-stage blending process is still required. In the second-stage of blending, there is only one intermediate product which is a blend of Scrap1 and Scrap2. The ratio of two scrap materials must be equal in two final products, Product A and Product B. This can be formulated by Eq (108).

In this section, it is assumed that the amount of Scrap 1 is less than the optimal amount of Scrap 1 when there is no availability constraint. In other words, the constraint in (104) becomes binding. This assumption allows us to change the inequality constraint (104) to the equality constraint (114).

$$x_{1A} + x_{1B} = A_1 \tag{114}$$

Also let the blending ratio of scrap 1 and scrap 2 be k

$$\frac{x_{2A}}{x_{1A}} = \frac{x_{2B}}{x_{1B}} = k \tag{115}$$

Then other variables can be expressed using two variables, $k \; and \; x_{1A}$, as

$$x_{1B} = A_1 - x_{1A} \tag{116}$$

$$x_{2A} = kx_{1A} \tag{117}$$

$$x_{2B} = k(A_1 - x_{1A}) \tag{118}$$

$$x_{3A} = \frac{D_A(e_4 - e_A) + (e_1 - e_4)x_{1A} + (e_2 - e_4)kx_{1A}}{e_4 - e_3} \quad (119)$$

$$x_{3B} = \frac{D_B(e_4 - e_B) + (e_1 - e_4)(A_1 - x_{1A}) + (e_2 - e_4)(A_1 - x_{1A})k}{e_4 - e_3} \quad (120)$$

$$x_{4A} = \frac{D_A(e_A - e_3) + (e_3 - e_1)x_{1A} + (e_3 - e_2)kx_{1A}}{e_4 - e_3} \quad (121)$$

$$x_{4B} = \frac{D_B(e_B - e_3) + (e_3 - e_1)(A_1 - x_{1A}) + (e_3 - e_2)(A_1 - x_{1A})k}{e_4 - e_3} \quad (122)$$

Since x_{1A}, x_{1B}, x_{2A}, x_{2B}, x_{3A}, x_{3B}, x_{4A}, x_{4B} ≥ 0, Eq (116) – (122) must also satisfy non-negativity as well. Using the relationships in Eq (116) – (122) and the characteristics of non-negativity, the feasible region can be plotted with two variables, k and x_{1A}. Let's consider the following simple example in Table 5-6.

Table 5-6 An example of operational parameters

	Example 9
Scrap 1 composition(e_1)	0.1%
Scrap 2 composition (e_2)	1.9%
Scrap 1 price(c_1)	\$1700/t
Scrap 2 price(c_2)	\$1700/t
Scrap 1 Availability (A_1)	40t
Scrap 2 Availability (A_2)	40t
Primary price (c_3)	\$2137/t
Alloying element price (c_4)	\$2689/t
Product A specification (e_A)	0.4%
Product B specification (e_B)	1.1%
Product A demand (D_A)	40t
Product B demand (D_B)	40t

Figure 5-9 shows the feasible region and the contour of the objective function when there is no availability constraint and when the availability of Scrap 1 is limited. The areas colored by blue and orange represent the feasible region of making Product A and Product B, respectively. Since the specifications and demands for both products must be satisfied, the intersection of two the colored areas is the feasible region for P2. The contours of the objective function are depicted by the red line. It should be noted that the contour of the objective function is parallel to the X-axis. This is because the objective function is not a function of the variable, x_{1A}. The objective function in Eq (103) can be rewritten using the relationships of (116)-(122) as

$$c_1 A_1 + c_2 k A_1$$

$$+c_3 \left(\frac{D_A(e_4 - e_A) + D_B(e_4 - e_B) + (e_1 - e_4)A_1 + (e_2 - e_4)A_1 k}{e_4 - e_3} \right)$$

$$+c_4 \left(\frac{D_A(e_A - e_3) + D_B(e_B - e_3) + (e_3 - e_1)A_1 + (e_3 - e_2)A_1 k}{e_4 - e_3} \right) \qquad (123)$$

Therefore, the objective function is a function of only k, the blending ratio of Scrap 1 and Scrap 2 in both products. The value of the objective function decreases as the k increases. Since the total amount of Scrap 1 used is fixed as A_1, the higher k means the higher relative scrap amount in finished alloy products which leads to the cheaper total production cost. Therefore, the optimal point will be form at the highest k among the intersection of blue and orange regions.

If there is no availability constraint, the optimal composition of the intermediate product (ε_p^*) is 0.4% which is Product A specification, similarly as in the previous section. The amount of Scrap1 and Scrap2 used in the intermediate product is 66.43 t and 13.29 t, respectively. The resulting blending ratio of Scrap 2 to Scrap 1, which is defined as k, is 0.2 As shown in Figure 5-9(a), the highest corner of the intersection of blue and orange regions is located where $k = 0.2$.

When there is an availability constraint ($A_1 = 40$), the optimal blending ratio of Scrap 2 to Scrap 1 is no longer 0.2 as shown in Figure 5-9(b). The top corner of the feasible region is formed at $k = 0.54$. The composition of the intermediate product with $k = 0.54$ is 0.733%. This value is neither the specification of Product A nor the specification of Product B. Although $k = 0.2$ is still a feasible point, the objective value at $k = 0.2$ is much higher than $k = 0.54$ as shown in Figure 5-9(b). This result clearly shows that the optimal blending behavior in the pool when there is only limited amount of scrap is different from when it is not. When the amount of relatively pure alloy scrap is limited, the reprocessing stage operator has two options: producing the intermediate product with the composition same as Product A ($\varepsilon_p = e_A$) with only limited amount, or producing relatively less pure intermediate product with the higher volume. The former option allows production of Product A at a cheaper price. However, the amount of intermediate product is not enough. Therefore, using primary aluminum to produce Product B is necessary to satisfy its demand. The latter certainly limits the maximum amount of the intermediate product that can be used in Product A. However, it can reduce the cost of producing Product B by reducing use of primary aluminum in Product B. In other words, having the intermediate products with the composition same as Product A (pure alloy product) is no longer the way to minimize the usage of primary aluminum, unlike the case without the availability constraint.

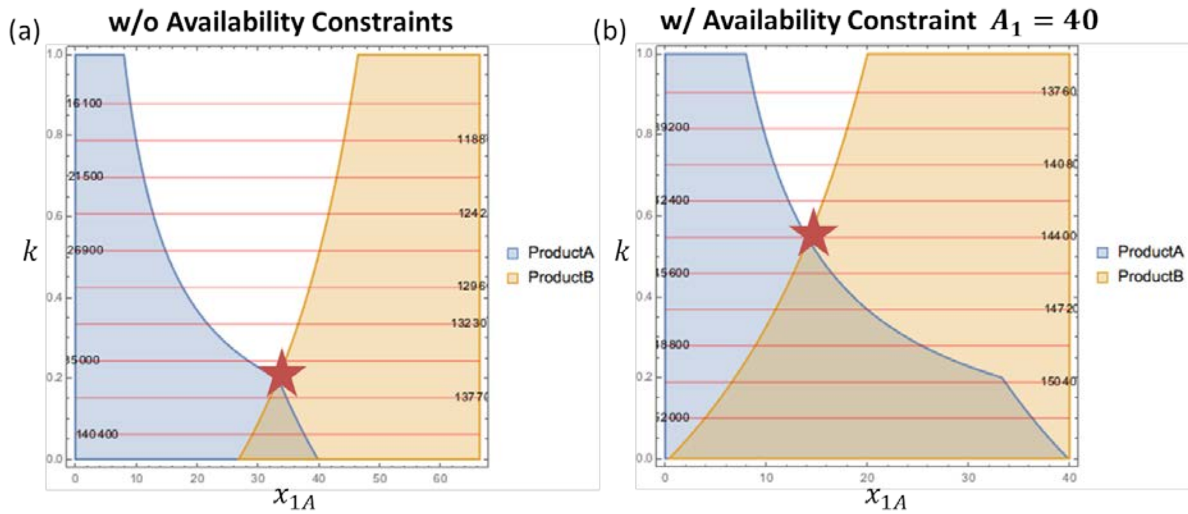


Figure 5-9. Feasible regions and contour of objective function of Example 9 (a) when there is no availability constraint of scrap materials (b) when the available amount of Scrap1 is limited ($A_1 = 40$). Red star represents the feasible point that minimizes the objective function.

The example presented in this section suggests that the optimal intermediate compositions can be very different depending on availability of raw materials even though all other conditions, such as product specifications and the compositions of scrap the same. This reflects the actual operational environment. Although the specification of final products and sources of raw materials are determined, the inventory of raw materials varies in daily basis. Consequently, the results in this section implies us the importance of designing the intermediate products using integrated production planning.

5.3 Two-stage recycling operation with product demand uncertainty (P3)

In two-stage aluminum recycling operations, decisions can be largely divided into two categories: decisions for the reprocessing stage and decisions for the re-melting stage. As discussed earlier, in integrated production planning, these two types of decisions are made simultaneously. Although decisions for the re-melting furnace and the reprocessing furnaces are made together, two-stage aluminum recycler operator often face issues due to demand uncertainty. This is because the demands for final products in re-melting stages can often change frequently. Intermediate products must be produced before a batch of final products which the intermediate products are targeted to be used start. Consequently, there is always time gap of operations for the reprocessing stage and the re-melting stage. If the batch list for the re-melting stage suddenly changes between this time period, it is impossible to reflect the demand information of final product in the production of intermediate products in the reprocessing furnace. Even if the changes in demand for final products are informed before starting a batch of intermediate product, solving the pooling problem require longer time than solving the linear programming. As a result, designing a new intermediate product may not be possible given the limited time. In general, unlike changing batch list in the reprocessing furnace, the changing batch list of the reprocessing furnace is more inflexible in integrated production planning. Therefore, within short time period, only possible options for a reprocessing furnace operator would be to adjust the amount of intermediate products produced. However, when the intermediate products are designed in advance, it is possible to take the demand uncertainty into account.

