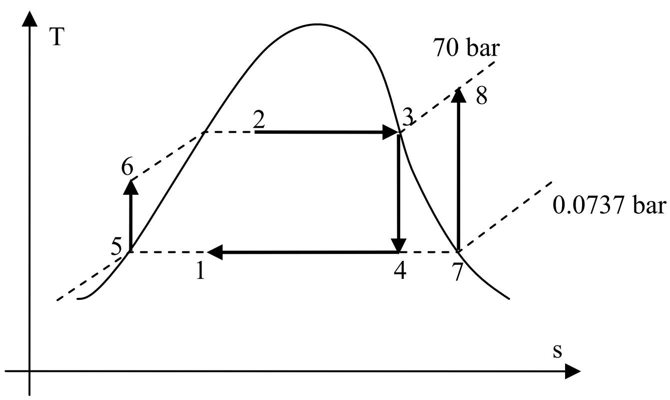
22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, October 17th, 2006, 2:30 – 4:00 p.m.

OPEN BOOK QUIZ 1 SOLUTION

Problem 1 (45%) – Assessment of a steam cycle with moisture separation and vapor compression

i) T-s diagram for the modified cycle



ii) The thermal efficiency of the modified cycle is:

$$\eta_{th} = \frac{\dot{W}_{Turbine} - \dot{W}_{Pump} - \dot{W}_{Compressor}}{\dot{Q}} = \frac{(h_3 - h_4) - (1 - x_1)(h_6 - h_5) - x_1(h_8 - h_7)}{(h_3 - h_2)} \approx 0.166$$

where x_1 is the steam quality at Point 1. Thus the fraction of flow diverted to the pump is $1\text{-}x_1$ and the fraction of flow diverted to the compressor is x_1. The enthalpies were clauclated as follows:

Turbine inlet (**Point 3**): T_3=285.7°C, P_3=70 bar, h_3=2772 kJ/kg, s_3=5.815 kJ/kg·K, x_3=1.0
Turbine outlet (**Point 4**): T_4=40°C, P_4=0.0737 bar, $s_4=s_3$=5.815 kJ/kg·K, $x_4=(s_4-s_f)/(s_g-s_f)$=0.6822, $h_4=h_f+x_4(h_g-h_f)$=1809 kJ/kg
Condenser outlet (**Point 1**): T_1=40°C, P_1=0.0737 bar, $s_1=s_f$(70 bar)=3.119 kJ/kg·K, $x_1=(s_1-s_f)/(s_g-s_f)$=0.33142, $h_1=h_f+x_1(h_g-h_f)$=965 kJ/kg
Pump inlet (**Point 5**): T_5=40°C, P_5=0.0737 bar, $h_5=h_f$=167 kJ/kg, x_5=0
Pump outlet (**Point 6**): P_6=70 bar, $h_6=h_5+(P_6-P_5)v_f \approx$ 174 kJ/kg
Vapor compressor inlet (**Point 7**): T_7=40°C, P_7=0.0737 bar, h_7=2574 kJ/kg, s_7=8.257 kJ/kg·K, x_7=1.0
Vapor compressor outlet (**Point 8**): P_8=70 bar, $s_8=s_7$=8.257 kJ/kg·K, h_8=4957 kJ/kg, T_8=1132°C
Feedwater heater outlet (**Point 2**): P_2=70 bar, $h_2=(1-x_1)\cdot h_6+x_1\cdot h_8$ =1759 kJ/kg

iii) The thermal efficiency of the Carnot cycle is:

$$\eta_{\mathrm{th,\,Carnot}} = 1 - \frac{\mathrm{T}_4}{\mathrm{T}_3} \approx 0.440$$

Therefore, the modified cycle does not achieve a thermal efficiency nearly as high as that of the Carnot cycle. This is due primarily to two reasons. First, the amount of work needed to compress the vapor from P_7 to P_8 is very large. Second, the temperature difference between the two streams mixed in the feedwater heater (T_8-T_6) is extremely high. This process is highly irreversible, which hurts the thermal efficiency.

