22.312 ENGINEERING OF NUCLEAR REACTORS

Due November 16, 2005 by 12:00 pm

TAKE HOME QUIZ #2 SOLUTIONS

Problem 1 (80%) – Coolant selection for an advanced high-temperature reactor

i) To calculate the mass flow rate, $\dot{m}$, we use the continuity equation:

$$\dot{m} = \rho V\left(\frac{\pi}{4}D^2\right) \quad (1)$$

where ρ, V and D are the coolant density, coolant velocity and channel diameter, respectively. The friction pressure drop in the coolant channel, ΔP_{fric}, can be calculated as:

$$\Delta P_{fric} = f \cdot \frac{L}{D} \cdot \frac{\rho V^2}{2} \quad (2)$$

where f and L, are the friction factor and channel length, respectively. The generic expression of the friction factor for fully-developed flow in smooth channels is:

$$f = C/\mathrm{Re}^n \quad (3)$$

where Re is the Reynolds number, $\mathrm{Re} \equiv (\rho VD)/\mu$, and C and n are numerical coefficients depending on the flow regime. Substituting Eq. (3) into Eq. (2), we get:

$$\Delta P_{fric} = \frac{C}{\mathrm{Re}^n} \cdot \frac{L}{D} \cdot \frac{\rho V^2}{2} = \frac{C}{2} \cdot \frac{\mu^n L \rho^{1-n} V^{2-n}}{D^{1+n}} \quad (4)$$

Since ΔP_{fric} is given, Eq. (4) can be solved to find the coolant velocity:

$$V = \left(\frac{2\Delta P_{fric} D^{1+n}}{C\mu^n L \rho^{1-n}}\right)^{1/(2-n)} \quad (5)$$

Because the flow regime is not known a priori, one has to guess it and then verify the accuracy of the guess. Let us assume turbulent flow with Re>30,000 for which C=0.184, n=0.2 (see Eq. 9-79 in the textbook). Then, Eq. (5) yields V=5.82 m/s for liquid sodium and V=2.95 m/s for the liquid salt. The corresponding Reynolds numbers are 267,250 and 28,656, respectively. So the flow regime is indeed turbulent, but it is not high-Re turbulent for the liquid salt, because Re<30,000. So, Eq. (5) has to be recomputed for the liquid salt with C=0.316 and n=0.25 (see Eq. 9-80 in the textbook), which gives V=2.91 m/s and Re=28,207.

The corresponding mass flow rates are calculated from Eq. (1) and are 0.357 kg/s and 0.443 kg/s for liquid sodium and liquid salt, respectively.

ii) The pumping power due to friction, $\dot{W}_p$, can be calculated as follows:

$$\dot{W}_p = \dot{m} \cdot \frac{\Delta P_{fric}}{\rho} \tag{6}$$

Thus, the pumping power is about 91 W and 46 W for liquid sodium and liquid salt, respectively.

iii) Since the power profile is axially uniform, the maximum temperature in the fuel occurs at the channel outlet (z=L). Per the hint, let us approximate the fuel around the coolant channel as an annulus of inner diameter D and outer diameter D_f, where D_f is calculated imposing the conservation of the fuel volume:

$$\left(\frac{\sqrt{3}}{2}w^2 - \frac{\pi}{4}D^2\right)L = \left(\frac{\pi}{4}D_f^2 - \frac{\pi}{4}D^2\right)L \quad \Rightarrow \quad D_f = \sqrt{\frac{2\sqrt{3}}{\pi}}w = 3.15 \text{ cm} \tag{7}$$

where w=3 cm is the width of the hexagonal unit cell of the core (see Figure 1).

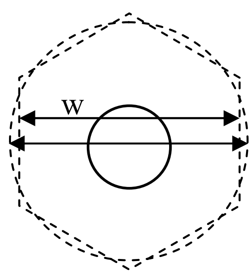

Figure 1. Equivalent annulus.

