22.312 **ENGINEERING OF NUCLEAR REACTORS**

Tuesday, December 14th, 2004, 1:30-4:30 pm

OPEN BOOK FINAL EXAM 3 HOURS

SOLUTIONS

Problem 1 (55%) – Structural and thermal-hydraulic analysis of a PWR pressurizer

i) The primary general membrane stresses for a thin cylindrical shell can be calculated as:

$\sigma_r = -(p_i+p_o)/2$
$\sigma_\theta = (p_i-p_o)R/t$
$\sigma_z = (p_i-p_o)R/2t$

So the primary general membrane stress intensity, P_m, is $(p_i-p_o)R/t -[-(p_i+p_o)/2]$, and the ASME code mandates that

$P_m < S_m$

Solving for t,

$t > R(p_i-p_o)/[S_m - (p_i+p_o)/2] = 9.6$ cm

with R=1 m, p_i=17 MPa, p_o=0.1 MPa, S_m=180 MPa.

ii) Taking the pressurizer as the control volume (CV), the conservation of mass equation gives the final coolant mass in the pressurizer, M_2:

$$\frac{\partial M_{CV}}{\partial t} = -\dot{m}_0 \Rightarrow \text{(integrating)} \quad M_2 - M_1 = -\int_{t_1}^{t_2} \dot{m}_o dt \qquad (1)$$

where $\int_{t_1}^{t_2} \dot{m}_o dt$ =M_o=3,500 kg/s, and M_1 can be easily found from the initial liquid and vapor volumes and densities (see Table 1 in the problem statement).
The conservation of energy equation (with negligible gravitational and kinetic terms) is:

$$\frac{\partial U_{CV}}{\partial t} = -\dot{m}_0 h_f + \dot{Q} \Rightarrow \text{(integrating)} \quad U_2 - U_1 = -h_f \int_{t_1}^{t_2} \dot{m}_o dt + \int_{t_1}^{t_2} \dot{Q} dt = -h_f M_o + Q \qquad (2)$$

where h_f (=1,631 kJ/kg) is the enthalpy of the water leaving the pressurizer, Q is the total heat supplied by the heaters, $U_1=M_1(u_f+x_1u_{fg})$ can be readily found from the initial conditions. The final internal energy, U_2, is:

$$U_2=M_2(u_f+x_2u_{fg}) \qquad (3)$$

where x_2 is the (unknown) final quality in the pressurizer. Also, the volume of the pressurizer, V_p, does not change during the transient, so:

$$V_p=M_2(v_f+x_2v_{fg}) \qquad (4)$$

Equations 1, 2, 3 and 4 constitute a system of four equations in four unknown (i.e., M_2, U_2, Q and x_2), from which Q can be found.

iii) The answer in "ii" does not depend on the duration of the process, but only on the initial and final conditions of the system.

iv) The total heater power, $\dot{Q}$ (=3 MW), is related to the heat flux as:

$$\dot{Q} = 30\,\pi\,d\,L\,q''$$

Thus the minimum heater length is:

$$L > \dot{Q}/(30\,\pi\,d\,q''_{DNB}) = 1.13 \text{ m}$$

where d=1 cm and q''_{DNB}=2.8 MW/m^2 from the boiling curve.

v) The DNB heat flux increases with pressure at low pressure, while it decreases at high pressure (e.g., see Figure 12-3, T&K, Vol. 1, page 527). Therefore, if the pressure in the pressurizer significantly decreased while the heaters are at full power, meeting the no-CHF criterion would depend on the magnitude of the pressure reduction. For example, for a final pressure of 10 MPa, the no-CHF criterion would still be met, because the DNB heat flux at 10 MPa is higher than at 15.5 MPa.

Problem 2 (35%) – Void fraction and pressure drop in an isolation condenser.

i) Saturated steam is completely condensed at a rate of 50 kg/s. Thus, the heat removal rate can be calculated from the energy balance as:

$$\dot{Q} = \dot{m}(h_o - h_i) = \dot{m}h_{fg} = 77.15 \text{ MW}$$

where the subscripts "o" and "i" refer to the tube outlet and inlet, respectively.

ii) The average heat flux at the inner surface of the tubes, q'', is:

$$q'' = \frac{\dot{Q}}{N_{tubes}\pi DL} \approx 341 \text{ kW/m}^2$$

The inner wall temperature can be readily found from Newton's law of cooling:

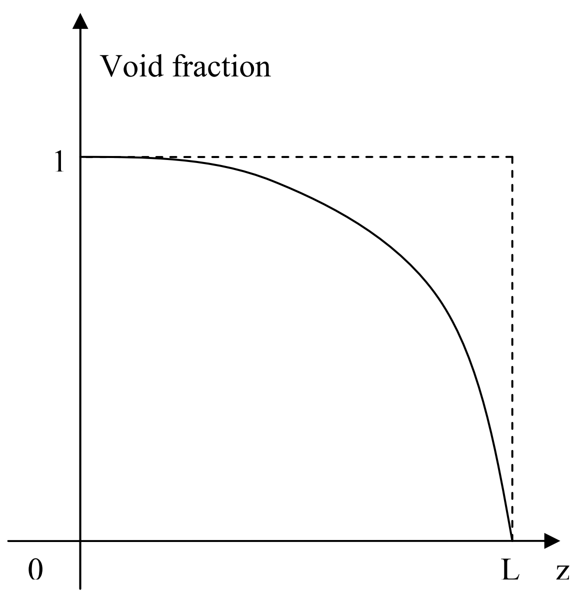
$$q'' = h_D(T_{sat} - T_w) = 0.555\left[\frac{g\rho_f(\rho_f - \rho_g)k_f^3 h_{fg}}{\mu_f D}\right]^{1/4}(T_{sat} - T_w)^{3/4}$$

where h_D is from the simplified Chato correlation. Solving for T_w, one gets $T_w \approx 216°C$.

