22.312 **ENGINEERING OF NUCLEAR REACTORS**

Due November 17, 2006 by 12:00 pm

TAKE HOME QUIZ 2 (SOLUTION)

Problem 1 (60%) – Hydraulic Analysis of the Emergency Core Spray System in a BWR

The numbering of the relevant locations within the system is shown in Figure 1 below.

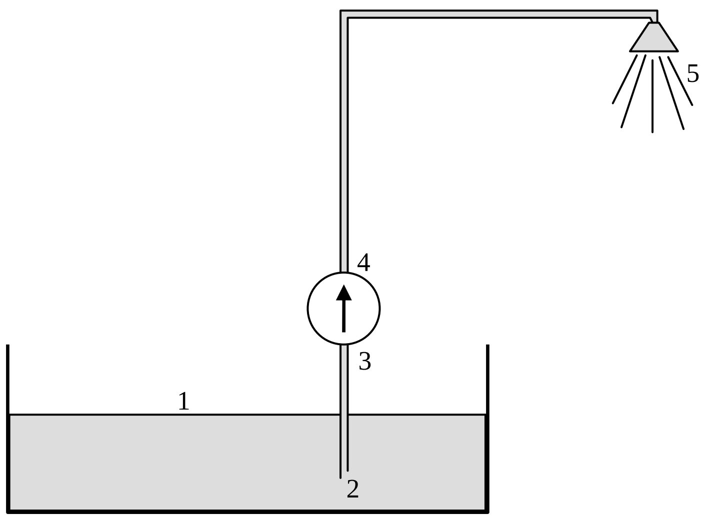

Figure 1. Numbering of the locations within the emergency spray system.

The pumping power, $\dot{W}_p$, is:

$$\dot{W}_p = \dot{m}\frac{\Delta P_{pump}}{\rho}\cdot\frac{1}{\eta_p} \qquad (1)$$

where $\dot{m}$=50 kg/s, ρ=997 kg/m^3, η_p=0.8 and ΔP_{pump} is the pressure head provided by the pump (i.e., $\Delta P_{pump}=P_4-P_3$). To find ΔP_{pump}, we have to solve the momentum equation for this system, which we can break down in the segments 1→2, 2→3, 3→4 and 4→5. First, let us calculate some parameters that will be used in the analysis. The mass flux in the pipes, G, is:

$$G = \frac{\dot{m}}{\frac{\pi}{4}D_i^2} \approx 6366 \text{ kg/m}^2\text{s} \qquad (2)$$

where D_i=0.1 m. Obviously, the velocity in the pipes is $V=G/\rho\approx 6.385$ m/s. The Reynolds number is the pipes, Re, is:

$$\mathrm{Re} = \frac{GD_i}{\mu} \approx 707376 \tag{3}$$

where $\mu=9\times10^{-4}$ Pa·s. The flow is turbulent and the friction factor in the (smooth) pipes, f, can be calculated as follows:

$$f = \frac{0.184}{\mathrm{Re}^{0.2}} \approx 0.01244 \tag{4}$$

Momentum equation for 1→2:

$$P_1 - P_2 = \rho g(z_2 - z_1) \tag{5}$$

where z_1 and z_2 are the elevation of location 1 and 2, respectively.

Momentum equation for 2→3:

$$P_2 - P_3 = f\frac{L_{23}}{D_i}\cdot\frac{G^2}{2\rho} + \rho g(z_3 - z_2) + \frac{G^2}{2\rho} + K_{entrance}\frac{G^2}{2\rho} \tag{6}$$

where L_{23}=2 m, $K_{entrance}$=0.5 and the last two terms on the right-hand side of the equation represent the form acceleration and form loss term, respectively.

