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Nonlinear Group IV photonics based on silicon and germanium: from near-infrared to mid-infrared

1 Introduction

Nonlinear optics [1–3] was born immediately after the first laser was demonstrated, with many nonlinear effects reported in 1961 and 1962. Those include second-harmonic generation (SHG) [4, 5], third-harmonic generation (THG) [5], two-photon absorption (TPA) [6], stimulated Raman scattering (SRS) [7], and phase matching in parametric wave mixing [8, 9]. As an important branch of optics, nonlinear optics is at the heart of frequency conversion, amplification, soliton and pulse compression, supercontinuum generation, and laser mode-locking and frequency comb generation, which have found a wide variety of applications in optical signal processing, communications, sensing, imaging, and metrology.

While nonlinear effects in atomic gases [10] have been extensively explored in research laboratories, other materials platforms based on nonlinear crystals [11] and optical fibers [12] have become more practical for real-world applications. Particularly, since the invention of photonic crystal fiber (PCF) [13], nonlinear fiber optics has experienced a revolutionary development [14, 15]. Supercontinuum generation as an example is one of nonlinear applications enabled by PCFs [14]. Obtaining a wideband or even octave-spanning supercontinuum does not only require an in-depth understanding of nonlinear phenomena in optical fibers but also provides numerous commercial opportunities (see e.g., “http://www.leukos-systems.com/”). The success of PCFs in refreshing nonlinear fiber optics mainly relies on a great enhancement of nonlinearity and flexible engineering of dispersion [15]. Both of these benefits are enabled by a large refractive index contrast between glass and air. PCFs allow us to confine light much more tightly than standard fibers and enhance the contribution of waveguide dispersion that is highly tailorable by designing fiber structures. The reasoning behind PCF’s success serves as one of motivations to explore new materials platforms with even higher index contrast for nonlinear optics. In this sense, silicon photonics that has an index contrast of 1.5–2 exhibits great potential to further enhance performance of nonlinear devices.
In the past two decades, silicon photonics [16–25] has become increasingly mature. What silicon photonics can contribute to nonlinear optics is much more than a high index contrast. Several fundamental advantages drive nonlinear silicon photonics [26–30].

First, as bulk material, crystalline silicon typically has stronger Kerr and Raman nonlinearities than silica by 2–3 orders of magnitude [26–30], depending on crystal orientation. Materials engineering further enhanced the nonlinearity, e.g., in amorphous silicon under certain deposition conditions [31–35] and in Si-rich silicon oxide (SRO) and nitride (SRN) based on the formation of silicon nano-crystals [36–41].

Second, the high index contrast not only allows sub-wavelength light confinement [42] but also makes it possible to form novel waveguiding structures. For example, when a nano-scale low-index layer is sandwiched between two high-index layers, a slot waveguide can be formed [43–46], with a large fraction of optical power trapped in the thin layer. Relative to many nonlinear materials such as SRO, SRN, chalcogenides, and polymers, silicon has a sufficiently large index to form a slot waveguide [47–51]. Moreover, adding a slot layer provides more design freedom to tailor chromatic dispersion, as reported in standard slot waveguides [51–58] and strip/slot hybrid waveguides [59–63]. A high refractive index is also beneficial to build a slow light element with reduced group velocity of light, which can effectively increase optical length for nonlinear interactions and reduce power requirement [64–66].

Third, CMOS compatibility in silicon device fabrication and fabrication can be utilized for nonlinear silicon photonics. Although optoelectronic integration of silicon devices has long been proposed [16], there have been few reports on the integration of nonlinear devices with electrical functionality. As silicon photonics becomes more mature, it is foreseeable that CMOS-compatible nonlinear silicon devices are seamlessly integrated with microelectronics on a single chip. For example, on-chip pumping using an integrated laser, data encoding and modulation before nonlinear signal processing, and on-chip signal detection and data analysis after nonlinear devices are all highly likely to be integrated together, which is achievable by building nonlinear photonics on a silicon platform.

Fourth, cost-effectiveness and portability of devices enabled by silicon photonics can be directly transferred to the nonlinear optics branch. Although probably not a key consideration in a very early stage of a research topic, these factors will finally drive the technological development and determine how widely this technology will be used. Nonlinear silicon photonic devices have unique competence, especially when a large amount of elements need to be incorporated in a (sub-) system.

The above motivations led to considerable research efforts on nonlinear silicon photonics in recent years [26–30]. Nonlinear effects such as SRS [67–83], self- and cross-phase modulation (SPM and XPM) [84–91], four-wave mixing (FWM) [92–102], and supercontinuum generation [60, 61, 103–108] are actively investigated. The main focus is on the near-IR wavelength range, around the telecom window. However, TPA in silicon is a detrimental effect in most cases [109, 110], significantly limiting optical power that is actually available in nonlinear interactions. TPA thus degrades nonlinear efficiency and bandwidth in the effects of interest, such as wavelength conversion and supercontinuum generation, and TPA-induced free carriers cause slow device responses [26]. It is noted that both the desired nonlinear effect and TPA are accumulative over a certain distance, so if one can engineer chromatic dispersion to make the desired nonlinear effect more efficient, the negative influence of TPA can be mitigated [60, 108]. One can also use silicon nitride [111–115] to eliminate TPA or even three photon absorption (3PA) for a pumping wavelength in the near-IR, because silicon nitride has a bandgap energy of ~5.3 eV [113].

It would be sometimes instructive to look at the trends in nonlinear optics based on non-integrated platforms, in order to gain a vision on how nonlinear silicon photonics can evolve. One of the recent trends in nonlinear optics is that a significantly larger portion of electromagnetic spectrum, from X-ray to mid-IR and even THz, is being exploited to acquire previously hardly accessible information [116–118]. For nonlinear silicon photonics, it is a natural step to extend to the mid-IR wavelength range [119–125]. The definition of the term “mid-IR” varies substantially in the literature, according to different research communities and organizations. Specific application scenarios correspond to different wavelength ranges. These include but are not limited to (i) mid-IR spectroscopy and sensing from 2.5 to 15 μm [126] or from 2.5 to 25 μm according to infrared spectroscopy correlation table [127] “http://en.wikipedia.org/wiki/Infrared_spectroscopy_correlation_table.”, (ii) free space communications, LADAR, and remote sensing in atmosphere transparent windows from 3 to 5 μm and from 8 to 12 μm [128, 129], and (iii) astronomical instrumentation over a bandwidth from 5 to 20 μm [130].