This section will investigate the differences of the optimal solutions of the integrated production planning model between deterministic approach and stochastic approach. The major difference between the deterministic approach and the stochastic approach is how to consider the different scenarios. In a deterministic approach, the optimal solution is determined by solving the model with the average of input parameters across the different scenarios. In other words, the model is solved for one scenario that consists of the expected value of the input parameters. In the other hand, a recourse approach finds the solution that satisfies all scenarios and determines the optimal that makes the expected value of objective function the minimum. To understand the difference of two approaches, let's consider the simple case in 5.2.1 where there is no availability constraint but two difference scenarios in terms of final products.

(P3)

Objective function

$$
\begin{aligned}
minimize\ \alpha_1\big(c_1 f_{1,p} + c_2 f_{2,p,1} + c_3(f_{3,A,1} + f_{3,B,1}) + c_4(f_{4,A,1} + f_{4,B,1})\big) \\
+\ \alpha_2\big(c_1 f_{1,p,2} + c_2 f_{2,p,2} + c_3(f_{3,A,2} + f_{3,B,2}) \\
+ c_4(f_{4,A,2} + f_{4,B,2})\big)
\end{aligned}
\tag{124}
$$

Subject to

$$f_{1,p,1} + f_{2,p,1} = f_{p,A,1} + f_{p,B,1} \tag{125}$$

$$e_1 f_{1,p,1} + e_2 f_{2,p,1} = \varepsilon_p\big(f_{1,p,1} + f_{2,p,1}\big) \tag{126}$$

$$f_{p,A} + f_{3,A,1} + f_{4,A,1} = 1 \tag{127}$$

$$f_{p,B} + f_{3,B,1} + f_{4,B,1} = 1 \tag{128}$$

$$\varepsilon_p f_{p,A,1} + e_3 f_{3,A,1} + e_4 f_{4,A,1} \leq e_{A,1}^U \tag{129}$$

$$\varepsilon_p f_{p,A,1} + e_3 f_{3,A,1} + e_4 f_{4,A,1} \geq e_{A,1}^L \tag{130}$$

$$\varepsilon_p f_{p,B,1} + e_3 f_{3,B,1} + e_4 f_{4,B,1} \leq e_{B,1}^U \tag{131}$$

$$\varepsilon_p f_{p,B,1} + e_3 f_{3,B,1} + e_4 f_{4,B,1} \geq e_{B,1}^L \tag{132}$$

$$f_{1,p,2} + f_{2,p,2} = f_{p,A,2} + f_{p,B,2} \tag{133}$$

$$e_1 f_{1,p,1} + e_2 f_{2,p,1} = \varepsilon_p\big(f_{1,p,1} + f_{2,p,1}\big) \tag{134}$$

$$f_{p,A,2} + f_{3,A,2} + f_{4,A,2} = 1 \tag{135}$$

$$f_{p,B,2} + f_{3,B,2} + f_{4,B,2} = 1 \tag{136}$$

$$\varepsilon_p f_{p,A,2} + e_3 f_{3,A,2} + e_4 f_{4,A,2} \leq e^U_{A,2} \quad (137)$$

$$\varepsilon_p f_{p,A,2} + e_3 f_{3,A,2} + e_4 f_{4,A,2} \geq e^L_{A,2} \quad (138)$$

$$\varepsilon_p f_{p,B,2} + e_3 f_{3,B,2} + e_4 f_{4,B,2} \leq e^U_{B,2} \quad (139)$$

$$\varepsilon_p f_{p,B,2} + e_3 f_{3,B,2} + e_4 f_{4,B,2} \geq e^L_{B,2} \quad (140)$$

$$f_{1,p,1}, f_{2,p,1}, f_{1,p,2}, f_{2,p,2} \geq 0 \quad (141)$$

$$f_{p,A,1}, f_{p,B,1}, f_{p,A,2}, f_{p,B,2} \geq 0 \quad (142)$$

$$f_{3,A,1}, f_{3,B,1}, f_{3,A,2}, f_{3,B,2}, f_{4,A,1}, f_{4,B,1}, f_{4,A,2}, f_{4,B,2} \geq 0 \quad (143)$$

It should be noted that all the constraints in 5.2.1 appear twice for Scenario 1 and Scenario 2. Eq (125)- (132) are associated with Scenario 1 and Eq (133)-(140) are associated with Scenario 2. Also the objective function Eq (124) consists of two objective functions for Scenario 1 and Scenario 2 weighted by their probability α_1 and α_2. However, in the deterministic formulation, the formulation will be exactly identical with Eq (78)-(86). However, the parameters in the right-hand side of constraint (83)-(86) will be replaced the expected value of parameters as

$$\varepsilon_p f_{p,A,1} + e_3 f_{3,A,1} + e_4 f_{4,A,1} \leq \alpha_1 e^U_{A,1} + \alpha_2 e^U_{A,2} \quad (144)$$

$$\varepsilon_p f_{p,A,1} + e_3 f_{3,A,1} + e_4 f_{4,A,1} \geq \alpha_1 e^L_{A,1} + \alpha_2 e^L_{A,2} \quad (145)$$

$$\varepsilon_p f_{p,B,1} + e_3 f_{3,B,1} + e_4 f_{4,B,1} \leq \alpha_1 e^U_{B,1} + \alpha_2 e^U_{B,2} \quad (146)$$

$$\varepsilon_p f_{p,B,1} + e_3 f_{3,B,1} + e_4 f_{4,B,1} \geq \alpha_1 e^L_{B,1} + \alpha_2 e^L_{B,2} \quad (147)$$

Now, let's look at the characteristics of the optimal composition of the intermediate products determined by two different approaches, stochastic and deterministic approach with the following Example in Table 5-7.

Table 5-7 Data of input parameters used in Example 9

Example 9	Scenario1	Scenario2
Scrap 1 composition(e_1)	0.2%	
Scrap 2 composition (e_2)	1.5%	
Scrap 1 price (c_1)	$1700/t	
Scrap 2 price (c_2)	$1700/t	
Primary price (c_3)	$2137/t	
Alloying element price (c_4)	$2689/t	
Product A Demand (D_A)	1	
Product B Demand (D_B)	1	
Product A min specification (e_A^L)	0.3%	0.5%
Product A max specification (e_A^U)	0.45%	0.65%
Product B min specification (e_B^L)	0.9%	1.1%
Product B max specification (e_B^U)	1.2%	1.3%
Probability of scenario(α)	50%	50%

The graphs Figure 5-10(a) and (b) are the objective value as the function of the composition of the intermediate product in stochastic integrated production planning model and deterministic integrated production planning model, respectively. In Figure 5-10(a), the blue line represents the total production cost in Scenario 1 and the red line represents the total production cost in Scenario 2. The green line is the weighted average of the total production costs of Scenario 1 and Scenario 2. The minimum of the green line exists at $\varepsilon_p = 0.45\%$. In the other hand, the optimal composition of the intermediate product obtained from deterministic approach is 0.55%, as shown in Figure 5-10(b). The optimal solutions obtained from two models are clearly different.

As we have seen in earlier sections, the penalty of adding primary aluminum to reduce the composition of the alloying element is more expensive than the penalty of adding alloying element to increase the composition when there is no availability constraint. Because of this asymmetric cost structure in aluminum alloys, the optimal composition of the intermediate product tends to be the minimum of the max specifications. Similar behavior of the composition of the intermediate product is found when the uncertainty is considered. In the recourse model, the optimal composition of the intermediate product is equivalent to the minimum of max specifications across all scenarios.

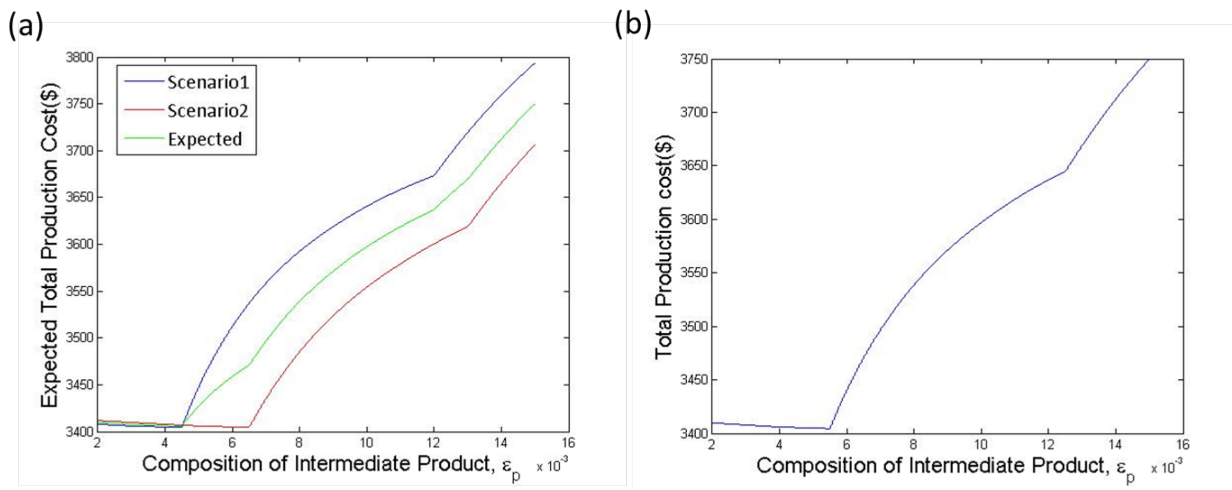


Figure 5-10. Objective value as the function of composition of the intermediate product in (a) stochastic model and (b) deterministic model.

However, in the deterministic model, the product specifications of different scenarios are averaged out. Therefore, the optimal composition obtained from the deterministic model is neither the minimum of Scenario 1 nor the minimum of Scenario 2, but the average max specification of Product A, 0.55%. The actual production cost can be read from Figure 5-10(a) when each scenario is realized. If Scenario 2 is realized, the total production cost of the deterministic model is slightly lower than the total production cost of the stochastic model. However, if Scenario1 is realized, the total production cost of the deterministic model is almost $100 higher than that of the stochastic model.

From this example, it is clear that the design of the intermediate product suggested by stochastic model is more robust than the intermediate product suggested by the deterministic model. Even though, in some scenarios, it may not offer the batch plan with the lowest production cost, it exhibits a much lower production cost when an unfavorable scenario is realized.