Momentum equation for 4→5:

$$P_4 - P_5 = f\frac{L_{45}}{D_i}\cdot\frac{G^2}{2\rho} + \rho g(z_5 - z_4) + (K_{elbow1} + K_{elbow2} + K_{spray})\frac{G^2}{2\rho} + \frac{G^2}{2\rho}\left[\left(\frac{\frac{\pi}{4}D_i^2}{A_{spray}}\right)^2 - 1\right] \tag{7}$$

where L_{45}=27 m, K_{elbow1}=K_{elbow2}=0.9, K_{spray}=15 and A_{spray}=26 cm^2. The last term on the right-hand side of the equation represents the form acceleration term for the spray nozzle, and was calculated using the definition of form acceleration plus the continuity equation for the nozzle:

$$\rho V\frac{\pi}{4}D_i^2 = \rho V_{spray}A_{spray} \tag{8}$$

where V_{spray} is the water velocity immediately outside the nozzle.
Adding Eq. (5), (6) and (7), one gets:

$$P_1 - P_3 + P_4 - P_5 = f\frac{L_{23}+L_{45}}{D_i}\cdot\frac{G^2}{2\rho} + \rho g(z_5 - z_1) + (K_{entrance} + K_{elbow1} + K_{elbow2} + K_{spray})\frac{G^2}{2\rho} + \frac{G^2}{2\rho}\left(\frac{\frac{\pi}{4}D_i^2}{A_{spray}}\right)^2 \tag{9}$$

where it was assumed $z_3 \approx z_4$, as per the problem statement. Note that $z_5 - z_1 = 16.5$ m. However, $P_1 = P_5 = 0.1$ MPa and $P_4 - P_3 = \Delta P_{pump}$, therefore Eq. (9) becomes:

$$\Delta P_{pump} = f\frac{L_{23}+L_{45}}{D_i}\cdot\frac{G^2}{2\rho} + \rho g(z_5 - z_1) + (K_{entrance} + K_{elbow1} + K_{elbow2} + K_{spray})\frac{G^2}{2\rho} + \frac{G^2}{2\rho}\left(\frac{\frac{\pi}{4}D_i^2}{A_{spray}}\right)^2 \tag{10}$$

Substituting all numerical values in Eq. (9), one gets $\Delta P_{pump} \approx 766.7$ kPa. Finally, Eq. (1) gives $\dot{W}_p \approx 48.06$ kW.

ii) There are three thermal resistances here, i.e., convection in the steam, conduction in the pipe wall and convection in the water. Thus the total temperature drop from steam to water, $T_{steam} - T_{water} = 200°C - 25°C = 175°C$, can be expressed as:

$$T_{steam} - T_{water} = q'\left[\frac{1}{\pi D_o h_o} + \frac{\ln(D_o/D_i)}{2\pi k_{ss}} + \frac{1}{\pi D_i h_i}\right] \tag{11}$$

where q' is the heat transfer rate per unit length of the pipe, $D_o = 11$ cm, $h_o = 5000$ W/m^{2}K (given in the problem statement), $k_{ss} = 14$ W/m·K and h_i is the heat transfer coefficient on the water side of the pipe. Note that Eq. (11) is very similar to the expression to calculate the temperature drop within a fuel pin. Solving for q', one gets:

$$q' = \frac{T_{steam} - T_{water}}{\frac{1}{\pi D_o h_o} + \frac{\ln(D_o/D_i)}{2\pi k_{ss}} + \frac{1}{\pi D_i h_i}} \tag{12}$$

In order to obtain q' from Eq. (12), one needs to know h_i. To find h_i, we recognize that:

- The heat transfer mode is internal forced convection
- The fluid of interest is non-metallic ($Pr = c_p\mu/k \approx 6.917$)
- The geometry is round tube
- The flow regime is turbulent, so the boundary condition does not matter much
- Entry region effects are neglected, as suggested by the problem statement

With these assumptions the Dittus-Boelter correlation is suitable to calculate hi:

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} \approx 2385 \qquad \Rightarrow \qquad h_i = Nu \cdot k/D_i \approx 14.55 \text{ kW/m}^2 \qquad (13)$$

Substituting the numerical values in Eq. (12), one gets $q' \approx 93$ kW/m. Because the length of pipe exposed to steam is 5 m, the total heat transfer rate is $\dot{Q} = 93 \times 5 \approx 465.1$ kW.

iii) The water temperature rise due to heating from the steam is small, $\Delta T_{water} = \frac{\dot{Q}}{\dot{m} c_p} \approx 2.2°C$, so the assumption of constant properties used in part 'i' is accurate.