These new opportunities in the mid-IR are open to silicon photonics from an application perspective, while nonlinear silicon photonics also gains additional advantages in this wavelength range in terms of materials and devices. First, silicon has no TPA beyond 2.2 μm, and nonlinear loss by 3PA is significantly less influential for
an optical intensity of below 5 GW/cm² [131, 132]. Second, more Group IV elements, e.g., germanium, become transparent and can be used for nonlinear purposes. In fact, germanium exhibits higher refractive index and Kerr index n₂ than silicon [133]. Third, at longer wavelengths, device dimensions need to be scaled up according to wavelength, and thus surface roughness induced in device fabrication causes relatively small scattering loss. Fourth, fabrication errors in device dimensions are a smaller fraction of target numbers, which is beneficial to obtain a higher yield in device manufacture.

However, nonlinear photonics in the mid-IR also meet some challenges. Although, going beyond half-bandgap wavelengths, one can remove TPA, the nonlinear Kerr index n₂, markedly decreases [134]. With waveguide dimensions scaled up, the effective mode area, A_eff increases almost quadratically. Therefore, the nonlinear coefficient, γ, equal to 2πn₂/(λA_eff) where λ is the wavelength in vacuum, decreases quickly with wavelength. This would require careful dispersion engineering for phase matching in nonlinear parametric processes in order to improve nonlinear efficiency [12].

Many nonlinear effects have been reported recently in silicon photonics touching the mid-IR wavelength range [60, 61, 106, 108, 114, 115, 135–145]. It is noted that most of them address the short-wavelength end of the mid-IR, from 2 to 2.5 μm, which is mainly the short-wave IR [144] or transition from near-IR to mid-IR. Little was reported in longer wavelength beyond 2.5 μm [136, 138, 142, 145]. In fact, there are about two octaves of bandwidth in the mid-IR (e.g., from 2.5 to 10 μm) available, much wider than that in the near-IR. As an approach to creating new frequencies, nonlinear optics is much more efficient than electro-optic modulation in terms of how far an optical spectrum can be extended. We believe that the mid-IR would be an exciting arena for ultrafast octave-spanning nonlinear applications.

In this paper, we discuss the materials properties of the Group IV platform for nonlinear applications. The waveguide-based devices are optimized for four different wavelength ranges from near-IR to mid-IR in terms of both nonlinearity and dispersion. We show by simulation that our dispersion-engineering approach based on a strip/slot hybrid structure is widely applicable and can dramatically enhance nonlinear interaction efficiency and spectrum broadening. Supercontinuum and frequency comb generations are predicted to be octave-spanning according to our numerical simulations, in which excellent spectral coherence of the generated wideband spectra is confirmed by the creation of ultrashort cycle-level optical pulses.

## 2 Materials

Loss, nonlinearity, and dispersion jointly determine the nonlinear performance of optical waveguides. All the three are both material- and device-dependent. In this section, we survey the major material choices for nonlinear Group IV photonics in the near- and mid-IR.

Figure 1 shows material transparency windows with an optical loss below 2 db/cm [122] for materials including silicon (Si, i.e., crystalline silicon unless otherwise specified), silicon nitride (Si₃N₄), silicon dioxide (SiO₂), germanium (Ge), arsenic sulfide (As₂S₃), and arsenic selenide (As₂Se₃). Since TPA plays an important role in nonlinear applications [109, 110], the color transition from red to green in Figure 1 is between the bandgap wavelength and the half-bandgap wavelength (two blue lines), where TPA decreases with wavelength. For silicon, almost two-octave bandwidth from 2.2 to 8.5 μm [146] is available for nonlinear applications without TPA, covering a large fraction of mid-IR range. It is important to note that amorphous silicon has a bandgap energy of 1.7 eV [32, 33] and thus has TPA diminishing at a much shorter wavelength (<1.55 μm) than crystalline silicon. Both silicon nitride and silicon dioxide have large bandgap energies, but silicon dioxide becomes highly lossy beyond 3 μm [119]. Germanium has an indirect bandgap energy of 0.67 eV and is transparent until up to 14 μm [146]. From Figure 1, one can see that germanium’s green bar without TPA has no overlap with silicon dioxide.

Chalcogenide glasses are actively investigated as photonic materials [147–149] and exhibit a wide transparency window in the near- and mid-IR. For example, As₂S₃ and As₂Se₃ have bandgap energies around 2.26 eV [150, 151] and 1.77 eV [152], and they are transparent up to 12 and 15 μm [153], respectively. Although in this paper

![Figure 1](image-url)
chalcogenide glasses are not considered a material for the core of a waveguide, one may use them for waveguide cladding and slot layer [53, 58].

Linear optical properties of the materials discussed here are collected in terms of refractive index and material dispersion, as shown in Figure 2. The two “X” on the curves for silicon and germanium in Figure 2(A) indicate the half-bandgap wavelengths. The refractive index is given by the Sellmeier equations for silicon [154], silicon nitride [155], silicon dioxide [156], SRO [51], germanium [157], arsenic sulfide [158], and arsenic selenide [159], as detailed in Appendix A. For SRN, there is no comprehensive measurement of material index found currently. As shown in Figure 2(A), the materials under our consideration have strong index contrasts, especially between germanium and chalcogenides in the mid-IR. The refractive index decreases with wavelength, and beyond the half-bandgap wavelength, these materials have a relatively small index change. One can properly choose materials for waveguide core and cladding and also a low-index slot layer, based on the information given in Figure 2(A).

Overall dispersion in an integrated waveguide consists of material dispersion and waveguide dispersion that is affected by index contrast and waveguide dimensions. First, material dispersion is shown in Figure 2(B), which is defined as $D = \frac{c}{\lambda} \cdot \frac{d^2n_{mat}}{d\lambda^2}$, where $n_{mat}$ is material index, and $\lambda$ and $c$ are wavelength and the speed of light in vacuum. It is important to note that, except for silicon dioxide, all the other materials have a flat and low dispersion within ±100 ps/(nm·km) at the long-wavelength end of the bandwidth of interest. This means that, if waveguides are not designed to tightly confine guided modes, one can reduce the contribution of the waveguide dispersion and have the overall dispersion close to the flat and low material dispersion. However, this will cause a large effective mode area and a small nonlinear coefficient.

On the other hand, we note from Figure 2(B) that, at the short-wavelength end of the spectrum, material dispersion changes quickly with wavelength for all the considered materials even if the material refractive index looks flat in Figure 2(A) beyond the half-bandgap wavelength. This is because the dispersion is the 2nd-order derivative of the index with respect to wavelength. To fully use the portion of the spectrum near the half-bandgap wavelength, dispersion engineering by tailoring waveguide dispersion is required.

