**22.312 ENGINEERING OF NUCLEAR REACTORS**

Monday, December 17th, 2007, 9:00am-12:00 pm

FINAL EXAM SOLUTIONS

**Problem 1 (45%) – Analysis of Decay Heat Removal during a Severe Accident**

i) The energy balance for the corium melt is as follows:

$$M_c C_c \frac{dT_c}{dt} = \dot{Q}_{dec} - \dot{Q}_{boil} - \dot{Q}_{cond} \quad (1)$$

where $M_c$, $C_c$ and $T_c$ are the mass, specific heat and temperature of the corium melt, respectively, while $\dot{Q}_{dec}$ is the decay power, $\dot{Q}_{boil}$ and $\dot{Q}_{cond}$ are the heat removal by water boiling above the corium and conduction through the vessel wall, respectively. If the right-hand term of Eq. (1) is positive, the corium melt temperature is increasing with time; vice versa, if the right-hand term is negative, the corium is cooling down. Therefore, we need to evaluate the three terms on the right-hand side of Eq. (1). The decay power term is simply:

$$\dot{Q}_{dec} = 0.066\dot{Q}_0 t^{-0.2} \approx 35.0 \text{ MW} \quad (2)$$

Where $\dot{Q}_{dec}$=3400 MW and t =3×3600 s=3 hours, and it was assumed that the reactor had operated for a long time before shutdown.

Because the thickness to diameter ratio for the vessel lower head shell is small (0.22/4.8=0.046<<1), we can approximate it to a flat wall with little loss of accuracy[1]. Therefore, the heat rate through the vessel is:

$$\dot{Q}_{cond} = k_v \frac{T_c - T_{v,o}}{\delta} \cdot \frac{\pi}{2} D^2 \approx 9.3 \text{ MW} \quad (3)$$

Where $k_v$=30 W/m°C is the vessel thermal conductivity, δ=22 cm is the vessel wall thickness, the corium temperature at the time of interest is $T_c$=2000°C and the vessel outer temperature is $T_{v,o}$=120°C (as per the problem statement). The term $\frac{\pi}{2}D^2 = 36.2$ m$^2$ represents the surface of the lower (hemispherical) head.

The boiling term is found as follows:

[1] Alternatively one can solve the heat conduction equation in spherical coordinates.

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$$\dot{Q}_{boil} = h_{FB}(T_c - T_{sat}) \cdot \frac{\pi}{4} D^2 \approx 13.8 \text{ MW} \tag{4}$$

Where the film boiling[2] heat transfer coefficient, $h_{FB}$ (≈400 W/m$^2$°C), is from the Berenson correlation given in the problem statement. The term $\frac{\pi}{4} D^2 = 18.1$ m$^2$ is the upper surface area of the corium melt.

The right-hand term of Eq. (1) is 35-9.3-13.8=11.9 MW>0. Therefore, at this time the corium melt is still heating up.

ii) The void fraction, α, in the drift-flux model is given as:

$$\alpha = \frac{j_v}{C_o j + V_{vj}} \tag{5}$$

Where (for churn flow) $C_o$=1 and $V_{vj} = 1.53\left[\sigma g(\rho_f - \rho_g)/\rho_f^2\right]^{0.25}$ ≈0.24 m/s. Now, in the pool of water above the corium, the liquid is stagnant, while the vapor flows upward (due to buoyancy). Therefore, one has $j_\ell$=0 and also $j=j_v$. The vapor superficial velocity, $j_v$, in general can be calculated as $xG/\rho_g$. In this case, x=1 (only vapor is flowing, so the flow quality is one) and G is equal to the vapor generation rate per unit area of the corium surface. Thus $G=q''/h_{fg}$=0.09 kg/m$^2$s, where $q''$=200 kW/m$^2$, and $j_v$=0.148 m/s. From Eq. (5) one finally gets α≈0.381.

iii) HEM ($S= V_v/V_\ell$ =1) would have clearly been a bad choice because the velocity of the liquid is zero, while the velocity of the vapor is greater than zero. In fact, in this case the slip ratio S is infinite.

**Problem 2 (15%) – Boiling Crisis on the Vessel Outer Surface during a Severe Accident**

i) DNB is typical of subcooled boiling or low-quality saturated boiling, while dryout is typical of high-quality (annular flow) saturated boiling.

