22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, October 16th, 2007, 2:30 – 4:00 p.m.

OPEN BOOK QUIZ 1 (solutions)

Problem 1 (55%) - Nuclear cogeneration plant

i) The T-s diagram for the cycle is shown in Figure 1.

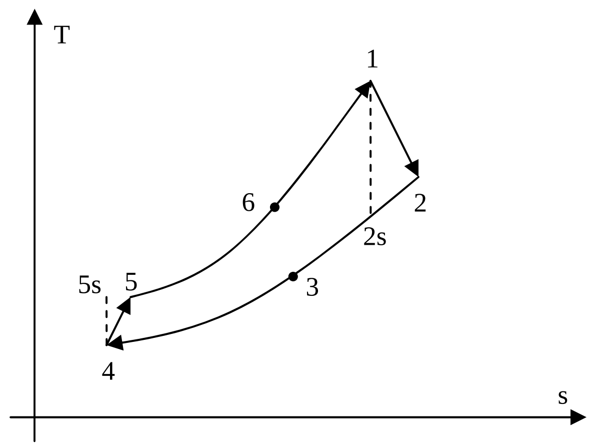

Figure 1. T-s diagram for the cogeneration Brayton cycle.

ii) The condition that the thermal efficiency has to be equal to 0.3 can be expressed mathematically as follows:

$$\eta_{th} \equiv \frac{W_T - W_C}{Q_{1\to6}} = 0.3 \qquad (1)$$

Turbine

The temperature of point 2s is $T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(\gamma-1)/\gamma} = T_1 r_p^{(1-\gamma)/\gamma}$ where T_1=1000 K and $r_p=P_1/P_{2s}=2$. From the definition of isentropic efficiency of the turbine (η_T=0.9) it is possible to calculate T_2 as

$$T_2 = T_1 - \eta_T(T_1 - T_{2s}) = T_1[1-\eta_T(1-r_p^{(1-\gamma)/\gamma})] \approx 782 \text{ K}$$

The turbine work is then:

$$W_T = c_p(T_1 - T_2) \approx 1132 \text{ kJ/kg}$$

Where c_p=5193 J/kg-K.

Compressor

The temperature of point 5s is $T_{5s} = T_4\left(\frac{P_{5s}}{P_4}\right)^{(\gamma-1)/\gamma} = T_4 r_p^{(\gamma-1)/\gamma}$ where T_4=373 K and r_p=P_{5s}/P_4=2. From the definition of isentropic efficiency of the compressor (η_C=0.9) it is possible to calculate T_5 as

$$T_5 = T_4 + \frac{(T_{5s} - T_4)}{\eta_C} = T_4[1 + \frac{r_p^{(\gamma-1)/\gamma} - 1}{\eta_C}] \approx 505 \text{ K}$$

The compressor work is then:

$$W_C = c_p(T_5 - T_4) \approx 688 \text{ kJ/kg}$$

Reactor and Regenerator

The reactor heat rate per unit mass of helium is:

$$Q_{1\to 6} = c_p(T_1 - T_6) \tag{2}$$

T_6 is the temperature at the outlet of the regenerator. To relate T_6 to T_3 (the unknown of the problem), one has to analyze the regenerator. The energy balance for the regenerator is $c_p(T_6 - T_5) = c_p(T_2 - T_3)$, which yields:

$$T_6 = T_5 + T_2 - T_3 = 1287 - T_3 \tag{3}$$

Back substituting Eq. (3) and (2) into Eq. (1) and solving for T_3, one gets T_3≈572 K and then (from Eq. 3) T_6≈715 K.

iii) The EUF of this cycle is obviously equal to one, because all heat produced by the reactor is either converted to (net) work in the turbine and compressor or utilized in the cogeneration heat exchanger, i.e., no heat is discharged to the environment. Mathematically:

$$EUF = \frac{W_T - W_C + Q_{3\to 4}}{Q_{6\to 1}} = \frac{c_p(T_1 - T_2) - c_p(T_5 - T_4) + c_p(T_3 - T_4)}{c_p(T_1 - T_6)} = 1$$

iv) The power of the cogeneration heat exchanger, i.e., the power required to generate 100 kg/s of saturated steam at 0.5 MPa from water at 80°C (also at 0.5 MPa), is:

$$\dot{Q}_{CGHX} = \dot{m}_w\left[c_{pw}(T_{sat} - T_{in}) + h_{fg}\right] \approx 241 \text{ MW}$$

Where T_{sat}=152°C is the saturation temperature at 0.5 MPa, c_{pw}= 4.24 kJ/kg-K is the specific heat of water and h_{fg}=2109 kJ/kg is the water enthalpy of vaporization.

Now, to find the reactor thermal power, $\dot{Q}_{6\to 1}$, one can proceed in two different ways:

1. The energy balance for the whole cycle is $\dot{Q}_{6\to 1} + \dot{W}_C = \dot{W}_T + \dot{Q}_{3\to 4}$. Recognizing that $\dot{Q}_{3\to 4} = \dot{Q}_{CGHX}$ and that $\eta_{th} \equiv \frac{\dot{W}_T - \dot{W}_C}{\dot{Q}_{1\to 6}}$, one gets $\dot{Q}_{1\to 6} = \frac{\dot{Q}_{3\to 4}}{1 - \eta_{th}} \approx 348$ MW

2. Recognizing that $\dot{Q}_{3\to4} = \dot{Q}_{CGHX}$, one has $\dot{Q}_{CGHX} = \dot{m}_{He} c_p (T_3 - T_4)$, from which the helium mass flow rate can be calculated, $\dot{m}_{He} \approx 233$ kg/s. Then, $\dot{Q}_{1\to6} = \dot{m}_{He} c_p (T_1 - T_6) \approx 348$ MW.

v) The main issues hindering development of nuclear cogeneration for residential heating on a large scale are as follows.