Problem 2 (40%) – Radial and Axial Temperature Distribution in a Restructured Fuel Pin

i) Restructuring will occur where the temperature exceeds 1600°C. To find the first axial location at which restructuring occurs, one needs to know the axial distribution of the coolant and fuel temperature. The coolant energy equation is:

$$\dot{m}c_p \frac{dT_b}{dz} = q'(z) = q'_m \sin\left(\frac{\pi z}{L}\right) \tag{14}$$

where T_b is the bulk coolant temperature, z is the axial coordinate measured from the channel inlet, $\dot{m}$ =0.38 kg/s, c_p=6.1 kJ/kg·K, q'_m =40 kW/m and L=4 m. Equation (14) can be integrated to give:

$$T_b(z) = T_{b0} + \frac{q'_m L}{\dot{m}c_p\pi}\left[1-\cos\left(\frac{\pi z}{L}\right)\right] \tag{15}$$

where T_{b0}=285°C. At any axial location before restructuring occurs, the max fuel temperature, T_{max}, can be calculated by means of Eq. (8-119) in the T&K textbook:

$$T_{\max}(z) = T_b(z) + q'(z)\left[\frac{1}{4\pi k_f} + \frac{1}{2\pi R_g h_g} + \frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co}h}\right] \tag{16}$$

where k_f=3 W/m·K (assumed to be independent of temperature, as per the problem statement), R_g=4.14 mm, h_g=5 kW/m^{2}K, k_c=13 W/m·K and h=25 kW/m^{2}K. Substituting Eq. (15) into Eq. (16), one gets:

$$T_{\max}(z) = T_{b0} + \frac{q'_m L}{\dot{m}c_p\pi}\left[1-\cos\left(\frac{\pi z}{L}\right)\right] + q'_m \sin\left(\frac{\pi z}{L}\right)\left[\frac{1}{4\pi k_f} + \frac{1}{2\pi R_g h_g} + \frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co}h}\right] \tag{17}$$

If T_{max} is set equal to 1600°C, Eq. (17) can be solved for z, to find the axial location at which restructuring first occurs. Note that Eq. (17) can be re-arranged as follows:

$$\frac{\frac{1}{4\pi k_f} + \frac{1}{2\pi R_g h_g} + \frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co}h}}{\frac{1600°C - T_{b0}}{q'_m} - \frac{L}{\dot{m}c_p\pi}}\sin\left(\frac{\pi z}{L}\right) - \frac{\frac{L}{\dot{m}c_p\pi}}{\frac{1600°C - T_{b0}}{q'_m} - \frac{L}{\dot{m}c_p\pi}}\cos\left(\frac{\pi z}{L}\right) = 1 \tag{18}$$

Equation (18) is in the form $a\cdot \sin x - b\cdot \cos x = 1$, with a=1.14834 and b=0.01699. Thus, the solution is (πz/L)=1.07147, or z≈1.364 m.

ii) The linear power at z=2 m is at its maximum q'_m =40 kW/m. The bulk coolant temperature at this axial location is T_b≈307°C, obtained from Eq. (15). The fuel outer temperature, T_{fo}, can be found from the following equation:

$$T_{fo} = T_b + q'_m\left[\frac{1}{2\pi R_g h_g} + \frac{1}{2\pi k_c}\ln\frac{R_{co}}{R_{ci}} + \frac{1}{2\pi R_{co} h}\right] \approx 730.7°\mathrm{C} \qquad (19)$$

