22.312 ENGINEERING OF NUCLEAR REACTORS

Friday, December 16th, 2005, 1:30-4:30 pm

OPEN BOOK	FINAL EXAM	3 HOURS

Problem 1 (50%) –Loss Of Flow Accident (LOFA) in a fast-spectrum BWR.

i) The mass flow rate can be calculated from the energy equation:

$$\dot{Q} = \dot{m}\left[C_{p,f}(T_{sat} - T_{b,in}) + x_{out}h_{fg}\right] \qquad \Rightarrow \dot{m} = \frac{\dot{Q}}{C_{p,f}(T_{sat} - T_{b,in}) + x_{out}h_{fg}} = 10.56 \text{ kg/s}$$

where the inlet temperature is $T_{b,i}$=260°C and the outlet quality is x_{out}=0.3. The corresponding mass flux is 10.56/0.005=2,112 kg/m^{2}s.

ii) The critical quality at the outlet is found to be x_{cr}=0.401 from the CISE-4 correlation with L_b=2.5 m, and the coefficients a=0.5539 and b=0.9543, calculated for P=6.4 MPa, P_c=22.1 MPa, G=2,112 kg/m^{2}s>G*=1,211 kg/m^{2}s, D_e=D_h=0.01 m. Since the heat flux is axially uniform, dryout would occur first at the channel outlet. Since the equilibrium quality at the outlet is lower than the critical quality at the outlet, dryout does not occur.

iii) When the mass flow rate is reduced to 0.5×10.56=5.28 kg/s, the corresponding mass flux is reduced to 5.28/0.005=1,056 kg/m^{2}s. Since the mass flux decreases, the critical quality increases.

The critical quality at the outlet is found to be x_{cr}=0.585 from the CISE-4 correlation with L_b=2.5 m, and the coefficients a=0.696 and b=0.477, calculated for P=6.4 MPa, P_c=22.1 MPa, G=1,056 kg/m^{2}s<G*=1,211 kg/m^{2}s, D_e=D_h=0.01 m. The equilibrium quality at the outlet can be calculated from the energy equation (at the reduced flow conditions) and is found to be 0.668. Since the equilibrium quality is higher than the critical quality, it can be concluded that dryout has in fact occurred in the fuel assembly at the reduced conditions.

iv) Given the conclusions in "ii" and "iii", two-phase forced convection (saturated boiling) and post-dryout heat transfer (forced convection to vapor + droplet evaporation at the wall) are the heat transfer mechanisms of interest at the channel outlet for the nominal and reduced-flow conditions, respectively. Thus, for example the Chen correlation (saturated boiling) and the Groeneveld correlation (post-dryout heat transfer) can be used, respectively.

v) Acceleration pressure drop. If the flow is reduced, while the power remains constant, there are two conflicting effects, i.e., G decreases, which tends to reduce the pressure drop, but the two-phase density at the outlet (ρ^+_{out}) also decreases, which tends to

J. Buongiorno, 2005

increase the acceleration pressure drop. (Note that the coolant density at the fuel assembly inlet does not change.) At high pressure typically the G effect is dominant.

Friction pressure drop. Once again, two conflicting effects. G decreases, which tends to reduce the friction pressure drop, but the average quality increases, which increases the two-phase multiplier ($\phi_{\ell o}^2$) and thus tends to increase the friction pressure drop. At high pressure typically the G effect is dominant.

Form pressure drop. Same as friction.

Gravity pressure drop. The gravity pressure drop is reduced because the average density in the fuel assembly is lower.

vi) Operation at reduced flow (and same power) is more susceptible to dynamic instabilities because more steam is generated within the fuel assembly, thus making the flow more compressible, which has a destabilizing effect.

Problem 2 (15%) – Sizing the pressure vessel for a high-pressure gas cooled reactor

The primary general membrane stresses for a thin cylindrical shell can be calculated as follows:

$$\sigma_r = -(p_i+p_o)/2 \qquad (1)$$
$$\sigma_\theta = (p_i-p_o)R/t$$
$$\sigma_z = (p_i-p_o)R/2t$$

with p_i=20 MPa and p_o=0.1 MPa, and R and t unknown.

The primary general membrane stress intensity, P_m, is then $(p_i-p_o)R/t -[-(p_i+p_o)/2]$, and the ASME code mandates that

$$P_m < S_m \qquad (2)$$

with S_m=220 MPa.

