22.312 **ENGINEERING OF NUCLEAR REACTORS**

Due November 17 by 12:00 pm

TAKE HOME QUIZ #2 SOLUTIONS

Problem 1 (60%)

i) To select a suitable heat transfer correlation for this problem, we must first calculate the Reynolds number and identify the flow regime. The flow area is $A_f = \frac{\sqrt{3}}{2}w^2 - 19\frac{\pi}{4}d^2$=1053 mm^2; w is the fuel assembly width, and d is the fuel pin outer diameter.
The wetted perimeter is $p_w = 2\sqrt{3}w + 19\pi d$ =714 mm

The equivalent diameter is $d_e = \frac{4A_f}{P_w}$ =5.9 mm

$\dot{m} = \frac{Q}{c_p \Delta T}$ =19.6 kg, cp is the Pb specific heat, Q the assembly power and T=150°C is the coolant temperature rise in the fuel assembly.

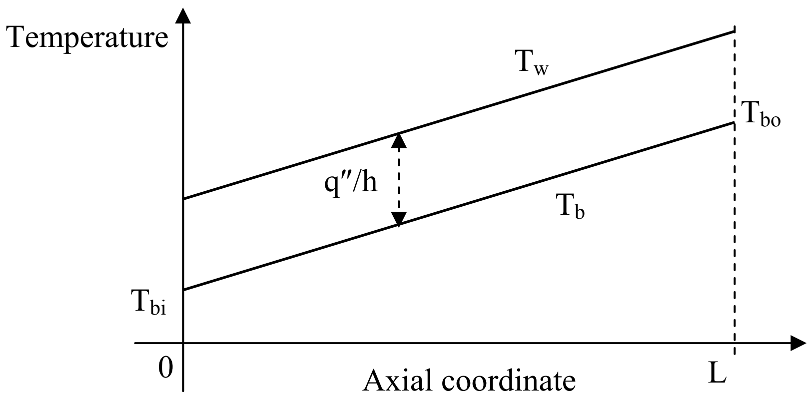
$Re = \frac{(\dot{m}/A_f)d_e}{\mu}$ =57,800. Therefore the flow regime is turbulent

$Pr = \frac{c_p \mu}{k}$ =0.0184 Pe=Re·Pr=1,064

For example, the Westinghouse correlation (Eq. 10-117, T&K) is for liquid metal fully-developed turbulent flow in triangular rod bundles, with 1.1<P/D<1.4 and 10<Pe<5000, and thus is suitable for this problem. The Westinghouse correlation yields Nu=9.842 and h=Nu·k/d_e=26.7 kW/m^2 (Note that sometime people use the "heated" diameter, d_h, instead of d_e for calculating the heat transfer coefficient).

ii) The length of the velocity entry region in turbulent flow can be as high as 40·d_e, or about 24 cm in our case. The length of the thermal entry region for a metallic fluid in turbulent flow is about 60·d_e (Chapter 10, Section A.IV.2) or ~35 cm in our case. Due to the larger radial momentum and temperature gradients, the heat transfer coefficient is higher in the entry region than in the fully-developed region. Therefore, using a fully-developed-flow correlation for the whole channel underestimates the actual heat transfer coefficient.

iii) If the axial power profile is flat and the transport properties are constant, the coolant bulk temperature and the wall temperature vary linearly with the axial coordinate (we are again neglecting the thermal entry region effects here).



iv) The cladding temperature reaches its peak value at the fuel assembly outlet (see Figure 1), and can be calculated as $T_{pc} = T_{bo} + \frac{q'}{\pi d\, h}$=576.5°C, where T_{bo}=550°C is the outlet bulk temperature, q'=20 kW/m is the linear power (calculated assuming local and axial peaking factors equal to unity), and h=26.7 kW/m^{2}K is the heat transfer coefficient (calculated with the Westinghouse correlation).
The fuel centerline temperature, T_{cl}, can be calculated using the concept of thermal resistance, and recognizing that there are three thermal resistances in series between the cladding outer position and the fuel centerline, i.e., cladding + thermal bond + fuel. Therefore:

$$T_{cl} = T_{pc} + q'\left[\frac{\ln\left(\frac{d}{d_i}\right)}{2\pi k_c} + \frac{\ln\left(\frac{d_i}{d_f}\right)}{2\pi\, k_{Na}} + \frac{1}{4\pi\, k_f}\right] = 696°C$$

where d_i=7.8 mm and d_f=6.8 mm are the cladding inner diameter and the fuel slug diameter, respectively, while k_c, k_{Na} and k_f are the stainless steel, sodium and U-Zr thermal conductivities, respectively.