The heat conduction equation in this annulus is:

$$\frac{1}{r} \cdot \frac{d}{dr}\left[k_f r \frac{dT}{dr}\right] + q''' = 0 \tag{8}$$

where r is the radial coordinate, k_f is the fuel thermal conductivity (=6 W/m·K, independent of temperature) and q''' is the volumetric heat generation rate within the fuel. The linear power, q', is related to q''' as follows:

$$q' = \frac{\pi}{4}\left(D_f^2 - D^2\right)q''' \tag{9}$$

The boundary condition for Eq. (8) is:

$$-k_f \frac{dT}{dr} = 0 \qquad \text{at } r = D_f/2 \tag{10}$$

The solution of Eq. (8) is then:

$$\therefore T_{\max} - T_{s,L} = \frac{q'}{2\pi k_f}\left[\frac{D_f^2}{D_f^2 - D^2}\ln\left(\frac{D_f}{D}\right) - \frac{1}{2}\right] \qquad (11)$$

where T_{max}=1000°C is the maximum temperature in the fuel, $T_{s,L}$ is the (unknown) temperature at the surface of the coolant channel at z=L. Newton's law of cooling provides a relationship between $T_{s,L}$ and $T_{b,L}$, i.e., the coolant bulk temperature at the channel outlet:

$$\frac{q'}{\pi D} = h(T_{s,L} - T_{b,L}) \qquad \Rightarrow \qquad T_{s,L} = T_{b,L} + \frac{q'}{\pi D h} \qquad (12)$$

where h is the heat transfer coefficient, which can be calculated from Eq. 10-113 in the textbook for the liquid sodium (Pr=0.037, Re=267,250 ⇒ Nu=13.2), and the Dittus-Boelter correlation for the liquid salt (Pr=4.82, Re=28,207 ⇒ Nu=156.7). The values of 'h' are then 79.2 kW/m^2K and 15.7 kW/m^2K for liquid sodium and liquid salt, respectively.

The bulk temperature at the channel outlet can be calculated from the energy equation as follows:

$$\dot{m}c_p(T_{b,L} - T_{b,o}) = q'L \qquad \Rightarrow \qquad T_{b,L} = T_{b,o} + \frac{q'L}{\dot{m}c_p} \qquad (13)$$

Where $T_{b,o}$=600°C is the inlet temperature. Combining Eq. (11), (12) and (13), one gets:

$$T_{\max} - T_{b,o} = \frac{q'}{2\pi k_f}\left[\frac{D_f^2}{D_f^2 - D^2}\ln\left(\frac{D_f}{D}\right) - \frac{1}{2}\right] + \frac{q'}{\pi D h} + \frac{q'L}{\dot{m}c_p} \qquad (14)$$

which can be solved for q′:

$$q' = \frac{T_{\max} - T_{b,o}}{\frac{1}{2\pi k_f}\left[\frac{D_f^2}{D_f^2 - D^2}\ln\left(\frac{D_f}{D}\right) - \frac{1}{2}\right] + \frac{1}{\pi D h} + \frac{L}{\dot{m}c_p}} \qquad (15)$$

Equation (15) gives q′=9.4 kW/m and q′=12.5 kW/m for liquid sodium and liquid salt, respectively.

iv) The thermal-hydraulic analysis indicates that a liquid salt coolant is the better choice, as it affords higher heat removal rates while requiring lower pumping power. The good thermal-hydraulic performance of the liquid salt is due mainly to its high heat capacity ($\dot{m}$ c_p).

Problem 2 (20%) – Flow split in downflow

Because the two channels are connected to the same inlet and outlet plena, the total pressure change in each channel, $-\Delta P_{tot}$, is the same:

$$-\Delta P_{tot} = f \cdot \frac{L}{D_e} \cdot \frac{G_1^2}{2\rho_1} - \rho_1 g L \quad (16)$$

$$-\Delta P_{tot} = f \cdot \frac{L}{D_e} \cdot \frac{G_2^2}{2\rho_2} - \rho_2 g L \quad (17)$$

where the form and acceleration terms were neglected, f is the friction factor (assumed equal in both channels, as per the problem statement), D_e is the hydraulic diameter of the channels, G_1 and G_2 are the mass fluxes in channel 1 and 2, respectively, and ρ_1 and ρ_2 are the average water densities in channel 1 and 2, respectively. Eliminating $-\Delta P_{tot}$ from Eq. (16) and (17) and recognizing that $\rho_1>\rho_2$ (i.e., channel 1 is cooled, while channel 2 is heated), one gets:

$$\frac{G_1^2}{\rho_1} > \frac{G_2^2}{\rho_2} \quad \Rightarrow \quad G_1 > \sqrt{\frac{\rho_1}{\rho_2}} G_2 > G_2 \quad (18)$$

Therefore, the mass flow rate in channel 1 is higher than the mass flow rate in channel 2. This result is also intuitive because cooling channel 1 and heating channel 2 creates a "chimney" effect that opposes downflow in channel 2.