Problem 2 (55%) – Analysis of a transient overpower in a PWR steam generator

i) The control volume selected to analyze the problem is the volume occupied by the secondary coolant in the steam generator. The conservation of mass at steady state is:

$$0 = \dot{m}_i - \dot{m}_o \quad \Rightarrow \quad \dot{m}_o = \dot{m}_i$$

where $\dot{m}_o$ is the secondary coolant outlet mass flow rate. The conservation of energy for steady-state yields the following equation:

$$0 = \dot{Q} + \dot{m}_i h_i - \dot{m}_o h_g \quad \Rightarrow \quad \dot{Q} = \dot{m}_i (h_g - h_i) = 737.8 \text{ MW}$$

where kinetic and gravitational terms were neglected and h_g is the specific enthalpy of dry saturated steam at 5.7 MPa.

ii) The conservation of mass equation is:

$$\frac{dM_{SC}}{dt} = \dot{m}_i - \dot{m}_o \tag{1}$$

The conservation of energy equation is:

$$\frac{dE_{SC}}{dt} = 1.2\dot{Q} + \dot{m}_i h_i - \dot{m}_o h_g \tag{2}$$

where the total energy of the secondary coolant is:

$$E_{SC} = M_{SC}(u_f + u_{fg}x) \tag{3}$$

and x is the steam quality of the secondary coolant. The total volume of the secondary coolant in the steam generator is V_{SC}=100 m^3 and can be written as:

$$V_{SC} = M_{SC}(v_f + v_{fg}x) \tag{4}$$

In Eqs. (1) through (4) $\dot{m}_i$, $\dot{Q}$, h_i, h_g, u_f, u_{fg}, V_{SC}, v_f and v_{fg} are all known. Therefore, these equations represent a system of four equations of the four unknown M_{SC}, $\dot{m}_o$, E_{SC} and x, which can be solved to find the variation of $M_{SC}(t)$ during the transient.

Note that for this particular problem it is possible to find $\frac{dM_{SC}}{dt}$ in close form, as follows. Solving Eq. (4) for x, substituting into Eq. (3), and eliminating E_{SC} from Eq. (2) one gets:

$$\frac{d}{dt}\left[M_{SC}u_f + \frac{u_{fg}}{v_{fg}}(V_{SC} - v_f M_{SC})\right] = 1.2\dot{Q} + \dot{m}_i h_i - \dot{m}_o h_g \qquad (5)$$

The left-hand side of Eq. (5) can be simplified to give:

$$(u_f - \frac{v_f}{v_{fg}}u_{fg})\frac{dM_{SC}}{dt} = 1.2\dot{Q} + \dot{m}_i h_i - \dot{m}_o h_g \qquad (6)$$

Eliminating $\dot{m}_o$ from Eqs. (1) and (6), and solving for $\frac{dM_{SC}}{dt}$, one gets:

$$\frac{dM_{SC}}{dt} = \frac{\dot{m}_i(h_g - h_i) - 1.2\dot{Q}}{h_g - u_f + \frac{v_f}{v_{fg}}u_{fg}} = \text{constant} = \text{-89.2 kg/s} \qquad (7)$$

Thus,

$$M_{SC}(t) = M_{SC}(0) - 89.2 \cdot t \qquad (8)$$

where $M_{SC}(0)$ is 54880 kg and t is in seconds.

iii) Since the secondary coolant receives more heat, the rate at which steam is produced and delivered to the turbine increases, which means $\dot{m}_o$ increases.

iv) If the inlet and outlet are closed shut and heat is still being supplied to the secondary coolant, the pressure will increase. The equations are as follows.
Mass:

$$\frac{dM_{SC}}{dt} = 0 \qquad \Rightarrow \qquad M_{sc} = \text{const}$$

That is, the secondary coolant mass in the steam generator does not change during the transient and can be treated as a constant, equal to 44176 kg from Eq. (8).

Energy:

$$\frac{dE_{SC}}{dt} = 1.2\dot{Q} \qquad (9)$$

where the stored energy is:

$$E_{SC} = M_{SC}[u_f(P) + u_{fg}(P)x] \qquad (10)$$

Volume:

$$V_{SC} = M_{SC}[v_f(P) + v_{fg}(P)x] \qquad (11)$$

Equations (9), (10) and (11) are three equations of the three unknown E_{sc}, P and x, which can be solved to find $P(t)$.