v) The reactor power (hence the linear power) is increased by 10% without changing the coolant mass flow rate. Then the ΔT in the coolant bulk, coolant film, cladding, thermal bond and fuel temperature also increase by 10%. This is because constant properties are assumed, thus the thermal resistances do not change with power. The result is T_{pc}=400+1.1×(576.5-400)=594.2°C, and T_{cl}=400+1.1×(696-400)=725.6°C

vi) Wire spacers would reduce the flow area and the equivalent diameter of the fuel assembly. Therefore, the coolant velocity would increase, which would result in higher turbulence and thus higher heat transfer coefficient, but also higher pressure drop. The bulk temperature would stay the same because the mass flow rate and power are the same.

Problem 2 (40%)

i) Because UO_2 has a higher melting point, it should be placed in Zone 1.

ii) The linear power for the duplex pellet can be expressed as:

$$q' = \pi R_1^2 q_1''' + \pi(R_{fo}^2 - R_1^2)q_2''' = \pi\left[R_1^2 + \frac{q_2'''}{q_1'''}(R_{fo}^2 - R_1^2)\right]q_1''' \qquad (1)$$

Where q_1''' and q_2''' are the volumetric heat generation rates in Zone 1 and 2, respectively, and $\frac{q_2'''}{q_1'''}$=1.5. To solve the problem, it is necessary to establish a relationship between q_1''' and the max temperature in the pellet. To do so, one needs to solve the heat conduction equation in both zones. Starting from Zone 2 (i.e., PuO_2):

$$\frac{1}{r}\frac{d}{dr}\left[rk_2\frac{dT}{dr}\right] + q_2''' = 0 \qquad (2)$$

where k_2 is the PuO_2 thermal conductivity. Integrating twice and setting the boundary conditions $-2\pi \cdot R_1 k_2 \left.\frac{dT}{dr}\right|_{R_1} = q_1'''\pi R_1^2$ and $T|_{R_{fo}} = T_{fo}$, one gets:

$$T_1 - T_{fo} = -\frac{R_1^2}{2k_2}(q_2''' - q_1''')\log\left(\frac{R_{fo}}{R_1}\right) + \frac{q_2'''}{4k_2}(R_{fo}^2 - R_1^2) \qquad (3)$$

where T_1 is the temperature at R_1.
For Zone 1 the heat conduction equation yields:

$$\frac{1}{r}\frac{d}{dr}\left[rk_1\frac{dT}{dr}\right] + q_1''' = 0 \qquad (4)$$

Integrating twice and setting the boundary condition $-k_1\left.\frac{dT}{dr}\right|_0 = 0$ and $T|_0 = T_{cl}$, one gets:

$$T_{cl} - T_1 = \frac{q_1''' R_1^2}{4k_1} \qquad (5)$$

From equations 3 and 4 it follows that:

$$T_{cl} - T_{fo} = q_1'''\left\{\frac{R_1^2}{4k_1} - \frac{R_1^2}{2k_2}\left(\frac{q_2'''}{q_1'''} - 1\right)\log\left(\frac{R_{fo}}{R_1}\right) + \frac{1}{4k_2}\frac{q_2'''}{q_1'''}(R_{fo}^2 - R_1^2)\right\} \qquad (6)$$

For T_{cl}=2,800°C and T_{fo}=400°C, equation 6 yields $q_1''' \approx 1{,}000$ MW/m^3 and from equation 1 it is easy to get the linear power $q' \approx 95.6$ kW/m.

iii) For the traditional pellet the linear power is:

$$q' = 4\pi \cdot k_1 (T_{cl} - T_{fo}) \approx 90.5 \text{ kW/m} \tag{7}$$

which is somewhat lower than for the duplex pellet.