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Abstract

We study the energy loss of a heavy quark propagating through the matter produced in the collision of two sheets of energy [1]. Even though this matter is initially far-from-equilibrium we find that, when written in terms of the energy density, the equilibrium expression for heavy quark energy loss describes most qualitative features of our results well. At later times, once a plasma described by viscous hydrodynamics has formed, the equilibrium expression describes the heavy quark energy loss quantitatively. In addition to the drag force that makes it lose energy, a quark moving through the out-of-equilibrium matter feels a force perpendicular to its velocity.

The discovery that strongly coupled quark-gluon plasma (QGP) is produced in ultrarelativistic heavy ion collisions has prompted much interest in the real-time dynamics of strongly coupled non-Abelian plasmas. For example, heavy quark energy loss has received substantial attention. If one shoots a heavy quark though a non-Abelian plasma how much energy does it lose as it propagates? This question has been answered [2] for equilibrium plasma in strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in the large number of colors N_c limit, where holography permits a semiclassical description of energy loss in terms of string dynamics in asymptotically AdS$_5$ spacetime [3, 4, 2]. One challenge (not the only one) in using these results to glean qualitative insights into heavy quark energy loss in heavy ion collisions is that a heavy quark produced at time zero must first propagate through the initially far-from-equilibrium matter produced in the collision before it later plows through the expanding, cooling, near-equilibrium strongly coupled QGP. In this contribution we sketch our work in progress toward obtaining guidance for how to meet this challenge. We want a toy model in which we can reliably calculate how the energy loss rate of a heavy quark moving through the far-from-equilibrium matter present just after a collision compares to that in strongly coupled plasma close to equilibrium.

We study the energy loss of a heavy quark moving through the debris produced by the collision of planar sheets of energy in strongly coupled SYM theory analyzed in Ref. [1]. The incident sheets of energy move at the speed of light in the $+z$ and $-z$ directions and collide at $z = 0$ at time $t = 0$. They each have a Gaussian profile in the z direction and are translationally invariant in the two directions $\boldsymbol{x}_\perp = \{x_1, x_2\}$ orthogonal to z. Their energy density per unit transverse area is $\mu^3 (N_c^2/2\pi^2)$, with μ an arbitrary scale with respect to which all dimensionful quantities in the conformal theory that we are working in can be measured. The width σ of the Gaussian energy-density profile of each sheet is chosen to be $\sigma = 1/(2\mu)$.

We study heavy quark energy loss by inserting a heavy quark moving at constant velocity $\vec{\beta}$ between the colliding sheets before the collision and calculating the force needed to keep its velocity constant throughout the collision. The energy density of the colliding sheets as well as

two sample trajectories of a quark getting sandwiched between them are shown in the left panels of Figs. 1 and 2. Via holography, the colliding planar sheets in SYM theory map into colliding planar gravitational waves in asymptotically AdS$_5$ spacetime [1]. The addition of a heavy quark moving at constant velocity $\vec{\beta}$ amounts to including a classical string attached to the boundary of the geometry [4] and dragging the string endpoint at constant velocity $\vec{\beta}$, pulling the string through the colliding gravitational wave geometry. The force needed to maintain the velocity of the string endpoint yields the energy and momentum loss rates of the heavy quark [2].

We employ infalling Eddington-Finkelstein coordinates to describe the asymptotically AdS$_5$ geometry. Translation invariance in the $\boldsymbol{x}_\perp$ plane allows one to write the metric G_{MN} in the form

$$ds^2 = G_{MN}dX^M dX^N = -A\,dv^2 + \Sigma^2\left(e^B d\boldsymbol{x}_\perp^2 + e^{-2B}dz^2\right) + 2\,F\,dv\,dz - 2\,du\,dv/u^2 \tag{1}$$

where A, B, Σ and F depend on coordinates v, z and u. Here u is the AdS radial coordinate with the AdS boundary located at $u = 0$. Lines of constant v and $(z, \boldsymbol{x}_\perp)$ are radially infalling null geodesics affinely parameterized by u. At $u = 0$ the time coordinate v coincides with the time coordinate t in SYM. For initial conditions consisting of two colliding gravitational waves, the functions A, B, F and Σ were obtained numerically in Ref. [1] by solving Einstein's equations.