6 The value of the integrated production planning

Chapter 5 uses an analytical approach to understand the pooling problem in an aluminum recycling context and the optimal blending behaviors in intermediate pools. However, the analytical approach is only tractable for a simplified problem. This simplified problem with assumptions, such as relaxing constraints on raw materials availability, cannot fully capture the actual operational environment. The actual operational environment can be more complex. The inventory of raw materials can be constrained or the design of the reprocessing furnace may be different.

The goal of this chapter is to expand the findings in Chapter 5 to more complex systems. A simulated screening analysis is introduced to achieve this goal. Using this method allows us to observe the behaviors of systems under more complex conditions as well as to identify the conditions under which a system behaves in a previously observed way. Since we are interested in the optimal blending behavior of intermediate products and the conditions under which the benefit of integrated production planning can be significant, two values are chosen as response variables: the optimal composition of intermediate products and the production cost difference between independent production planning and integrated production planning. The significant parameters or key drivers of these two response variables are screened.

Finally, the value of the integrated production planning for two-stage recycling operations will be demonstrated at an industrial scale. Furthermore, the impacts of parameters identified as significant from the simulated screening analysis on the value of the integrated production planning are evaluated.

6.1 Identifying the significant parameters to determine optimal composition of intermediate products in integrated production planning

6.1.1 Optimal composition without constraints of scrap availability or capacity of reprocessing furnace

As we have seen in Chapter 5, the optimal composition of the intermediate products is highly dependent on the range of scrap composition relative to product specifications. This is because the feasible points in terms of the composition of the intermediate product are defined by the range of scrap composition. Depending on the composition of scrap relative to product specification, the geometry of the total production cost varies. For this reason, data obtained from simulations are divided into three sets. The first set is when the composition of Scrap 1, the one with lower alloying element, is higher than the minimum of max specifications. The second set is when the composition of Scrap 2, the one with higher alloying element, is less than the minimum of max specifications. Lastly, the third set is when the range of scrap compositions contains the minimum of max specifications. The criteria to partition data are summarized in Table 6-1.

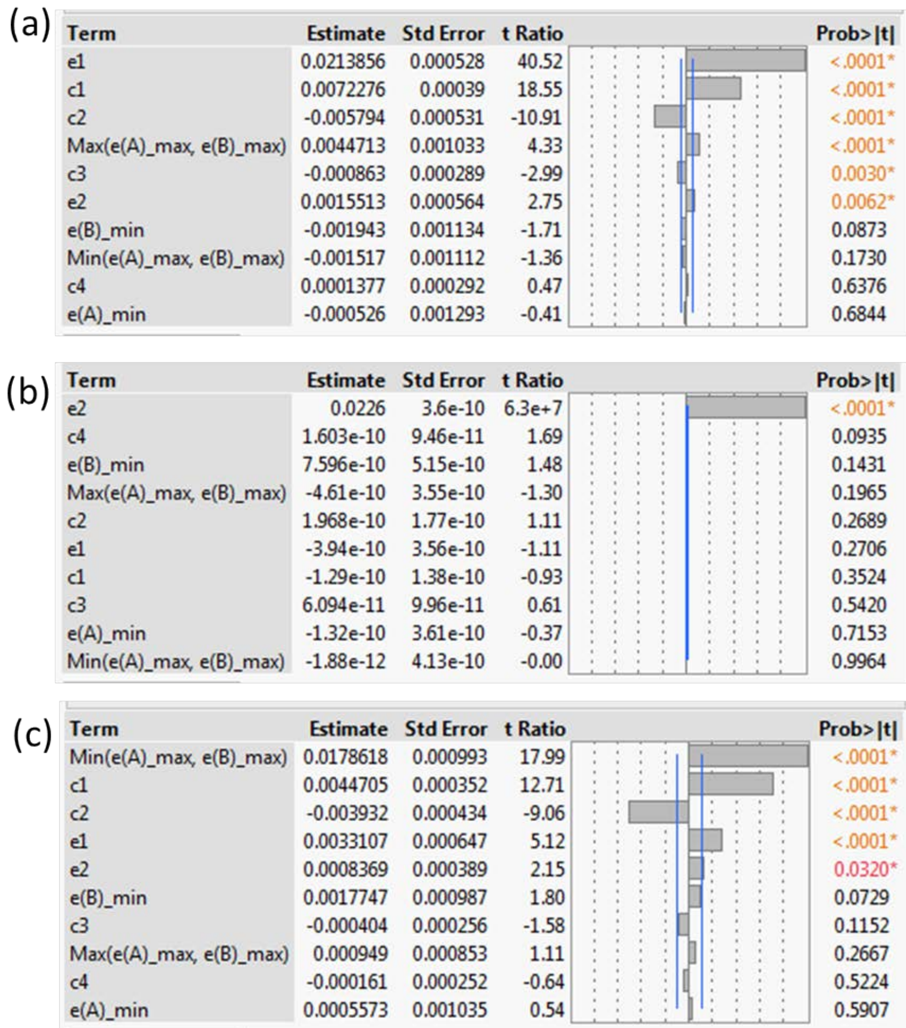
Table 6-1. Criteria to partition data from simulation results based on the composition of scrap relative to the specification of products.

Group1	$e_1 > \min(e_A^U, e_B^U)$
Group2	$e_2 < \min(e_A^U, e_B^U)$
Group3	$\min\left(e_A^U, e_B^U\right) \in [e_1, e_2]$

Figure 6-1 shows the results of regression analysis for data obtained from solving the optimization model 1000 times based on randomly sampled input parameters. Figure 6-1(a) is the report of the regression result for the first group of data in which the compositions of both scrap materials are higher than the minimum of max specifications of Product A and Product B. In this table, parameters are sorted in decreasing order of significance. In the right column of the table, the value of Prob>[t] means the probability of obtaining the estimated value of the parameters if the actual parameter value is zero. Thus, the smaller the Prob>|t|, the more significant the parameter and the less likely the actual parameter value is zero. The most influential parameter is Scrap 1 composition (e_1). This result is not surprising. As seen in the previous chapter, when the compositions of two scrap materials are higher than the minimum of max specifications, the total production cost increases monotonically with the composition of the intermediate product. Therefore, the minimum possible composition of the intermediate product, which is the composition of Scrap 1, becomes the optimal composition that minimizes the total production cost unless the price of Scrap 2 is significantly lower than the price of Scrap 1. Therefore, in this case, the optimal composition of the intermediate products is highly dependent on the composition of Scrap 1. The second influential parameter is the price of Scrap 1(c_1). In Chapter 5, we have seen that the price of Scrap 1 relative to Scrap 2 is high enough, the optimal solution is to blend more Scrap 2, resulting in higher optimal intermediate composition. When the price of Scrap 1 is high enough, then using Scrap 2 is cheap enough to compensate for the penalty for using primary aluminum in the second stage. The third influential parameter is the price of Scrap 2(c_2), and it is negatively correlated to the optimal composition of the intermediate product. It can be interpreted similarly to the price of Scrap 1. As the price of Scrap 2 increases, there is no incentive to use Scrap 2. As a result, the optimal composition decreases as the price of Scrap 2 increases. The fourth influential parameter is the maximum of max specifications of two products. Chapter 5 demonstrated that the total production cost significantly increases at two different points: when the intermediate composition is the minimum of max specifications and when the intermediate composition is the maximum of max specifications. Since the composition of Scrap 1 is higher than the minimum of max specifications, the first point is not within the feasible range. If the price difference of two scrap materials is not big, the optimal composition will be the minimum of the max specifications. However, if the price of Scrap 2 is significantly lower than that of Scrap 1, the optimal solution would be the maximum of the max specifications. The next significant parameter is the price of primary aluminum. The optimal composition of the intermediate product is negatively dependent on the price of primary aluminum. If the price of primary aluminum increases, the amount of primary aluminum used in the second-stage of the blending process should be minimized. As a result, the optimal composition of the intermediate product should decrease to reduce the amount of primary aluminum used.

Figure 6-1(b) shows the result of regression analysis for the second group of data, where the compositions of both scrap materials is less than the minimum of max specifications of two products. Only one parameter, the composition of Scrap 2(e_2), is identified as the significant parameter for the optimal composition of the intermediate product. This result is consistent with the analytic results in Section 5.2.2.1. The composition of Scrap 2 is relatively close to both products' specifications. As a result, making the intermediate product by using only Scrap 2 is a way to minimize the addition of alloying element in the second-stage, as discussed in Chapter 5.

Lastly, when the range of the compositions of scrap covers the minimum of max specifications of two products (group3), we found that the optimal composition of the intermediate product generally becomes the minimum of max specification unless the price of Scrap 2 is much lower than that of Scrap 1. As shown in Figure 6-1(c), the minimum of max specifications is identified as the most significant parameter followed by the price of Scrap 1 and Scrap 2. The second and the third most parameters are the prices of Scrap 1 and Scrap 2. The reason behind this result is similar to that of the first case, $e_1 > \min(e_A^U, e_B^U)$. In some cases, the price of Scrap 2 can be significantly lower than the price of Scrap 1. This situation can lead to different optimal compositions as we saw in Chapter 5. Stochastically speaking, however, the composition of the intermediate product is the most dependent on the minimum of max specifications of two products.



