Tuesday, December 19th, 2006, 9:00am-12:00 pm

FINAL EXAM SOLUTIONS

Problem 1 (35%) – Analysis of a Liquid-Metal Reactor Vessel

i) The primary general membrane stresses for a thin cylindrical shell can be calculated as follows:

$$\sigma_r = -(P_i+P_b)/2 \qquad (1)$$
$$\sigma_\theta = (P_i-P_b)R/t$$
$$\sigma_z = (P_i-P_b)R/2t$$

where P_b=0.1 MPa is the external pressure, R=2.5 m and t=4 cm are the vessel radius and thickness, respectively, and P_i is the internal pressure, which is a function of the elevation within the vessel, as follows:

$$P_i=P_{gas}+\rho_{Pb}gz \qquad (2)$$

where P_{gas}=0.5 MPa is the cover gas pressure, ρ_{Pb}=10500 kg/m^3 is the lead density, g=9.81 m/s^2 is the acceleration of gravity and z is the depth measured from the free surface of the lead.
The primary general membrane stress intensity in the cylindrical shell, P_m, is:

$$P_m=(P_i-P_b)R/t -[-(P_i+P_b)/2] \qquad (3)$$

Eq. (3) shows that the stress intensity increases with increasing internal pressure, so the maximum stress intensity in the cylindrical shell is reached at z=15 m, and is equal to about 122 MPa. Thus, the margin to the ASME code limit is S_m/P_m=138/122≈1.13, or 13%. Note that, while the internal pressure further increases in the lower vessel head, the stress intensity in the lower vessel head is relatively low because of its spherical geometry.

ii) First case. The pressure at the bottom of the vessel is P_i=2.3 MPa, calculated from Eq. (2) for z=17.5 m. Assuming steady-state, incompressible, inviscid and adiabatic flow, the mass flow rate through the break, $\dot{m}$, can be calculated as:

$$\dot{m} = GA = A\sqrt{2\rho_{Pb}(P_i - P_b)} \approx 215 \text{ kg/s} \qquad (4)$$

where G is the mass flux through the break and A=10 cm^2 is the break size.

Second case. Here we can use the critical flow model for a perfect gas. The critical pressure, P_{cr}, is given by Eq. (11-126b) in the textbook:

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$$P_{cr} = P_{gas}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \approx 0.264 \text{ MPa} \tag{5}$$

where γ=1.4. Since P_b<P_{cr}, critical flow is achieved in the break. To calculate the value of the critical flow we can use Eq. (11-125) in the textbook, which can be re-rewritten for our situation as:

$$\dot{m} = GA = A\rho_i\sqrt{2c_pT_i\left[\left(\frac{P_{cr}}{P_{gas}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{cr}}{P_{gas}}\right)^{\frac{\gamma+1}{\gamma}}\right]} \approx 0.765 \text{ kg/s} \tag{6}$$

where T_i=673 K (400°C) is the temperature of the cover gas in the vessel, c_p=1039 J/kg·K is the nitrogen specific heat and ρ_i is the density of the cover gas in the vessel, which can be found from the equation of state for a perfect gas as follows:

$$\rho_i = \frac{P_{gas}}{R^*T_i} \approx 2.5 \text{ kg/m}^3 \tag{7}$$

iii) The first case (i.e., break at the bottom of the vessel) is far more dangerous from a safety viewpoint, because it has the potential to empty the vessel and uncover the core. The second case merely results in a depressurization of the reactor without any loss of coolant (the boiling point of lead is very high, so it does not flash upon depressurization).

Problem 2 (30%) – Boiling Crisis in the Plasma Divertor of a Fusion Reactor

i) Because the heat flux is axially uniform, the MDNBR will occur at the tube outlet:

$$\text{MDNBR} = \frac{q''_{DNB,out}}{q''} \tag{8}$$

The Tong-68 correlation provides the value of the DNB heat flux at the tube outlet, $q''_{DNB,out}$, as follows:

$$q''_{DNB,out} = K_{Tong}(x_{e,out})\frac{G^{0.4}\mu_f^{0.6}h_{fg}}{D^{0.6}} \tag{9}$$

where G is the mass flux and D=8 mm. Substituting Eq.(9) into Eq. (8), one gets:

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$$\text{MDNBR}=K_{Tong}(x_{e,out})\frac{\mu_f^{0.6}h_{fg}}{D^{0.6}}\frac{G^{0.4}}{q''} \qquad (10)$$