We obtain the string equations of motion from the Polyakov action

$$S_{\mathrm{P}} = \int d\sigma d\tau \mathcal{L}_P = -\frac{T_0 L^2}{2} \int d\sigma d\tau\, \sqrt{-\eta}\eta^{ab}G_{MN}\partial_a X^M \partial_b X^N \tag{2}$$

where T_0 and L are the string tension and AdS radius and where η_{ab} is the worldsheet metric with τ and σ being worldsheet coordinates chosen such that

$$\eta_{ab} = \begin{pmatrix} -\alpha(\tau,\sigma) & -1 \\ -1 & 0 \end{pmatrix} \tag{3}$$

with $\alpha(\tau,\sigma)$ to be determined. With this choice of worldsheet coordinates, which are the worldsheet analogue of infalling Eddington-Finkelstein coordinates, lines of constant τ are infalling null worldsheet geodesics and the string equations of motion take the particularly simple form

$$\partial_\sigma \dot{X}^N + \Gamma^N_{AB}\dot{X}^A \partial_\sigma X^B = 0 \quad \text{with} \quad \dot{X}^M \equiv \partial_\tau X^M - \tfrac{1}{2}\,\alpha(\tau,\sigma)\,\partial_\sigma X^M\,, \tag{4}$$

where Γ^N_{AB} are Christoffel symbols. $\dot{X}^M$ is the directional derivative along outgoing null worldsheet geodesics. The constraint equations are $(\partial_\sigma X)^2 = 0$ and $\dot{X}^2 = 0$. The first is a temporal constraint: if it is satisfied at one value of τ and the other equations are satisfied, it remains satisfied at all times. $\dot{X}^2 = 0$ is a boundary constraint: if it is satisfied at one value of σ and all other equations are satisfied, it is satisfied at all values of σ. Residual worldsheet diffeomorphism invariance allows us to choose $u = \sigma$. Eqs. (4) now prescribe the τ-evolution of X^M, as follows: given X^M at one τ, use (4) to compute $\dot{X}^M$; with $u = \sigma$ the definition of $\dot{X}^5$ becomes $\alpha = -2\,\dot{X}^5$; with α in hand, construct $\partial_\tau X^M$; finally, determine X^M at the next τ. To execute this procedure and obtain $v(\tau,\sigma)$, $z(\tau,\sigma)$ and $\boldsymbol{x}_\perp(\tau,\sigma)$ we first need an initial profile of the string that satisfies $(\partial_\sigma X)^2 = 0$ and we need to impose boundary conditions at $u = 0$, requiring that the endpoint of the string at $u = 0$ moves at constant velocity $\vec{\beta}$ and that $\dot{X}^2 = 0$ remains satisfied at $u = 0$. After we have solved (4) and discovered how the string is buffeted by the colliding sheets, we compute the canonical momentum flux π^σ_μ down the string at $u = 0$. This flux is minus the force required to maintain the velocity of the string endpoint and yields the four momentum lost by the quark per unit time