(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e1	0.0213856	0.000528	40.52	<.0001*
c1	0.0072276	0.00039	18.55	<.0001*
c2	-0.005794	0.000531	-10.91	<.0001*
Max(e(A)_max, e(B)_max)	0.0044713	0.001033	4.33	<.0001*
c3	-0.000863	0.000289	-2.99	0.0030*
e2	0.0015513	0.000564	2.75	0.0062*
e(B)_min	-0.001943	0.001134	-1.71	0.0873
Min(e(A)_max, e(B)_max)	-0.001517	0.001112	-1.36	0.1730
c4	0.0001377	0.000292	0.47	0.6376
e(A)_min	-0.000526	0.001293	-0.41	0.6844

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.0226	3.6e-10	6.3e+7	<.0001*
c4	1.603e-10	9.46e-11	1.69	0.0935
e(B)_min	7.596e-10	5.15e-10	1.48	0.1431
Max(e(A)_max, e(B)_max)	-4.61e-10	3.55e-10	-1.30	0.1965
c2	1.968e-10	1.77e-10	1.11	0.2689
e1	-3.94e-10	3.56e-10	-1.11	0.2706
c1	-1.29e-10	1.38e-10	-0.93	0.3524
c3	6.094e-11	9.96e-11	0.61	0.5420
e(A)_min	-1.32e-10	3.61e-10	-0.37	0.7153
Min(e(A)_max, e(B)_max)	-1.88e-12	4.13e-10	-0.00	0.9964

(c)

Term	Estimate	Std Error	t Ratio	Prob>|t|
Min(e(A)_max, e(B)_max)	0.0178618	0.000993	17.99	<.0001*
c1	0.0044705	0.000352	12.71	<.0001*
c2	-0.003932	0.000434	-9.06	<.0001*
e1	0.0033107	0.000647	5.12	<.0001*
e2	0.0008369	0.000389	2.15	0.0320*
e(B)_min	0.0017747	0.000987	1.80	0.0729
c3	-0.000404	0.000256	-1.58	0.1152
Max(e(A)_max, e(B)_max)	0.000949	0.000853	1.11	0.2667
c4	-0.000161	0.000252	-0.64	0.5224
e(A)_min	0.0005573	0.001035	0.54	0.5907

Figure 6-1. Sorted parameter estimates from regression analysis for the optimal composition of the intermediate product when there is no constraint of scrap availability or the capacity of the reprocessing furnace for three different cases: (a) $e_1 > min(e_A^U, e_B^U)$ (b) $e_2 < min(e_A^U, e_B^U)$ and (c) $min(e_A^U, e_B^U) \in [e_1, e_2]$. R-squared values for each regression model are (a) 0.8812 (b) 1 and (c) 0.8460.

6.1.2 Optimal composition with the constraints of scrap availability and capacity of reprocessing furnace

The regression analysis results in the previous section are consistent with the results of the analytic approach in Chapter 5. Therefore, we can also apply this approach for more complicated operational situations to understand blending behaviors. Two major constraints in integrated production planning as well as independent production planning are the availability of raw materials and the capacity limit of the reprocessing furnace. Depending on the situation of the scrap market, or daily inventory, the amount of available scrap can often be limited. Also the design of reprocessing furnaces may vary. In this section, the scope of the regression analysis in the previous section is extended to include these parameters, and the impact of them on the optimal composition of the intermediate products will be investigated.

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.0076363	0.00041	18.62	<.0001*
A1	-0.004418	0.000274	-16.14	<.0001*
e1	0.009242	0.000696	13.28	<.0001*
A2	0.0035142	0.000285	12.34	<.0001*
c1	0.0033323	0.00076	4.38	<.0001*
Min(e(A)_max, e(B)_max)	0.0040497	0.000992	4.08	<.0001*
c2	-0.002593	0.000989	-2.62	0.0091*
Max(e(A)_max, e(B)_max)	0.0016018	0.000908	1.76	0.0784
c3	-0.000584	0.00048	-1.22	0.2245
e(A)_min	0.0004255	0.000951	0.45	0.6546
e(B)_min	-0.000349	0.001058	-0.33	0.7420
c4	2.2433e-5	0.000278	0.08	0.9358

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.0128393	0.000633	20.30	<.0001*
A2	0.0083615	0.000686	12.18	<.0001*
V	-0.006007	0.000556	-10.81	<.0001*
e1	0.0093256	0.001097	8.50	<.0001*
A1	-0.003299	0.000704	-4.68	<.0001*
e(A)_min	-0.002651	0.001491	-1.78	0.0767
c2	-0.001986	0.001537	-1.29	0.1976
Min(e(A)_max, e(B)_max)	0.0019278	0.00154	1.25	0.2118
c1	0.0014118	0.001185	1.19	0.2348
e(B)_min	0.0014246	0.00148	0.96	0.3368
c4	0.0002861	0.00042	0.68	0.4961
Max(e(A)_max, e(B)_max)	0.0006527	0.001357	0.48	0.6311
c3	0.0003435	0.000735	0.47	0.6405

Figure 6-2. Sorted parameter estimates from regression analysis of group 3 for the optimal composition of the intermediate product (a) when the constraint of scrap availability is considered and (b) when constraints for both scrap availability and capacity of the reprocessing furnace are considered. R-squared values for each regression model are (a) 0.8128 and (b) 0.8248.

The appearance of compositions of two scrap materials as the significant parameters in Figure 6-2 is not a surprising result since the feasible composition of the intermediate product is bounded by the two scrap compositions. When the availability of scrap is constrained, Scrap 1 availability is one of the most significant parameters for determining the optimal composition of the intermediate product. The optimal composition increases as the availability of Scrap 1 decreases. This negative dependency between the optimal composition of the intermediate product and Scrap 1 availability is also consistent with the analytic approach in Chapter 5. In the previous chapter, the optimal composition increases when the amount of Scrap 1 is limited. Making an intermediate product whose composition is equivalent to the minimum of max specifications is also a feasible solution. However, only a limited amount of the intermediate product can be produced to satisfy this composition due to the limited availability of Scrap 1. This feasible solution actually leads to more primary usage in the re-melting stage. As a result, the optimal composition of the intermediate product becomes higher than the min of the max specifications.

The capacity of the reprocessing furnace plays the same role as the availability of scrap materials. If the capacity of the reprocessing furnace is smaller than the theoretical maximum amount of the intermediate product that can be incorporated in two alloy products, primary usage is inevitable to meet the demand for final products even if enough scrap is available. As a result, even if the composition of the intermediate product is higher than the minimum of max specifications, it can be diluted by the primary aluminum used in the re-melting stage. This logic explains the negative dependency of the optimal composition on the capacity of the reprocessing furnace.

6.2 Identifying the significant parameters that maximize the value of integrated production planning over independent production planning

In the previous section, the interaction between the optimal blending behavior of intermediate product and input parameters is investigated. Another important research question in this thesis is to identify the operational conditions that make integrated production planning most beneficial. A similar approach used in the previous section can be used to determine when integrated production planning is more beneficial than independent production planning. Instead of the optimal composition of the intermediate product, the production cost difference between independent production planning (fixed composition model) and integrated production planning is chosen as the response variable for regression analysis.

Figure 6-3 shows the results of regression analysis for the production cost difference of two models. Figure 6-3 includes regression analysis results only from scrap compositional range group 3. Other regression analysis results from group1 and 2 can be found in the Appendix. Unlike the regression for the optimal composition of the intermediate product, the R-squared values are low. However, the goal of this study is to determine the relative importance or the significance of parameters in the benefit of integrated production planning rather than developing a regression model to predict the accurate value of the benefit. Regardless of the R-squared values, the significant coefficients still represents the information about how changes in the parameters are associated with changes in the response value which is the difference of performances of two models. Therefore, we can still draw conclusions about the characteristics of parameters that make the integrated production planning more beneficial. The directionality (positive or negative signs) for the estimates of parameters is particularly of interest.

Figure 6-3(a) shows the results when there is no constraint of either availability of scrap or capacity of the reprocessing furnace. The first most significant parameter is the minimum of max specifications. As the minimum of max specifications increases, the benefit of using integrated production planning model over the independent production planning model decreases. The benefit of integrated production planning, compared to independent production planning, is to design the intermediate product that minimizes the total production cost. In Chapter 5, the total production cost curve generally exhibits an asymmetric structure due to difference between slopes to primary aluminum side and alloying element side. If the predetermined composition in independent production planning is higher than the minimum of max specifications, the penalty of adjusting composition with primary aluminum is very sensitive to the minimum of max specifications. On the contrary, if the predetermined composition of the independent planning is less than the minimum of max specifications, changes in the penalty of adding alloying elements is minor with a change of the minimum of max specifications according to the slope of alloying element side in the composition-cost graph. Statistically, both cases can be present. However, the impact of the minimum of max specifications on the production cost difference in the former case is significant.

The second most significant parameter is the difference in scrap compositions. The feasible region of the composition of the intermediate product is defined by the range of scrap composition. Thus, when the scrap composition range becomes narrow, the pre-determined composition for independent production planning and the optimal composition is likely similar. This is because the solution space in terms of the composition of the intermediate products becomes very limited when two scrap materials are compositionally similar.

Figure 6-3(b) shows the result from the regression analysis of simulation results when the constraint of scrap availability is considered. This result is quite different from the result when there is no availability constraint. The availability of both Scrap 1 and Scrap 2 are positively correlated with the benefit of the integrated production planning. As studied in Chapter 5, when the amounts of scrap materials are limited, both of them tend to be used to satisfy the demand even though the resulting composition of the intermediate product does not exactly match to the final product specification. However, when the availability of both scrap materials is high, integrated production planning can take advantage of designing the intermediate products that are customized to the final products. The second significant parameter is the price of Scrap 1. The regression analysis result suggests that the value of integrated production planning has a negative dependency on the price of Scrap 1. In other words, the benefit of integrated production planning increases as the price of Scrap 1 decreases.

When both constraints of scrap availability and capacity of the reprocessing furnace are considered, the capacity of the furnace (V) is the most significant factor as shown in Figure 6-3(c). If the capacity of the reprocessing furnace is small, the maximum amount of intermediate products that can be used in the production of finished alloy products is limited by this capacity. Even if the composition of the intermediate products is exactly matched to final products' specifications, only limited amounts of intermediate products can be produced. Therefore, the impact of designing different intermediate products is not significant when a recycling facility has a limited reprocessing furnace capacity. Since there are other operational conditions such as product portfolio, scrap availability or scrap price, the impact of capacity may be different facility by facility. However, as the capacity of the reprocessing furnace becomes bigger, the potential benefit of integrated production planning can also be significant if all other constraints remain the same. In addition to the capacity of the reprocessing furnace, the availability of Scrap 2 is identified as the second most significant parameter.