So the minimum allowable thickness is:

$$t = R(p_i-p_o)/[S_m - (p_i+p_o)/2] \qquad (3)$$

The total mass of the shell, M, can be as high as 300,000 kg:

$$M=2\pi RtL\rho \qquad (4)$$

with M=300,000 kg, L=4 m and ρ=7,000 kg/m^3. Substituting Eq. (3) into Eq. (4) and solving for R, one gets:

$$R = \sqrt{\frac{M(S_m - \frac{P_i + P_o}{2})}{\rho(P_i - P_o)2\pi \cdot L}} \approx 4.24 \text{ m, or } D \approx 8.48 \text{ m} \quad (5)$$

The corresponding thickness is t=0.40 m, from Eq. (3). Note that the thin-shell approximation is indeed acceptable in this case because R/t>10.

Problem 3 (10%) – Thermodynamic analysis of a gas turbine

Consider the entropy equation for transformation 1→2:

$$0 = \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} + \frac{\dot{Q}}{T_s} \qquad \Rightarrow \qquad s_2 = s_1 + \frac{\dot{S}_{gen}}{\dot{m}} + \frac{\dot{Q}}{\dot{m}T_s} \quad (6)$$

where $\dot{m}$ is the mass flow rate, $\dot{S}_{gen}$ (>0) is the entropy generation due to irreversibilities (e.g., friction), $\dot{Q}$ is the heat rate exchanged between the turbine and the surroundings and T_s is the temperature at which that exchange occurs.

Normally, for a turbine it is assumed that $\dot{Q}$=0 (the turbine is adiabatic). Then Eq. (6) gives:

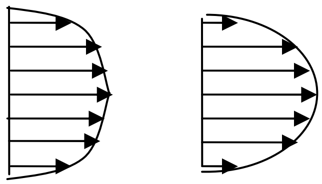
$$s_2 = s_1 + \frac{\dot{S}_{gen}}{\dot{m}} > s_1 \quad (7)$$

and one has to conclude that transformation 1→2 is not thermodynamically possible.
However, the transformation becomes thermodynamically possible if one assumes that $\dot{Q}$<0, i.e., the turbine is cooled.

Problem 4 (25%) – Effect of geometry on single-phase heat transfer in straight tubes

i) Both tubes have the same equivalent diameter, D_e=2 cm. The Reynolds number $\mathrm{Re} = \frac{\rho V D_e}{\mu} = \frac{\dot{m} D_e}{A\mu}$ is 1996 for the round tube and 1568 for the square tube. Since Re<2100, the flow regime is laminar for both tubes.

ii) We must first calculate if Points b and c are in the entry or fully developed region. For laminar flow, the length of the entry region, z_e, can be calculated as $\frac{z_e}{D_e} \approx 0.05\,\mathrm{Re}$. Thus, z_e=1.996 m (round tube) and z_e=1.568 m (square tube), and it can be concluded that Points b and c are in the entry region and fully-developed region, respectively, for both tubes. The qualitative velocity profile at Point b and c is:



Point b Point c

The profile develops from uniform to parabolic because of the radial momentum transfer due to viscosity.

iii) In order to select a heat transfer coefficient correlation, we note that:

- $Pr \sim 0.97$, thus the fluid is non-metallic.
- The flow regime is laminar.
- The geometry is round tube and square tube.
- The boundary condition is constant wall temperature.
- Point c is in the fully-developed region (for both velocity and temperature profiles)

Thus, the Nusselt number for laminar flow in a round tube with constant wall temperature is 3.66, while for a square tube is 2.98. The corresponding heat transfer coefficients are ~99.8 $W/m^2{}^\circ C$ and ~80.5 $W/m^2{}^\circ C$, respectively, i.e., the round tube has a higher heat transfer coefficient.

iv) If the flow rate triples, the Reynolds number triples (i.e., Re~5998 for round tube and Re~4704 for the square tube), taking the flow regime from laminar to turbulent. The situation is still one of fully-developed flow ($z_e=40D_e=0.8m<z_c$). For turbulent flow in the fully developed region, the heat transfer is insensitive to geometry and boundary conditions. Since the round tube has a higher Reynolds number than the square tube, its heat transfer coefficient will also be higher. Note that the Dittus-Boelter correlation cannot be used because it is valid for Re>10,000 (see Page 443 in the textbook).

J. Buongiorno, 2005