The equilibrium quality at the outlet, $x_{e,out}$, can be calculated starting from the energy equation:

$$G\frac{dh}{dz}=\frac{q''P_h}{A} \qquad (11)$$

where h is the enthalpy, q″ is the heat flux, $P_h=\pi D/2$ is the heated perimeter and $A=\pi D^2/4$ is the flow area. Using the definition of equilibrium quality ($x_e=(h-h_f)/h_{fg}$) and integrating Eq. (11), one gets:

$$x_{e,out}=x_{e,in}+\frac{2q''L}{Gh_{fg}D} \qquad (12)$$

where $x_{e,in}=(h_{in}-h_f)/h_{fg}=-0.326$ and L=0.2 m. Equation (12) yields $x_{e,out}=-0.268$ at 100% power. Equation (12) also shows that $x_{e,out}$ depends only on the q″/G ratio, which is constant because both heat flux and mass flux scale linearly with power, as per the problem statement. Thus, $x_{e,out}$ is also constant in the power range 20-100%. Then Eq. (10) suggests that the MDNBR depends only on the parameters G and q″, and can be re-written as follows:

$$\text{MDNBR}=K_{Tong}(x_{e,out})\frac{\mu_f^{0.6}h_{fg}}{D^{0.6}}\left(\frac{G}{q''}\right)^{0.4}\frac{1}{q''^{0.6}} \qquad (13)$$

Because the G/q″ ratio is constant, Eq. (13) clearly indicates that the MDNBR will have its minimum value when the heat flux is maximum, i.e., at 100% power. The minimum value of the MDNBR, calculated from Eq. (13) is 1.138.

ii) The equilibrium quality at the outlet is negative (-0.268), indicating that the flow conditions are subcooled. However, the flow quality is defined positive, so assuming $x_e=x$ would be inaccurate.

Problem 3 (35%) – Flow Dynamics of Nanofluids

i) Since the slip ratio is one, the volumetric flow rate of the nanoparticles is simply $\dot{Q}_s=\alpha\dot{Q}_{tot}$=20 cm^3/s, and that of water is $\dot{Q}_\ell=(1-\alpha)\dot{Q}_{tot}$=380 cm^3/s, where $\dot{Q}_{tot}$=400 cm^3/s and α=0.05. Their respective mass flow rates are $\dot{m}_s=\rho_s\dot{Q}_s$=80 g/s and $\dot{m}_\ell=\rho_\ell\dot{Q}_\ell$=380 g/s. Therefore, the total mass flow rate of the nanofluid is $\dot{m}_{tot}=\dot{m}_s+\dot{m}_\ell$=460 g/s. The mass flux, G, is then:

$$G=\frac{\dot{m}_{tot}}{A}=\frac{[\alpha\rho_s+(1-\alpha)\rho_\ell]}{A}\dot{Q}_{tot}=93.7\ \text{g/cm}^2\text{s} \qquad (14)$$

where $A=\pi D^2/4=4.91\ cm^2$ and $D=2.5$ cm.

ii) The pressure gradient within the tube is provided by the momentum equation for downflow at steady-state with no acceleration term (i.e., no change in density of the nanofluid within the tube) and no form term (i.e., the tube has no change in flow area):

$$-\left.\frac{dP}{dz}\right)_{tot} = \left.\frac{dP}{dz}\right)_{fric} + \left.\frac{dP}{dz}\right)_{grav} = f_{TP}\frac{1}{D}\frac{G^2}{2\rho_m} - \rho_m g \qquad (15)$$

where $\rho_m=\alpha\rho_s+(1-\alpha)\rho_\ell=1.15\ g/cm^3$ is the mixture density, and f_{TP} is the nanofluid friction factor, which is equal to the liquid-only friction factor, $f_{\ell o}$, as per the problem statement. To calculate $f_{\ell o}$, one needs to find the Reynolds number for the liquid:

$$Re = \frac{GD}{\mu_\ell} = 23425 \qquad (16)$$

where $\mu_\ell=10^{-3}$ Pa·s. Thus the friction factor can be calculated with the correlation $f_{\ell o}=0.316/Re^{0.25}\approx 0.0255$ (valid for fully-developed turbulent flow in smooth tubes and $Re<30000$). Thus, from Eq. (15) the friction pressure gradient is about 389 Pa/m, while the gravity pressure gradient is about -11281 Pa/m (negative because the direction of the flow is downward). The total pressure gradient in the tube is then -10892 Pa/m, i.e., the pressure increases in the direction of the flow.

iii) Assume that $\dot{Q}_{tot}$ and α are the same, as per the problem statement. If the nanoparticles were made of a material with higher density than alumina, the mixture density ρ_m would be higher, and thus the gravity pressure gradient would be larger (more negative). As for the friction pressure gradient, Eq. (15) and the friction factor correlation suggest that:

$$\left.\frac{dP}{dz}\right)_{fric} \propto \frac{G^{1.75}}{\rho_m} \qquad (17)$$

Using Eq. (14) and the definition of ρ_m, one gets:

$$\left.\frac{dP}{dz}\right)_{fric} \propto \frac{\rho_m^{1.75}}{\rho_m} = \rho_m^{0.75} \qquad (18)$$

Equation (18) suggests that the pressure gradient would be higher, if the nanoparticles were made of a material heavier than alumina.

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