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
Min of Max in Mn	-4.767696	0.783755	-6.08	<.0001*
e2-e1	1.3311164	0.324942	4.10	<.0001*
e1	2.5846297	0.643065	4.02	<.0001*
c2	-2.129212	0.783743	-2.72	0.0069*
Prod.B. Min	-1.98158	0.837241	-2.37	0.0184*
c3	0.7271749	0.379772	1.91	0.0562
Prod.A Min	1.228284	0.753539	1.63	0.1039
Max of Max in Mn	0.9940074	0.719225	1.38	0.1677
c1	0.213246	0.602502	0.35	0.7236
c4	0.0304536	0.220135	0.14	0.8900

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
A1	1.5317361	0.485667	3.15	0.0019*
c1	-2.558927	0.845031	-3.03	0.0029*
A2	1.4668672	0.511132	2.87	0.0047*
e1	-2.202679	0.90351	-2.44	0.0159*
e2-e1	-1.013072	0.444437	-2.28	0.0240*
e(A)_min	2.4970925	1.107282	2.26	0.0256*
c2	1.3920975	1.098715	1.27	0.2071
c4	-0.370302	0.30351	-1.22	0.2243
Max(e(A)_max, e(B)_max)	-0.764609	1.010963	-0.76	0.4506
e(B)_min	-0.531584	1.196927	-0.44	0.6576
Min(e(A)_max, e(B)_max)	0.4236161	1.116222	0.38	0.7048
c3	0.0605955	0.502632	0.12	0.9042

(c)

Term	Estimate	Std Error	t Ratio	Prob>|t|
V	2.5435513	0.373751	6.81	<.0001*
A2	1.4128349	0.461693	3.06	0.0025*
c4	0.5435183	0.282388	1.92	0.0555
c2	-1.35744	1.034185	-1.31	0.1906
e2-e1	0.390004	0.425633	0.92	0.3604
Min(e(A)_max, e(B)_max)	0.8883967	1.036045	0.86	0.3920
e(B)_min	-0.734653	0.995945	-0.74	0.4615
c1	0.4397156	0.797455	0.55	0.5819
A1	-0.254172	0.473728	-0.54	0.5921
c3	0.1661086	0.494253	0.34	0.7371
Max(e(A)_max, e(B)_max)	0.1902486	0.913174	0.21	0.8351
e(A)_min	0.1789338	1.003185	0.18	0.8586
e1	-0.081774	0.844938	-0.10	0.9230

Figure 6-3. Sorted parameter estimates from regression analysis for the difference of total production cost between independent production plan (FC) and integrated production plan for scrap composition group3 (a) when there is no constraint of scrap availability or the capacity of reprocessing furnace (b) when the constraint of scrap availability is considered and (c) when constraints for both scrap availability and capacity of reprocessing furnace are considered. R-squared values for each regression model are (a) 0.2111 (b) 0.2520 and (c) 0.2515.

In this section, regression analysis for numerical simulation of many different systems allowed us to identify which parameters are most likely the influential parameters in determining whether integrated production planning is more beneficial than independent production planning in the complex system. In the next section, we investigate how these identified parameters impact the benefit of integrated production planning in actual industrial problems.

6.3 Scaled-up to industrial size problems

In this section, the benefit of the integrated production planning model will be evaluated and compared to two independent production planning models for an industrial size case. In addition to the difference of the total production cost between independent production planning and integrated production planning, two other metrics are introduced to evaluate the value of integrated production planning: the amount of intermediate products produced but not used and the relative recycled content in the final alloy products. The product specifications and the compositions of scrap, and the batch list of final products are normalized from production data obtained from an aluminum recycling facility that has the two-stage recycling operation system.

6.3.1 Scrap availability

Figure 6-4 shows the relative production cost savings of independent production planning compared to integrated production planning as a function of the availability of a particularly pure scrap compositionally. Due to its pure composition, the availability of this scrap can be often constrained.

In Figure 6-4, the first thing to notice is that the relative production cost savings are positive in all cases. This positive difference indicates that the integrated production planning model is always outperforming two independent production planning models. In other words, the production cost can be reduced when the reprocessing furnace and the re-melting furnaces are planned together compared to when they are separately planned. It should be noted that production costs do not include costs of intermediate products produced but not used. Therefore, if those costs are considered, the benefit of integrated production planning model is even larger than the values appeared in Figure 6-4.

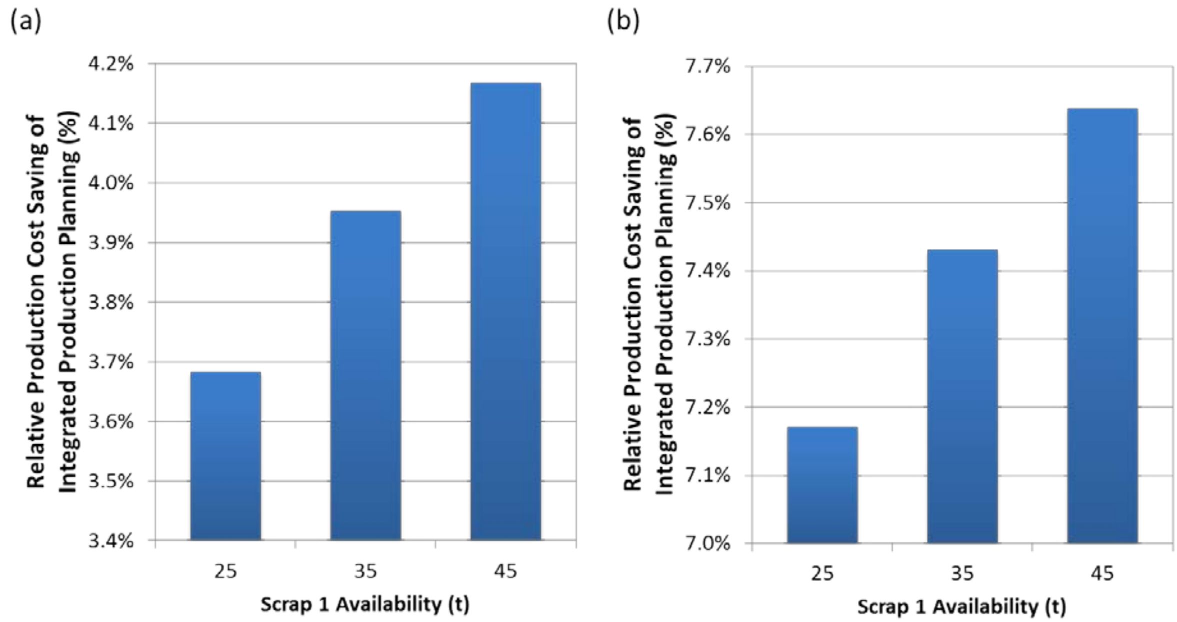


Figure 6-4. The relative production cost saving of integrated production planning as a function of the availability of higher-purity scrap (Scrap 1), compared to independent production planning (a) fixed recipe model (FR) and (b) fixed composition model (FC).

Figure 6-4(a) presents the relative production cost saving of the integrated production planning model to the production cost of the fixed recipe model of independent production planning. Figure 6-4(b) presents the relative production cost saving of integrated production planning to production cost of the fixed composition model of independent production planning. Compared to either of two independent production planning model, the benefit of integrated production planning becomes larger when the availability of Scrap 1 increases. This result is consistent with results of the simulated screening study in Section 6.2 which is the availability of Scrap 1 was identified as the significant parameter. Also the value of integrated production planning has the positive dependency on the availability of Scrap1.

Figure 6-5(a) shows the total amount of total intermediate products produced and leftover during one day. The blue bars in the Figure 6-5(a) represent the amount of intermediate produced and red bars represent the amount of intermediate products leftover. Therefore, the differences between the blue bar and the red bar can be viewed as the amount of intermediate products successfully delivered as liquid metal and used in the production of final alloy products. The amounts of intermediate products produced during a day by the fixed recipe model of independent production planning and by the integrated production planning model are almost similar as 213t and 223t, respectively as presented in Figure 6-5(a). Among these, 83t of the intermediate products are not used in final alloy productions in the fixed recipe model of independent production planning whereas all intermediate products produced are successfully incorporated in finished alloy productions without casting them as sows in integrated production planning. The fixed composition model does not produce the intermediate product as much as two different models. In the fixed composition model, if the predetermined composition does not match the specification of final products, the model has freedom not to produce a batch of intermediate products rather than casting most of intermediate product unlike the fixed recipe model. Therefore, the amount of leftover intermediate products is very low compared to the fixed recipe model. However the amount of intermediate products used in the finished alloy production is low as well.

Figure 6-5(b) shows the relative recycled content in final alloy products when the availability of Scrap 1 is 35t. In other words, this value indicates how much of this alloy made of undervalued raw materials such as scrap and dross. Although all three models provide the batch plans for the exactly same products, the relative recycled content in final products are significantly different. The relative recycled content of the alloy products produced by the integrated model, 60%, is much higher than the values by the fixed recipe model (38.3%) or by the fixed composition model (13.6%). As expected from the amount of intermediate product produced in Figure 6-5(a), the relative recycled content of the fixed composition model is very low. This means that the large portion of final alloy products is made of primary aluminum and alloying elements. Consequently, the total production cost is higher than two other models.

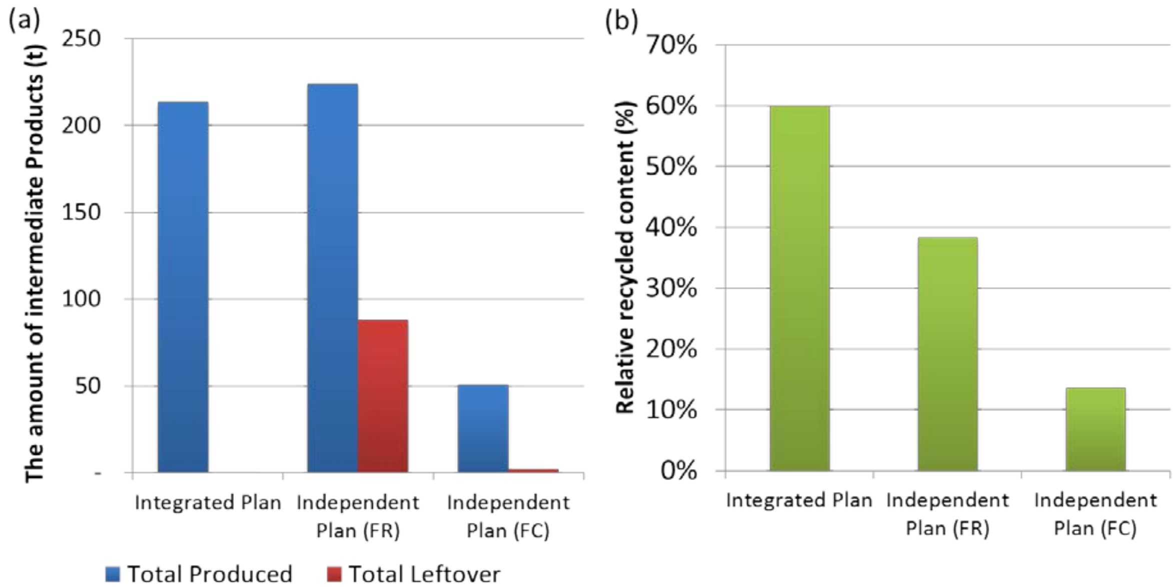


Figure 6-5. (a) The amount of the intermediate products produced(blue) and not used in production of final alloy products(red) (b) relative recycled content in finished alloy products for three different planning models when Scrap 1 availability is 35t.