The energy balance for the water in the gap is:

$$\dot{Q} = \dot{m}C_{p,f}(T_{out} - T_{in}) \Rightarrow \qquad T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_{p,f}} \approx 92°\text{C} \tag{6}$$

[2] The heat transfer mechanism of interest here is obviously film boiling, as the corium surface is at 2000°C.

Where $\dot{Q} = q''\frac{\pi}{2}D^2$=14.86 MW, q″=350 kW/m$^2$, D=5.2 m, $\dot{m}$=300 kg/s, $C_{p,f}$=4.2 kJ/kg°C, $T_{in}$=80°C. Because the bulk temperature is below $T_{sat}$ (=100°C) throughout the channel, this is a subcooled boiling situation, for which DNB would be the relevant boiling crisis.

ii) It is well known that the DNB heat flux ($q''_{DNB}$) decreases with increasing equilibrium quality and increases with increasing mass flux. Therefore, in the situation of interest here the minimum DNB heat flux is at the outlet of the hemispherical section (θ=90°), where the equilibrium quality is highest and the mass flux lowest.

**Problem 3 (25%) – Entropy generation in a steam turbine system**

The entropy equation for a generic control volume is:

$$\frac{\partial S_{CV}}{\partial t} = \sum_i \dot{m}_i s_i + \frac{\dot{Q}}{T_s} + \dot{S}_{gen} \qquad (7)$$

Turbines can be considered adiabatic machines operating at steady state, thus Eq. (7) becomes:

$$\dot{S}_{gen} = \dot{m}(s_{out} - s_{in}) \qquad (8)$$

where $\dot{m}$ is the steam flow rate processed by the turbine, and $s_{in}$ and $s_{out}$ are the specific entropy at the turbine inlet and outlet, respectively.

i) In this case $\dot{m}$=600 kg/s. Let Point 1 be the turbine inlet and Point 2 be the turbine outlet. We need to find $s_1$ and $s_2$.

Turbine inlet (**Point 1**): $T_1$=280°C, $P_1$=64 bar, $h_1$=2780 kJ/kg, $s_1$=5.9 kJ/kg·K, $x_1$=1.0
Turbine outlet (**Point 2**): $T_2$=30°C, $P_2$=0.04 bar, $s_{2s}=s_1$=5.9 kJ/kg·K, $x_{2s}=(s_{2s}-s_{f2})/(s_{g2}-s_{f2})\approx0.688$, $h_{2s}=h_{f2}+x_{2s}(h_{g2}-h_{f2})\approx1797$ kJ/kg, $h_2=h_1-\eta_T(h_1-h_{2s})\approx1865$ kJ/kg (where $\eta_T$=0.93), $x_2=(h_2-h_{f2})/(h_{g2}-h_{f2})\approx0.716$, $s_2=s_{f2}+x_2(s_{g2}-s_{f2})\approx6.127$ kJ/kg·K, and (from Eq.8) $\dot{S}_{gen}\approx136$ kW/K.

ii) Let Point 1 be the high-pressure turbine inlet, Point 2 be the high-pressure turbine outlet, Point 3 the low-pressure turbine inlet and Point 4 the low-pressure turbine outlet. Then the total entropy generation rate is:

$$\dot{S}_{gen} = \dot{m}_{HP}(s_2 - s_1) + \dot{m}_{LP}(s_4 - s_3) \qquad (9)$$

where $\dot{m}_{HP}$=600 kg/s, but $\dot{m}_{LP}$, $s_1$, $s_2$, $s_3$ and $s_4$ are unknown.

HP turbine inlet (**Point 1**): $T_1$=280°C, $P_1$=64 bar, $h_1$=2780 kJ/kg, $s_1$=5.9 kJ/kg·K, $x_1$=1.0
HP turbine outlet (**Point 2**): $T_2$=180°C, $P_2$=10 bar, $s_{2s}=s_1$=5.9 kJ/kg·K, $x_{2s}=(s_{2s}-s_{f2})/(s_{g2}-s_{f2})\approx0.826$, $h_{2s}=h_{f2}+x_{2s}(h_{g2}-h_{f2})\approx2427$ kJ/kg, $h_2=h_1-\eta_T(h_1-h_{2s})\approx2451$ kJ/kg (where $\eta_T$=0.93), $x_2=(h_2-h_{f2})/(h_{g2}-h_{f2})\approx0.838$, $s_2=s_{f2}+x_2(s_{g2}-s_{f2})\approx5.956$ kJ/kg·K