$$\frac{dp_\mu}{dt} = \pi^\sigma_\mu = \frac{\sqrt{\lambda}}{2\pi L^2}G_{\mu\nu}\Big(\partial_\tau X^\nu - \alpha(\tau,\sigma)\,\partial_\sigma X^\nu\Big)\Big|_{u=0}\,, \tag{5}$$
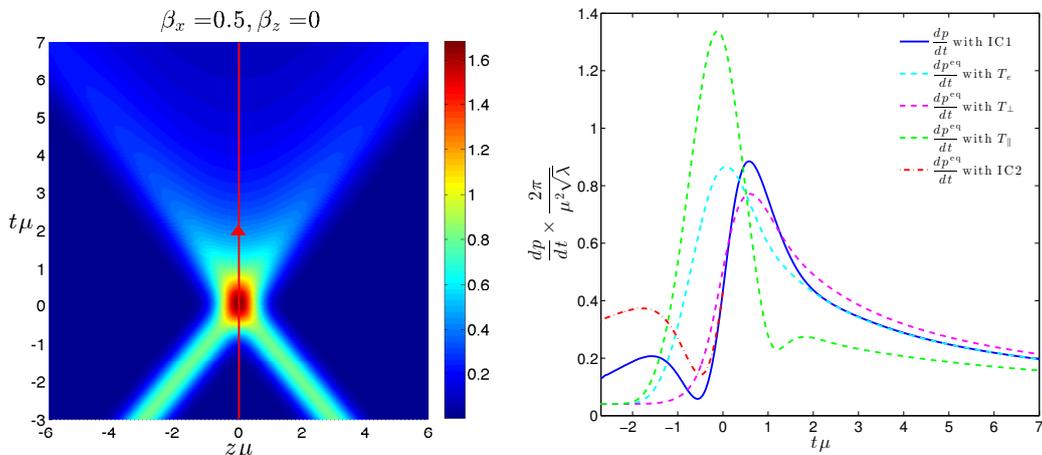
Figure 1: Left: The path in (t, z) of a quark moving in the x-direction with $\beta = 0.5$ along $z = 0$. Energy density of the colliding sheets (in units of $N_c^2 \mu^4/(2\pi^2)$) is shown by the color. Right: Instantaneous momentum loss of a heavy quark with two different initial conditions for the string profile (solid blue and dot-dashed red curves). The results agree for $t > 0$, illustrating that our calculation of the momentum loss after the collision is insensitive to the initial string profile. The dashed cyan, magenta or green curves show what the momentum loss would be in an equilibrium plasma with temperature T_e, $T_\perp$ or $T_\parallel$ determined from the instantaneous energy density, transverse pressure or parallel pressure.

where $\lambda = g^2 N_c$ is the 't Hooft coupling in the gauge theory. We choose several sets of initial conditions for the string and demonstrate that they all yield the same dp_μ/dt at suitably late times.

The instantaneous momentum loss $d\vec{p}/dt$ experienced by the heavy quark is given by the solid curves in Figs. 1 and 2. These curves are the central results of our calculation. Fig. 1 illustrates that we are initializing the string profile early enough that our results for the drag force are sensitive to how we do this only before the collision happens, not after the sheets collide ($t > 0$). In Fig. 2, the quark is at $z = 0$ at $t = 0$ but moves off the $z = 0$ plane with some nonzero rapidity. In this case, in the frame in which we do the calculation the fluid is not at rest at the location of the quark for $t > 0$. However, in Fig. 2 we plot $d\vec{p}/dt$ in the local fluid rest frame, meaning that at each instant in time we transform to a frame in which the fluid at the location of the quark is static, but of course not in equilibrium. In particular, when the quark is moving with nonzero rapidity the gradient of the fluid velocity at the location of the quark is nonvanishing. The blue curve shows the component of $d\vec{p}/dt$ in the direction opposite to the velocity vector of the quark in the local fluid rest frame. This is a drag force, just as in the case of motion with zero rapidity in Fig. 1. The red curve in Fig. 2 shows the component of $d\vec{p}/dt$ perpendicular to the quark velocity in the local fluid rest frame. A perpendicular force like this can arise because of the presence of gradients in the fluid velocity. (We define the sign of the perpendicular component such that $dp_\perp/dt > 0$ when $\left| dp_z^{\text{eq}}/dt \right|$, see below, is greater than $|dp_z/dt|$.)

We are interested in comparing our result to expectations based upon the classic result for the drag force required to move a heavy quark with constant velocity $\vec{\beta}$ through the *equilibrium* plasma in strongly coupled SYM theory, namely [2]

$$\left. \frac{d\vec{p}}{dt} \right|_{\text{eq}} = \frac{\pi}{2} \sqrt{\lambda} T^2 \frac{\vec{\beta}}{\sqrt{1 - \beta^2}} \tag{6}$$