In the Integrated production planning model, most intermediate products are successfully incorporated in final alloy productions and the relative recycled content in final alloy products is high, compared to two independent planning models.

6.3.2 Price of scrap

The impact of price of Scrap 1 on the value of integrated production planning is evaluated in this section. As explained in the previous section, Scrap 1 is relatively pure alloy product. Figure 6-6(a) presents the relative production cost saving of the integrated production planning model to the production cost of the fixed recipe model of independent production planning. Figure 6-6(b) shows the relative production cost saving of the integrated production planning model to the production cost of the fixed composition model of independent production planning. As shown in both graphs, the benefit of integrated production planning decreases as the price of Scrap 1 increases.

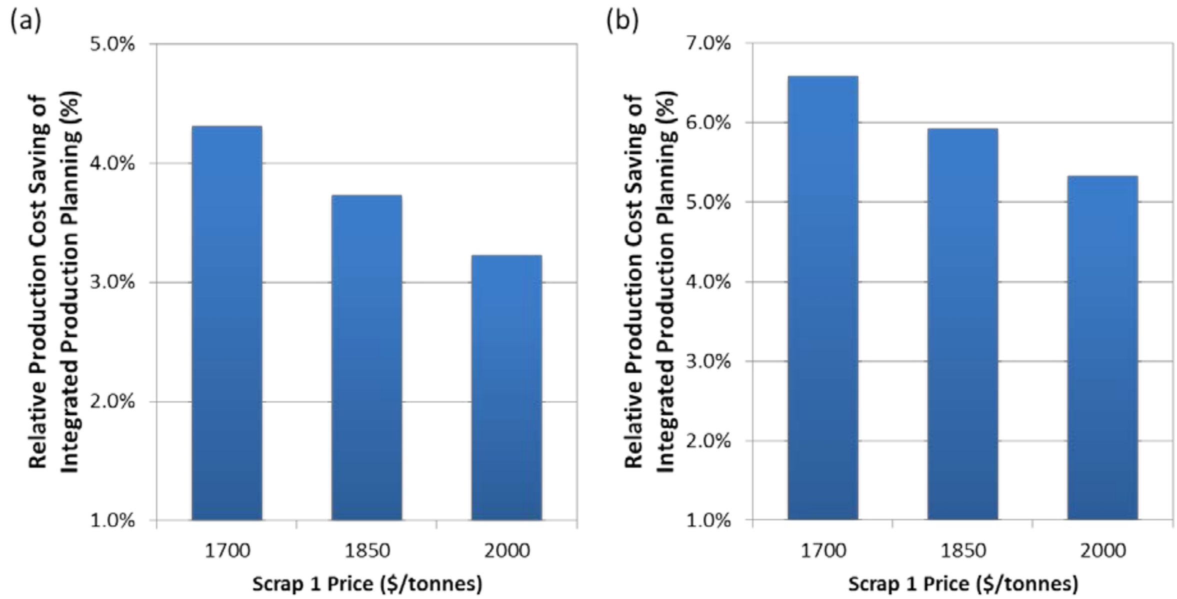


Figure 6-6. The relative production cost saving of integrated production planning as a function of the price of higher-purity scrap, compared to independent production planning (a) fixed recipe model(FR) and (b) fixed composition model (FC).

In other words, the benefit of integrated production planning can be largest when high-purity scrap is relatively cheap. In the integrated production planning, the model decides which scrap should be used based on the demand information of the re-melting stage as well as the price or availability of scrap. For independent production planning, since batch plans or the specifications of intermediate products are predetermined, it is more rigid under the circumstance where the price or the availability of the scrap materials changes. In the other hand, integrated production planning can adjust production plans for intermediate products depending on the given situations. Hence, in this particular example, when the price of Scrap 1 becomes relatively cheap, the integrated production planning can provide the batch plans that use high-purity scrap more aggressively for intermediate products.

Figure 6-7(a) shows the amount of intermediate product produced and the amount of leftover intermediate products in three different planning models. The batch list used in this section includes some high purity alloy products. As a result, the amount of intermediate products that can be used in final alloy products is relatively limited, compared to the example in the previous section. For integrated production planning, however, it still incorporates the most of intermediate products produced into the finished alloy productions. Also the resulting final alloy products consist of more scrap and dross in integrated production planning as shown in Figure 6-7(b). On the other hand, in the fixed recipe model, more than 50% of intermediate products produced are not used in the production of finished alloy products. Both models of independent production planning have significantly lower values of the relative recycled content in final alloy products.

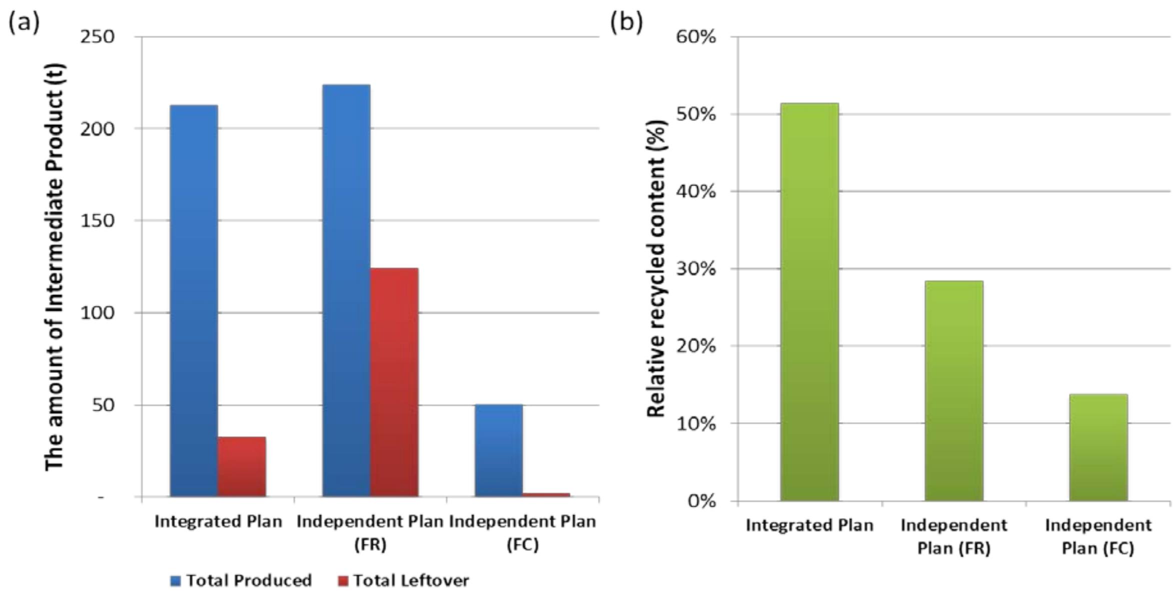


Figure 6-7. (a) The amount of the intermediate products produced(blue) and not used in production of final alloy products (b) the relative recycled content in finished alloy products for three different planning models for a particular price of Scrap 1.

6.3.3 The capacity of the reprocessing furnace

In this section, the impact of the capacity of the reprocessing furnace on the value of integrated production planning will be investigated. Figure 6-8 shows the examples of the relative production cost saving of the integrated production planning model compared to two independent production planning models. The integrated production planning model can save production cost by 3.5% and 5.3% when the capacity of the reprocessing furnace is 40t and 80t, respectively. This result is consistent with the regression analysis in Section 6.2 where the capacity of the reprocessing furnace is the most significant parameter. A similar trend is also found for comparison between the fixed composition model and the integrated production plans.

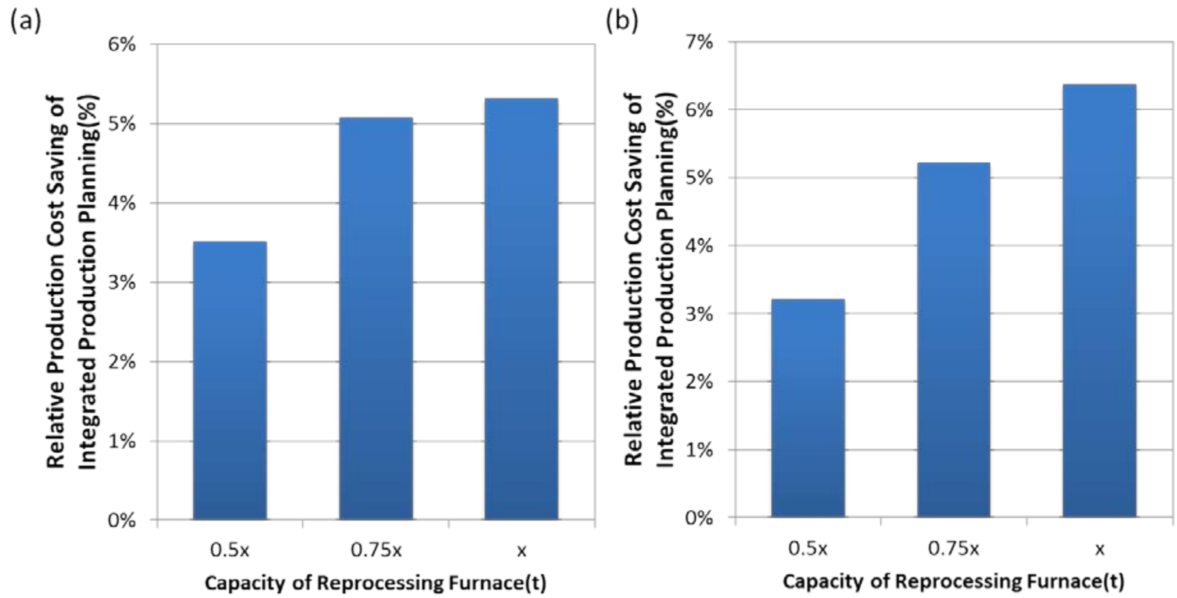


Figure 6-8. The relative production cost saving of integrated production planning as a function of the capacity of the reprocessing furnace, compared to independent production planning (a) fixed recipe model(FR) and (b) fixed composition model (FC).