As a measure of nonlinear material property, the nonlinear index $n_2$ is shown in Figure 3 for silicon, silicon nitride, SRO, germanium, and arsenic sulfide. Looking at broadband nonlinear applications, one needs to take the wavelength dependence of $n_2$ into account. Unfortunately, there is often a lack of complete measurement data at a wavelength range of interest, and also measurement results from different groups could vary widely. For silicon, data from several sources are available [160, 161]. A recently published review paper [133] shows a prediction of third-order nonlinear susceptibility $\chi^{(3)}_{III}$ for silicon and germanium in the mid-IR range, based on a two-band model, which is used to fit wavelength-dependent

![Figure 2](image1.png)  
**Figure 2** (A) Refractive indices and (B) material dispersion curves of the materials considered for nonlinear Group IV photonics in the near- and mid-IR wavelength ranges.

![Figure 3](image2.png)  
**Figure 3** The Kerr nonlinear index $n_2$ values of the considered materials in near- and mid-IR ranges.
measurements. We obtain the Kerr nonlinear index \( n_2 \) as a function of wavelength, based on \( \chi^{(3)} \) using the results from [133]. As shown in Figure 3, the \( n_2 \) value peaks at 1.9 and 2.7 \( \mu \text{m} \) for silicon and germanium, respectively, and changes slightly from 3.3 to 4.8 \( \mu \text{m} \) beyond which both TPA and 3PA disappear. The TPA coefficient \( \beta_{\text{TPA}} \) varies with wavelength and is also extracted for silicon and germanium from [133], as detailed in Tables 1–4 in Appendix B.

Hydrogenated amorphous silicon has been identified as a potentially good nonlinear material, not only because of its large bandgap energy of 1.7 eV but also because of large nonlinear index, \( n_2 \), and nonlinear figure of merit (FOM), \( n_2/\beta_{\text{TPA}} \). We did not include specific data in Figure 3 for amorphous silicon, since different groups reported highly variable \( n_2 \) and nonlinear FOM values in the near-IR [31–35]. The \( n_2 \) value could be one order of magnitude higher than that in silicon [33], while the nonlinear FOM can be as high as 5 [35], although these may not be obtained simultaneously [34]. Moreover, linear properties of amorphous silicon may also vary when fabrication conditions and its nonlinear characteristics change.

Silicon nano-crystals in silicon dioxide and silicon nitride have also been investigated as a nonlinear material, exhibiting higher nonlinear indices than crystalline silicon by an order of magnitude or more [36–41]. The values of \( n_2, \beta_{\text{TPA}}, \) and nonlinear FOM are also highly variable, if silicon excess, annealing temperature and wavelength change. We include one data point (\( n_2 = 4.8 \times 10^{-19} \text{m}^2/\text{W} \)) from [38] in Figure 3. Extremely high \( n_2 \) and FOM by 3–4 orders have been obtained experimentally [41] with large silicon excess (note that the FOM in [41] is defined as the reciprocal of ours here).

Both amorphous silicon and silicon nanocrystals exhibit great potential as a nonlinear material in the mid-IR, which can be used to compensate for the reduction of the nonlinear coefficient due to a large mode area at long wavelengths. In particular, with a small linear refractive index, SRO is often chosen as a slot material to enhance nonlinearity in the near-IR [51, 53, 55, 56, 59], while SRN exhibits a great potential for nonlinear applications beyond 3 \( \mu \text{m} \). Typically, strong nonlinearity in bulk materials is associated with a high linear refractive index, which is known as Miller rule. However, silicon nano-crystals exhibit unique properties to simultaneously possess strong nonlinearity and low linear index. It is important to mention that the silicon nano-crystals (i.e., nano-clusters) could act as scattering centers of light, causing an increased propagation loss in SRO slot waveguides. Nevertheless, relatively low propagation loss has been achieved, which is 3–5 dB/cm [163].
compatibility in device fabrication. On the other hand, compared to the material choices, device design (mainly on waveguide and resonator) can also produce widely variable dispersion and nonlinearity properties. In the next section, we will discuss enhanced waveguide properties using improved designs.

3 Devices

Optical waveguides form the backbone of photonic devices. Light propagation properties in a waveguide could be remarkably different from those in corresponding bulk materials, especially when there is a high index contrast between waveguide core and surrounding cladding. Therefore, understanding and optimizing the waveguide properties including loss, dispersion, and nonlinearity are essential in nonlinear photonics.

Propagation loss in a waveguide includes material loss, confinement loss, scattering loss, and nonlinear loss. Working at a transparency window of a material, especially beyond the half-bandgap wavelength, one can primarily have low material and nonlinear loss caused by TPA. Note that, benefiting from the wide multi-octave bandwidth in the mid-IR, one can even eliminate the impact of 3PA in silicon and germanium by pumping at >3.3 μm and >4.8 μm, respectively. Since the substrate index in a silicon wafer is higher than or equal to that in most of materials we consider for a waveguide core, confinement loss exists due to mode leakage to the silicon substrate. This loss can be markedly reduced by increasing the spacing between waveguide core and the substrate or choosing low-index material between them. In general, scattering loss due to sidewall roughness of a waveguide is dominant in high-index-contrast silicon photonics, which is mainly caused in device fabrication and can thus be reduced by improving the fabrication processes [182–184].

Compared to propagation loss, chromatic dispersion and nonlinearity in integrated waveguides are more designable. Since the dispersion is the second-order derivative of the effective index with respect to wavelength, it is particularly tailorable by changing waveguide shape and dimension. Moreover, dispersion has been recognized to be critical for broadband nonlinear effects [12, 14, 15, 60, 61, 92–108, 112–115, 137–145], which is true especially for ultrafast octave-spanning applications [185]. Spectral characteristics in a dispersion profile, including the number and positions of zero-dispersion wavelengths (ZDWs) and dispersion slope, greatly affect and often set the limit on the bandwidth of optical spectra, the temporal widths of pulses, and conversion efficiency in nonlinear interactions [185]. Generally speaking, a flat dispersion profile (i.e., third- and higher-order dispersion terms are small) with low dispersion values is preferred.

In conventional ultrafast nonlinear optics in a free-space setup, many components were developed to control dispersion over a wide bandwidth [185, 186], such as prisms, gratings, chirped mirrors, and so on. However, in a waveguiding system, e.g., in fiber-based ultrafast optics, the dispersion-control toolkit is smaller, and engineering waveguide dispersion becomes critical. In particular, when waveguides are built on a silicon platform with a much higher index contrast than optical fibers, dispersion in a highly nonlinear waveguide [187–190] often shows strong wavelength dependence, which is not preferable for wideband nonlinear applications. In [187, 190], the ZDW in silicon rib and strip waveguides is mapped by scanning waveguide dimensions. It is shown that tight confinement of a guided mode produces a ZDW in its dispersion profile around 1.2–1.4 μm close to the bandgap wavelength. Moreover, even if the waveguide size is increased to move the ZDW to longer wavelength, the dispersion slope near the ZDW is not small, as shown in [187–189], causing a limited low-dispersion bandwidth.