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LP turbine inlet (**Point 3**): $T_3$=180°C, $P_3$=10 bar, $h_3$=2777 kJ/kg, $s_3$=6.7 kJ/kg·K, (moisture separation) $x_3$=1.0, $\dot{m}_{LP} = x_2\dot{m}$ ≈503 kg/s
LP turbine outlet (**Point 4**): $T_4$=30°C, $P_4$=0.04 bar, $s_{4s}$=$s_3$=6.7 kJ/kg·K, $x_{4s}$=($s_{4s}$-$s_{f4}$)/($s_{g4}$-$s_{f4}$)≈0.7875, $h_{4s}$=$h_{f4}$+$x_{4s}$($h_{g4}$-$h_{f4}$)≈2039 kJ/kg, $h_4$=$h_3$-$\eta_T$($h_3$-$h_{4s}$)≈2091 kJ/kg (where $\eta_T$=0.93), $x_4$=($h_4$-$h_{f4}$)/($h_{g4}$-$h_{f4}$)≈0.809, $s_4$=$s_{f4}$+$x_4$($s_{g4}$-$s_{f4}$)≈6.870 kJ/kg·K

From Eq. (9), one gets $\dot{S}_{gen}$ ≈119 kW/K.

iii) Let Point 5 be the liquid outlet of the moisture separator. If the moisture separator is considered adiabatic (no heat exchange with the surroundings), then Eq. (7), applied to the moisture separator, yields:

$$\dot{S}_{gen} = \dot{m}_{HP}s_2 - x_2\dot{m}_{HP}s_3 - (1-x_2)\dot{m}_{HP}s_5 = \dot{m}_{HP}[s_2 - x_2s_3 - (1-x_2)s_5] \tag{10}$$

But, $s_3$=$s_{g2}$ and $s_5$=$s_{f2}$, thus the bracketed term in Eq. (10) is $s_2 - x_2s_3 - (1-x_2)s_5$=0, and the entropy generation in the moisture separator is zero.

iv)
Advantages of moisture separator:
- Increases the quality in the turbine, thus lengthens the lifetime of the turbine. (Reliability)
- The separated moisture can be used for regeneration, thus increasing the thermal efficiency of the cycle. (Efficiency)
- The increased in thermal efficiency reduces the operating costs. (Economics)

Disadvantages
- Added capital cost of the moisture separator and connecting piping. (Economics)

**Problem 4 (15%) - Sizing the Silicon Carbide Layer in a TRISO Fuel Particle**

The stresses for a thin spherical shell of radius $R_s$ can be calculated as follows:

$$\sigma_r = -(p_i+p_o)/2 \tag{14}$$
$$\sigma_\theta = \sigma_\varphi = (p_i-p_o)R_s/(2\ t)$$

Where $R_s$=300 μm, $p_o$=9 MPa, t is the (unknown) thickness of the shell, and $p_i$ is the internal pressure due to the fission gases. The fission gas pressure can be calculated from the perfect gas equation as follows:

$$p_i=NRT/V_{FG}=31.2\text{ MPa} \tag{15}$$

Where N=$10^{-7}$ mol, R=8.31 J/mol-K, T=1273 K (1000°C) and $V_{FG} = 0.3\ \frac{4}{3}\pi R_s^3$=$3.4\times10^{-11}$ $m^3$.
The Von Mises failure criterion is expressed by the following inequality:

$$\sqrt{\frac{1}{2}\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_\varphi)^2 + (\sigma_\theta - \sigma_\varphi)^2\right]} < S_y \qquad (16)$$

Where Sy=200 MPa for SiC at 1000°C. Noting that $\sigma_\theta = \sigma_\varphi$ , Eq. (16) becomes:

$$(\sigma_\theta - \sigma_r) < S_y \qquad (17)$$

Substituting Eqs. (14) into Eq. (17), one gets:

$$(p_i-p_o)R_s/(2\ t)+(p_i+p_o)/2<S_y \qquad (18)$$

Solving Eq. (18) for t, one gets the minimum required value of the shell thickness to prevent failure:

$$t_{\min} = R_s \frac{p_i - p_o}{2S_y - (p_i + p_o)} = 18.5\ \mu\text{m}$$

Note that the use of the thin-shell theory is justified because $R_s/t_{min}>10$.

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