As discussed in Section 6.2, the capacity of the reprocessing furnace determines the maximum volume of intermediate products that can be produced. When the capacity of the reprocessing furnace is small, the impact of designing the intermediate products by considering the demand for final products is limited accordingly. For example, if the size of batches for finished alloy products in the re-melting furnace is x tonnes and the size of the batches for the reprocessing furnace is 0.25x tonnes, 0.75x tonnes of the re-melting furnace must be filled with the second-stage raw materials. Even though the composition of the intermediate products is out of specification, there is 0.75x tonnes of other raw materials to adjust the composition. In other words, there is a buffer to incorporate unsuccessful design of the intermediate products. Therefore, it can be concluded that, the benefit of integrated production planning can be relatively insignificant when the capacity of the reprocessing stage is small.

Actually, the capacity of the reprocessing furnace is different from other operational parameters such as scrap price and scrap availability. Since the capacity of the furnace is the part of design of the facility, this value is unchangeable unlike other input parameters. Thus, the capacity of the reprocessing furnace can be a measure of the potential impact of integrated production planning. If the aluminum producers try to newly introduce the reprocessing furnace, this analysis can be also guideline to determine the optimal design of the reprocessing furnace.

7 Conclusion

Recycling is one of the key strategies to achieve a sustainable society. Recycling enables materials to recirculate within the system. It can reduce materials extracted from nature and waste discarded to landfills. In addition, replacing primary materials with secondary materials can save the energy required in production. Considering the growing demand for aluminum as well as significant energy savings, recycling aluminum can alleviate the impact of the intensive material consumption on the environment.

In the past, using secondary raw materials in alloy production was limited to low-quality alloy products, and only very high-quality scrap such as prompt scrap was incorporated in high-quality alloy production. However, recent advances in various reprocessing technologies allow aluminum producers to make use of undervalued secondary raw materials in higher value products. In spite of development of reprocessing technologies, incorporation of recovered low-quality scrap materials used in finished alloy production is still limited.

The main reasons for this limited usage of reprocessed low-quality raw materials are different depending on the capability of delivering the reprocessed dross and scrap as liquid metal to a downstream re-melter. When the reprocessed dross and scrap must be delivered as sows, the coordination between the reprocessing and the re-melting stages is either unnecessary or impossible. In this case, dross and scrap are processed without any planning in the reprocessing stage. Consequently, the compositions of outputs from the reprocessing furnace are highly variable. In the second case, where the reprocessed dross and scrap can be delivered as liquid metal, the benefit can be maximized when the intermediate products are immediately incorporated in the final alloy production as liquid without casting them. Therefore, coordination between the reprocessing and the re-melting stage is critical.

Given these issues, the following question is raised; how can we improve the usage of undervalued raw materials in the final alloy products by taking advantages of advances in reprocessing technologies? This thesis has attempted to answer this question for both situations.

Chapter 4 suggests grouping sows by their compositional similarity to increase their usage in the re-melting stage. A clustering analysis can identify compositional patterns of recovered dross and scrap. Such patterns provide criteria for separating raw materials to increase their homogeneity. Binning cast sows by clustering analysis allows for full utilization of sows in alloy production without the need to maintain compositional information of all individual outputs from the rotary furnace. Therefore, clustering analysis is an effective method to separate raw materials. The results in this thesis suggest a new opportunity for material recyclers to maximize the usage of low-quality raw materials in alloy production using existing data when the coordination between the reprocessing stage and the re-melting stage is either impossible or unnecessary. This new approach can be used not as an alternative but as a complement to the existing modeling tools and recycling technology.

Chapter 5 and Chapter 6 investigate the situation when the intermediate products are transferred as liquid metals from the reprocessing furnace to the re-melting furnace. In these two chapters, integrated production planning is suggested to maximize the incorporation of intermediate products as liquid metal in the re-melting stage. The mathematical pooling problem is used to model two-stage blending processes simultaneously.

In Chapter 5, an analytical approach is used to understand the interaction between parameters and the solution of the pooling problem in the aluminum recycling context. Two simple case studies are introduced for this purpose. The first case study is designed to investigate the fundamental differences between integrated production planning approaches and a typical one-stage blending process. The analytical approach enables deeper understanding of the pooling problem in the aluminum recycling context. This study suggests that the strategies to design intermediate products that minimize the alloy production cost can be very contextual. The impacts of raw material compositions compared to product specifications, and the relative price of scrap on the optimal designs of intermediate products as well as on the structure of total production cost function have been demonstrated. The second case study is the extension of the first. In this study, the benefit of considering the demand uncertainty in the integrated production planning is considered. The batch plans determined by the stochastic approach are not necessarily the minimum production cost in all scenarios but they can reduce the significant cost increase when the demand for high purity alloy product occurs.

The analytical approach in Chapter 5 enables deeper understanding of the relationships between parameters and the optimal solutions. However, the scope of this approach can be limited to the simplified case study due to the complicated nature of the pooling problem. To extend our understanding of two-stage recycling operations, a simulated screening study is performed to understand behaviors of more complex systems in Chapter 6.

Based on the results obtained from the simulations, the regression analysis is performed to identify the most significant parameters in determining the optimal design of intermediate products as well as maximizing the benefit of integrated production planning versus independent production planning. The results are quite different depending on the operational conditions. When there is no constraint of either availability or capacity of furnace, the optimal compositions of intermediate products are highly dependent on the minimum of max specifications of finished products, which is consistent with the analytical approach in Chapter 5. When the constraint of either availability of scrap or the capacity of a reprocessing furnace is considered, the composition of two scrap materials, the availability of scrap and the capacity of reprocessing furnace become relatively more influential to the optimal composition than the specifications of final products. For the benefits of integrated production planning versus independent production planning, the minimum of max specifications is also the most significant parameter when other constraints are not considered. However, when the availability constraint is included, the price and the availability of scrap become the most significant parameters. When the capacity of a reprocessing furnace is limited, it becomes the dominant factor that determines the value of the integrated production planning.

Lastly, the benefits of integrated production planning are demonstrated in a real industrial case study. The results show that the integrated production planning model outweighs two independent planning models in terms of all performance metrics. In all examples, the majority of the intermediate products designed by integrated production planning are successfully incorporated into the production of final alloy products. In the fixed recipe model of independent production planning, only limited amounts of intermediate products produced without any consideration of final product demand information can be used in alloy production. Although the amounts of liquid metals cast as sow are very minimal in the fixed composition model of independent production planning, the relative recycled content in final alloy products is the lowest, compared to the two other models.

Also the impacts of changes in operational parameters, such as scrap availability, prices of scrap, and the capacity of reprocessing furnace, on the benefit of the integrated production are studied. These results are also consistent with the results from the simulated screening study.

As scrap materials with higher purity are more abundant and cheaper, the value of integrated production planning can be more significant, compared to the independent production planning. These results suggest that integrated production planning can adjust the optimal batch plans by reflecting changes in operational conditions whereas two models of independent planning provide relatively rigid batch plans for the reprocessing stage. The capacity of reprocessing furnaces is also a key parameter that determines the potential impact of designing intermediate products in re-melting furnace operations. These results in this study also can be used as guidelines for aluminum producers to estimate the potential benefit of adapting integrated production planning, given their operational conditions.

This thesis demonstrates the significant opportunities for increasing the usage of undervalued raw materials in two-stage recycling operations. However, room exists for improvement in current approaches and models. Proposed future work will be discussed in the next chapter.

8 Future Work

8.1 Different clustering algorithms to group uncertain raw materials

Although we use a hierarchical clustering algorithm in Chapter 4 due to limited knowledge about the number of bins, different methods can be adapted depending on production environment. For instance, if a recycling firm knows the maximum amount of resources it has available to devote to expanding raw material inventory, it can start with that number using popular partitioning algorithms such as k-means.

Another potential way to use clustering analysis in order to increase the usage of reprocessed dross and sows in alloy production is to use information of product specification. It is possible to group alloy products into a few categories based on their similarities in the specification if the recycling facility produces multiple products share similar specification. Then the average specification of products in each category can be used as the starting points for clustering analysis. Sows are assigned into each cluster based on their compositional distance. Each bin of sows created by this method can be used as dedicated raw materials for each group of final products.

However, the goal of this study is to address a new opportunity presented by clustering analysis to increase the usage of low-quality raw materials while accounting for uncertainty. Also, clustering methods are heuristic in nature. A suitable clustering method can be changed based on characteristics of data or the goal of a study. Therefore, the effect of choosing different cluster algorithms is not the scope of this study. The relationship between clustering methods and data are well reviewed in (Xu and Wunsch 2005). The researcher can choose appropriate clustering methods depending on the structure of targeting data and the context of manufacturing environment.

8.2 Strategies for stochastic pooling problem in two-stage aluminum recycling operations

In Chapter 5, the benefit of considering the demand uncertainty for final products has been demonstrated. As the number of scenarios increase, solving times for stochastic pooling problem become computationally intractable. Therefore, only a limited number of scenarios can be implemented. The existing stochastic pooling problem algorithms model the situation where the first-stage decision variables are discrete. For example, the recent study by Li et al. suggests decomposition strategies to solve the stochastic pooling problem. The first-stage decision variables are associated with decisions of opening of nodes and arcs in the network. Each scenario of the second stage is actually one pooling problem. This situation is obviously different from two-stage aluminum recycling operations. The first-stage decision is determining the compositions of the intermediate products. As discussed earlier, the quality of the pools (intermediate products) is not only a complicated variable itself that introduces the bilinear terms in the problem when there is no uncertainty factor in the system, but also a variable representing the decisions made at the different times from other decisions when the uncertainty is considered. Currently, there is no study on the stochastic pooling problem when the first-stage decision variables are continuous variables. As a result, there is no global algorithm to solve our problem in the reasonable times. Therefore, new strategies are required to resolve the current major issue, the inability to implement more scenarios into stochastic models.