Recently, a dispersion engineering technique for integrated high-index-contrast waveguides has been proposed, in which an off-center nano-scale slot controls modal distribution at different wavelengths [59, 60]. The guided mode experiences a transition from strip-mode like to slot-mode like as wavelength increases. This approach can produce a very flat dispersion profile over an ultra-wide bandwidth, with dispersion flatness improved by 1–2 orders in terms of dispersion variation divided by low-dispersion bandwidth. More importantly, it is applicable to different material combinations and wavelength ranges [59–63].

Towards mid-IR applications, different types of Group IV waveguides have been reported recently, based on silicon-on-insulator (SOI) [191–193], silicon-on-sapphire [142, 194, 195], silicon-on-nitride [196, 197], suspended membrane silicon [198], silicon pedestal [199], and germanium-on-silicon [200]. Most of the waveguides are not aimed specifically at nonlinear applications, and little attention has been paid to dispersion engineering [196].

In this section, we survey different structures of Group IV waveguides for broadband nonlinear applications from the near- to mid-IR. There are three main goals in waveguide designs: (i) we consider joint optimization on both dispersion and nonlinearity properties; (ii) we tend to fully utilize the available bandwidth brought by
the materials in Figure 1; and (iii) we emphasize dispersion engineering naturally as a result of aiming at octave-spanning broadband applications.

Figure 4 shows a general illustration of various types of integrated waveguides for nonlinear Group IV photonics. Looking at a specific wavelength range, one can accordingly choose a materials combination for an appropriate index contrast and a desired level of nonlinearity. Note that one may need low nonlinearity in some cases, when high-power output is required. Here we discuss waveguide design at four different wavelength ranges as follows.

First, we consider SOI waveguides for a wavelength range from the telecom window in the near-IR to the shortwave end in the mid-IR, i.e., roughly from 1.4 to 2.5 μm. This is the wavelength range that many of the current research efforts have been addressing [60, 61, 106, 108, 114, 115, 135, 137, 139–141, 143, 144]. In this wavelength range, a SOI strip waveguide as shown in Figure 4 can be used with air as an upper cladding (see e.g., [140]). One can change the width of the waveguide to tailor its dispersion profile, while the height of the waveguide is 220 nm set by SOI wafers. From Figure 5(A), we note that a relatively small width, W = 800 nm, is corresponding to a dispersion profile with two ZDWs at 1.585 and 2.345 μm and a peak value of anomalous dispersion, 532 ps/(nm·km) at 2.05 μm, for the quasi-TE mode. The anomalous dispersion is typically useful for parametric amplification and oscillation, soliton and soliton-based supercontinuum generation [12]. With W = 900 nm, one can have a flatter dispersion profile, but the anomalous band is smaller. When W is increased to 1000 nm, the dispersion is even flatter, but no anomalous dispersion occurs. Figure 5(A) shows a good example that tight mode confinement in a strip waveguide moves ZDW to short wavelengths, and near ZDWs dispersion changes quickly with a large slope.

![Figure 4](https://example.com/figure4.png)

**Figure 4** Different types of Group IV waveguides (WGs) for dispersion and nonlinearity engineering in the near- and mid-IR ranges.

One can calculate the nonlinear coefficient, γ, as a function of wavelength, with the nonlinear Kerr index, \( n_2 \), given in Tables 1–4 in Appendix B. We show in Figure 5(B) that the nonlinear coefficient in the silicon strip waveguide with W = 900 nm first increases to 187 / (m·W) with wavelength until 1.7 μm and then decreases to 56 / (m·W) at 2.5 μm. This is caused by both the peaking of the silicon \( n_2 \) value near 1.9 μm and the gradual increase of wavelength and mode area beyond that.

A silicon strip/slot hybrid waveguide exhibits very flat dispersion as presented in Figure 5(A). The SOI waveguide has crystalline silicon at the bottom, a thin SRO slot, and amorphous silicon at the top. The upper cladding is silicon dioxide. When setting the lower Si height to \( H_L = 430 \) nm, slot height to \( H_s = 54 \) nm, upper Si height to \( H_U = 160 \) nm, and width to \( W = 660 \) nm, we obtain an extremely flat dispersion profile for the quasi-TM mode over a wide bandwidth, between two ZDWs at 1.545 and 2.448 μm. From 1.605 to 2.38 μm, the value of anomalous dispersion changes between 30 and 46 ps/(nm·km). In this way, one can have a flatter and low anomalous dispersion between two far apart ZDWs. The average dispersion value can be shifted by increasing \( H_U \) to move dispersion between normal and anomalous regimes. Detailed explanation on how the flattened and saddle-shaped dispersion profile is produced is given in [59, 60]. Briefly, the mode transition over wavelength for the quasi-TM mode is responsible for this behavior. Due to the off-center slot, the mode is mostly confined in the crystalline silicon at short wavelengths, while the mode becomes more like a slot mode at long wavelengths. As shown in Figure 5(C), we plot the mode power distribution at wavelengths of 1.5, 1.83, 2.17, and 2.5 μm. The mode transition adds negative dispersion in the middle of the low-dispersion bandwidth, as explained in [60, 201].

Having a slot, one has an opportunity to fill the slot with highly nonlinear materials into it [49, 51, 53, 55, 56, 58, 59], which can overcome the decrease of the nonlinear coefficient over wavelength. In Figure 5(B), we show the \( \gamma \) value increasing to 306 / (m·W) with wavelength from 1.4 to 2.5 μm. This is because the guided mode extends more to the highly nonlinear thin slot layer. Note that the used \( n_2 \) value in SRO [38] is currently the one measured at 1.55 μm, so the \( n_2 \) and \( \gamma \) values may vary in the mid-IR, but the trend is general.

Next, we explore the short-wavelength end of the near-IR spectrum. Silicon-based devices become unusable for nonlinear photonics as wavelength decreases to 1.1 μm, and we thus look at silicon nitride for near-IR nonlinear applications extending to the visible light spectrum. Again, a strip waveguide based on silicon nitride is examined first. Figure 6(A) shows dispersion curves of the quasi-TE mode in two waveguides sized to be 1300 × 540
and 1400×800 nm². The upper cladding is air and the lower cladding is silicon dioxide. The anomalous dispersion region in the dispersion curves shrinks when the waveguide is made smaller. This is because of a relatively small index contrast between silicon nitride and silicon dioxide, which makes the guided mode leak quickly to the substrate as wavelength increases. For the strip waveguide with a cross-section of 1400×800 nm², there are two ZDWs near 1.0 and 2.3 µm, but one can see a strong dispersion of 250 ps/(nm·km) between the two ZDWs in Figure 6(A). The nonlinear coefficient in the second silicon nitride waveguide is shown in Figure 6(B), which is much smaller than that in silicon waveguides because of a one-order smaller n² value and larger Aeff in the silicon nitride waveguide. At 1.6 µm, γ is about 1.23/(m·W).