One can find R_s, the boundary of the restructured region, from Eq. (8-99) in the T&K textbook for the two-zone restructuring situation:

$$k_f(1600°C - T_{fo}) = \frac{q'_m}{4\pi}\left[1-\left(\frac{R_s}{R_{fo}}\right)^2\right] \qquad (20)$$

Eq. (20) yields R_s/R_{fo}≈0.4251, or R_s≈1.743 mm. Then, the radius of the void region, R_v, can be found from Eq. (8-98) in the textbook:

$$R_v^2 = \frac{\rho_s - \rho}{\rho_s} R_s^2 \qquad \Rightarrow \qquad R_v \approx 0.25 \text{ mm} \qquad (21)$$

where ρ_s=97 %TD and ρ=95 %TD. Finally, the maximum temperature in the fuel at this axial location, T_{max}, can be found from Eq. (8-100) in the textbook:

$$k_{f,s}(T_{\max} - 1600°C) = \frac{q'_m}{4\pi}\cdot\frac{\rho_s}{\rho}\cdot\left(\frac{R_s}{R_{fo}}\right)^2\left\{1-\left(\frac{R_v}{R_s}\right)^2\left[1+\ln\left(\frac{R_s}{R_v}\right)^2\right]\right\} \qquad (22)$$

where $k_{f,s}$≈3.093 W/m·K is the thermal conductivity of the restructured fuel (97 %TD), obtained scaling the value at 95 %TD with the Biancharia's correlation (spherical pores). Equation (22) yields T_{max}≈1770.8°C.

iii) There is no void region for z<1.364 m because the temperature in the fuel does not reach 1600°C. For z>1.364 m the void region expands as the linear power increases and contracts as the linear power decreases above the fuel pin midplane. Note that there exists an axial location where the fuel temperature drops again under 1600°C and no restructuring occurs. The (quantitative) R_v vs. z plot is shown in Figure 2 below. While it is hard to see from this figure, the maximum R_v is reached at a location slightly above the midplane.

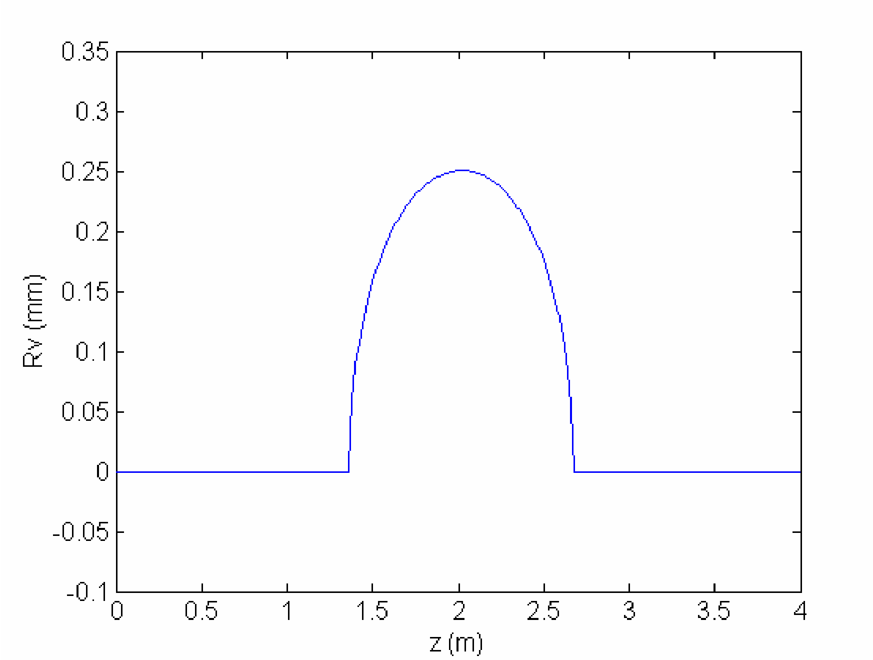

Figure 2. Axial variation of the void region radius.