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Appendix. Additional data

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e1	0.0147362	0.00049	30.05	<.0001*
e2	0.007415	0.000516	14.38	<.0001*
A1	-0.002926	0.000256	-11.44	<.0001*
c1	0.006958	0.00074	9.40	<.0001*
c2	-0.007684	0.001023	-7.51	<.0001*
A2	0.0013123	0.000268	4.89	<.0001*
Max(e(A)_max, e(B)_max)	0.0025568	0.000797	3.21	0.0014*
Min(e(A)_max, e(B)_max)	-0.001504	0.000851	-1.77	0.0780
e(B)_min	0.0011064	0.000878	1.26	0.2081
c3	0.000357	0.000498	0.72	0.4742
e(A)_min	0.0002125	0.000981	0.22	0.8286
c4	-4.615e-5	0.000268	-0.17	0.8631

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.011953	0.000519	23.02	<.0001*
e1	0.0101171	0.000577	17.54	<.0001*
A2	0.0032203	0.000426	7.57	<.0001*
V	-0.002793	0.000386	-7.23	<.0001*
A1	-0.001548	0.000425	-3.65	0.0003*
Max(e(A)_max, e(B)_max)	0.0022258	0.00086	2.59	0.0101*
c1	0.0017133	0.000782	2.19	0.0293*
e(A)_min	-0.001661	0.001055	-1.57	0.1164
Min(e(A)_max, e(B)_max)	0.0013434	0.000941	1.43	0.1547
c4	-0.000363	0.000281	-1.29	0.1976
c2	-0.001104	0.001069	-1.03	0.3024
c3	-0.000261	0.000528	-0.50	0.6208
e(B)_min	0.0000459	0.000998	0.05	0.9634

Sorted parameter estimates from regression analysis of scrap composition group1 for the optimal composition of the intermediate product (a) when the constraint of scrap availability is considered and (b) when constraints for both scrap availability and capacity of reprocessing furnace are considered. R-squared values for each regression model are (a) 0.8730 and (b) 0.9043

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.0124157	0.000733	16.94	<.0001*
e1	0.0100623	0.000722	13.94	<.0001*
A2	0.0023528	0.000202	11.66	<.0001*
A1	-0.000728	0.000227	-3.21	0.0018*
c3	-0.000968	0.000417	-2.32	0.0223*
c2	0.0011704	0.000752	1.56	0.1229
e(A)_min	-0.000829	0.000705	-1.18	0.2426
Min(e(A)_max, e(B)_max)	0.0005763	0.000733	0.79	0.4338
c4	0.0001488	0.000226	0.66	0.5124
Max(e(A)_max, e(B)_max)	0.0004509	0.000716	0.63	0.5305
e(B)_min	-0.000468	0.000966	-0.48	0.6288
c1	0.0001371	0.000565	0.24	0.8086

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e2	0.0147991	0.001475	10.03	<.0001*
e1	0.0084067	0.001347	6.24	<.0001*
A2	0.0024851	0.000618	4.02	0.0003*
V	-0.001747	0.00053	-3.29	0.0021*
A1	-0.001599	0.000556	-2.87	0.0065*
Max(e(A)_max, e(B)_max)	-0.002048	0.0012	-1.71	0.0956
c2	-0.002214	0.001452	-1.52	0.1352
c3	0.0009509	0.000888	1.07	0.2905
e(A)_min	0.0012459	0.001379	0.90	0.3715
c4	-0.000176	0.000354	-0.50	0.6213
Min(e(A)_max, e(B)_max)	-0.000657	0.001389	-0.47	0.6390
e(B)_min	0.0003155	0.001549	0.20	0.8396
c1	-0.000105	0.001086	-0.10	0.9235

Sorted parameter estimates from regression analysis of scrap composition group2 for the optimal composition of the intermediate product (a) when the constraint of scrap availability is considered and (b) when constraints for both scrap availability and capacity of reprocessing furnace are considered. R-squared values for each regression model are (a) 0.9483 and (b) 0.9320

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e1	-1.615476	0.363605	-4.44	<.0001*
e2-e1	1.4844391	0.379867	3.91	0.0001*
c2	-2.805402	0.75412	-3.72	0.0002*
c3	1.097004	0.367363	2.99	0.0030*
Min(e(A)_max, e(B)_max)	1.4683635	0.625864	2.35	0.0194*
c1	0.7950174	0.544572	1.46	0.1450
Max(e(A)_max, e(B)_max)	-0.566873	0.586393	-0.97	0.3342
c4	0.1434985	0.197024	0.73	0.4668
e(B)_min	-0.417889	0.646783	-0.65	0.5185
e(A)_min	0.4018937	0.721896	0.56	0.5780

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
e1	-2.437455	0.721113	-3.38	0.0009*
Max(e(A)_max, e(B)_max)	1.7123861	1.15102	1.49	0.1386
c3	0.9469163	0.718686	1.32	0.1893
c4	0.4618129	0.375213	1.23	0.2200
e(A)_min	1.6492101	1.471327	1.12	0.2638
Min(e(A)_max, e(B)_max)	1.2873584	1.275178	1.01	0.3141
c1	-0.873964	1.007668	-0.87	0.3869
e(B)_min	-0.979247	1.292805	-0.76	0.4498
A2	0.3971844	0.584419	0.68	0.4976
e2-e1	-0.440331	0.729922	-0.60	0.5471
A1	-0.327405	0.580957	-0.56	0.5738
c2	-0.457359	1.475646	-0.31	0.7570

(c)

Term	Estimate	Std Error	t Ratio	Prob>|t|
V	1.6196071	0.464898	3.48	0.0006*
c2	-3.217216	1.285664	-2.50	0.0129*
e1	-1.465647	0.63026	-2.33	0.0208*
c4	0.7091577	0.338433	2.10	0.0370*
c1	1.6865375	0.940899	1.79	0.0741
c3	1.0974591	0.634641	1.73	0.0849
e(A)_min	1.9464443	1.269015	1.53	0.1262
A1	-0.773118	0.510851	-1.51	0.1313
e2-e1	-0.694851	0.624772	-1.11	0.2670
Max(e(A)_max, e(B)_max)	1.0282927	1.034516	0.99	0.3211
A2	-0.504234	0.511965	-0.98	0.3255
e(B)_min	-0.170854	1.200749	-0.14	0.8870
Min(e(A)_max, e(B)_max)	0.1212898	1.132586	0.11	0.9148

Sorted parameter estimates from regression analysis for the difference of total production cost between independent production plan (FC) and integrated production plan for scrap composition group1 (a) when there is no constraint of scrap availability or the capacity of reprocessing furnace (b) when the constraint of scrap availability is considered and (c) when constraints for both scrap availability and capacity of reprocessing furnace are considered. R-squared values for each regression model are (a) 0.1674 (b) 0.1261 and (c) 0.1346

(a)

Term	Estimate	Std Error	t Ratio	Prob>|t|
c1	9.7530324	0.794493	12.28	<.0001*
c2	-8.125955	1.058634	-7.68	<.0001*
c4	0.9083751	0.316411	2.87	0.0050*
e(A)_min	1.2167588	0.979842	1.24	0.2172
c3	-0.64322	0.583636	-1.10	0.2730
e(B)_min	-1.467754	1.357469	-1.08	0.2822
Max(e(A)_max, e(B)_max)	0.7857161	1.008612	0.78	0.4378
Min(e(A)_max, e(B)_max)	-0.332489	1.02692	-0.32	0.7468
e2-e1	0.2655277	1.010977	0.26	0.7934
e1	-0.033094	0.975097	-0.03	0.9730

(b)

Term	Estimate	Std Error	t Ratio	Prob>|t|
A1	3.5321527	1.126725	3.13	0.0041*
c3	3.293919	1.56244	2.11	0.0444*
A2	2.7714231	1.401766	1.98	0.0583
e(A)_min	4.8960452	2.600968	1.88	0.0706
c1	-3.265408	2.436646	-1.34	0.1914
Min(e(A)_max, e(B)_max)	-4.271555	3.412172	-1.25	0.2214
c4	1.0098302	0.85727	1.18	0.2491
Max(e(A)_max, e(B)_max)	-3.136208	2.804973	-1.12	0.2734
e1	-2.43043	3.284796	-0.74	0.4657
e(B)_min	2.7691208	3.964033	0.70	0.4908
c2	-1.803498	3.071068	-0.59	0.5619
e2-e1	-0.221779	3.209707	-0.07	0.9454

Term	Estimate	Std Error	t Ratio	Prob>|t|
V	4.9885621	1.358389	3.67	0.0007*
c1	5.9570391	2.781782	2.14	0.0384*
c2	-7.907151	3.719755	-2.13	0.0398*
e(A)_min	5.7189505	3.530752	1.62	0.1131
c4	1.2240104	0.907704	1.35	0.1851
A2	1.5966489	1.583445	1.01	0.3194
Max(e(A)_max, e(B)_max)	-2.988314	3.072921	-0.97	0.3367
e1	-3.15633	3.905015	-0.81	0.4237
A1	0.42325	1.425258	0.30	0.7680
Min(e(A)_max, e(B)_max)	-1.018474	3.558351	-0.29	0.7762
c3	0.4292736	2.273445	0.19	0.8512
e2-e1	0.5327383	3.778107	0.14	0.8886
e(B)_min	-0.160882	3.967425	-0.04	0.9679

Sorted parameter estimates from regression analysis for the difference of total production cost between independent production plan (FC) and integrated production plan for scrap composition group2 (a) when there is no constraint of scrap availability or the capacity of reprocessing furnace (b) when the constraint of scrap availability is considered and (c) when constraints for both scrap availability and capacity of reprocessing furnace are considered. R-squared values for each regression model are (a) 0.6629 (b) 0.5268 and (c) 0.4649