One can also use a strip/slot hybrid structure to tailor the dispersion profile in silicon nitride waveguides. For example, the slot and lower cladding are silicon dioxide, and the upper cladding is air. In Figure 6(A), we show the dispersion curves in two silicon nitride strip/slot hybrid waveguides for comparison. The waveguide #1 has Hl=900 nm, Hs=124 nm, Hu=340 nm, and W=1000 nm, and the waveguide #2 has Hl=920 nm, Hs=154 nm, Hu=480 nm, and W=1300 nm. These two waveguides produce increasingly flatter dispersion profiles, as shown in Figure 6(B). The first waveguide has two ZDWs located at 1.06 and 2.2 µm, with the peak dispersion of 67 ps/(nm·km). The second waveguide has two ZDWs at 1.15 and 2.35 µm, with the dispersion varying within 0~20 ps/(nm·km). This octave-spanning dispersion flattening with different levels
of dispersions can be used for multiple applications, as detailed in the next section. The nonlinear coefficients for the two waveguides are shown in Figure 6(B). We note that the strip/slot hybrid waveguides have similar nonlinear coefficients as the strip waveguide, which shows that the dispersion profile is much more tailorable by waveguide designs.

Then, we move to the mid-IR, using silicon and silicon nitride for waveguiding. A comparison of different types of silicon-on-nitride waveguides have been presented in [196], where rib waveguides were preferred due to the wideband low dispersion over an octave-spanning bandwidth from 2.4 to 6.6 µm for the quasi-TE mode. This is a spectral range from silicon's half-bandgap wavelength to the cut-off wavelength of silicon nitride. In Figure 7(A), we plot the dispersion curve for a silicon-on-nitride rib waveguide, with air as the upper cladding, the rib width of 2000 nm, the total height of 1200 nm, and the slab height of 1000 nm, which are the same parameters used in [196]. It is shown that less confinement of optical modes reduces the contribution of waveguide dispersion and makes the overall dispersion profile closer to the material dispersion, which is flat and low at long wavelengths as in Figure 2(B).

Accordingly, the nonlinear coefficient is small, 2.85 / (m·W) at 3 µm, as shown in Figure 7(B).

On the other hand, if one needs a small $A_{\text{eff}}$ to enhance nonlinearity, additional dispersion tailoring (e.g., based on strip/slot hybrid waveguides) would be beneficial. Pursuing a higher nonlinear coefficient, we use a 500-nm silicon nitride suspended membrane, as illustrated in Figure 4, to support a silicon strip/slot hybrid waveguide. This helps confine light in the waveguide core. Using $W=880$ nm, $H_u=550$ nm, $H_s=87$ nm, and $H_l=840$ nm, we obtain a saddle-shaped anomalous dispersion from 1.9 to 4.49 µm within 0~60 ps/(nm·km) for the quasi-TM mode, as shown in Figure 7(A). This structure produces much tighter mode confinement than the rib waveguide and exhibits a 3 times larger nonlinear coefficient in Figure 7(B) while having similar dispersion flatness.

Finally, we consider germanium-on-silicon waveguides over a wavelength range from 3.3 to 8.5 µm, between the half-bandgap wavelength of germanium and the cut-off wavelength of silicon. This type of waveguide has been demonstrated with strain-free mono-crystalline germanium [200]. Here, we assume that the germanium waveguide has a 10-nm silicon nitride layer on its
surface for passivation. The upper cladding could be air or silicon, which provides significantly different dispersion properties due to a varied index contrast. For comparison only, we also have silicon nitride as the upper cladding, although silicon nitride becomes lossy for wavelengths longer than 6.7 µm. Figure 8(A) shows the dispersion profiles of four germanium-on-silicon strip waveguides with equal size, 3000×1600 nm², for the quasi-TE mode. Air and silicon nitride as an upper cladding result in similar shape and bandwidth in the dispersion profiles. Thus, the air-cladded waveguide is chosen and discussed further. The waveguide with silicon upper cladding has normal dispersion at all wavelengths, since there is a relatively small index contrast between germanium and silicon and thus weak mode confinement. A germanium strip waveguide on a 600-nm-thick silicon suspended membrane is also considered to increase light confinement, with an air upper cladding to maximize light confinement. However, as mentioned earlier, strong confinement typically causes strong dispersion, as shown in Figure 8(A), and therefore the germanium waveguide on a silicon membrane is not chosen for broadband nonlinear applications. In contrast, the germanium strip waveguide with air upper cladding exhibits a flat and low dispersion.

The dimensions of the air-cladded germanium waveguide are varied by simultaneously changing its height and width with a step of 200 nm for both polarization states. It is interesting to see from Figure 8(B) and 8(C) that the dispersion peak value remains nearly unchanged for all the waveguide sizes, although we have a widely tunable ZDW at long wavelengths. For the quasi-TE mode, the right ZDW moves from 6.05 µm to 8.41 µm, while the left ZDW is always near 4 µm. We can thus obtain an octave-spanning anomalous dispersion band with the peak value below 100 ps/(nm·km). For the quasi-TM mode, one can see similar dispersion properties, but the anomalous dispersion band is smaller. Thus, we choose the quasi-TE mode for further discussion in next section.

The nonlinear coefficient in the germanium waveguides for the quasi-TE mode is shown in Figure 8(D), which is about 10/(m·W) at 5 µm with a small variation for different waveguide sizes. This is quite high, considering that both wavelength and effective mode area become much larger over this wavelength range, compared to the near-IR.

From above, we can see that the strip/slot hybrid waveguides enable unique controllability of dispersion, and that this concept is applicable to different wavelength ranges. However, their performance may be sensitive to fabrication errors, especially for inaccuracies in slot height, Hₜ, [59, 60]. A higher yield in device fabrication is expected using advanced fabrication technologies and facilities.

Figure 8 In a wavelength range covering the main part of the mid-IR spectrum, (A) on silicon substrate or on suspended silicon membrane are analyzed in terms of dispersion. Germanium-on-silicon strip waveguides with an air upper cladding and different dimensions are characterized by (B) dispersion for the quasi-TE mode, (C) dispersion for the quasi-TM mode, and (D) nonlinearity for the quasi-TE mode.
Besides photonic waveguides, another important category of nonlinear devices is integrated resonators. In the scope of this paper, we consider relatively large resonators for frequency comb generation, in which the bending radius of a ring resonator is varied from 50 µm to 100 µm, depending on free spectral range (FSR) and the group index in the waveguide. In these cases, the waveguide-bending-induced dispersion is small, and we would not discuss intra-cavity dispersion [202, 203] in details here.