iii) For HEM the void fraction can be calculated as:

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_f} \cdot \frac{1-x}{x}}$$

where x is the flow quality, assumed to vary linearly with the axial location, z, from 1 (inlet) to 0 (outlet), i.e., x(z)=1-z/L. This equation can be used to sketch α vs. z, as shown below.



iv) The gravity pressure drop is zero, because the tubes are horizontal.

Acceleration:

$$\Delta P_{acc} = G^2\left(\frac{1}{\rho_o^+} - \frac{1}{\rho_i^+}\right) \approx \text{-3,594 Pa}$$

where the subscripts "o" and "i" refer to the tube outlet and inlet, respectively. Note that in this case, $\rho_i^+ = \rho_g$=33.3 kg/m^3, $\rho_o^+ = \rho_f$=769.2 kg/m^3, and G=50/200/(π/4×0.03^2) ≈ 354 kg/m^{2}s.

Friction:

$$\left(\frac{dP}{dz}\right)_{fric} = f_{TP}\frac{1}{D}\cdot\frac{G^2}{2\rho_{TP}} \qquad (5)$$

where D=3cm, $f_{TP} = f_{\ell o} = \frac{0.184}{\mathrm{Re}_{\ell o}^{0.2}}$ ≈0.018 ($\mathrm{Re}_{\ell o} = \frac{GD}{\mu_f}$≈108,400). Also, for HEM, $\frac{1}{\rho_{TP}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_f}$. Thus, equation 5 can be integrated:

$$\Delta P_{fric} = \int_0^L f_{TP}\frac{1}{D}\frac{G^2}{2\rho_{TP}}dz = f_{\ell o}\frac{L}{D}\frac{G^2}{2}\int_0^1\left[\frac{x}{\rho_g} + \frac{1-x}{\rho_f}\right]dx = f_{\ell o}\frac{L}{D}\frac{G^2}{2}\left(\frac{x^2/2}{\rho_g} + \frac{x - x^2/2}{\rho_f}\right)\Bigg|_0^1 \approx 7{,}060 \text{ Pa}$$

where again a linear variation of x along the tubes has been assumed (i.e., x(z)=1-z/L).

Total:

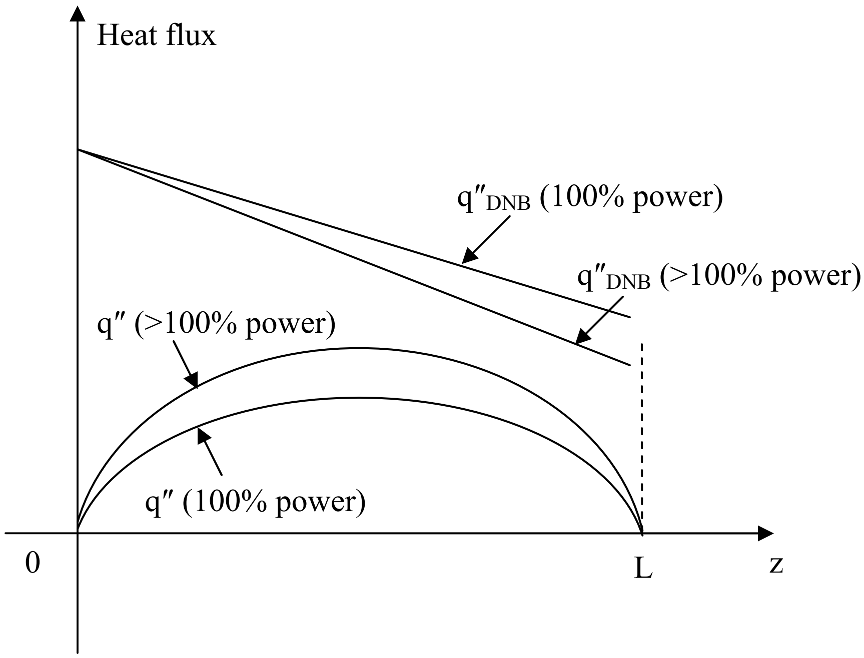
$-\Delta P_{tot} = \Delta P_{acc} + \Delta P_{fric}$ ≈ -3,594+7,060 = 3,466 Pa

I.e., the pressure at the tube outlet is lower than at the inlet.

v) The acceleration and friction pressure drops would not change. The gravity pressure drop would be negative because the flow direction is downward.

Problem 3 (10%) – Departure from Nucleate Boiling in a PWR

The mass flow rate, pressure and inlet bulk temperature (and thus the inlet equilibrium quality) are fixed. If the reactor power increased, the equilibrium quality would increase in the channel, thus, the DNB heat flux would decrease. The heat flux and DNB curves for the higher power level are shown below. Note that obviously the MDNBR is lower at the higher power.


Heat flux
q″DNB (100% power)
q″DNB (>100% power)
q″ (>100% power)
q″ (100% power)
0
L
z