22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, October 18th, 2005, 9:30 – 11:00 a.m.

OPEN BOOK QUIZ #1 SOLUTIONS 1.5 HOURS

Problem 1 (45%) – Two-unit nuclear plant with single containment building

i) The containment building is the control volume of choice.

The energy equation is:

$$\frac{\partial E}{\partial t} = \dot{m}_{ws} h_{ws} \qquad \Rightarrow \qquad E_2 - E_1 = h_{ws} \int_{t_1}^{t_2} \dot{m}_{ws} dt = h_{ws} 0.2 M_{ws} \qquad (1)$$

where E is the total energy of the control volume, and $0.2M_{ws}$ is the mass of water flown into the containment building. Expanding the energy terms on the left-hand side of Eq. (1), one gets:

$$(0.2M_{ws} + M_{wsp})u_{w2}(T_2, P_{w2}) + M_a u_{a2} - M_{wsp} u_{wsp} - M_a u_{a1} = h_{ws} 0.2 M_{ws} \qquad (2)$$

where superheated steam conditions were assumed in the containment at t_2. Equation (2) can be re-written as follows:

$$(0.2M_{ws} + M_{wsp})u_{w2}(T_2, P_{w2}) + M_a c_{va}(T_2 - T_1) = M_{wsp} u_{wsp} + h_{ws} 0.2 M_{ws} \qquad (3)$$

The equation to calculate the mass of air in the containment building (with zero initial humidity) is:

$$M_a = \frac{P_1 V_{a1}}{R_a T_1} \qquad (4)$$

The total control volume is:

$$V_c = V_{a1} + \frac{M_{wsp}}{\rho_{wsp}} \qquad (5)$$

The equation for the water volume at t_2 is:

$$V_c = (0.2M_{ws} + M_{wsp}) v_{w2}(T_2, P_{w2}) \qquad (6)$$

Finally, the equation to calculate the containment pressure at t_2 is:

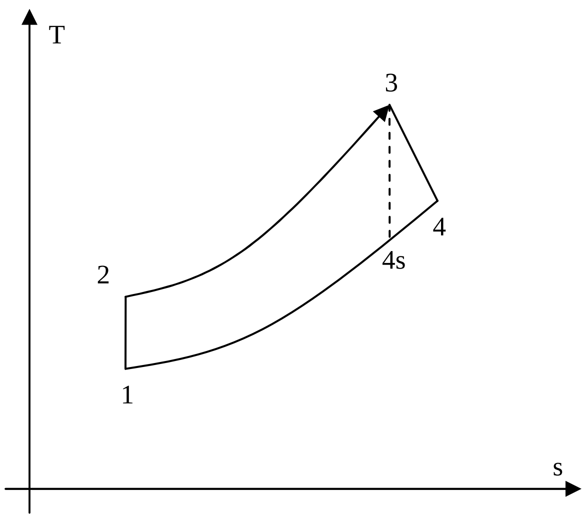
$$P_2 = P_{w2} + P_{a2} = P_{w2} + \frac{M_a R_a T_2}{V_c} \qquad (7)$$

Equations (3) through (7) constitute a system of 5 equations in the 5 unknowns T_2, P_{w2}, M_a, V_c and P_2.

ii) Advantages of the single-containment approach include lower capital cost and greater ease of access and inspection. The disadvantages include reduced redundancy (failure of the containment building would result in a leak path for two reactors instead of one), and possibly larger plant footprint (depending on the building design).

Problem 2 (55%) –Power cycle for a High Temperature Gas-Cooled Reactor

i)



ii) The net electric power of the plant, $\dot{W}_{net}$, is:

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_C = \dot{m}c_p\left[(T_3 - T_4) - (T_2 - T_1)\right] = \dot{m}c_p\left[\eta_T(T_3 - T_{4s}) - (T_2 - T_1)\right] =$$
$$= \dot{m}c_p\left[\eta_T T_3\left(1 - \frac{T_{4s}}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)\right] \qquad (8)$$

where $\dot{m}$, c_p, η_T and T_1 are all given in the problem statement. The ratios T_{4s}/T_3 and T_2/T_1 can be readily calculated from the equation of the isentropic transformation for a perfect gas:

$$\frac{T_3}{T_{4s}} = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \approx 1.904 \qquad (9)$$

Then $T_2 = 1.904 \cdot T_1 \approx 710.2$ K.

T_3 can be calculated from the energy equation for the reactor:

$$\dot{Q}_0 = \dot{m}c_p(T_3 - T_2) \quad \Rightarrow \quad T_3 = T_2 + \frac{\dot{Q}_0}{\dot{m}c_p} \approx 1{,}017.9 \text{ K} \qquad (10)$$

Substituting the numerical values into Eq. (8), one gets $\dot{W}_{net} \approx 1{,}017$ MW.

iii) The energy equation for the fuel is:

$$\frac{\partial E}{\partial t} = \dot{Q} \quad \Rightarrow \quad E_2 - E_1 = \int_{t_1}^{t_2} \dot{Q}dt = \int_{60\,\text{min}}^{65\,\text{min}} 0.066\dot{Q}_0 t^{-0.2} dt = 1.22\times10^{10} \text{ J} \qquad (11)$$

where $\dot{Q}$ is the decay power. The energy variation on the left-hand term of Eq. (11) can be rewritten in terms of the temperature rise, ΔT_F, as follows:

$$E_2 - E_1 = M_F c_F \Delta T_F \qquad (12)$$

where M_F is the fuel mass (4.405×11,000=44,495 kg) and c_F is the fuel specific heat. Therefore the temperature rise is:

$$\Delta T_F = 1.22\times10^{10}/(44{,}495\times230) \approx 1{,}200 \text{ K} \qquad (13)$$