4 Applications

Benefiting from the unique dispersion engineering over an octave-spanning bandwidth as described above, one can develop ultra-wideband nonlinear applications that could hardly be attained in an integrated platform previously. These include octave-spanning supercontinuum generation, pulse compression to a few-cycle or even sub-cycle level, octave-spanning Kerr frequency comb generation, and the associated mode-locked ultrashort pulse generation using microresonators. In this section, we review our recent work on these topics.

First, we discuss the supercontinuum generation and pulse compression in a straight waveguide. The nonlinear envelope equation used here to simulate supercontinuum generation is the following:

\[ \frac{\partial}{\partial z}A(z,t) = K(A) + R(A) \]  

where

\[ K(A) = \sum_{n=0}^{\infty} \frac{i^n}{n!} \left( \frac{i}{2} \right)^n \left[ \frac{i}{\omega_0} \frac{\partial}{\partial t} \right]^{n} \left[ A^{*} \frac{\partial^n}{\partial t^n} \left( A^{2} \right) \right] \]

and

\[ R(A) = i\gamma_{\text{R}} \left( 1-i\tau_{\text{shock}} \frac{\partial}{\partial t} \right) \left[ A^{*} \int_{-\infty}^{t} h_{\text{shock}}(t-t') |A|^{2} dt' \right]. \]

We denote \( A = A(z,t) \) as the complex amplitude of an optical pulse. Note that its Fourier transform is

\[ \tilde{A}(z, \omega) = \left( \frac{1}{2\pi} \right) \int_{-\infty}^{\infty} A(z, t) \exp(-i\omega t) dt. \]

In Eq. (1), \( \alpha \) is the total propagation loss, and \( \beta_{m} \) is the \( m \)th-order dispersion coefficient. The frequency dependence of nonlinearity parameters including the nonlinear index \( n_{2} \), the TPA coefficient \( \beta_{\text{TPA}} \), and the effective mode area \( A_{\text{eff}} \) is included in the \( n \)th-order dispersion coefficient \( \gamma_{n} \) of nonlinearity, which is defined as

\[ \gamma_{n} = \alpha_{n} \frac{\partial^{n}}{\partial \omega_{0}^{n}} \left[ \gamma(\omega_{0})/\omega_{0} \right], \]

where \( \omega_{0} \) is the angular frequency of the carrier. Therefore, we can consider all-order linear dispersion terms and all-order dispersion of the nonlinear coefficient in Eq. (1). Specifically in the simulations for a silicon and silicon nitride waveguides, we have all-order linear dispersion and up to 6th-order and 2nd-order of the nonlinear coefficient dispersion included. A detailed derivation of Eq. (1) is given in [204]. For the quasi-TM mode that experiences the engineered dispersion due to the mode transition, SRS in silicon waveguides fabricated on the (001) surface can be ignored [27, 96]. For silicon nitride waveguide, we include the SRS term in Eq. (1), where \( \gamma_{\text{R}} \) represents the Raman gain coefficient, the full width at half maximum of the gain spectrum, and the Raman shift, respectively. The Raman shock time \( \tau_{\text{shock}, R} \) is calculated to be

\[ \tau_{\text{shock}, R} = \frac{\Omega_{R}}{2}\left( \frac{\omega_{0}}{\gamma_{\text{R}}} \right)/\Omega_{R}, \]

where \( \Omega_{R} = \frac{\gamma_{\text{R}}}{\Gamma_{R}} / (A_{\text{eff}}) \), and \( \gamma_{\text{R}}, \Gamma_{R}, \) and \( \Omega_{R} \) represent the Raman gain coefficient, the full width at half maximum of the gain spectrum, and the Raman shift, respectively. The Raman shock time \( \tau_{\text{shock}, R} \) is associated with \( \gamma_{R} \)'s frequency dependence, which is \( 1/\Omega_{R} = 1/(\gamma_{\text{R}}/A_{\text{eff}})[dA_{\text{eff}}(\omega)/d\omega] \).

Similarly as in [14], if we ignore frequency dependent \( g_{\text{R}}, \Gamma_{R}, \) and \( \Omega_{R} \), \( h_{\text{R}}(t) \) is the Raman response function, and it corresponds to the Raman gain spectrum

\[ H_{\text{R}}(\omega) = \frac{\Omega_{R}^{2}}{\Omega_{R}^{2} (\omega-\omega_{0})^{2} + 2\Gamma_{R} (\omega-\omega_{0})}. \]

Note that the sign before the imaginary unit is different from that in [27] to be consistent with the expression of the Fourier transform that we used.

We have considered supercontinuum generation in both silicon and silicon nitride waveguides with the slot-assisted dispersion tailoring. Octave-spanning supercontinua in a silicon-based strip/slot hybrid waveguide have been investigated in detail in [108], in which two-cycle optical pulses are obtained. The main results in that work are shown in Figure 9 for comparison purposes.

Here, we mainly focus on the supercontinuum generated in the silicon nitride strip/slot hybrid waveguide (i.e., the slot WG #1 in Section III), whose dispersion and nonlinearity properties are shown in Figure 6. In the nonlinear simulations, we set the total propagation loss to be 1 dB/cm. The SRS parameters used here are the following [205]: \( \Omega_{R}/2\pi = 14.3 \) THz, \( \Gamma_{R}/2\pi = 1.72 \) THz, and \( g_{\text{R}} = 1 \times 10^{-12} \) m/W. \( \tau_{\text{shock}, R} \) is calculated to be 1.56 fs.

In our simulations, we use a time step of 0.25 fs, which is corresponding to a bandwidth of 4000 THz in the frequency domain. For a femtosecond input pulse, we set the time window length to 50 ps (i.e., frequency resolution \( \Delta f = 20 \) GHz).

We simulate the nonlinear propagation of a chirp-free hyperbolic secant pulse in the silicon nitride waveguide.
The pulse center wavelength is at 1610 nm, and its full width at half-maximum (FWHM) is 120 fs. Its peak power is 1200 W, corresponding to pulse energy of 0.16 nJ.