- For safety reasons nuclear plants are sited far from major residential areas. As such, heat transport losses and costs from the plant to the end users would be high.
- The residential heating load varies greatly throughout the year and even during a single day. This may force frequent changes in the operating conditions of the plant, something nuclear plants are not particularly suitable for.

Note that questions (iii), (iv) and (v) could be answered without answering question (ii).

Problem 2 (45%) – Pressure rise in a BWR suppression pool during a LOCA

i) As the steam coming from the reactor is condensed in the pool, the pool temperature increases, which leads to a higher partial pressure of nitrogen and steam in the wet well. When the wet well pressure reaches 0.68 MPa, the safety/relief valve opens. Since the steam discharge rate, $\dot{m}_i$, is constant in time, the time at which the safety/relief valve opens, t_2, is simply:

$$t_2 = M_i / \dot{m}_i \qquad (4)$$

Where M_i is the mass of steam that will cause the pressure to rise to 0.68 MPa. To find M_i, we need to use the conservation of mass, energy, volume and the definition of the total pressure in the wet well. But first let us identify the initial conditions (t_1=0) for the system. The initial mass of water in the pool is 240×996=239000 kg, where 996 kg/m^3 is the density of water at 30°C (from the steam tables). The mass of nitrogen in the system, M_N, is found from the equation of state:

$$M_N = \frac{P_{N1} V_{N1}}{R_N T_1} = 215 \text{ kg}$$

WhereV_{N1}=160 m^3, T_1=303 K (30°C) and P_{N1} is the partial pressure of nitrogen:

$$P_{N1} = P_1 - P_{\text{sat}(30°C)} = 96.8 \text{ kPa}$$

Where P_1=0.101 MPa is the initial wet well pressure and, as the initial humidity in the wet well is 100%, $P_{sat(30°C)}$=4.2 kPa is the saturation pressure of steam at 30°C (obtained from the steam tables). Obviously, the mass of steam initially in the nitrogen is $V_{N1}/v_{sat(30°C)}$=6 kg, where $v_{sat(30°C)}$=32.9 m^3/kg is the specific volume of saturated steam at 30°C (from the steam tables). Therefore the total mass of water initially present in the system is M_{w1}=239000+6=239006 kg. The total water internal energy at t_1=0 is E_{w1}=239000×125+6×2146≈3×10^{10} J with 125 kJ/kg and

2146 kJ/kg being the specific internal energy of the pool water and steam in the wet well, respectively.

Now, we can write the equations to find M_i.

Mass conservation

$$\frac{\partial M_{CV}}{\partial t} = \dot{m}_i \quad \Rightarrow \quad (M_{w2} + M_N) - (M_{w1} + M_N) = M_i \quad \Rightarrow \quad M_{w2} = M_{w1} + M_i \tag{5}$$

Energy conservation

$$\frac{\partial E}{\partial t} = \dot{m}_i h_i \quad \Rightarrow \quad E_2 - E_1 = M_i h_i \quad \Rightarrow$$

$$M_{w2}[u_f(T_2) + x_2 u_{fg}(T_2)] + M_N u_{N2} - E_{w1} - M_N u_{N1} = M_i h_i \quad \Rightarrow$$

$$M_{w2}[u_f(T_2) + x_2 u_{fg}(T_2)] + M_N c_{vN}(T_2 - T_1) - E_{w1} = M_i h_i \tag{6}$$

Where h_i=2600 kJ/kg is the enthalpy of the steam coming from the reactor, x_2 is the steam quality at t_2 and c_{vN}=742 J/kg-K.

Volume

The total volume of the suppression pool does not change during the accident. Therefore:

$$V_{tot} = 440\,m^3 = M_{w2}[v_f(T_2) + x_2 v_{fg}(T_2)] \tag{7}$$

Finally, the total pressure at t_2 is equal to 0.68 MPa and also equal to the sum of the steam and nitrogen partial pressures:

$$P_2 = 0.68\,MPa = P_{w2}(T_2) + P_{N2} = P_{w2}(T_2) + \frac{M_N R_N T_2}{[V_{tot} - M_{w2}(1 - x_2) v_f(T_2)]} \tag{8}$$

Equations (5) through (8) are four equations in the four unknowns M_i, M_{w2}, T_2 and x_2. When M_i is found, Eq. (4) provides t_2. The numerical values for these parameters can be found by iteration and are M_i=60000 kg, M_{w2}=299006 kg, T_2=147.4°C, x_2=0.0009, t_2=60 s.

ii)

Effects that tend to increase the level:

1. water heat up leads to lower density
2. condensation of steam from reactor leading to more liquid water in the pool

Effects that tend to decrease the level:

3. water heat up leads to water evaporation into the wet well (this is also the main effect causing the wet well pressure to rise)

The dominant effect is number 2, so the level during the LOCA actually increases.