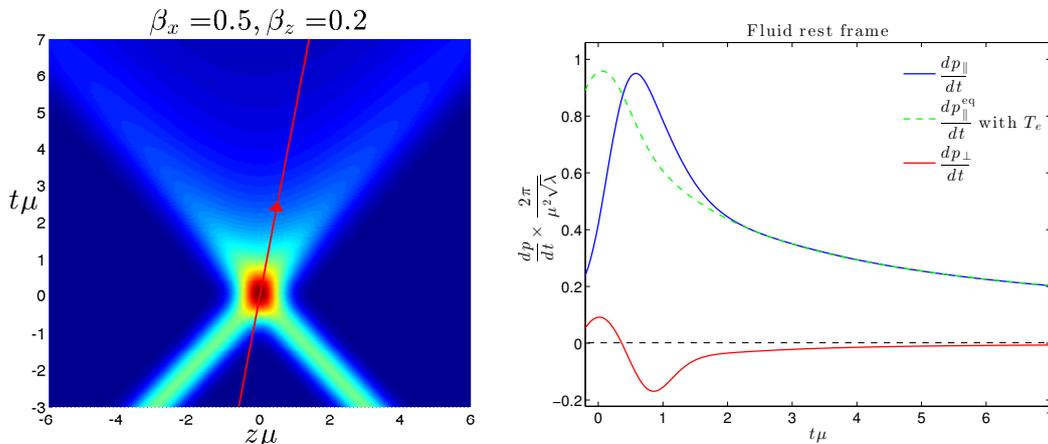where T is the temperature of the equilibrium plasma. Out of equilibrium, the matter does not

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Figure 2: Left: The path in (t, z) of a quark with nonzero rapidity, specifically $\beta_z = 0.2$ and $\beta_x = 0.5$. Right: Instantaneous momentum loss in the fluid rest frame, antiparallel to the quark velocity (solid blue) and perpendicular to it (solid red). Green dashed curve shows the drag force in an equilibrium plasma with temperature T_e.

have a single temperature. We can nevertheless use (6) to frame expectations for $d\vec{p}/dt$ at any point in spacetime, as follows. We transform to the local fluid rest frame and define three "temperatures" from the stress-energy tensor by writing it as $T_{\mu\nu} = (\pi^2 N_c^2/8) \operatorname{diag}\left(T_e^4, T_\perp^4, T_\perp^4, T_\parallel^4\right)$. (In static equilibrium, $T_e = T_\perp = T_\parallel = T$.) We can then use each of these three "temperatures" in the equilibrium expression (6) and see to what degree the three curves that result bracket our result for the energy loss of a heavy quark in far-from-equilibrium matter. We do this in Fig. 1. The green dashed curve in Fig. 2 shows the force (6) in an equilibrated plasma with $T = T_e$. In an equilibrated plasma, the quark only feels a drag force, opposing its velocity; the component of $d\vec{p}/dt$ perpendicular to the velocity of the quark vanishes. The red curve in Fig. 2 arises because the heavy quark is propagating through out-of-equilibrium matter featuring velocity gradients.

With the exception of the (small) component of $d\vec{p}/dt$ depicted by the red curve in Fig. 2, our results are rather well described by the equilibrium expression (6) with $T = T_e$. At late times, we see from Figs. 1 and 2 that the drag force that we have calculated agrees quantitatively with this equilibrium expectation. This agreement begins at the same time that viscous hydrodynamics starts to describe the still far-from-isotropic fluid well [1]. At early times, when the fluid is far from equilibrium, (6) with $T = T_e$ does a reasonable job of characterizing our results, although perhaps using $T = T_\perp$ does even better. Certainly there is no sign of any significant "extra" energy loss arising by virtue of being far from equilibrium. The message of our calculation seems to be that if we want to use (6) to learn about heavy quark energy loss in heavy ion collisions, it is reasonable to apply it throughout the collision, even before equilibration, defining the T that appears in it through the energy density. The error that one would make by treating the far-from equilibrium energy loss in this way is likely to be much smaller than other uncertainties.

References

[1] P. M. Chesler and L. G. Yaffe, Phys. Rev. Lett. **106**, 021601 (2011) [arXiv:1011.3562 [hep-th]].

[2] C. P. Herzog *et al.*, JHEP **0607**, 013 (2006) [hep-th/0605158]; S. S. Gubser, Phys. Rev. D **74**, 126005 (2006) [hep-th/0605182]; J. Casalderrey-Solana and D. Teaney, *ibid.* 085012 [hep-ph/0605199].

[3] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200]; E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].

[4] A. Karch and E. Katz, JHEP **0206**, 043 (2002) [hep-th/0205236].