Figure 10(A) shows the supercontinua at different propagation distances. At 4.8 mm, the spectrum is greatly broadened at the -30 dB level, covering a wavelength range from 0.585 to 2.833 µm, which is more than two octaves. The spectrum evolution in Figure 10(A) shows a similar spectrum shape as that in Figure 9(A), both featuring a “triangular” central spectrum bounded by two dispersive waves at the edges. However, it is important to note that the absence of TPA and 3PA in silicon nitride at the telecom window leads to a much more efficient spectrum broadening than that in silicon [108]. The generated spectrum extends from the visible light to the mid-IR, with excellent spectral coherence, which is confirmed by the pulse waveform shown in Figure 10(B). In the time domain, the pulse is greatly compressed from 120 to 4.08 fs, corresponding to 0.76 optical cycles at 1.61 µm wavelength.

We examine the pulsewidth as a function of propagation distance. Figure 11 shows that the pulse becomes increasingly narrower until the propagation distance reaches 4.7 mm. For longer distances, the pulsewidth remains almost constant. However, it is important to mention that, after 4.8 mm, the dispersive waves become increasingly stronger as shown in Figure 10(A), causing larger pedestals.

Comparing the results in Figures 9 and 10, we note that the mid-IR wavelength range for silicon would be in analogy to the near-IR for silicon nitride in terms of nonlinear optics operations. Pumping at or beyond 3.3 µm, one can use the waveguide designs shown in Figure 7 to produce very efficient nonlinear interactions without TPA and 3PA in silicon. Ultrashort pulses in the mid-IR from parametric amplifiers [206, 207] could be used to pump the Group IV waveguides.

Another nonlinear application of the dispersion-engineered Group IV waveguides is micro-resonator-based Kerr frequency comb generation. When such a waveguide is curved to form a microring resonator, input CW light travels around the cavity and amplifies the noise in the source located at the frequencies with a high parametric gain. As a result of modulation instability and cascaded FWM in the cavity, a frequency comb can be generated [208–211]. Mode-locked frequency combs have been reported, producing low-noise pulse trains in time domain [212–214].
The formation of cavity solitons is identified as the main reason for the mode-locking in the Kerr frequency combs [215]. This is instructive, because one can thus predict the spectral bandwidth and temporal pulsewidth in the generated low-noise combs. The 3-dB comb bandwidth is inversely proportional to the square root of the 2nd-order dispersion coefficient, $|\beta_2|$, as given in [213, 215]. It is desirable that all comb lines that constitute the soliton spectrum experience the same $|\beta_2|$. In this sense, the strip/slot hybrid waveguides with flattened dispersion are preferably suitable for supporting broadband Kerr comb generation and ultra-short cavity soliton generation.

The Kerr frequency comb generation can be modeled using the generalized Lugiato-Lefever equation (LLE) [216–219]:

$$
\frac{d}{dt} E(t, \tau) + \frac{\alpha}{2} E(t, \tau) + \frac{k}{2} \frac{d}{d\tau} E(t, \tau) + j \sum_{m=2}^{\infty} \left( \frac{j}{m!} \beta_m \frac{d^m}{d\tau^m} E(t, \tau) \right) = \sqrt{k} E(t, \tau) |E(t, \tau)|^2 E(t, \tau) 
$$

(2)

where $t_R$ is the round-trip time, $E=E(t, \tau)$ and $E_{in}$ are intra-cavity field and input field (pump power $P_{in} = |E_{in}|^2$), $t$ and $\tau$ are the slow and fast times. $\delta_0$ is the cavity phase detuning defined as $\delta_0 = t_R (\omega_0 - \omega_n)$, where $\omega_0$ and $\omega_n$ are the pump’s angular frequency and the $n$th angular resonance frequency that is pumped. Other resonator parameters include the power loss per round trip $\alpha$, the power coupling coefficient $\kappa$, the nonlinear coefficient $\gamma$, and the $m$th dispersion coefficient $\beta_m$. Since a flattened dispersion profile has a small $\beta_2$ over a wide bandwidth, it is important to take the influence of higher-order dispersion into account. We include all-order dispersion terms in Eq. (2), as we did in solving Eq. (1).

To enhance the Kerr comb bandwidth in the near-IR, we use the ultra-flattened dispersion profile in Figure 6(A), which is obtained in the slot WG #2 based on silicon nitride. The ring resonator has a bending radius of 104 µm, corresponding to a FSR of 200 GHz. Pumping near 1.55 µm with a pump power of 2 W, the resonance peak is red-shifted, and we need to red-shift the pump wavelength accordingly and tune it into the resonance from the short-wavelength side. When the pump is step-by-step tuned by up to 63 resonance linewidths, we obtain the comb spectrum and the mode-locked pulse waveform as shown in Figure 12. One can see that, over an octave-spanning bandwidth from 133 to 268 THz, the comb lines have a power drop by 20 dB from the center of the spectrum. The spectral flatness of this comb is relatively good, compared to the previously reported results [208–211]. The comb bandwidth at -40 dB is as wide as two octaves. There are two dispersive peaks in normal dispersion regions beyond the low-dispersion band. Such a mode-locked broadband comb produces a train of sub-two-cycle optical pulses, as shown in Figure 12(B), with one pulse per round trip. The peak power of the pulse is up to 600 W. Nonlinear conversion efficiency is estimated to be -26.7 dB.

To generate frequency combs in the mid-IR, the germanium-on-silicon strip waveguide is chosen. We choose...
A cross-section of 3200×1800 nm², which has an octave-spanning low-dispersion band from 4 to 7.67 µm, as shown in Figure 8(B). A germanium ring resonator is formed with a bending radius of 56.4 µm, corresponding to a FSR of 200 GHz. Pumping at 6 µm with a CW power of 1.4 W and detuning the pump wavelength by 10 resonance linewidth, one can see that a mode-locked wideband mid-IR comb is generated from 35.8 to 64.4 THZ (i.e., from 4.66 to 8.38 µm) at -40 dB level. The FWHM of the produced pulses is 69 fs, which corresponds to ~3.5 optical cycles. The pulse peak power is 184 W, and the nonlinear conversion efficiency is estimated to be -14.3 dB. Since the pumping frequency is not at the center of the low-dispersion band, we only see one peak in the comb spectrum caused by the dispersive wave in the normal dispersion region from Figure 13(A). There is another peak at higher frequencies beyond what is shown in the figure.

As shown above, broadband dispersion engineering is critical for octave-spanning nonlinear applications in both near- and mid-IR wavelength ranges, which enables us to fully utilize the bandwidth allowed by the materials transparency windows. Generally speaking, the nonlinear applications mentioned here, such as supercontinuum generation, ultrafast pulse compression, and frequency comb generation, are often the intermediate steps towards higher-level system applications. In the frequency domain, a wide spectrum can serve as an electromagnetic carrier to acquire high-volume of information, e.g., for sensing [220] and imaging [221]. In the time domain, an ultrashort pulse can be used as probe to sample ultrafast phenomena [222].

5 Summary and Outlook

We have presented a review of our recent work on nonlinear photonics based on silicon and germanium. Various types of Group IV waveguides are analyzed and optimized for four different wavelength ranges, from near- to mid-IR. The recently proposed dispersion engineering technique based on strip/slot hybrid waveguide structures is used for different material combinations and wavelength ranges. Numerical simulations show that the dispersion-flattened Group IV waveguides are preferably suitable for octave-spanning nonlinear applications, including on-chip supercontinuum generation, ultrashort pulse compression, and mode-locked wideband frequency comb generation based on micro-resonators.

The presented approach to achieving octave-spanning nonlinear applications on an integrated, CMOS-compatible Group IV platform holds great potential for realizing chip-scale sensing, imaging, communications, and signal processing system. The ultrawide transparency windows in the mid-IR, allowed by Group IV elements and compounds potentially together with other materials [223], provide an exciting arena for building powerful information acquisition and processing units, enabled by nonlinear optics, nano-photonics, and ultrafast optics.

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Appendix

A. Material index and dispersion

In this section, we give the wavelength-dependent material index expressed as Sellmeier equations, where wavelength, \( \lambda \), is in \( \mu m \).

For silicon, we use the following material index that is a fit curve from measurement results at room temperature (293 K), with 184 data points in total, from 1.12 to 588 \( \mu m \) [154]:

\[
n^2(\lambda) = \varepsilon + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2}
\]

where \( \varepsilon = 11.6858 \), \( C_1 = 0.939816 \) \( \mu m^2 \), \( C_2 = 0.00810461 \), and \( \lambda_1 = 1.071 \) \( \mu m \).

For silicon nitride, the material index is affected by deposition conditions using plasma-enhanced chemical vapor deposition (PECVD), low-pressure chemical vapor deposition (LPCVD), and so on. We use the following Sellmeier equation [155], which predicts the material index close to that in LPCVD silicon nitride films measured by a few groups [224]:

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2}
\]

where \( C_1 = 2.8939 \) and \( \lambda_1 = 0.13967 \) \( \mu m \).

For silicon dioxide, we use the following Sellmeier equation for fused silica [156]:

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2}
\]

where \( C_1 = 0.6961663 \), \( C_2 = 0.4079426 \), \( C_3 = 0.8974794 \), \( \lambda_1 = 0.0684043 \) \( \mu m \), \( \lambda_2 = 0.1162414 \) \( \mu m \), and \( \lambda_3 = 9.896161 \) \( \mu m \).

For SRO, the material index is affected by deposition conditions such as silicon excess, annealing temperature, and so on. Here we choose the one with silicon excess of 8% and annealed at 1250°C [51]:

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2}
\]

where \( C_1 = 0.01 \), \( C_2 = 1.96 \), \( C_3 = 1.41 \), \( \lambda_1 = 0.3 \) \( \mu m \), \( \lambda_2 = 0.07071 \) \( \mu m \), and \( \lambda_3 = 27.75968 \) \( \mu m \).

For germanium, the temperature-dependent material index was measured [157]. Here we choose the one for room temperature (293 K):

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2}
\]

where \( C_1 = 9.28156 \), \( C_2 = 6.7288 \), \( C_3 = 0.21307 \), \( \lambda_1 = 0.664116 \) \( \mu m \), and \( \lambda_2 = 62.21013 \) \( \mu m \).

For arsenic sulfide, we use the material index provided in [158]:

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2} + \frac{C_4 \lambda^2}{\lambda^2 - \lambda_4^2} + \frac{C_5 \lambda^2}{\lambda^2 - \lambda_5^2}
\]

where \( C_1 = 1.8983678 \), \( C_2 = 1.9222979 \), \( C_3 = 0.8765134 \), \( C_4 = 0.1188704 \), \( C_5 = 0.9569903 \), \( \lambda_1 = 0.15 \) \( \mu m \), \( \lambda_2 = 0.25 \) \( \mu m \), \( \lambda_3 = 0.35 \) \( \mu m \), \( \lambda_4 = 0.45 \) \( \mu m \), and \( \lambda_5 = 27.386128 \) \( \mu m \).

For arsenic selenide, we use the material index provided by Prof. Kathleen A. Richardson group:

\[
n^2(\lambda) = 1 + \frac{C_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{C_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{C_3 \lambda^2}{\lambda^2 - \lambda_3^2}
\]

where \( C_1 = 2.98463 \), \( C_2 = 3.21011 \), \( C_3 = 100.182 \), \( \lambda_1 = 0.44118 \) \( \mu m \), \( \lambda_2 = 0.000354953 \) \( \mu m \), and \( \lambda_3 = 384.13 \) \( \mu m \).

B. Nonlinear Kerr index \( n_2 \)

The third-order nonlinear susceptibility \( \chi^{(3)} \) for silicon and germanium is predicted over the mid-IR range [133], based on a two-band model. The effective nonlinear susceptibility \( \chi^{(3)} \) is dependent on polarization and crystallographic orientation [225]. For strong nonlinearity, we consider a single-polarization incident light aligned to the crystallographic axis, and we have \( \chi^{(3)} = \chi_{\text{III}}^{(3)} \). To investigate the octave-spanning nonlinear phenomena, one need to take the wavelength-dependent nonlinear Kerr index \( n_2 \) and TPA coefficient \( \beta_{\text{TPA}} \) into account, which are expressed as

\[
n_2(\lambda) = \frac{3}{4 \varepsilon_0 c n^2(\lambda)} \chi^{(3)}(\lambda)
\]

\[
\beta_{\text{TPA}}(\lambda) = \frac{3 \pi}{4 \varepsilon_0 c n^2(\lambda)} \chi^{(3)}(\lambda)
\]

where \( \varepsilon_0 \) and \( c \) are the vacuum permittivity and the speed of light in vacuum. Using the material index given in Appendix A and \( \chi^{(3)} \) value from [133], we obtain the \( n_2 \) and \( \beta_{\text{TPA}} \) values tabulated as follows.
Table 1 Nonlinear Kerr index $n_2$ in silicon.

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<th>$n_2$ (10^{-18} m^2/W-TPA)</th>
<th>$\lambda$ (μm)</th>
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Table 2 TPA coefficient $\beta_{\text{TPA}}$ in silicon.

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Table 3 Nonlinear Kerr index $n_2$ in germanium.

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Table 4 TPA coefficient $\beta_{\text{TPA}}$ in germanium.

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<th>$\lambda$ (μm)</th>
<th>$\beta_{\text{TPA}}$ (10^{-18} m/W)</th>
<th>$\lambda$ (μm)</th>
<th>$\beta_{\text{TPA}}$ (10^{-18} m/W)</th>
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References


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